Likelihood for Gaussian Markov Bot

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First-order Markov Bot (Gaussian noise)

$$
\begin{aligned}
a_{t+1} &= -Aa_t + \pi \exp(-B/d_t) + \eta_t \\
d_{t+1} &= \frac{C}{1+|a_t|} + \frac{D}{1+d_t} + 60 + \gamma_t \\
&\text{where} \quad \eta_t \sim \mathcal{N}(0, \sigma) \\
&\gamma_t \sim \text{Uniform}(0, 50) \\
&t \leftarrow t+1.
\end{aligned}
$$

Sigma is fixed to $\sigma = \sqrt{\pi/10}$
Infer the parameters $\theta = (A, B, C, D)$

$$
\begin{aligned}
\mathcal{L}(\{d_t, a_t\}|A, B, C, D, \sigma^2) &= \sum_t \log p(d_{t+1}, a_{t+1}|d_t, a_t) \\
&= \sum_t -\frac{1}{2\sigma^2}\left(a_{t+1} + Aa_t - \pi \exp(-B/d_t)\right)^2 + \text{const.} \\
&\quad + \log\left(\mathbf{I}_{[min_t < d_{t+1} < max_t]} \frac{1}{\max_t - \min_t}\right)
\end{aligned}
$$

where

$$
\begin{aligned}
\min_t &= \frac{C}{1+|a_t|} + \frac{D}{1+d_t} + 60 \\
\max_t &= \frac{C}{1+|a_t|} + \frac{D}{1+d_t} + 110
\end{aligned}
$$