Part A:

Exercise 1:

Exercise 1:

$$y = -\frac{1}{2}gt^2 + ut + c$$

$$\frac{dy}{dt} = -\frac{1}{2}2gt + u$$

$$= -gt + u$$

Max height is reached when $\frac{dy}{dt} = 0$ therefore we must substitute with our values to get t_{max}.

$g = 10$, $u = 5$

$$-gt + u = 0$$

$$-10t + 5 = 0$$

$$t = \frac{-5}{-10}$$

$$t = \frac{5}{10}$$

$$\boxed{t = \frac{1}{2}}$$

Our t_{max} is 0.5 seconds

To find the height reached at t_{max}:

$$y = -\frac{1}{2} \cdot 10 \cdot (0.5)^2 + 5 \cdot 0.5 + 1$$

$$y = -5 \cdot 0,25 + 2.5 + 1$$

$$y = -1,25 + 2.5 + 1$$

$$\boxed{y = 2.25}$$

The maximum height reached by the ball is 2.25 meters.

Exercise 2:

Exercise 2:

$f(x) = 2x^3 - 3x^2 - 36x + 2$

We first need to calculate the 1st and the 2nd derivative so we can find the stationary points using the equation $\frac{df}{dx} = 0$

1st $\frac{df}{dx} = 6x^2 - 6x - 36$

2nd $\frac{d^2f}{dx^2} = 12x - 6$

$$6x^2 - 6x - 36 = 0$$
$$x^2 - x - 6 = 0$$
$$(x+2)(x-3) = 0$$
$$x - 3 = 0$$
$$\Rightarrow x = 3$$

$$x + 2 = 0$$
$$\Rightarrow x = -2$$

Stationary points are $x_1 = 3$ and $x_2 = -2$

$\frac{d^2f}{dx^2}$ when $x = 3 = 12.3 - 6 = 30$

$30 > 0$ therefore stationary point $x_1 = 3$ is the minimum

$\frac{d^2f}{dx^2}$ when $x = -2 = 12.(-2) - 6 = -30$

$-30 < 0$ therefore stationary point $x_2 = -2$ is the maximum

Exercise 3:

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$$f(x) = \frac{1}{1+\exp(-m(x-x_0))}$$

a) $$\lim_{x\to\infty} \frac{1}{1+e^{-m(x-x_0)}} = \frac{1}{1+e^{-\infty}} = \frac{1}{1+\frac{1}{e^{\infty}}} = \frac{1}{1+0} = 1$$

$$\lim_{x\to-\infty} \frac{1}{1+e^{-m(x-x_0)}} = \frac{1}{1+e^{\infty}} = \frac{1}{e^{\infty}} = 0$$

b) 1st $$\frac{d}{dx} \frac{1}{1+e^{-m(x-x_0)}} = \frac{1}{(1+e^{-m(x-x_0)})^2} \left(-me^{-m(x-x_0)}\right)$$

$$= \frac{me^{-m(x-x_0)}}{(1+e^{-m(x-x_0)})^2}$$

2nd $$\frac{d^2}{dx^2} = \frac{me^{-m(x-x_0)}}{(1+e^{-m(x-x_0)})^2} = (me^{-m(x-x_0)}) \cdot (1-e^{-m(x-x_0)})^{-2}$$

$$= -2 \cdot me^{-m(x-x_0)} \cdot (-me^{-m(x-x_0)}) \cdot (1+e^{-m(x-x_0)})^{-3} + ((1+e^{-m(x-x_0)})^{-2} \cdot (-m) \cdot me^{-m(x-x_0)})$$

$$= \frac{2m^2(e^{-m(x-x_0)})^2}{(1+e^{-m(x-x_0)})^3} + \frac{m^2e^{-m(x-x_0)}}{(1+e^{-m(x-x_0)})^2}$$

$$= \frac{2m^2(e^{-m(x-x_0)})^2}{(1+e^{-m(x-x_0)})^3} - \frac{m^2e^{-m(x-x_0)}}{(1+e^{-m(x-x_0)})^2} \cdot \frac{1+e^{-m(x-x_0)}}{1+e^{-m(x-x_0)}}$$

$$= \frac{2m^2(e^{-m(x-x_0)})^2 - m^2e^{-m(x-x_0)} - m^2(e^{-m(x-x_0)})^2}{(1+e^{-m(x-x_0)})^3}$$

$$= \frac{m^2(2(e^{-m(x-x_0)})^2 - (e^{-m(x-x_0)})^2 - (e^{-m(x-x_0)}))}{(1+e^{-m(x-x_0)})^3}$$

$$= \frac{m^2((e^{-m(x-x_0)})^2 - e^{-m(x-x_0)})}{(1+e^{-m(x-x_0)})^3}$$

$$\frac{m^2((e^{-m(x-x_0)})^2 - e^{-m(x-x_0)})}{(1+e^{-m(x-x_0)})^3} = 0$$

$$\Rightarrow \quad m^2 e^{-m(x-x_0)}(e^{-m(x-x_0)} - 1) = 0$$

$e^{-m(x-x_0)}$ can't be zero since x on e^x can't be zero.
Therefore the answer is 1.

$$e^{-m(x-x_0)} = 1$$

$$-m(x-x_0) = 0$$

$$x = x_0$$

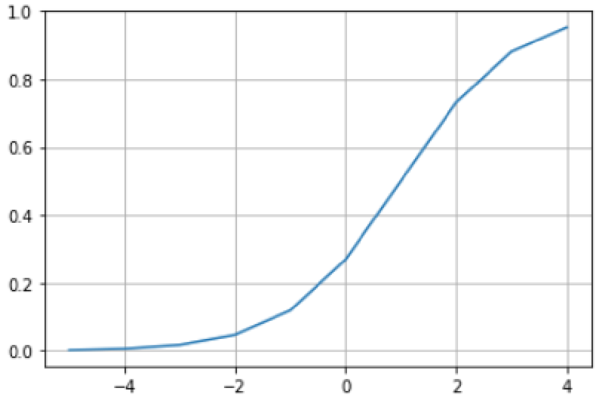
Therefore the derivative of the sigmoid function has x_0 as a stationary point and takes the value of x_0.

$$\Rightarrow \frac{df}{dx}\Big|_{x=x_0} = \frac{m}{(1+1)^2} = \frac{m}{4}$$

In [1]:
```
# Exercise 3.C.
import numpy as np
from matplotlib import pyplot as plt

x = np.arange( -5, 5, 1)
c = np.exp( -1 * ( x - 1))
y = 1 / (1 + c)
```

In [2]:
```
plt.plot(x,y)
plt.grid()
```



The plot confirms what was found on 3b as m=1 therefore xo = 1/4 which can be seen on the plot.

Exercise 4:

<u>Exercise 4:</u>

$$E = -\sum_{i=1}^{2} \sum_{j=1}^{2} w_{ij} x_i x_j = -(w_{11} x^2 + w_{12} x_1 x_2 + w_{21} x_2 x_1 + w_{22} x_2^2$$

$$\frac{\partial E}{\partial x_1} = -(2 w_{11} x_1 + w_{12} x_2 + w_{21} x_2)$$

$$\frac{\partial E}{\partial w_{12}} = -x_1 x_2$$