Solutions: Exercise 2

Q1.

$$\left.\begin{aligned} FPR &= 0.01, \\ FPR + TNR &= 1, \\ \Pr(\text{All negative} \mid \text{no drug}) &= TNR_1 \cdot TNR_2 \cdot \ldots \cdot TNR_{100} \end{aligned}\right\} 0.99^{100} = 0.37$$

$\Pr(\text{All negative} \mid \text{no drug}) = 0.37$. Not so surprising ($\Pr = 0.63$) that some tests would be positive.

Q2.

$$\begin{aligned} \text{Sensitivity (TPR)} &= \frac{TP}{TP+FN} = \frac{80}{100} = 0.80 \\ \text{Specificity (TNR)} &= \frac{TN}{TN+FP} = \frac{400-8}{400} = 0.98 \\ \text{False positive rate (FPR)} &= \frac{FP}{TN+FP} = \frac{8}{400} = 0.02 \\ \text{False negative rate (FNR)} &= \frac{FN}{TP+FN} = \frac{20}{100} = 0.20 \end{aligned}$$

Q3.

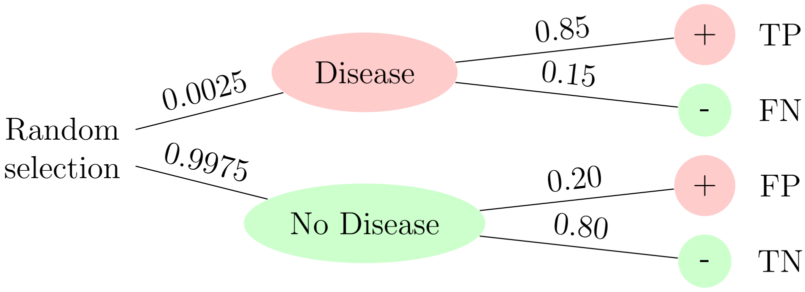
$$\begin{aligned} \text{Sensitivity (TPR)} &= \frac{TP}{TP+FN} = \frac{133}{133+1} \approx 0.99 \\ \text{Specificity (TNR)} &= \frac{TN}{TN+FP} = \frac{35}{35+12} \approx 0.74 \\ \text{Positive predictive value (PPV)} &= \frac{TP}{TP+FP} = \frac{133}{133+12} \approx 0.92 \\ \text{Negative predictive value (NPV)} &= \frac{TN}{TN+FN} = \frac{35}{35+1} \approx 0.97 \end{aligned}$$

Q4. (a)

$$\left.\begin{aligned} \Pr(FN) &= FNR, \\ FNR + TPR &= 1 \end{aligned}\right\} \quad 1 - 0.85 = 0.15$$

(b)

$$\left.\begin{aligned} \Pr(FP) &= FPR, \\ FPR + TNR &= 1 \end{aligned}\right\} \quad 1 - 0.80 = 0.20$$



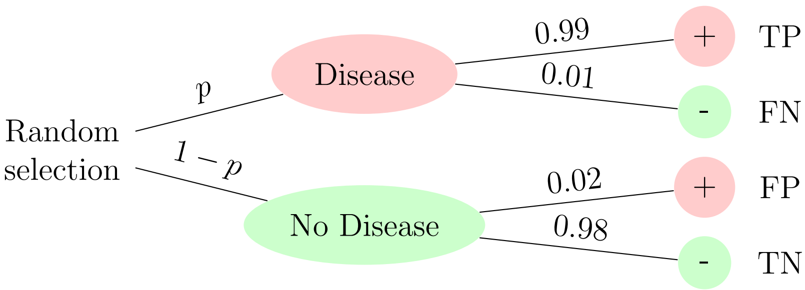
(c)

$$\Pr(+) = \underbrace{0.0025 \cdot 0.85}_{TP} + \underbrace{0.9975 \cdot 0.20}_{FP},$$

$$\Pr(\text{Disease} \cap +) = \underbrace{0.0025 \cdot 0.85}_{TP},$$

$$\Pr(\text{Disease} \mid +) = PPV = \frac{\Pr(\text{Disease} \cap +)}{\Pr(+)} \approx 0.01$$

Q5. Well done if you solved this!



(a) Assume a prevalence “p”, then:

$$\Pr(+) = \underbrace{p \cdot 0.99}_{TP} + \underbrace{(1-p) \cdot 0.02}_{FP},$$

$$\Pr(\text{Disease} \cap +) = \underbrace{p \cdot 0.99}_{TP},$$

$$\Pr(\text{Disease} \mid +) = \frac{\Pr(\text{Disease} \cap +)}{\Pr(+)}$$

Now can $\Pr(\text{Disease} \mid +) < 1\%$?

$$\Pr(\text{Disease} \mid +) = \frac{p \cdot 0.99}{p \cdot 0.99 + (1-p) \cdot 0.02} < 0.01$$

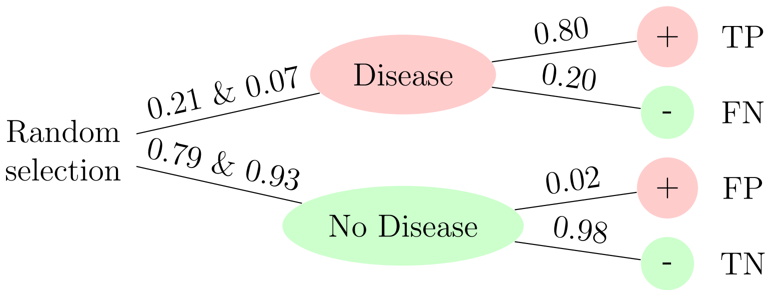
$$\frac{0.01 \cdot 0.02}{0.01 \cdot 0.02 + 0.99 - 0.01 \cdot 0.99} \approx \frac{204}{10^6} < p$$

So, yes, it is possible if the disease is more rare than 204 cases in a million.

(b) The implication is that for rare diseases a test needs to have very high validity.

Q6. (a) Low sensitivity $\Rightarrow$ prevalence underestimated (if $FN > FP$).

(b) Low specificity $\Rightarrow$ prevalence may be overestimated (if $FP > FN$).



(c) By the diagnostic test reported prevalence:

$$\Pr(+)_{\text{First Pop}} = \underbrace{0.21 \cdot 0.80}_{TP} + \underbrace{0.79 \cdot 0.02}_{FP} \approx 0.18,$$
$$\Pr(+)_{\text{Second Pop}} = \underbrace{0.07 \cdot 0.80}_{TP} + \underbrace{0.93 \cdot 0.02}_{FP} \approx 0.075$$

(d) The true rate ratio as stated in the question, and the rate ratio as reported by the diagnostic test:

$$\text{Rate ratio}_{\text{ True}} = \frac{0.21}{0.07} = 3,$$
$$\text{Rate ratio}_{\text{ Diagnostic test}} = \frac{0.18}{0.075} \approx 2.4$$