Program construction in C++ for Scientific Computing
Teacher: Michael Hanke

Ilian Häggmark
mail ilianh@kth.se

Andreas Karlsson
mail andreas.a.karlsson@ki.se

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Project 4

Task 1 - Redesign Domain class

Task 2 - GFkt class

The discrete differential operator $\frac{\partial}{\partial x}$ can be approximated as simply $u(x+1,y) - u(x-1,y)$ and analogously for y. The second order derivative can be approximated with $u(x+1,y) - 2 \cdot u(x,y) + u(x-1,y)$ for the x-direction. The Laplacian can be expressed as

$$\begin{bmatrix} 0 & 1 & 0 \\ 1 & -4 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

The code below shows these approximate derivatives in Matlab:

```
% Approximate partial derivatives

x = -30:0.1:30;
y = -5:0.1:5;

ux = sin((x/10).^2).*cos(x/10); % x-comp of u(x,y)
[X,Y] = meshgrid(ux,y);
u = X+Y;

Dx = zeros(size(u));    % first partial derivative in x
Dy = zeros(size(u));    % first partial derivative in y
L = zeros(size(u));     % laplacian

for i = 2:(length(x)-1)
    for j = 2:(length(y)-1)
        Dx(j,i) = u(j,i+1)-u(j,i-1);
        Dy(j,i) = u(j+1,i)-u(j-1,i);
        L(j,i) = -4*u(j,i)+u(j-1,i)+u(j+1,i)+u(j,i-1)+u(j,i+1);
    end
end
```

Task 3 - Discrete differential operators

The function $u(x,y)$ and its derivatives are shown below

$$
\begin{aligned}
u(x,y) &= \sin\left(\left(\frac{x}{10}\right)^2\right)\cos(x/10) + y \\
\frac{\partial}{\partial x}u(x,y) &= \frac{2}{100}x\cos\left(\left(\frac{x}{10}\right)^2\right)\cos\left(\frac{x}{10}\right) - \frac{1}{10}\sin\left(\left(\frac{x}{10}\right)^2\right)\sin\left(\frac{x}{10}\right) \\
\frac{\partial}{\partial y}u(x,y) &= \frac{\partial}{\partial y}y = 1 \\
\frac{\partial^2}{\partial x^2}u(x,y) &= \frac{2}{100}\cos\left(\left(\frac{x}{10}\right)^2\right)\cos\left(\frac{x}{10}\right) - \left(\frac{2}{100}x\right)^2\sin\left(\left(\frac{x}{10}\right)^2\right)\cos\left(\frac{x}{10}\right) \\
&\quad - \frac{2}{100}\frac{1}{10}x\cos\left(\left(\frac{x}{10}\right)^2\right)\sin\left(\frac{x}{10}\right) - \frac{1}{10}\frac{2}{100}x\cos\left(\left(\frac{x}{10}\right)^2\right)\sin\left(\frac{x}{10}\right) \\
&\quad - \frac{1}{100}\sin\left(\left(\frac{x}{10}\right)^2\right)\cos\left(\frac{x}{10}\right) \\
\frac{\partial^2}{\partial y^2}u(x,y) &= 0 \\
\Delta u(x,y) &= \frac{\partial^2}{\partial x^2}u(x,y) + \frac{\partial^2}{\partial y^2}u(x,y) = \frac{\partial^2}{\partial x^2}u(x,y)
\end{aligned}
$$

The code below plot the algebraic expressions in MATLAB

```matlab
% Algebraic expressions of first and second partial derivatives in x

x = -50:0.1:50;

figID = figure(101);                                % open figure

ux = sin((x./10).^2).*cos(x./10);                   % x-component of u(x,y)

dxu = 2/100.*x.*cos((x./10).^2).*cos(x./10) ...   % first parital derivative
      - 1/10*sin((x./10).^2).*sin(x./10);

dxxu = 2/100*cos((x./10).^2).*cos(x./10) ...       % second partial derivative
       - (2/100.*x).^2.*sin((x./10).^2).*cos(x./10) ...
       - 1/250*cos((x./10).^2).*x.*sin(x./10) ...
       - 1/100*sin((x./10).^2).*cos(x./10);

subplot(1,3,1); plot(x,ux);
subplot(1,3,2); plot(x,dxu);                        % plot result in subplot
subplot(1,3,3); plot(x,dxxu);
```

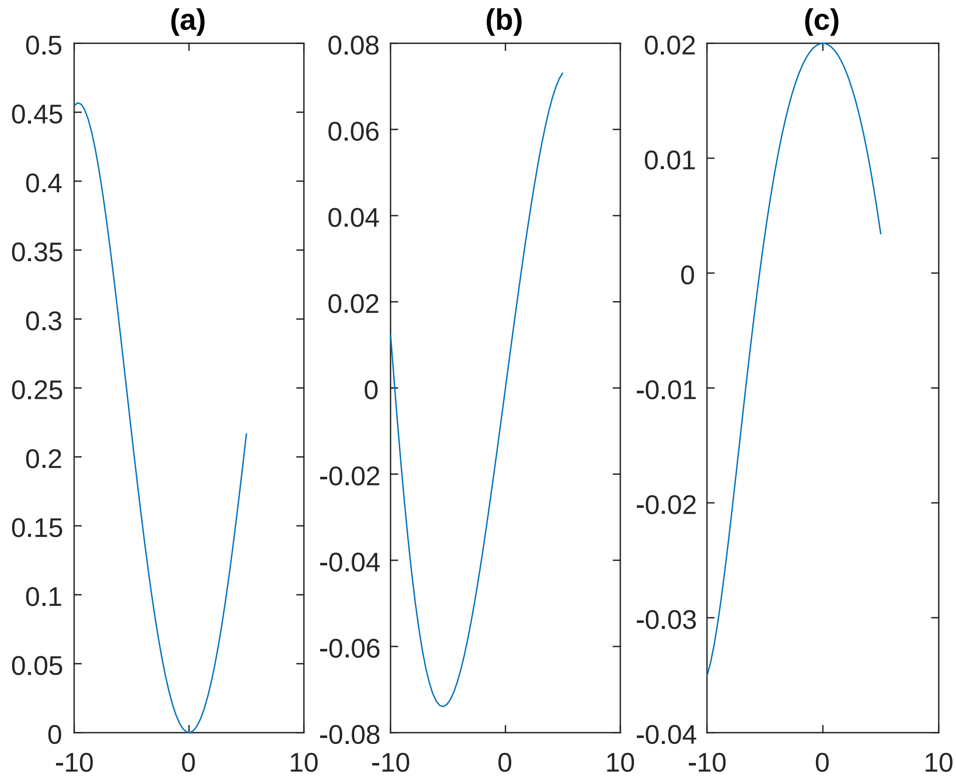

Figure 1: (a) x-component of $u(x, y)$. (b) First partial derivative of $u(x, y)$ in x. (c) Second partial derivative of $u(x, y)$ in x.