

Program construction in C++ for Scientific Computing

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Project 4

Task 1 - Redesign Domain class

Task 2 - GFkt class

The discrete differential operator $\frac{\partial}{\partial x}$ can be approximated as simply $u(x+1, y) - u(x-1, y)$ and analogously for y . The second order derivative can be approximated with $u(x+1, y) - 2 \cdot u(x, y) + u(x-1, y)$ for the x -direction. The Laplacian can be expressed as

$$\begin{bmatrix} 0 & 1 & 0 \\ 1 & -4 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

The code below shows these approximate derivatives in MATLAB:

```

1  % Approximate partial derivatives
2
3  x = -30:0.1:30;
4  y = -5:0.1:5;
5
6  ux = sin((x/10).^2).*cos(x/10); % x-comp of u(x,y)
7  [X,Y] = meshgrid(ux,y);
8  u = X+Y;
9
10 Dx = zeros(size(u)); % first partial derivative in x
11 Dy = zeros(size(u)); % first partial derivative in y
12 L = zeros(size(u)); % laplacian
13
14 for i = 2:(length(x)-1)
15     for j = 2:(length(y)-1)
16         Dx(j,i) = u(j,i+1)-u(j,i-1);
17         Dy(j,i) = u(j+1,i)-u(j-1,i);
18         L(j,i) = -4*u(j,i)+u(j-1,i)+u(j+1,i)+u(j,i-1)+u(j,i+1);
19     end
20 end

```

Task 3 - Discrete differential operators

The function $u(x, y)$ and its derivatives are shown below

$$\begin{aligned}
u(x, y) &= \sin\left(\left(\frac{x}{10}\right)^2\right) \cos(x/10) + y \\
\frac{\partial}{\partial x} u(x, y) &= \frac{2}{100} x \cos\left(\left(\frac{x}{10}\right)^2\right) \cos\left(\frac{x}{10}\right) - \frac{1}{10} \sin\left(\left(\frac{x}{10}\right)^2\right) \sin\left(\frac{x}{10}\right) \\
\frac{\partial}{\partial y} u(x, y) &= \frac{\partial}{\partial y} y = 1 \\
\frac{\partial^2}{\partial x^2} u(x, y) &= \frac{2}{100} \cos\left(\left(\frac{x}{10}\right)^2\right) \cos\left(\frac{x}{10}\right) - \left(\frac{2}{100} x\right)^2 \sin\left(\left(\frac{x}{10}\right)^2\right) \cos\left(\frac{x}{10}\right) \\
&\quad - \frac{2}{100} \frac{1}{10} \cos\left(\left(\frac{x}{10}\right)^2\right) \sin\left(\frac{x}{10}\right) - \frac{1}{10} \frac{2}{100} x \cos\left(\left(\frac{x}{10}\right)^2\right) \sin\left(\frac{x}{10}\right) \\
&\quad - \frac{1}{100} \sin\left(\left(\frac{x}{10}\right)^2\right) \cos\left(\frac{x}{10}\right) \\
\frac{\partial^2}{\partial y^2} u(x, y) &= 0 \\
\Delta u(x, y) &= \frac{\partial^2}{\partial x^2} u(x, y) + \frac{\partial^2}{\partial y^2} u(x, y) = \frac{\partial^2}{\partial x^2} u(x, y)
\end{aligned}$$

The code below plot the algebraic expressions in MATLAB

```

1  % Algebraic expressions of first and second partial derivatives in x
2
3  x = -50:0.1:50;
4
5  figID = figure(101);           % open figure
6
7  ux = sin((x./10).^2).*cos(x./10); % x-component of u(x,y)
8
9  dxu = 2/100.*x.*cos((x./10).^2).*cos(x./10) ... % first partial derivative
10      - 1/10*sin((x./10).^2).*cos(x./10);
11
12  dxxu = 2/100*cos((x./10).^2).*cos(x./10) ... % second partial derivative
13      - (2/100.*x).^2.*sin((x./10).^2).*cos(x./10) ...
14      - 1/250*cos((x./10).^2).*sin(x./10) ...
15      - 1/100*sin((x./10).^2).*cos(x./10);
16
17  subplot(1,3,1); plot(x,ux);
18  subplot(1,3,2); plot(x,dxu); % plot result in subplot
19  subplot(1,3,3); plot(x,dxxu);

```

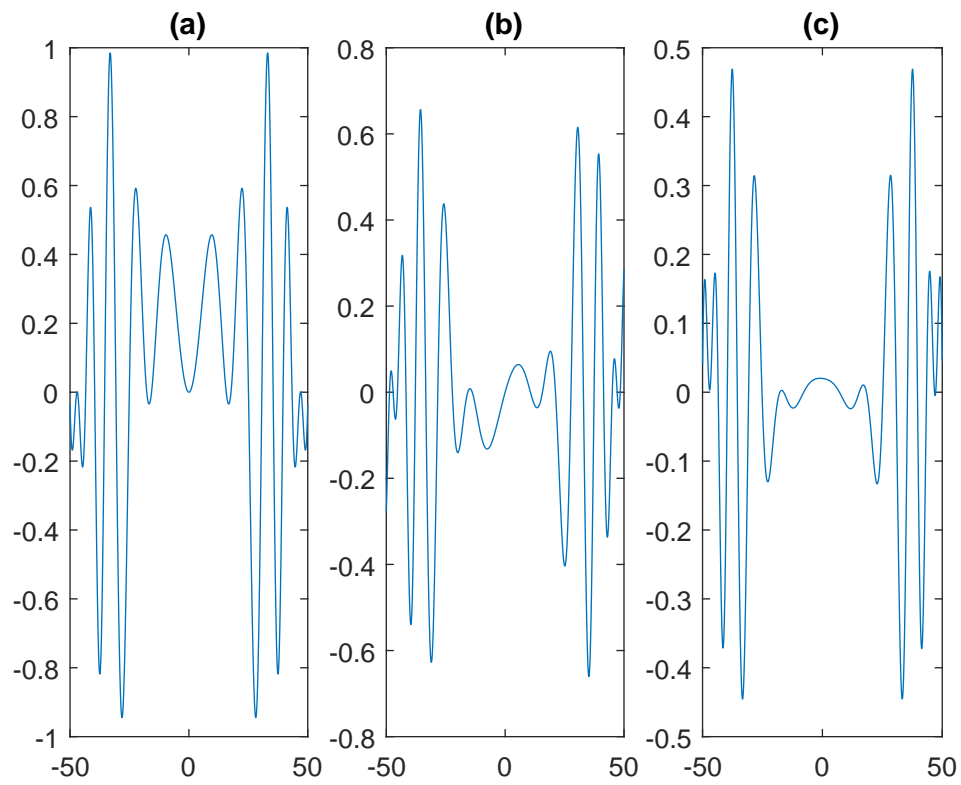


Figure 1: (a) x-component of $u(x, y)$. (b) First partial derivative of $u(x, y)$ in x . (c) Second partial derivative of $u(x, y)$ in x .