Image Analysis and Object Recognition

Exercise 2

Summer Semester 2025

(Course materials for internal use only!)

Computer Vision in Engineering – Prof. Dr. Rodehorst
M.Sc. Mariya Kaisheva
mariya.kaisheva@uni-weimar.de

Agenda

	Topics:	Submission Dates:
Assignment 1.	Image enhancement, Binarization, Morphological operators	30.04.25
Assignment 2.	**Gradient of Gaussian filtering, Förstner interest operator**	**21.05.25**
Assignment 3.	Shape detection based on Hough-voting	04.06.25
Assignment 4.	Filtering in the frequency domain, Fourier descriptors	18.06.25
Assignment 5.	Image segmentation using clustering	02.07.25
Final Project.	**-** *Will be announced during the last exercise class* **-**	10.08.25

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Assignment 1:
Sample Solution

Assignment 1: Overview

Topics:

- Image enhancement → histogram stretching
- Global Thresholding → binary mask
- Morphological operators → opening, closing

Goal:

- Extracting image pixels representing foreground objects
  → e.g. extraction of the water regions

Assignment 1: Workflow

Low-contrast input image

Enhanced grayscale image

Binary image

Refined binary image

Final overlay image

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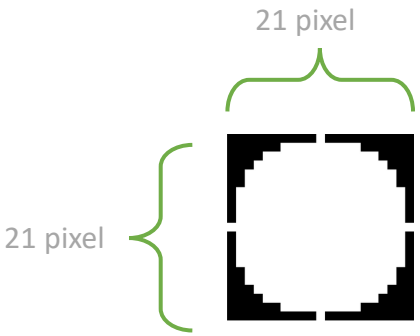
helper functions

```python
import numpy as np
import matplotlib.pyplot as plt
from PIL import Image
from utils import rgb2gray, imopen, imclose

#Task 1
def enhancing(input_img):
    """ apply contrast stretching """
    min_val, max_val = np.min(input_img), np.max(input_img)
    return ((input_img - min_val) / (max_val - min_val) * 255).astype(np.uint8)

#Task 2
def thresholding(input_img, threshold):
    """ generate binary mask"""
    return (input_img < threshold).astype(np.uint8)

#Tanks 3
def filtering(binary_mask):
    """ apply morfological filtering """
    radius = 10
    y, x = np.ogrid[-radius:radius+1, -radius:radius+1]
    structuring_element = (x**2 + y**2 <= radius**2).astype(np.uint8)
    return imclose(imopen(binary_mask, structuring_element), structuring_element)
```



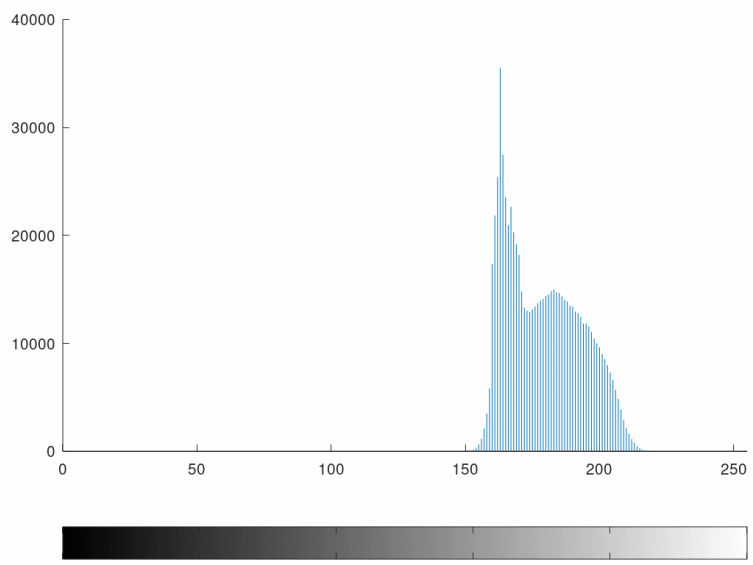
used to generate centered coordinate grid

main function

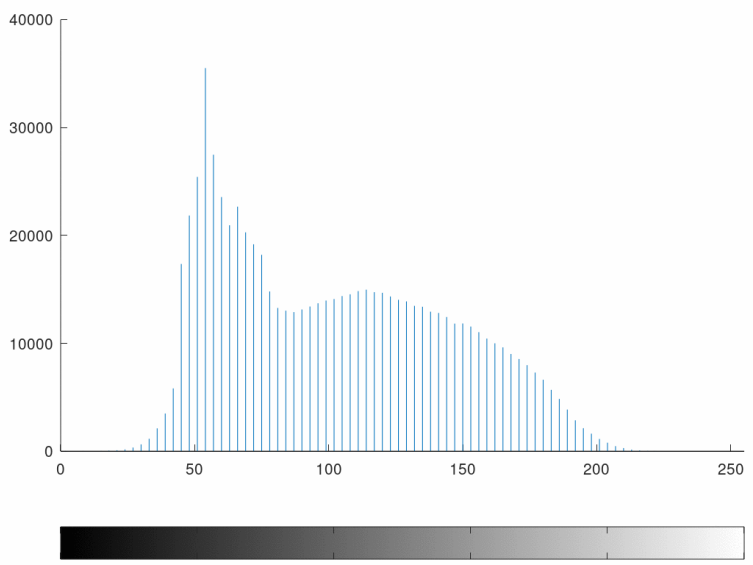
```
def assignment1():
    input_img = np.array(Image.open('input_sat_image.jpg')) # load input image as a NumPy array
    input_gray = rgb2gray(input_img) # convert to grayscale iamge
    plt.figure(); plt.imshow(input_img); plt.title('Input image'); # display RGB input image
    plt.figure(); plt.hist(input_img.flatten(), bins=256, range=(0, 255), density=True); plt.title('Inital Histogram')

    improved = enhancing(input_gray) # apply contrast stretching
    plt.figure(); plt.hist(improved.flatten(), bins=256, range=(0, 255), density=True); plt.title('Histogram after Enhancement')
    plt.figure(); plt.imshow(improved, cmap='gray'); plt.title('Enhanced image');

    binary_mask = thresholding(improved, 90) # create binary mask
    plt.figure(); plt.imshow(binary_mask, cmap='gray'); plt.title('Binary mask');

    filtered_mask = filtering(binary_mask) # apply morphological filtering
    plt.figure(); plt.imshow(filtered_mask, cmap='gray'); plt.title('Morphological Filtering')

    output = improved.copy() # final overlay iamge
    output[filtered_mask > 0] = 255
    plt.figure(); plt.imshow(output, cmap='gray'); plt.title('Output image')
    plt.show()
    return output
```

Assignment 1 – sample results

Initial Histogram



Histogram after Enhancement



Assignment 1 – sample results

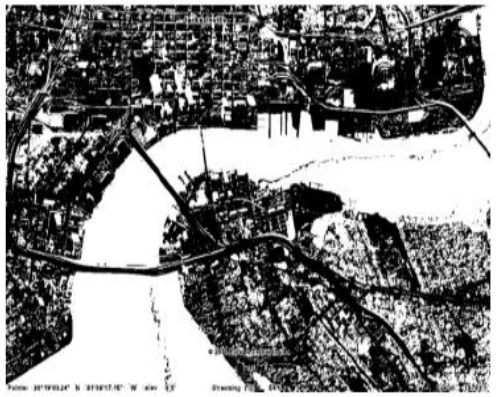
Enhanced grayscale image

Influence of the binarization threshold
on the initial binary mask

threshold = 50

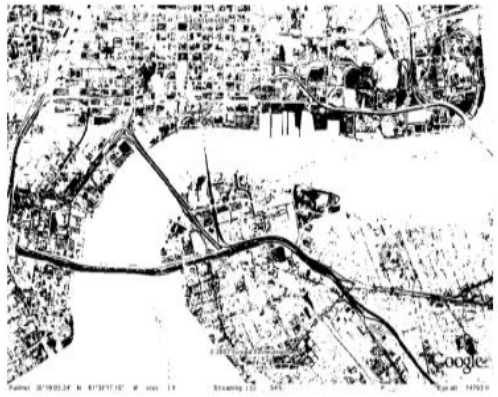
threshold = 80

threshold = 100



threshold = 120

threshold = 150



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Assignment 1 – sample results

Enhanced grayscale image

Influence of the size of the structuring element
on the refined binary mask

SE: disk, 5

SE: disk, 7

SE: disk, 10

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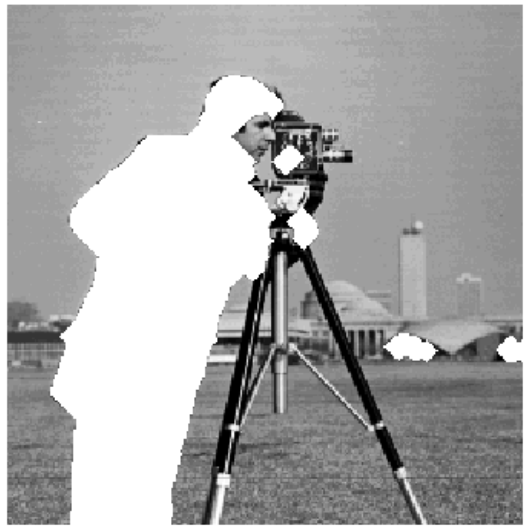
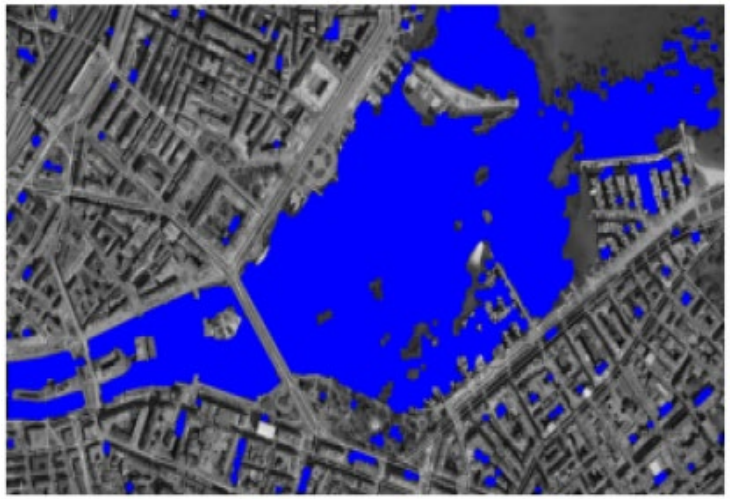
Assignment 1 – selected submission results







Assignment 1 – selected submission results



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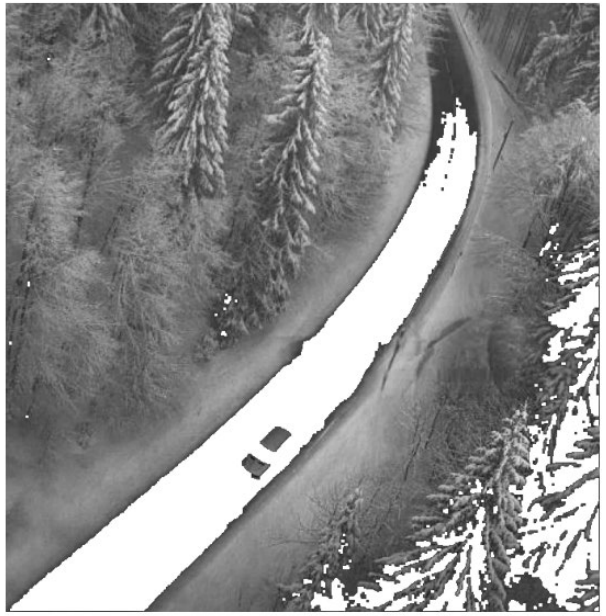
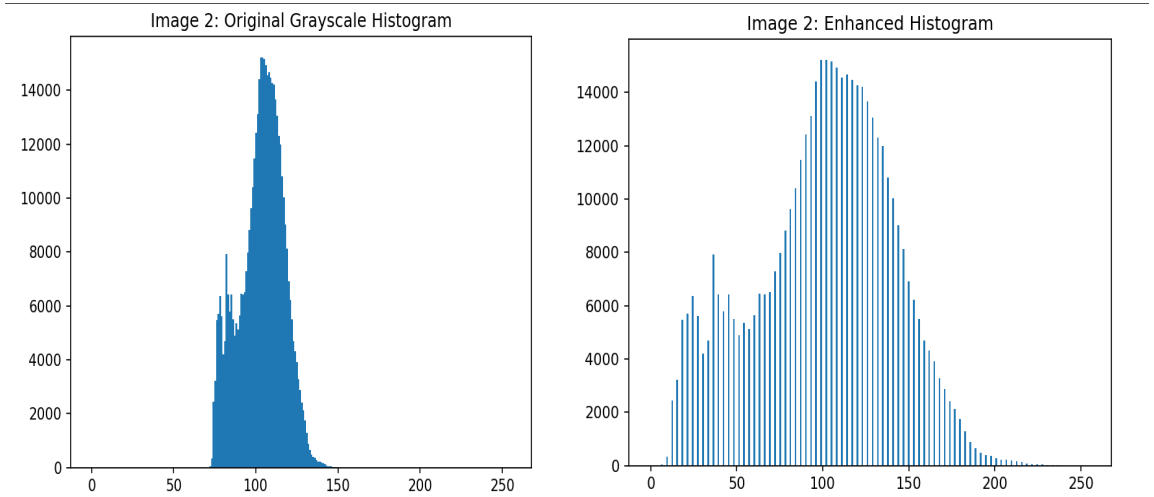
Assignment 1 – selected submission results





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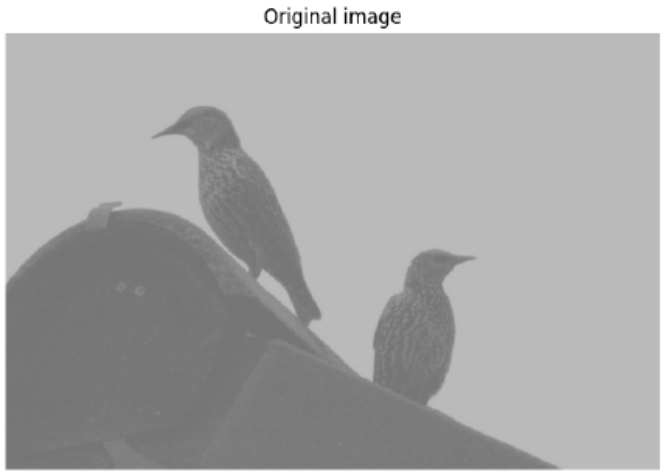
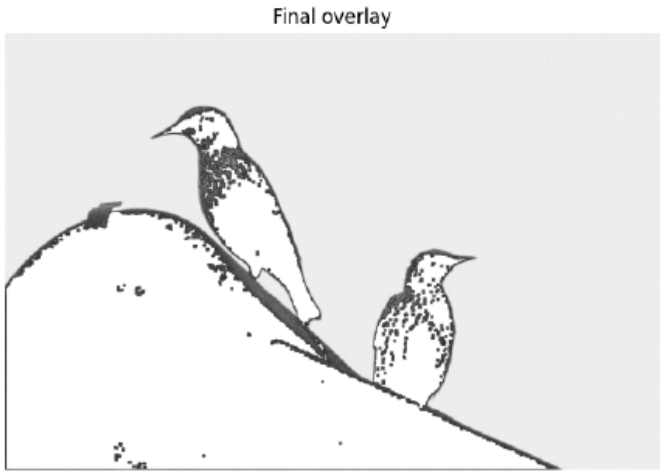
Assignment 1 – selected submission results





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Assignment 2

Assignment 2: Overview

Topics:

- Image filtering with Gradient of Gaussian (GoG)
- Interest points detection

Goal:

- Learn how to perform image filtering
- Practice reducing noise and **simultaneously** deriving image gradients (intensity changes)
- Practice identifying points of interest with the help of image gradients

Assignment 2: Overview

Input:

- Provided image → *ampelmaennchen.png*
- Or a different image of your own choice

Tasks:

- **A:** Gradient of Gaussian (GoG) filtering
- **B:** Förstner operator

Provided input image

Assignment 2: Task A

GoG filtering:

a. Compute **GoG-filter kernels** for filtering in *x*- and *y*- direction

b. Apply the two filters G_x and G_y on the input image *I* using convolution to derive **two gradient images I_x and I_y**

c. Compute and visualize the **gradient magnitude**

$$G = \sqrt{{I_x}^2 + {I_y}^2}$$

Input image I

Grayscale converted I

Note:
Compute grayscale image and scale it to double [0.0 , 1.0].

Assignment 2: Task A

GoG filtering:

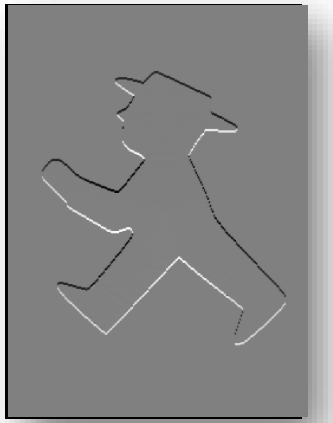
a. Compute **GoG-filter kernels** for filtering in *x*- and *y*- direction

b. Apply the two filters G_x and G_y on the input image *I* using convolution to derive **two gradient images I_x and I_y**

c. Compute and visualize the **gradient magnitude**

$$G = \sqrt{{I_x}^2 + {I_y}^2}$$

Gradient image I_x



Gradient image I_y

Assignment 2: Task A

GoG filtering:

a. Compute **GoG-filter kernels** for filtering in *x*- and *y*- direction

b. Apply the two filters G_x and G_y on the input image *I* using convolution to derive **two gradient images I_x and I_y**

c. Compute and visualize the **gradient magnitude**

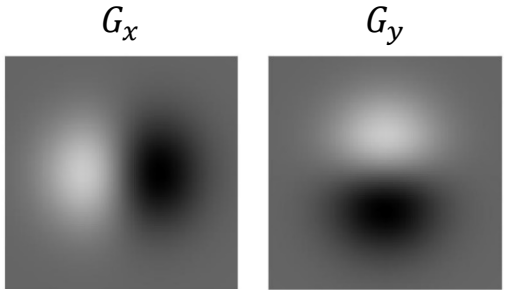
$$G = \sqrt{{I_x}^2 + {I_y}^2}$$

Gradient magnitude

2D GoG filter computation

$$G_x = \frac{\partial G(x,y,\sigma)}{\partial x} = -\frac{x}{2\pi\sigma^4} exp\left(-\frac{(x^2+y^2)}{2\sigma^2}\right)$$

general formula for computation of G_x

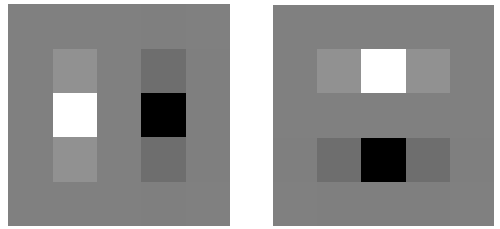


general GoG filters in x and y directions

$$G_y = {G_x}^T$$

$$G_x = \begin{bmatrix} 0.0000 & 0.0001 & 0.0000 & -0.0001 & -0.0000 \\ 0.0002 & 0.0466 & 0.0000 & -0.0466 & -0.0002 \\ 0.0017 & 0.3446 & 0.0000 & -0.3446 & -0.0017 \\ 0.0002 & 0.0466 & 0.0000 & -0.0466 & -0.0002 \\ 0.0000 & 0.0001 & 0.0000 & -0.0001 & -0.0000 \end{bmatrix}$$

numerical example with $\sigma = 0.5$



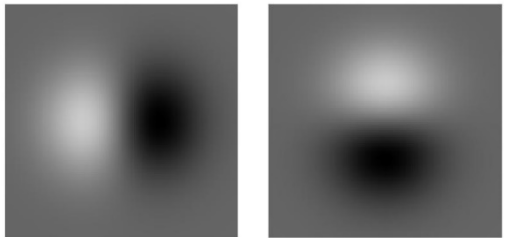
5x5 pixel GoG filters in x and y directions

2D GoG filter computation

$$G_x = \frac{\partial G(x,y,\sigma)}{\partial x} = -\frac{x}{2\pi\sigma^4} exp\left(-\frac{(x^2+y^2)}{2\sigma^2}\right)$$

general formula for computation of G_x

G_x G_y



$$G_y = {G_x}^T$$

1) Define standard deviation, e.g. $\sigma = 0.5$
2) Filter kernel radius: $r = |3 \cdot \sigma| = 2.0$
3) Define two arrays c_x and c_y with $(r \cdot 2 + 1)$ columns and rows for centered local coordinates

$$c_x = \begin{bmatrix} -2 & -1 & 0 & 1 & 2 \\ -2 & -1 & 0 & 1 & 2 \\ -2 & -1 & 0 & 1 & 2 \\ -2 & -1 & 0 & 1 & 2 \\ -2 & -1 & 0 & 1 & 2 \end{bmatrix}; \; c_y = c_x^T$$

4) Compute filter using c_x and c_y for x and y $\rightarrow \frac{\partial G(x,y,\sigma)}{\partial x} = -\frac{c_x}{2\pi\sigma^4} exp\left(-\frac{({c_x}^2 + {c_y}^2)}{2\sigma^2}\right)$

Assignment 2: Task B

Förstner interest operator:

a. Compute the **autocorrelation matrix $\boldsymbol{M}$** for each pixel using a 5×5 moving window

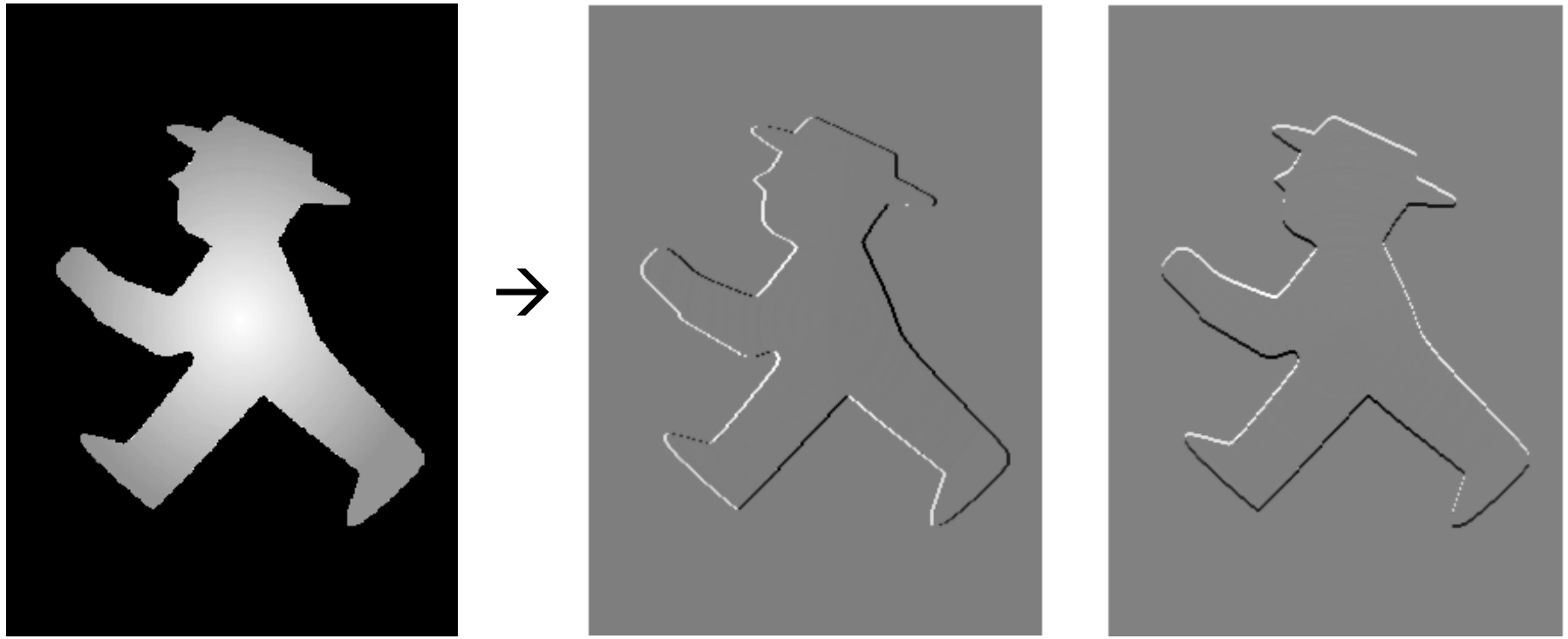
b. Instead of storing M for each pixel, compute the **cornerness $\boldsymbol{w}$** and **roundness $\boldsymbol{q}$** from M and store these values in matrices W and Q. Plot these arrays.

c. Derive a **binary mask $\boldsymbol{M_c}$** of potential interest points by simultaneously applying thresholds, e.g. $t_w = 0.004$ and $t_q = 0.5$, on W and Q

d. Plot an overlay of the initial input image and the **detected points**

Auto-correlation Matrix M

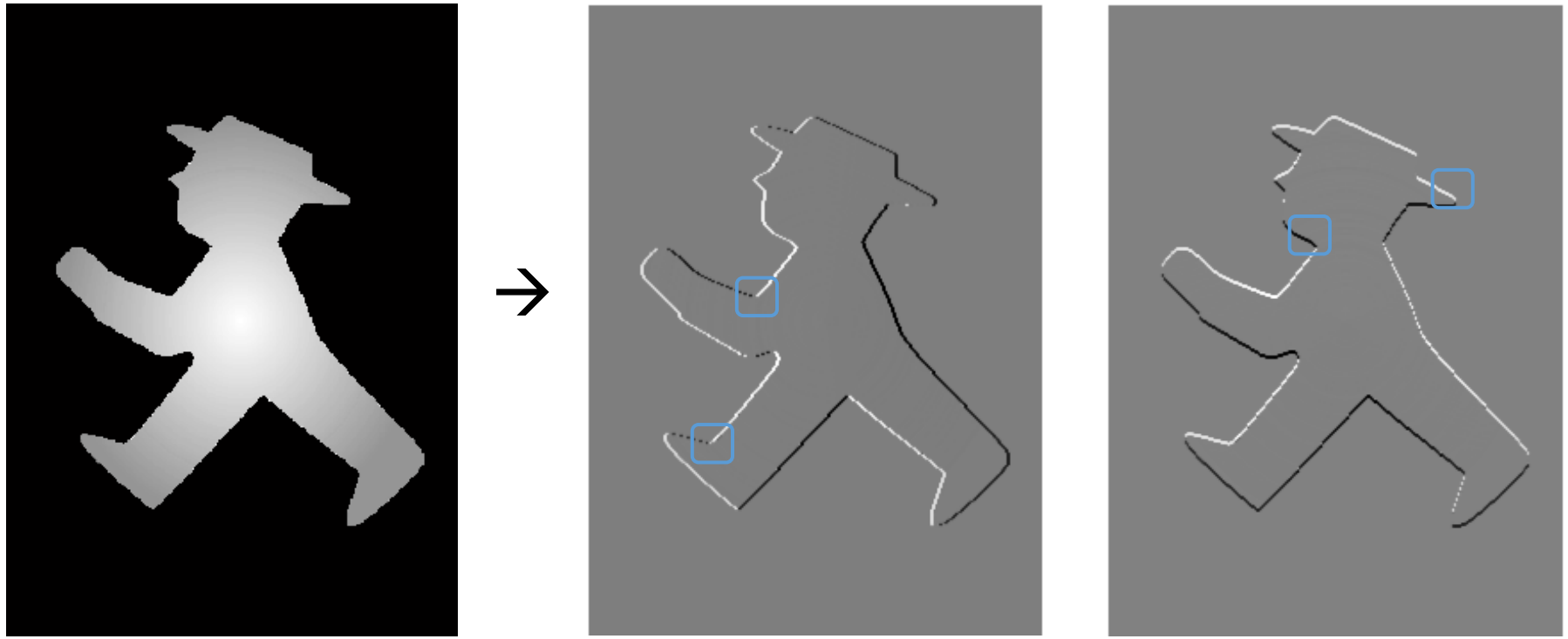
- Identification of corners
- Input: First order derivatives in x- and y-direction I_x and I_y (i.e. the result of Task A.b.)



Grayscale image I_x (GoG) I_y (GoG)

Auto-correlation Matrix M

- Identification of corners
- Input: First order derivatives in x- and y-direction I_x and I_y (i.e. the result of Task A.b.)

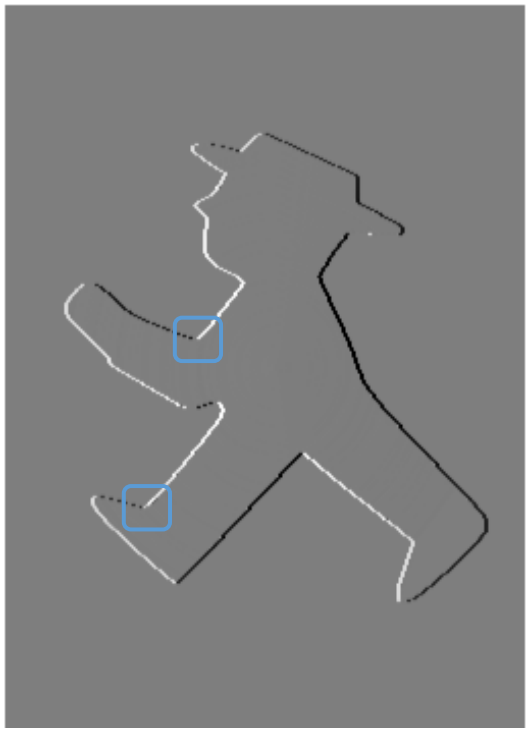


Grayscale image I_x (GoG) I_y (GoG)

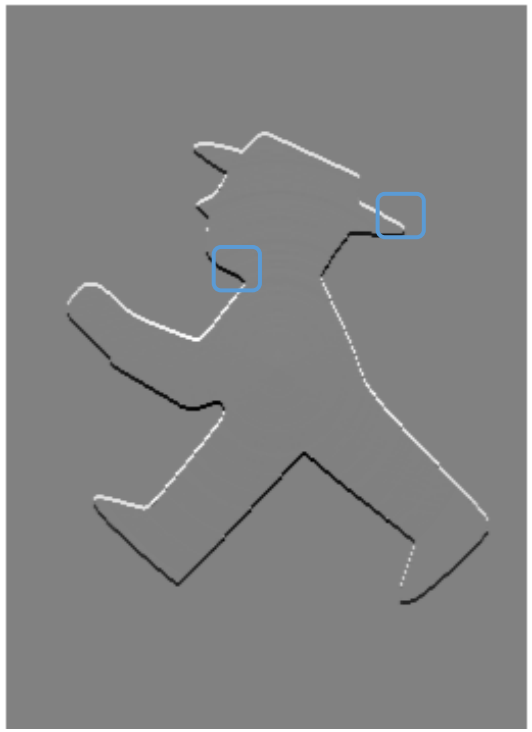
Auto-correlation Matrix M

- Identification of corners
- Input: First order derivatives in x- and y-direction I_x and I_y (i.e. the result of Task A.b.)

Grayscale image



I_x (GoG)



I_y (GoG)

I_x^2, I_y^2
and
$I_x I_y$
3 arrays
necessary for the next steps

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Auto-correlation Matrix M

Computation of M for each pixel:

Definition: $w_N = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \end{bmatrix}$ → weights for the local neighborhood N

$$M = \sum_{x,y \in N} w_N(x,y) \cdot \begin{bmatrix} I_x^2 & I_x I_y \\ I_x I_y & I_y^2 \end{bmatrix} = w_N \star \begin{bmatrix} I_x^2 & I_x I_y \\ I_x I_y & I_y^2 \end{bmatrix}$$

→ $M = \sum_{x,y \in N} \begin{bmatrix} I_x^2 & I_x I_y \\ I_x I_y & I_y^2 \end{bmatrix}$ M contains the sum of all values of I_x^2, I_y^2 and $I_x I_y$ in the local neighborhood N of 5x5 pixels

Auto-correlation Matrix M

For each pixel in the image (except boundaries)**:**

1) Extract local image window w_N for I_x^2, I_y^2 and $I_x I_y$

2) Compute M :
   → sum up extracted values I_x^2, I_y^2 and $I_x I_y$
   → $\bar{I}_x^2 = \sum_N I_x^2$, also for $\bar{I}_y^2$ and $\bar{I}_x \bar{I}_y$

3) Build M (2×2 matrix) for each pixel

$$M = \begin{bmatrix} \bar{I}_x^2 & \bar{I}_x \bar{I}_y \\ \bar{I}_x \bar{I}_y & \bar{I}_y^2 \end{bmatrix}$$

Or: convolve I_x^2, I_y^2 and $I_x I_y$ with w_N and then compute M for each pixel

Auto-correlation Matrix M

Cornerness:

$$w = \frac{trace(M)}{2} - \sqrt{\left(\frac{trace(M)}{2}\right)^2 - det(M)}, \qquad w > 0$$

Roundness:

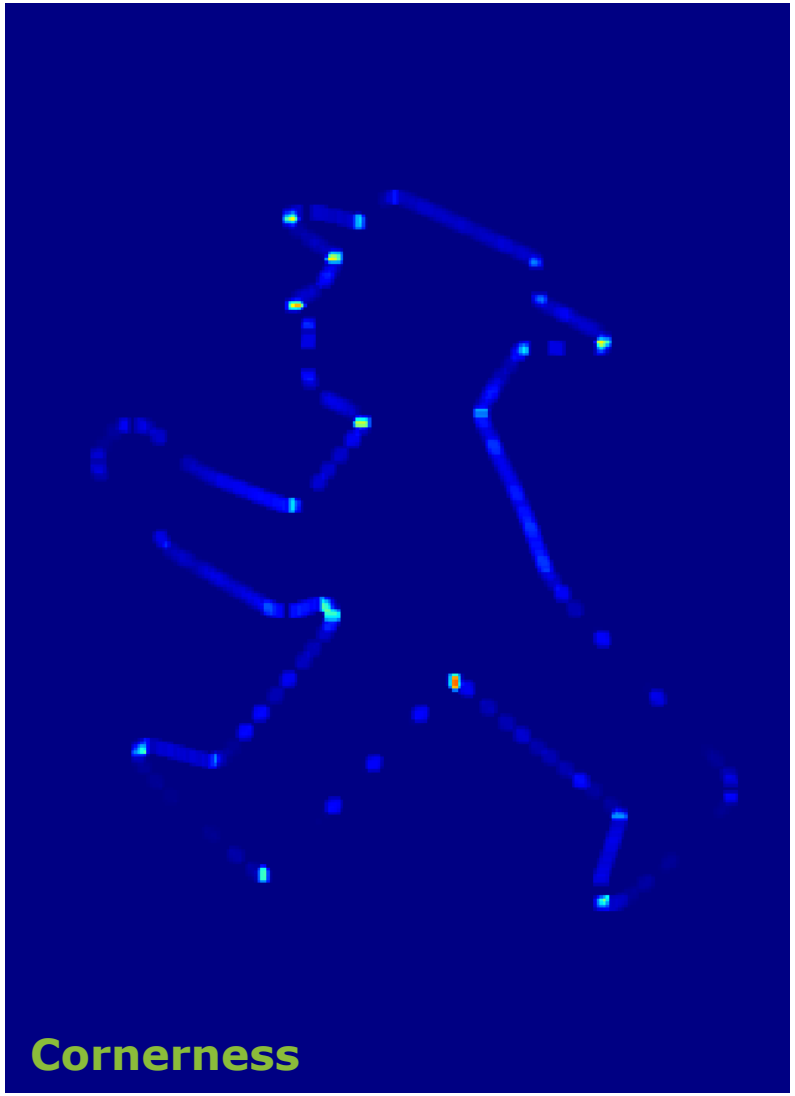
$$q = \frac{4 \cdot det(M)}{trace(M)^2}, \qquad 0 \leq q \leq 1$$

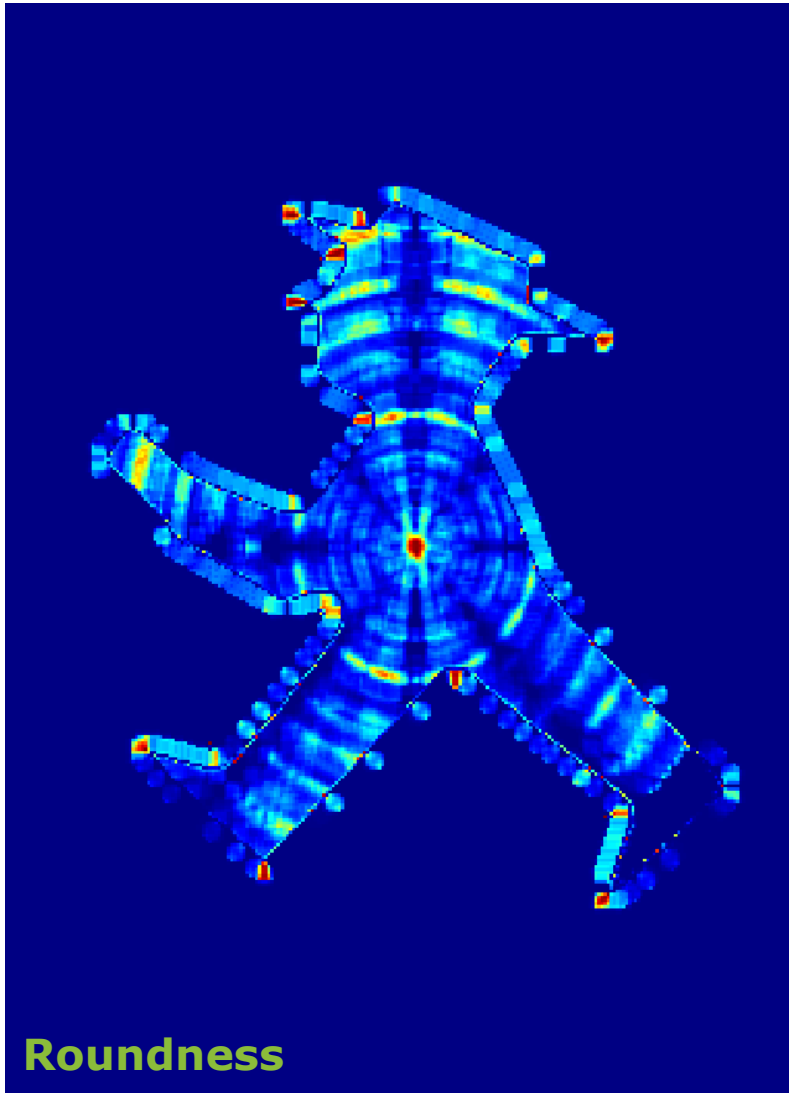
Find **corner point candidates $\boldsymbol{M_c}$**, if

$$w > t_w \quad \text{and} \quad q > t_q$$

$$t_w = [0.001 \ldots 0.01], t_q = [0.5 \ldots 0.75]$$

Thresholded regions of w and q





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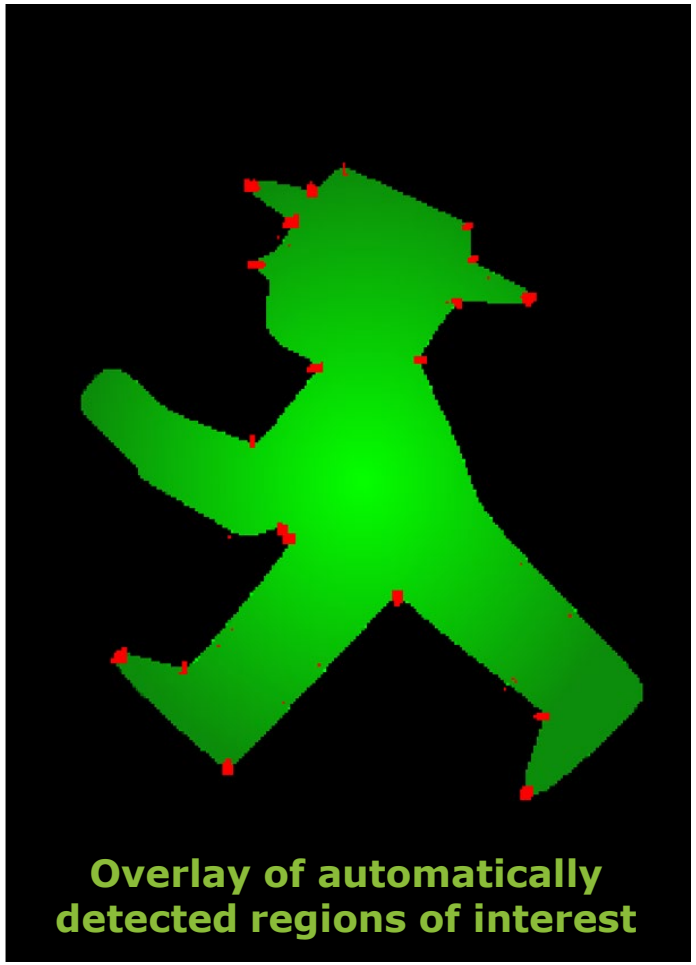
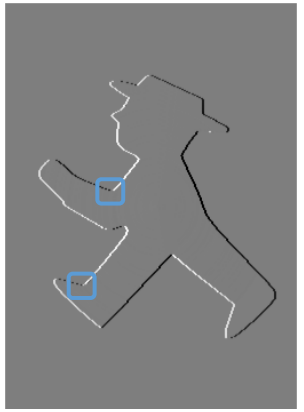
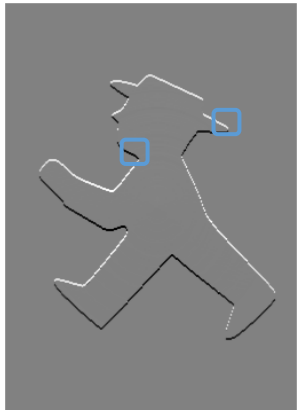
Overlay of I with M_c



Overlay of automatically detected regions of interest





Gradient images I_x and I_y

- areas expected to be detected as interest regions

Assignment 2: Task B

Förstner interest operator:

a. Compute the **autocorrelation matrix $\boldsymbol{M}$** for each pixel using a 5×5 moving window

b. Instead of storing M for each pixel, compute the **cornerness $\boldsymbol{w}$** and **roundness $\boldsymbol{q}$** from M and store these values in matrices W and Q. Plot these arrays

c. Derive a **binary mask $\boldsymbol{M_c}$** of potential interest points by simultaneously applying thresholds, e.g. $t_w = 0.004$ and $t_q = 0.5$, on W and Q

d. Plot an overlay of the initial input image and the **detected points**

Original input image I overplayed with detected interest points