Business Analytics

Course Overview

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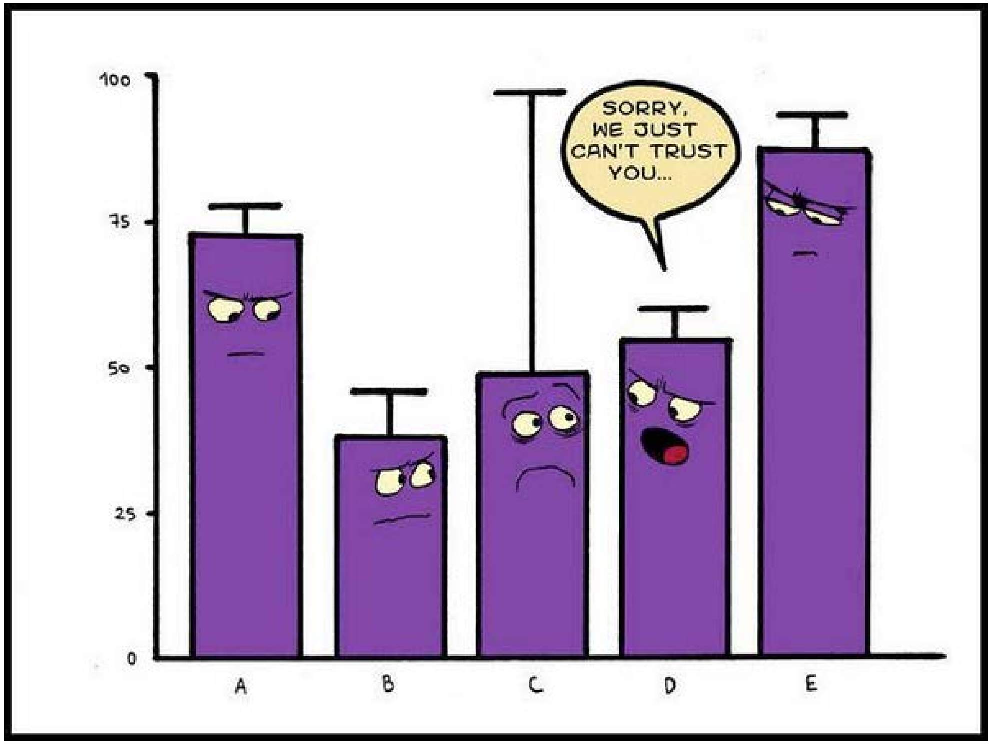
Agenda for Today

1. Understand what this course is all about
2. Refresh basic statistical concepts
3. Learn about organization, grading, and tutor groups (separate video)



Business Analytics

Business analytics makes extensive use of statistical analysis, including explanatory and predictive modeling, and fact-based management to drive decision making. It is therefore closely related to management science. Analytics may be used as input for human decisions or may drive fully automated decisions.

http://en.wikipedia.org/wiki/Business_analytics

Descriptive Analytics

What has occurred?

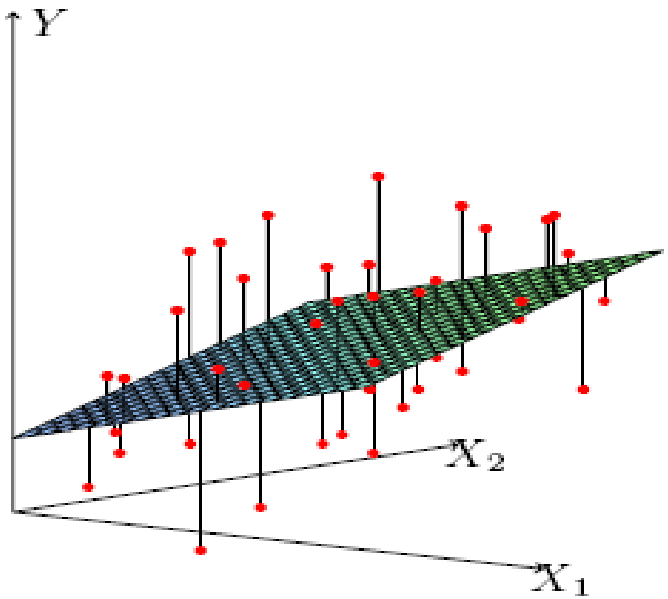
	A	B	C	D	E	F	G	H	I	J
1	Category	(All)								
2										
3	**Sum of Amount**	**Column**								
4	**Row Labels**	**Apple**	**Banana**	**Beans**	**Broccoli**	**Carrots**	**Mango**	**Orange**	**Grand Total**	
5	Australia	20634	52721	14433	17953	8106	9186	8680	131713	
6	Canada	24867	33775		12407		3767	19929	94745	
7	France	80193	36094	680	5341	9104	7388	2256	141056	
8	Germany	9082	39686	29905	37197	21636	8775	8887	155168	
9	New Zealand	10332	40050		4390			12010	66782	
10	United Kingdom	17534	42908	5100	38436	41815	5600	21744	173137	
11	United States	28615	95061	7163	26715	56284	22363	30932	267133	
12	**Grand Total**	**191257**	**340295**	**57281**	**142439**	**136945**	**57079**	**104438**	**1029734**	
13										

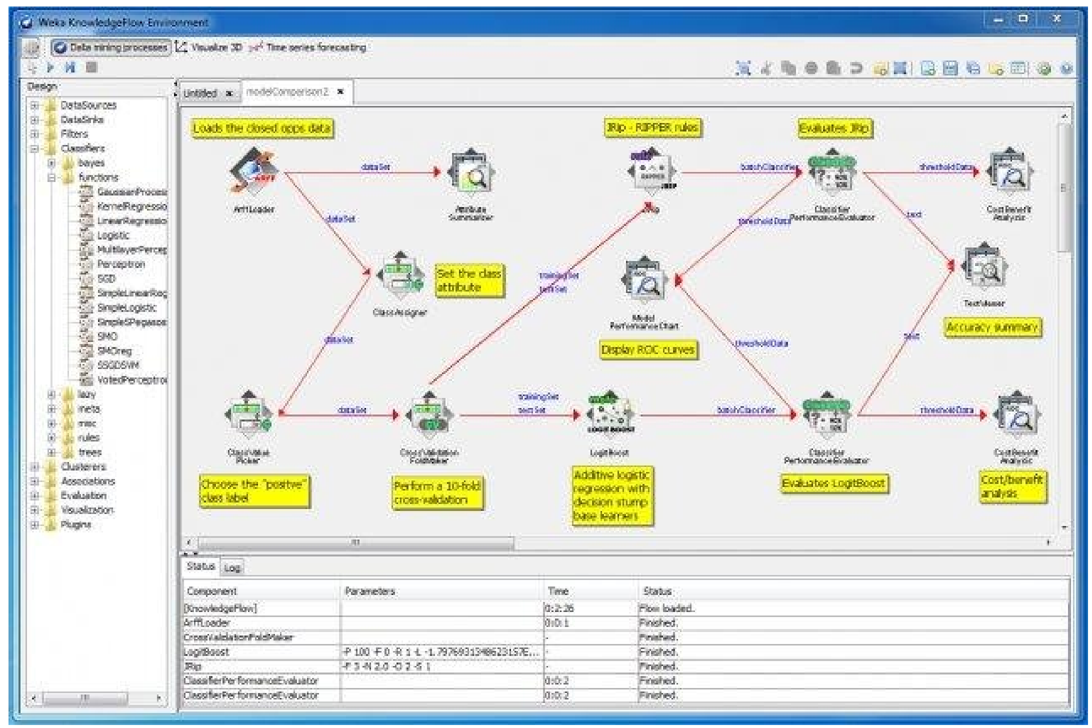
Data Engineering and Statistics

Organize data, execute large queries, describe means, trends, and test hypotheses

Predictive Analytics

What is likely to occur?

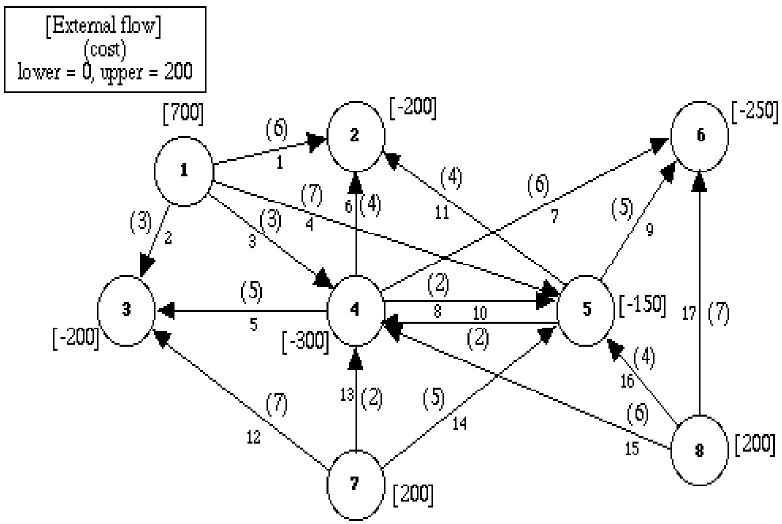


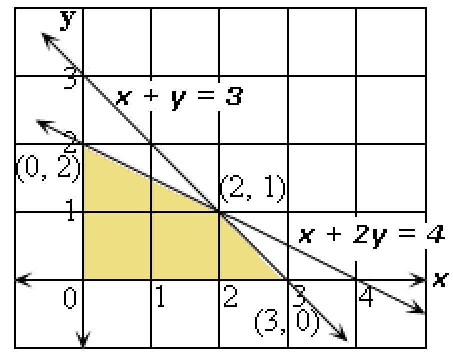


Machine Learning and Econometrics

Forecast events,
predict time series, or discrete choice decisions of customers

Prescriptive Analytics



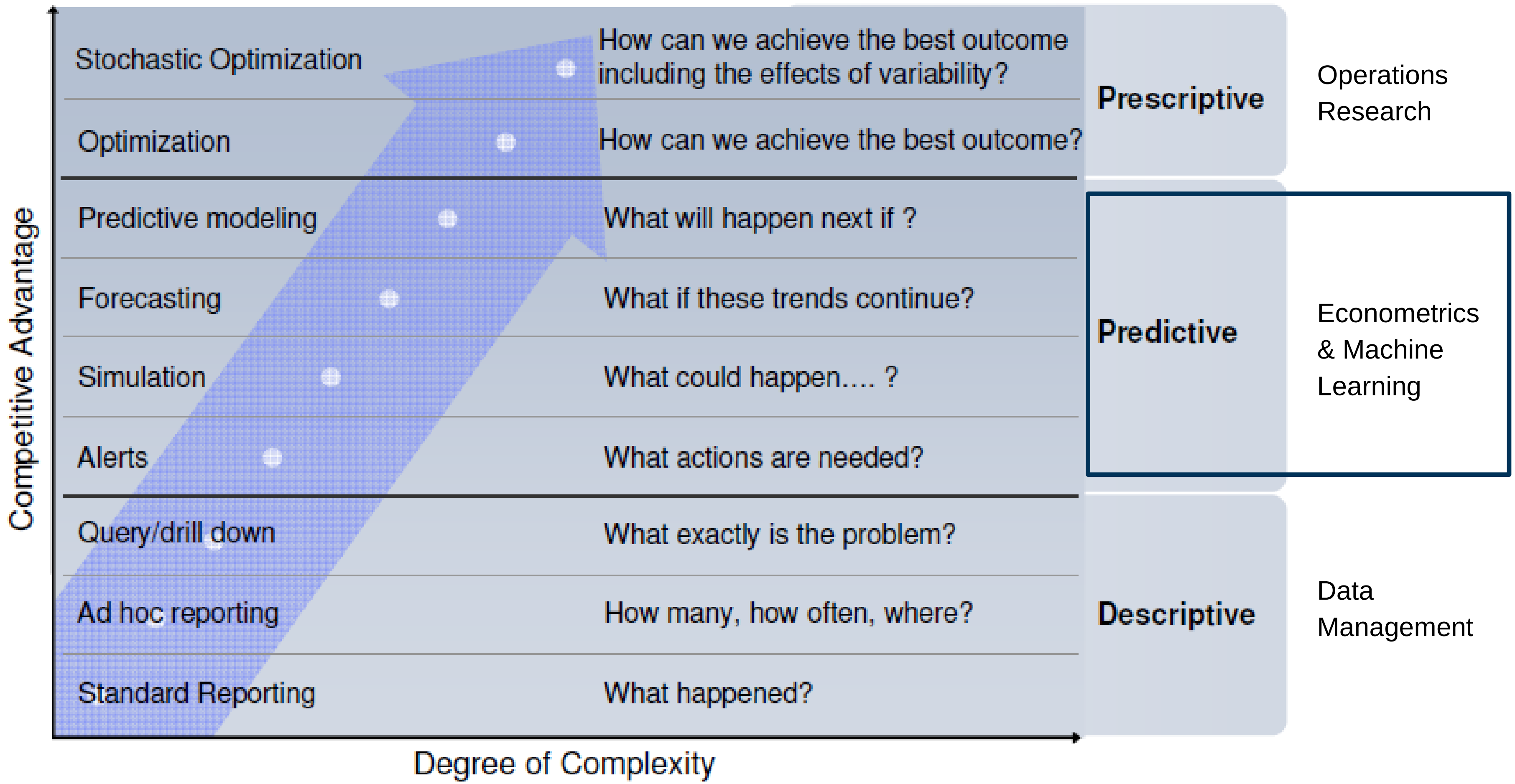


Algorithms and Optimization

Develop algorithms and optimization models for planning, scheduling, pricing, and revenue mgt.

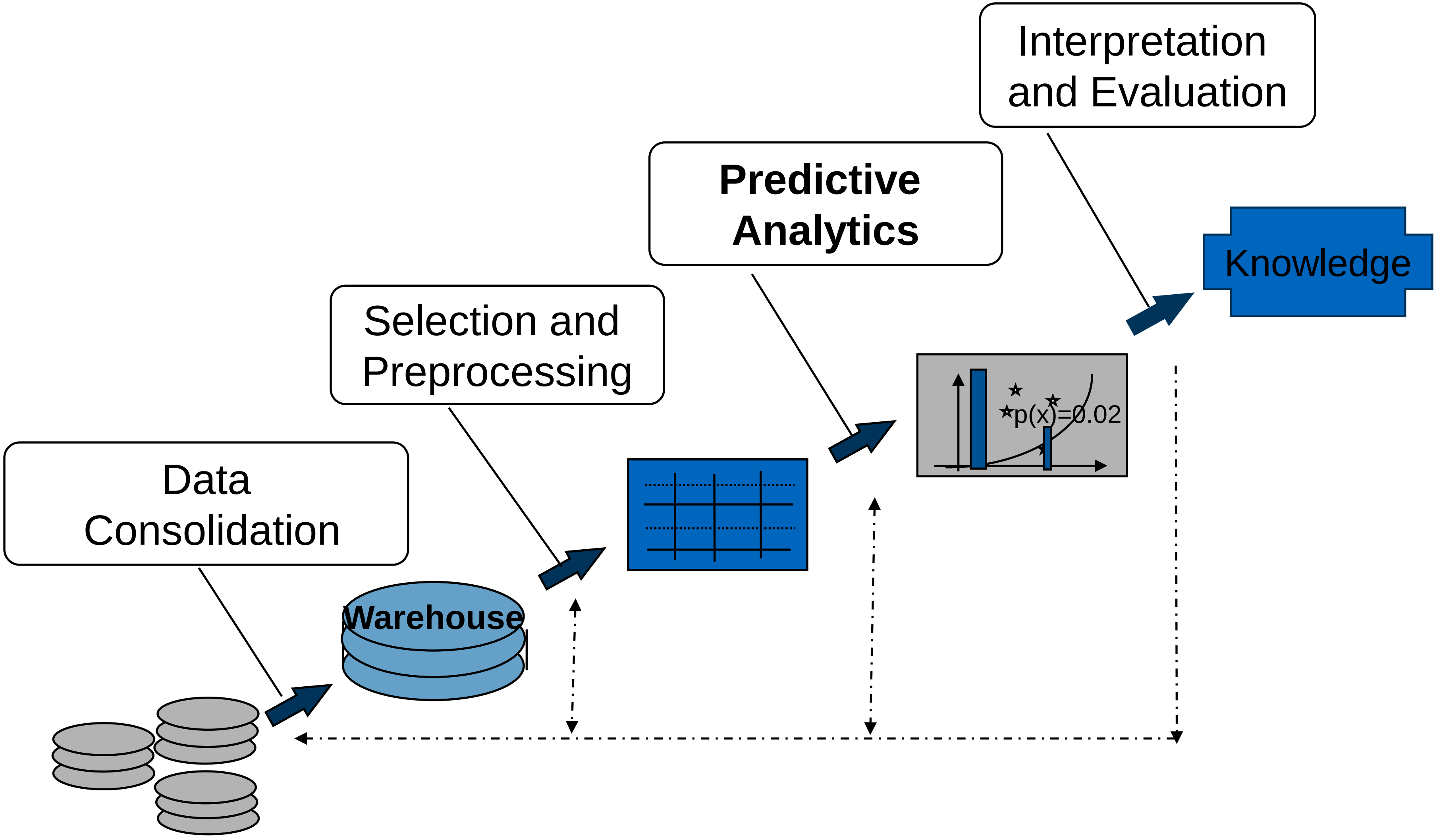
Analytics Landscape

Academic Fields



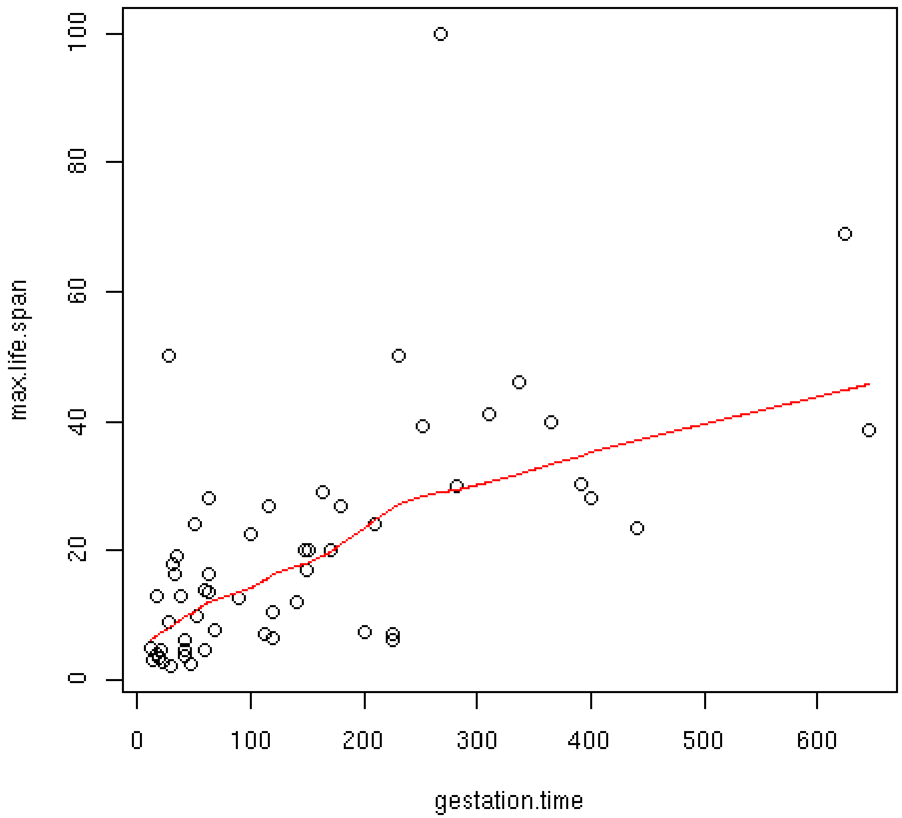
Based on: Competing on Analytics, Davenport and Harris, 2007

From Data to Information



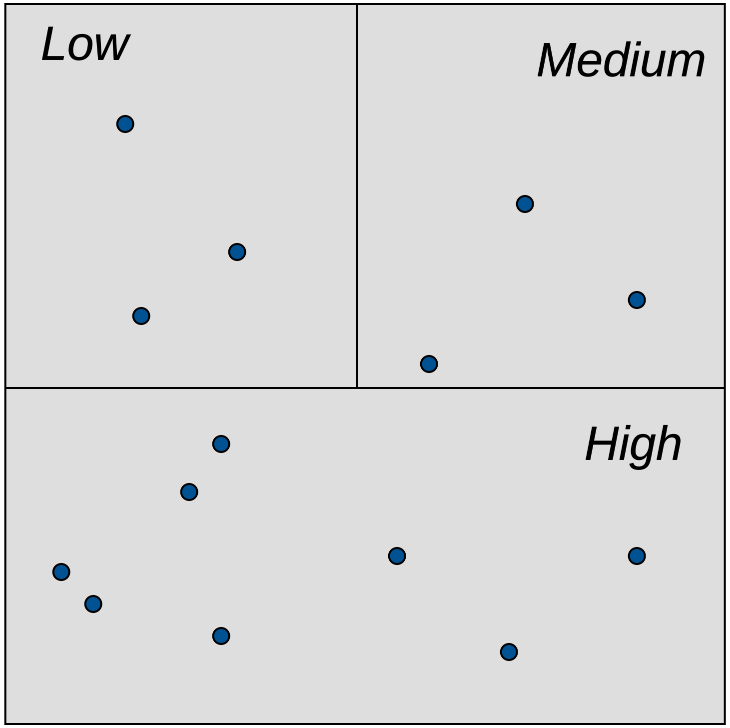
Numerical Prediction

- Given a collection of data with known numeric outputs, create a function that outputs a predicted value from a new set of inputs
- E.g. given gestation time of an animal, predict its maximum life span



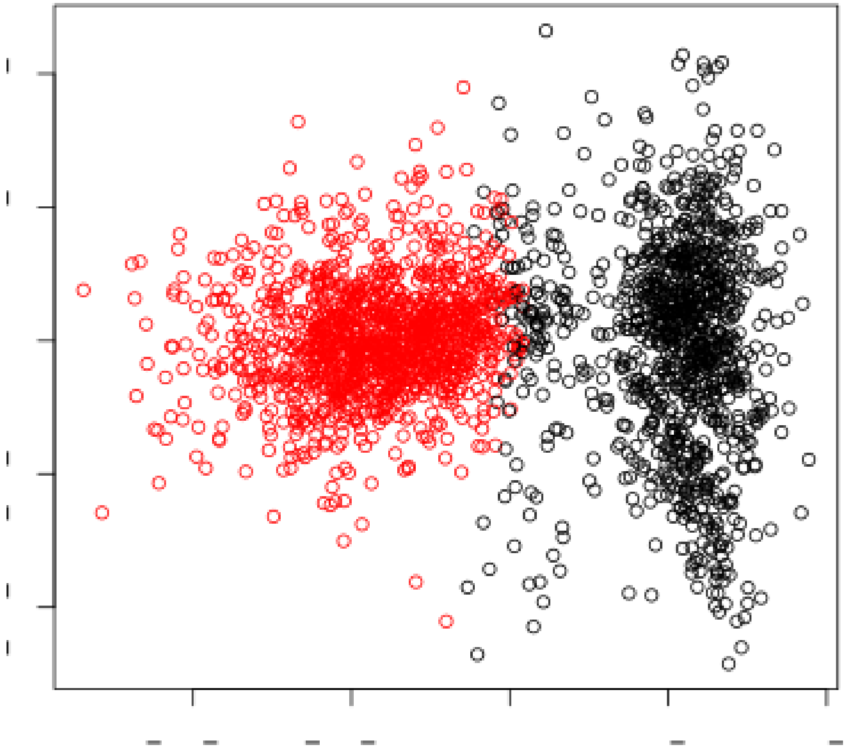
Classification

- From data with known labels, create a *classifier* that determines which label to apply to a new observation
- E.g. Identify new loan applicants as low, medium, or high risk based on existing applicant behavior



Clustering

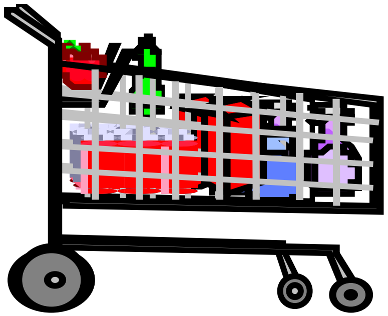
- Identify "natural" groupings in data
- Unsupervised learning, no predefined groups
- E.g. Identify clusters of "similar" customers



Association Rule Analysis

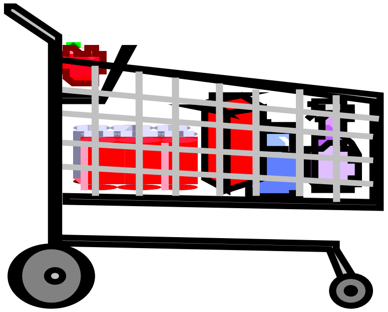
- Identify relationships in data from co-occuring terms or items
- E.g., analyze grocery store purchases to identify items most commonly purchased together

Milk, eggs, sugar, bread



Customer1

Milk, eggs, cereal, bread



Customer2

Eggs, sugar

Customer3

Machine Learning Terminology

In machine learning these tasks are categorized as supervised and usupervised learning:

Supervised learning:

- Supervised methods are thought to attempt the discovery of the relationships between input attributes and a target attribute.
- A training set is given and the objective is to form a description that can be used to predict unseen examples.
- Examples: Classification, numerical prediction

Unsupervised learning:

- There is no supervisor and only input data is available.
- The aim is now to find regularities, irregularities, relationships, similarities and associations in the input.
- Examples: Clustering, association rules or pattern mining

What is a Model?

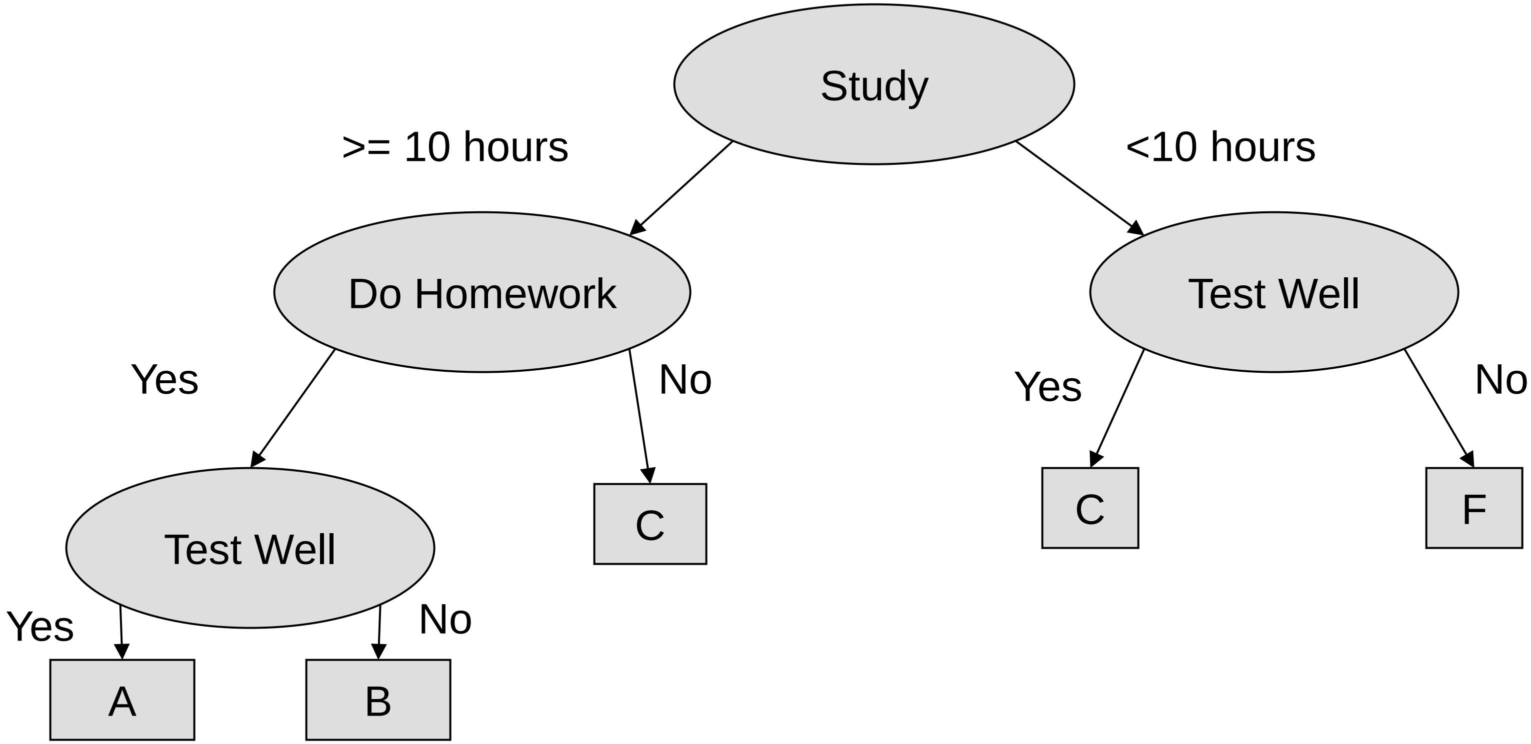
- „A representation of a system that allows for investigation of the properties of the system and, in some cases, prediction of future outcomes."

- Linear functions as a well-known example
  - Mathematical combination of attribute values
  - E.g. linear model, non-linear model
  - CPU performance prediction

$$
\begin{aligned}
PRP = &-55.9 + 0.489 MYCT + 0.0153 MMIN + 0.0056 MMAX \\
&+ 0.6410 CACH - 0.2700 CHMIN + 1.480 CHMAX
\end{aligned}
$$

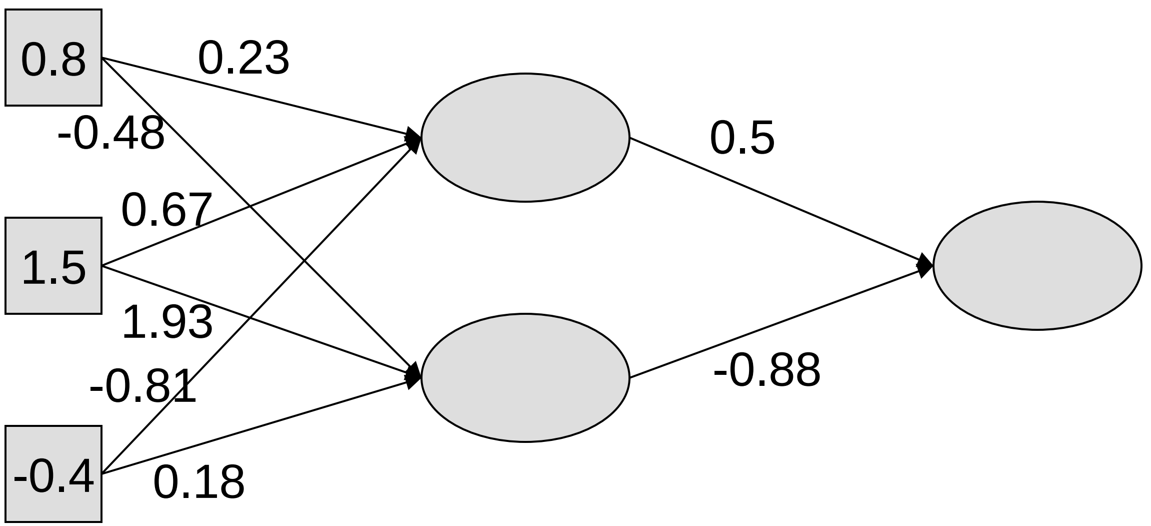
Models

Decision Trees



Models

Neural Networks



2017 Data Science Survey by Rexter

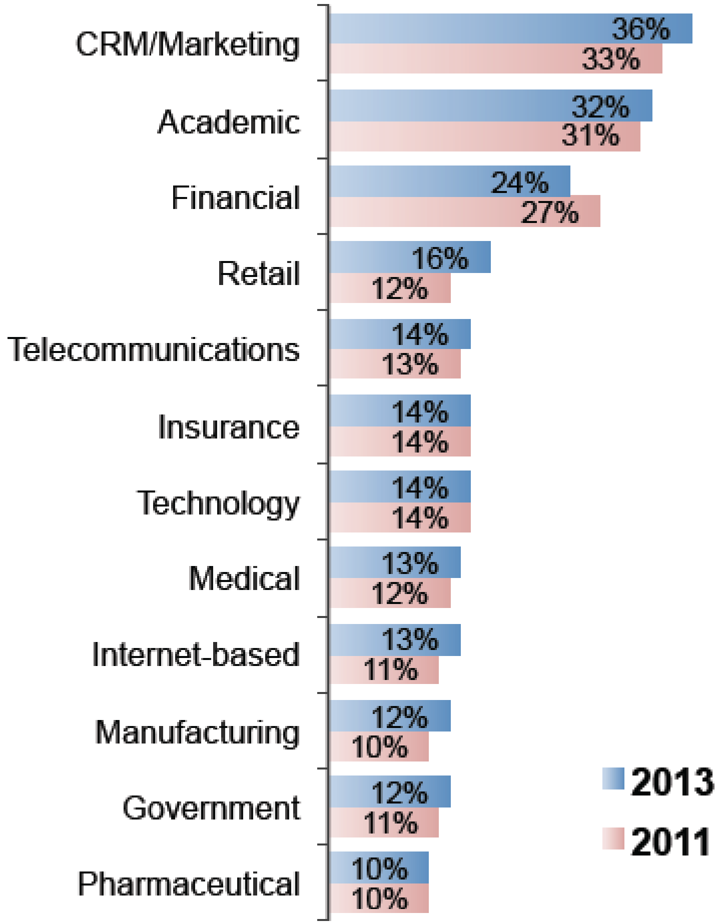
- Formal data science training is important to respondents (75% or so) with particular concerns about data preparation and misinterpreting results when people don’t have formal training.
- Most data scientists use multiple tools – an average of 5, still – with SQL, R and Python dominating across the board outside of academia.
- R has shown rapid growth over the last few years with more usage and more primary usage every year and RStudio is now the dominant environment.
- While there’s lots of interest in “deep learning”, 2/3 have not used deep learning at all with only 2% using it a lot.
- Job satisfaction is good and most data scientists are confident they could find a new job.
- People agree that a wide range of skills are needed with domain knowledge scoring very highly as important.

http://www.rexeranalytics.com/data-science-survey.html

CRM / Marketing: #1 Place for Data Miners

CRM / Marketing remains the #1 area to which data mining is applied.

The roots of data mining in customer focused analytics are strong. In each of the 6 Data Miner Surveys, more people report applying their analytics in the field of CRM / Marketing than any other field. In 2013, 36% of data miners indicated that they are commonly involved in CRM / Marketing data mining, up slightly from 2011. The number of data miners working in the overlapping area of Retail analytics is also increasing.



Data miners also report working in Non-profit (5%), Hospitality / Entertainment / Sports (4%), Military / Security (2%), and Other (10%).

Question: In what fields do you TYPICALLY apply data mining? (Select all that apply)

Why Focus on Business and Economics?

	2011	2013	2015
Improving understanding of customers	33%	45%	46%
Retaining customers	30%	36%	37%
Improving customer experiences	22%	36%	36%
Selling products / services to existing customers	23%	33%	35%
Market research / survey analysis	29%	36%	34%
Acquiring customers	23%	32%	32%
Improving direct marketing programs	22%	27%	30%
Sales forecasting	19%	27%	27%
Fraud detection or prevention	21%	23%	26%
Risk management / credit scoring	22%	26%	25%
Price optimization	14%	22%	23%
Manufacturing improvement	10%	15%	17%
Medical advancement / drug discovery / biotech / genomics	12%	17%	17%
Supply chain optimization	7%	11%	15%
Investment planning / optimization	11%	13%	14%
Software optimization	7%	9%	11%
Website or search optimization	8%	12%	10%
Human resource applications	4%	8%	9%
Collections	6%	7%	8%
Language understanding	4%	7%	8%
Criminal or terrorist detection	4%	4%	7%

Source: http://www.rexeranalytics.com/files/Rexer_Data_Science_Survey_Highlights_Apr-2016.pdf

Example of a Business Application: Churn Prediction

- **Churn**: the proportion of contractual customers or subscribers who leave a supplier during a given time period
- **High churn rates** ≅ 2.6% a month
- **Causes:** increased competition, lack of differentiation, market saturation
- **Cost:** $300 to $700 cost of replacement of a lost customer in terms of sales support, marketing, advertising, etc.
- **Response:** Targeted retention strategies

Churn Prediction as Classification

Churn as a Classification problem:

Classify a customer *i* characterized by *p* variables

$x_i = (x_{i1}, x_{i2}, \ldots, x_{ip})$ as

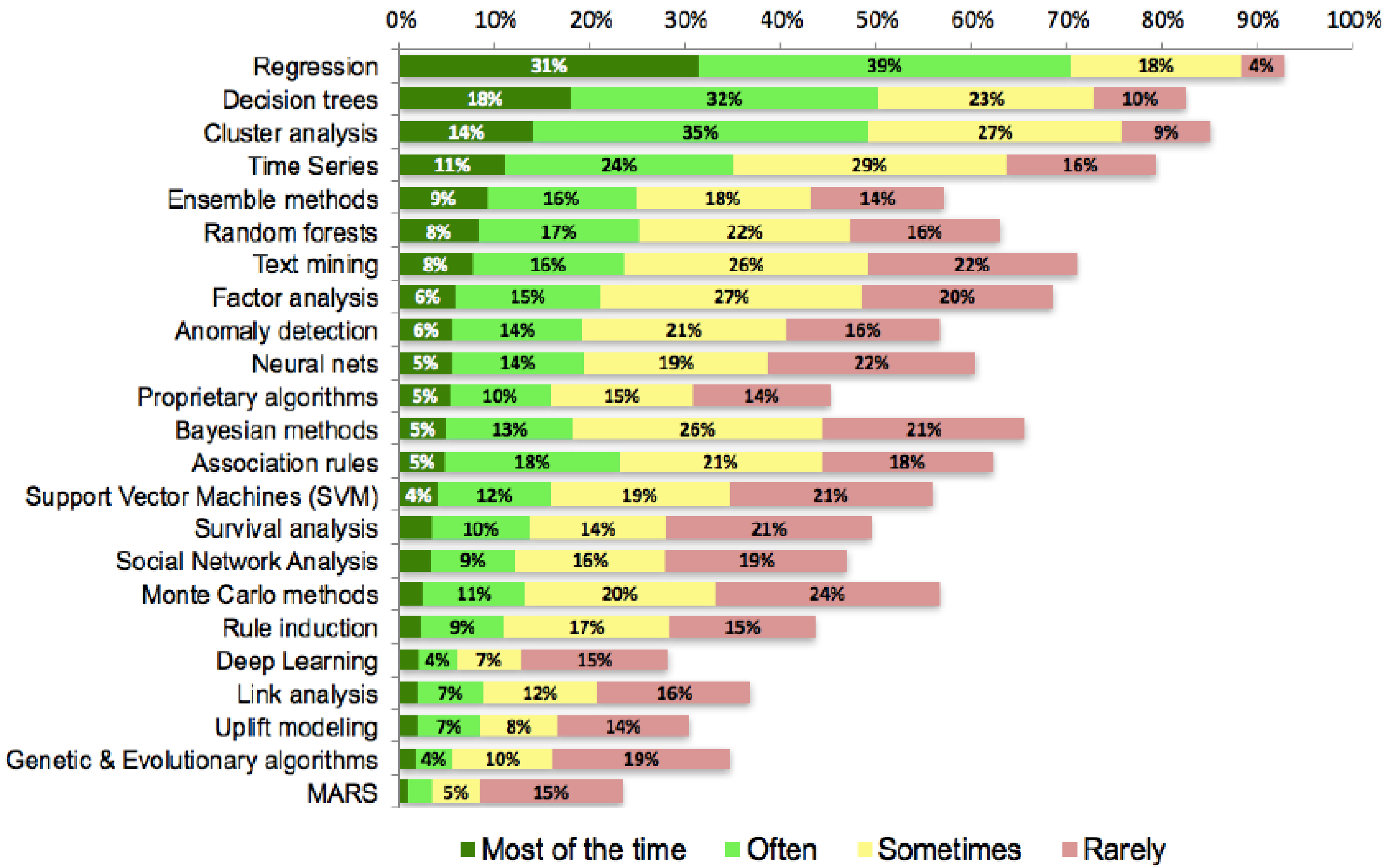
- Churner $y_i = +1$
- Non-churner $y_i = -1$

Churn is the response binary variable to predict: $y_i = f(x_i)$

Choice of the binary choice model $f(\,.\,)$?

Our Core Algorithms Remain the Same

- Regression, decision trees, and cluster analysis continue to form a triad of core algorithms for most data miners. This has been consistent since the first Data Miner Survey in 2007.



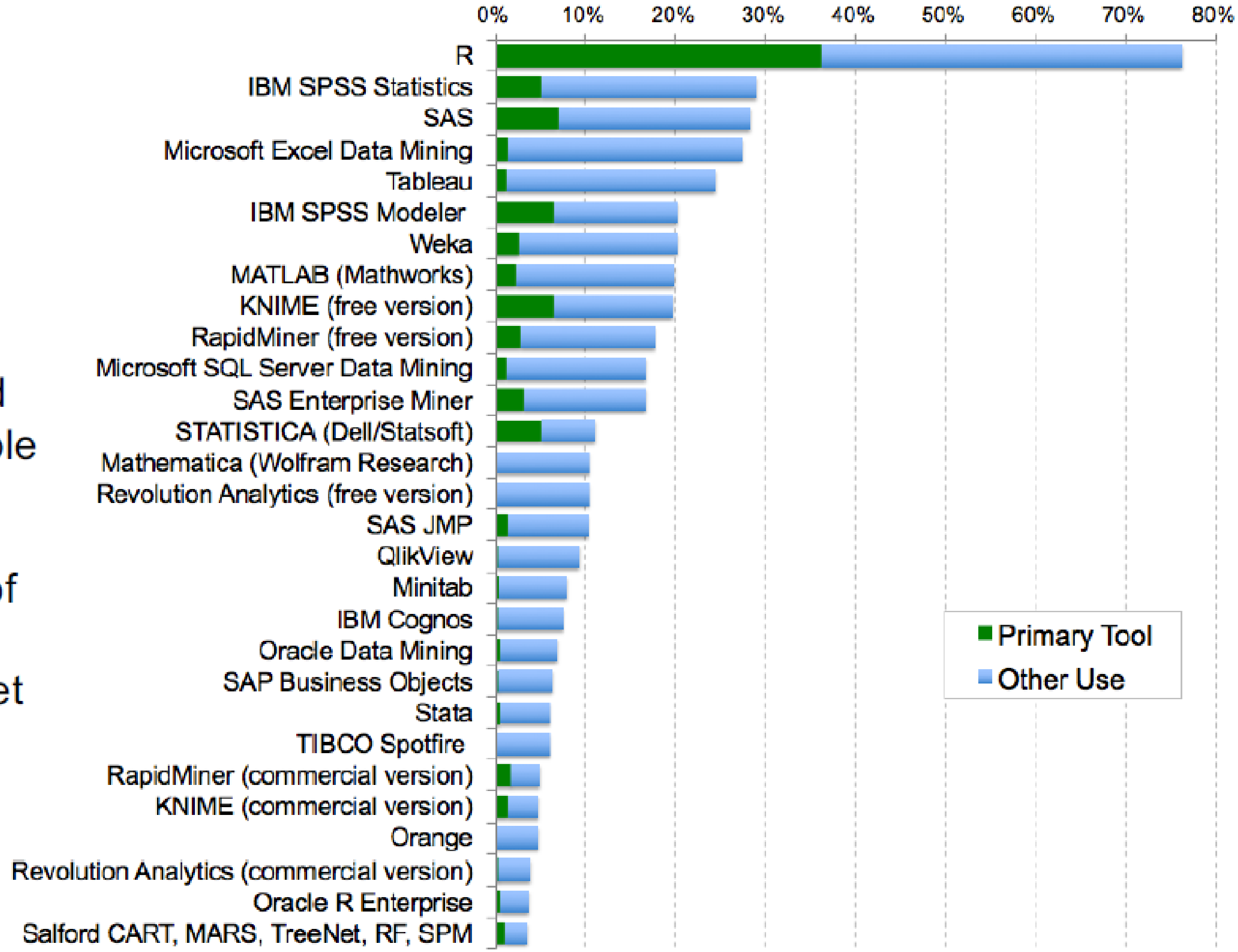
Question: What algorithms / analytic methods do you TYPICALLY use? (Select all that apply)

Source: Rexter Analytics

The Tools We're Using

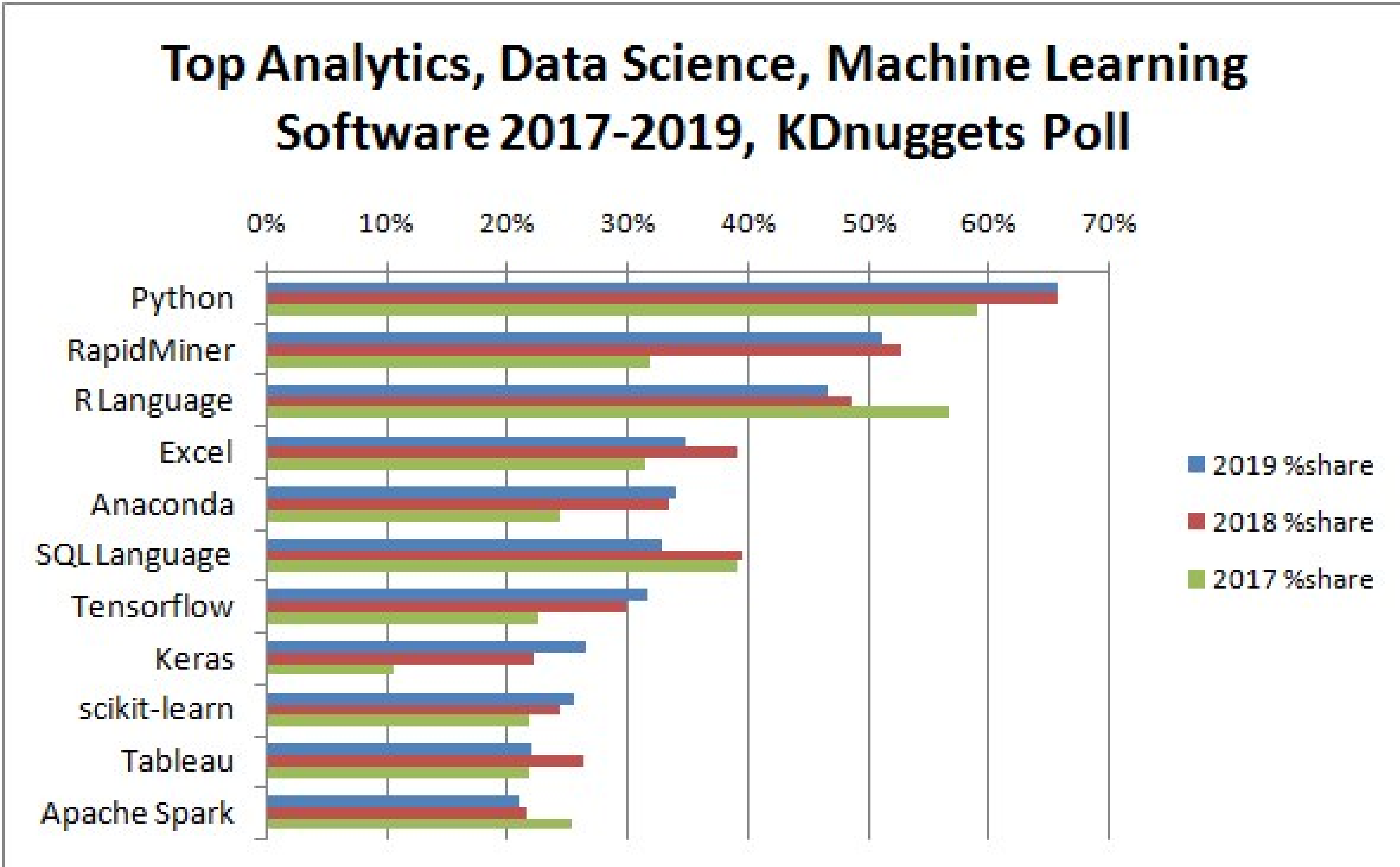
Vendors are excluded from tool-use analyses

- The average analytics professional reports using 5 software tools
- R is the tool used by the most people (76%)
- A large number of tools have substantial market penetration



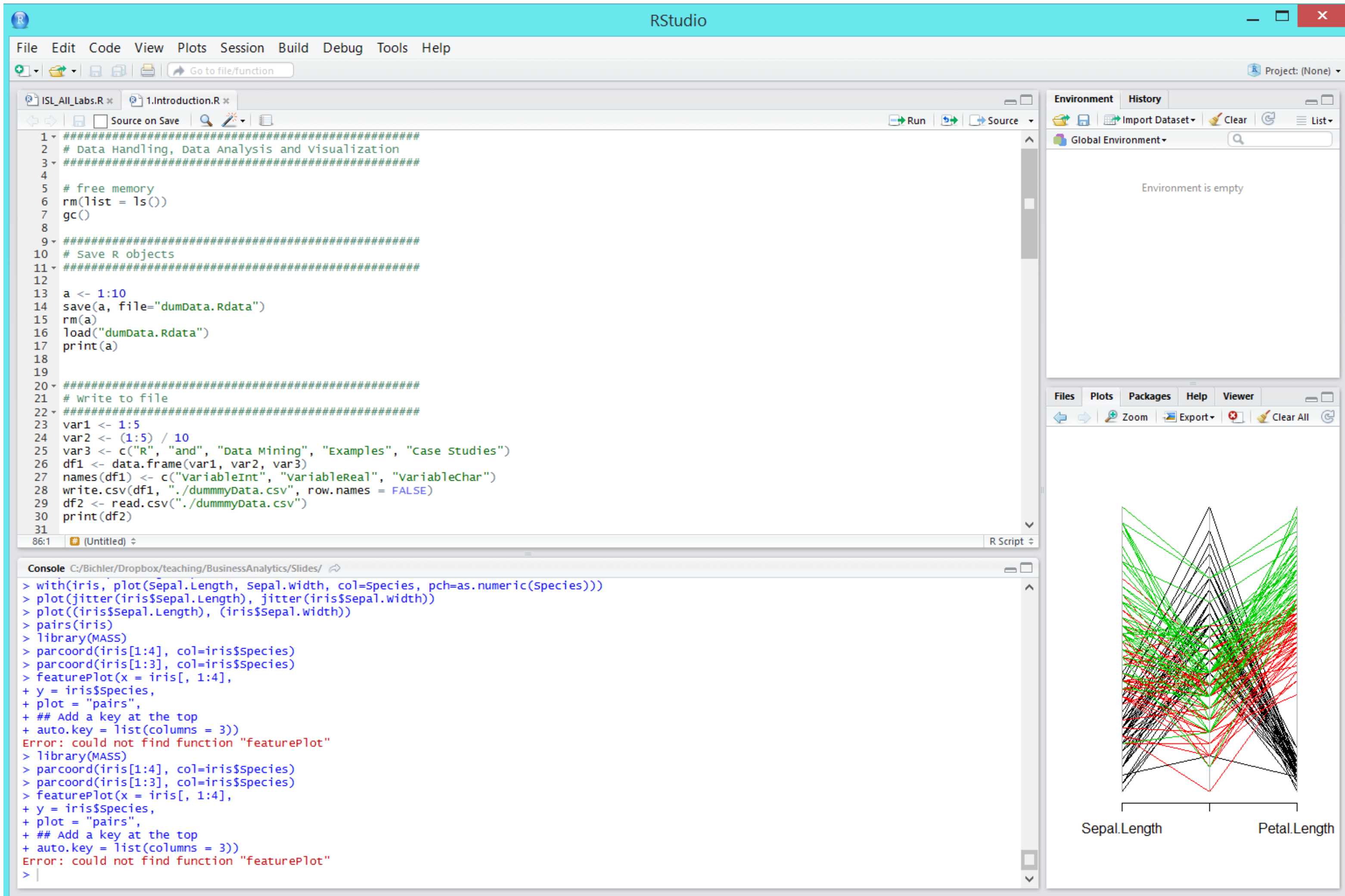
Question: What Data mining / analytic tools did you use in the past year?
Question: What one data mining / analytic software package do you use most frequently in the past year?

Source: Rexter Analytics



Source: KDnuggets Analytics/Data Science 2019 Software Poll

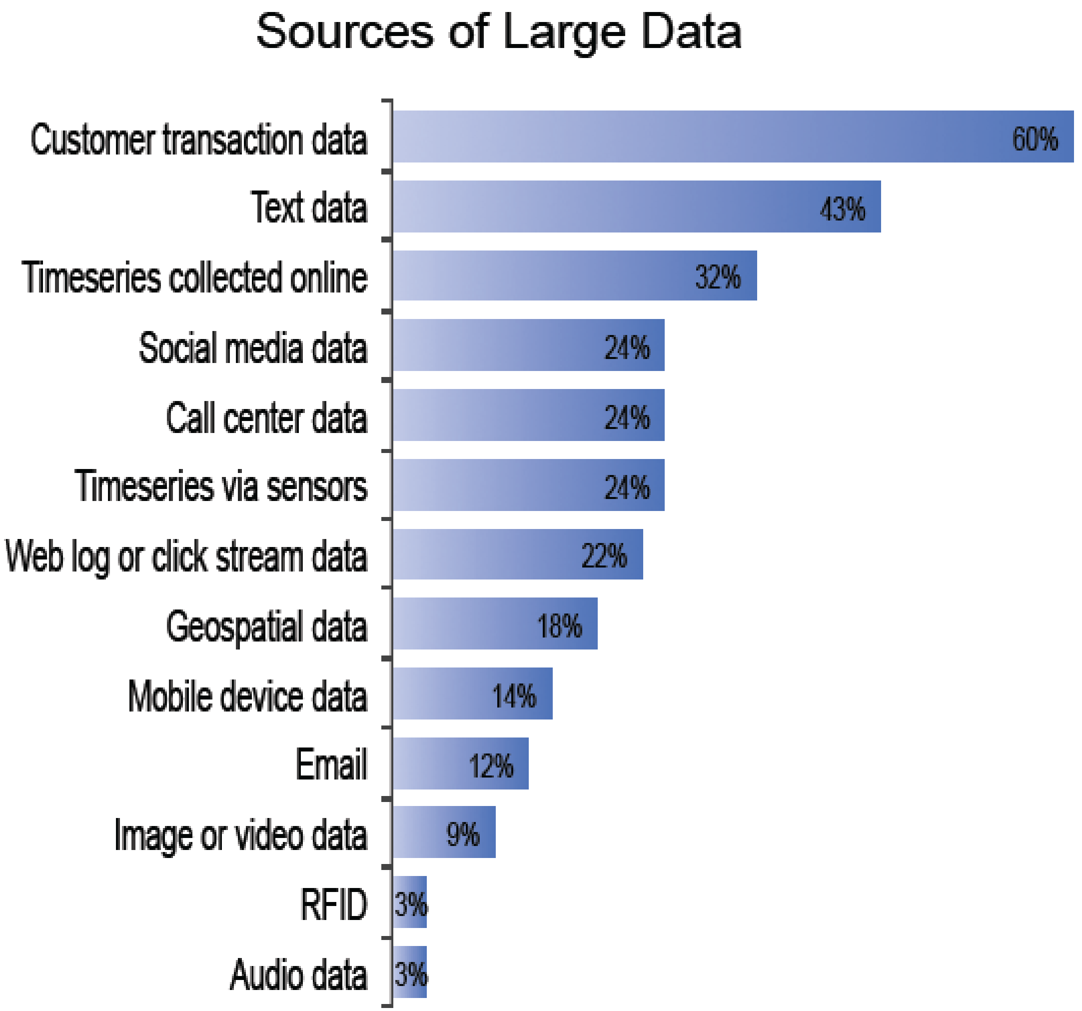
R Studio


RStudio
File Edit Code View Plots Session Build Debug Tools Help
Go to file/function
Project: (None)
ISL_All_Labs.R
1.Introduction.R
Source on Save
Run
Source
##
Data Handling, Data Analysis and Visualization
##
# free memory
rm(list = ls())
gc()
##
Save R objects
##
a <- 1:10
save(a, file="dumData.Rdata")
rm(a)
load("dumData.Rdata")
print(a)
##
Write to file
##
var1 <- 1:5
var2 <- (1:5) / 10
var3 <- c("R", "and", "Data Mining", "Examples", "Case Studies")
df1 <- data.frame(var1, var2, var3)
names(df1) <- c("VariableInt", "VariableReal", "VariableChar")
write.csv(df1, "./dummmyData.csv", row.names = FALSE)
df2 <- read.csv("./dummmyData.csv")
print(df2)
86:1
(Untitled)
R Script
Console C:/Bichler/Dropbox/teaching/BusinessAnalytics/Slides/
> with(iris, plot(Sepal.Length, Sepal.Width, col=Species, pch=as.numeric(Species)))
> plot(jitter(iris$Sepal.Length), jitter(iris$Sepal.Width))
> plot((iris$Sepal.Length), (iris$Sepal.Width))
> pairs(iris)
> library(MASS)
> parcoord(iris[1:4], col=iris$Species)
> parcoord(iris[1:3], col=iris$Species)
> featurePlot(x = iris[, 1:4],
+ y = iris$Species,
+ plot = "pairs",
+ ## Add a key at the top
+ auto.key = list(columns = 3))
Error: could not find function "featurePlot"
> library(MASS)
> parcoord(iris[1:4], col=iris$Species)
> parcoord(iris[1:3], col=iris$Species)
> featurePlot(x = iris[, 1:4],
+ y = iris$Species,
+ plot = "pairs",
+ ## Add a key at the top
+ auto.key = list(columns = 3))
Error: could not find function "featurePlot"
Environment History
Import Dataset
Clear
List
Global Environment
Environment is empty
Files Plots Packages Help Viewer
Zoom
Export
Clear All
Sepal.Length
Petal.Length

Customer Transactions: #1 Source of Large Data

Customer transactional data often affords the opportunity for a wide range of analytics due to the depth and scope of available data.

Among respondents who reported increases in data volume, 60% identified customer transaction data as a source of their large data sets.

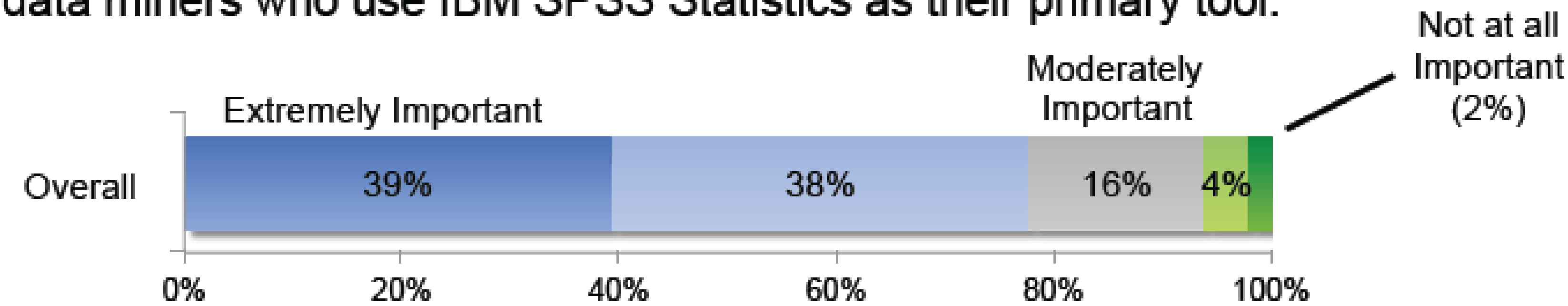


Question: What are the sources of data for your large datasets? (select all that apply)

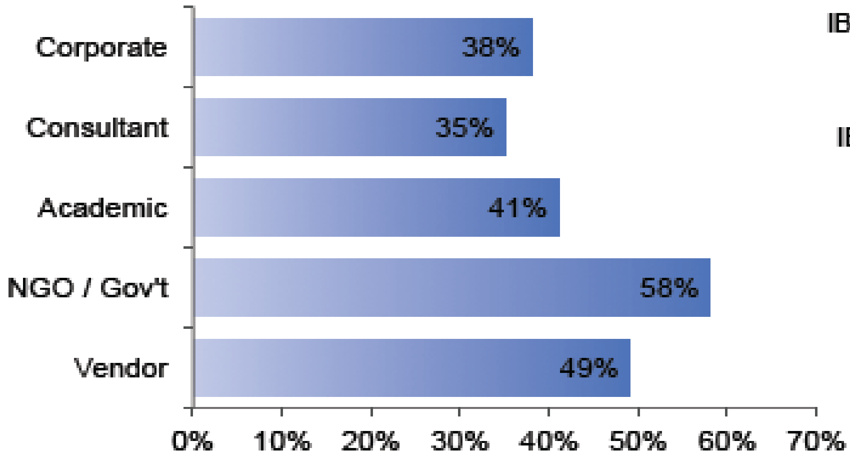
Source: Rexter Analytics

Importance of Model Explainability

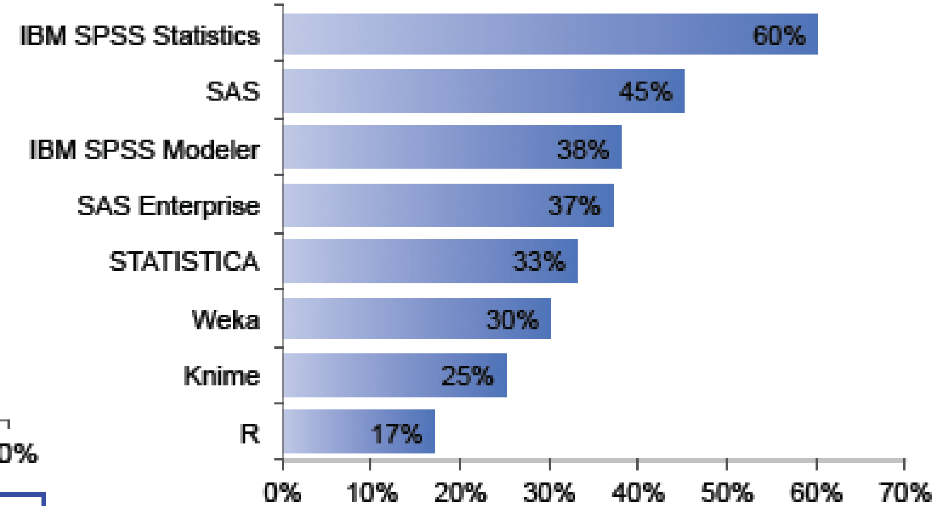
- Model explainability / transparency is important to most data miners.
- It is particularly important to data miners working in NGO / Gov't settings and to data miners who use IBM SPSS Statistics as their primary tool.



Percent indicating Model Explainability is "Extremely Important" by Employment Type



Percent indicating Model Explainability is "Extremely Important" by Primary Tool Used



Question: How important is model explainability / transparency to you?

Goals of this Course

- Learn data analysis methods with a focus on
  - problems in business and economics, and
  - causal inference, which is particularly challenging when analyzing human decision behavior

- Learn to know techniques for
  - *numerical prediction*
  - *classification*
  - *clustering* and *dimensionality reduction*

- Learn to analyze data with the *R programming language*
  - during the Analytics Cup you analyze data sets as part of the tutorials in small groups

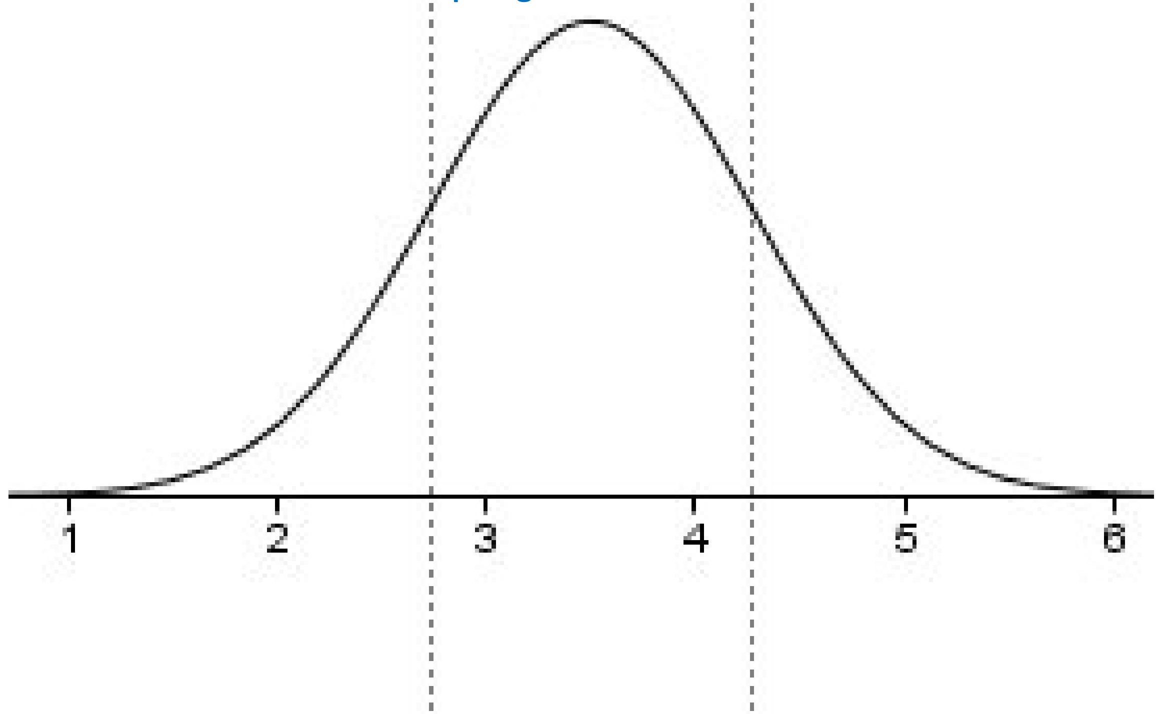
Please note:
- This is an introductory course on data analysis.
- We expect that you had an introductory course in statistics and algorithms.
- Students in this class have very different prerequisites.

This Course is For

I know very little about statistics and programming

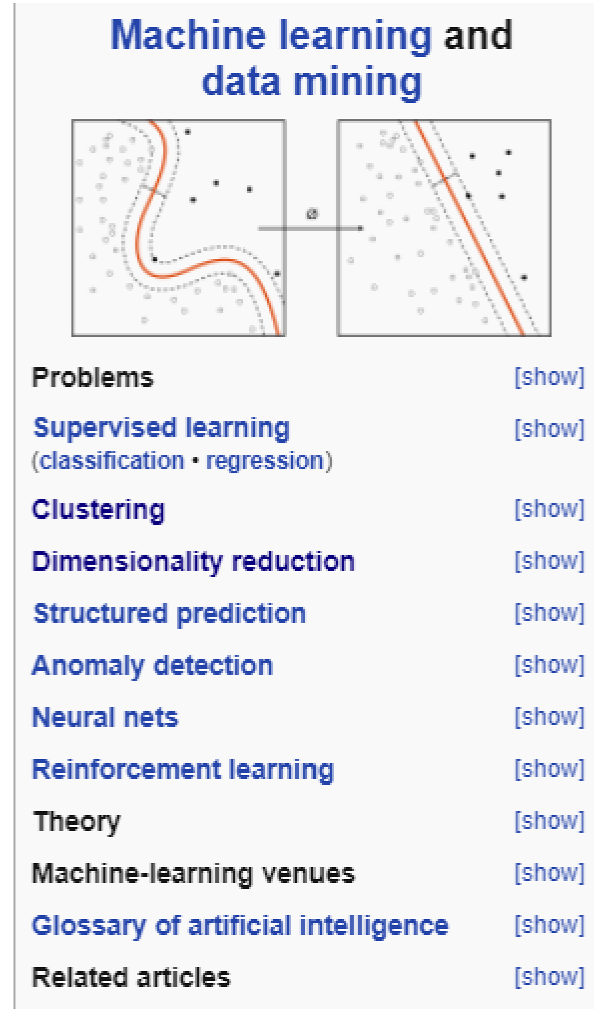
I have heared statistics and algorithms in my bachelor program

I have already heared a machine learning or deep learning course



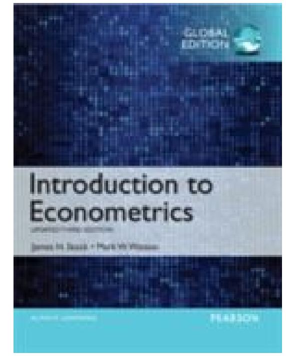
Course Content

- Introduction
- Regression Analysis
- Regression Diagnostics
- Logistic and Poisson Regression
- Naive Bayes and Bayesian Networks
- Decision Tree Classifiers
- Data Preparation and Causal Inference
- Model Selection and Learning Theory
- Ensemble Methods and Clustering
- High-Dimensional Problems
- Association Rules and Recommenders
- Neural Networks

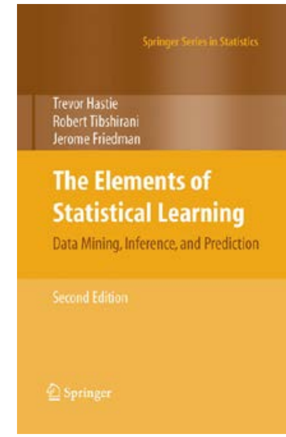


Check out related entries on wikipedia.org!

Primary Literature



- **Introduction to Econometrics**
  - Stock, James H., and Mark W. Watson



- **The Elements of Statistical Learning**
  - Trevor Hastie, Robert Tibshirani, Jerome Friedman, 2014
  - https://web.stanford.edu/~hastie/ElemStatLearn/

- **Data Mining: Practical Machine Learning Tools and Techniques**
  - Ian H. Witten, Eibe Frank, Mark A. Hall, Christopher Pal, 2016
  - http://www.cs.waikato.ac.nz/ml/weka/book.html

- **An Introduction to Statistical Learning: With Applications in R**
  - Gareth James, Trevor Hastie, Robert Tibshirani, 2014
  - http://www-bcf.usc.edu/~gareth/ISL/

Parts of the slides have kindly been provided by
Prof. Dr. Gregory Piatetsky-Shapiro and Prof. Gary Parker
(Univ. of Connecticut).

This Course is Available Students from …

- MSc Information Systems (*mandatory*)
- MSc Informatics, MSc Games Engineering, MSc Data Engineering & Analytics
- MSc Management and Technology, MSc Consumer Affairs
- MSc Mathematics, MSc Mathematics in Operations Research

Students from IN, GE, and DE&A can choose one class in Analytics and one class in Machine Learning:

Analytics

* Data Mining, IN2030, 2V, WS, Prof. Runkler
* Business Analytics, IN2028, 2V+2Ü, WS, Prof. Bichler
* Data Analysis and Visualization in R, IN2339, 2V+4Ü, WS, Prof. Gagneur

Machine Learning

* Statistical Modeling & Machine Learning, IN2332, 4V+4Ü, SS, Prof. Gagneur
* Machine Learning, IN2064, 4V+2Ü, WS, Prof. Günnemann

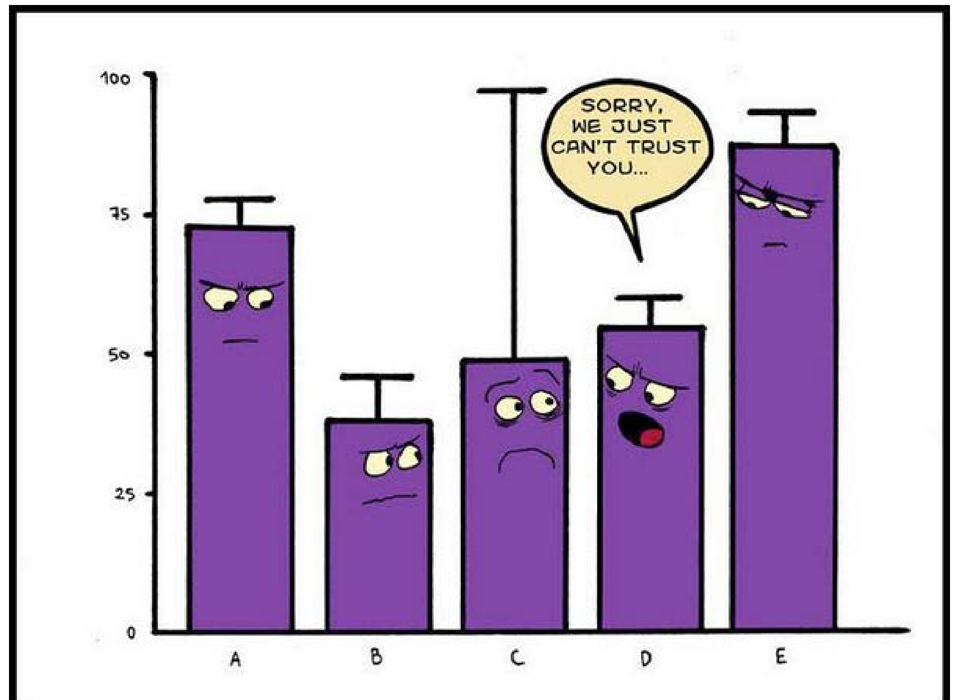
Agenda for Today

1. Understand what this course is all about
2. Learn about organization, grading, and tutor groups
3. Homework: refresh basic statistical concepts

In the first tutorials, we will recap important concepts from inferential statistics and introduce the R programming language, required for the rest of the course.

Check out http://onlinestatbook.com as an online source.

For this week please revisit the concepts on the following slides. Slides are only meant as a refresher.



Statistics

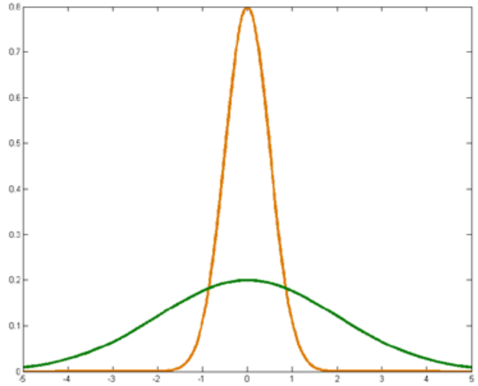
- **Descriptive statistics** can be used to summarize the data, either numerically or graphically, to describe the sample (e.g., mean and standard deviation)

- **Inferential statistics** is used to model patterns in the data, accounting for randomness and drawing inferences about the larger population. These inferences may take the form of
  - estimates of numerical characteristics (estimation),
  - answers to yes/no questions (hypothesis testing),
  - forecasting of future observations (forecasting),
  - descriptions of association (correlation), or
  - modeling of relationships (regression).

Random Variables

- X is a random variable if it represents a random draw from some population and is associated with a probability distribution
  - a <u>discrete</u> random variable can take on only selected values (e.g., Binomial or Poisson distributed)
  - a <u>continuous</u> random variable can take on any value in a real interval (e. g., uniform, Normal or Chi-Square distributions)
- For example, a Normal distribution, with mean μ and variance σ^2 is written as $N(\mu, \sigma^2)$ has a probability density function (pdf) of:

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$



The Standard Normal

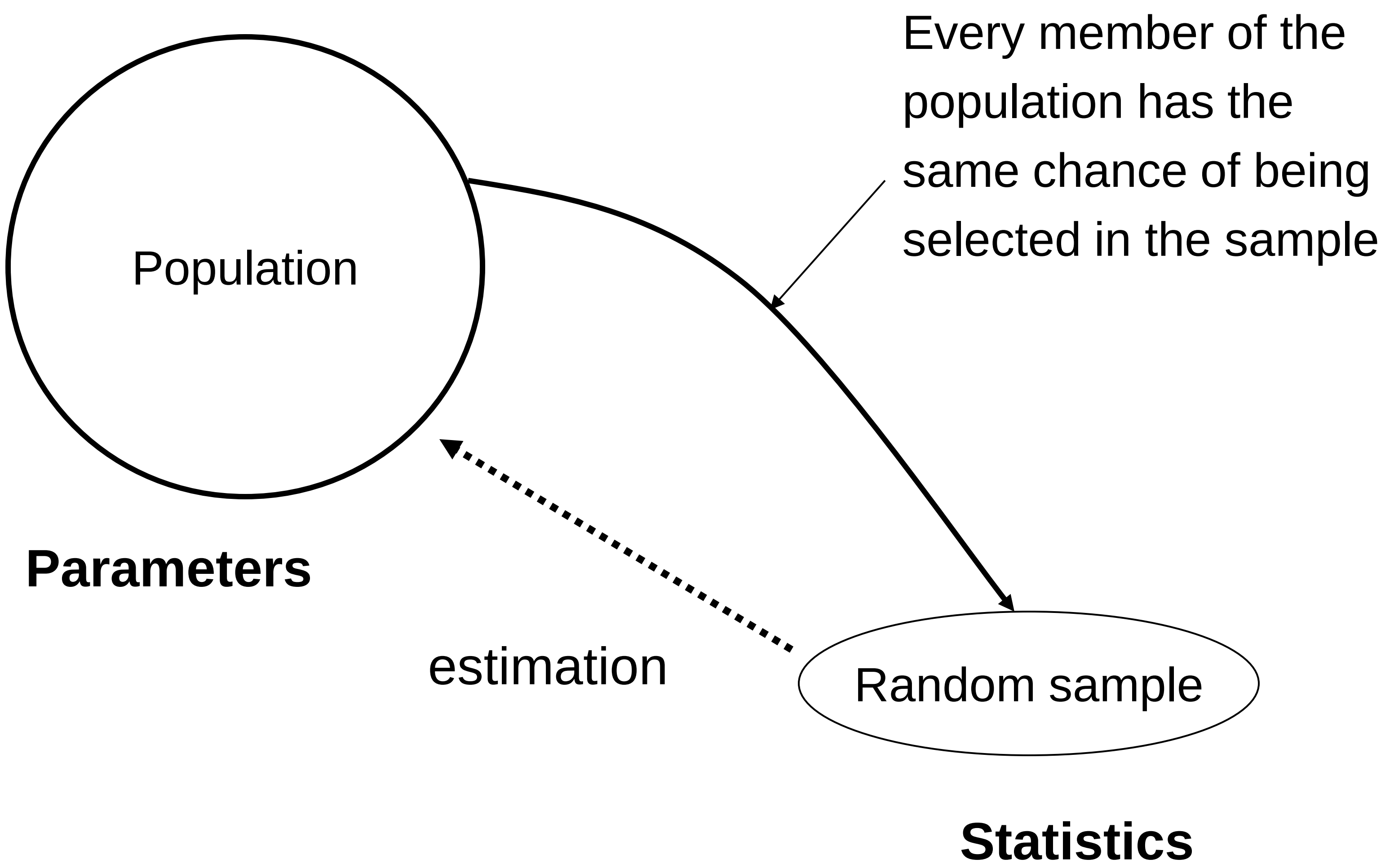
- Any random variable can be “standardized” by subtracting the mean, μ, and dividing by the standard deviation, σ , so $E(Z) = 0$, $Var(Z) = 1$
- Thus, the standard normal, $N(0,1)$, has the probability density function (pdf)

$$\varphi(z) = \frac{1}{\sqrt{2\pi}} e^{\frac{-z^2}{2}}$$

- For a pdf, $f(x)$, where $f(x)$ is $P(X = x)$, the cumulative distribution function (cdf), $F(x)$, is $P(X \leq x)$; $P(X > x) = 1 - F(x) = P(X < -x)$
- For the standard normal, $\varphi(z)$, the cdf is

$$\Phi(z) = P(Z \leq z) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{z} e^{\frac{-t^2}{2}} dt$$

Statistical Estimation



Expected Value of X: Population Mean $E(X)$

- The expected value of a probability weighted average of X, $E(X)$, is the mean or expected value of the distribution of X, denoted by μ_x
- Let $f(x_i)$ be the (discrete) probability that $X = x_i$, then

$$\mu_x = E(X) = \sum_{i=1}^{n} x_i f(x_i) \; or \int_{-\infty}^{\infty} x f(x) dx$$

Example: Expected Value

Students were surveyed and told to pick the number of hours that they play online games each day. The probability distribution is given below.

# of Hours *x*	Probability *P*(*x*)
0	.3
1	.4
2	.2
3	.1

Compute a “weighted average” by multiplying each possible time value by its probability and then adding the products

$$E(X) = 0(.3) + 1(.4) + 2(.2) + 3(.1) = 1.1$$

Random Samples and Sampling

- For a random variable X, repeated draws from the same population can be labeled as $X_1, X_2, \ldots, X_n$
- If every combination of n sample points has an equal chance of being selected, this is a random sample
- A random sample is a set of independent, identically distributed (i.i.d) random variables

Examples of Estimators

- Suppose we want to estimate the **population mean**
- Suppose we use the formula for $E(X)$, but substitute $1/n$ for $f(x_i)$ as the probability weight since each point has an equal chance of being included in the sample, then we can calculate the sample mean:

$$\bar{X} = \frac{1}{n}\sum_{1}^{n} X_i$$

- $\bar{X}$ describes the random variable for the **arithmetic mean of the sample**, while $\bar{x}$ is the mean of a particular realization of a sample

Estimators Should be Unbiased

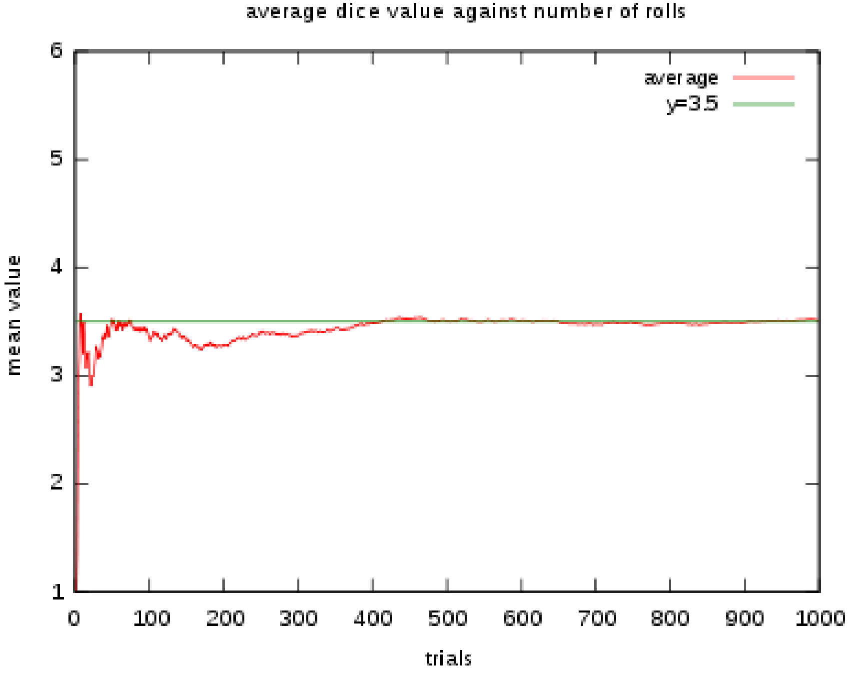
- An estimator (e.g., the arithmetic sample mean) is a statistic (a function of the observable sample data) that is used to estimate an unknown population parameter (e.g., the expected value)
- We want the estimator to be right, on average, i.e. unbiased.
- For our example, this means we want

$$E(\overline{X}) = E\left(\frac{1}{n}\sum_{i=1}^{n} X_i\right) = \frac{1}{n}\sum_{i=1}^{n} E(X_i)$$

$$= \frac{1}{n}\sum_{i=1}^{n} \mu_X = \frac{1}{n} n\mu_X = \mu_X$$

Rolling a Dice

Wikipedia: Expected value of 3.5 as the
number of die rolls grows.



According to the **law of large numbers**, the sample mean converges to the expected value of the population distribution.

Law of Large Numbers

Proposition (Weak Law of Large Numbers)

$$\lim_{\{n\to\infty\}} \Pr(|\bar{X}_n - \mu| > \varepsilon) = 0$$

In other words: $n \to \infty$, $\bar{X}_n \to \mu$

Proof

Remember Chebyshev's inequality: No more than a certain fraction of values can be more than a certain distance from the mean.

$$\Pr(|X - \mu| > \varepsilon) \leq \frac{Var(X)}{\varepsilon^2}$$

Replace X by the sample mean of n i.i.d random variables $\bar{X}_n$

$$\left.\Pr(|\bar{X}_n - \mu| > \varepsilon) \leq \frac{Var(\bar{X}_n)}{\varepsilon^2} = \frac{\sigma^2}{n\varepsilon^2}\right\} \to 0 \text{ with } n \to \infty$$

$$\bar{X}_n = \frac{1}{n}\Sigma_{i=1}^{n} X_i,\ Var(\bar{X}_n) = \frac{1}{n^2}\Sigma_{i=1}^{n} Var(X_i) = \frac{n\sigma^2}{n^2} = \frac{\sigma^2}{n}$$

Standard Error of the Sample Mean

$$Var(\bar{X}) = Var\left(\frac{1}{n}\sum_{i=0}^{n}(X_i)\right) = \frac{1}{n^2}\sum_{i=1}^{n}Var(X_i)$$

$$= \frac{1}{n^2}\sum_{i=1}^{n}\sigma^2 = \frac{1}{n^2}(n\sigma^2) = \frac{\sigma^2}{n}$$

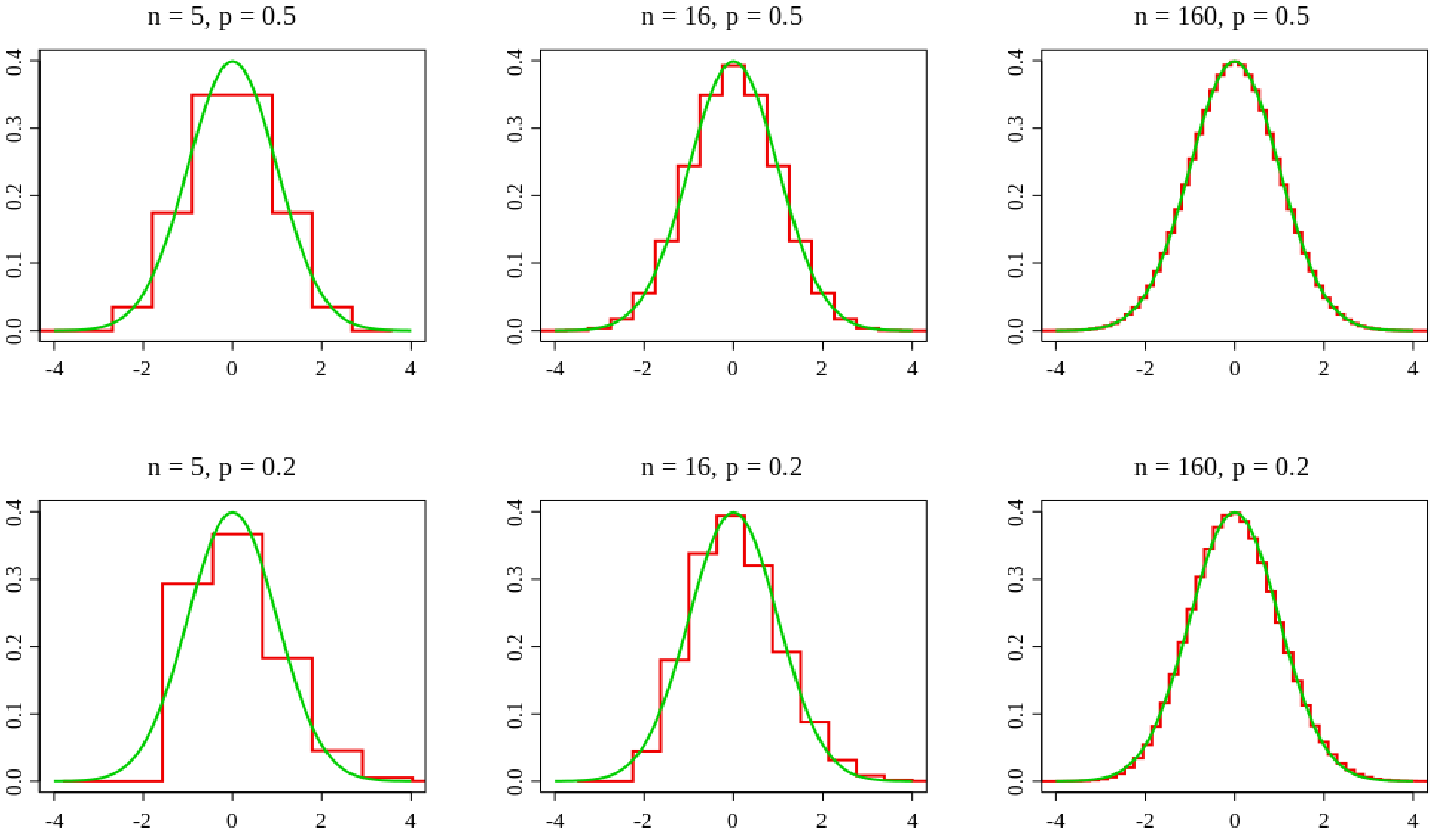
$$\sigma_{\bar{X}} = SD(\bar{X}) = \sqrt{Var(\bar{X})} = \frac{\sigma}{\sqrt{n}}$$

Rule: $Var[aX + b] = a^2Var[X]$

The **standard error of the sample mean** is an estimate of how far the sample mean is likely to be from the population mean. This means, the standard error of the mean tells you how accurate your estimate of the mean is likely to be.
The **standard deviation** of the sample is the degree to which individuals within the sample differ from the sample mean

- The *central limit theorem* states that the standardized average of any population of i.i.d. random variables X_i with mean μ_X and variance σ^2 is asymptotically $\sim N(0,1)$ as n

Binomial Distributions and the Normal Distribution

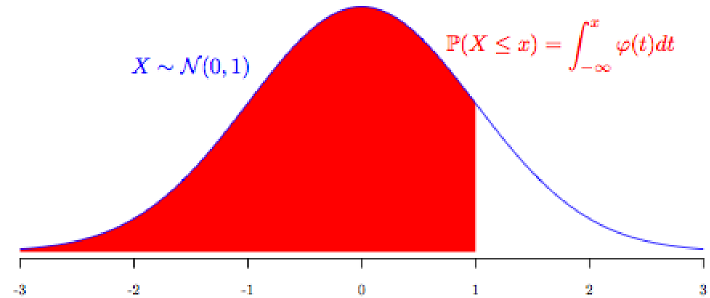


Source: Wikipedia

Summary: Sampling Distribution of the Mean

- We can say something about the distribution of sample statistics (such as the sample mean)
- The sample mean is a random variable, and consequently it has its own distribution and variance
- The distribution of sample means for different samples of a population is centered on the population mean
- The mean of the sample means is equal to the population mean
- If the population is normally distributed or when the sample size is large, sample means are distributed normally (Central Limit Theorem)

Question



What is the probability that a sample of 100 randomly selected elements with a mean of 300 or more gets selected if the true population mean is 288 and the population standard deviation is 60?

Please check if you can answer this question and you're familiar with the topics on the following slides.

	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990