Tutorial Business Analytics

Exercise 8 – Solution

Exercise 8.1

Create a 3-Fold Cross-validation for the specified instances

- Partition the data set into complementary subsets
- Decide which subsets will be used for training and which for testing

Additionally, create a *stratified* 3-Fold Cross-validation for specified instances

Instance	Class
1	+
2	+
3	+
4	+
5	-
6	+
7	+
8	-
9	-
10	-
11	+
12	+
13	+
14	-
15	-

Solution 8.1

3-Fold Cross-validation:

(Note: This is only one of many possible solutions!)

P1 = {1, … ,5}, P2 = {6, … ,10}, P3 = {11, … ,15}

Fold 1: Train: P2 & P3, Test: P1, classes: [4,1]
Fold 2: Train: P1 & P3, Test: P2, classes: [2,3]
Fold 3: Train: P1 & P2, Test: P3, classes: [3,2]

Classes (initial data set): [9,6]

Stratified 3-Fold Cross-validation:

We need to keep the distribution of + and – classes in balance for each set, i.e., three + classes and two - classes in each set.

P1 = {1,2,3,5,8}, P2 = {4,6,7,9,10}, P3 = {11,12,13,14,15}

Fold 1: Train: P2 & P3, Test: P1, classes: [**3,2**]
Fold 2: Train: P1 & P3, Test: P2, classes: [**3,2**]
Fold 3: Train: P1 & P2, Test: P3, classes: [**3,2**]

Classes (initial data set): [9,6]

Exercise 8.2

True Class	Predicted Class
0	0
0	1
1	1
1	0
0	0
1	0
0	0
1	1
0	1
1	0

Calculate Recall, False Alarm Rate, Specificity and Accuracy.

Solution 8.2

True Class	Predicted Class	
0	0	**TN**
0	1	**FP**
1	1	**TP**
1	0	**FN**
0	0	**TN**
1	0	**FN**
0	0	**TN**
1	1	**TP**
0	1	**FP**
1	0	**FN**

Recall (True Positive Rate, Sensitivity, Hit Rate)
"How many positive instances have been predicted to be positive"

$$tpr = \frac{tp}{tp + fn} = \frac{2}{2+3} = 0.4$$

False Alarm Rate (False Positive Rate)
"How many negative instances have been predicted to be positive"

$$fpr = \frac{fp}{fp + tn} = \frac{2}{2+3} = 0.4$$

Specificity (True Negative Rate)
"How many negative instances have been predicted to be negative"

$$tnr = \frac{tn}{fp + tn} = \frac{3}{2+3} = 0.6$$

Accuracy
"How many instances have been predicted correctly"

$$acc = \frac{tp + tn}{tp + fp + tn + fn} = \frac{2+3}{2+2+3+3} = 0.5$$

Exercise 8.3

The table below contains 3 classifier's **accuracy values**. Evaluate whether the results obtained by the new classifiers 1 and 2 are significantly different from the baseline classifier 0 (two-sided test, significance level 5%).

Classifier 0	Classifier 1	Δ	Classifier 2	Δ
0.67	0.98	-0.31	0.67	0.00
0.63	0.91	-0.28	0.69	-0.06
0.95	0.93	0.02	0.90	0.05
0.75	0.86	-0.11	0.91	-0.16
0.75	0.95	-0.20	0.86	-0.11
0.79	0.85	-0.06	0.75	0.04
0.79	0.90	-0.11	0.68	0.11
0.82	0.87	-0.05	0.83	-0.01
0.83	0.98	-0.15	0.82	0.01
0.78	0.91	-0.13	0.80	-0.02

$$t = \frac{\bar{d}}{s_d / \sqrt{k}} \sim t_{k-1}$$

$$\bar{d} = \frac{1}{k}\sum_i d_i$$

$$s_d = \sqrt{\frac{1}{k-1}\sum_i \left(d_i - \bar{d}\right)^2}$$

Solution 8.3

$H_0\colon \mu_0 - \mu_1 = 0$ vs $H_1\colon \mu_0 - \mu_1 \neq 0$

$\bar{d}_1$ = - 0.138

s_d = 0.102

T_1 = - 4.267

$$t^{c}_{1-\frac{\alpha}{2};\, n-1} = t^{c}_{0.975;\, 9} = 2.262$$

$|T_1| > t^{c}_{1-\frac{\alpha}{2};\, n-1}$

⇒ **Reject H_0 . Classifier 1 is significantly different from Classifier 0**

$H_0\colon \mu_0 - \mu_2 = 0$ vs $H_1\colon \mu_0 - \mu_1 \neq 0$

$\bar{d}_2$ = -0.015

s_d = 0.079

T_2 = -0.602

$|T_2|$ **<** $t^{c}_{1-\frac{\alpha}{2};\, n-1}$

⇒ **Do not reject H_0. Classifier 2 is not significantly different from Classifier 0**

Exercise 8.4

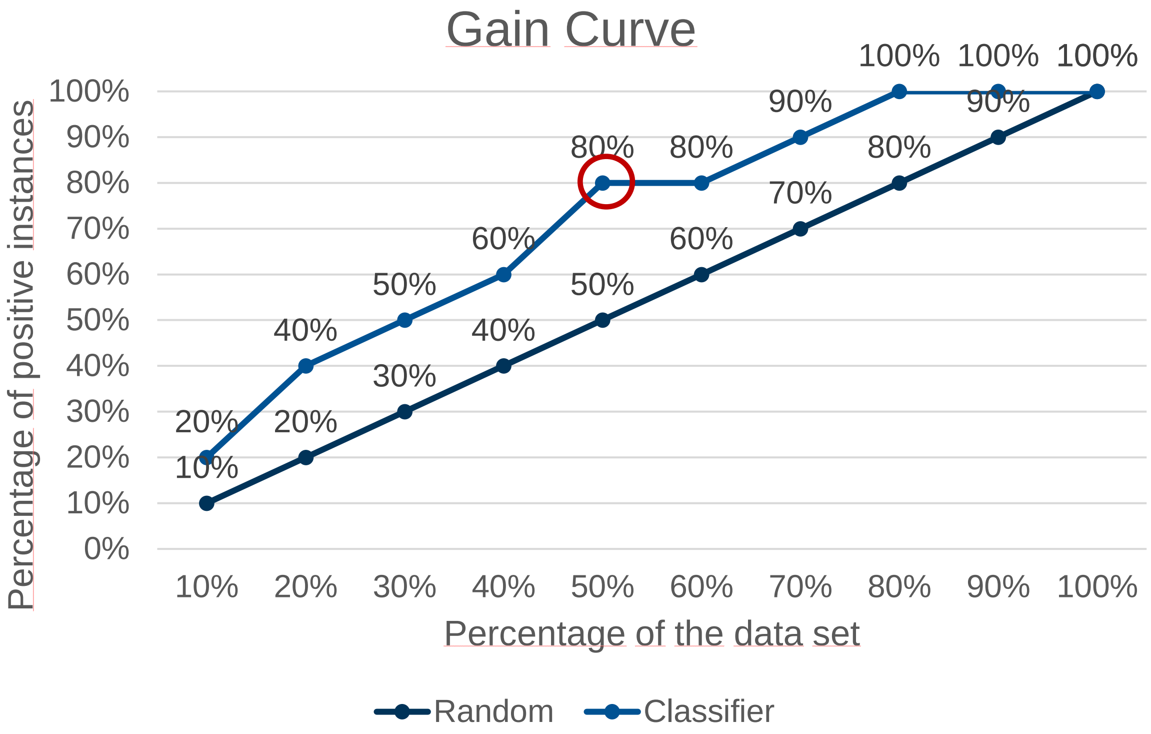
Use the given result of an evaluation (Cutoff = 0.87) to construct:

- a gain curve (10% steps)
- a lift curve
- an ROC curve

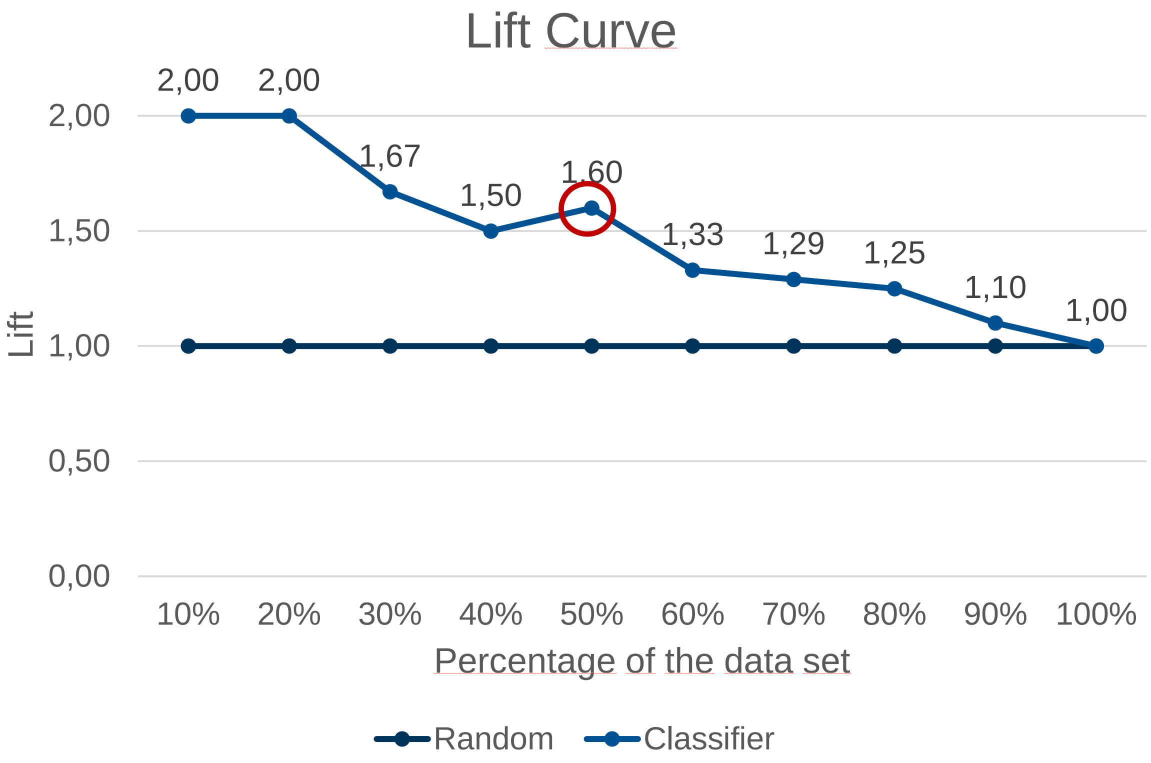
Remember: A Cutoff value of 0.87 means, we will classify an instance as positive until its probability falls below 0.87

Number	Probability	Class
1	0.991	+
2	0.977	+
3	0.973	+
4	0.945	+
5	0.918	+
6	0.915	-
7	0.906	+
8	0.889	-
9	0.873	+
10	0.871	+
11	0.869	-
12	0.866	-
13	0.862	+
14	0.852	-
15	0.837	+
16	0.831	-
17	0.829	-
18	0.811	-
19	0.787	-
20	0.779	-

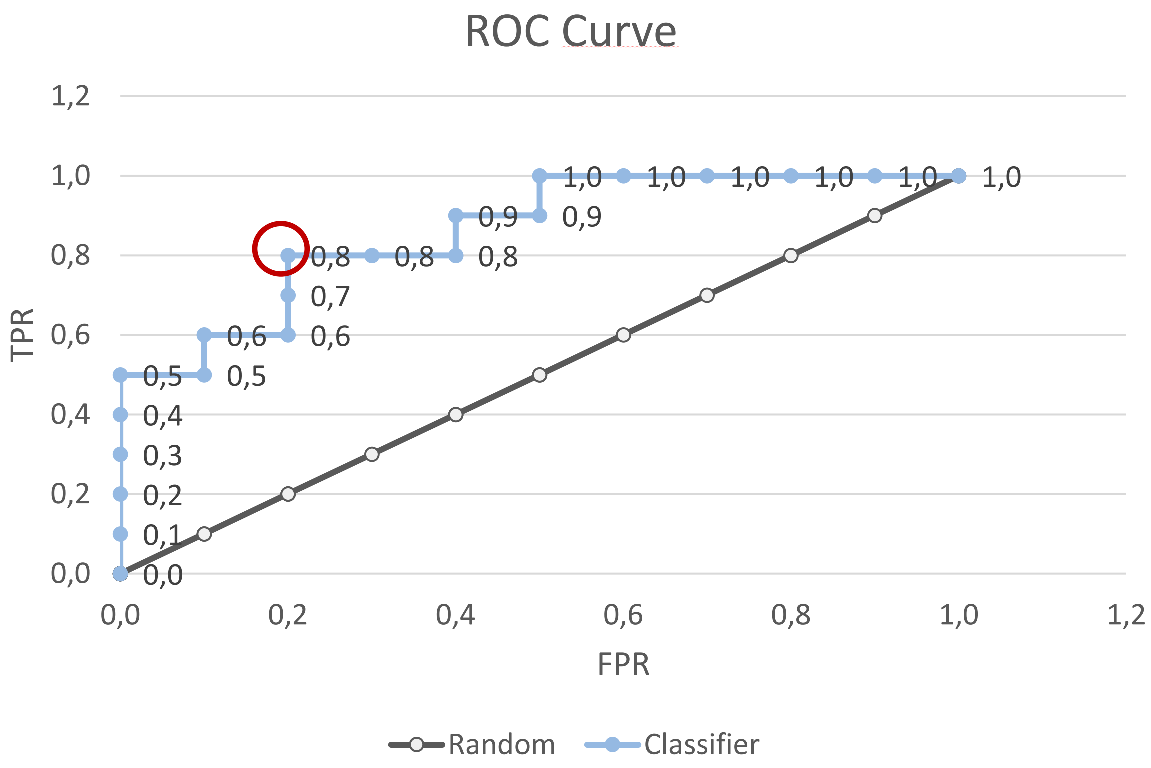
Solution 8.4



Gain Curve
Percentage of positive instances
100%
90%
80%
70%
60%
50%
40%
30%
20%
10%
0%
10%
20%
30%
40%
50%
60%
70%
80%
90%
100%
Percentage of the data set
Random: 10%, 20%, 30%, 40%, 50%, 60%, 70%, 80%, 90%, 100%
Classifier: 20%, 40%, 50%, 60%, 80%, 80%, 90%, 100%, 100%, 100%
Random
Classifier



Lift Curve
Lift
2,00
1,50
1,00
0,50
0,00
10%
20%
30%
40%
50%
60%
70%
80%
90%
100%
Percentage of the data set
Classifier: 2,00, 2,00, 1,67, 1,50, 1,60, 1,33, 1,29, 1,25, 1,10, 1,00
Random
Classifier


ROC Curve
TPR
FPR
1,2
1,0
0,8
0,6
0,4
0,2
0,0
0,0
0,2
0,4
0,6
0,8
1,0
1,2
0,0
0,1
0,2
0,3
0,4
0,5
0,5
0,6
0,6
0,7
0,8
0,8
0,8
0,9
0,9
1,0
1,0
1,0
1,0
1,0
1,0
Random
Classifier

Annex

t-table

df	α = 0.1	α = 0.05	α = 0.025	α = 0.01	α = 0.005
1	3.078	6.314	12.706	31.821	63.657
2	1.886	2.920	4.303	6.965	9.925
3	1.638	2.353	3.182	4.541	5.841
4	1.533	2.132	2.776	3.747	4.604
5	1.476	2.015	2.571	3.365	4.032
6	1.440	1.943	2.447	3.143	3.707
7	1.415	1.895	2.365	2.998	3.499
8	1.397	1.860	2.306	2.896	3.355
9	1.383	1.833	2.262	2.821	3.250
10	1.372	1.812	2.228	2.764	3.169
11	1.363	1.796	2.201	2.718	3.106
12	1.356	1.782	2.179	2.681	3.055
13	1.350	1.771	2.160	2.650	3.012
14	1.345	1.761	2.145	2.624	2.977
15	1.341	1.753	2.131	2.602	2.947
16	1.337	1.746	2.120	2.583	2.921
17	1.333	1.740	2.110	2.567	2.898
18	1.330	1.734	2.101	2.552	2.878
19	1.328	1.729	2.093	2.539	2.861
20	1.325	1.725	2.086	2.528	2.845
21	1.323	1.721	2.080	2.518	2.831
22	1.321	1.717	2.074	2.508	2.819
23	1.319	1.714	2.069	2.500	2.807