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Chair of Mathematical Finance
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MA9712 Statistics for Business Administration
Summer Semester 2017
Exercise Sheet 12

14 July 2017

Tutorial Exercises

Exercise T.17

A producer of chocolate suspects irregularities in his production. For quality control, he randomly takes out 15 bars and looks at the weight in grams. The variable `x` contains the observations (sample realizations). The production is optimised in such a way that the standard deviation from the average weight is 2g. The producer, however, sees a problem with the average weight, which should be 100g. The normal distribution is assumed to be an appropriate model for the weight.

a) Formulate null hypothesis and alternative hypothesis for a two-sided test for the average weight ϑ.

b) Which test would you use and why?

c) Perform the selected test at the significance level $\alpha = 0.05$. What conclusion can be drawn from the results of the test?

d) What do the null hypothesis and alternative hypothesis need to be if you want to show that the average weight is below 100g?

e) Perform an appropriate test for the test problem from (d) at significance level $\alpha = 0.05$.

Hint:

```
c(sum(x), qnorm(0.9), qt(0.9, df = 14), qnorm(0.95), qt(0.95, df = 14),
  qnorm(0.975), qt(0.975, df = 14))
```

```
## [1] 1471.200000    1.281552    1.345030    1.644854    1.761310    1.959964
## [7]    2.144787
```

Homework Exercises

Exercise H.18

Please complete this exercise **online in the Moodle course**. The eTest is accessible from 14 July, 18:00 to 21 July, 6:00. Once you have started, you have a time limit of 90 minutes to complete the test.

Note: This eTest is relevant for the **bonus**.

Example exercise:

Note: Do **not** round any intermediate results in your calculations. Round your (numerical) non-integer end results to **3 decimal places**. Points as well as commas are accepted as decimal separators. Provide only numerical results without any extra symbols such as % signs, etc.

a) For scientific studies the body weight of 16 antelopes was measured (in kg):

```
## [1] 620 700 840 530 820 830 710 690 470 520 510 850 580 890 600 670
```

It is assumed that the body weight of antelopes can be represented by a normally distributed variable X_i, i.e. $X_1, \ldots, X_{16} \overset{iid}{\sim} N(\vartheta_1, \vartheta_2)$.

- Calculate the maximum likelihood estimator of ϑ_1.
- Calculate the maximum likelihood estimator of ϑ_2.

b) For a lecture in statistics, there are 106 students registered. The professor would like to know the proportion ϑ of participants who actually attend the lecture. A randomly selected lecture is actually attended by 45 students.

- Calculate an unbiased point estimator of the proportion ϑ.
- Calculate an exact two-sided confidence interval for ϑ at the level 0.95. Provide the length of the interval as final solution.

c) We want to analyse the daily expenditures of (adult) summer tourists in Lutenblag (Molwanîen). It is assumed that the expenditures are normally distributed, where neither the expectation ϑ_1 nor the variance ϑ_2 are known. In a survey, 76 tourists were asked about their expenditures. From this data, it was found that on average 155.64 Euro were spent per day with a sample variance of 102.15. Determine a two-sided confidence interval at the level 0.99 for ϑ_1. Provide the length of the interval as final solution.

d) This part of the exercise consists of questions on the topic, where one or more than one answer may be correct and you have to choose the correct answer(s). The concrete questions will only be released during the eTest.

Exercise H.19

Let $X_1, \ldots, X_n \overset{iid}{\sim} N(\vartheta_1, \vartheta_2)$ be a sample with size $n = 12$. An empirical mean and empirical variance for a given sample realization are given by

$$\overline{x}_{12} = 22.45 \text{ and } s_{12}^2 = 27.31\,.$$

a) Calculate a two-sided confidence interval for ϑ_1 at the confidence level 0.95.

b) Test the null hypothesis $H_0 : \vartheta_1 = 23$ against $H_1 : \vartheta_1 \neq 23$ at the significance level (niveau) $\alpha = 0.05$.