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Chapter 6
Vector Semantics and Embeddings

- content is based on [1]
- certain elements (e.g. equations or tables) were taken over or taken over in a modified form from [1]
- citations of [1] or from [1] are omitted for legibility
- errors are fully in the responsibility of Georg Groh
- BIG thanks to Dan and James for a great book!

Lexical Semantics

- "meaning" of a word == ? aspects of "meaning" desirable to model:
  - general conceptual relatedness / similarity (dog, cat) (buy, sell, pay)
  - specific forms of conceptual relations: antonmy (hot, cold), hyponymy (cat, mammal)
  - distinguish between word senses (bank^1, bank^2) (mouse^1, mouse^2)
  - ….

Word Senses

see also [3]

- Lemma is homonymous: it has multiple semantically unrelated word senses e.g.
  - lemma jaguar: jaguar^1 (the cat), jaguar^2 (the car) jaguar^3 (the Fender guitar model)
  - lemma bank: bank^1 (“financial institution”), bank^2 (“sloping mound”)
  - lemma bat: bat^1 (“club for hitting a ball”), bat^2 (“nocturnal flying animal”):
  - each of those: homonyms and homographs (same writing)

- *write* - *right*; *piece* - *peace:* homophones. (←→ spelling errors)

- homographs that are not homophones:
  bass^1 (“fish”) - bass^2 (“instrument”)
  (←→ speech synthesis errors)

Relations Between Senses: Synonymy

see also [3]

- (nearly) identical senses of different lemmas: synonymy:
  - substitutable one for the other in any sentence without changing the truth conditions of the sentence (same propositional meaning)
  - *couch/sofa vomit/throw up filbert/hazelnut car/automobile*

- synonymy: actually between senses of words:
  synonyms may replace one another in a sentence:
  - example: big / large replaceable (big = big^1):
    - *How big is that plane?*
    - *Would I be flying on a large or small plane?*
  - big / large not replaceable (big = big^2):
    - *Miss Nelson became a kind of big sister to Benjamin.*

- synonyms: principle of contrast: different word forms → at least SLIGHTLY different meaning / different pragmatics.
  examples: water, H_20 ;   car, automobile

Relations Between Senses: Hyponymy, Meronymy, and Antonymy

see also [3]

- Hyponmy: sub-class - class relation;
  Hypernymy (Superordinate): super-class – class relation;
  - sense A hyponym of sense B ("B subsumes A" "A is-a B") if $\forall x\colon A(x) \rightarrow B(x)$
  - examples:

Hypernym:	vehicle	fruit	furniture	mammal
Hyponym:	car	mango	chair	dog

- Meronym: part-of - whole relation;
  Holonym: whole – part-of relation;
  - examples:

Meronym:	leg	leg	wheel	CPU
Holonym:	chair	human	car	computer

- Antonymy:
  - *long/short big/little fast/slow cold/hot* (opposite ends of a scale)
  - *rise/fall up/down in/out* (reversed direction (reversives))

Word Similarity, Word Relatedness

- general similarity between words (abstracting from word senses)
  - e.g. measure via human judgement (e.g. via crowd sourcing)

vanish	disappear	9.8
behave	obey	7.3
belief	impression	5.95
muscle	bone	3.65
modest	flexible	0.98
hole	agreement	0.3

- word relatedness (association): words belong to the same semantic field, e.g. semantic field of
  - hospitals (surgeon , scalpel , nurse , anaesthetic , hospital),
  - restaurants (waiter , menu , plate , food , chef),
  - houses (door , roof , kitchen , family , bed).

semantic fields ←→ topics in Topic Models (Latent Dirchlet Allocation (LDA))

Semantic Frames

- Semantic Frame: special form of semantic field:
  set of words that denote perspectives or participants in a particular type of event.

- in semantic frame exist distinct Semantic Roles:
  - e.g. in semantic frame ‘transaction of goods or services’:
    buy or sell → buyer, seller; pay ←→ buyer etc.

- identifying association between words and roles → better sentence understanding (e.g. for question answering):

  Sam bought the book from Ling
  → == Ling sold the book to Sam,

Connotation

- words can have affective meaning (connotation):
  sentiment, opinions, evaluations
  e.g. *happy* ←→ *sad*; *great, love* ←→ *terrible, hate*

- dimensions of affective meaning:
  - valence: the pleasantness of the stimulus,
    e.g. *happy* ←→ *sad*
  - arousal: the intensity of emotion provoked by the stimulus,
    e.g. *excited* ←→ *relaxed*
  - dominance: the degree of control exerted by the stimulus
    e.g. *important, controlling* ←→ *awed, influenced*

	Valence	Arousal	Dominance
courageous	8.05	5.5	7.38
music	7.67	5.57	6.5
heartbreak	2.45	5.65	3.58
cub	6.71	3.95	4.24
life	6.68	5.59	5.89

→ word semantics as vectors

Distributional Semantics

- "words that occur in similar contexts tend to have similar meanings"
- "you shall know a word by the company it keeps"

- *A bottle of tesgüino is on the table.*
  Everybody likes tesgüino.
  Tesgüino makes you drunk.
  We make tesgüino out of corn.

beer tequila tesgüino

- occurs around words *bottle drunk table*
- occurs as object of likes
- occurs after bottle

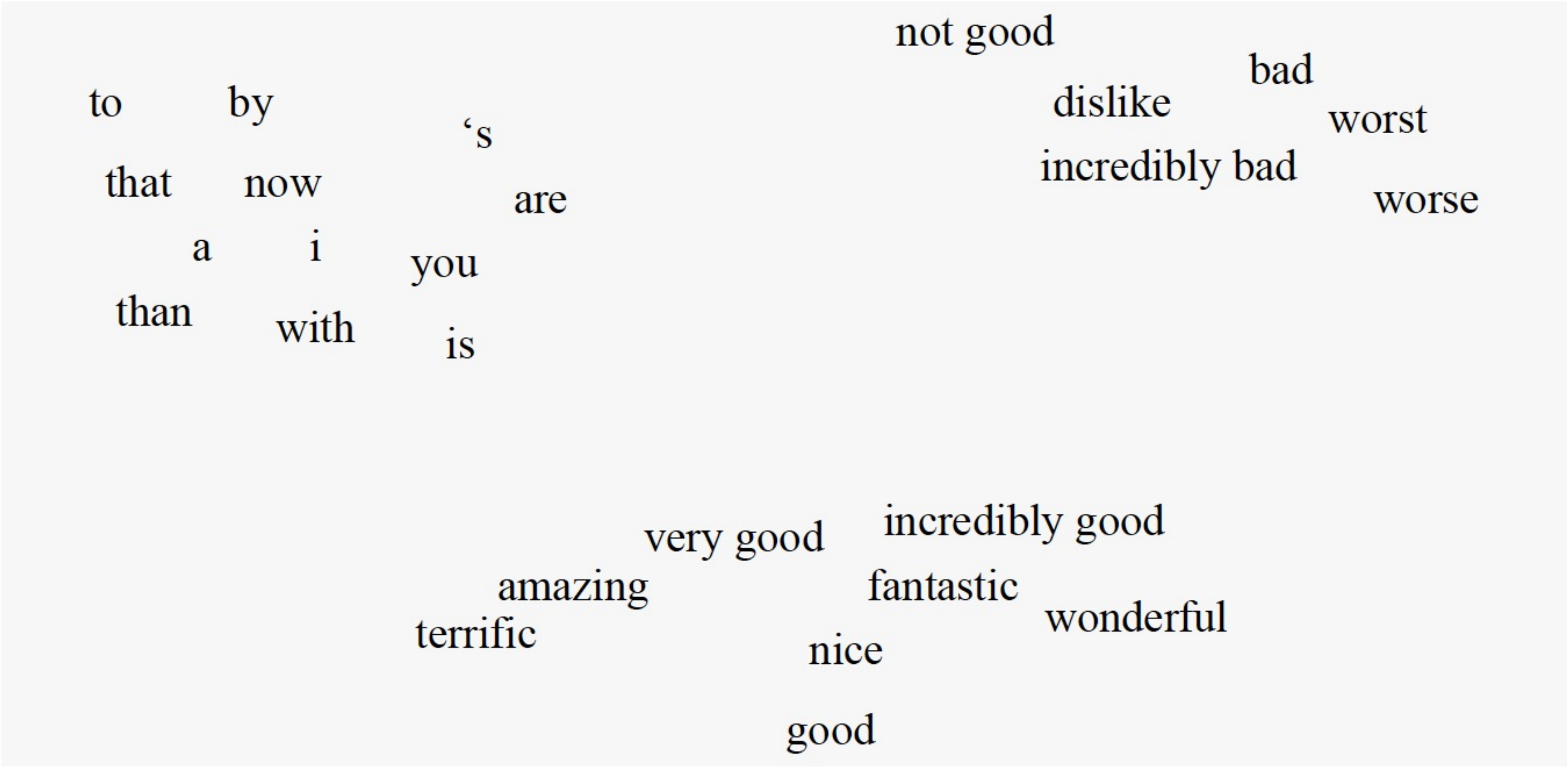
Ongchoi is delicious sauteed with garlic.
Ongchoi is superb over rice.
...ongchoi leaves with salty sauces...

...spinach sauteed with garlic over rice...
...chard stems and leaves are delicious...
...collard greens and other salty leafy greens

- distributional semantics: meaning of a word is computed from the distribution of words around it

Embeddings

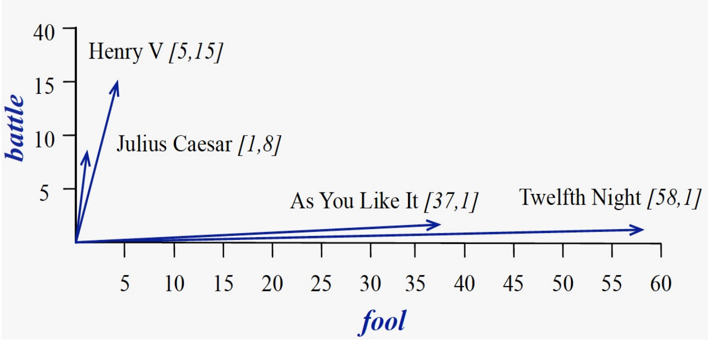
- combine distributionalist intuition with vector representation idea → word embeddings



Words and Vectors

- vector space models of semantics: model word by embedding it into a vector space → vector representations of a word: embedding
- semantic similarity of words ←→ similarity of vectors
- $|V| \times D$ term (word)-document matrix with D document vectors:

	As You Like It	Twelfth Night	Julius Caesar	Henry V
battle	1	0	7	13
good	114	80	62	89
fool	36	58	1	4
wit	20	15	2	3



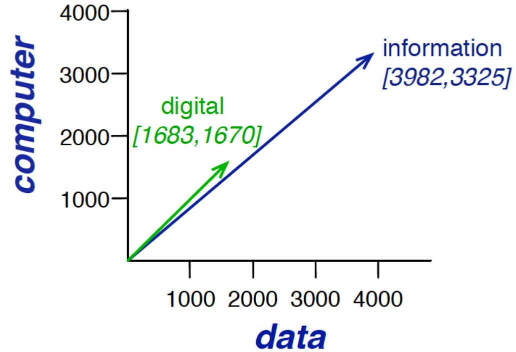
- Information retrieval: query q, document collection $D = \{d_1, d_2, \ldots\}$: find $\{d_i\}$ with high $sim(d_i, q)$. → simple application of tf-idf document (column) vectors : $sim(d_i, q) = \cos(d_i, q) = (||d_i||\ ||q||)^{-1} \mathbf{d}_i^T \boldsymbol{q}$

Words as Vectors

- words as vectors: e.g. rows of word-document matrix:
  fool:[37,58,1,5], *clown*:[5, 117, 0, 0], *battle*:[1,1,8,15]

- more common: word-vectors: rows of $|V|\times|V|$ word-word matrix (term-term matrix, term-context matrix)
  - matrix elements: feature-values (e.g. counts) of co-occurences in certain contexts: documents, n-word window around word,...

	aardvark	...	computer	data	result	pie	sugar	...
cherry	0	...	2	8	9	442	25	
strawberry	0	...	0	0	1	60	19	
digital	0	...	1670	1683	85	5	4	
information	0	...	3325	3982	378	5	13	



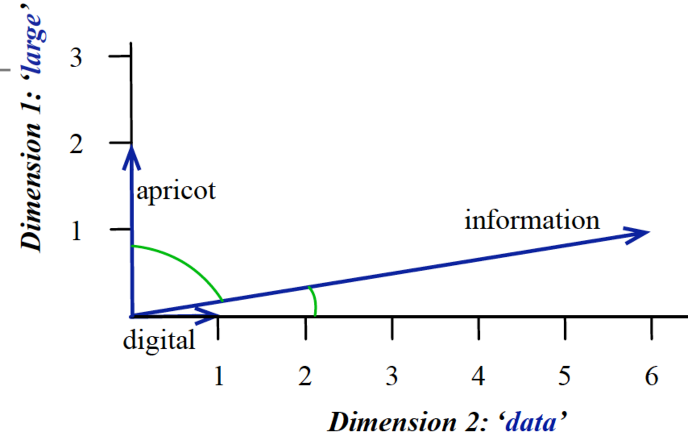
Words as Vectors

- question: instead of counts (which may not be very discriminative / informative): what features can we use? → TF-IDF, Pointwise Mutual Information etc.

- other question: how to compute similarity in a vector space? → usually use cosine similarity

$$\text{cosine}(\mathbf{v},\mathbf{w}) = \frac{\mathbf{v}\cdot\mathbf{w}}{|\mathbf{v}||\mathbf{w}|} = \frac{\sum_{i=1}^{N} v_i w_i}{\sqrt{\sum_{i=1}^{N} v_i^2}\sqrt{\sum_{i=1}^{N} w_i^2}}$$

Similarity Measures on Word-Vectors



$$\mathrm{sim}_{\text{Cosine}}(\vec{v},\vec{w}) = \frac{\vec{v}\cdot\vec{w}}{|\vec{v}||\vec{w}|}$$

$$\mathrm{sim}_{\text{Jaccard}}(\vec{v},\vec{w}) = \frac{\sum_{i=1}^{N}\min(v_i,w_i)}{\sum_{i=1}^{N}\max(v_i,w_i)}$$

$$\mathrm{sim}_{\text{Dice}}(\vec{v},\vec{w}) = \frac{2\times\sum_{i=1}^{N}\min(v_i,w_i)}{\sum_{i=1}^{N}(v_i+w_i)}$$

$$\mathrm{dist}_{\text{Jensen-Shannon}}(\vec{v},\vec{w}) = D(\vec{v}\|\frac{\vec{v}+\vec{w}}{2}) + D(\vec{w}\|\frac{\vec{v}+\vec{w}}{2})$$

JS∈[0,1] → e.g. $\mathrm{sim}_{JS}=1-\mathrm{dist}_{JS}$

KL-divergence: $$D(P||Q) = \sum_x P(x)\log\frac{P(x)}{Q(x)}$$

$$JS(P||Q) = D(P\|\frac{P+Q}{2}) + D(Q\|\frac{P+Q}{2})$$

TF-IDF

- term frequencies alone: not very discriminative
  (e.g. regard frequent words such as the, I, they, good

- → alternative term-document matrix:
  tf-idf: term frequency in a document $\mathrm{tf}_{t,d} = \mathrm{count}(t,d)$

  * inverse document frequency $\mathrm{idf}_t = \log_{10}\left(\frac{N}{\mathrm{df}_t}\right)$

N : number of documents

df_t : number of documents in which t occurs

→ matrix element for word *t* and document *d*:

$$w_{t,d} = \mathrm{tf}_{t,d} \times \mathrm{idf}_t$$

Word	df	idf
Romeo	1	1.57
salad	2	1.27
Falstaff	4	0.967
forest	12	0.489
battle	21	0.074
fool	36	0.012
good	37	0
sweet	37	0

	As You Like It	Twelfth Night	Julius Caesar	Henry V
battle	0.074	0	0.22	0.28
good	0	0	0	0
fool	0.019	0.021	0.0036	0.0083
wit	0.049	0.044	0.018	0.022

Applications of TF-IDF

- applications of tf-idf + cosine similarity:
  e.g. information retrieval, plagiarism detection, recommender systems:

- idea: represent document by centroids of its word vectors:

$$d = \frac{w_1 + w_2 + \ldots + w_k}{k}$$

then compute $sim(d_1, d_2) = \cos(d_1, d_2)$

Pointwise Mutual Information

- Specific Conditional Entropy:

$$H(X|Y=y) = -\sum_x P(X=x|Y=y)\log_2 P(X=x|Y=y)$$

- Conditional Entropy:

$$H(X|Y) = -\sum_y P(Y=y)\ H(X|Y=y)$$

- Mutual Information:

$$I(X,Y) = H(X) - H(X|Y) = H(Y) - H(Y|X)$$

$$= \sum_x \sum_y P(X=x,Y=y)\log_2 \frac{P(X=x,Y=y)}{P(X=x)P(Y=y)}$$

- Pointwise Mutual Information (PMI):

$$I(x,y) = \log_2 \frac{P(x,y)}{P(x)P(y)}$$

how often two events x and y occur, compared with what we would expect if they were independent

Pointwise Mutual Information

- PMI between a word w and a word c in context:

$$I(w,c) = \log_2 \frac{P(w,c)}{P(w)P(c)}$$

co-occurrence of words (MLE)

expectation of co-occurrence if words were independent

- $I(w,c) \in [-\infty, \infty]$ but negative values (co-occurrence *less* likely than independent occurrences) not very reliable → Positive PMI:

$$\mathrm{PPMI}(w,c) = \max(\log_2 \frac{P(w,c)}{P(w)P(c)}, 0)$$

- Assuming a co-occurrence count matrix F, we have

$$p_{ij} = \frac{f_{ij}}{\sum_{i=1}^{W} \sum_{j=1}^{C} f_{ij}} \quad p_{i*} = \frac{\sum_{j=1}^{C} f_{ij}}{\sum_{i=1}^{W} \sum_{j=1}^{C} f_{ij}} \quad p_{*j} = \frac{\sum_{i=1}^{W} f_{ij}}{\sum_{i=1}^{W} \sum_{j=1}^{C} f_{ij}}$$

$$\mathrm{PPMI}_{ij} = \max(\log_2 \frac{p_{ij}}{p_{i*}p_{*j}}, 0)$$

Pointwise Mutual Information

	computer	data	result	pie	sugar	count(w)
cherry	2	8	9	442	25	486
strawberry	0	0	1	60	19	80
digital	1670	1683	85	5	4	3447
information	3325	3982	378	5	13	7703
count(context)	4997	5673	473	512	61	11716

	p(w,context)					p(w)
	computer	**data**	**result**	**pie**	**sugar**	**p(w)**
cherry	0.0002	0.0007	0.0008	0.0377	0.0021	0.0415
strawberry	0.0000	0.0000	0.0001	0.0051	0.0016	0.0068
digital	0.1425	0.1436	0.0073	0.0004	0.0003	0.2942
information	0.2838	0.3399	0.0323	0.0004	0.0011	0.6575
p(context)	0.4265	0.4842	0.0404	0.0437	0.0052	

$$
\begin{aligned}
P(\text{w=information,c=data}) &= \frac{3982}{11716} = .3399 \\
P(\text{w=information}) &= \frac{7703}{11716} = .6575 \\
P(\text{c=data}) &= \frac{5673}{11716} = .4842 \\
\text{ppmi}(\text{information,data}) &= \log 2(.3399/(.6575 * .4842)) = .0944
\end{aligned}
$$

PPMI	computer	data	result	pie	sugar
cherry	0	0	0	4.38	3.30
strawberry	0	0	0	4.10	5.51
digital	0.18	0.01	0	0	0
information	0.02	0.09	0.28	0	0

Pointwise Mutual Information

- PMI biased towards infrequent events →
  - use PPMI$_\alpha$:

$$\text{PPMI}_\alpha(w,c) = \max(\log_2 \frac{P(w,c)}{P(w)P_\alpha(c)}, 0) \qquad P_\alpha(c) = \frac{count(c)^\alpha}{\sum_c count(c)^\alpha}$$

e.g. with $\alpha = 0.75$ → $P_\alpha(c) > P(c)$ → PPMI smaller

  - use Laplace Smoothing:

	computer	data	pinch	result	sugar
apricot	0	0	1	0	1
pineapple	0	0	1	0	1
digital	2	1	0	1	0
information	1	6	0	4	0

⇩

	computer	data	pinch	result	sugar
apricot	2	2	3	2	3
pineapple	2	2	3	2	3
digital	4	3	2	3	2
information	3	8	2	6	2

PPMI	computer	data	pinch	result	sugar
apricot	0	0	2.25	0	2.25
pineapple	0	0	2.25	0	2.25
digital	1.66	0	0	0	0
information	0	0.57	0	0.47	0

⇩

PPMI	computer	data	pinch	result	sugar
apricot	0	0	0.56	0	0.56
pineapple	0	0	0.56	0	0.56
digital	0.62	0	0	0	0
information	0	0.58	0	0.37	0

Dense Word-Vectors

- in a $|V| \times |V|$ word-word-matrix, each $|V|$-dim. word vector (with $|V| \propto 10^5$) will be sparse → use more dense, lower dim. vectors with dim $\propto 10^3$:
  - models will have to learn fewer parameters, less overfitting, better modelling of semantic relations
  - if dimensions (say for car and automobile) are distinct: sparse, high-dim. vectors may fail to model the semantic similarity between words often occurring together with car and automobile

- first idea: use dimensionality reduction techniques (SVD, PCA, pPCA, Factor-Analysis, Auto-Encoders etc.) → see [5] and / or slides in appendix in this slide-set

- modern idea: use prediction based embeddings (word2vec, GloVe etc.)

Embeddings from Prediction: Skip-Gram w. Neg. Sampling (word2vec)

- intuition of word2vec: instead of counting how often each word w occurs near e.g. *apricot* , train a classifier on a binary prediction task: “Is word w likely to show up near apricot ?”
- use classifier weights as dense embeddings for words
- use large corpora as “supervised” data sources for classifier

1. Treat the target word and a neighboring context word as positive examples.
2. Randomly sample other words in the lexicon to get negative samples
3. Use logistic regression to train a classifier to distinguish those two cases
4. Use the regression weights as the embeddings

(Log. Regression) Classifier

```
... lemon,  a [tablespoon of apricot jam,      a] pinch ...
                c1          c2    t      c3       c4
```

$$Similarity(t,c) \approx t \cdot c$$

word t

any word c in context window

vector repre-sentation of t

vector repre-sentation of c

$$P(+|t,c) = \frac{1}{1+e^{-t \cdot c}}$$

$$P(-|t,c) = 1 - P(+|t,c)$$

- assuming independence between context words, we get

$$P(+|t,c_{1:k}) = \prod_{i=1}^{k} \frac{1}{1+e^{-t \cdot c_i}}$$

$$\log P(+|t,c_{1:k}) = \sum_{i=1}^{k} \log \frac{1}{1+e^{-t \cdot c_i}}$$

Learning Skip-Gram Embeddings

```
... lemon,   a [tablespoon of apricot jam,      a] pinch ...
               c1         c2     t      c3       c4
```

positive examples +

t	c
apricot	tablespoon
apricot	of
apricot	preserves
apricot	or

negative examples -

t	c	t	c
apricot	aardvark	apricot	twelve
apricot	puddle	apricot	hello
apricot	where	apricot	dear
apricot	coaxial	apricot	forever

for each positive example, k negative examples (here k=2)

- sample negative examples from modified unigram distribution:

$$P_\alpha(w) = \frac{count(w)^\alpha}{\sum_{w'} count(w')^\alpha}$$

usually with $\alpha = 0.75$ (give rare words higher probability)

Learning Skip-Gram Embeddings

- → objective function: $$L(\theta) = \sum_{(t,c)\in+} \log P(+|t,c) + \sum_{(t,c)\in-} \log P(-|t,c)$$

  for **one** word/ context pair (t,c) and k noise words:

$$\begin{aligned} L(\theta) &= \log P(+|t,c) + \sum_{i=1}^{k} \log P(-|t,n_i) \\ &= \log \sigma(c \cdot t) + \sum_{i=1}^{k} \log \sigma(-n_i \cdot t) \\ &= \log \frac{1}{1+e^{-c \cdot t}} + \sum_{i=1}^{k} \log \frac{1}{1+e^{n_i \cdot t}} \end{aligned}$$

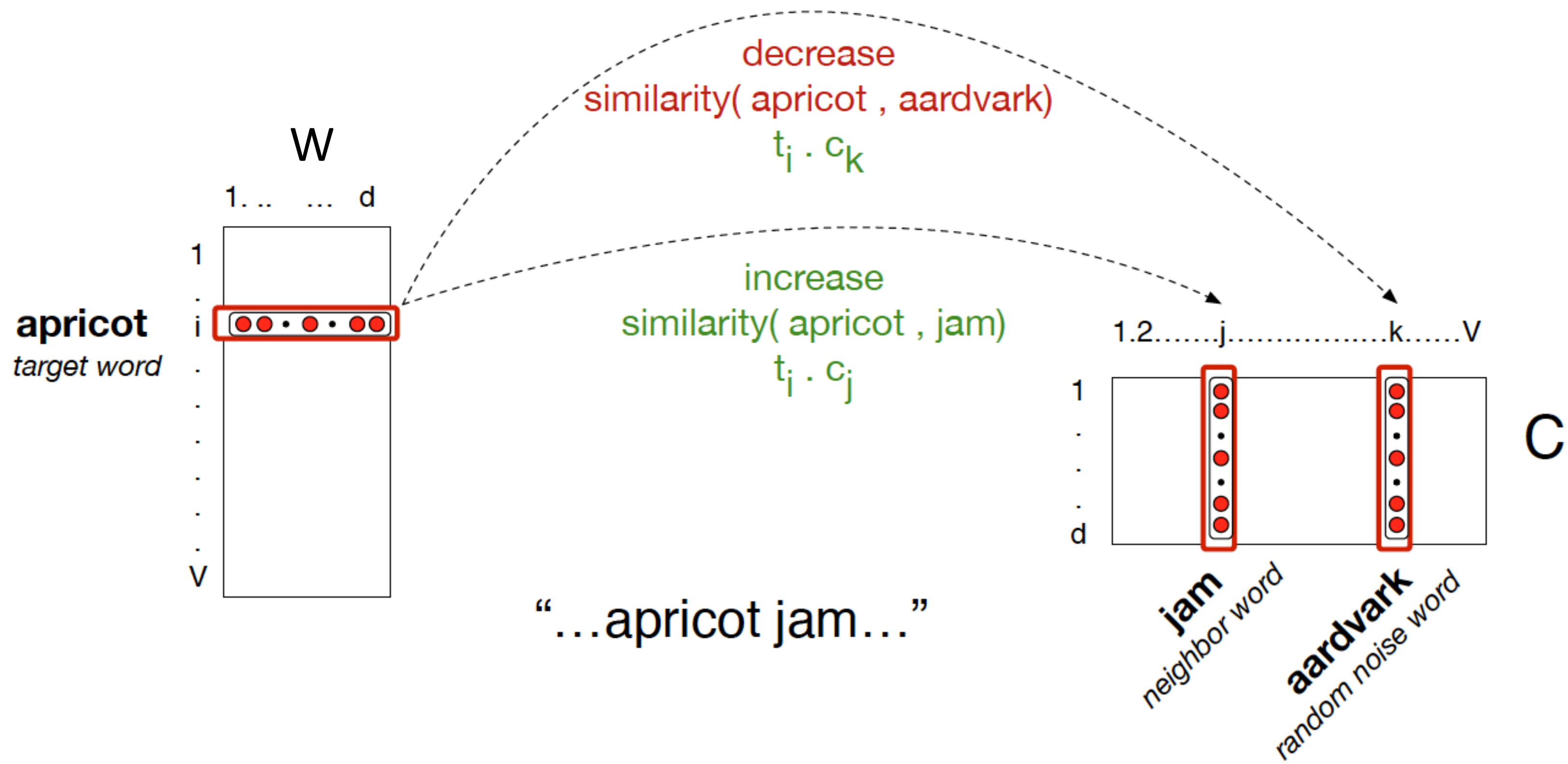
  ←→ maximize vector similarity of (t,c) drawn from positive examples, minimize for negative examples

- start with random embeddings;
  use stochastic gradient descent as usual for log. regression;
  k, dimension d, and window size L: meta parameters

Learning Skip-Gram Embeddings

- → we get two different embeddings for each word:
  - target embedding t (matrix W)
  - context embedding c (matrix C)

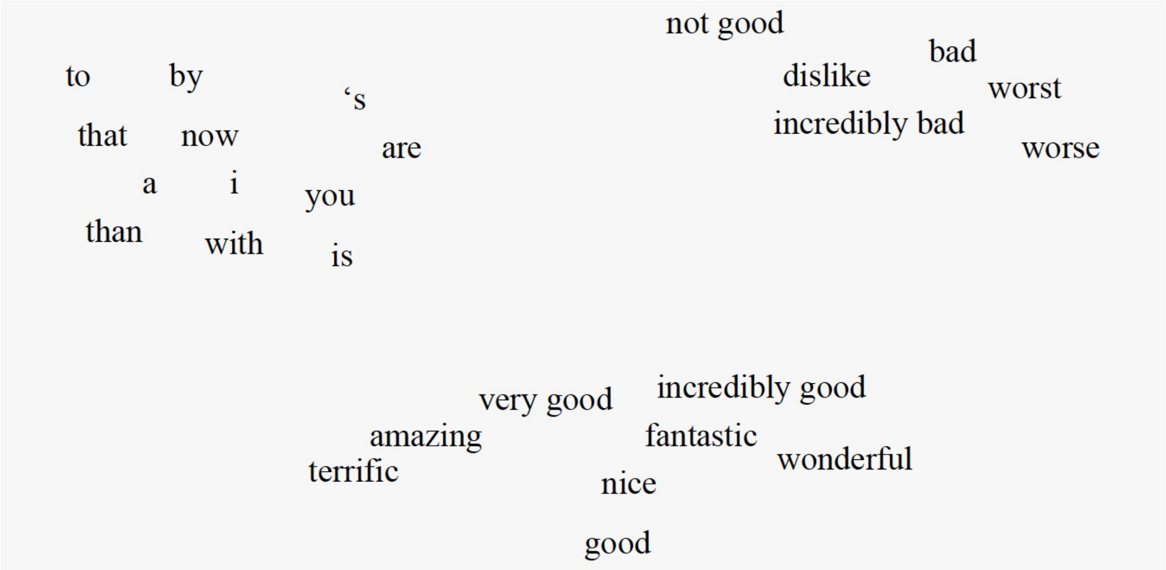
In Jurafksy sometimes also called T



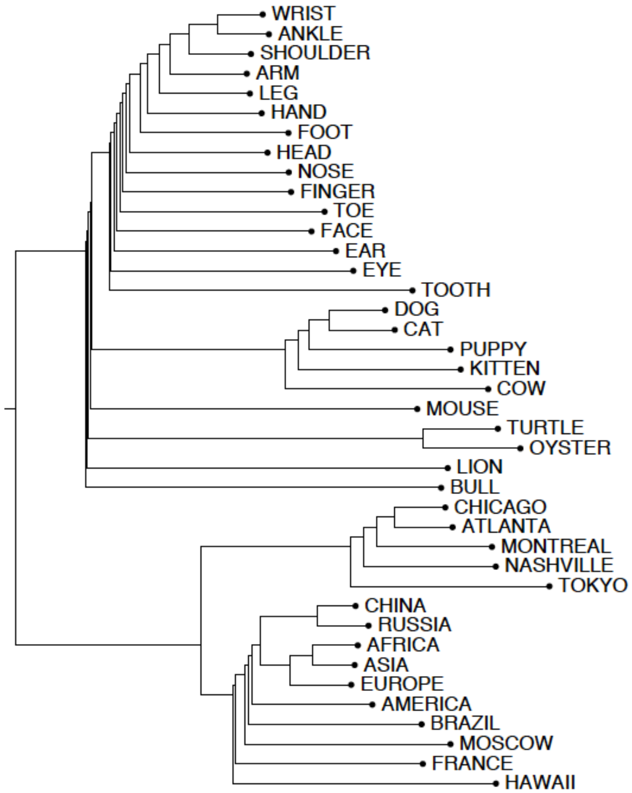
- final embeddings: use only t, or add t and c, or stack them

Visualizing Embeddings

- use word clouds of most similar words
- or use projection technique (PCA or t-SNE)



- or use hierarchical clustering



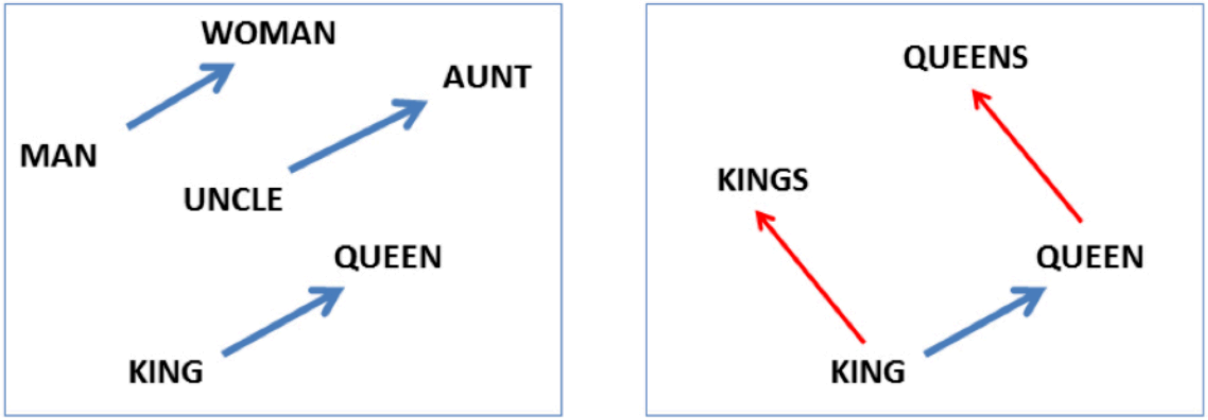
Properties of Embeddings ←→ Window Size

- short context windows: most similar words: semantically similar words; long context windows: most similar words: topically related words
  - *Hogwarts* and $L = \pm 2$ → other „schools“: *Sunnydale* (from Buffy the Vampire Slayer) or *Evernight* (from a vampire series)
  - *Hogwarts* and $L = \pm 5$ → *Dumbledore*, *Malfoy*, and *half-blood*

- first order co-occurence (syntagmatic co-occurrence): words that typically occur near each other: *wrote* ←→ *book, poem*
  second order co-occurrence (paradigmatic co-occurrence): words that have similar neighbours: *wrote* ←→ *said, remarked*

Properties of Embeddings: Analogy / Compositionality

- skip-gram embeddings show semantic and other forms of compositionality:

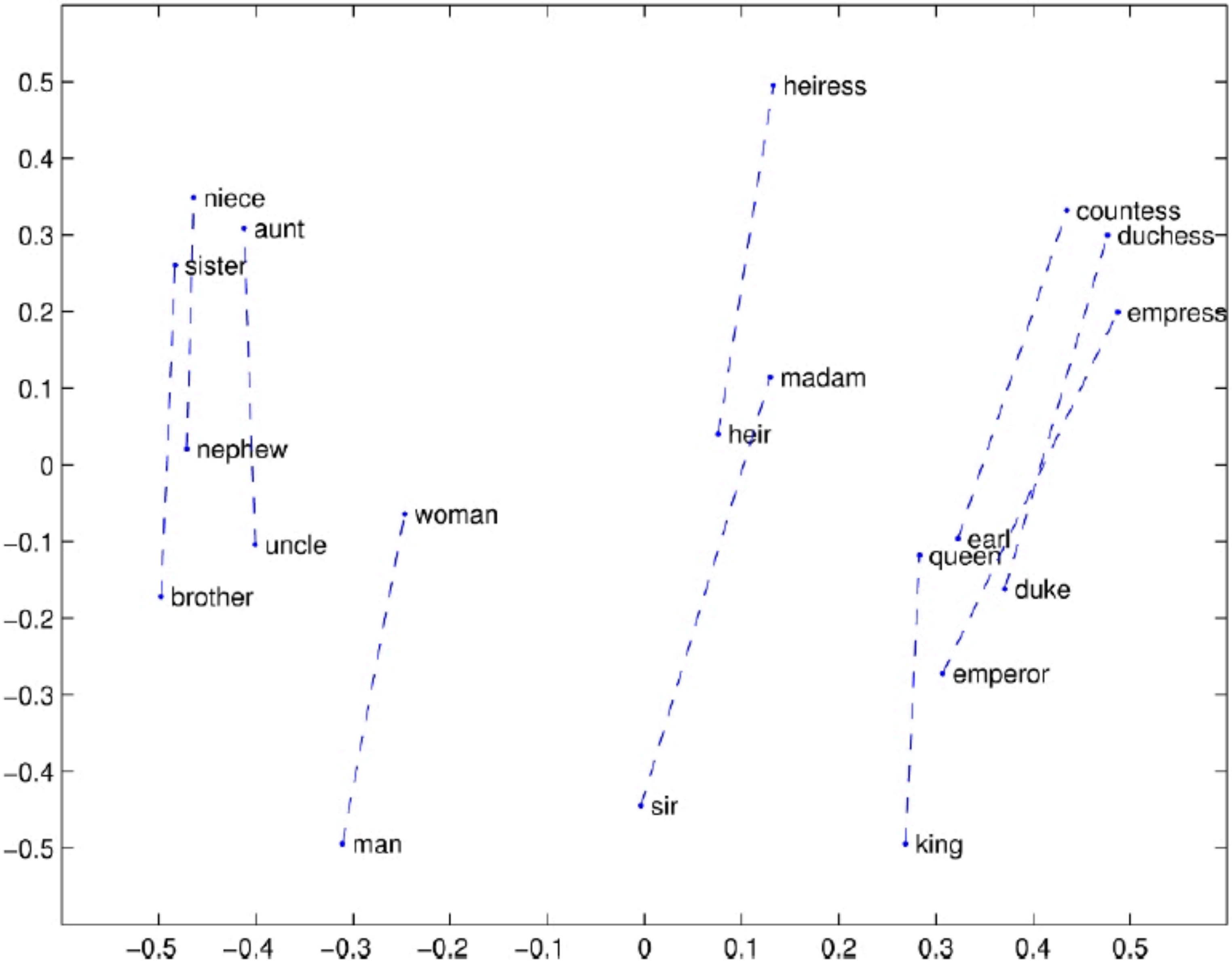


2D PCA projection of 1000-dim. embeddings:
“king” – “man” + “woman” ≈ “queen”
“paris” – “France” + “Italy” ≈ “Rome”

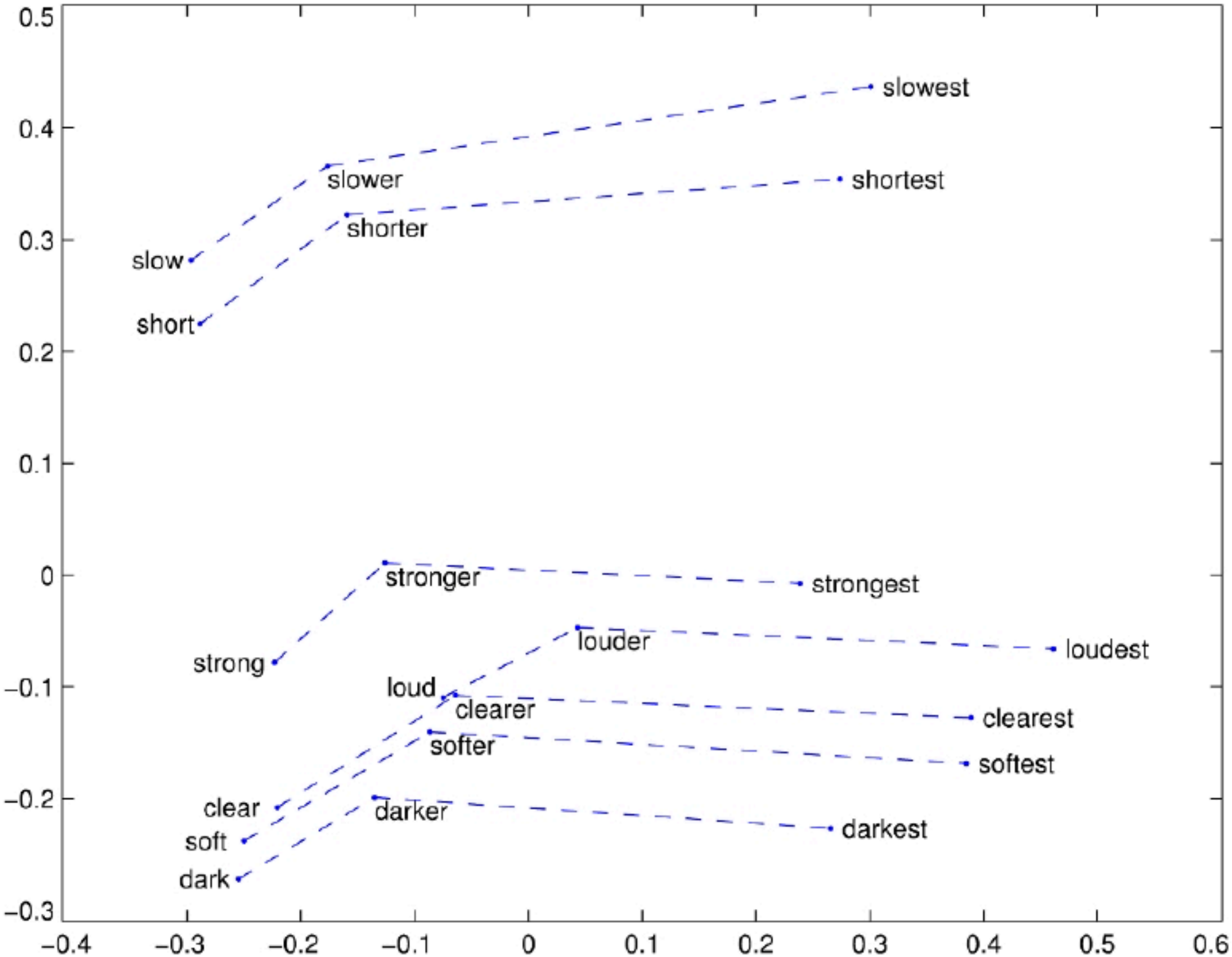
Czech + currency	Vietnam + capital	German + airlines	Russian + river	French + actress
koruna	Hanoi	airline Lufthansa	Moscow	Juliette Binoche
Check crown	Ho Chi Minh City	carrier Lufthansa	Volga River	Vanessa Paradis
Polish zolty	Viet Nam	flag carrier Lufthansa	upriver	Charlotte Gainsbourg
CTK	Vietnamese	Lufthansa	Russia	Cecile De

closest 4 tokens of sum of skip-gram embeddings [6]

Properties of Embeddings: Analogy / Compositionality

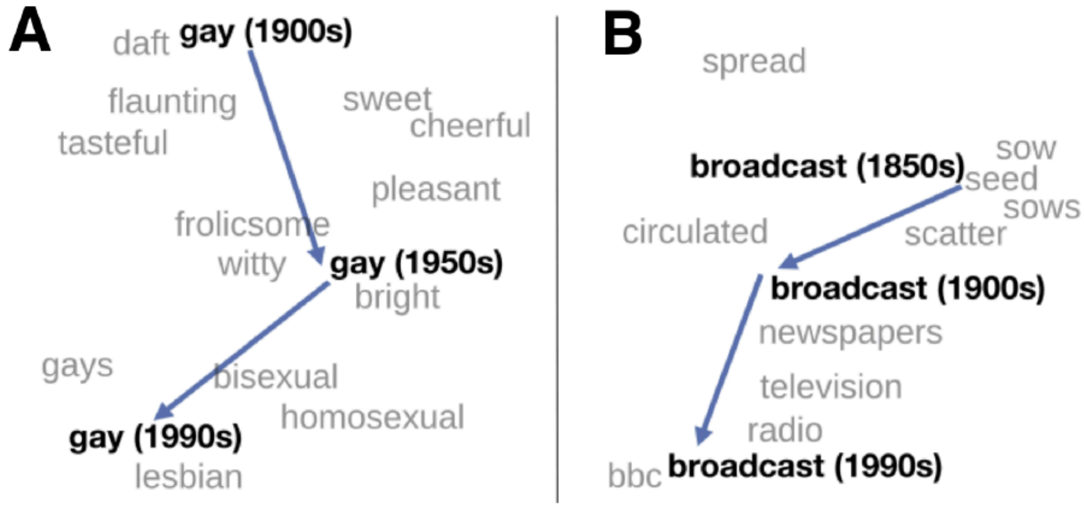


Properties of Embeddings: Analogy / Compositionality



Properties of Embeddings: Historical Semantics & Cultural Bias

- historical drift in semantics (t-SNE projections)



- gender bias: father ←→ doctor → mother ←→ nurse

- racist bias: Implicit Association Test: USA:
  African American names ←→ unpleasant words;
  European American names ←→ pleasant words
  replicated with GloVe embeddings

Evaluating Word Vectors

- compare vector similarity against human similarity judgement of raw word pairs (without sentences)
  - Word-sim-353 (2002): 353 word pairs, rated on a 0-10 similarity scale
  - SimLex-999 (2015)
  - TOEFL dataset (1997): 4 proposed synonyms per words: choose closest

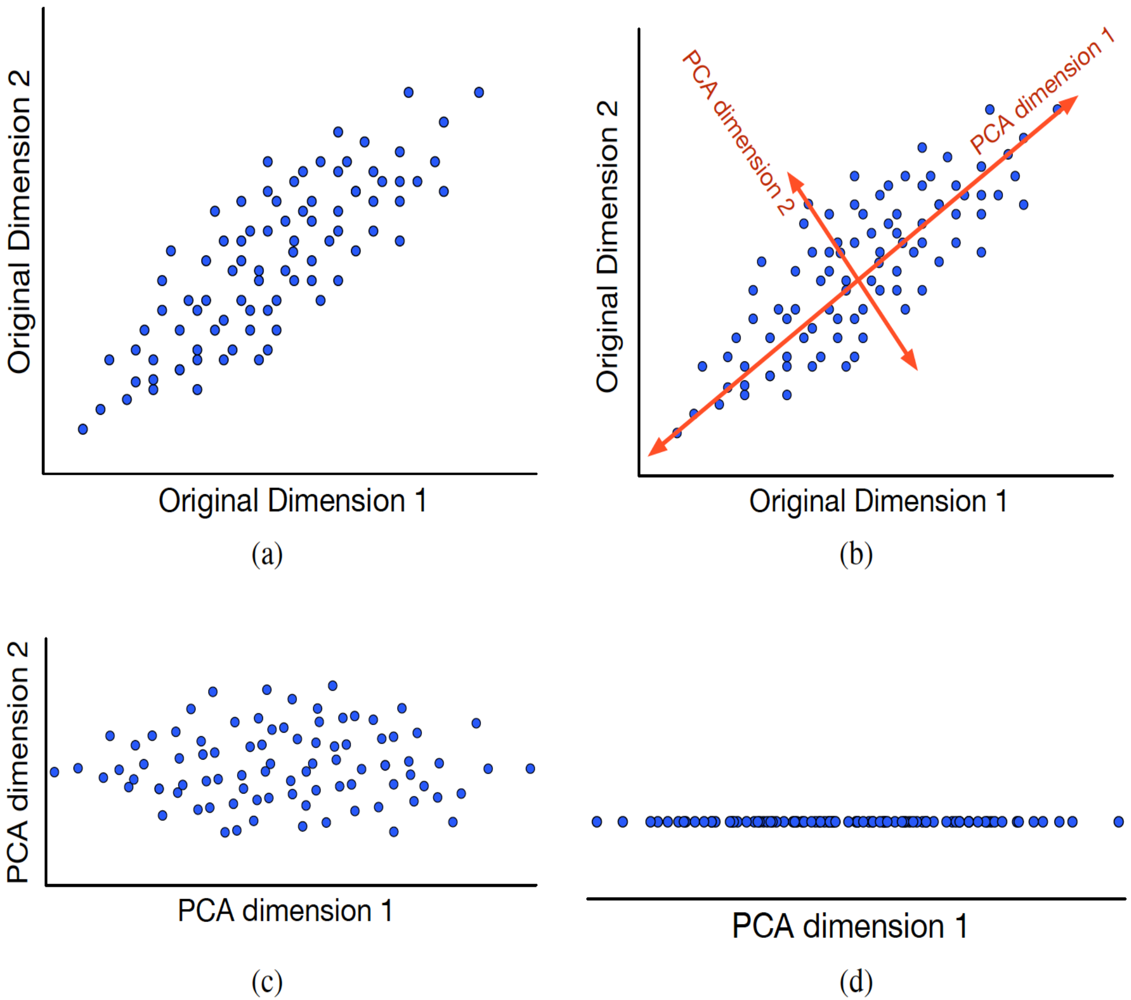
- compare vector similarity against human similarity judgement of raw word pairs (with sentences)
  - Stanford Contextual Word Similarity (SCWS) (2012): 2003 pairs with sentential context
  - semantic textual similarity task (2012)

- analogy task: a is to b as c is to d; given a, b, and c: find d.
  example: *Athens, Greece, Oslo* → *Norway*
  large datasets exist (2013)

Appendix to this slide set: Latent Semantic Indexing and Embeddings via SVD (see [5])

Repetition from ML1: Principal Component Analysis

- for $N \times D$ pattern matrix X (here e.g. a $|V| \times |V|$ word-word-matrix) compute $D \times D$ covariance matrix $S = \frac{1}{N}(X^T X - \bar{x}\,\bar{x}^T)$
- compute Eigendecomposition of S (e.g. with van Mises power iteration): $S = \Gamma \Lambda \Gamma^T = \sum_{i=1}^{D} \lambda_i \, \gamma_i \gamma_i^T$ where $S\gamma_i = \lambda_i \gamma_i$
- restrict $D \times D$ Γ to $D \times k$ $\tilde{\Gamma}$ with only the k eigenvectors as columns that correspond k largest eigenvalues λ_i
- project pattern matrix: $X' = X\,\tilde{\Gamma}$



Repetition from ML1: Singular Value Decomposition

- Decompose $X = U\,\Sigma\,V^T$ with dimensions $N{\times}D = N{\times}N \;\; N{\times}D \;\; D{\times}D$ and
  - $U^T U = I_N$ (columns of U are left singular vectors),
  - $V^T V = V V^T = I_D$ (columns of V are right singular vectors)
  - Σ diagonal matrix with at most $\min(N, D)$ many singular values $\sigma_i > 0$ on main diagonal and 0 elsewhere
  - relation to eigen-decomposition: assuming $N \geq D$: $X^T X = V\,\Sigma^T U^T U\,\Sigma\,V^T$ = $V\,\Sigma^T \Sigma\,V^T$ = $V\;diag(\sigma_1^2, \ldots \sigma_D^2)\;V^T$, so $X^T X v_i = \sigma_i^2 v_i$ and $\Gamma = V$
  - assuming $N \geq D$: Truncated SVD: restrict
    - $N{\times}N$ matrix U to $N{\times}(N-k)$ matrix $\tilde{U}$ and
    - $N{\times}D$ diag. matrix Σ to $(N-k){\times}(D-k)$ diag. matrix $\tilde{\Sigma}$ and
    - $D{\times}D$ matrix V to $(D-k){\times}D$ matrix $\tilde{V}$ and

    dropping the k smallest singular values and their associated singular vectors, effectively producing a low rank approximation $\sum_{i=1}^{D-k} \sigma_i\, u_i v_i^T$ of X
  - project $\mathrm{X}' = \mathrm{X}\,\tilde{V}$

Repetition from ML1: Singular Value Decomposition

- example [2]: 7x5 user-movie-rating matrix (rank 3 → columns of U corresponding to zero singular values and respective columns of Σ and rows of V^T are dropped remark due to X having rank = 3 this goes beyond economy-sized SVD here)

$$
\begin{array}{ccccc}
\text{Matrix} & \text{Alien} & \text{Serenity} & \text{Casablanca} & \text{Amelie}
\end{array}
\qquad
\begin{array}{ccc}
\text{SciFi-concept} & \text{Romance-concept} & \\
\end{array}
$$

$$
\underbrace{\begin{bmatrix}
\mathbf{1} & \mathbf{1} & \mathbf{1} & 0 & 0 \\
\mathbf{3} & \mathbf{3} & \mathbf{3} & 0 & 0 \\
\mathbf{4} & \mathbf{4} & \mathbf{4} & 0 & 0 \\
\mathbf{5} & \mathbf{5} & \mathbf{5} & 0 & 0 \\
0 & \mathbf{2} & 0 & \mathbf{4} & \mathbf{4} \\
0 & 0 & 0 & \mathbf{5} & \mathbf{5} \\
0 & \mathbf{1} & 0 & \mathbf{2} & \mathbf{2}
\end{bmatrix}}_{X}
=
\underbrace{\begin{bmatrix}
\mathbf{0.13} & -0.02 & -0.01 \\
\mathbf{0.41} & -0.07 & -0.03 \\
\mathbf{0.55} & -0.09 & -0.04 \\
\mathbf{0.68} & -0.11 & -0.05 \\
0.15 & \mathbf{0.59} & \mathbf{0.65} \\
0.07 & \mathbf{0.73} & \mathbf{-0.67} \\
0.07 & \mathbf{0.29} & \mathbf{0.32}
\end{bmatrix}}_{U}
\times
\underbrace{\begin{bmatrix}
\mathbf{12.4} & 0 & 0 \\
0 & \mathbf{9.5} & 0 \\
0 & 0 & \mathbf{1.3}
\end{bmatrix}}_{\Sigma}
\times
\underbrace{\begin{bmatrix}
\mathbf{0.56} & \mathbf{0.59} & \mathbf{0.56} & 0.09 & 0.09 \\
-0.12 & 0.02 & -0.12 & \mathbf{0.69} & \mathbf{0.69} \\
0.40 & \mathbf{-0.80} & 0.40 & 0.09 & 0.09
\end{bmatrix}}_{V^T}
$$

[2]

X	U	Σ	V^T
user movie rating matrix	user to movie genre similarity matrix	strength of movie genre	movie genre to movie similarity matrix

possible: drop third column of U and Σ and third row of V^T → low rank (rank = 2) approximation of X

Latent Semantic Analysis / Latent Semantic Indexing

Apply SVD to $|V|\times c$ word-document (co-)occurrence matrix X (e.g. using tf-idf or other features). Let rank(X) = m → drop c-m columns of U and rows of V^T corresponding to zero singular values:

$$\underbrace{\begin{bmatrix} \\ \\ & X & \\ \\ \\ \end{bmatrix}}_{|V|\times c} = \underbrace{\begin{bmatrix} \\ \\ & W & \\ \\ \\ \end{bmatrix}}_{|V|\times m} \underbrace{\begin{bmatrix} \sigma_1 & 0 & 0 & \dots & 0 \\ 0 & \sigma_2 & 0 & \dots & 0 \\ 0 & 0 & \sigma_3 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \dots & \sigma_m \end{bmatrix}}_{m\times m} \underbrace{\begin{bmatrix} \\ & C & \\ \\ \end{bmatrix}}_{m\times c}$$

SVD notation in Jurafsky:
$X = W\,\Sigma\,C^T$

Taking only the top $k, k \leq m$ dimensions after the SVD is applied to the co-occurrence matrix X:

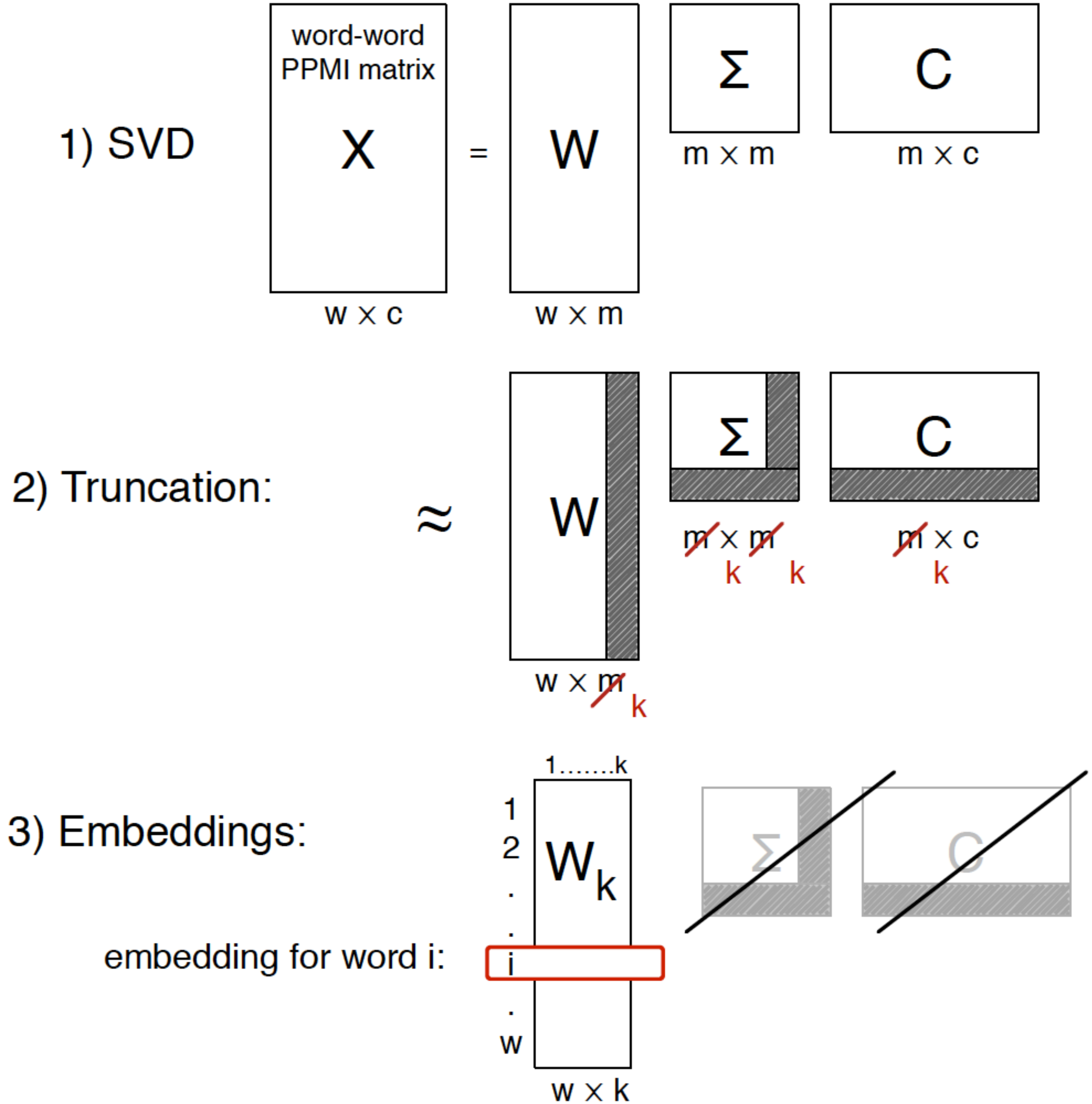
$$\underbrace{\begin{bmatrix} \\ \\ & X & \\ \\ \\ \end{bmatrix}}_{|V|\times c} = \underbrace{\begin{bmatrix} \\ \\ & W_k & \\ \\ \\ \end{bmatrix}}_{|V|\times k} \underbrace{\begin{bmatrix} \sigma_1 & 0 & 0 & \dots & 0 \\ 0 & \sigma_2 & 0 & \dots & 0 \\ 0 & 0 & \sigma_3 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \dots & \sigma_k \end{bmatrix}}_{k\times k} \underbrace{\begin{bmatrix} & C & \end{bmatrix}}_{k\times c}$$

k or m many document genres / document classes

SVD on Word-Word-Co-Occurrence Matrices

- SVD of a PPMI-weighted word-word matrix

- → c is larger than for word-doc case → use k = 500…5000



remark: for some applications the less “compact” more noisy word-vectors / embeddings (rows) of the raw PPMI matrix X may even perform better

Bibliography

(1) Dan Jurafsky and James Martin: Speech and Language Processing (3rd ed. draft, version Oct 2019); Online: https://web.stanford.edu/~jurafsky/slp3/ (URL, Oct 2019) (this slide set is especially based on chapter6)

(2) Günnemann, Groh, Bojchevski, Shchur: Machine Learning 1, TUM-IN2064, WS 2017/18: Lecture + Tutorial Material on Dimensionality Reduction (2018)

(3) Dan Jurafsky and James Martin: Speech and Language Processing (3rd ed. draft, version Oct. 2019); Online: https://web.stanford.edu/~jurafsky/slp3/ (URL, Oct 2019): chapter 19

(4) Dan Jurafsky and James Martin: Speech and Language Processing (3rd ed. draft, version Aug 2017), section 16.1. “Dense Vectors with SVD”

(5) Murphy: Machine Learning, MIT Press (2012), Chapter 12, esp. Section 12.2.3

(6) Mikolov et al.: "Distributed Representations of Words and Phrases and their Compositionality." In NIPS2013 , pp. 3111--3119. 2013

Recommendations for Studying

- minimal approach:

  work with the slides and understand their contents! Think beyond instead of merely memorizing the contents

- standard approach:

  minimal approach + read the corresponding pages in Jurafsky [1]

- interested students

  == standard approach