

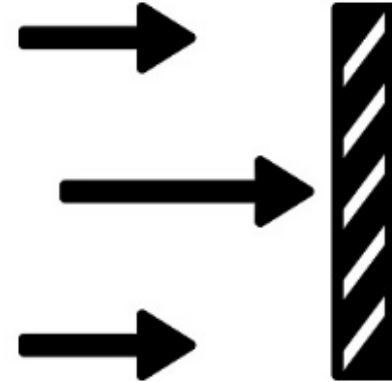


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CRDT – CONFLICT-FREE REPLICATED DATA TYPES

CRDTs Units

- Eventual consistency, informally
- State-based objects
- Eventual consistency, more formally
- Conflict-free replicated data types



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EVENTUAL CONSISTENCY, INFORMALLY

Eventual Consistency

- Eventual consistency is desirable for **large-scale distributed systems** where **high availability** is important
- Tends to be cheap to implement (e.g., via gossip) but may serve stale data
- Constitutes a **challenge** for environments where **stronger consistency is important**

Handling Concurrent Writes

- Premise for eventual consistency **were scenarios with few (no) concurrent writes** to the same key (cf. client-centric consistency)
- However, we do need a mechanism to **handle concurrent writes** should they so happen
- **If there were** a way to **handle concurrent writes**, we could support **eventual consistency** more broadly
- Would “only” need to guarantee that **after processing all writes for a key, all replicas converge, no matter what order** the writes are processed (e.g., assuming gossip)

Examples

Growth-only counter (G-counter)

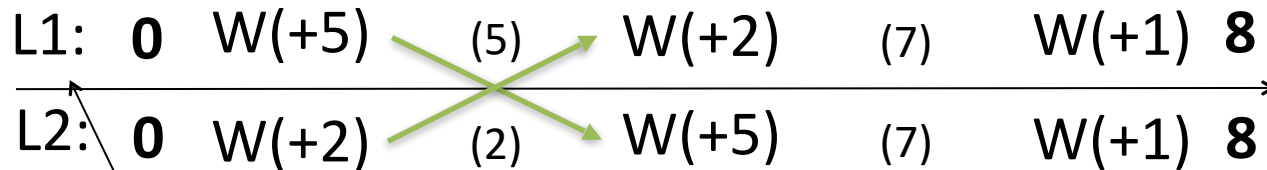
L1:	0	W(+5)	(5)	W(+2)	(7)	W(+1)	8
L2:	0	W(+2)	(2)	W(+5)	(7)	W(+1)	8

The diagram illustrates two replicas, L1 and L2, of a Growth-only counter. A horizontal arrow spans the operations of both replicas. Two arrows point from the text 'Different locations (replicas)' to the initial values '0' in the first column of each row.

Different locations (replicas)

Examples

Growth-only counter (G-counter)



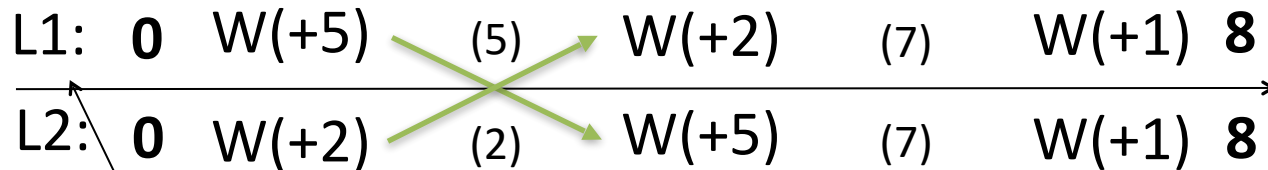
Writes propagate to L2, L1, respectively



Different locations (replicas)

Examples

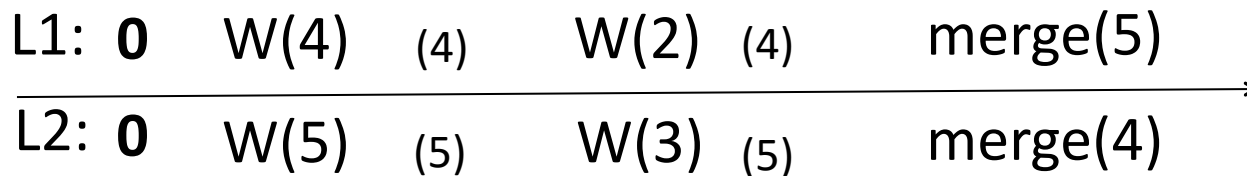
Growth-only counter (G-counter)



Writes propagate to L2, L1, respectively

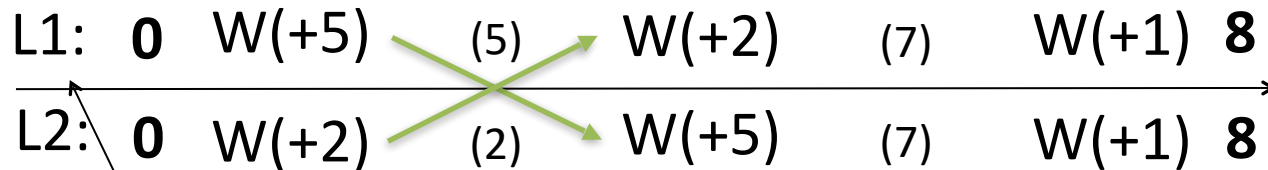
Different locations (replicas)

Max register



Examples

Growth-only counter (G-counter)

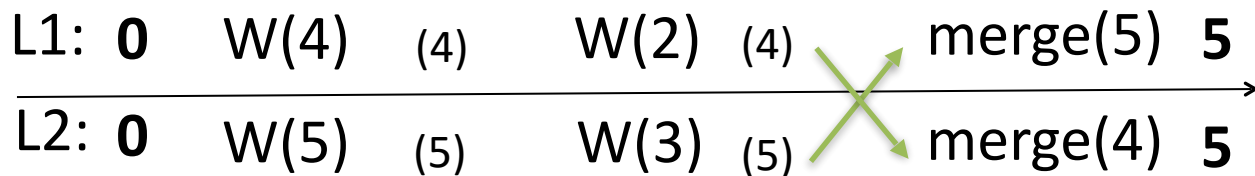


Writes propagate to L2, L1, respectively



Different locations (replicas)

Max register



State propagate to L2, L1 via periodic merging



Self-study Questions

- Think of a few basic data structures, like lists, sets, counters, binary trees, heaps, maps, etc., and visualize for yourself what happens if replicated instances of these structures are updated via gossip.
- Does their state converge, no matter the update sequence?
- What happens if update operations are lost or duplicated?
- What mechanisms we know other than gossip could be used to keep these replicated structures updated without violating their convergence.
- What are pros and cons of these mechanisms?



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CRDT – FROM STATE-BASED OBJECTS TO REPLICATED STATE-BASED OBJECTS

State-based objects

Mostly plain old objects

- Offer update and query requests to clients
- Maintain internal state
- Process client requests
- Perform merge requests amongst each other
- Periodically merge (support infrastructure)

State-based Object

- What we commonly know as object
- Comprised of
 - Internal state
 - One or more `query` methods
 - One or more `update` methods
 - A `merge` method

Class Average

Running Example

```
class Avg(object):  
def __init__(self):  
    self.sum = 0  
    self.cnt = 0
```

```
def query(self):  
    if self.cnt != 0:  
        return  
        self.sum /  
        self.cnt  
else:  
    return 0
```

```
def update(self, x):  
    self.sum += x  
    self.cnt += 1
```

```
def merge(self, avg):  
    self.sum += avg.sum  
    self.cnt += avg.cnt
```

Average

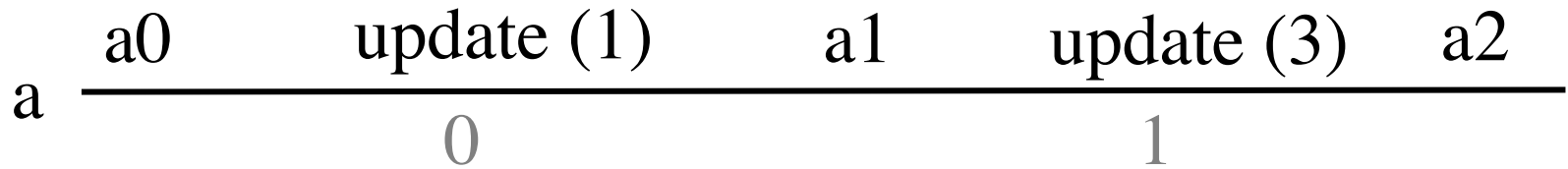
State-based object representing a running average

- Internal state
 - `self.sum` and `self.cnt`
- `Query` returns average
- `Update` updates average with a new value `x`
- `Merge` merges one `Avg` instance into another one

Replicated State-based Object

- State-based object replicated across multiple nodes
- E.g., replicate Avg across two nodes
- Both nodes have a copy of state-based object
- Clients send query and update to a single node
- Nodes periodically send their copy of state-based object to other nodes for merging

Timeline

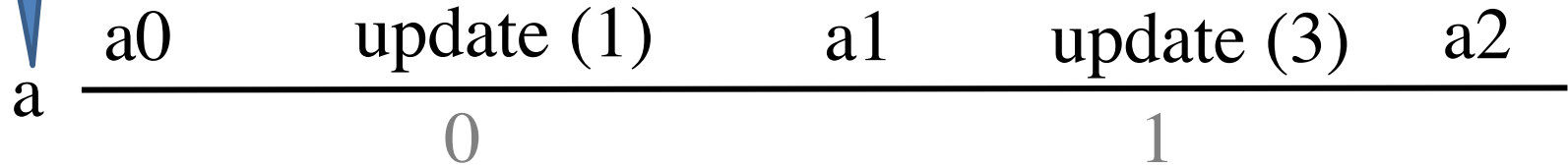


Each state represents a snapshot of object in time that results from updates applied

	state	query ()	history
a0	sum:0, cnt:0	0	{ }
a1	sum:1, cnt:1	1	{0}
a2	sum:4, cnt:2	2	{0,1}

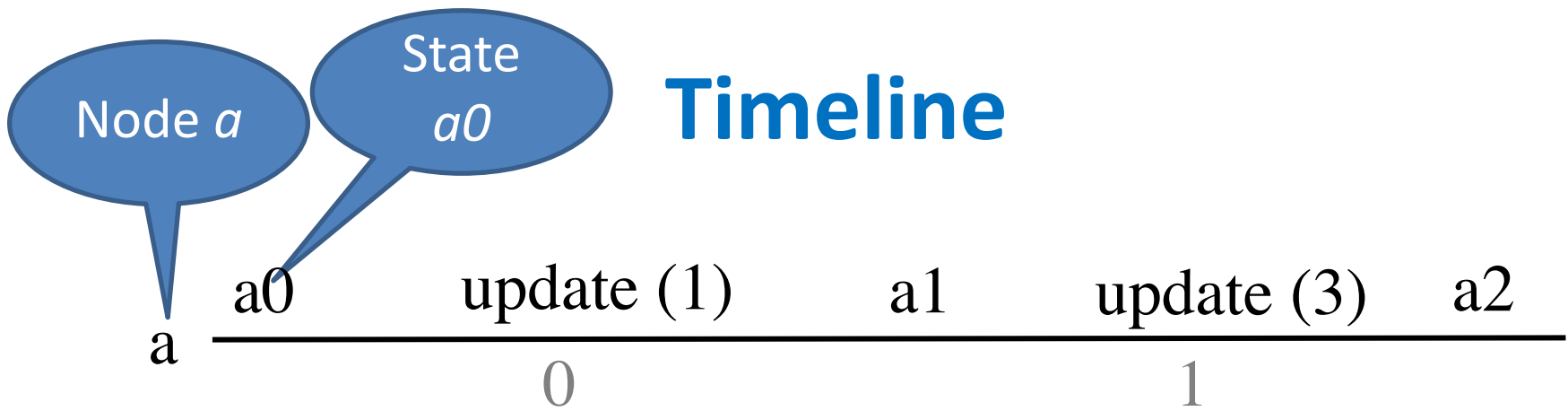
Node *a*

Timeline



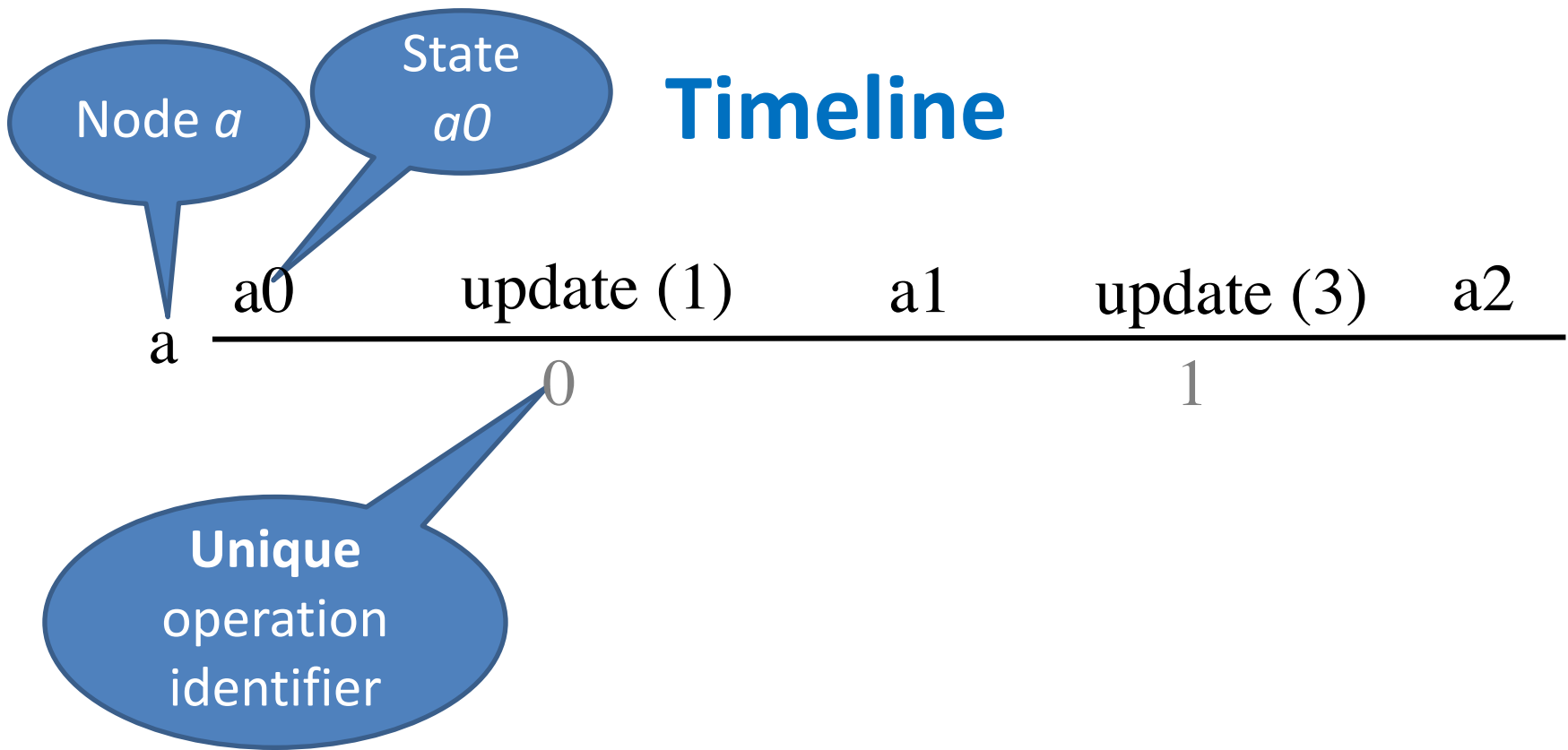
Each state represents a snapshot of object in time that results from updates applied

	state	query ()	history
<i>a0</i>	sum:0, cnt:0	0	{ }
<i>a1</i>	sum:1, cnt:1	1	{0}
<i>a2</i>	sum:4, cnt:2	2	{0,1}



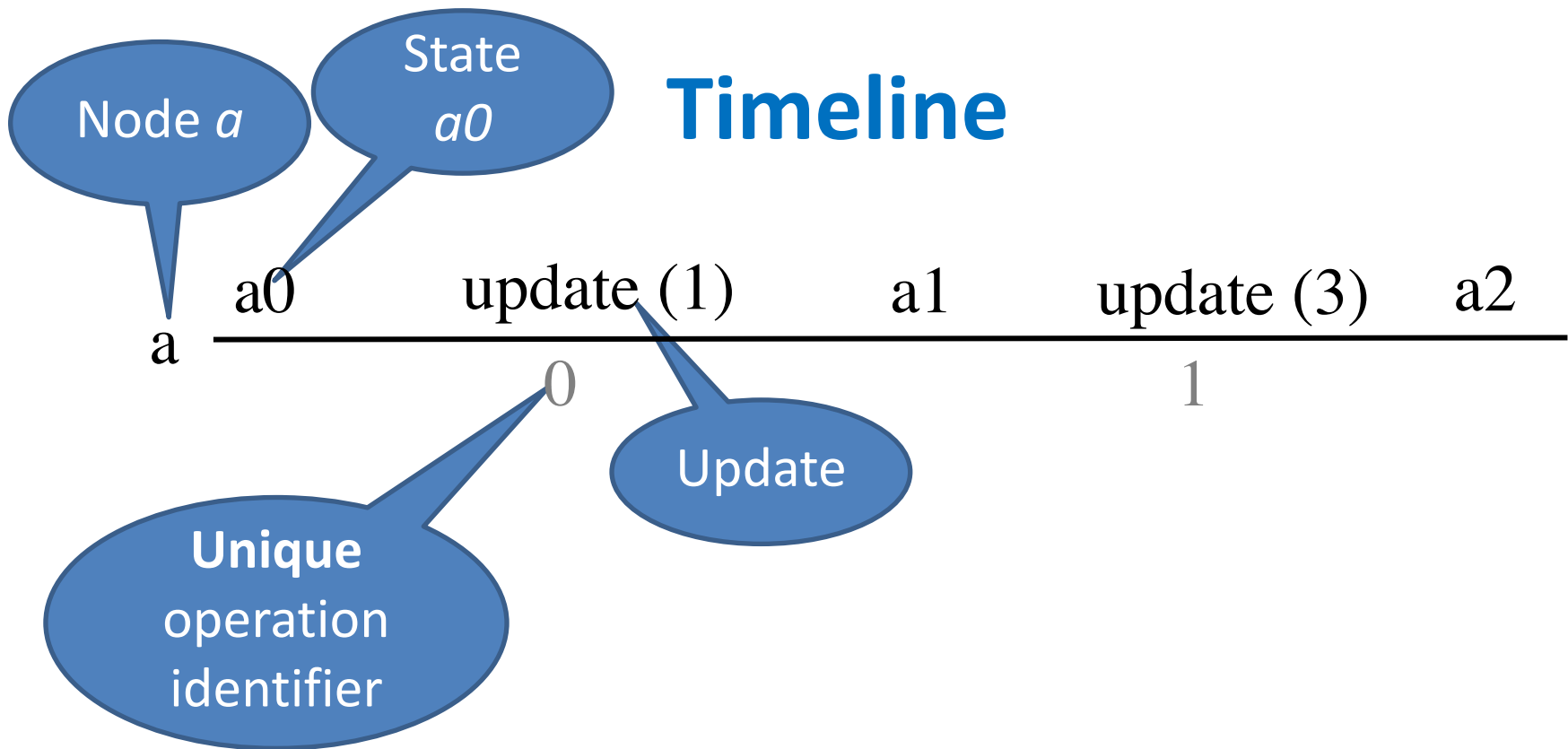
Each state represents a snapshot of object in time that results from updates applied

	state	query ()	history
<i>a0</i>	sum:0, cnt:0	0	{ }
<i>a1</i>	sum:1, cnt:1	1	{0}
<i>a2</i>	sum:4, cnt:2	2	{0,1}



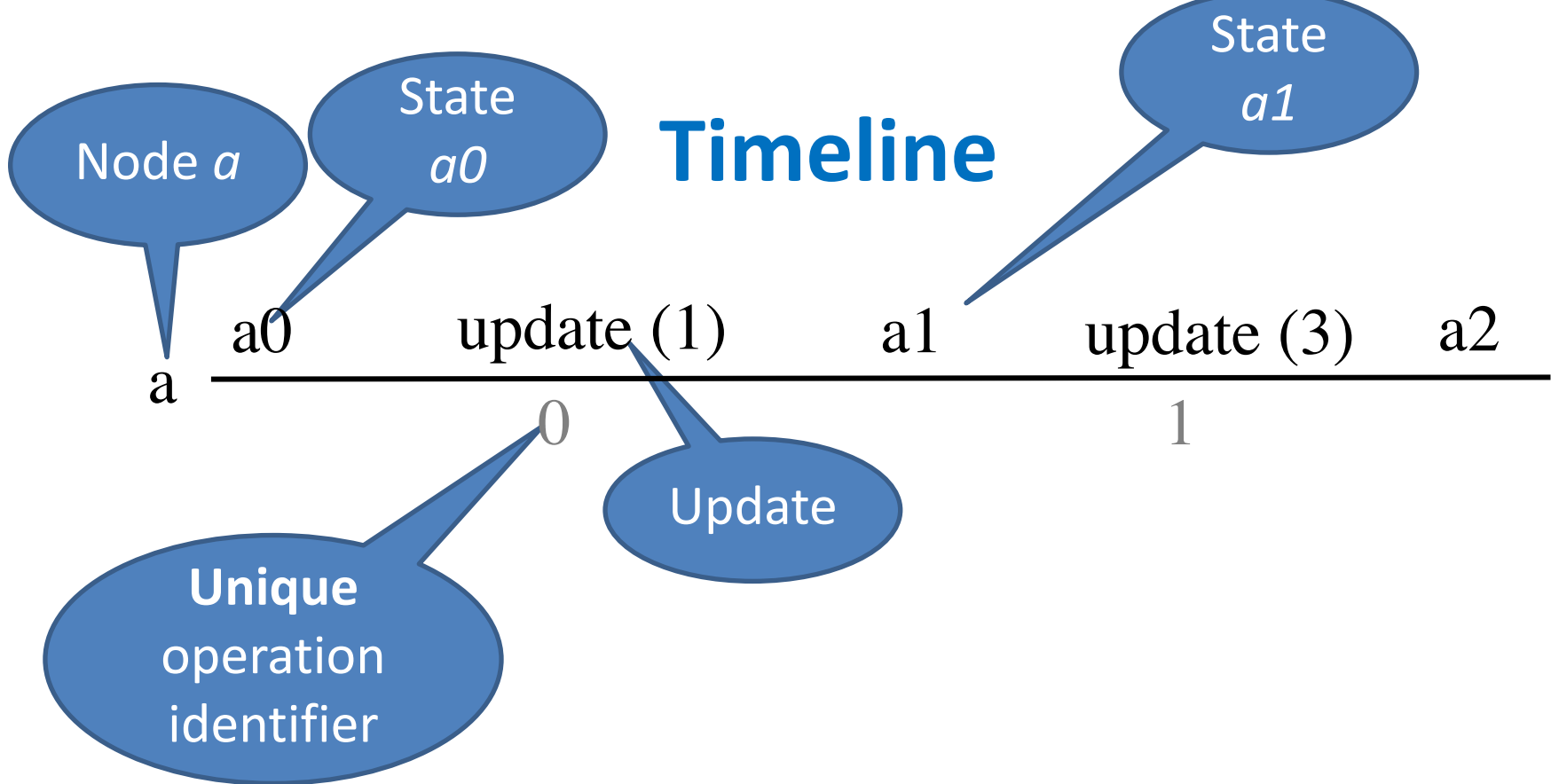
Each state represents a snapshot of object in time that results from updates applied

	state	query ()	history
a0	sum:0, cnt:0	0	{ }
a1	sum:1, cnt:1	1	{0}
a2	sum:4, cnt:2	2	{0,1}



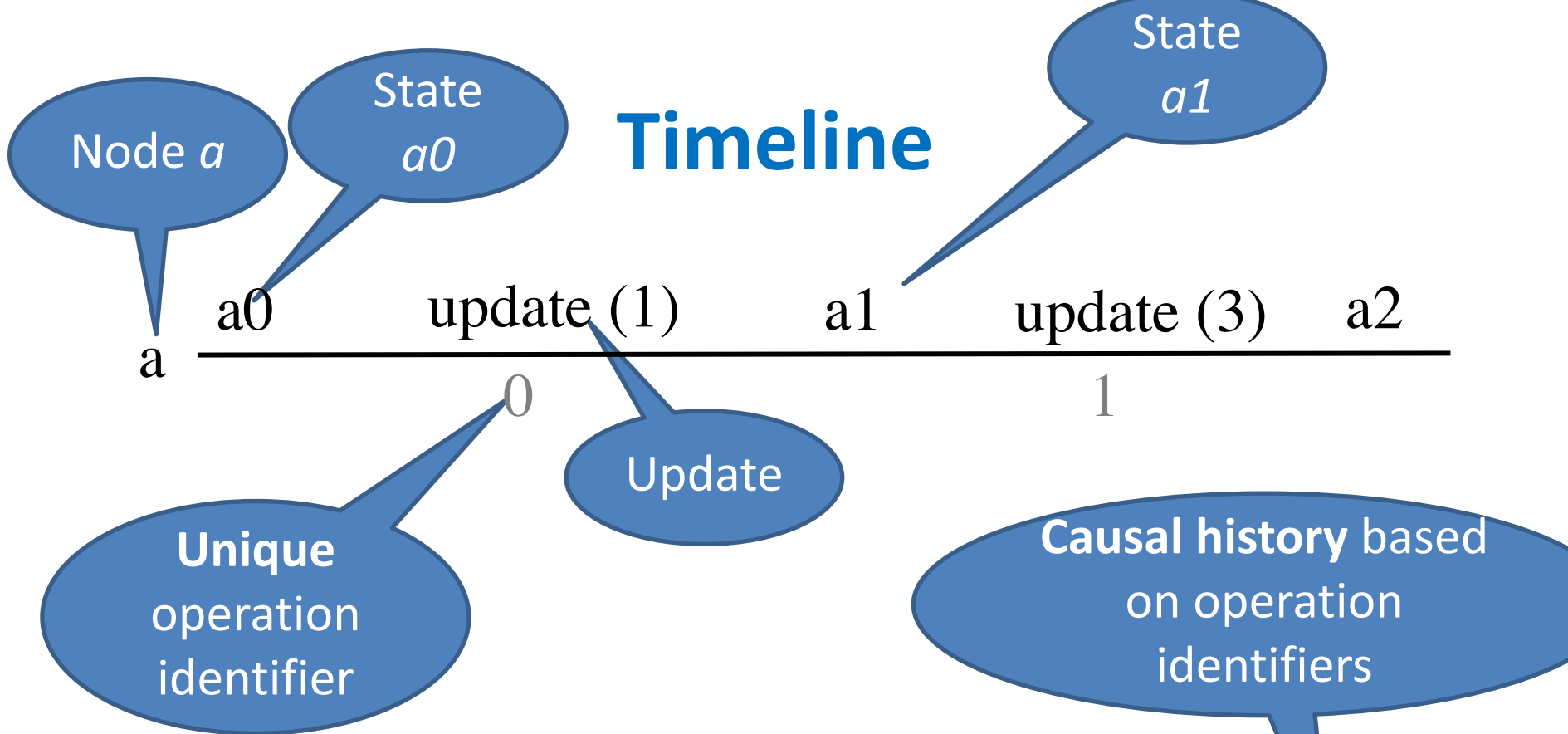
Each state represents a snapshot of object in time that results from updates applied

	state	query ()	history
<i>a0</i>	sum:0, cnt:0	0	{ }
<i>a1</i>	sum:1, cnt:1	1	{0}
<i>a2</i>	sum:4, cnt:2	2	{0,1}



Each state represents a snapshot of object in time that results from updates applied

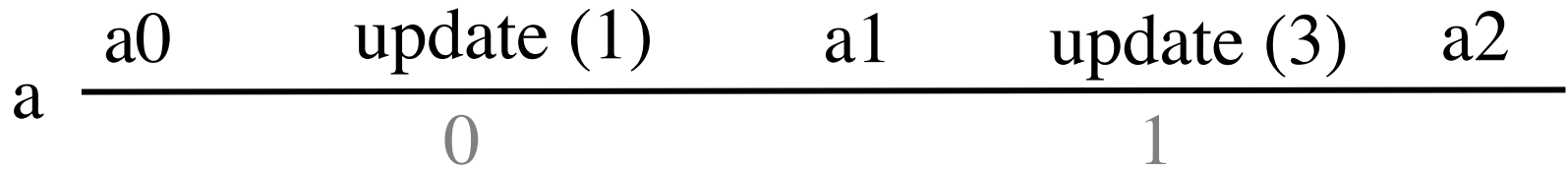
	state	query ()	history
a0	sum:0, cnt:0	0	{ }
a1	sum:1, cnt:1	1	{0}
a2	sum:4, cnt:2	2	{0,1}



Each state represents a snapshot of object in time that results from updates applied

	state	query ()	history
a0	sum:0, cnt:0	0	{ }
a1	sum:1, cnt:1	1	{0}
a2	sum:4, cnt:2	2	{0,1}

Timeline

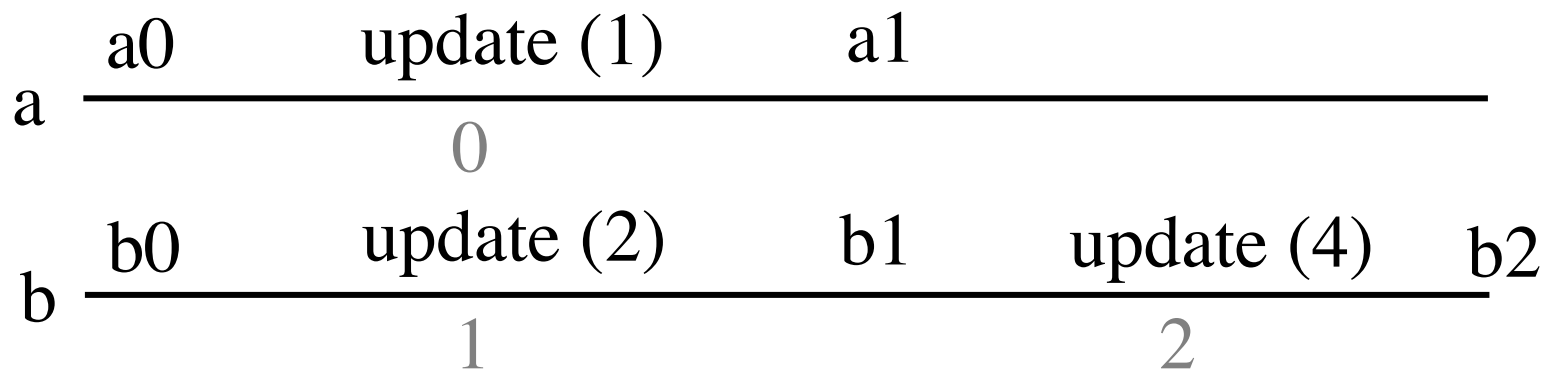


Operation identifier is unique across replicas

Each state represents a snapshot of object in time that results from updates applied

	state	query ()	history
a0	sum:0, cnt:0	0	{ }
a1	sum:1, cnt:1	1	{0}
a2	sum:4, cnt:2	2	{0,1}

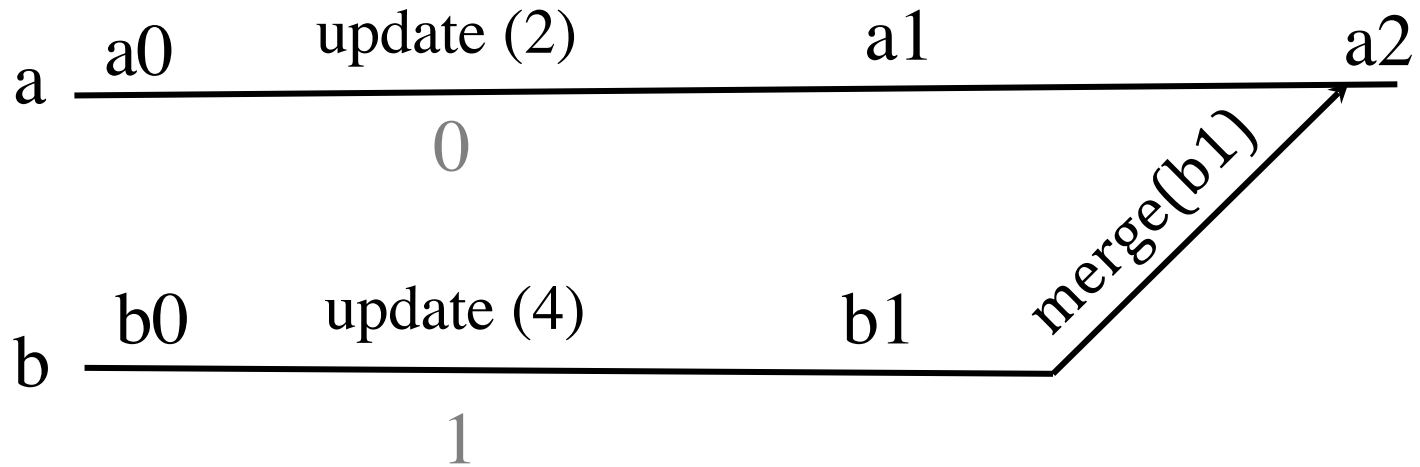
States and Causal Histories



If $y = x.\text{update}(\dots)$ where the update has identifier i , then the causal history of y is the causal history of x union $\{i\}$.

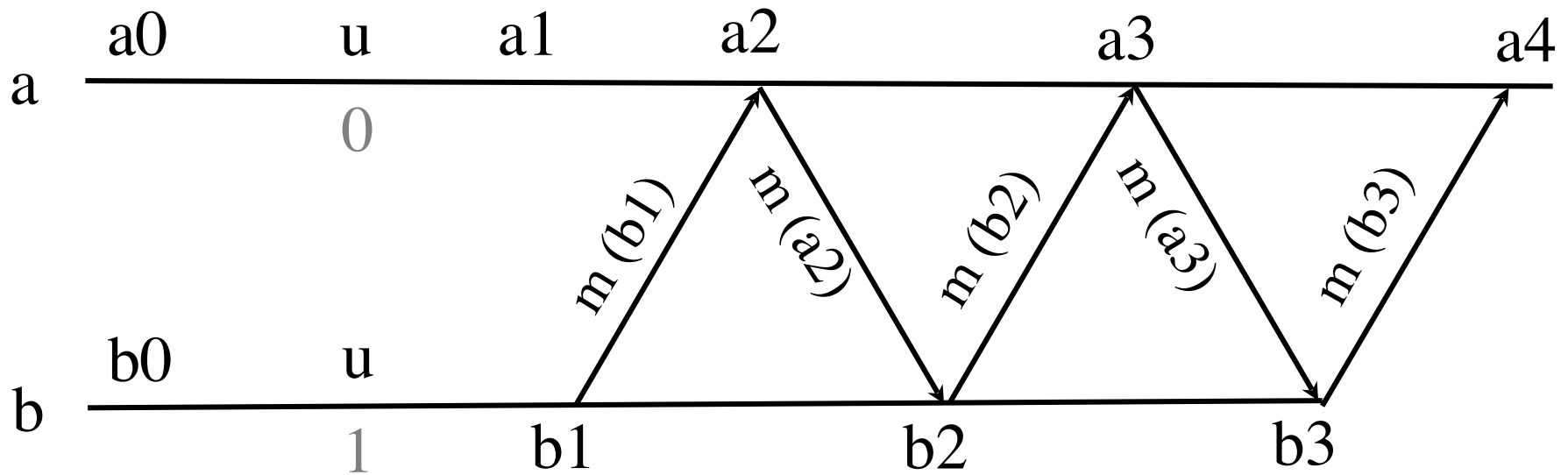
	state	query ()	history
a0	sum:0, cnt:0	0	{ }
a1	sum:1, cnt:1	1	{0}
b0	sum:0, cnt:0	0	{ }
b1	sum:2, cnt:1	2	{1}
b2	sum:6, cnt:2	3	{1,2}

Merge



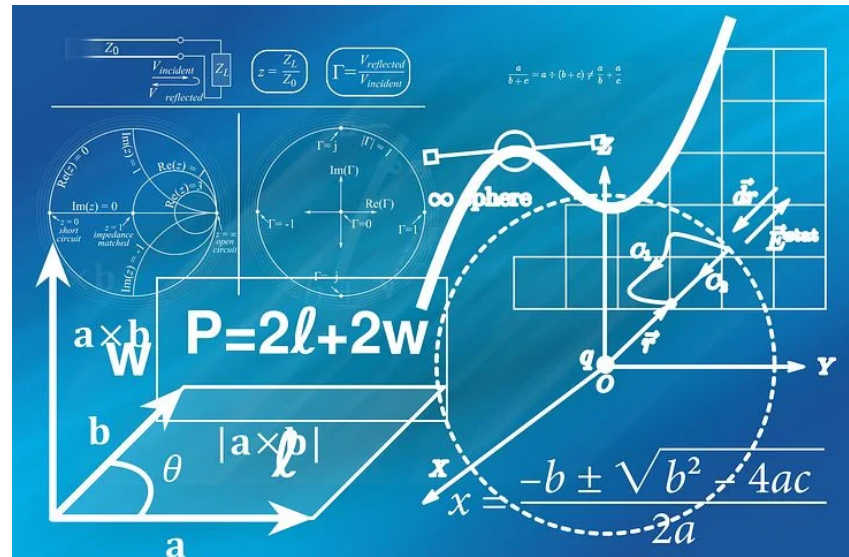
	state	query ()	history
a0	sum:0, cnt:0	0	{ }
a1	sum:2, cnt:1	2	{0}
b0	sum:0, cnt:0	0	{ }
b1	sum:4, cnt:1	4	{1}
a2	sum:6, cnt:2	3	{0,1}

Nodes Periodically Propagate Their State



Self-study Questions

- Think of a few basic data structures, like lists, sets, counters, binary trees, heaps, maps, etc., and visualize for yourself what happens if replicated instances of these structures are updated via gossip.
- For the above data structures, specify merge operations that merge the state of two instances of a given structure.
- Assume merge happens periodically, does your replicated structures' state converge?



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CRDT – EVENTUAL CONSISTENCY, MORE FORMALLY

Eventual Consistency

- A replicated state-based object is
 - **eventually consistent** if whenever two replicas of the state-based object have the same causal history, **they eventually** (not necessarily immediately) converge to the same internal state

Strong Eventual Consistency

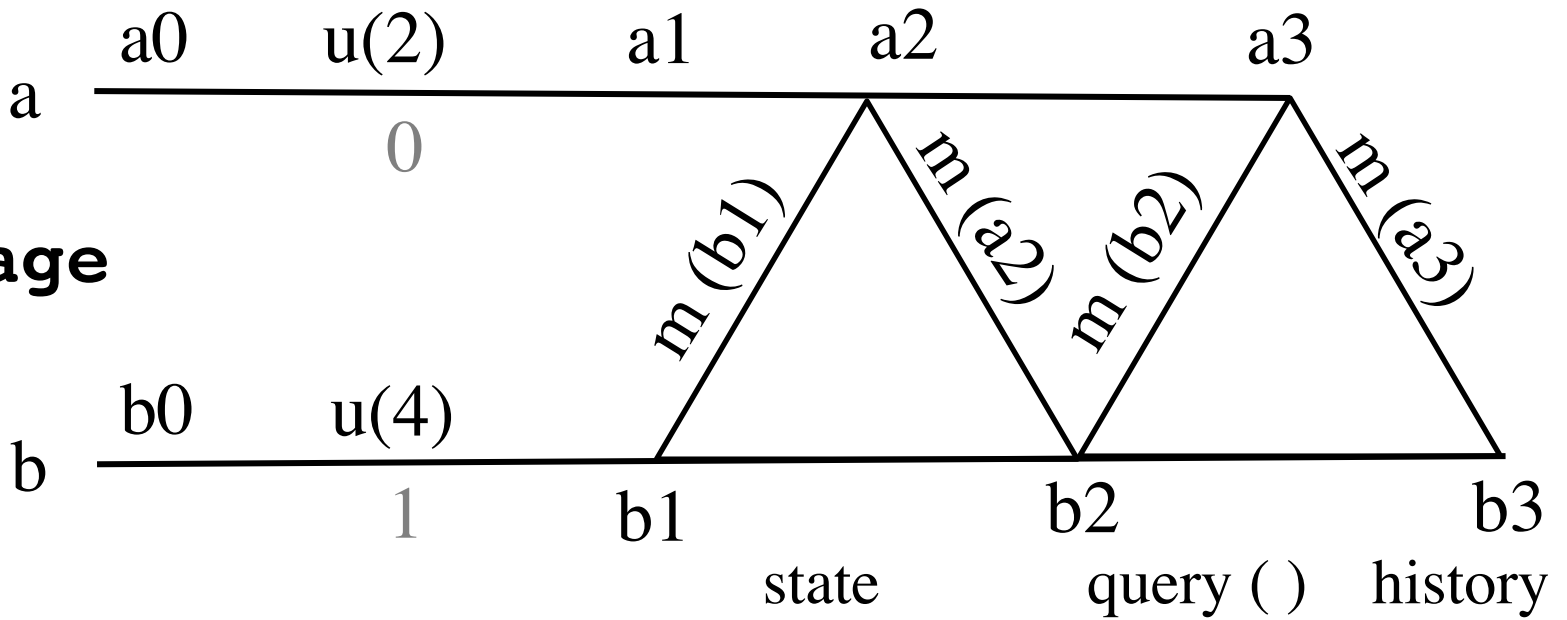
- A replicated state-based object is
 - **strongly eventually consistent** if whenever two replicas of the state-based object have the same causal history, **they** (immediately) **have the same internal state**
- Strong eventual consistency implies eventual consistency

EC or SEC

That is the question?

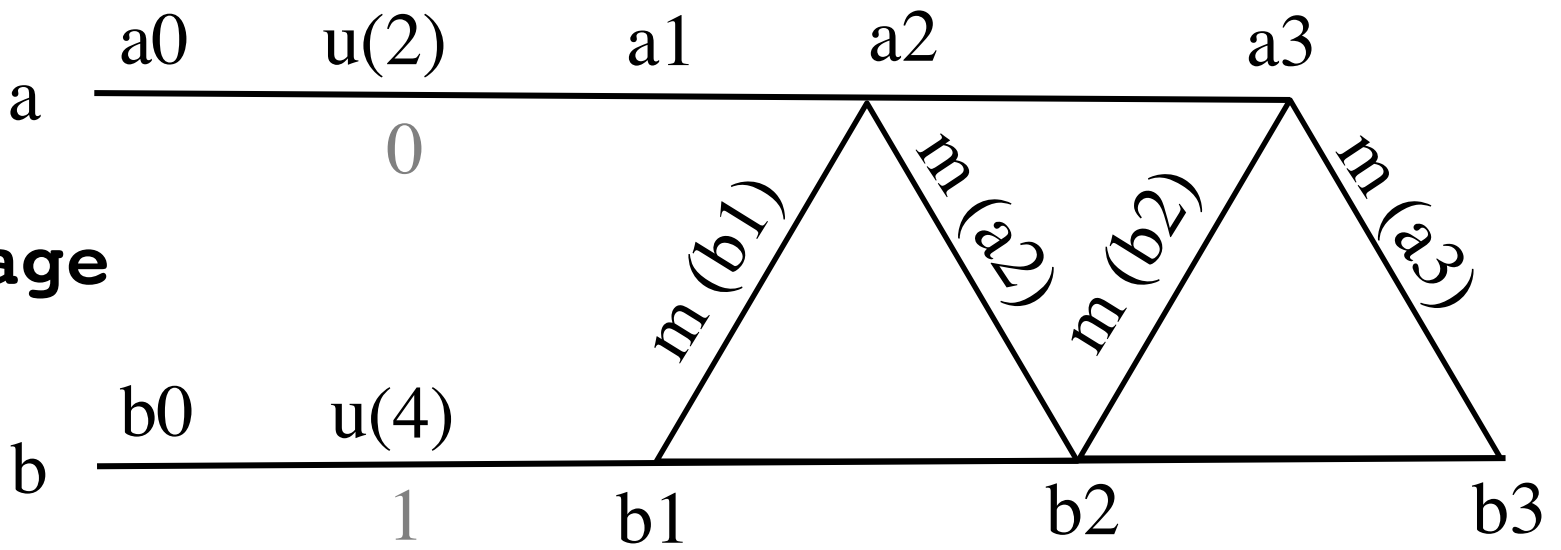
- Variants of our Average object, defined next
 - Average
 - NoMergeAverage
 - BMergeAverage
 - MaxAverage
- Note that some of these objects do not represent realistic functionality (i.e., needed functionality)
- These objects are meant to illustrate convergence concepts only

Average



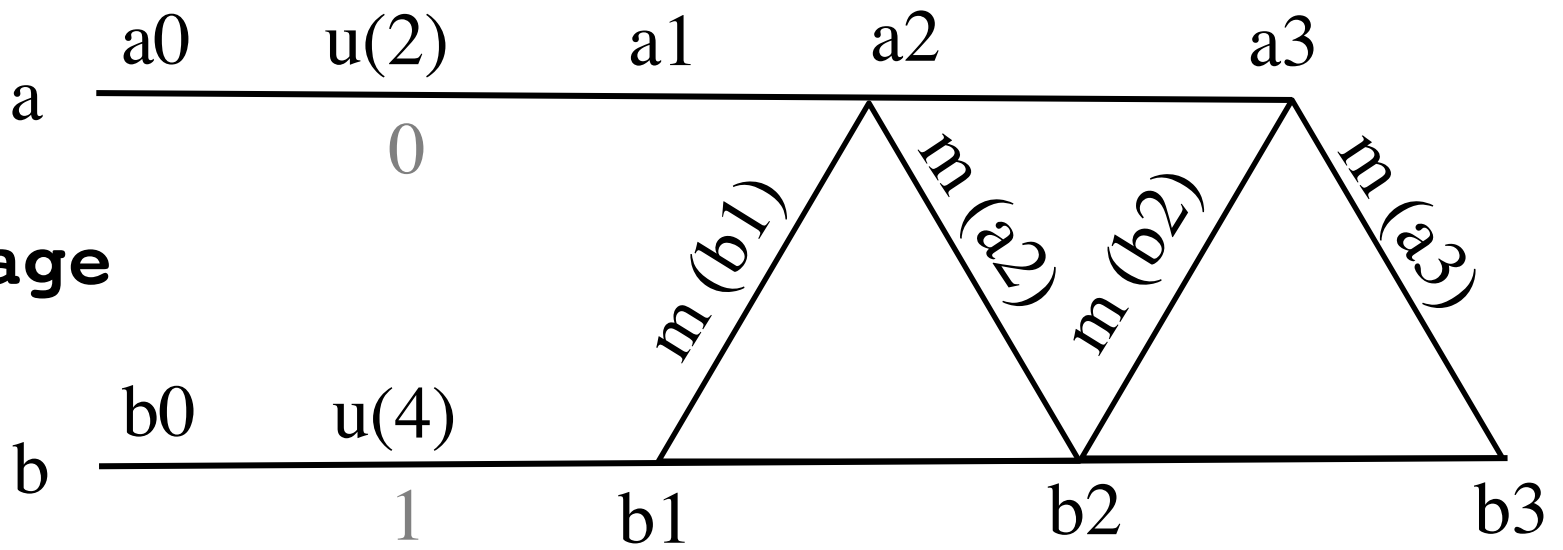
	state	query ()	history
<i>a0</i>			
<i>a1</i>			
<i>a2</i>			
<i>a3</i>			
<i>b0</i>			
<i>b1</i>			
<i>b2</i>			
<i>b3</i>			

Average



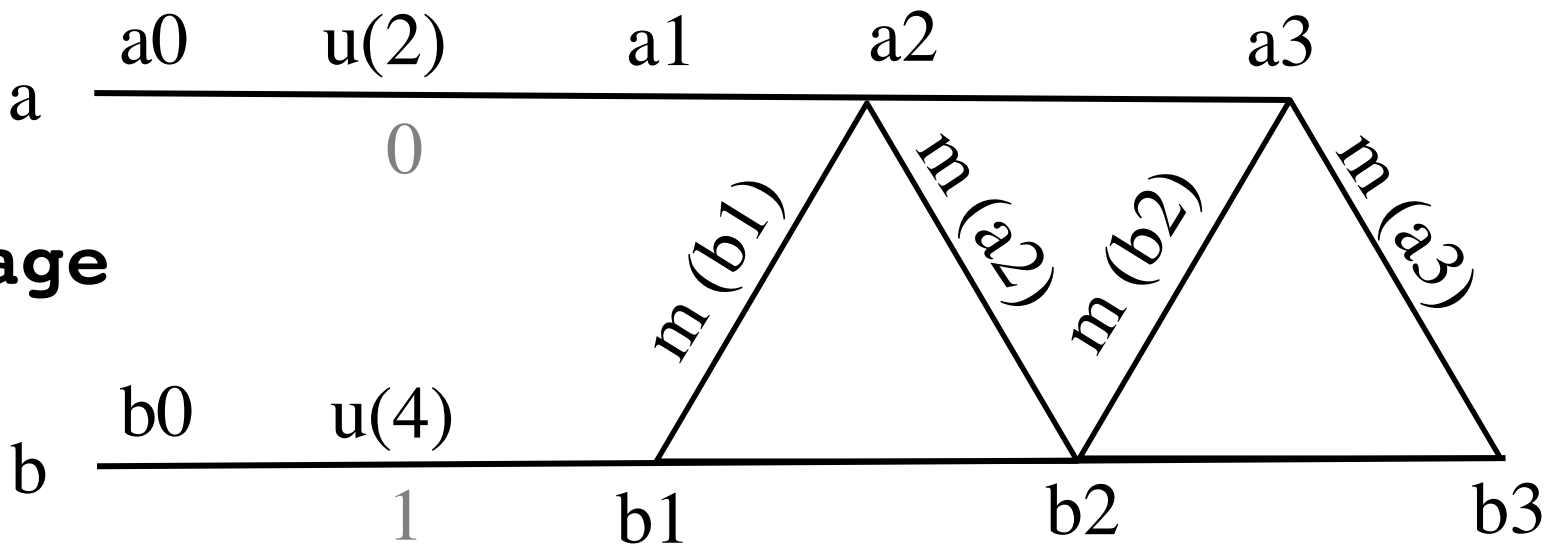
	state	query ()	history
a0	sum:0, cnt:0	0	{ }
a1			
a2			
a3			
b0			
b1			
b2			
b3			

Average



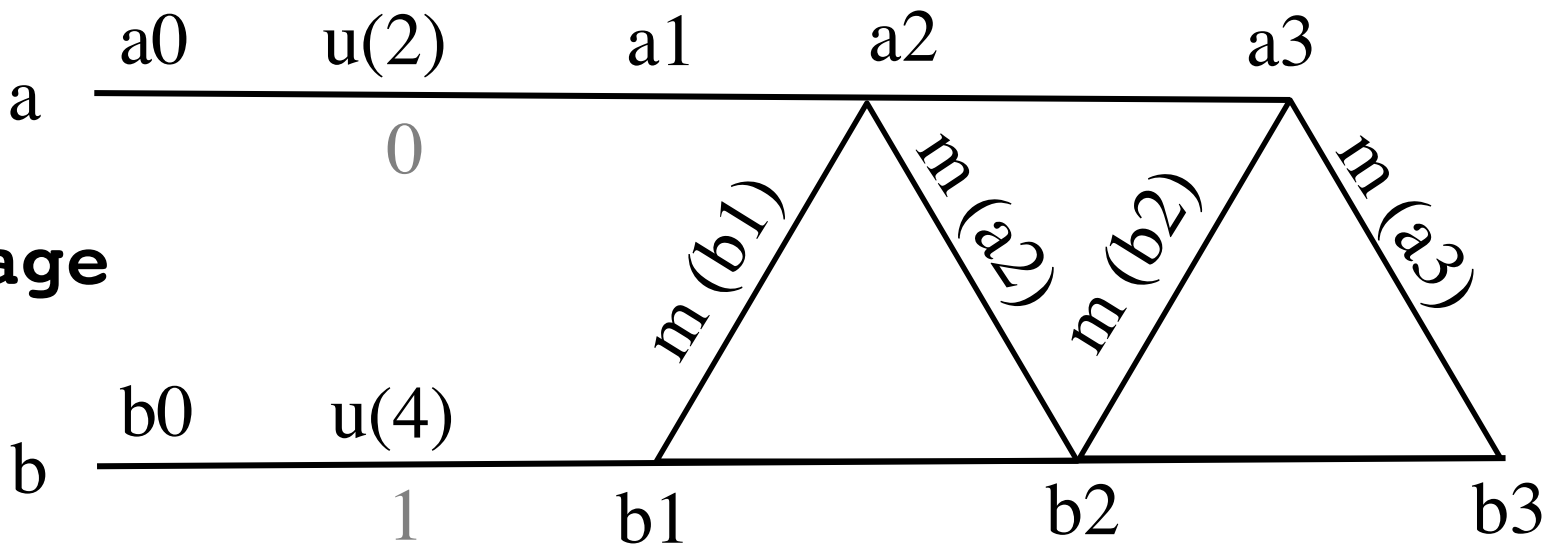
	state	query ()	history
a0	sum:0, cnt:0	0	{ }
a1	sum:2, cnt:1	2	{0}
a2			
a3			
b0			
b1			
b2			
b3			

Average



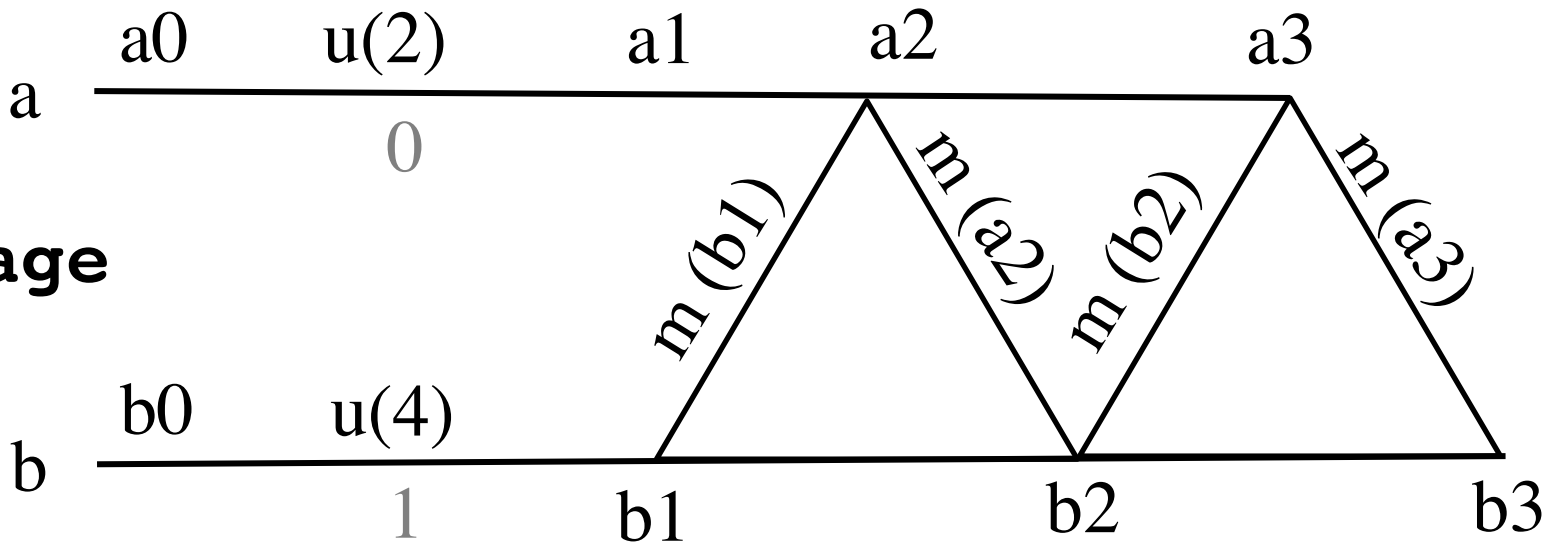
	state	query ()	history
a0	sum:0, cnt:0	0	{ }
a1	sum:2, cnt:1	2	{0}
a2			
a3			
b0	sum:0, cnt:0	0	{ }
b1			
b2			
b3			

Average



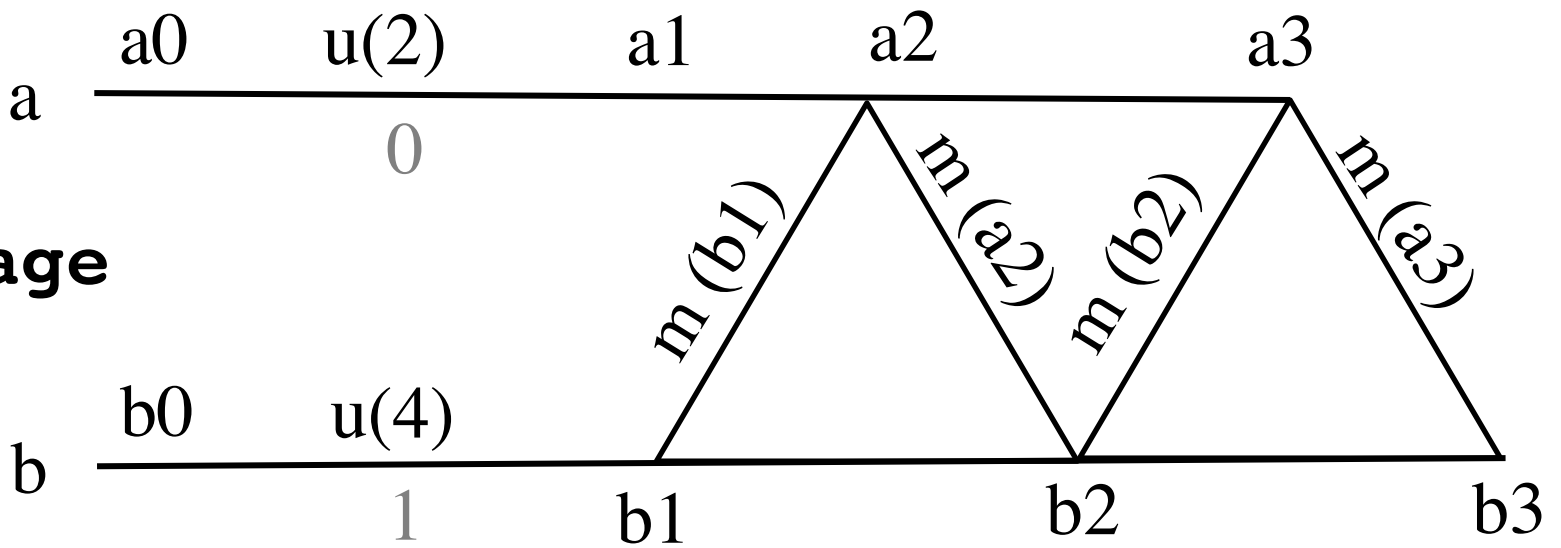
	state	query ()	history
a_0	sum:0, cnt:0	0	{ }
a_1	sum:2, cnt:1	2	{0}
a_2			
a_3			
b_0	sum:0, cnt:0	0	{ }
b_1	sum:4, cnt:1	4	{1}
b_2			
b_3			

Average



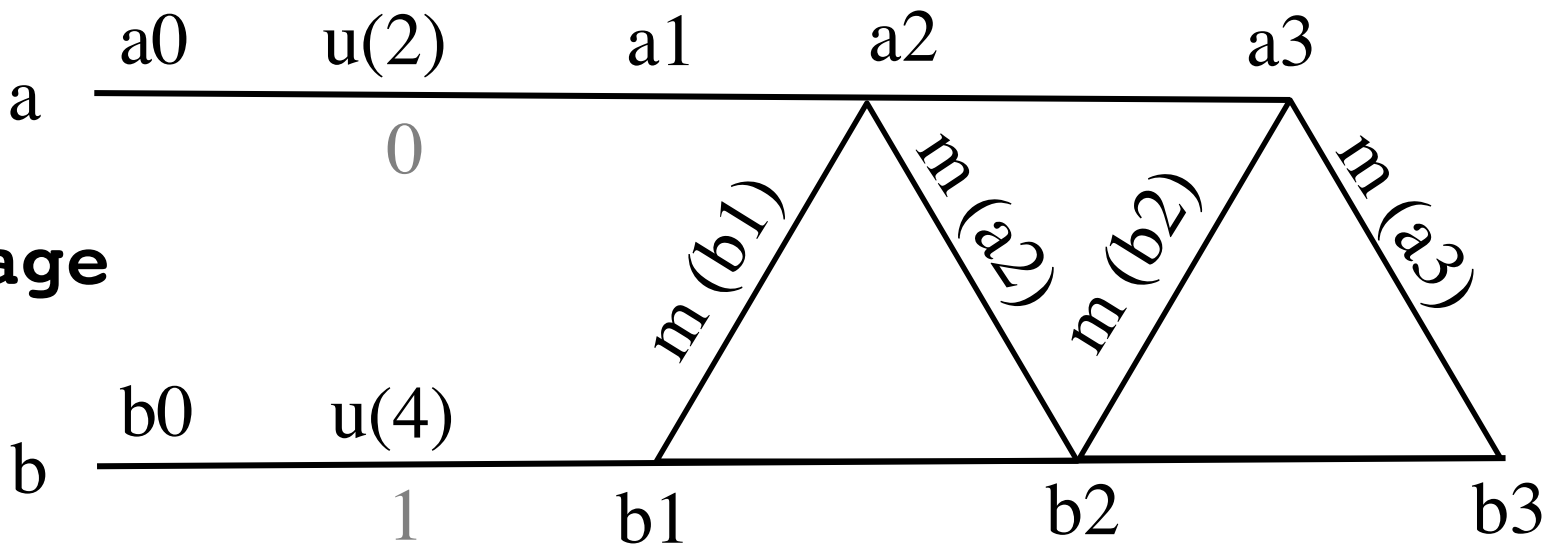
	state	query ()	history
a0	sum:0, cnt:0	0	{ }
a1	sum:2, cnt:1	2	{0}
a2	sum:6, cnt:2	3	{0,1}
a3			
b0	sum:0, cnt:0	0	{ }
b1	sum:4, cnt:1	4	{1}
b2			
b3			

Average



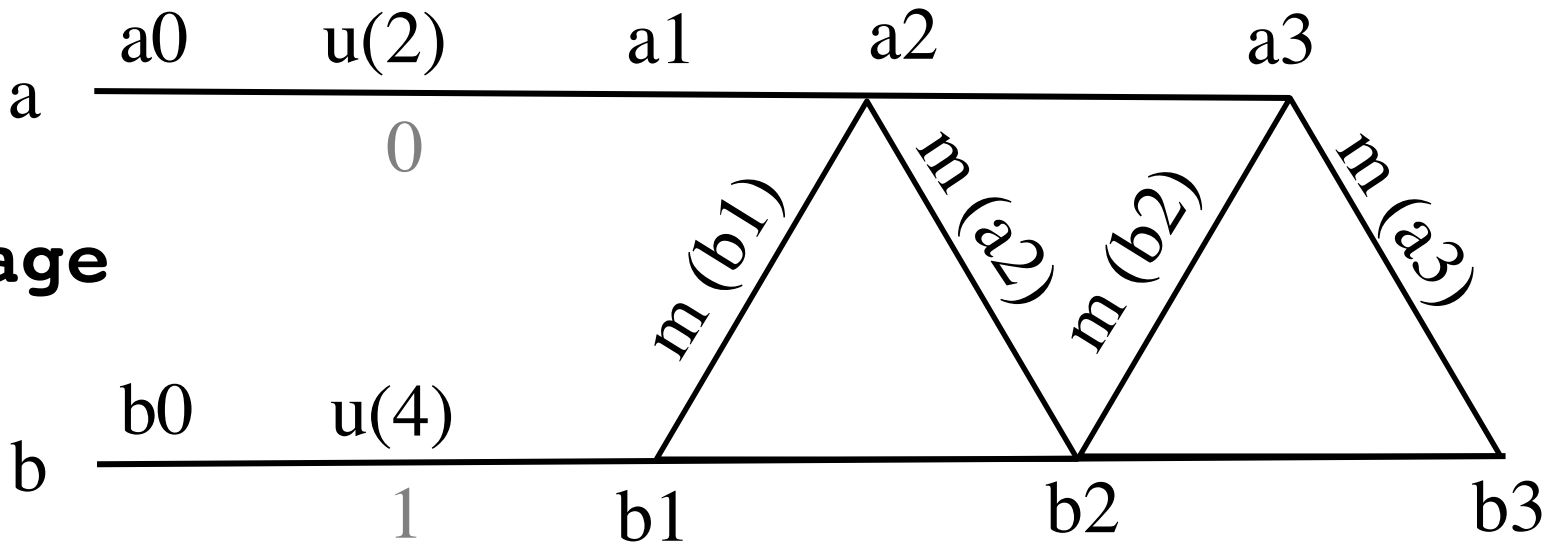
	state	query ()	history
a0	sum:0, cnt:0	0	{ }
a1	sum:2, cnt:1	2	{0}
a2	sum:6, cnt:2	3	{0,1}
a3			
b0	sum:0, cnt:0	0	{ }
b1	sum:4, cnt:1	4	{1}
b2	sum:10, cnt:3	3.3	{0,1}
b3			

Average



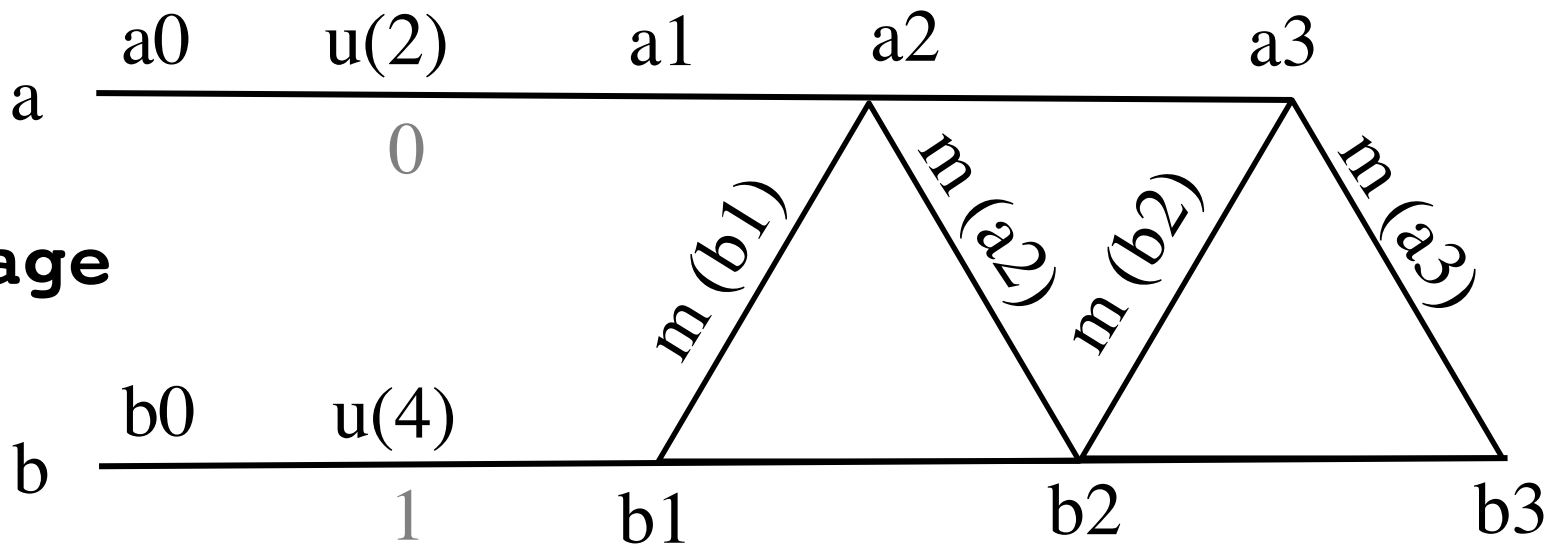
	state	query ()	history
a0	sum:0, cnt:0	0	{ }
a1	sum:2, cnt:1	2	{0}
a2	sum:6, cnt:2	3	{0,1}
a3	sum:16, cnt:5	3.2	{0,1}
b0	sum:0, cnt:0	0	{ }
b1	sum:4, cnt:1	4	{1}
b2	sum:10, cnt:3	3.3	{0,1}
b3			

Average



	state	query ()	history
a0	sum:0, cnt:0	0	{ }
a1	sum:2, cnt:1	2	{0}
a2	sum:6, cnt:2	3	{0,1}
a3	sum:16, cnt:5	3.2	{0,1}
b0	sum:0, cnt:0	0	{ }
b1	sum:4, cnt:1	4	{1}
b2	sum:10, cnt:3	3.3	{0,1}
b3	sum:26, cnt:8	3.25	{0,1}

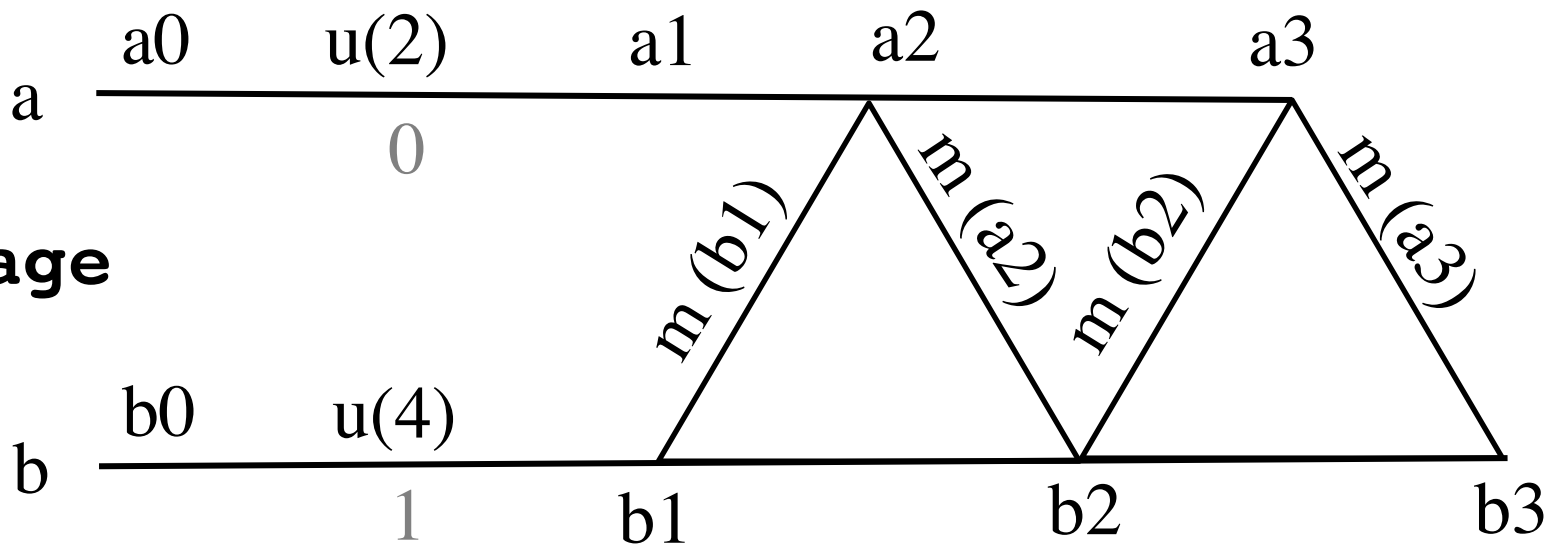
Average



a, b attain the **same causal history** but **do not converge** to the **same internal state** – they **do not converge** at all!

	state	query ()	history
a0	sum:0, cnt:0	0	{ }
a1	sum:2, cnt:1	2	{0}
a2	sum:6, cnt:2	3	{0,1}
a3	sum:16, cnt:5	3.2	{0,1}
b0	sum:0, cnt:0	0	{ }
b1	sum:4, cnt:1	4	{1}
b2	sum:10, cnt:3	3.3	{0,1}
b3	sum:26, cnt:8	3.25	{0,1}

Average



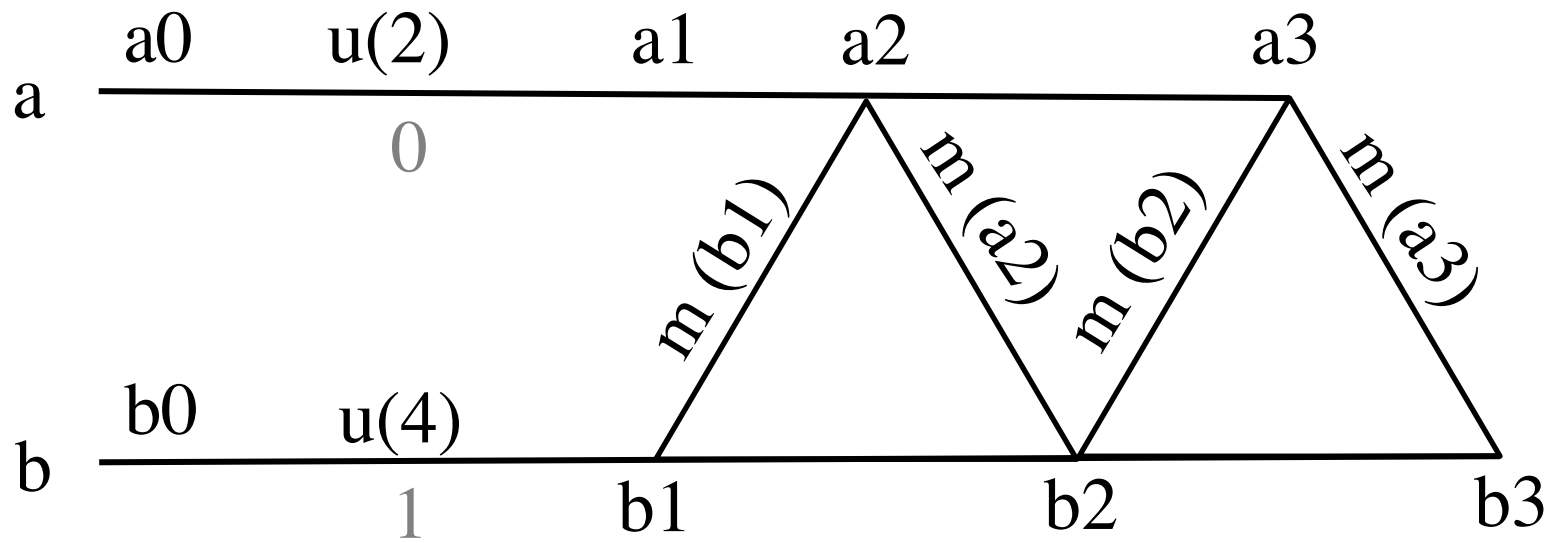
a, *b* attain the **same causal history** but **do not converge** to the **same internal state** – they **do not converge** at all!

Neither eventually consistent, nor strongly eventually consistent

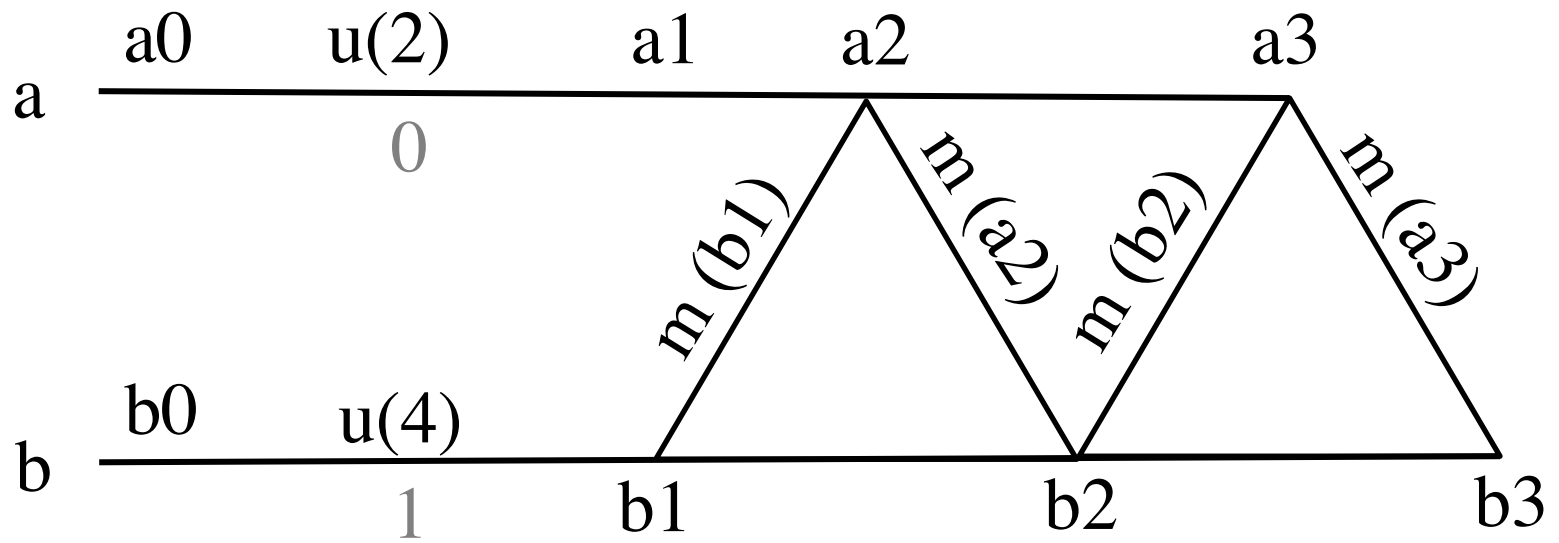
	state	query ()	history
<i>a0</i>	sum:0, cnt:0	0	{ }
<i>a1</i>	sum:2, cnt:1	2	{0}
<i>a2</i>	sum:6, cnt:2	3	{0,1}
<i>a3</i>	sum:16, cnt:5	3.2	{0,1}
<i>b0</i>	sum:0, cnt:0	0	{ }
<i>b1</i>	sum:4, cnt:1	4	{1}
<i>b2</i>	sum:10, cnt:3	3.3	{0,1}
<i>b3</i>	sum:26, cnt:8	3.25	{0,1}

NoMergeAverage

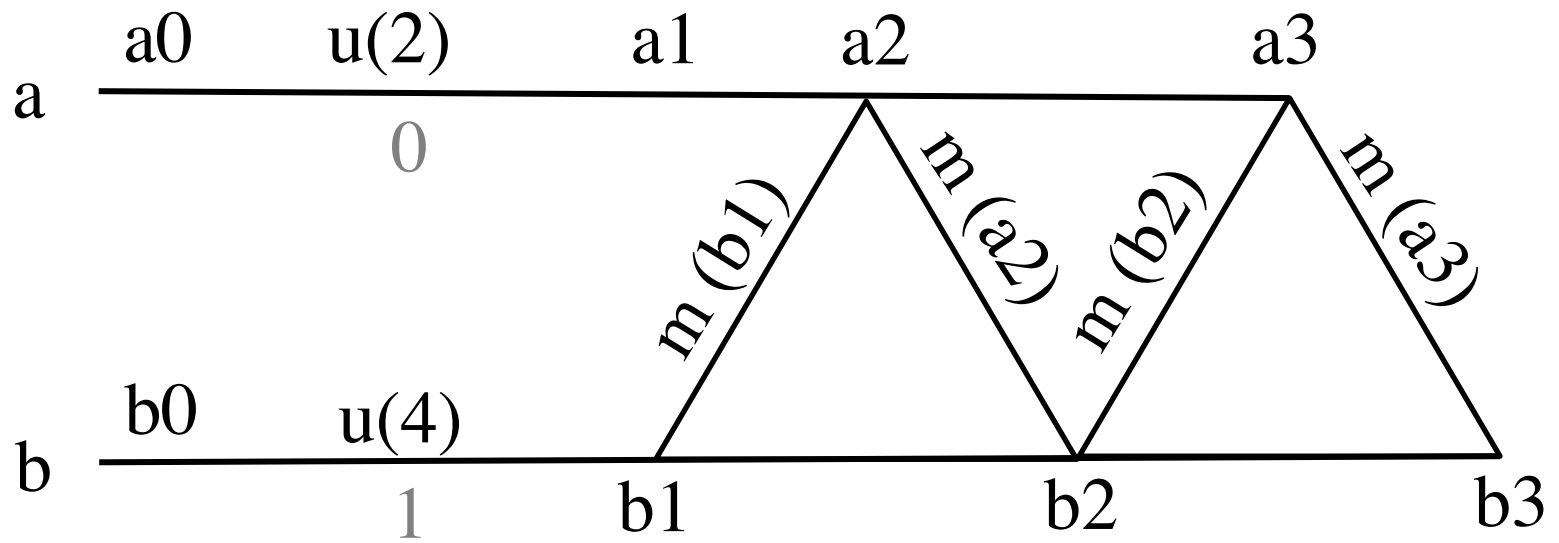
- **Object's merge does nothing**
- All else is the same as for Average



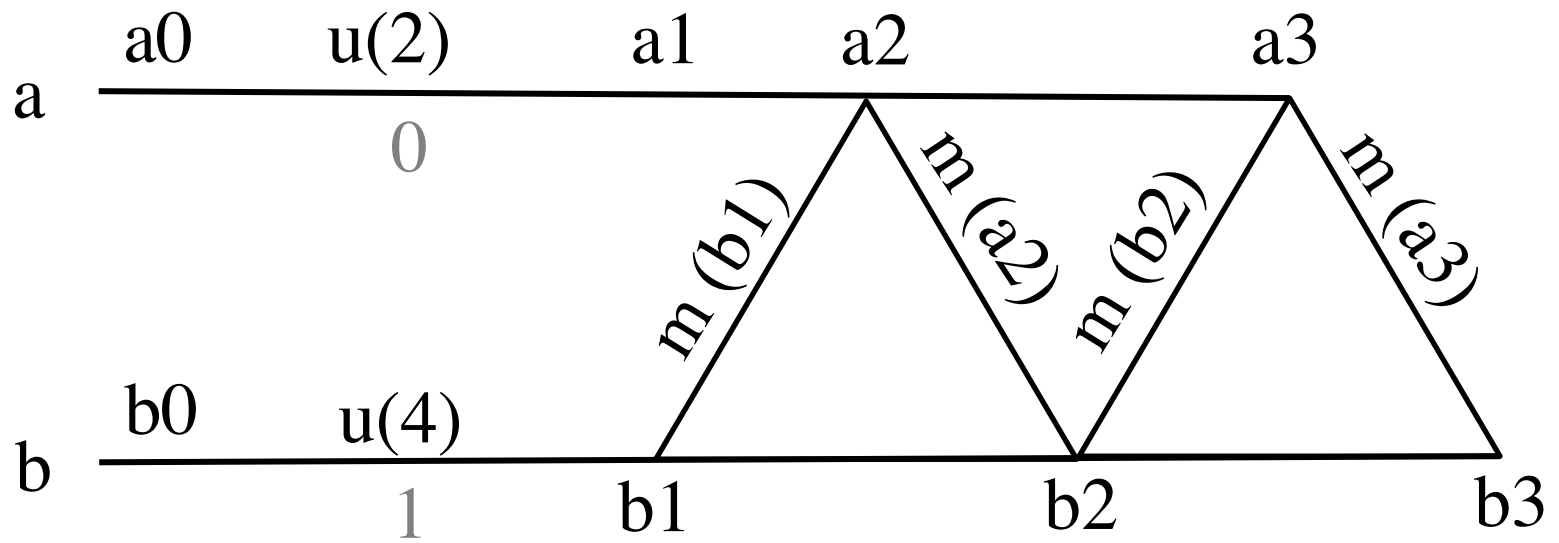
	state	query ()	history
a0	sum:0, cnt:0	0	{ }
a1	sum:2, cnt:1	2	{0}
a2	sum:2, cnt:1	2	{0,1}
a3	sum:2, cnt:1	2	{0,1}
b0	sum:0, cnt:0	0	{ }
b1	sum:4, cnt:1	4	{1}
b2	sum:4, cnt:1	4	{0,1}
b3	sum:4, cnt:1	4	{0,1}



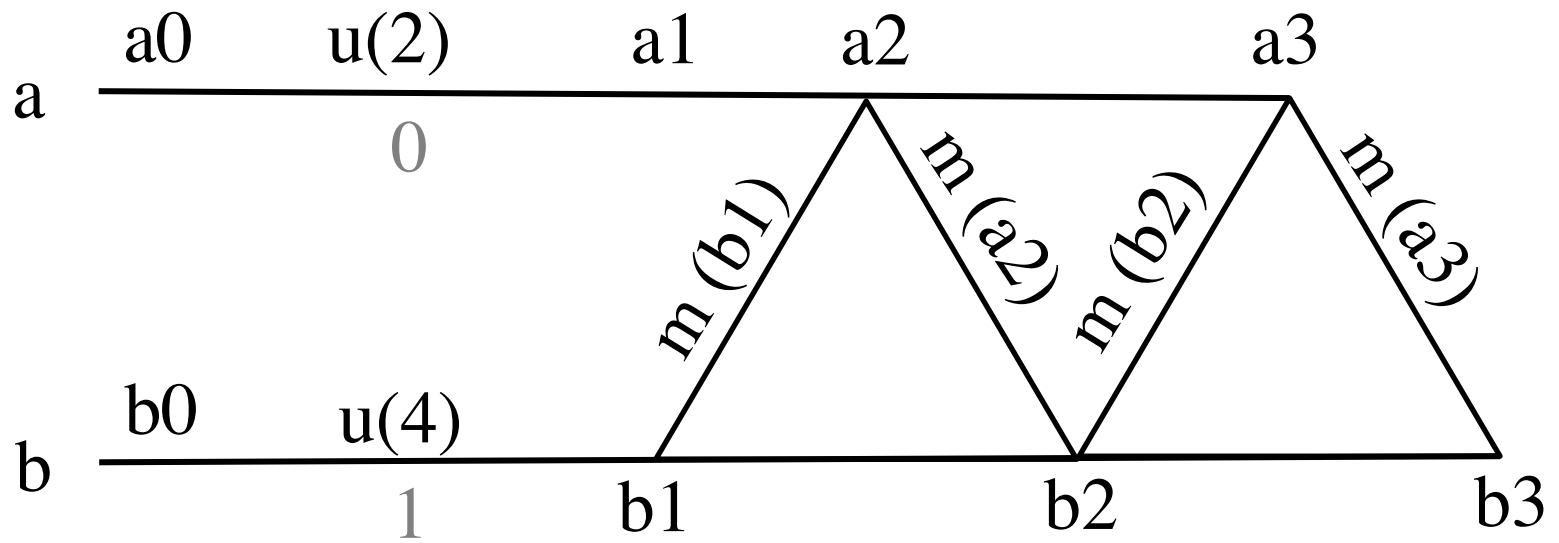
	state	query ()	history
a0	sum:0, cnt:0	0	{ }
→ a1	sum:2, cnt:1	2	{0}
a2	sum:2, cnt:1	2	{0,1}
a3	sum:2, cnt:1	2	{0,1}
b0	sum:0, cnt:0	0	{ }
→ b1	sum:4, cnt:1	4	{1}
b2	sum:4, cnt:1	4	{0,1}
b3	sum:4, cnt:1	4	{0,1}



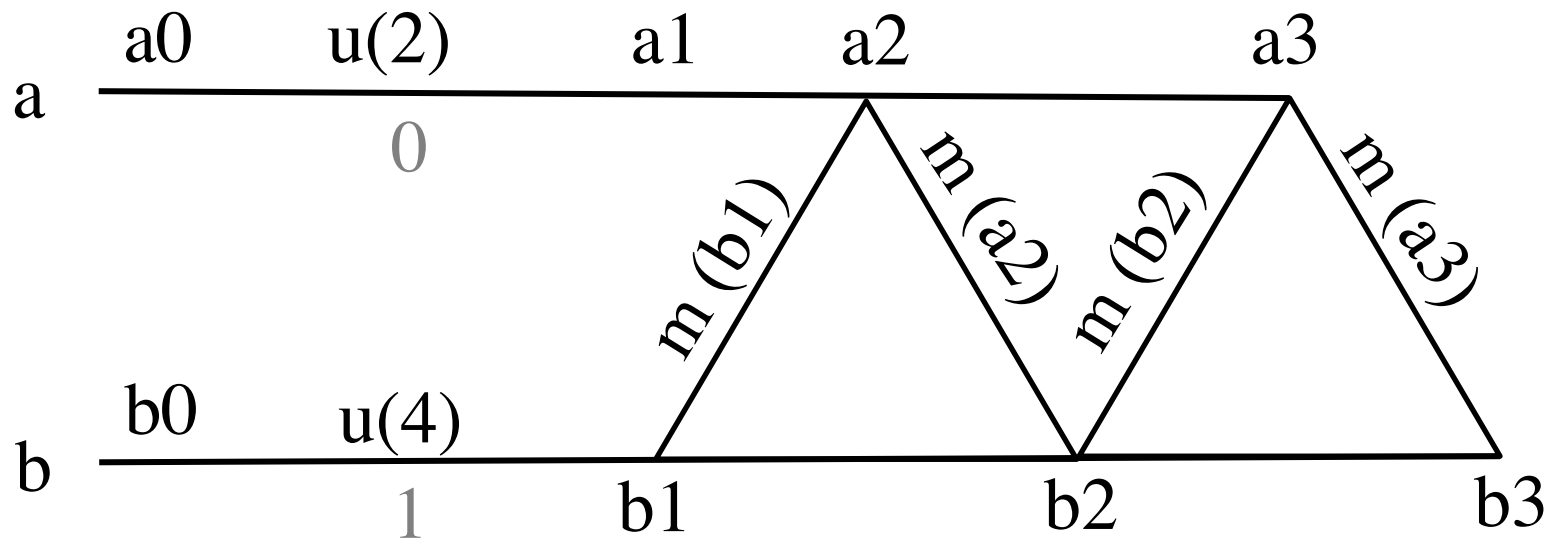
	state	query ()	history
a0	sum:0, cnt:0	0	{ }
a1	sum:2, cnt:1	2	{0}
a2	sum:2, cnt:1	2	{0,1}
a3	sum:2, cnt:1	2	{0,1}
b0	sum:0, cnt:0	0	{ }
b1	sum:4, cnt:1	4	{1}
b2	sum:4, cnt:1	4	{0,1}
b3	sum:4, cnt:1	4	{0,1}



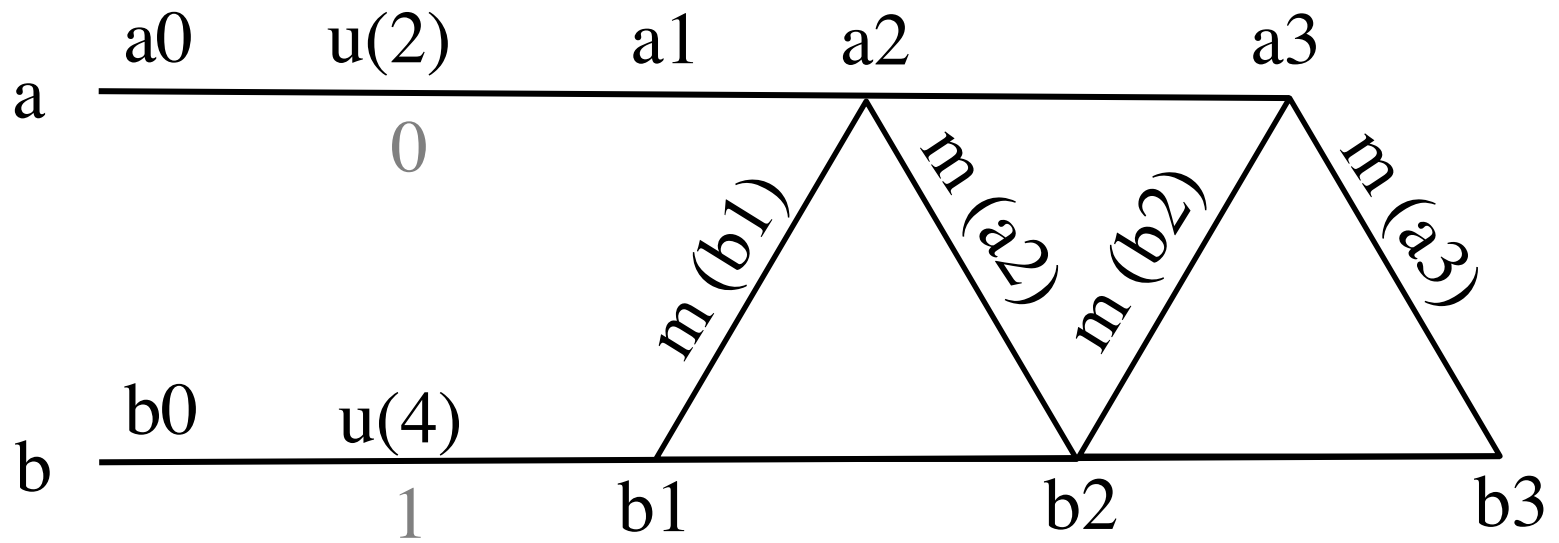
	state	query ()	history
<i>a</i> 0	sum:0, cnt:0	0	{ }
<i>a</i> 1	sum:2, cnt:1	2	{0}
→ <i>a</i> 2	sum:2, cnt:1	2	{0,1}
<i>a</i> 3	sum:2, cnt:1	2	{0,1}
<i>b</i> 0	sum:0, cnt:0	0	{ }
→ <i>b</i> 1	sum:4, cnt:1	4	{1}
<i>b</i> 2	sum:4, cnt:1	4	{0,1}
<i>b</i> 3	sum:4, cnt:1	4	{0,1}



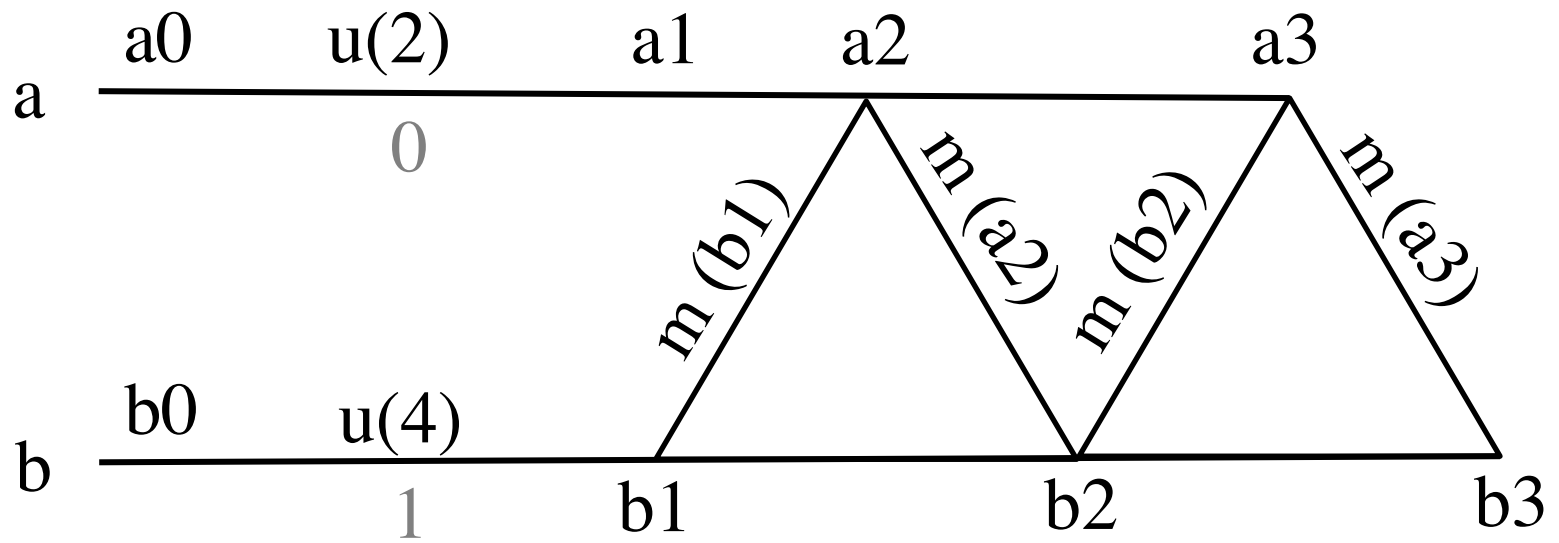
	state	query ()	history
a0	sum:0, cnt:0	0	{ }
a1	sum:2, cnt:1	2	{0}
a2	sum:2, cnt:1	2	{0,1}
a3	sum:2, cnt:1	2	{0,1}
b0	sum:0, cnt:0	0	{ }
b1	sum:4, cnt:1	4	{1}
b2	sum:4, cnt:1	4	{0,1}
b3	sum:4, cnt:1	4	{0,1}



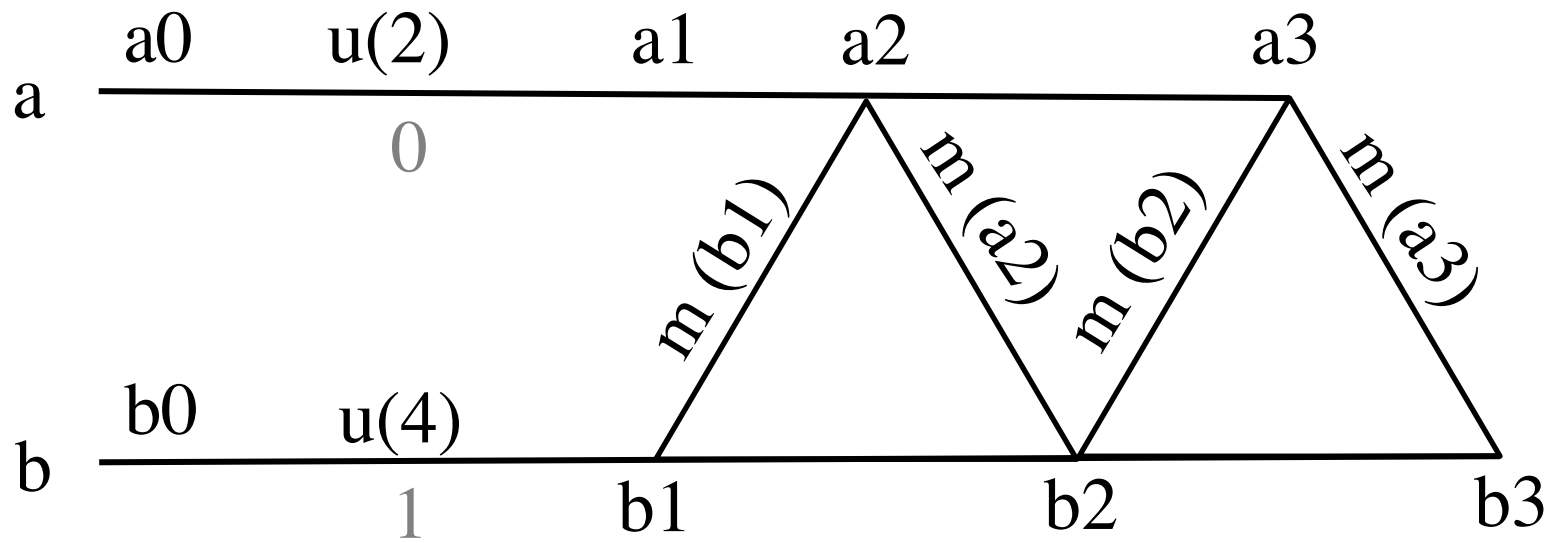
	state	query ()	history
a0	sum:0, cnt:0	0	{ }
a1	sum:2, cnt:1	2	{0}
→ a2	sum:2, cnt:1	2	{0,1}
a3	sum:2, cnt:1	2	{0,1}
b0	sum:0, cnt:0	0	{ }
b1	sum:4, cnt:1	4	{1}
→ b2	sum:4, cnt:1	4	{0,1}
b3	sum:4, cnt:1	4	{0,1}



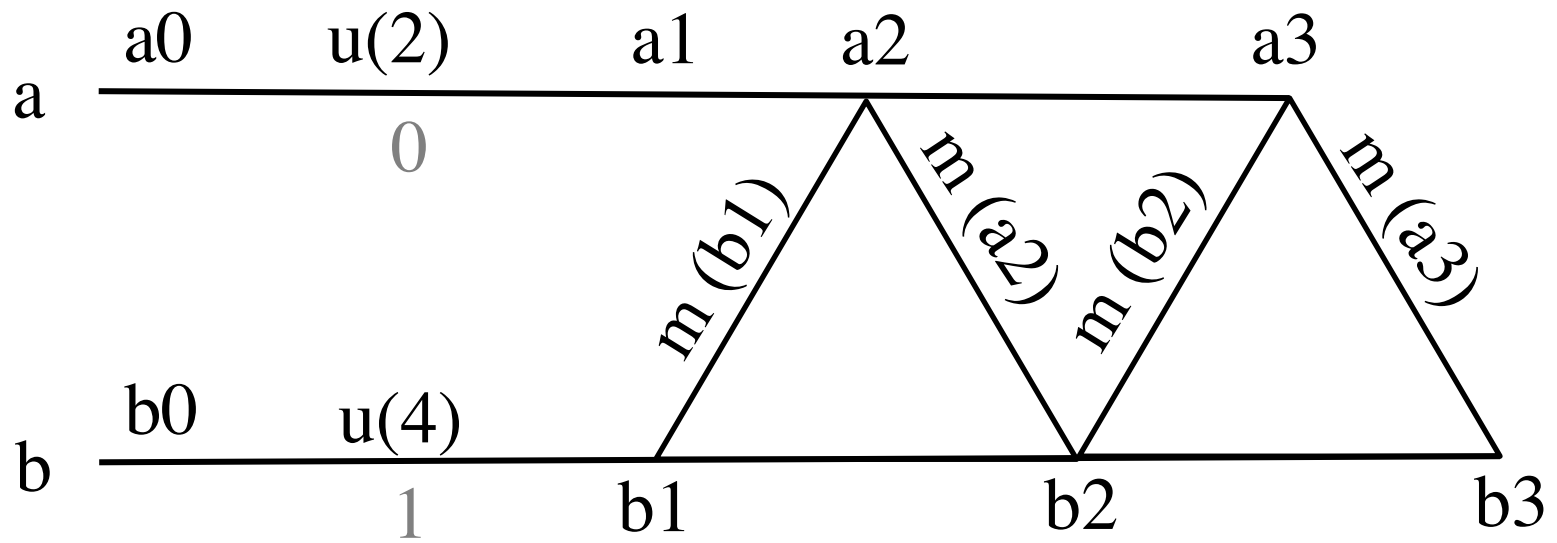
	state	query ()	history
a0	sum:0, cnt:0	0	{ }
a1	sum:2, cnt:1	2	{0}
a2	sum:2, cnt:1	2	{0,1}
a3	sum:2, cnt:1	2	{0,1}
b0	sum:0, cnt:0	0	{ }
b1	sum:4, cnt:1	4	{1}
b2	sum:4, cnt:1	4	{0,1}
b3	sum:4, cnt:1	4	{0,1}



	state	query ()	history
<i>a</i> 0	sum:0, cnt:0	0	{ }
<i>a</i> 1	sum:2, cnt:1	2	{0}
<i>a</i> 2	sum:2, cnt:1	2	{0,1}
→ <i>a</i> 3	sum:2, cnt:1	2	{0,1}
<i>b</i> 0	sum:0, cnt:0	0	{ }
<i>b</i> 1	sum:4, cnt:1	4	{1}
→ <i>b</i> 2	sum:4, cnt:1	4	{0,1}
<i>b</i> 3	sum:4, cnt:1	4	{0,1}

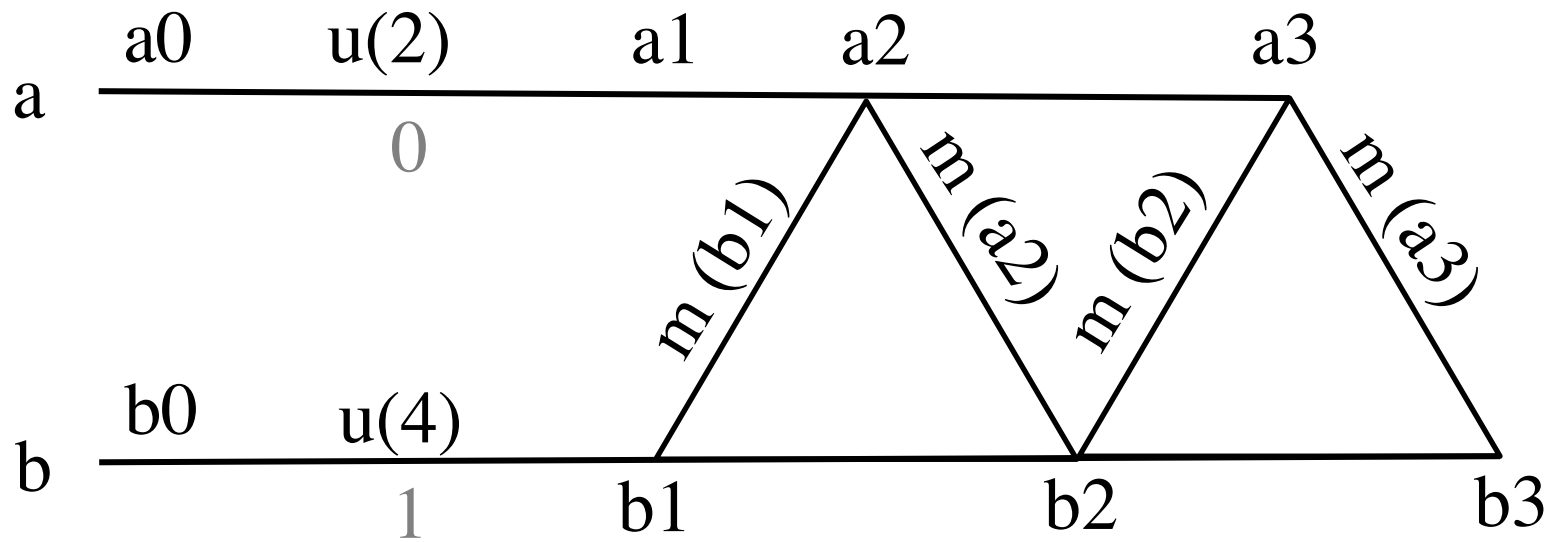


	state	query ()	history
a0	sum:0, cnt:0	0	{ }
a1	sum:2, cnt:1	2	{0}
a2	sum:2, cnt:1	2	{0,1}
a3	sum:2, cnt:1	2	{0,1}
b0	sum:0, cnt:0	0	{ }
b1	sum:4, cnt:1	4	{1}
b2	sum:4, cnt:1	4	{0,1}
b3	sum:4, cnt:1	4	{0,1}



a, b have **same causal history**, both **converge** to a stable but *different internal state*.

	state	query ()	history
a0	sum:0, cnt:0	0	{ }
a1	sum:2, cnt:1	2	{0}
a2	sum:2, cnt:1	2	{0,1}
a3	sum:2, cnt:1	2	{0,1}
b0	sum:0, cnt:0	0	{ }
b1	sum:4, cnt:1	4	{1}
b2	sum:4, cnt:1	4	{0,1}
b3	sum:4, cnt:1	4	{0,1}



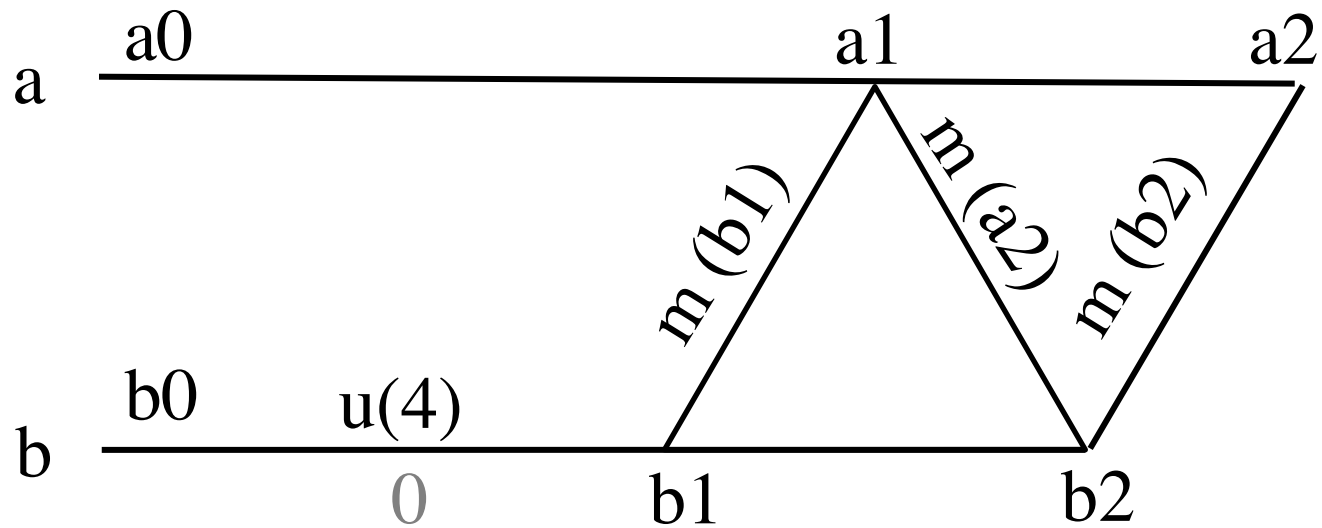
a, *b* have **same causal history**, both **converge** to a stable but *different internal state*.

Neither eventually consistent, nor strongly eventually consistent.

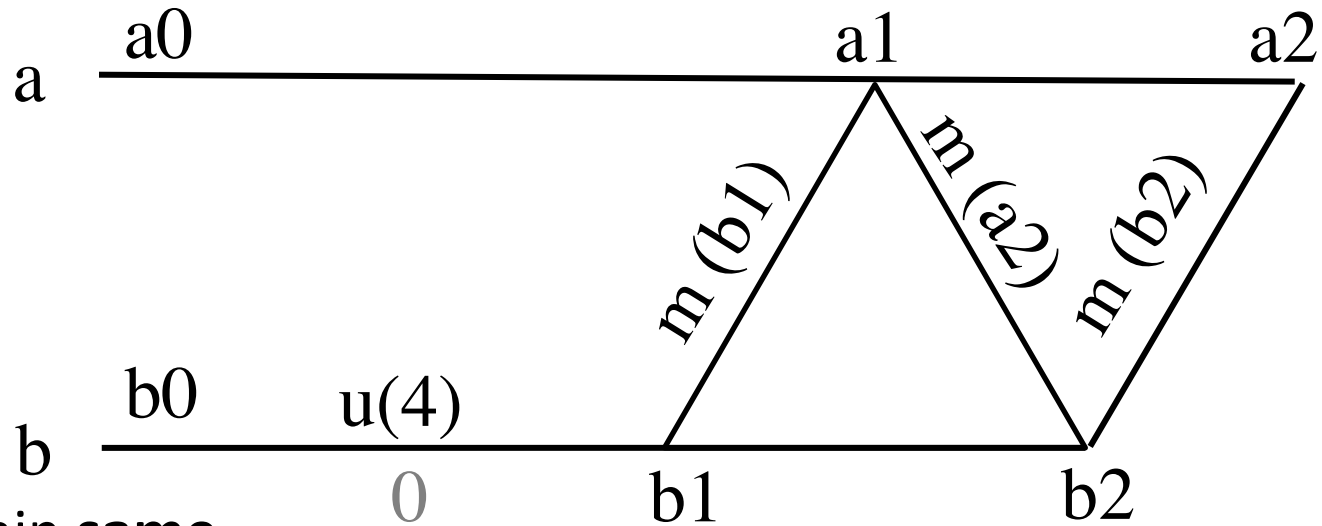
	state	query ()	history
<i>a0</i>	sum:0, cnt:0	0	{ }
<i>a1</i>	sum:2, cnt:1	2	{0}
<i>a2</i>	sum:2, cnt:1	2	{0,1}
<i>a3</i>	sum:2, cnt:1	2	{0,1}
<i>b0</i>	sum:0, cnt:0	0	{ }
<i>b1</i>	sum:4, cnt:1	4	{1}
<i>b2</i>	sum:4, cnt:1	4	{0,1}
<i>b3</i>	sum:4, cnt:1	4	{0,1}

BMergeAverage

- Object's merge
 - At b – overwrite state with state at a
 - At a – do nothing
- All else is the same as for Average

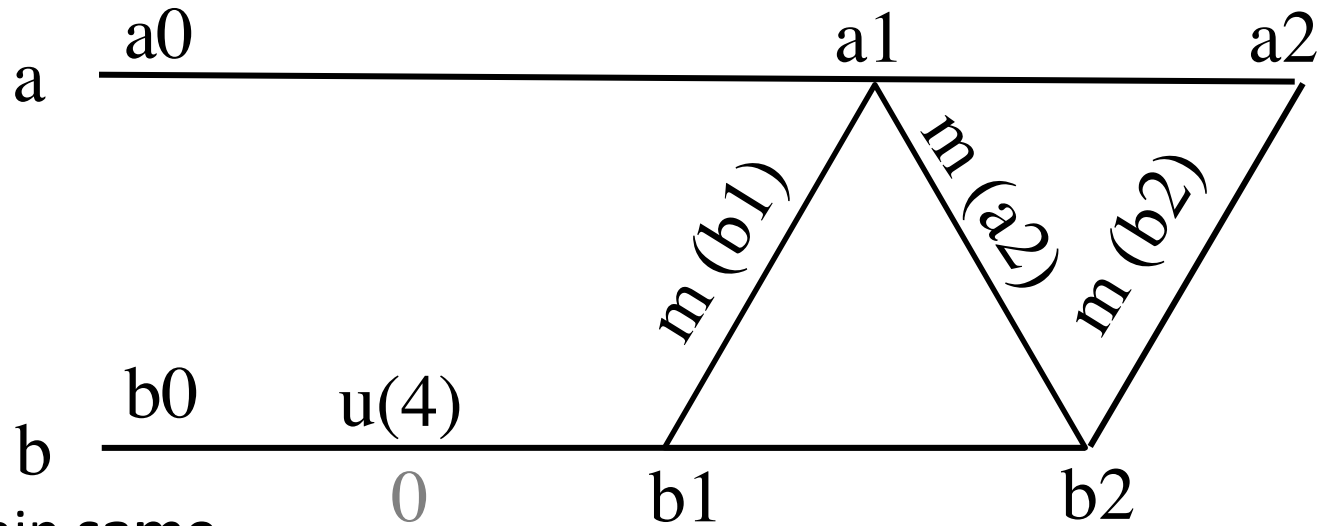


	state	query ()	history
a0	sum:0, cnt:0	0	{ }
a1	sum:0, cnt:0	0	{0}
a2	sum:0, cnt:0	0	{0}
b0	sum:0, cnt:0	0	{ }
b1	sum:4, cnt:1	4	{0}
b2	sum:0, cnt:0	0	{0}



a, b attain **same causal history**, both eventually **converge** to the same **internal state** – **eventual consistent**.

	state	query ()	history
a0	sum:0, cnt:0	0	{ }
a1	sum:0, cnt:0	0	{0}
a2	sum:0, cnt:0	0	{0}
b0	sum:0, cnt:0	0	{ }
b1	sum:4, cnt:1	4	{0}
b2	sum:0, cnt:0	0	{0}



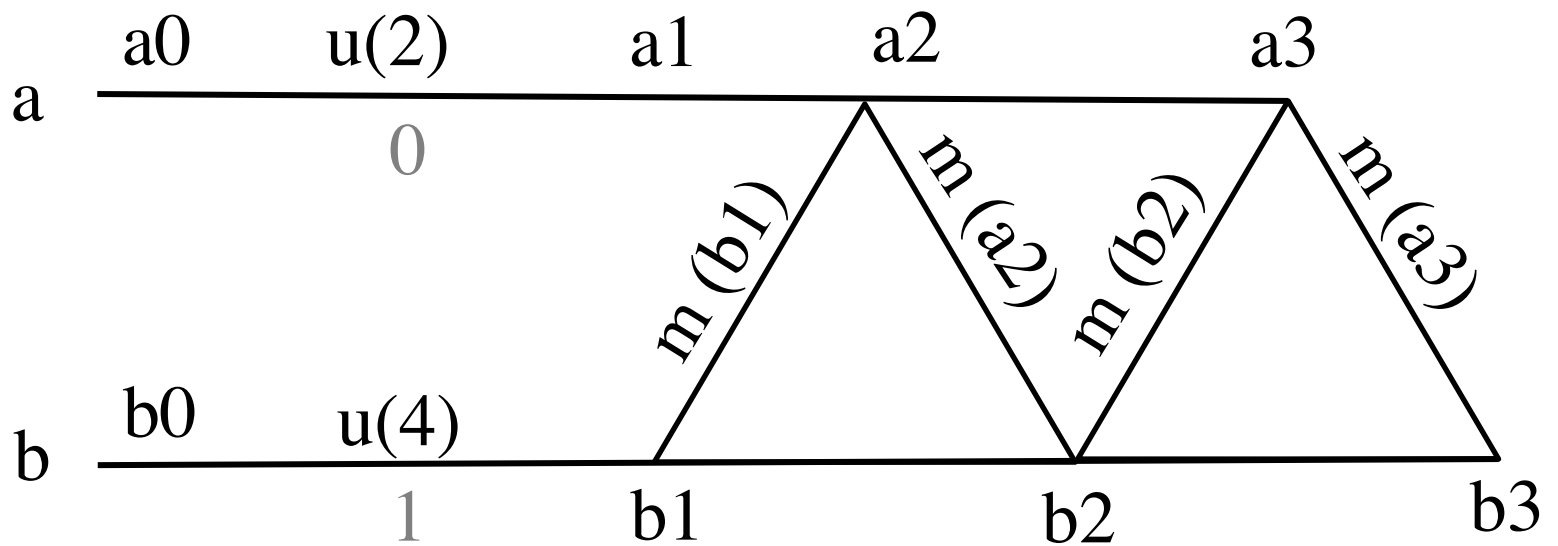
a, b attain **same causal history**, both eventually **converge** to the same **internal state** – **eventual consistent**.

a1, b1 have same causal history but different internal state – **not strongly eventually consistent**

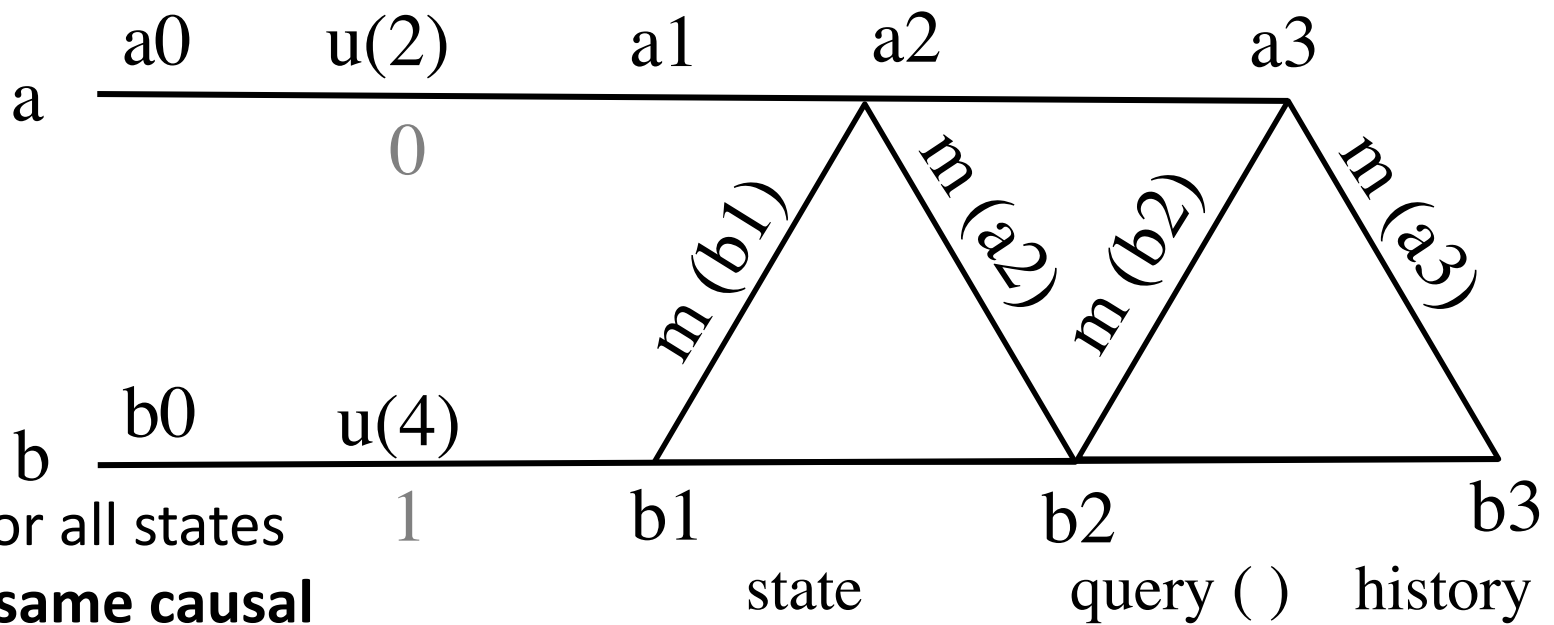
	state	query ()	history
a0	sum:0, cnt:0	0	{ }
a1	sum:0, cnt:0	0	{0}
a2	sum:0, cnt:0	0	{0}
b0	sum:0, cnt:0	0	{ }
b1	sum:4, cnt:1	4	{0}
b2	sum:0, cnt:0	0	{0}

MaxAverage

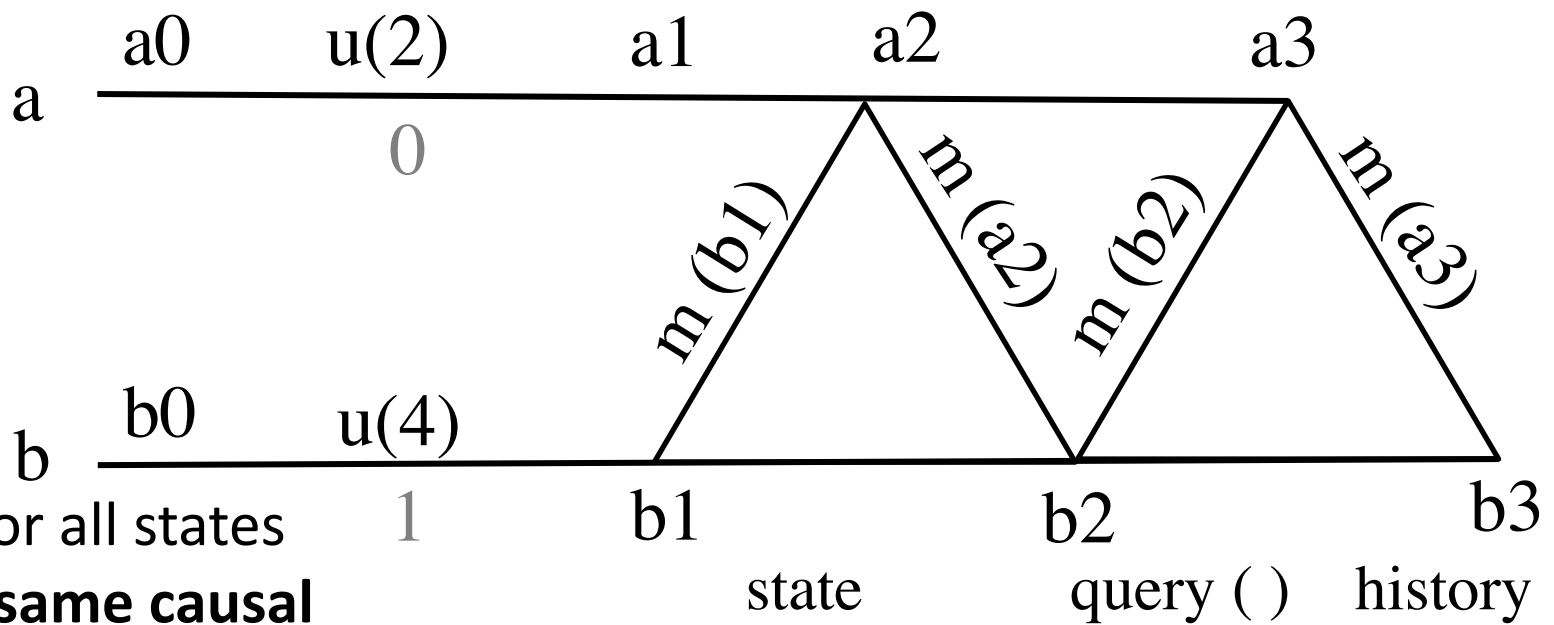
- Object's merge
 - **Pair-wise max of sum and cnt**
- All else is the same as for Average



	state	query ()	history
<i>a</i> 0	sum:0, cnt:0	0	{ }
<i>a</i> 1	sum:2, cnt:1	2	{0}
<i>a</i> 2	sum:4, cnt:1	4	{0,1}
<i>a</i> 3	sum:4, cnt:1	4	{0,1}
<i>b</i> 0	sum:0, cnt:0	0	{ }
<i>b</i> 1	sum:4, cnt:1	4	{1}
<i>b</i> 2	sum:4, cnt:1	4	{0,1}
<i>b</i> 3	sum:4, cnt:1	4	{0,1}



	state	query ()	history
<i>a0</i>	sum:0, cnt:0	0	{ }
<i>a1</i>	sum:2, cnt:1	2	{0}
<i>a2</i>	sum:4, cnt:1	4	{0,1}
<i>a3</i>	sum:4, cnt:1	4	{0,1}
<i>b0</i>	sum:0, cnt:0	0	{ }
<i>b1</i>	sum:4, cnt:1	4	{1}
<i>b2</i>	sum:4, cnt:1	4	{0,1}
<i>b3</i>	sum:4, cnt:1	4	{0,1}



At **a**, **b** for all states with the **same causal history**, they have the **same internal state** – **strongly eventually consistent**.

Great!!! But, what does it actually compute? Here, update(2) overwritten by update(4)! ☹

	state	query ()	history
a0	sum:0, cnt:0	0	{ }
a1	sum:2, cnt:1	2	{0}
a2	sum:4, cnt:1	4	{0,1}
a3	sum:4, cnt:1	4	{0,1}
b0	sum:0, cnt:0	0	{ }
b1	sum:4, cnt:1	4	{1}
b2	sum:4, cnt:1	4	{0,1}
b3	sum:4, cnt:1	4	{0,1}

Lessons Learned I

- Same causal history, different internal state
- Same causal history, same state but different internal state
- Same causal history, eventually same internal state
- Same causal history, always same internal state – SEC

	C?	EC?	SEC?
Average	no	no	no
NoMergeAverage	yes	no	no
BMergeAverage	yes	yes	no
MaxAverage	yes	yes	yes

Designing a **strongly eventually consistent state-based object** with intuitive semantics is challenging!

Lessons Learned II

- Replicated state-based object
- No convergence
- Convergence
- Eventual consistency in this model
- Strong eventual consistency in this model

Self-study Questions

- Can you design Average such that it becomes EC or SEC as well as offers correct averaging semantics?
- Think of other data structures and design update, query, and merge operations with reasonable semantics.
- Always draw timelines and state diagrams for your designs and proof EC or SEC, if possible.
- Think of data structures that support multiple update operations and one or more query operations.



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CRDT – CONFLICT-FREE REPLICATED DATA TYPES

Conflict-Free Replicated Data Types

- CRDT is a conflict-free replicated state-based object
- CRDT handles concurrent writes
- **Intuition:**
 - Do not allow writes with arbitrary values, limit to write operations which are **guaranteed not to conflict**
 - CRDTs are data structures with **special** write operations; they guarantee **strong eventual consistency** and are monotonic (no rollbacks)
- CRDTs are no panacea but a great solution when they apply!

Conflict-Free Replicated Data Types

- CRDTs can be **commutative / op-based (CmRDT)**:
 - **Example**: A growth-only counter, which can only process *increment* operations
 - Propagate operations among replicas (**duplicate-free, no-loss messaging**)
- CRDTs can be **convergent / state-based (CvRDT)**:
 - **Example**: A max register, which stores the maximum value written
 - Propagate and merge states (idempotent)
- Therefore, the value of a CRDT depends on **multiple write operations or states**, not just the latest one

State-based CRDTs

- A CRDT is a replicated state-based object
- Supports
 - Query
 - Update
 - Merge

CRDT Properties

A CRDT is a replicated state-based object that satisfies

- Merge is **associative** (e.g., $(A + (B + C)) = ((A + B) + C)$)
 - For any three state-based objects x , y , and z ,
 $\text{merge}(\text{merge}(x, y), z)$ is equal to $\text{merge}(x, \text{merge}(y, z))$
- Merge is **commutative** (e.g., $A + B = B + A$)
 - For any two state-based objects, x and y , $\text{merge}(x, y)$ is equal to $\text{merge}(y, x)$
- Merge is **idempotent**
 - For any state-based object x , $\text{merge}(x, x)$ is equal to x
- Every **update is increasing**
 - Let x be a state-based object and let $y = \text{update}(x, \dots)$ be the result of applying an update to x
 - Then, update is increasing if $\text{merge}(x, y)$ is equal to y

Max Register is a CRDT

The state-based object IntMax is a CRDT

- IntMax wraps an integer
- Merge (a, b) is the max of a, b
- Update (x) adds x to the wrapped integer
- Prove that IntMax is associative, commutative, idempotent, increasing

```
class IntMax(object):  
    def __init__(self):  
        self.x = 0  
    def query(self):  
        return self.x  
    def update(self, x):  
        assert x >= 0  
        self.x += x  
    def merge(self,  
        other):  
        self.x =  
            max(self.x,  
                other.x)
```

Establish Four Properties of CRDT

- Associativity

```
merge(merge(a, b), c)
= max(max(a.x, b.x), c.x)
= max(a.x, max(b.x, c.x))
= merge(a, merge(b, c))
```

- Commutativity

```
merge(a, b)
= max(a.x, b.x)
= max(b.x, a.x)
= merge(b, a)
```

- Idempotence

```
merge(a, a)
= max(a.x, a.x)
= a.x
= a
```

- Update is increasing

```
merge(a, update(a, x))
= max(a.x, a.x + x)
= a.x + x
= update(a, x)
```

G-Counter CRDT

Replicated growth-only counter

- Internal state of a G-Counter replicated on n nodes is an n -length array of non-negative integers
- `query` returns sum of every element in n -length array
- `add(x)` when invoked on the i -th server, increments the i -th entry of the n -length array by x
 - E.g., Server 0 increments 0th entry, Server 1 increments 1st entry of array, and so on
- `merge` performs a pairwise maximum of the two arrays

PN-Counter CRDT

Replicated counter supporting addition & subtraction

- Internal state of a PN-Counter
 - pair of two G-Counters named p and n .
 - p represents total value added to PN-Counter
 - n represents total value subtracted from PN-Counter.
- query method returns difference $p.query() - n.query()$
- add(x) –first of two updates– invokes $p.add(x)$
- sub(x) –second of two updates– invokes $n.add(x)$
- merge performs a pairwise merge of p and n

G-Set CRDT

Replicated growth-only set

A G-Set CRDT represents a replicated set which can be added to but not removed from

- Internal state of a G-Set is just a set
- `query` returns the set
- `add(x)` adds x to the set
- `merge` performs a set union

2P-Set CRDT

Replicated set supporting addition and subtraction

- Internal state of a 2P-Set is a
 - pair of two G-Sets named a and r
 - a represents set of values added to the 2P-Set
 - r represents set of values removed from the 2P-Set
- `query` method returns the set difference
 $a.query() - r.query()$
- `add(x)` is the first of two updates
 - invokes `a.add(x)`.
- `sub(x)` is the second of two updates
 - invokes `r.add(x)`
- `merge` performs a pairwise merge of a and r

Summary on CRDTs

- Formalized and introduced in 2014
- CmCRDTs and CvCRDTs are equivalent!
- Really neat solution if it applies
- Challenge is to design new CRDTs

Self-study Questions

- For the CRDTs introduced, establish its four properties.
- Create example executions for each CRDT and complete a timeline and a state table.
- Find use cases where the introduced CRDTs apply and show how they are used.
- Think of new CRDTs and repeat the above.

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