



Tutorial 5: Naïve Bayes

Decision Sciences & Systems (DSS)

Department of Informatics

TU München





#### Classification

- Given a dataset  $D = \{x_1, ..., x_n\}$  of tuples and a set of classes  $C = \{C_1, ..., C_m\}$
- Each instance  $x_i$  consistis of k features (e.g. categorical or numerical)
- The classification problem is to define a mapping  $f: D \to C$  where each instance  $x_i$  is assigned to one class





### 0-Rule

## Algorithm:

- i. For each class count its absolute frequency
- ii. Choose the most frequent one





#### 1-Rule

Algorithm: for each attribute

- i. Count the frequency of each class per attribute value
- ii. Pick the most frequent class
- iii. Define a rule that assigns this most frequent class to the attribute value (rule set)
- iv. Calculate error rate

⇒ Choose the attribute with the smallest error rate!





### Naïve Bayes - Prerequisites

Bayes Rule is

$$Pr(h_l|E) = \frac{Pr(E|h_l) \cdot Pr(h_l)}{Pr(E)}$$

and with  $E = (e_1, ..., e_k)$  we have

$$\Pr(h_l|e_1, \dots, e_k) = \frac{\Pr(e_1, \dots, e_k|h_l) \cdot \Pr(h_l)}{\Pr(e_1, \dots, e_k)}$$





## Naïve Bayes - Usage and Assumptions

- It is especially appropriate when the dimension of the feature space is high, making density estimation unattractive
- Assumption: Attributes independent and equally important

$$Pr(h_l|E) = \frac{Pr(e_1|h_l) \cdot Pr(e_2|h_l) \cdots Pr(e_k|h_l) \cdot Pr(h_l)}{Pr(E)}$$
$$= \frac{\prod_{i=1}^k Pr(e_i|h_l) \cdot Pr(h_l)}{Pr(E)}$$

 These assumptions are rather optimistic, however, Naïve Bayes classifiers often outperform more sophisticated alternatives





## Naïve Bayes - Algorithm

- For each attribute, count the frequency of each class per attribute value (and resolve zero-frequency problem if needed)
- ii. Calculate prior  $Pr(h_l)$  and likelihood  $Pr(e_i|h_l)$
- iii. Find  $\prod_{i=1}^k \Pr(e_i|h_i) \cdot \Pr(h_i)$
- iv. Normalize the results

$$\Pr(h_l|E) = \frac{\prod_{i=1}^k \Pr(e_i|h_l) \cdot \Pr(h_l)}{\Pr(E)}$$

⇒ Choose the class with the highest probability