Q&A Linear Regression

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$$\nabla_w w^T \underbrace{X^T X}_{B} w = [X^T X + (X^T X)^T] w = 2 X^T X w$$

$$\underset{\text{posterior}}{\boxed{p(y \mid x)}} = \frac{\overset{\text{likelihood}}{p(x \mid y)} \overset{\text{prior}}{p(y)}}{\underset{\text{evidence}}{p(x)}}$$

MCMC

$$p(x) = \sum_y p(x \mid y) \cdot p(y) = \sum_y p(x, y)$$

$$= \int_y p(x, y) \, dy$$

$y = 0.0, 0.1, \ldots, 1.0$

$$w^* = \hat{\Phi}^{\dagger} \hat{y} = (\hat{\Phi}^T \hat{\Phi})^{-1} \hat{\Phi}^T \hat{y}$$

$$= (\Phi^T \Phi + \lambda I_m)^{-1} \begin{pmatrix} \Phi \\ \sqrt{\lambda} I_m \end{pmatrix}^T \begin{pmatrix} y \\ 0_m \end{pmatrix}$$

$$\lambda = \frac{\alpha}{\beta}$$

$$= (\underline{\Phi^T \Phi} + \underline{\lambda I_m})^{-1} \Phi^T y$$

$$\Phi^T \Phi \approx \sum_{\text{samples}}$$

$$\boldsymbol{\mu} = \beta \boldsymbol{\Sigma} \boldsymbol{\Phi}^\top \boldsymbol{y}$$

$$\beta \Sigma = (\Phi^T \Phi + \lambda I_m)^{-1}$$

$$\boldsymbol{\Sigma}^{-1} = \alpha \boldsymbol{I} + \beta \boldsymbol{\Phi}^\top \boldsymbol{\Phi}.$$

$$\Sigma = (\alpha I + \beta \Phi^T \Phi)^{-1} = (\alpha [I + \lambda^{-1} \Phi^T \Phi])^{-1}$$

$$p(\boldsymbol{w}, \beta) = \mathcal{N}(\boldsymbol{w} \mid \boldsymbol{m}_0, \beta^{-1}\boldsymbol{S}_0)\mathrm{Gamma}(\beta \mid a_0, b_0)$$

$$\ldots \cdot \exp\left[-\frac{1}{2}(w-m_0)^T(\beta^{-1}S_0)^{-1}(w-m_0)\right) \cdot$$