

**Machine Learning for Graphs and Sequential Data Exercise Sheet 09****Temporal Point Processes**

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**Problem 1:** Consider a temporal point process, where all the inter-event times  $\tau_i = t_i - t_{i-1}$  are sampled i.i.d. from the distribution with the survival function

$$S(\tau) = \exp\left(-(e^{b\tau} - 1)\right)$$

with a parameter  $b > 0$ .

- a) Write down the closed-form expression for the conditional intensity function  $\lambda^*(t)$  of this TPP. Simplify as far as you can.
- b) Write down the closed-form expression for the log-likelihood of a sequence  $\{t_1, \dots, t_N\}$  generated from this TPP on the interval  $[0, T]$ . Simplify as far as you can.

**Problem 2:** Consider an inhomogeneous Poisson process (IPP) on  $[0, 1]$  with the intensity function  $\lambda(t) = 2t$ . We simulate a sample from this IPP using thinning. For this, we first simulate a *homogeneous* Poisson process (HPP) with intensity  $\mu = 4$  and apply the thinning procedure described in the lecture. What is the expected number of events from the HPP that will be rejected when using this procedure?

**Problem 3:** Consider an inhomogeneous Poisson process on  $[0, 4]$  with the intensity function  $\lambda(t) = \beta t$ , where  $\beta > 0$  is a parameter that has to be estimated. You have observed a single sequence  $\{1, 2.1, 3.3, 3.8\}$  generated from this IPP. What is the maximum likelihood estimate of the parameter  $\beta$ ?

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