

Machine Learning for Graphs and Sequential Data Exercise Sheet 07

Hidden Markov Models

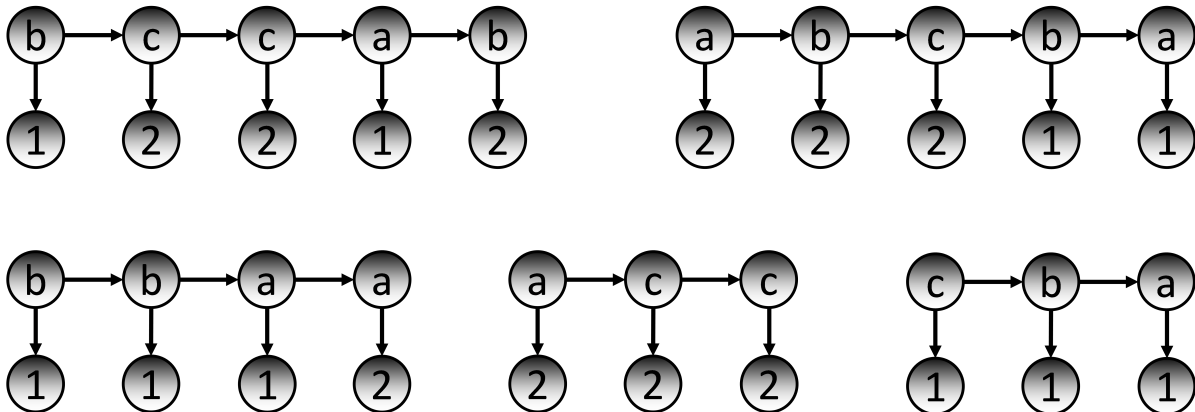
Problem 1: Consider an HMM where hidden variables are in $\{1, 2\}$ and observed variables are in $\{a, b, c\}$. Let the model parameters be as follows:

$$A = \begin{matrix} & \begin{matrix} 1 & 2 \end{matrix} \\ \begin{matrix} 1 \\ 2 \end{matrix} & \begin{bmatrix} 0.2 & 0.8 \\ 0.5 & 0.5 \end{bmatrix} \end{matrix} \quad B = \begin{matrix} & \begin{matrix} a & b & c \end{matrix} \\ \begin{matrix} 1 \\ 2 \end{matrix} & \begin{bmatrix} 0.2 & 0 & 0.8 \\ 0.4 & 0.6 & 0 \end{bmatrix} \end{matrix} \quad \pi = \begin{matrix} 1 \\ 2 \end{matrix} \begin{bmatrix} 0.5 \\ 0.5 \end{bmatrix}$$

Assume that the sequence $X_{1:5} = [cabac]$ is observed.

1. Filtering: find the distribution $P(Z_3|X_{1:3})$.
2. Smoothing: find the distribution $P(Z_3|X_{1:5})$.
3. Viterbi algorithm: find the most probable sequence $[Z_1, \dots, Z_5]$.

Problem 2: Consider an HMM where states Z_t are in $\{a, b, c\}$ and emissions X_t are in $\{1, 2\}$. Given is the following set of fully-observed instances (two sequences of length 5, one sequence of length 4, and two sequences of length 3):



Learn the parameters of the HMM (i.e. $\pi \in \mathbb{R}^3$, $\mathbf{A} \in \mathbb{R}^{3 \times 3}$, and $\mathbf{B} \in \mathbb{R}^{3 \times 2}$) using maximum-likelihood estimation.