

Example 25 Using Semantic Tableaux, check if $\forall x(Q \rightarrow P(x)) \models Q \rightarrow \forall xP(x)$, where x does not occur free in Q .

$$\begin{array}{c}
 \forall x(Q \rightarrow P(x)) : \mathbf{T}, Q \rightarrow \forall xP(x) : \mathbf{F} \\
 \downarrow \\
 \forall x(Q \rightarrow P(x)) : \mathbf{T}, Q : \mathbf{T}, \forall xP(x) : \mathbf{F} \\
 \downarrow \\
 P(c) : \mathbf{F} \\
 \downarrow \\
 Q \rightarrow P(c) : \mathbf{T} \\
 \swarrow \quad \searrow \\
 Q : \mathbf{F} \quad P(c) : \mathbf{T} \\
 \times \quad \times
 \end{array}$$

The tableau above closes, implying that $\forall x(Q \rightarrow P(x)) \vdash_{\mathbf{ST}} Q \rightarrow \forall xP(x)$, hence $\forall x(Q \rightarrow P(x)) \models Q \rightarrow \forall xP(x)$ holds.

Example 26 Using Semantic Tableaux, check if $\exists x(A(x) \rightarrow B(x)), \exists xA(x) \models \exists xB(x)$.

$$\begin{array}{c}
 \exists x(A(x) \rightarrow B(x)) : \mathbf{T}, \exists xA(x) : \mathbf{T}, \exists xB(x) : \mathbf{F} \\
 \downarrow \\
 A(c_1) \rightarrow B(c_1) : \mathbf{T} \\
 \downarrow \\
 A(c_2) : \mathbf{T} \\
 \downarrow \\
 B(c_1) : \mathbf{F} \\
 \downarrow \\
 B(c_2) : \mathbf{F} \\
 \swarrow \quad \searrow \\
 A(c_1) : \mathbf{F} \quad B(c_1) : \mathbf{T} \\
 \bigcirc \quad \times
 \end{array}$$