

Example 35 Let $A = \exists z(\exists xQ(x, z) \vee \exists xP(x)) \rightarrow \neg(\neg\exists xP(x) \wedge \forall x\exists zQ(z, x))$.

1. *Eliminating \rightarrow :* $A \equiv \neg\exists z(\exists xQ(x, z) \vee \exists xP(x)) \vee \neg(\neg\exists xP(x) \wedge \forall x\exists zQ(z, x))$
2. *Importing the negation:* $A \equiv \forall z(\neg\exists xQ(x, z) \wedge \neg\exists xP(x)) \vee (\neg\neg\exists xP(x) \vee \neg\forall x\exists zQ(z, x))$
 $\equiv \forall z(\forall x\neg Q(x, z) \wedge \forall x\neg P(x)) \vee (\exists xP(x) \vee \exists x\forall z\neg Q(z, x))$.
3. *Using the equivalences (a) and (b):* $A \equiv \forall z\forall x(\neg Q(x, z) \wedge \neg P(x)) \vee \exists x(P(x) \vee \forall z\neg Q(z, x))$.
4. *Renaming:* $A \equiv \forall z\forall x(\neg Q(x, z) \wedge \neg P(x)) \vee \exists y(P(y) \vee \forall w\neg Q(w, y))$.
5. *Using the equivalences (c)-(f) and pulling the quantifiers in front:*
 $A \equiv \forall z\forall x\exists y\forall w((\neg Q(x, z) \wedge \neg P(x)) \vee P(y) \vee \neg Q(w, y))$.
6. *The resulting formula is in a prenex DNF. For a prenex CNF we have to distribute the \vee over \wedge :*
 $A \equiv \forall z\forall x\exists y\forall w((\neg Q(x, z) \vee P(y) \vee \neg Q(w, y)) \wedge (\neg P(x) \vee P(y) \vee \neg Q(w, y)))$.