

Example 36

1. The result of Skolemization of the formula

$$\exists x \forall y \forall z (P(x, y) \rightarrow Q(x, z))$$

is

$$\forall y \forall z (P(c, y) \rightarrow Q(c, z)),$$

where c is a new constant symbol, called **Skolem constant**.

2. More generally, the result of Skolemization of the formula

$$\exists x_1 \cdots \exists x_k \forall y_1 \cdots \forall y_n A(x_1, \dots, x_k, y_1, \dots, y_n)$$

is

$$\forall y_1 \cdots \forall y_n A(c_1, \dots, c_k, y_1, \dots, y_n),$$

where c_1, \dots, c_k are new Skolem constants.

Note that the resulting formula is not equivalent to the original one, but is equally satisfiable with it.

3. The result of Skolemization of the formula

$$\exists x \forall y \exists z (P(x, y) \rightarrow Q(x, z))$$

is

$$\forall y (P(c, y) \rightarrow Q(c, f(y))),$$

where c is a new Skolem constant and f is a new unary function, called **Skolem function**.

4. More generally, the result of Skolemization of the formula

$$\forall y \exists x_1 \cdots \exists x_k \forall y_1 \cdots \forall y_n A(y, x_1, \dots, x_k, y_1, \dots, y_n)$$

is

$$\forall y \forall y_1 \cdots \forall y_n A(y, f_1(y), \dots, f_k(y), y_1, \dots, y_n),$$

where f_1, \dots, f_k are new Skolem functions.