Example 25 Using Semantic Tableaux, check if $\forall x(Q \rightarrow P(x)) \models Q \rightarrow \forall x P(x)$, where x does not occur free in Q. $\begin{aligned} \forall x(Q \to P(x)) : \mathbf{T}, Q \to \forall x P(x) : \mathbf{F} \\ \downarrow \\ \forall x(Q \to P(x)) : \mathbf{T}, Q : \mathbf{T}, \forall x P(x) : \mathbf{F} \end{aligned}$

$$\forall x(Q \to X)$$

$$\forall x(Q \to X)$$

$$\forall x(Q \to P(x)) : T, Q : T, \forall x P(x) : F$$

$$\downarrow P(c) : F$$

$$\downarrow \downarrow$$

$$Q o P(c)$$
: 1
 $Q: \mathsf{F} \quad P(c): \mathsf{T}$
 $\times \quad \times$

The tableau above closes, implying that $\forall x(Q \to P(x)) \vdash_{\mathbf{ST}} Q \to \forall x P(x)$, hence $\forall x(Q \to P(x)) \vDash Q \to \forall x P(x) \text{ holds.}$

Example 26 Using Semantic Tableaux, check if
$$\exists x (A(x) \to B(x)), \exists x A(x) \models \exists x B(x)$$
.

$$\exists x (A(x) \to B(x)) : \mathtt{T}, \exists x A(x) : \mathtt{T}, \exists x B(x) : \mathtt{F}$$

$$\exists x (A(x) \to B(x)) : \mathtt{T}, \exists x A(x) : \mathtt{T}, \exists x B(x) : \mathtt{F}$$

$$\downarrow \\ A(c_1) \to B(c_1) : \mathtt{T}$$

$$\downarrow \\ A(c_2) : \mathtt{T}$$

$$(x) \rightarrow B(x)$$
): T, $\exists x A(x)$: T, $\exists x B(x)$: F
$$A(c_1) \rightarrow B(c_1)$$
: T

$$A(c_1) \stackrel{\downarrow}{
ightarrow} B(c_1) : \mathtt{T}$$

$$\downarrow A(c_2) : \mathtt{T}$$

$$A(c_1) \stackrel{\downarrow}{
ightarrow} B(c_1): \mathtt{T} \ \downarrow \ A(c_2): \mathtt{T}$$

$$A(c_1) \stackrel{\downarrow}{
ightarrow} B(c_1) : \mathtt{T} \ \downarrow \ A(c_2) : \mathtt{T}$$

$$A(c_1)
ightarrow B(c_1): extsf{T} \ A(c_2): extsf{T}$$

$$A(c_1) o B(c_1): \mathtt{T}$$
 \downarrow $A(c_2): \mathtt{T}$

$$A(c_1)$$
 \downarrow $A(c_2)$: T

$$A(c_2): extsf{T} \ \downarrow$$

$$A(c_2): \mathsf{T} \ \downarrow \ B(c_2): \mathsf{F}$$

$$A(c_2)$$
 : T \downarrow $B(c_1)$: F

$$egin{aligned} A(c_2): \mathtt{T} \ \downarrow \ B(c_1): \mathtt{F} \end{aligned}$$

$$B(c_1): \mathsf{F}$$

$$B(c_1): \mathtt{F}$$

$$B(c_1): \mathtt{F}$$
 \downarrow
 $B(c_2): \mathtt{F}$
 $A(c_1): \mathtt{F} \quad B(c_1): \mathtt{T}$
 \bigcirc
 \times