

Example 39 Consider the formula

$$A = \exists z(\exists xQ(x, z) \vee \exists xP(x)) \rightarrow \neg(\neg\exists xP(x) \wedge \forall x\exists zQ(z, x)).$$

from example 35.

1. Transforming A to a prenex CNF:

$$A \equiv \forall z\forall x\exists y\forall w((\neg Q(x, z) \vee P(y) \vee \neg Q(w, y)) \wedge (\neg P(x) \vee P(y) \vee \neg Q(w, y))).$$

2. Skolemize all existential quantifiers:

$$\text{Skolem}(A) =$$

$$\forall z\forall x\forall w((\neg Q(x, z) \vee P(f(z, x)) \vee \neg Q(w, f(z, x))) \wedge (\neg P(x) \vee P(f(z, x)) \vee \neg Q(w, f(z, x)))).$$

3. Remove all universal quantifiers and write the matrix as a set of clauses:

$$\text{clausal}(A) = \{\neg Q(x, z), P(f(z, x)), \neg Q(w, f(z, x))\}, \{\neg P(x), P(f(z, x)), \neg Q(w, f(z, x))\}.$$