

1 The system

The vessel is modelled by

$$\mathbf{M}_{RB}\dot{\boldsymbol{\nu}} + \mathbf{M}_A\dot{\boldsymbol{\nu}}_r + \mathbf{C}_{RB}(\boldsymbol{\nu})\boldsymbol{\nu} + \mathbf{C}_A(\boldsymbol{\nu}_r)\boldsymbol{\nu}_r + \mathbf{D}(\boldsymbol{\nu}_r)\boldsymbol{\nu}_r + \mathbf{G}\boldsymbol{\eta} = \boldsymbol{\tau} + \mathbf{w}(t) \quad (1)$$

$$\dot{\boldsymbol{\eta}} = \mathbf{J}(\boldsymbol{\eta})\boldsymbol{\nu} \quad (2)$$

where

$$\mathbf{M}_{RB} = \begin{bmatrix} m & 0 & 0 \\ 0 & m & 0 \\ 0 & 0 & I_z \end{bmatrix} \quad \mathbf{M}_A = \begin{bmatrix} -X_{\dot{u}} & 0 & 0 \\ 0 & -Y_{\dot{v}} & -Y_{\dot{r}} \\ 0 & -N_{\dot{v}} & -N_{\dot{r}} \end{bmatrix} \quad (3)$$

$$\mathbf{C}_{RB} = \begin{bmatrix} 0 & 0 & -m(x_g r + v) \\ 0 & 0 & mu \\ m(x_g r + v) & -mu & 0 \end{bmatrix} \quad \mathbf{C}_A = \begin{bmatrix} 0 & 0 & Y_{\dot{v}}v_r + \frac{1}{2}(N_{\dot{v}}Y_{\dot{r}})r \\ 0 & 0 & -X_{\dot{u}}u_r \\ -Y_{\dot{v}}v_r + \frac{1}{2}(N_{\dot{v}}Y_{\dot{r}})r & X_{\dot{u}}u_r & 0 \end{bmatrix} \quad (4)$$

$$\mathbf{D}_L = \begin{bmatrix} -X_u & 0 & 0 \\ 0 & -Y_v & -Y_r \\ 0 & -N_v & -N_r \end{bmatrix} \quad \mathbf{D}_{NL} = \begin{bmatrix} -X_{|u|u}|u| - X_{uuu}u^2 & 0 & 0 \\ 0 & -Y_{|v|v}|v_r| - Y_{|r|v}|r| & -Y_{|v|r}|v_r| - Y_{|r|r}|r| \\ 0 & -N_{|v|v}|v_r| - N_{|r|v}|r| & -N_{|v|r}|v_r| - N_{|r|r}|r| \end{bmatrix} \quad (5)$$

Using property 8.1 of [Fossen] we can define the dynamics using only the water relative velocity $\boldsymbol{\nu}_r$

$$\mathbf{M}\dot{\boldsymbol{\nu}}_r + \mathbf{C}(\boldsymbol{\nu}_r)\boldsymbol{\nu}_r + \mathbf{D}(\boldsymbol{\nu}_r)\boldsymbol{\nu}_r = \boldsymbol{\tau} \quad (6)$$

$$\dot{\boldsymbol{\eta}} = \mathbf{J}(\boldsymbol{\eta})\boldsymbol{\nu}_r + \boldsymbol{\nu}_c \quad (7)$$

Where $\boldsymbol{\nu}_c$ is the velocity of the water current in global coordinates. And

$$\mathbf{M} = \mathbf{M}_{RB} + \mathbf{M}_A \quad (8)$$

$$\mathbf{C}(\boldsymbol{\nu}_r) = \mathbf{C}_{RB}(\boldsymbol{\nu}_r) + \mathbf{C}_A(\boldsymbol{\nu}_r) \quad (9)$$

Additionally $\mathbf{G}\boldsymbol{\eta} = \mathbf{0}$ as there exists no restoring forces in surge, sway and yaw. Isolating the acceleration $\dot{\boldsymbol{\nu}}_r$ in (6) we have

$$\dot{\boldsymbol{\nu}}_r = \mathbf{M}^{-1}(\boldsymbol{\tau} - \mathbf{C}(\boldsymbol{\nu}_r)\boldsymbol{\nu}_r - \mathbf{D}(\boldsymbol{\nu}_r)\boldsymbol{\nu}_r) \quad (10)$$

$$\dot{\boldsymbol{\eta}} = \mathbf{J}(\boldsymbol{\eta})\boldsymbol{\nu}_r + \boldsymbol{\nu}_c \quad (11)$$

Assuming that \mathbf{M} and \mathbf{C} as well as the linear damping coefficients X_u and N_r are known.

$$\mathbf{D}(\boldsymbol{\nu}_r)\boldsymbol{\nu}_r = -\mathbf{d}\boldsymbol{\nu}_r - \Phi(\boldsymbol{\nu}_r)\boldsymbol{\vartheta} \quad (12)$$

The system equation can then be split into a known part and a unknown or uncertain part. This can be written as

$$\dot{\boldsymbol{\nu}}_r = \mathbf{M}^{-1}(\boldsymbol{\tau} - \mathbf{C}(\boldsymbol{\nu}_r)\boldsymbol{\nu}_r + \mathbf{d}\boldsymbol{\nu}_r + \Phi(\boldsymbol{\nu}_r)\boldsymbol{\vartheta}) \quad (13)$$

$$\dot{\boldsymbol{\eta}} = \mathbf{J}(\boldsymbol{\eta})\boldsymbol{\nu}_r + \boldsymbol{\nu}_c \quad (14)$$

Where

$$\mathbf{d} = \begin{bmatrix} X_u & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & N_r \end{bmatrix} \quad (15)$$

And the uncertain part is described by

$$\Phi(\boldsymbol{\nu}_r)^T = \begin{bmatrix} |u|u & 0 & 0 \\ u^3 & 0 & 0 \\ 0 & v & 0 \\ 0 & r & 0 \\ 0 & |v|v & 0 \\ 0 & |r|v & 0 \\ 0 & |v|r & 0 \\ 0 & |r|r & 0 \\ 0 & 0 & v \\ 0 & 0 & |v|v \\ 0 & 0 & |r|v \\ 0 & 0 & |v|r \\ 0 & 0 & |r|r \end{bmatrix} \quad \boldsymbol{\vartheta} = \begin{bmatrix} X_{|u|u} \\ X_{uuu} \\ Y_v \\ Y_r \\ Y_{|v|v} \\ Y_{|r|v} \\ Y_{|v|r} \\ Y_{|r|r} \\ N_v \\ N_{|v|v} \\ N_{|r|v} \\ N_{|v|r} \\ N_{|r|r} \end{bmatrix} \quad (16)$$

Lastly the known part is collected in a function $\mathbf{f}(\boldsymbol{\nu}_r)$.

$$\dot{\boldsymbol{\nu}}_r = \mathbf{M}^{-1}(\mathbf{d}\boldsymbol{\nu}_r - \mathbf{C}(\boldsymbol{\nu}_r)\boldsymbol{\nu}_r) + \mathbf{M}^{-1}(\boldsymbol{\tau} + \Phi(\boldsymbol{\nu}_r)\boldsymbol{\vartheta}) \quad (17)$$

$$= \mathbf{f}(\boldsymbol{\nu}_r) + \mathbf{M}^{-1}(\boldsymbol{\tau} + \Phi(\boldsymbol{\nu}_r)\boldsymbol{\vartheta}) \quad (18)$$

$$(19)$$

The system is now in a form where the known part $\mathbf{f}(\boldsymbol{\nu}_r)$, the uncertain part $\Phi(\boldsymbol{\nu}_r)\boldsymbol{\vartheta}$ and the input $\boldsymbol{\tau}$ are clear to distinguish from one another.

2 Adaptive Velocity Control

2.1 Control Design

The control objective is to minimize $\tilde{\nu}(t) \triangleq \nu(t) - r_\nu(t)$. That is to minimize the difference (error) between the velocity of the vessel $\nu(t)$ and the reference velocity $r_\nu(t)$. A new variable \mathbf{z} is introduced to describe this difference.

$$\mathbf{z} \triangleq \nu - r_\nu \quad (20)$$

Choosing the Lyapunov function

$$V \triangleq \frac{1}{2}\mathbf{z}^T\mathbf{z} + \frac{1}{2}\tilde{\boldsymbol{\vartheta}}^T\boldsymbol{\Gamma}^{-1}\tilde{\boldsymbol{\vartheta}} \quad (21)$$

Which has the derivative

$$\dot{V} = \mathbf{z}^T\dot{\mathbf{z}} + \tilde{\boldsymbol{\vartheta}}^T\boldsymbol{\Gamma}^{-1}\dot{\tilde{\boldsymbol{\vartheta}}} \quad (22)$$

$$= \mathbf{z}^T(\dot{\boldsymbol{\nu}}_r - \dot{r}_\nu) + \tilde{\boldsymbol{\vartheta}}^T\boldsymbol{\Gamma}^{-1}\dot{\tilde{\boldsymbol{\vartheta}}} \quad (23)$$

$$= \mathbf{z}^T(\mathbf{f}(\boldsymbol{\nu}_r) + \mathbf{M}^{-1}(\boldsymbol{\tau} + \Phi(\boldsymbol{\nu}_r)\boldsymbol{\vartheta}) - \dot{r}_\nu) + \tilde{\boldsymbol{\vartheta}}^T\boldsymbol{\Gamma}^{-1}\dot{\tilde{\boldsymbol{\vartheta}}} \quad (24)$$

$$= \mathbf{z}^T\left(\mathbf{f}(\boldsymbol{\nu}_r) + \mathbf{M}^{-1}\left(\boldsymbol{\tau} + \Phi(\boldsymbol{\nu}_r)\hat{\boldsymbol{\vartheta}}\right) - \dot{r}_\nu\right) + \mathbf{z}^T\mathbf{M}^{-1}\Phi(\boldsymbol{\nu}_r)\tilde{\boldsymbol{\vartheta}} + \tilde{\boldsymbol{\vartheta}}^T\boldsymbol{\Gamma}^{-1}\dot{\tilde{\boldsymbol{\vartheta}}} \quad (25)$$

$$= \mathbf{z}^T\left(\mathbf{f}(\boldsymbol{\nu}_r) + \mathbf{M}^{-1}\left(\boldsymbol{\tau} + \Phi(\boldsymbol{\nu}_r)\hat{\boldsymbol{\vartheta}}\right) - \dot{r}_\nu\right) + \tilde{\boldsymbol{\vartheta}}^T\left(\boldsymbol{\Gamma}^{-1}\dot{\tilde{\boldsymbol{\vartheta}}} + \Phi(\boldsymbol{\nu}_r)^T\mathbf{M}^{-T}\mathbf{z}\right) \quad (26)$$

$$(27)$$

It is seen that choosing the control law

$$\boldsymbol{\tau} = \mathbf{M}(-\mathbf{K}\mathbf{z} - \mathbf{f}(\boldsymbol{\nu}_r) + \dot{r}_\nu) - \Phi(\boldsymbol{\nu}_r)\hat{\boldsymbol{\vartheta}} \quad (28)$$

And the adaptation law

$$\dot{\hat{\boldsymbol{\vartheta}}} = -\Gamma\Phi(\boldsymbol{\nu}_r)^T\mathbf{M}^{-T}\mathbf{z} \quad (29)$$

Will makes the Lyapunov function derivative become

$$\dot{V} = -\mathbf{z}^T\mathbf{K}\mathbf{z} \quad (30)$$

Which is negative semi-definite if \mathbf{K} is positive definite.

Barbalat's lemma

3 Adaptive Position Control

3.1 The system

As in the previous section the system is

$$\dot{\boldsymbol{\nu}}_r = \mathbf{f}(\boldsymbol{\nu}_r) + \mathbf{M}^{-1}\boldsymbol{\tau} + \mathbf{M}^{-1}\Phi(\boldsymbol{\nu}_r)\boldsymbol{\vartheta} \quad (31)$$

$$\dot{\boldsymbol{\eta}} = \mathbf{R}(\psi)\boldsymbol{\nu}_r + \boldsymbol{\nu}_c \quad (32)$$

where the known part $\mathbf{f}(\boldsymbol{\nu}_r)$, the uncertain part $\Phi(\boldsymbol{\nu}_r)\boldsymbol{\vartheta}$ and the input $\boldsymbol{\tau}$ of the system is separated. Considering the water currents as a unknown quantity

$$\dot{\boldsymbol{\nu}}_r = \mathbf{f}(\boldsymbol{\nu}_r) + \mathbf{M}^{-1}\boldsymbol{\tau} + \mathbf{M}^{-1}\Phi_2(\boldsymbol{\nu}_r)\boldsymbol{\vartheta} \quad (33)$$

$$\dot{\boldsymbol{\eta}} = \mathbf{R}(\psi)\boldsymbol{\nu}_r + \Phi_1\boldsymbol{\vartheta} \quad (34)$$

3.2 Control objective

The control objective is to minimize $\tilde{\eta}(t) \triangleq \eta(t) - r_\eta(t)$. That is to minimize the difference (error) between the position of the vessel $\eta(t)$ and the reference position $r_\eta(t)$. A new variable \mathbf{z} is introduced to describe this difference.

$$\mathbf{z}_1 \triangleq \eta - r_\eta \quad (35)$$

$$\dot{\mathbf{z}}_1 = \dot{\boldsymbol{\eta}} - \dot{r}_\eta = \mathbf{R}(\psi)\boldsymbol{\nu}_r + \Phi_1\boldsymbol{\vartheta} - \dot{r}_\eta \quad (36)$$

Since $\dot{\mathbf{z}}_1$ does not contain the input $\boldsymbol{\tau}$ it is necessary to consider the velocity $\boldsymbol{\nu}_r$ as a virtual input α . This is done in order that backstep though the system equations until we find the real input $\boldsymbol{\tau}$. For now $\boldsymbol{\nu}_r$ is considered a virtual input, that is, a state the can be controlled to act as the input of a subsystem.

3.3 Step 1

Choosing the Lyapunov function

$$V \triangleq \frac{1}{2}\mathbf{z}_1^T\mathbf{z}_1 + \frac{1}{2}\tilde{\boldsymbol{\vartheta}}^T\Gamma^{-1}\tilde{\boldsymbol{\vartheta}} \quad (37)$$

Which has the derivative

$$\dot{V} = \mathbf{z}_1^T \dot{\mathbf{z}}_1 + \tilde{\boldsymbol{\vartheta}}^T \Gamma^{-1} \dot{\tilde{\boldsymbol{\vartheta}}} \quad (38)$$

$$= \mathbf{z}_1^T (\mathbf{R}(\psi)\alpha + \Phi_1 \boldsymbol{\vartheta} - \dot{r}_\eta) + \tilde{\boldsymbol{\vartheta}}^T \Gamma^{-1} \dot{\tilde{\boldsymbol{\vartheta}}} \quad (39)$$

$$= \mathbf{z}_1^T \left(\mathbf{R}(\psi)\alpha + \Phi_1 \hat{\boldsymbol{\vartheta}} - \dot{r}_\eta \right) + \mathbf{z}_1^T \Phi_1 \tilde{\boldsymbol{\vartheta}} + \tilde{\boldsymbol{\vartheta}}^T \Gamma^{-1} \dot{\tilde{\boldsymbol{\vartheta}}} \quad (40)$$

$$= \mathbf{z}_1^T \left(\mathbf{R}(\psi)\alpha + \Phi_1 \hat{\boldsymbol{\vartheta}} - \dot{r}_\eta \right) + \tilde{\boldsymbol{\vartheta}}^T \left(\Phi_1^T \mathbf{z}_1 + \Gamma^{-1} \dot{\tilde{\boldsymbol{\vartheta}}} \right) \quad (41)$$

It is seen that choosing the control law for the virtual input α

$$\alpha = \mathbf{R}(\psi)^T \left(-\Phi_1 \hat{\boldsymbol{\vartheta}} + \dot{r}_\eta - \mathbf{K}_1 \mathbf{z}_1 \right) \quad (42)$$

$$= \mathbf{R}(\psi)^T \mathbf{f}_\alpha \quad (43)$$

Inserting α in (41) we get

$$\dot{V}_1 = -\mathbf{z}_1^T \mathbf{K}_1 \mathbf{z}_1 + \tilde{\boldsymbol{\vartheta}}^T \left(\Phi_1^T \mathbf{z}_1 + \Gamma^{-1} \dot{\tilde{\boldsymbol{\vartheta}}} \right) \quad (44)$$

And the adaptation law

$$\dot{\tilde{\boldsymbol{\vartheta}}} = -\Gamma \Phi_1^T \mathbf{z}_1 = -\Gamma \tau(\mathbf{z}_1) \quad (45)$$

where $\tau(\mathbf{z}_1) = \Phi_1^T \mathbf{z}_1$ is a tuning function, that must not to be confused with the input $\boldsymbol{\tau}$.

3.4 Step 1.2

$$\mathbf{z}_1 \triangleq \mathbf{R}(\psi)^T (\eta - r_\eta) \quad (46)$$

$$\dot{\mathbf{z}}_1 = \mathbf{R}(\psi)^T \mathbf{S}(r)^T (\eta - r_\eta) + \mathbf{R}(\psi)^T (\dot{\eta} - \dot{r}_\eta) \quad (47)$$

$$= \mathbf{S}(r)^T \mathbf{z}_1 + \mathbf{R}(\psi)^T (\dot{\eta} - \dot{r}_\eta) \quad (48)$$

$$= \mathbf{S}(r)^T \mathbf{z}_1 + \boldsymbol{\nu}_r + \mathbf{R}(\psi)^T (\Phi_1 \boldsymbol{\vartheta} - \dot{r}_\eta) \quad (49)$$

Choosing the Lyapunov function

$$V \triangleq \frac{1}{2} \mathbf{z}_1^T \mathbf{z}_1 + \frac{1}{2} \tilde{\boldsymbol{\vartheta}}^T \Gamma^{-1} \tilde{\boldsymbol{\vartheta}} \quad (50)$$

Which has the derivative

$$\dot{V} = \mathbf{z}_1^T \dot{\mathbf{z}}_1 + \tilde{\boldsymbol{\vartheta}}^T \Gamma^{-1} \dot{\tilde{\boldsymbol{\vartheta}}} \quad (51)$$

$$= \mathbf{z}_1^T \left(\mathbf{S}(r)^T \mathbf{z}_1 + \boldsymbol{\nu}_r + \mathbf{R}(\psi)^T (\Phi_1 \boldsymbol{\vartheta} - \dot{r}_\eta) \right) + \tilde{\boldsymbol{\vartheta}}^T \Gamma^{-1} \dot{\tilde{\boldsymbol{\vartheta}}} \quad (52)$$

$$= \mathbf{z}_1^T \left(\boldsymbol{\nu}_r + \mathbf{R}(\psi)^T (\Phi_1 \hat{\boldsymbol{\vartheta}} - \dot{r}_\eta) \right) + \mathbf{z}_1^T \mathbf{R}(\psi)^T \Phi_1 \tilde{\boldsymbol{\vartheta}} + \tilde{\boldsymbol{\vartheta}}^T \Gamma^{-1} \dot{\tilde{\boldsymbol{\vartheta}}} \quad (53)$$

$$= \mathbf{z}_1^T \left(\boldsymbol{\nu}_r + \mathbf{R}(\psi)^T (\Phi_1 \hat{\boldsymbol{\vartheta}} - \dot{r}_\eta) \right) + \tilde{\boldsymbol{\vartheta}}^T \left(\Gamma^{-1} \dot{\tilde{\boldsymbol{\vartheta}}} + \Phi_1^T \mathbf{R}(\psi) \mathbf{z}_1 \right) \quad (54)$$

It is seen that choosing the control law for the virtual input α

$$\alpha = -\mathbf{K}_1 \mathbf{z}_1 - \mathbf{R}(\psi)^T \left(\Phi_1 \hat{\boldsymbol{\vartheta}} - \dot{r}_\eta \right) \quad (55)$$

$$= \mathbf{R}(\psi)^T \mathbf{f}_\alpha \quad (56)$$

Inserting α in (41) we get

$$\dot{V}_1 = -\mathbf{z}_1^T \mathbf{K}_1 \mathbf{z}_1 + \tilde{\boldsymbol{\vartheta}}^T \left(\Phi_1^T \mathbf{z}_1 + \Gamma^{-1} \dot{\hat{\boldsymbol{\vartheta}}} \right) \quad (57)$$

And the adaptation law

$$\dot{\hat{\boldsymbol{\vartheta}}} = -\Gamma \Phi_1^T \mathbf{z}_1 = -\Gamma \tau(\mathbf{z}_1) \quad (58)$$

where $\tau(\mathbf{z}_1) = \Phi_1^T \mathbf{z}_1$ is a tuning function, that must not to be confused with the input $\boldsymbol{\tau}$.

3.5 Derivative of a rotation matrix

Consider the rotation matrix $R(\psi)$

$$\mathbf{R}(\psi) = \begin{bmatrix} \cos(\psi) & -\sin(\psi) & 0 \\ \sin(\psi) & \cos(\psi) & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (59)$$

And its derivative

$$\dot{\mathbf{R}}(\psi) = \frac{d}{d\psi} \mathbf{R}(\psi) \dot{\psi} = \frac{d}{d\psi} \mathbf{R}(\psi) r \quad (60)$$

$$\dot{\mathbf{R}}(\psi) = \begin{bmatrix} -\sin(\psi)r & -\cos(\psi)r & 0 \\ \cos(\psi)r & -\sin(\psi)r & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad (61)$$

Which can be written as

$$\dot{\mathbf{R}}(\psi) = \mathbf{S}(r) \mathbf{R}(\psi) \quad (62)$$

With

$$\mathbf{S}(r) = \begin{bmatrix} 0 & -r & 0 \\ r & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad (63)$$

This is easily verified by doing the matrix multiplication.

3.6 Step 2

Now we must ensure that $\boldsymbol{\nu}_r$ takes the value α . Thus the control objective of the second step is

$$\mathbf{z}_2 \triangleq \boldsymbol{\nu}_r - \alpha \quad (64)$$

$$\dot{\mathbf{z}}_2 = \dot{\boldsymbol{\nu}}_r - \dot{\alpha} \quad (65)$$

$$\dot{\alpha} = \mathbf{R}(\psi)^T \mathbf{S}(r)^T - \Phi_1 \dot{\hat{\boldsymbol{\vartheta}}} + \ddot{r}_\eta - \mathbf{K}_1 \dot{\mathbf{z}}_1 \quad (66)$$

$$\dot{\alpha} = \mathbf{R}(\psi)^T \mathbf{S}(r)^T \mathbf{f}_\alpha + \mathbf{R}(\psi)^T \dot{\mathbf{f}}_\alpha \quad (67)$$

$$= \mathbf{R}(\psi)^T \mathbf{S}(r)^T \mathbf{f}_\alpha + \mathbf{R}(\psi)^T \left(-\Phi_1 \dot{\hat{\boldsymbol{\vartheta}}} + \ddot{r}_\eta - \mathbf{K}_1 \dot{\mathbf{z}}_1 \right) \quad (68)$$

$$= \mathbf{R}(\psi)^T \mathbf{S}(r)^T \mathbf{f}_\alpha + \mathbf{R}(\psi)^T \left(-\Phi_1 \dot{\hat{\boldsymbol{\vartheta}}} + \ddot{r}_\eta - \mathbf{K}_1 (\mathbf{R}(\psi) \boldsymbol{\nu}_r + \Phi_1 \boldsymbol{\vartheta} - \dot{r}_\eta) \right) \quad (69)$$

$$= \mathbf{R}(\psi)^T \mathbf{S}(r)^T \mathbf{f}_\alpha + \mathbf{R}(\psi)^T \left(-\Phi_1 \dot{\hat{\boldsymbol{\vartheta}}} + \ddot{r}_\eta - \mathbf{K}_1 \Phi_1 \boldsymbol{\vartheta} + \mathbf{K}_1 \dot{r}_\eta \right) + \mathbf{K}_1 \boldsymbol{\nu}_r \quad (70)$$

Augmenting the Lyapunov function with a term for \mathbf{z}_2 we have

$$V_2 \triangleq V_1 + \frac{1}{2} \mathbf{z}_2^T \mathbf{z}_2 \quad (71)$$

Which has the derivative

$$\dot{V}_1 = \mathbf{z}_1^T \left(\mathbf{R}(\psi)(\mathbf{z}_2 + \boldsymbol{\alpha}) + \Phi_1 \hat{\boldsymbol{\vartheta}} - \dot{r}_\eta \right) + \tilde{\boldsymbol{\vartheta}}^T \left(\Phi_1^T \mathbf{z}_1 + \Gamma^{-1} \dot{\hat{\boldsymbol{\vartheta}}} \right) \quad (72)$$

$$\dot{V}_1 = \mathbf{z}_1^T \mathbf{R}(\psi) \mathbf{z}_2 - \mathbf{z}_1^T \mathbf{K}_1 \mathbf{z}_1 + \tilde{\boldsymbol{\vartheta}}^T \left(\Phi_1^T \mathbf{z}_1 + \Gamma^{-1} \dot{\hat{\boldsymbol{\vartheta}}} \right) \quad (73)$$

$$\dot{V} = \dot{V}_1 + \mathbf{z}_2^T \dot{\mathbf{z}}_2 \quad (74)$$

$$= \dot{V}_1 + \mathbf{z}_2^T \left(\mathbf{f}(\boldsymbol{\nu}_r) + \mathbf{M}^{-1} \boldsymbol{\tau} + \mathbf{M}^{-1} \Phi_2(\boldsymbol{\nu}_r) \boldsymbol{\vartheta} - \mathbf{R}(\psi)^T \mathbf{S}(r)^T + \Phi_1 \dot{\hat{\boldsymbol{\vartheta}}} - \ddot{r}_\eta + \mathbf{K}_1 \dot{\mathbf{z}}_1 \right) \quad (75)$$

$$= \dot{V}_1 + \mathbf{z}_2^T \left(\mathbf{f}(\boldsymbol{\nu}_r) + \mathbf{M}^{-1} \boldsymbol{\tau} + \mathbf{M}^{-1} \Phi_2(\boldsymbol{\nu}_r) \hat{\boldsymbol{\vartheta}} - \mathbf{R}(\psi)^T \mathbf{S}(r)^T + \Phi_1 \dot{\hat{\boldsymbol{\vartheta}}} - \ddot{r}_\eta + \mathbf{K}_1 \dot{\mathbf{z}}_1 \right) + \mathbf{z}_2^T \mathbf{M}^{-1} \Phi_2(\boldsymbol{\nu}_r) \tilde{\boldsymbol{\vartheta}} \quad (76)$$

$$(77)$$

Choosing the control law

$$\boldsymbol{\tau} = -\Phi_2(\boldsymbol{\nu}_r) \hat{\boldsymbol{\vartheta}} + \mathbf{M} \left(-\mathbf{K}_2 \mathbf{z}_2 - \mathbf{f}(\boldsymbol{\nu}_r) + \mathbf{R}(\psi)^T \mathbf{S}(r)^T - \Phi_1 \dot{\hat{\boldsymbol{\vartheta}}} + \ddot{r}_\eta - \mathbf{K}_1 \dot{\mathbf{z}}_1 - \mathbf{z}_1^T \mathbf{R}(\psi) \mathbf{z}_2 \right) \quad (78)$$

$$\dot{V} = -\mathbf{z}_1^T \mathbf{K}_1 \mathbf{z}_1 - \mathbf{z}_2^T \mathbf{K}_2 \mathbf{z}_2 + \tilde{\boldsymbol{\vartheta}}^T \left(\Phi_1^T \mathbf{z}_1 + \Phi_2(\boldsymbol{\nu}_r)^T \mathbf{M}^{-1} \mathbf{z}_2 + \Gamma^{-1} \dot{\hat{\boldsymbol{\vartheta}}} \right) \quad (79)$$

The adaptation law

$$\dot{\hat{\boldsymbol{\vartheta}}} = -\Gamma \left(\Phi_1^T \mathbf{z}_1 + \Phi_2(\boldsymbol{\nu}_r)^T \mathbf{M}^{-1} \mathbf{z}_2 \right) \quad (80)$$