1 The system

The vessel is modelled by

$$\boldsymbol{M}_{RB}\dot{\boldsymbol{\nu}} + \boldsymbol{M}_A\dot{\boldsymbol{\nu}}_{\boldsymbol{r}} + \boldsymbol{C}_{RB}(\boldsymbol{\nu})\boldsymbol{\nu} + \boldsymbol{C}_A(\boldsymbol{\nu}_r)\boldsymbol{\nu}_r + \boldsymbol{D}(\boldsymbol{\nu}_r)\boldsymbol{\nu}_r + \boldsymbol{G}\boldsymbol{\eta} = \boldsymbol{\tau} + \boldsymbol{w}(t) \tag{1}$$

$$\dot{\boldsymbol{\eta}} = \boldsymbol{J}(\eta)\boldsymbol{\nu} \tag{2}$$

where

$$\boldsymbol{M}_{RB} = \begin{bmatrix} m & 0 & 0 \\ 0 & m & 0 \\ 0 & 0 & I_z \end{bmatrix} \qquad \boldsymbol{M}_A = \begin{bmatrix} -X_{\dot{u}} & 0 & 0 \\ 0 & -Y_{\dot{v}} & -Y_{\dot{r}} \\ 0 & -N_{\dot{v}} & -N_{\dot{r}} \end{bmatrix} \tag{3}$$

$$\boldsymbol{C}_{RB} = \begin{bmatrix} 0 & 0 & -m(x_g r + v) \\ 0 & 0 & mu \\ m(x_g r + v) & -mu & 0 \end{bmatrix} \quad \boldsymbol{C}_A = \begin{bmatrix} 0 & 0 & Y_{\dot{v}}v_r + \frac{1}{2}(N_{\dot{v}}Y_{\dot{r}})r \\ 0 & 0 & -X_{\dot{u}}u_r \\ -Y_{\dot{v}}v_r + \frac{1}{2}(N_{\dot{v}}Y_{\dot{r}})r & X_{\dot{u}}u_r & 0 \end{bmatrix} \tag{4}$$

$$\boldsymbol{D}_L = \begin{bmatrix} -X_u & 0 & 0 \\ 0 & -Y_v & -Y_r \\ 0 & -N_v & -N_r \end{bmatrix} \quad \boldsymbol{D}_{NL} = \begin{bmatrix} -X_{|u|u}|u| - X_{uuu}u^2 & 0 & 0 \\ 0 & -Y_{|v|v}|v_r| - Y_{|r|v}|r| & -Y_{|v|r}|v_r| - Y_{|r|r}|r| \\ 0 & -N_{|v|v}|v_r| - N_{|r|v}|r| & -N_{|v|r}|v_r| - N_{|r|r}|r| \end{bmatrix} \tag{5}$$

Using property 8.1 of [Fossen] we can define the dynamics using only the water relative velocity $\boldsymbol{\nu}_r$

$$\boldsymbol{M}\dot{\boldsymbol{\nu}}_r + \boldsymbol{C}(\boldsymbol{\nu}_r)\boldsymbol{\nu}_r + \boldsymbol{D}(\boldsymbol{\nu}_r)\boldsymbol{\nu}_r = \boldsymbol{\tau} \tag{6}$$

$$\dot{\boldsymbol{\eta}} = \boldsymbol{J}(\eta)\boldsymbol{\nu}_r + \boldsymbol{\nu}_c \tag{7}$$

Where $\boldsymbol{\nu}_c$ is the velocity of the water current in global coordinates. And

$$\boldsymbol{M} = \boldsymbol{M}_{RB} + \boldsymbol{M}_A \tag{8}$$

$$\boldsymbol{C}(\boldsymbol{\nu}_r) = \boldsymbol{C}_{RB}(\boldsymbol{\nu}_r) + \boldsymbol{C}_A(\boldsymbol{\nu}_r) \tag{9}$$

Additionally $\boldsymbol{G}\boldsymbol{\eta} = \boldsymbol{0}$ as there exists no restoring forces in surge, sway and yaw. Isolating the acceleration $\dot{\boldsymbol{\nu}}_r$ in (6) we have

$$\dot{\boldsymbol{\nu}}_r = \boldsymbol{M}^{-1}(\boldsymbol{\tau} - \boldsymbol{C}(\boldsymbol{\nu}_r)\boldsymbol{\nu}_r - \boldsymbol{D}(\boldsymbol{\nu}_r)\boldsymbol{\nu}_r) \tag{10}$$

$$\dot{\boldsymbol{\eta}} = \boldsymbol{J}(\eta)\boldsymbol{\nu}_r + \boldsymbol{\nu}_c \tag{11}$$

Assuming that $\boldsymbol{M}$ and $\boldsymbol{C}$ as well as the linear damping coefficients X_u and N_r are known. The system equation can be split into a known part and a unknown or uncertain part. This can be written as

$$\dot{\boldsymbol{\nu}}_r = \boldsymbol{M}^{-1}(\boldsymbol{\tau} - \boldsymbol{C}(\boldsymbol{\nu}_r)\boldsymbol{\nu}_r + \boldsymbol{d}\boldsymbol{\nu}_r + \Phi(\boldsymbol{\nu}_r)\boldsymbol{\vartheta}) \tag{12}$$

$$\dot{\boldsymbol{\eta}} = \boldsymbol{J}(\eta)\boldsymbol{\nu}_r + \boldsymbol{\nu}_c \tag{13}$$

Where

$$\boldsymbol{d} = \begin{bmatrix} X_u & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & N_r \end{bmatrix} \tag{14}$$

And the uncertain part is described by

$$\Phi(\boldsymbol{\nu}_r)^T = \begin{bmatrix} |u|u & 0 & 0 \\ u^3 & 0 & 0 \\ 0 & v & 0 \\ 0 & r & 0 \\ 0 & |v|v & 0 \\ 0 & |r|v & 0 \\ 0 & |v|r & 0 \\ 0 & |r|r & 0 \\ 0 & 0 & v \\ 0 & 0 & |v|v \\ 0 & 0 & |r|v \\ 0 & 0 & |v|r \\ 0 & 0 & |r|r \end{bmatrix} \qquad \boldsymbol{\vartheta} = \begin{bmatrix} X_{|u|u} \\ X_{uuu} \\ Y_v \\ Y_r \\ Y_{|v|v} \\ Y_{|r|v} \\ Y_{|v|r} \\ Y_{|r|r} \\ N_v \\ N_{|v|v} \\ N_{|r|v} \\ N_{|v|r} \\ N_{|r|r} \end{bmatrix} \tag{15}$$

Lastly the known part is collected in a function $\boldsymbol{f}(\boldsymbol{\nu}_r)$.

$$\begin{aligned} \dot{\boldsymbol{\nu}}_r &= \boldsymbol{M}^{-1}(\boldsymbol{d}\boldsymbol{\nu}_r - \boldsymbol{C}(\boldsymbol{\nu}_r)\boldsymbol{\nu}_r) + \boldsymbol{M}^{-1}(\boldsymbol{\tau} + \Phi(\boldsymbol{\nu}_r)\boldsymbol{\vartheta}) && (16) \\ &= \boldsymbol{f}(\boldsymbol{\nu}_r) + \boldsymbol{M}^{-1}(\boldsymbol{\tau} + \Phi(\boldsymbol{\nu}_r)\boldsymbol{\vartheta}) && (17) \\ & && (18) \end{aligned}$$

The system is now in a form where the known part $\boldsymbol{f}(\boldsymbol{\nu}_r)$, the uncertain part $\Phi(\boldsymbol{\nu}_r)\boldsymbol{\vartheta}$ and the input $\boldsymbol{\tau}$ are clear to distinguish from one another.

2 Adaptive Control

2.1 Control Design

The control objective is to minimize $\tilde{\nu}(t) \triangleq \nu(t) - r_\nu(t)$. That is to minimize the difference (error) between the velocity of the vessel $\nu(t)$ and the reference velocity $r_\nu(t)$. A new variable $\boldsymbol{z}$ is introduced to describe this difference.

$$\boldsymbol{z} \triangleq \nu - r_\nu \tag{19}$$

Choosing the Lyapunov function

$$V \triangleq \frac{1}{2}\boldsymbol{z}^T\boldsymbol{z} + \frac{1}{2}\tilde{\boldsymbol{\vartheta}}^T\boldsymbol{\Gamma}^{-1}\tilde{\boldsymbol{\vartheta}} \tag{20}$$

Which

$$\begin{aligned} \dot{V} &= \boldsymbol{z}^T\dot{\boldsymbol{z}} + \tilde{\boldsymbol{\vartheta}}^T\boldsymbol{\Gamma}^{-1}\dot{\tilde{\boldsymbol{\vartheta}}} && (21) \\ &= \boldsymbol{z}^T\left(\dot{\boldsymbol{\nu}}_r - \dot{r_\nu}\right) + \tilde{\boldsymbol{\vartheta}}^T\boldsymbol{\Gamma}^{-1}\dot{\hat{\boldsymbol{\vartheta}}} && (22) \\ &= \boldsymbol{z}^T\left(\boldsymbol{f}(\boldsymbol{\nu}_r) + \boldsymbol{M}^{-1}\left(\boldsymbol{\tau} + \Phi(\boldsymbol{\nu}_r)\boldsymbol{\vartheta}\right) - \dot{r_\nu}\right) + \tilde{\boldsymbol{\vartheta}}^T\boldsymbol{\Gamma}^{-1}\dot{\hat{\boldsymbol{\vartheta}}} && (23) \\ &= \boldsymbol{z}^T\left(\boldsymbol{f}(\boldsymbol{\nu}_r) + \boldsymbol{M}^{-1}\left(\boldsymbol{\tau} + \Phi(\boldsymbol{\nu}_r)\hat{\boldsymbol{\vartheta}}\right) - \dot{r_\nu}\right) + \boldsymbol{z}^T\boldsymbol{M}^{-1}\Phi(\boldsymbol{\nu}_r)\tilde{\boldsymbol{\vartheta}} + \tilde{\boldsymbol{\vartheta}}^T\boldsymbol{\Gamma}^{-1}\dot{\hat{\boldsymbol{\vartheta}}} && (24) \\ &= \boldsymbol{z}^T\left(\boldsymbol{f}(\boldsymbol{\nu}_r) + \boldsymbol{M}^{-1}\left(\boldsymbol{\tau} + \Phi(\boldsymbol{\nu}_r)\hat{\boldsymbol{\vartheta}}\right) - \dot{r_\nu}\right) + \tilde{\boldsymbol{\vartheta}}^T\left(\boldsymbol{\Gamma}^{-1}\dot{\hat{\boldsymbol{\vartheta}}} + \Phi(\boldsymbol{\nu}_r)^T\boldsymbol{M}^{-T}\boldsymbol{z}\right) && (25) \\ & && (26) \end{aligned}$$

It is seen that choosing the control law

$$\boldsymbol{\tau} = \boldsymbol{M}\left(-\boldsymbol{K}\boldsymbol{z} - \boldsymbol{f}(\boldsymbol{\nu}_r) + \dot{r_\nu}\right) - \Phi(\boldsymbol{\nu}_r)\hat{\boldsymbol{\vartheta}} \tag{27}$$

And the adaptation law

$$\dot{\hat{\boldsymbol{\vartheta}}} = -\boldsymbol{\Gamma}\Phi(\boldsymbol{\nu}_r)^T \boldsymbol{M}^{-T}\boldsymbol{z} \tag{28}$$

Will makes the Lyapunov function derivative become

$$\dot{V} = -\boldsymbol{z}^T\boldsymbol{K}\boldsymbol{z} \tag{29}$$

Which is negative definite if $\boldsymbol{K}$ is positive definite.