1 Mass- and Coriolis matrices of the 6 DOF model

In the case of no payload and no current we have

$$M_{RB} = \begin{bmatrix} 55 & 0 & 0 & 0 & -11 & 0 \\ 0 & 55 & 0 & 11 & 0 & 11 \\ 0 & 0 & 55 & 0 & -11 & 0 \\ 0 & 11 & 0 & 14.6643 & 0 & 4.4000 \\ -11 & 0 & -11 & 0 & 22.5500 & 0 \\ 0 & 11 & 0 & 4.4000 & 0 & 18.1500 \end{bmatrix}$$

$C_{RB} =$

$$\begin{bmatrix} 0 & -55\,r & 55\,q & -11\,r & -11\,q & -11\,r \\ 55\,r & 0 & -55\,p & 0 & 11\,p - 11\,r & 0 \\ -55\,q & 55\,p & 0 & 11\,p & 11\,q & 11\,p \\ 11\,r & 0 & -11\,p & 0 & 4.4000\,p + 13.7500\,r & -18.1500\,q \\ 11\,q & 11\,r - 11\,p & -11\,q & -4.4000\,p - 13.7500\,r & 0 & 10.2643\,p + 4.4000\,r \\ 11\,r & 0 & -11\,p & 18.1500\,q & -10.2643\,p - 4.4000\,r & 0 \end{bmatrix}$$

$$M_A = \begin{bmatrix} 5.5000 & 0 & 0 & 0 & 0 & 0 \\ 0 & 82.5000 & 0 & 0 & 0 & 0 \\ 0 & 0 & 55 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2.4929 & 0 & 0 \\ 0 & 0 & 0 & 0 & 14.5200 & 0 \\ 0 & 0 & 0 & 0 & 0 & 27.1150 \end{bmatrix}$$

$$C_A = \begin{bmatrix} 0 & 0 & 0 & 0 & 55\,w & -82.5000\,v \\ 0 & 0 & 0 & -55\,w & 0 & 5.5000\,u \\ 0 & 0 & 0 & 82.5000\,v & -5.5000\,u & 0 \\ 0 & 55\,w & -82.5000\,v & 0 & 27.1150\,r & -14.5200\,q \\ -55\,w & 0 & 5.5000\,u & -27.1150\,r & 0 & 2.4929\,p \\ 0 & 0 & 0 & 14.5200\,q & -2.4929\,p & 0 \end{bmatrix}$$

$$D = \begin{bmatrix} 77.5544\,u & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 546.4805\,w & 0 & 0 & 0 \\ 0 & 0 & 0 & 54.3823\,p & 0 & 0 \\ 0 & 0 & 0 & 0 & 246.0496\,q & 0 \\ 0 & 0 & 0 & 0 & 0 & r\,(452.6500\,|r| + 45.2650) \end{bmatrix}$$

2 Simple state truncation

$$\dot{\nu} = A\nu + B\tau$$

$A = -(M_{RB} + M_A)^{-1}(C_{RB} + C_A + D) =$

$$\begin{bmatrix} 0.2944\,w - 1.3574\,u - 0.0294\,q & 0.0294\,p + 0.9037\,r & 0.1690\,r - 0.0742\,p + 1.4439\,v \\ -0.3601\,r & 0.2532\,w & 0.0651\,r - 0.1504\,q - 0.0427\,u + 0.6506\,r\,|r| \\ -0.1186\,r & 0.2726\,w & 0.0079\,u - 1.0376\,r - 0.1620\,q - 10.3762\,r\,|r| \end{bmatrix}$$

$$B = (M_{RB} + M_A)^{-1} = \begin{bmatrix} 0.0175 & 0 & 0 \\ 0 & 0.0078 & -0.0014 \\ 0 & -0.0014 & 0.0229 \end{bmatrix}$$

where the heave, roll and pitch dimensions has been removed from A and B

2.1 Reduction

Assiming that

$$\begin{aligned} w &= 0 \\ p &= 0 \\ q &= 0 \end{aligned}$$

The system equations simplify to

$$A(u, v, r) = \begin{bmatrix} -1.3574\,u & 0.9037\,r & 0.1690\,r + 1.4439\,v \\ -0.3601\,r & 0 & 0.0651\,r - 0.0427\,u + 0.6506\,r\,|r| \\ -0.1186\,r & 0 & 0.0079\,u - 1.0376\,r - 10.3762\,r\,|r| \end{bmatrix} \tag{1}$$

$$B = \begin{bmatrix} 0.0175 & 0 & 0 \\ 0 & 0.0078 & -0.0014 \\ 0 & -0.0014 & 0.0229 \end{bmatrix} \tag{2}$$

3 Selection matrix approch

$$C^*_{RB} = U M_{RB} L \qquad\qquad C^*_A = U M_A L \tag{3}$$

Where U is the surge of the vessel at the linarization point and L is a selection matrix seen in HMCHMC (3.63). For this leariztion $U = 3$

$$M_{RB} = \begin{bmatrix} 55 & 0 & 0 \\ 0 & 55 & 11 \\ 0 & 11 & 18.1500 \end{bmatrix} \qquad M_A = \begin{bmatrix} 5.5000 & 0 & 0 \\ 0 & 82.5000 & 0 \\ 0 & 0 & 27.1150 \end{bmatrix} \tag{4}$$

$$C^*_{RB} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 165 \\ 0 & 0 & 33 \end{bmatrix} \qquad C^*_A = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 247.5000 \\ 0 & 0 & 0 \end{bmatrix} \tag{5}$$

$$D^* = \begin{bmatrix} 232.6633 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 497.9150 \end{bmatrix} \tag{6}$$

$D*$ has been linearized in yaw around the point $r = 1$

$$\dot{\nu} = A\nu + B\tau \tag{7}$$

$$M = M_{RB} + M_A \tag{8}$$

$$N = C^*_{RB} + C^*_A + D \tag{9}$$

$$A = -M^{-1}N \tag{10}$$

$$B = M^{-1} \tag{11}$$

$$A = \begin{bmatrix} -3.8457 & 0 & 0 \\ 0 & 0 & -2.1026 \\ 0 & 0 & -11.2181 \end{bmatrix} \qquad B = \begin{bmatrix} 0.0165 & 0 & 0 \\ 0 & 0.0074 & -0.0018 \\ 0 & -0.0018 & 0.0225 \end{bmatrix} \tag{12}$$