1 Kinematics

$$M\dot{\nu} + C(\nu)\nu + D(\nu)\nu + g(\eta) + g_0 = \tau \tag{1}$$

$$M = M_{RB} + M_A$$
$$C = C_{RB} + C_A$$

$$\tau = \tau_{hydrodynamics} + \tau_{hydrostatics} + \tau_{wind} + \tau_{wave} + \tau_{control} \tag{2}$$

1.1 Control

$$\tau_{control} = \begin{bmatrix} \tau_{control,linear} \\ \tau_{control,torque} \end{bmatrix} \tag{3}$$

Each propeller can rotate with an angle ξ_p. The thrust of each propeller can be describe as a vector by spilting it into components that align with the ship coordinate frame

$$T_p = \begin{bmatrix} \cos(\xi_p) \\ \sin(\xi_p) \\ 0 \end{bmatrix} t_p \tag{4}$$

where t_p is the magnitude of the thrust and T_p is the thrust vector. The linear control force is found as the sum of forces from each propeller p.

$$\tau_{control,linear} = \sum^{P} T_p \tag{5}$$

$$\tau_{control,torque} = \sum^{P} r_p \times T_p = \sum^{P} S(r_p)T_p \tag{6}$$

where r_p is the position vector of thruster p and $S(r_p)$ is the skew-symetric matrix of vector r_p.

In the case of two propellers we have

$$\tau_{control} = \begin{bmatrix} I & I \\ S(r_1) & S(r_2) \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \end{bmatrix} \tag{7}$$

2 Thruster model

$$\begin{bmatrix} T_1 & T_2 \end{bmatrix} = \begin{bmatrix} \cos(\xi_1) & \cos(\xi_2) \\ \sin(\xi_1) & \sin(\xi_2) \\ 0 & 0 \end{bmatrix} \begin{bmatrix} t_1 & 0 \\ 0 & t_2 \end{bmatrix} \tag{8}$$

$$\begin{bmatrix} t_1 \\ t_2 \end{bmatrix} = \begin{bmatrix} T_{nn}n_1 & T_{nv}V_A \\ T_{nn}n_2 & T_{nv}V_A \end{bmatrix} \begin{bmatrix} n_1 \\ n_2 \end{bmatrix} \tag{9}$$

$$\tau_{control} = \begin{bmatrix} I & I \\ S(r_1) & S(r_2) \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \end{bmatrix} \tag{10}$$