

1 Mass- and Coriolis matrices of the 6 DOF model

In the case of no payload and no current we have

$$M_{RB} = \begin{bmatrix} 55 & 0 & 0 & 0 & -11 & 0 \\ 0 & 55 & 0 & 11 & 0 & 11 \\ 0 & 0 & 55 & 0 & -11 & 0 \\ 0 & 11 & 0 & 14.6643 & 0 & 4.4000 \\ -11 & 0 & -11 & 0 & 22.5500 & 0 \\ 0 & 11 & 0 & 4.4000 & 0 & 18.1500 \end{bmatrix}$$

$$C_{RB} =$$

$$\begin{bmatrix} 0 & -55r & 55q & -11r & -11q & -11r \\ 55r & 0 & -55p & 0 & 11p - 11r & 0 \\ -55q & 55p & 0 & 11p & 11q & 11p \\ 11r & 0 & -11p & 0 & 4.4000p + 13.7500r & -18.1500q \\ 11q & 11r - 11p & -11q & -4.4000p - 13.7500r & 0 & 10.2643p + 4.4000r \\ 11r & 0 & -11p & 18.1500q & -10.2643p - 4.4000r & 0 \end{bmatrix}$$

$$M_A = \begin{bmatrix} 5.5000 & 0 & 0 & 0 & 0 & 0 \\ 0 & 82.5000 & 0 & 0 & 0 & 0 \\ 0 & 0 & 55 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2.4929 & 0 & 0 \\ 0 & 0 & 0 & 0 & 14.5200 & 0 \\ 0 & 0 & 0 & 0 & 0 & 27.1150 \end{bmatrix}$$

$$C_A = \begin{bmatrix} 0 & 0 & 0 & 0 & 55w & -82.5000v \\ 0 & 0 & 0 & -55w & 0 & 5.5000u \\ 0 & 0 & 0 & 82.5000v & -5.5000u & 0 \\ 0 & 55w & -82.5000v & 0 & 27.1150r & -14.5200q \\ -55w & 0 & 5.5000u & -27.1150r & 0 & 2.4929p \\ 0 & 0 & 0 & 14.5200q & -2.4929p & 0 \end{bmatrix}$$

$$D = \begin{bmatrix} 77.5544u & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 546.4805w & 0 & 0 & 0 \\ 0 & 0 & 0 & 54.3823p & 0 & 0 \\ 0 & 0 & 0 & 0 & 246.0496q & 0 \\ 0 & 0 & 0 & 0 & 0 & r(452.6500|r| + 45.2650) \end{bmatrix}$$

2 Sytem equations

$$\dot{\boldsymbol{\nu}} = (\mathbf{M}_{RB} + \mathbf{M}_A)^{-1}(-\mathbf{C}_{RB} - \mathbf{C}_A - \mathbf{D})\boldsymbol{\nu} \quad (1)$$

$$\dot{\boldsymbol{\nu}} = f(\boldsymbol{\nu}) = f(u, v, w, p, q, r) \quad (2)$$

$$\dot{\boldsymbol{\nu}} = [\dot{u} \ \dot{v} \ \dot{w} \ \dot{p} \ \dot{q} \ \dot{r}]^T \quad (3)$$

$$\begin{aligned} \dot{u} = & 0.0177p^2 + 0.3371pr - 0.0147vp - 1.1303q^2 - 0.0265qu - 1.8664qw + \\ & 0.1690r^2 + 2.3477vr - 1.3574u^2 + 0.2649uw - 0.2925w^2 \end{aligned} \quad (4)$$

$$\dot{v} = 0.6506 r^2 |r| - 0.0364 p q + 0.1167 q r + 0.7867 p w - 0.4028 r u - 0.1266 v w + 0.2504 p^2 + 0.0651 r^2 \quad (5)$$

$$\dot{w} = -0.0903 p^2 - 0.0146 p r - 1.2581 v p - 0.8217 q^2 + 0.5354 q u - 0.0265 q w - 0.0071 r^2 + 0.0412 v r - 0.0415 u^2 + 0.1457 u w - 5.1289 w^2 \quad (6)$$

$$\dot{p} = 2.2439 r^2 |r| - 0.1257 p q - 0.5847 q r + 0.1266 p w - 0.3545 r u + 1.7190 v w - 3.3993 p^2 + 0.2244 r^2 \quad (7)$$

$$\dot{q} = 0.0971 p^2 + 0.8539 p r - 0.0810 v p - 7.2166 q^2 - 0.1457 q u - 0.2649 q w - 0.0707 r^2 + 0.4121 v r - 0.4151 u^2 + 1.4572 u w - 1.6087 w^2 \quad (8)$$

$$\dot{r} = 0.1257 q r - 0.4188 p q - 10.3762 r^2 |r| + 0.0395 p w - 0.1107 r u - 0.1363 v w + 0.2696 p^2 - 1.0376 r^2 \quad (9)$$

$$A = \frac{df}{d\nu} \begin{bmatrix} 0.2649 w - 2.7147 u - 0.0265 q & 2.3477 r - 0.0147 p & 0.3371 p + 0.3379 r \\ -0.4028 r & -0.1266 w & 0.1167 q + 0.0651 r - 0.4028 u + r (0.6506 |r| + 0.2504 p) \\ -0.1107 r & -0.1363 w & 0.1257 q - 1.0376 r - 0.1107 u - r (10.3762 |r| + 1.0376 p) \end{bmatrix}$$

3 Simple state truncation

$$\dot{\nu} = A\nu + B\tau$$

$$A = -(M_{RB} + M_A)^{-1}(C_{RB} + C_A + D) = \begin{bmatrix} 0.2944 w - 1.3574 u - 0.0294 q & 0.0294 p + 0.9037 r & 0.1690 r - 0.0742 p + 1.4439 v \\ -0.3601 r & 0.2532 w & 0.0651 r - 0.1504 q - 0.0427 u + 0.6506 r |r| \\ -0.1186 r & 0.2726 w & 0.0079 u - 1.0376 r - 0.1620 q - 10.3762 r |r| \end{bmatrix}$$

$$B = (M_{RB} + M_A)^{-1} = \begin{bmatrix} 0.0175 & 0 & 0 \\ 0 & 0.0078 & -0.0014 \\ 0 & -0.0014 & 0.0229 \end{bmatrix}$$

where the heave, roll and pitch dimensions has been removed from A and B

3.1 Reduction

Assuming that

$$\begin{aligned}w &= 0 \\p &= 0 \\q &= 0\end{aligned}$$

The system equations simplify to

$$A(u, v, r) = \begin{bmatrix} -1.3574 u & 0.9037 r & 0.1690 r + 1.4439 v \\ -0.3601 r & 0 & 0.0651 r - 0.0427 u + 0.6506 r |r| \\ -0.1186 r & 0 & 0.0079 u - 1.0376 r - 10.3762 r |r| \end{bmatrix} \quad (11)$$

$$B = \begin{bmatrix} 0.0175 & 0 & 0 \\ 0 & 0.0078 & -0.0014 \\ 0 & -0.0014 & 0.0229 \end{bmatrix} \quad (12)$$

4 Selection matrix approach

$$C_{RB}^* = U M_{RB} L \quad C_A^* = U M_A L \quad (13)$$

Where U is the surge of the vessel at the linearization point and L is a selection matrix seen in HMCHMC (3.63). For this linearization $U = 3$

$$M_{RB} = \begin{bmatrix} 55 & 0 & 0 \\ 0 & 55 & 11 \\ 0 & 11 & 18.1500 \end{bmatrix} \quad M_A = \begin{bmatrix} 5.5000 & 0 & 0 \\ 0 & 82.5000 & 0 \\ 0 & 0 & 27.1150 \end{bmatrix} \quad (14)$$

$$C_{RB}^* = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 165 \\ 0 & 0 & 33 \end{bmatrix} \quad C_A^* = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 247.5000 \\ 0 & 0 & 0 \end{bmatrix} \quad (15)$$

$$D^* = \begin{bmatrix} 232.6633 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 497.9150 \end{bmatrix} \quad (16)$$

D^* has been linearized in yaw around the point $r = 1$

$$\dot{\nu} = A\nu + B\tau \quad (17)$$

$$M = M_{RB} + M_A \quad (18)$$

$$N = C_{RB}^* + C_A^* + D \quad (19)$$

$$A = -M^{-1}N \quad (20)$$

$$B = M^{-1} \quad (21)$$

$$A = \begin{bmatrix} -3.8457 & 0 & 0 \\ 0 & 0 & -2.1026 \\ 0 & 0 & -11.2181 \end{bmatrix} \quad B = \begin{bmatrix} 0.0165 & 0 & 0 \\ 0 & 0.0074 & -0.0018 \\ 0 & -0.0018 & 0.0225 \end{bmatrix} \quad (22)$$