1 Kinematics

$$\boldsymbol{M}_{RB}\dot{\boldsymbol{\nu}} + \boldsymbol{M}_A\dot{\boldsymbol{\nu}}_{\boldsymbol{r}} + \boldsymbol{C}_A(\boldsymbol{\nu})\boldsymbol{\nu} + \boldsymbol{C}_{RB}(\boldsymbol{\nu}_r)\boldsymbol{\nu}_r + \boldsymbol{D}(\boldsymbol{\nu}_r)\boldsymbol{\nu}_r + \boldsymbol{G}\boldsymbol{\eta} = \boldsymbol{\tau} + \boldsymbol{w}(t) \tag{1}$$

$$\dot{\boldsymbol{\eta}} = \boldsymbol{J}(\eta)\boldsymbol{\nu} \tag{2}$$

2 State space

$$\dot{\boldsymbol{\nu}} = \boldsymbol{A}\boldsymbol{\nu} + \boldsymbol{G}\boldsymbol{\eta} + \boldsymbol{B}\boldsymbol{\tau} \tag{3}$$

$$\dot{\boldsymbol{\eta}} = \boldsymbol{J}(\eta)\boldsymbol{\nu} \tag{4}$$

Where

$$\boldsymbol{A} = -\boldsymbol{M}^{-1}(\boldsymbol{C} + \boldsymbol{D}) \tag{5}$$

$$\boldsymbol{B} = \boldsymbol{M}^{-1} \tag{6}$$

$$\boldsymbol{M} = \boldsymbol{M}_{RB} + \boldsymbol{M}_A \tag{7}$$

$$\boldsymbol{C} = \boldsymbol{C}_{RB} + \boldsymbol{C}_A \tag{8}$$

Notice that $\boldsymbol{A}$ still depends on the $\boldsymbol{\nu}$ and $\boldsymbol{\nu}_r$ even though it is not shown. Combining (3) and (4) gives us a state space description of the whole system

$$\dot{\boldsymbol{x}} = \begin{bmatrix} \boldsymbol{A} & \boldsymbol{G} \\ \boldsymbol{J}(\eta) & \boldsymbol{0} \end{bmatrix} \boldsymbol{x} + \begin{bmatrix} \boldsymbol{B} \\ \boldsymbol{0} \end{bmatrix} \boldsymbol{\tau} \tag{9}$$

where

$$\boldsymbol{x} = \begin{bmatrix} \boldsymbol{\nu} \\ \boldsymbol{\eta} \end{bmatrix} \tag{10}$$

2.1 Measurements

Assuming that the vessel is fitted with a compass, a GPS and a speedometer in surge.

$$\boldsymbol{y} = \boldsymbol{C}_m\boldsymbol{x} \tag{11}$$

$$\boldsymbol{C}_m = \begin{bmatrix} 1 & 0 & 0 & \dots \\ 0 & 0 & 0 & \dots \\ 0 & 0 & 0 & \dots \\ \vdots & \vdots & \vdots & I \end{bmatrix} \tag{12}$$

this is above

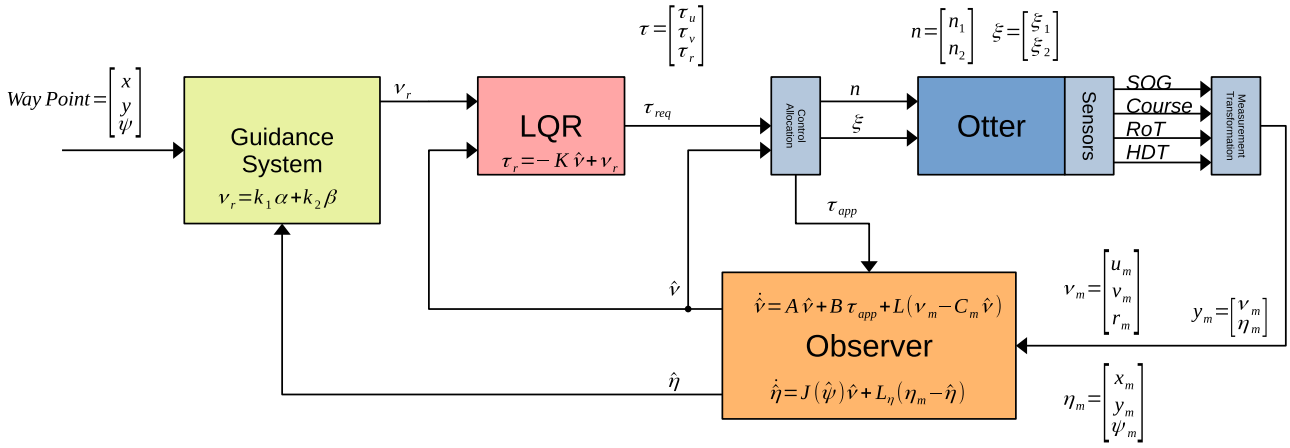

Figure 1:

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