1 Constants and Dimensions

Symbol	Value	Unit	Description
g	9.81	$\frac{m}{s^2}$	acceleration of gravity
ρ	1025	$\frac{kg}{m^3}$	density of water
L	2.0	m	length of hull
B	1.08	m	beam of hull
m	55.0	kg	mass of hull
r_g^{hull}	$\begin{bmatrix}0.2 & 0 & -0.2\end{bmatrix}^T$	m	CG of hull
R_{44}	$0.4 \cdot B$	m	radius of gyration
R_{55}	$0.25 \cdot L$	m	radius of gyration
R_{66}	$0.25 \cdot L$	m	radius of gyration
T_{yaw}	1	s	time constant in yaw
U_{max}	6	$knot$	max forward speed
B_{pont}	0.25	m	beam of one pontoon
y_{pont}	0.395	m	distance from centerline to waterline area center
Cw_{pont}	0.75	$-$	waterline area coefficient
Cb_{pont}	0.4	$-$	block coefficient

Waterline area of one pontoon

$$Aw_{pont} = Cw_{pont} L B_{pont} \tag{1}$$

2 Skew symetric matrix

$$S\left(\begin{bmatrix}a_1\\a_2\\a_3\end{bmatrix}\right) = \begin{bmatrix}0 & -a_3 & a_2\\ a_3 & 0 & -a_1\\ -a_2 & a_1 & 0\end{bmatrix} \tag{2}$$

3 Kinetics

$$\nu_1 = \begin{bmatrix}u & v & w\end{bmatrix}^T \qquad \nu_2 = \begin{bmatrix}p & q & r\end{bmatrix}^T \tag{3}$$

Inertia matrix of hull in CG

$$I_g^{CG} = m \cdot \mathrm{diag}\left[R_{44}^2, R_{55}^2, R_{66}^2\right] \tag{4}$$

CG location corrected for payload

$$r_g = \frac{m \cdot r_g^{hull} + m_p \cdot r_p}{m + mp} \tag{5}$$

Inertia matrix of hull and payload in CO

$$I_g = I_g^{CG} - m \cdot S(r_g)^2 - m_p \cdot S(r_p)^2 \tag{6}$$

$$M_{RB}^{CG} = \begin{bmatrix}(m+m_p)I & 0\\ 0 & I_g\end{bmatrix} \qquad C_{RB}^{CG}(\nu_2) = \begin{bmatrix}(m+m_p)S(\nu_2) & 0\\ 0 & -S(I_g \nu_2)\end{bmatrix} \tag{7}$$

Transform M_{RB} and C_{RB} from the C_G to the C_O

$$H = \begin{bmatrix} I & S(r_g)^T \\ 0 & I \end{bmatrix} \tag{8}$$

$$\begin{aligned} M_{RB} &= H^T M_{RB}^{CG} H && (9) \\ C_{RB}(\nu_2) &= H^T C_{RB}^{CG}(\nu_2) H && (10) \\ & && (11) \end{aligned}$$

4 Relative velocity

Water current surge and sway velocity

$$\begin{aligned} u_c &= v_{cur} \cos(\beta_{cur} - \psi) && (12) \\ v_c &= v_{cur} \sin(\beta_{cur} - \psi) && (13) \end{aligned}$$

Where v_{cur} is the current velocity, β_{cur} is the current direction in *rad* and ψ is the yaw of the vessel. Relative velocity vector

$$\nu_r = \nu - \begin{bmatrix} u_c & v_c & 0 & 0 & 0 & 0 \end{bmatrix}^T \tag{14}$$

In the case of no current we have

$$\nu_r = \nu \tag{15}$$

5 Hydrodynamics

Hydrodynamic added mass

$$M_A = \begin{bmatrix} mI & 0 \\ 0 & I_g \end{bmatrix} M_{A,coef} \tag{16}$$

$$M_{A,coef} = \mathrm{diag}\left(\begin{bmatrix} 0.1 & 1.5 & 1.0 & 0.2 & 0.8 & 1.7 \end{bmatrix}\right) \tag{17}$$

$$\begin{aligned} C_A(\nu_{r,1}, \nu_{r,2}) &= \begin{bmatrix} 0 & -S(M_{A,11}\nu_{r,1} + M_{A,12}\nu_{r,2}) \\ -S(M_{A,11}\nu_{r,1} + M_{A,12}\nu_{r,2}) & -S(M_{A,21}\nu_{r,1} + M_{A,22}\nu_{r,2}) \end{bmatrix} && (18) \\ &= \begin{bmatrix} 0 & -S(0.1 \cdot m\nu_{r,1}) \\ -S(0.1 \cdot m\nu_{r,1}) & -S(1.5 \cdot m\nu_{r,2}) \end{bmatrix} && (19) \end{aligned}$$

System mass and Coriolis-centripetal matrices

$$\begin{aligned} M &= M_{RB} + M_A && (20) \\ C &= C_{RB}(\nu_2) + C_A(\nu_{r,1}, \nu_{r,2}) && (21) \end{aligned}$$

6 Hydro statics

Water volume displacement

$$\nabla = \frac{m + m_p}{\rho} \tag{22}$$

Draft

$$T = \frac{\nabla}{2Cb_{pont}B_{pont}L} \tag{23}$$

$$KB = \frac{1}{3}(5\frac{T}{2} - \frac{\nabla}{2LB_{pont}}); \tag{24}$$

$$I_T = \frac{2}{12}LB_{pont}^3 \frac{6 \cdot Cw_{pont}^3}{(1 + Cw_{pont})(1 + 2Cw_{pont})} + 2 \cdot Aw_{pont}y_{pont}^2 \tag{25}$$

$$I_L = \frac{0.8 \cdot 2}{12}B_{pont}L^3 \tag{26}$$

$$GM_T = KB + \frac{I_T}{\nabla} - T + r_{g,z} \tag{27}$$

$$GM_L = KB + \frac{I_L}{\nabla} - T + r_{g,z} \tag{28}$$

$$\tag{29}$$

$$G_{CF} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \rho g(2Aw_{pont}) & 0 & 0 & 0 \\ 0 & 0 & 0 & \rho g \nabla GM_T & 0 & 0 \\ 0 & 0 & 0 & 0 & \rho g \nabla GM_L & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{30}$$

$$G = H^T G_{CF} H \tag{31}$$

$$\omega_3 = \sqrt{G_{33}/M_{33}} \tag{32}$$

$$\omega_4 = \sqrt{G_{44}/M_{44}} \tag{33}$$

$$\omega_5 = \sqrt{G_{55}/M_{55}} \tag{34}$$

7 Linear Damping

$$h(r) = \begin{bmatrix} -24.4\frac{g}{U_{max}} \\ 0 \\ -2 \cdot 0.3 \cdot \omega_3 M_{33} \\ -2 \cdot 0.2 \cdot \omega_4 M_{44} \\ -2 \cdot 0.4 \cdot \omega_5 M_{55} \\ \frac{-M_{66}}{T_{yaw}} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \frac{-M_{66}}{T_{yaw}} 10 \cdot abs(r) \end{bmatrix} \tag{35}$$

The matrix to the right includes non-linear dampimg for yaw

$$\tau_{damp}(r) = h(r) \bullet \nu_r \tag{36}$$

8 Crossflow Drag

The crossflow is computed using strip theory and is a function of ν_r

$$\tau_{cf} = \begin{bmatrix} 0 \\ Yh \\ 0 \\ 0 \\ 0 \\ Nh \end{bmatrix} \tag{37}$$

τ_{cf} Has components only in v and r

First-order fitting

$$\boldsymbol{\tau}_{cf} = \begin{bmatrix} 0 \\ 1.2907e-16\,r - 12.2363\,v - 4.8735e-17 \\ 0 \\ 0 \\ 0 \\ -5.9740\,r - 5.1780e-16\,v - 3.6373e-19 \end{bmatrix} \approx \begin{bmatrix} 0 \\ -12.2363\,v \\ 0 \\ 0 \\ 0 \\ -5.9740\,r \end{bmatrix}$$

Second-order fitting

$$\boldsymbol{\tau}_{cf} = \begin{bmatrix} 0 \\ 0.1824\,r - 0.0666\,v - 41.8276\,r\,|r| - 1.6409\,r\,|v| - 122.0516\,v\,|v| + 0.0019 \\ 0 \\ 0 \\ 0 \\ 0.0535\,v - 0.1471\,r - 2.5043\,r\,|r| - 88.3910\,r\,|v| - 0.2423\,v\,|v| - 0.0015 \end{bmatrix}$$

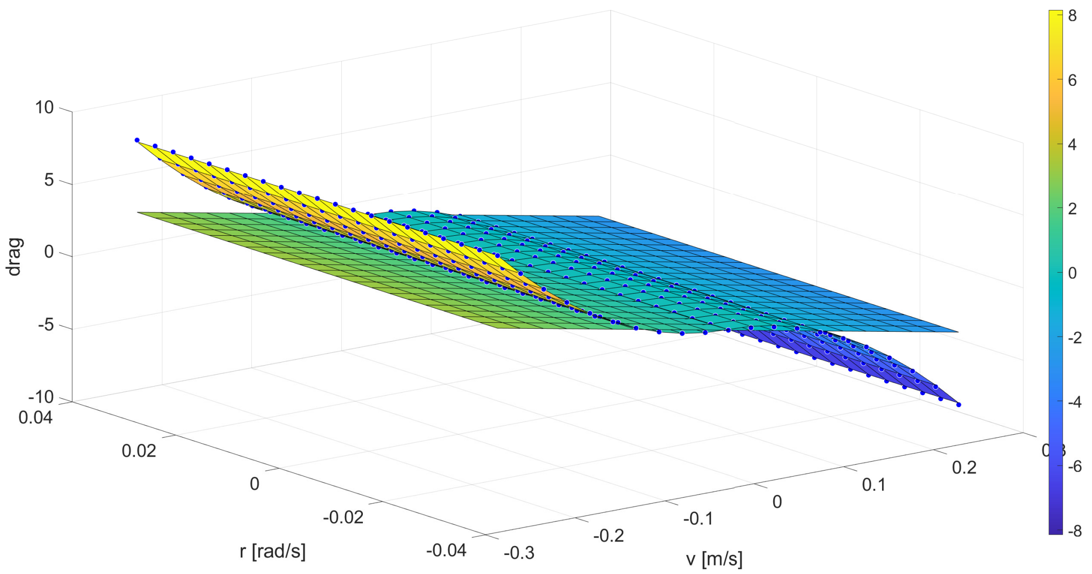

Figure 1: Linearized crossflow damping in sway

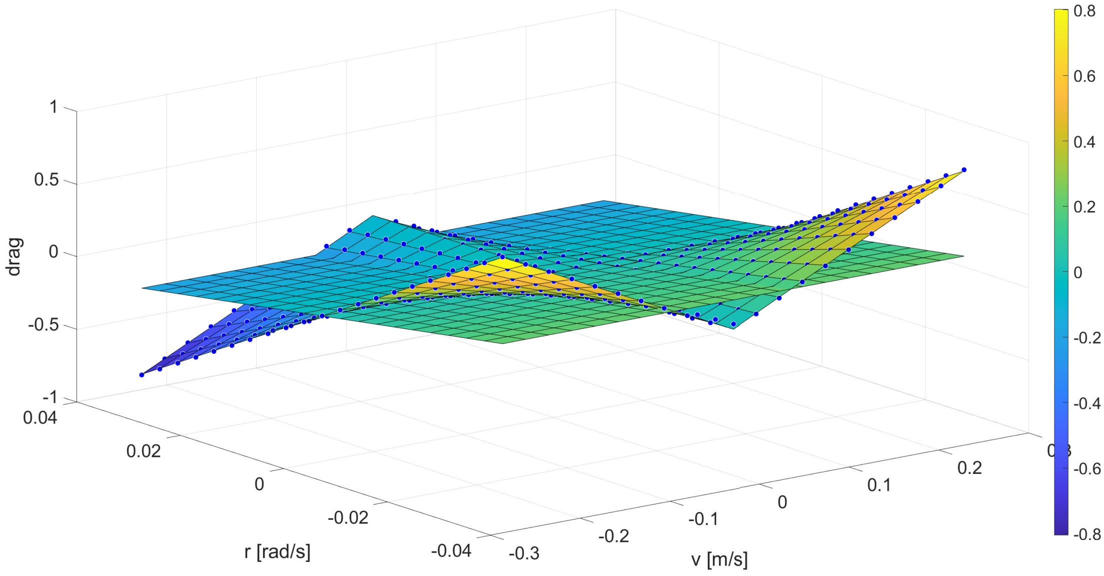

Figure 2: Linearized crossflow damping in yaw

9 Ballast

```
trim_setpoint = 280;
persistent trim_moment;
if isempty(trim_moment)
trim_moment = 0;
end
.
.
g_0 = [0 0 0 0 trim_moment 0]';
.
.
trim_moment = trim_moment + 0.05 * (trim_setpoint - trim_moment);
```

$$g_0 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ trim\_moment \\ 0 \end{bmatrix} \tag{38}$$

g_0 represents a dynamic torque in q (pitch)

10 Transformation

$$R = R_z R_y R_x = \begin{bmatrix} \cos(\psi) & -\sin(\psi) & 0 \\ \sin(\psi) & \cos(\psi) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos(\theta) & 0 & \sin(\theta) \\ 0 & 1 & 0 \\ -\sin(\theta) & & \cos(\theta) \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(\phi) & -\sin(\phi) \\ 0 & \sin(\phi) & \cos(\phi) \end{bmatrix} \tag{39}$$

$$T = \begin{bmatrix} 1 & \sin(\phi)\tan(\theta) & \cos(\phi)\tan(\theta) \\ 0 & \cos(\phi) & -\sin(\phi) \\ 0 & \dfrac{\sin(\phi)}{\cos(\theta)} & \dfrac{\cos(\phi)}{\cos(\theta)} \end{bmatrix} \tag{40}$$

$$J = \begin{bmatrix} R & 0 \\ 0 & T \end{bmatrix} \tag{41}$$

11 State derivative

$$M\dot{\nu} + C\nu_r + G\eta + g_0 = \tau + \tau_{damp} + \tau_{cf} \tag{42}$$

$$\dot{\eta} = J(\eta)\nu \tag{43}$$

$$\dot{x} = \begin{bmatrix} \dot{\nu} \\ \dot{\eta} \end{bmatrix} = \begin{bmatrix} M^{-1}(\tau + \tau_{damp}(r) + \tau_{cf}(\nu_r) - C\nu_r - G\eta - g_0) \\ J\nu \end{bmatrix} \tag{44}$$

$$\begin{aligned} M &= H^T M_{RB}^{CG} H + M_A \\ &= \begin{bmatrix} I & 0 \\ S(r_g) & I \end{bmatrix} \begin{bmatrix} (m+m_p)I & 0 \\ 0 & I_g \end{bmatrix} \begin{bmatrix} I & S(r_g)^T \\ 0 & I \end{bmatrix} + \begin{bmatrix} mI & 0 \\ 0 & I_g \end{bmatrix} M_{A,coef} \\ &= (m+m_p) \begin{bmatrix} I & S(r_g)^T \\ S(r_g) & S(r_g)I_g S(r_g)^T \end{bmatrix} + \begin{bmatrix} mI & 0 \\ 0 & I_g \end{bmatrix} M_{A,coef} \end{aligned}$$

$$\begin{aligned} C &= H^T C_{RB}^{CG}(\nu_2) H + C_A(\nu_{r,1}, \nu_{r,2}) \\ &= \begin{bmatrix} I & 0 \\ S(r_g) & I \end{bmatrix} \begin{bmatrix} (m+m_p)S(\nu_2) & 0 \\ 0 & -S(I_g\nu_2) \end{bmatrix} \begin{bmatrix} I & S(r_g)^T \\ 0 & I \end{bmatrix} + \begin{bmatrix} 0 & -S(0.1\cdot m\nu_{r,1}) \\ -S(0.1\cdot m\nu_{r,1}) & -S(1.5\cdot m\nu_{r,2}) \end{bmatrix} \\ &= (m+m_p) \begin{bmatrix} S(\nu_2) & S(\nu_2)S(r_g)^T \\ S(r_g)S(\nu_2) & -S(r_g)S(I_g\nu_2)S(r_g)^T \end{bmatrix} + \begin{bmatrix} 0 & -S(0.1\cdot m\nu_{r,1}) \\ -S(0.1\cdot m\nu_{r,1}) & -S(1.5\cdot m\nu_{r,2}) \end{bmatrix} \end{aligned}$$

12 All system equations

The equations can be found in **AutoDocking/Models/Primitive/SysEq6DOF.mat**

$$\begin{aligned} \dot{u} = {} & 0.0054\,\tau_q - 1.3169\,q + 0.0175\,\tau_u + 5.3523e-04\,\tau_w - 16.0798\,\theta \\ & - 1.3574\,u - 0.2925\,w - 12.1092\,z + 0.3371\,p\,r - 0.0147\,p\,v - 0.0265\,q\,u - 1.8664\,q\,w \\ & + 2.3477\,r\,v + 0.2649\,u\,w + 0.0177\,p^2 + 0.1866\,q^2 + 0.1690\,r^2 \end{aligned} \tag{45}$$

$$\begin{aligned} \dot{v} = {} & 0.2504\,p + 4.9601\,\phi + 0.0667\,r - 0.0046\,\tau_p - 0.0014\,\tau_r + 0.0078\,\tau_v \\ & - 5.9306e-04\,v - 0.0364\,p\,q + 0.1167\,q\,r + 0.7867\,p\,w - 0.4028\,r\,u - 0.1266\,v\,w + 0.3298\,r\,|r| \\ & + 0.1143\,r\,|v| - 0.9463\,v\,|v| + 1.6913e-05 \end{aligned} \tag{46}$$

$$\begin{aligned} \dot{w} = {} & 0.0029\,\tau_q - 0.7243\,q + 5.3523e-04\,\tau_u + 0.0094\,\tau_w - 22.5556\,\theta \\ & - 0.0415\,u - 5.1289\,w - 75.2186\,z - 0.0146\,p\,r - 1.2581\,p\,v + 0.5354\,q\,u - 0.0265\,q\,w \\ & + 0.0412\,r\,v + 0.1457\,u\,w - 0.0903\,p^2 - 0.0974\,q^2 - 0.0071\,r^2 \end{aligned} \tag{47}$$

$$\dot{p} = 0.2243\,r - 67.3414\,\phi - 3.3993\,p + 0.0625\,\tau_p - 0.0050\,\tau_r - 0.0046\,\tau_v + 4.1383e-05\,v - 0.1257\,p\,q - 0.5847\,q\,r + 0.1266\,p\,w - 0.3545\,r\,u + 1.7190\,v\,w + 2.4489\,r\ |r| + 0.4457\,r\ |v| + 0.5631\,v\ |v| - 1.2384e-06 \quad (48)$$

$$\dot{q} = 0.0294\,\tau_q - 7.2431\,q + 0.0054\,\tau_u + 0.0029\,\tau_w - 88.4387\,\theta - 0.4151\,u - 1.6087\,w - 66.6009\,z + 0.8539\,p\,r - 0.0810\,p\,v - 0.1457\,q\,u - 0.2649\,q\,w + 0.4121\,r\,v + 1.4572\,u\,w + 0.0971\,p^2 + 0.0265\,q^2 - 0.0707\,r^2 \quad (49)$$

$$\dot{r} = 0.2696\,p + 5.3406\,\phi - 1.0413\,r - 0.0050\,\tau_p + 0.0229\,\tau_r - 0.0014\,\tau_v + 0.0013\,v - 0.4188\,p\,q + 0.1257\,q\,r + 0.0395\,p\,w - 0.1107\,r\,u - 0.1363\,v\,w - 10.3735\,r\ |r| - 2.0239\,r\ |v| + 0.1699\,v\ |v| - 3.7453e-05 \quad (50)$$

$$\dot{x} = w\ (\sin(\phi)\ \sin(\psi) + \cos(\phi)\ \cos(\psi)\ \sin(\theta)) - v\ (\cos(\phi)\ \sin(\psi) - \cos(\psi)\ \sin(\phi)\ \sin(\theta)) + u\ \cos(\psi)\ \cos(\theta) \quad (51)$$

$$\dot{y} = v\ (\cos(\phi)\ \cos(\psi) + \sin(\phi)\ \sin(\psi)\ \sin(\theta)) - w\ (\cos(\psi)\ \sin(\phi) - \cos(\phi)\ \sin(\psi)\ \sin(\theta)) + u\ \cos(\theta)\ \sin(\psi) \quad (52)$$

$$\dot{z} = w\ \cos(\phi)\ \cos(\theta) - u\ \sin(\theta) + v\ \cos(\theta)\ \sin(\phi)$$

$$\dot{\phi} = p + \frac{\sin(\theta)\ (r\ \cos(\phi) + q\ \sin(\phi))}{\cos(\theta)}$$

$$\dot{\theta} = q\ \cos(\phi) - r\ \sin(\phi)$$

$$\dot{\psi} = \frac{r\ \cos(\phi) + q\ \sin(\phi)}{\cos(\theta)}$$