

1 Constants and Dimensions

Symbol	Value	Unit	Description
g	9.81	$\frac{m}{s^2}$	acceleration of gravity
ρ	1025	$\frac{kg}{m^3}$	density of water
L	2.0	m	length of hull
B	1.08	m	beam of hull
m	55.0	kg	mass of hull
r_g^{hull}	$[0.2 \ 0 \ -0.2]^T$	m	CG of hull
R_{44}	$0.4 \cdot B$	m	radius of gyration
R_{55}	$0.25 \cdot L$	m	radius of gyration
R_{66}	$0.25 \cdot L$	m	radius of gyration
T_{yaw}	1	s	time constant in yaw
U_{max}	6	$knot$	max forward speed
B_{pont}	0.25	m	beam of one pontoon
y_{pont}	0.395	m	distance from centerline to waterline area center
Cw_{pont}	0.75	—	waterline area coefficient
Cb_{pont}	0.4	—	block coefficient

Waterline area of one pontoon

$$Aw_{pont} = Cw_{pont}LB_{pont} \quad (1)$$

2 Skew symetric matrix

$$S \left(\begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} \right) = \begin{bmatrix} 0 & -a_3 & a_2 \\ a_3 & 0 & -a_1 \\ -a_2 & a_1 & 0 \end{bmatrix} \quad (2)$$

3 Kinetics

$$\nu_1 = [u \ v \ w]^T \quad \nu_2 = [p \ q \ r]^T \quad (3)$$

Inertia matrix of hull in CG

$$I_g^{CG} = m \cdot \text{diag} [R_{44}^2, R_{55}^2, R_{66}^2] \quad (4)$$

CG location corrected for payload

$$r_g = \frac{m \cdot r_g^{hull} + m_p \cdot r_p}{m + m_p} \quad (5)$$

Inertia matrix of hull and payload in CO

$$I_g = I_g^{CG} - m \cdot S(r_g)^2 - m_p \cdot S(r_p)^2 \quad (6)$$

$$M_{RB}^{CG} = \begin{bmatrix} (m + m_p)I & 0 \\ 0 & I_g \end{bmatrix} \quad C_{RB}^{CG}(\nu_2) = \begin{bmatrix} (m + m_p)S(\nu_2) & 0 \\ 0 & -S(I_g \nu_2) \end{bmatrix} \quad (7)$$

Transform M_{RB} and C_{RB} from the C_G to the C_O

$$H = \begin{bmatrix} I & S(r_g)^T \\ 0 & I \end{bmatrix} \quad (8)$$

$$M_{RB} = H^T M_{RB}^{CG} H \quad (9)$$

$$C_{RB}(\nu_2) = H^T C_{RB}^{CG}(\nu_2) H \quad (10)$$

$$(11)$$

4 Relative velocity

Water current surge and sway velocity

$$u_c = v_{cur} \cos(\beta_{cur} - \psi) \quad (12)$$

$$v_c = v_{cur} \sin(\beta_{cur} - \psi) \quad (13)$$

Where v_{cur} is the current velocity, β_{cur} is the current direction in *rad* and ψ is the yaw of the vessel. Relative velocity vector

$$\nu_r = \nu - [u_c \ v_c \ 0 \ 0 \ 0 \ 0]^T \quad (14)$$

In the case of no current we have

$$\nu_r = \nu \quad (15)$$

5 Hydrodynamics

Hydrodynamic added mass

$$M_A = \begin{bmatrix} mI & 0 \\ 0 & I_g \end{bmatrix} M_{A,coef} \quad (16)$$

$$M_{A,coef} = \text{diag}([0.1 \ 1.5 \ 1.0 \ 0.2 \ 0.8 \ 1.7]) \quad (17)$$

$$C_A(\nu_{r,1}, \nu_{r,2}) = \begin{bmatrix} 0 & -S(M_{A,11}\nu_{r,1} + M_{A,12}\nu_{r,2}) \\ -S(M_{A,11}\nu_{r,1} + M_{A,12}\nu_{r,2}) & -S(M_{A,21}\nu_{r,1} + M_{A,22}\nu_{r,2}) \end{bmatrix} \quad (18)$$

$$= \begin{bmatrix} 0 & -S(0.1 \cdot m\nu_{r,1}) \\ -S(0.1 \cdot m\nu_{r,1}) & -S(1.5 \cdot m\nu_{r,2}) \end{bmatrix} \quad (19)$$

System mass and Coriolis-centripetal matrices

$$M = M_{RB} + M_A \quad (20)$$

$$C = C_{RB}(\nu_2) + C_A(\nu_{r,1}, \nu_{r,2}) \quad (21)$$

6 Hydro statics

Water volume displacement

$$\nabla = \frac{m + m_p}{\rho} \quad (22)$$

Draft

$$T = \frac{\nabla}{2Cb_{pont}B_{pont}L} \quad (23)$$

$$KB = \frac{1}{3}\left(5\frac{T}{2} - \frac{\nabla}{2LB_{pont}}\right); \quad (24)$$

$$I_T = \frac{2}{12}LB_{pont}^3 \frac{6 \cdot Cw_{pont}^3}{(1 + Cw_{pont})(1 + 2Cw_{pont})} + 2 \cdot Aw_{pont}y_{pont}^2 \quad (25)$$

$$I_L = \frac{0.8 \cdot 2}{12}B_{pont}L^3 \quad (26)$$

$$GM_T = KB + \frac{I_T}{\nabla} - T + r_{g,z} \quad (27)$$

$$GM_L = KB + \frac{I_L}{\nabla} - T + r_{g,z} \quad (28)$$

$$(29)$$

$$G_{CF} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \rho g(2Aw_{pont}) & 0 & 0 & 0 \\ 0 & 0 & 0 & \rho g \nabla GM_T & 0 & 0 \\ 0 & 0 & 0 & 0 & \rho g \nabla GM_L & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad (30)$$

$$G = H^T G_{CF} H \quad (31)$$

$$\omega_3 = \sqrt{G_{33}/M_{33}} \quad (32)$$

$$\omega_4 = \sqrt{G_{44}/M_{44}} \quad (33)$$

$$\omega_5 = \sqrt{G_{55}/M_{55}} \quad (34)$$

7 Linear Damping

$$h(r) = \begin{bmatrix} -24.4 \frac{g}{U_{max}} \\ 0 \\ -2 \cdot 0.3 \cdot \omega_3 M_{33} \\ -2 \cdot 0.2 \cdot \omega_4 M_{44} \\ -2 \cdot 0.4 \cdot \omega_5 M_{55} \\ \frac{-M_{66}}{T_{yaw}} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \frac{-M_{66}}{T_{yaw}} 10 \cdot abs(r) \end{bmatrix} \quad (35)$$

The matrix to the right includes non-linear damping for yaw

$$\tau_{damp}(r) = h(r) \bullet \nu_r \quad (36)$$

8 Crossflow Drag

The crossflow is computed using strip theory and is a function of ν_r

$$\tau_{cf} = \begin{bmatrix} 0 \\ Yh \\ 0 \\ 0 \\ 0 \\ Nh \end{bmatrix} \quad (37)$$

τ_{cf} Has components only in v and r
First-order fitting

$$\tau_{cf} = \begin{bmatrix} 0 \\ 1.2907e - 16 r - 12.2363 v - 4.8735e - 17 \\ 0 \\ 0 \\ 0 \\ -5.9740 r - 5.1780e - 16 v - 3.6373e - 19 \end{bmatrix} \approx \begin{bmatrix} 0 \\ -12.2363 v \\ 0 \\ 0 \\ 0 \\ -5.9740 r \end{bmatrix}$$

Second-order fitting

$$\tau_{cf} = \begin{bmatrix} 0 \\ 0.1824 r - 0.0666 v - 41.8276 r |r| - 1.6409 r |v| - 122.0516 v |v| + 0.0019 \\ 0 \\ 0 \\ 0 \\ 0.0535 v - 0.1471 r - 2.5043 r |r| - 88.3910 r |v| - 0.2423 v |v| - 0.0015 \end{bmatrix}$$

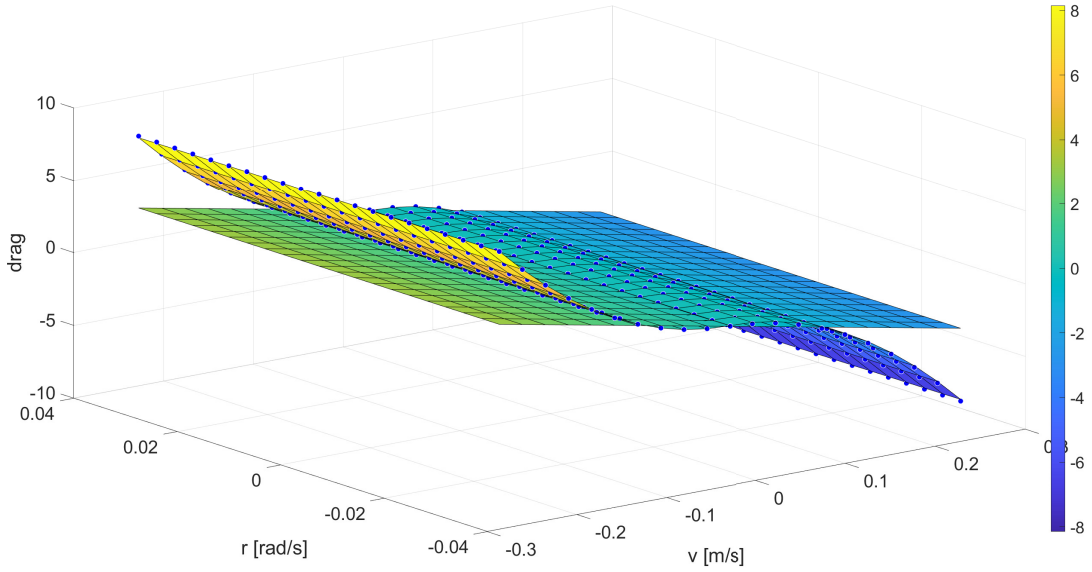


Figure 1: Linearized crossflow damping in sway

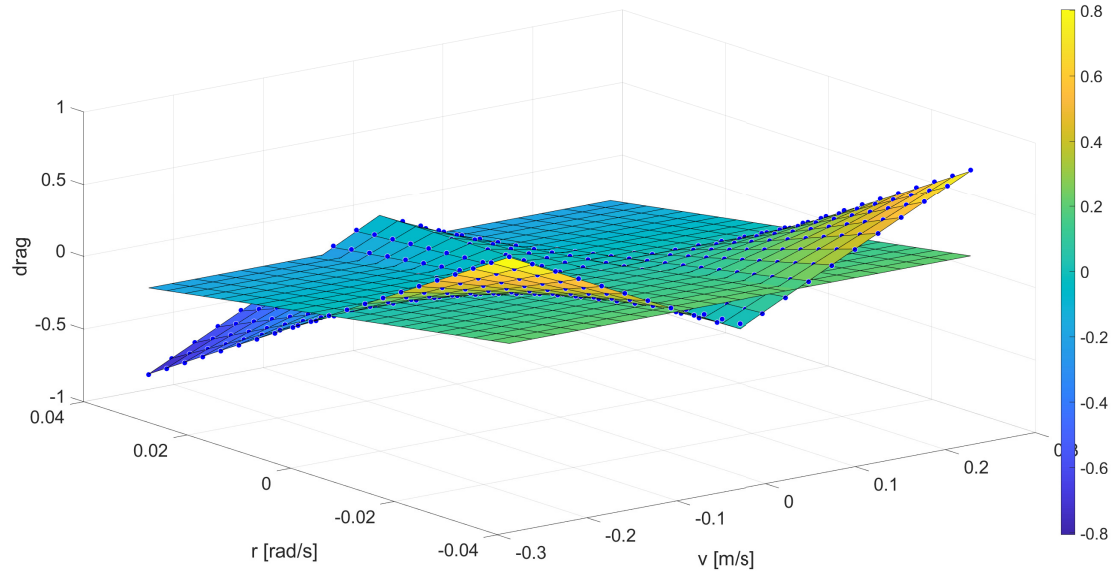


Figure 2: Linearized crossflow damping in yaw

9 Ballast

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1  trim_setpoint = 280;
2  persistent trim_moment;
3  if isempty(trim_moment)
4  trim_moment = 0;
5  end
6  .
7  .
8  g_0 = [0 0 0 0 trim_moment 0]';
9  .
10 .
11 trim_moment = trim_moment + 0.05 * (trim_setpoint - trim_moment);

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$$g_0 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ trim_moment \\ 0 \end{bmatrix} \quad (38)$$

g_0 represents a dynamic torque in q (pitch)

10 Transformation

$$R = R_z R_y R_x = \begin{bmatrix} \cos(\psi) & -\sin(\psi) & 0 \\ \sin(\psi) & \cos(\psi) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos(\theta) & 0 & \sin(\theta) \\ 0 & 1 & 0 \\ -\sin(\theta) & \cos(\theta) & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(\phi) & -\sin(\phi) \\ 0 & \sin(\phi) & \cos(\phi) \end{bmatrix} \quad (39)$$

$$T = \begin{bmatrix} 1 & \sin(\phi) \tan(\theta) & \cos(\phi) \tan(\theta) \\ 0 & \cos(\phi) & -\sin(\phi) \\ 0 & \frac{\sin(\phi)}{\cos(\theta)} & \frac{\cos(\phi)}{\cos(\theta)} \end{bmatrix} \quad (40)$$

$$J = \begin{bmatrix} R & 0 \\ 0 & T \end{bmatrix} \quad (41)$$

11 State derivative

$$M\dot{\nu} + C\nu_r + G\eta + g_0 = \tau + \tau_{damp} + \tau_{cf} \quad (42)$$

$$\dot{\eta} = J(\eta)\nu \quad (43)$$

$$\dot{x} = \begin{bmatrix} \dot{\nu} \\ \dot{\eta} \end{bmatrix} = \begin{bmatrix} M^{-1}(\tau + \tau_{damp}(r) + \tau_{cf}(\nu_r) - C\nu_r - G\eta - g_0) \\ J\nu \end{bmatrix} \quad (44)$$

$$\begin{aligned} M &= H^T M_{RB}^{CG} H + M_A \\ &= \begin{bmatrix} I & 0 \\ S(r_g) & I \end{bmatrix} \begin{bmatrix} (m + m_p)I & 0 \\ 0 & I_g \end{bmatrix} \begin{bmatrix} I & S(r_g)^T \\ 0 & I \end{bmatrix} + \begin{bmatrix} mI & 0 \\ 0 & I_g \end{bmatrix} M_{A,coef} \\ &= (m + m_p) \begin{bmatrix} I & S(r_g)^T \\ S(r_g) & S(r_g)I_g S(r_g)^T \end{bmatrix} + \begin{bmatrix} mI & 0 \\ 0 & I_g \end{bmatrix} M_{A,coef} \end{aligned}$$

$$\begin{aligned} C &= H^T C_{RB}^{CG}(\nu_2) H + C_A(\nu_{r,1}, \nu_{r,2}) \\ &= \begin{bmatrix} I & 0 \\ S(r_g) & I \end{bmatrix} \begin{bmatrix} (m + m_p)S(\nu_2) & 0 \\ 0 & -S(I_g \nu_2) \end{bmatrix} \begin{bmatrix} I & S(r_g)^T \\ 0 & I \end{bmatrix} + \begin{bmatrix} 0 & -S(0.1 \cdot m\nu_{r,1}) \\ -S(0.1 \cdot m\nu_{r,1}) & -S(1.5 \cdot m\nu_{r,2}) \end{bmatrix} \\ &= (m + m_p) \begin{bmatrix} S(\nu_2) & S(\nu_2)S(r_g)^T \\ S(r_g)S(\nu_2) & -S(r_g)S(I_g \nu_2)S(r_g)^T \end{bmatrix} + \begin{bmatrix} 0 & -S(0.1 \cdot m\nu_{r,1}) \\ -S(0.1 \cdot m\nu_{r,1}) & -S(1.5 \cdot m\nu_{r,2}) \end{bmatrix} \end{aligned}$$

12 All system equations

The equations can be found in **AutoDocking/Models/Primitive/SysEq6DOF.mat**

$$\begin{aligned} \dot{u} &= 0.0054 \tau_q - 1.3169 q + 0.0175 \tau_u + 5.3523 e - 04 \tau_w - 16.0798 \theta \\ &\quad - 1.3574 u - 0.2925 w - 12.1092 z + 0.3371 pr - 0.0147 pv - 0.0265 qu - 1.8664 qw \\ &\quad + 2.3477 rv + 0.2649 uw + 0.0177 p^2 + 0.1866 q^2 + 0.1690 r^2 \end{aligned} \quad (45)$$

$$\begin{aligned} \dot{v} &= 0.2504 p + 4.9601 \phi + 0.0629 r - 0.0046 \tau_p - 0.0014 \tau_r + 0.0078 \tau_v \\ &\quad - 7.8984 e - 04 v - 0.0364 pq + 0.1167 qr + 0.7867 pw - 0.4028 ru - 0.1266 vw + 0.3215 r |r| \\ &\quad + 0.1389 r |v| - 0.9472 v |v| + 4.4851 e - 05 \end{aligned} \quad (46)$$

$$\begin{aligned} \dot{w} &= 0.0029 \tau_q - 0.7243 q + 5.3523 e - 04 \tau_u + 0.0094 \tau_w - 22.5556 \theta \\ &\quad - 0.0415 u - 5.1289 w - 75.2186 z - 0.0146 pr - 1.2581 pv + 0.5354 qu - 0.0265 qw \\ &\quad + 0.0412 rv + 0.1457 uw - 0.0903 p^2 - 0.0974 q^2 - 0.0071 r^2 \end{aligned} \quad (47)$$

$$\begin{aligned}\dot{p} = & 0.2272 r - 67.3414 \phi - 3.3993 p + 0.0625 \tau_p - 0.0050 \tau_r - 0.0046 \tau_v \\ & + 0.0010 v - 0.1257 p q - 0.5847 q r + 0.1266 p w - 0.3545 r u + 1.7190 v w + 2.4254 r |r| \\ & + 0.4318 r |v| + 0.5610 v |v| - 5.8328 e - 05 \quad (48)\end{aligned}$$

$$\begin{aligned}\dot{q} = & 0.0294 \tau_q - 7.2431 q + 0.0054 \tau_u + 0.0029 \tau_w - 88.4387 \theta \\ & - 0.4151 u - 1.6087 w - 66.6009 z + 0.8539 p r - 0.0810 p v - 0.1457 q u - 0.2649 q w \\ & + 0.4121 r v + 1.4572 u w + 0.0971 p^2 + 0.0265 q^2 - 0.0707 r^2 \quad (49)\end{aligned}$$

$$\begin{aligned}\dot{r} = & 0.2696 p + 5.3406 \phi - 1.0432 r - 0.0050 \tau_p + 0.0229 \tau_r - 0.0014 \tau_v \\ & - 0.0020 v - 0.4188 p q + 0.1257 q r + 0.0395 p w - 0.1107 r u - 0.1363 v w - 10.2613 r |r| \\ & - 2.0308 r |v| + 0.1805 v |v| + 1.1532 e - 04 \quad (50)\end{aligned}$$

$$\begin{aligned}\dot{x} = & w (\sin(\phi) \sin(\psi) + \cos(\phi) \cos(\psi) \sin(\theta)) \\ & - v (\cos(\phi) \sin(\psi) - \cos(\psi) \sin(\phi) \sin(\theta)) + u \cos(\psi) \cos(\theta) \quad (51)\end{aligned}$$

$$\begin{aligned}\dot{y} = & v (\cos(\phi) \cos(\psi) + \sin(\phi) \sin(\psi) \sin(\theta)) \\ & - w (\cos(\psi) \sin(\phi) - \cos(\phi) \sin(\psi) \sin(\theta)) + u \cos(\theta) \sin(\psi) \quad (52)\end{aligned}$$

$$\begin{aligned}\dot{z} = & w \cos(\phi) \cos(\theta) - u \sin(\theta) + v \cos(\theta) \sin(\phi) \\ \dot{\phi} = & p + \frac{\sin(\theta) (r \cos(\phi) + q \sin(\phi))}{\cos(\theta)} \\ \dot{\theta} = & q \cos(\phi) - r \sin(\phi) \\ \dot{\psi} = & \frac{r \cos(\phi) + q \sin(\phi)}{\cos(\theta)}\end{aligned}$$

$$\begin{aligned}\dot{u} = & 0.0054 \tau_q - 1.3169 q + 0.0175 \tau_u + 5.3523 e - 04 \tau_w - 16.0798 \theta \\ & - 1.3574 u - 0.2925 w - 12.1092 z + 0.3371 p r - 0.0147 p v - 0.0265 q u - 1.8664 q w \\ & + 2.3477 r v + 0.2649 u w + 0.0177 p^2 + 0.1866 q^2 + 0.1690 r^2 \quad (53)\end{aligned}$$

$$\begin{aligned}\dot{v} = & 0.2504 p + 4.9601 \phi + 0.0667 r - 0.0046 \tau_p - 0.0014 \tau_r + 0.0078 \tau_v \\ & - 5.9306 e - 04 v - 0.0364 p q + 0.1167 q r + 0.7867 p w - 0.4028 r u - 0.1266 v w + 0.3298 r |r| \\ & + 0.1143 r |v| - 0.9463 v |v| + 1.6913 e - 05 \quad (54)\end{aligned}$$

$$\begin{aligned}\dot{w} = & 0.0029 \tau_q - 0.7243 q + 5.3523 e - 04 \tau_u + 0.0094 \tau_w - 22.5556 \theta \\ & - 0.0415 u - 5.1289 w - 75.2186 z - 0.0146 p r - 1.2581 p v + 0.5354 q u - 0.0265 q w \\ & + 0.0412 r v + 0.1457 u w - 0.0903 p^2 - 0.0974 q^2 - 0.0071 r^2 \quad (55)\end{aligned}$$

$$\begin{aligned}\dot{p} = & 0.2243 r - 67.3414 \phi - 3.3993 p + 0.0625 \tau_p - 0.0050 \tau_r - 0.0046 \tau_v \\ & + 4.1383 e - 05 v - 0.1257 p q - 0.5847 q r + 0.1266 p w - 0.3545 r u + 1.7190 v w + 2.4489 r |r| \\ & + 0.4457 r |v| + 0.5631 v |v| - 1.2384 e - 06 \quad (56)\end{aligned}$$

$$\begin{aligned}\dot{q} = & 0.0294 \tau_q - 7.2431 q + 0.0054 \tau_u + 0.0029 \tau_w - 88.4387 \theta \\ & - 0.4151 u - 1.6087 w - 66.6009 z + 0.8539 p r - 0.0810 p v - 0.1457 q u - 0.2649 q w \\ & + 0.4121 r v + 1.4572 u w + 0.0971 p^2 + 0.0265 q^2 - 0.0707 r^2 \quad (57)\end{aligned}$$

$$\begin{aligned}\dot{r} = & 0.2696 p + 5.3406 \phi - 1.0413 r - 0.0050 \tau_p + 0.0229 \tau_r - 0.0014 \tau_v \\ & + 0.0013 v - 0.4188 p q + 0.1257 q r + 0.0395 p w - 0.1107 r u - 0.1363 v w - 10.3735 r |r| \\ & - 2.0239 r |v| + 0.1699 v |v| - 3.7453 e - 05 \quad (58)\end{aligned}$$

$$\begin{aligned}\dot{x} = & w (\sin(\phi) \sin(\psi) + \cos(\phi) \cos(\psi) \sin(\theta)) \\ & - v (\cos(\phi) \sin(\psi) - \cos(\psi) \sin(\phi) \sin(\theta)) + u \cos(\psi) \cos(\theta) \quad (59)\end{aligned}$$

$$\begin{aligned}\dot{y} = & v (\cos(\phi) \cos(\psi) + \sin(\phi) \sin(\psi) \sin(\theta)) \\ & - w (\cos(\psi) \sin(\phi) - \cos(\phi) \sin(\psi) \sin(\theta)) + u \cos(\theta) \sin(\psi) \quad (60)\end{aligned}$$

$$\dot{z} = w \cos(\phi) \cos(\theta) - u \sin(\theta) + v \cos(\theta) \sin(\phi)$$

$$\dot{\phi} = p + \frac{\sin(\theta) (r \cos(\phi) + q \sin(\phi))}{\cos(\theta)}$$

$$\dot{\theta} = q \cos(\phi) - r \sin(\phi)$$

$$\dot{\psi} = \frac{r \cos(\phi) + q \sin(\phi)}{\cos(\theta)}$$