

1 Kinematics

$$M\dot{\nu} + C(\nu)\nu + D(\nu)\nu + g(\eta) + g_0 = \tau \quad (1)$$

$$\begin{aligned} M &= M_{RB} + M_A \\ C &= C_{RB} + C_A \end{aligned}$$

$$\tau = \tau_{hydrodynamics} + \tau_{hydrostatics} + \tau_{wind} + \tau_{wave} + \tau_{control} \quad (2)$$

1.1 Control

$$\tau_{control} = \begin{bmatrix} \tau_{control,linear} \\ \tau_{control,torque} \end{bmatrix} \quad (3)$$

Each propeller can rotate with an angle ξ_p . The thrust of each propeller can be describe as a vector by spilting it into components that align with the ship coordinate frame

$$T_p = \begin{bmatrix} \cos(\xi_p) \\ \sin(\xi_p) \\ 0 \end{bmatrix} t_p \quad (4)$$

where t_p is the magnitude of the thrust and T_p is the thrust vector. The linear control force is found as the sum of forces from each propeller p.

$$\tau_{control,linear} = \sum^P T_p \quad (5)$$

$$\tau_{control,torque} = \sum^P r_p \times T_p = \sum^P S(r_p)T_p \quad (6)$$

where r_p is the position vector of thruster p and $S(r_p)$ is the skew-symetric matrix of vector r_p .

In the case of two propellers we have

$$\tau_{control} = \begin{bmatrix} I & I \\ S(r_1) & S(r_2) \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \end{bmatrix} \quad (7)$$