1 System matrices of the 6 DOF model

$$\boldsymbol{M}\dot{\boldsymbol{\nu}} + \boldsymbol{C}(\boldsymbol{\nu})\boldsymbol{\nu} + \boldsymbol{D}(\boldsymbol{\nu})\boldsymbol{\nu} = \boldsymbol{\tau} \tag{1}$$

$$\boldsymbol{M} = \boldsymbol{M}_{RB} + \boldsymbol{M}_{A} \tag{2}$$

$$\boldsymbol{C} = \boldsymbol{C}_{RB} + \boldsymbol{C}_{A} \tag{3}$$

$$\dot{\boldsymbol{\nu}} = \boldsymbol{A}\boldsymbol{\nu} + \boldsymbol{B}\boldsymbol{\tau}$$

In the case of no payload, no current and no external force τ we have

$$\boldsymbol{M}_{RB} = \begin{bmatrix} 55 & 0 & 0 & 0 & -11 & 0 \\ 0 & 55 & 0 & 11 & 0 & 11 \\ 0 & 0 & 55 & 0 & -11 & 0 \\ 0 & 11 & 0 & 14.6643 & 0 & 4.4000 \\ -11 & 0 & -11 & 0 & 22.5500 & 0 \\ 0 & 11 & 0 & 4.4000 & 0 & 18.1500 \end{bmatrix}$$

$$\boldsymbol{C}_{RB} =$$
$$\begin{bmatrix} 0 & -55\,r & 55\,q & -11\,r & -11\,q & -11\,r \\ 55\,r & 0 & -55\,p & 0 & 11\,p - 11\,r & 0 \\ -55\,q & 55\,p & 0 & 11\,p & 11\,q & 11\,p \\ 11\,r & 0 & -11\,p & 0 & 4.4000\,p + 13.7500\,r & -18.1500\,q \\ 11\,q & 11\,r - 11\,p & -11\,q & -4.4000\,p - 13.7500\,r & 0 & 10.2643\,p + 4.4000\,r \\ 11\,r & 0 & -11\,p & 18.1500\,q & -10.2643\,p - 4.4000\,r & 0 \end{bmatrix}$$

$$\boldsymbol{M}_{A} = \begin{bmatrix} 5.5000 & 0 & 0 & 0 & 0 & 0 \\ 0 & 82.5000 & 0 & 0 & 0 & 0 \\ 0 & 0 & 55 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2.4929 & 0 & 0 \\ 0 & 0 & 0 & 0 & 14.5200 & 0 \\ 0 & 0 & 0 & 0 & 0 & 27.1150 \end{bmatrix}$$

$$\boldsymbol{C}_{A} = \begin{bmatrix} 0 & 0 & 0 & 0 & 55\,w & -82.5000\,v \\ 0 & 0 & 0 & -55\,w & 0 & 5.5000\,u \\ 0 & 0 & 0 & 82.5000\,v & -5.5000\,u & 0 \\ 0 & 55\,w & -82.5000\,v & 0 & 27.1150\,r & -14.5200\,q \\ -55\,w & 0 & 5.5000\,u & -27.1150\,r & 0 & 2.4929\,p \\ 0 & 0 & 0 & 14.5200\,q & -2.4929\,p & 0 \end{bmatrix}$$

$$\boldsymbol{D} = \begin{bmatrix} 77.5544 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 546.4805 & 0 & 0 & 0 \\ 0 & 0 & 0 & 54.3823 & 0 & 0 \\ 0 & 0 & 0 & 0 & 246.0496 & 0 \\ 0 & 0 & 0 & 0 & 0 & 452.6500\,|r| + 45.2650 \end{bmatrix}$$

2 Reduction from system equations

$$\dot{\boldsymbol{\nu}} = (\boldsymbol{M}_{RB} + \boldsymbol{M}_{A})^{-1}(-\boldsymbol{C}_{RB} - \boldsymbol{C}_{A} - \boldsymbol{D})\boldsymbol{\nu} \tag{4}$$

$$\dot{\boldsymbol{\nu}} = f(\boldsymbol{\nu}) = f(u, v, w, p, q, r) \tag{5}$$

$$\dot{\boldsymbol{\nu}} = \begin{bmatrix} \dot{u} & \dot{v} & \dot{w} & \dot{p} & \dot{q} & \dot{r} \end{bmatrix}^T \tag{6}$$

$$\begin{aligned} \dot{u} = {} & 0.3371\,p\,r - 1.3574\,u - 0.2925\,w - 1.3169\,q - 0.0147\,p\,v - 0.0265\,q\,u \\ & - 1.8664\,q\,w + 2.3477\,r\,v + 0.2649\,u\,w + 0.0177\,p^2 + 0.1866\,q^2 + 0.1690\,r^2 \end{aligned} \tag{7}$$

$$\begin{aligned} \dot{v} = {} & 0.2504\,p + 0.0651\,r - 0.0364\,p\,q + 0.1167\,q\,r + 0.7867\,p\,w - 0.4028\,r\,u \\ & - 0.1266\,v\,w + 0.6506\,r\,\,|r| \end{aligned} \tag{8}$$

$$\begin{aligned} \dot{w} = {} & 0.5354\,q\,u - 0.0415\,u - 5.1289\,w - 0.0146\,p\,r - 1.2581\,p\,v - 0.7243\,q \\ & - 0.0265\,q\,w + 0.0412\,r\,v + 0.1457\,u\,w - 0.0903\,p^2 - 0.0974\,q^2 - 0.0071\,r^2 \end{aligned} \tag{9}$$

$$\begin{aligned} \dot{p} = {} & 0.2244\,r - 3.3993\,p - 0.1257\,p\,q - 0.5847\,q\,r + 0.1266\,p\,w - 0.3545\,r\,u \\ & + 1.7190\,v\,w + 2.2439\,r\,\,|r| \end{aligned} \tag{10}$$

$$\begin{aligned} \dot{q} = {} & 0.8539\,p\,r - 0.4151\,u - 1.6087\,w - 7.2431\,q - 0.0810\,p\,v - 0.1457\,q\,u \\ & - 0.2649\,q\,w + 0.4121\,r\,v + 1.4572\,u\,w + 0.0971\,p^2 + 0.0265\,q^2 - 0.0707\,r^2 \end{aligned} \tag{11}$$

$$\begin{aligned} \dot{r} = {} & 0.2696\,p - 1.0376\,r - 0.4188\,p\,q + 0.1257\,q\,r + 0.0395\,p\,w - 0.1107\,r\,u \\ & - 0.1363\,v\,w - 10.3762\,r\,\,|r| \end{aligned} \tag{12}$$

$$\boldsymbol{A} = \frac{df}{d\boldsymbol{\nu}} = \begin{bmatrix} 0.2649\,w - 0.0265\,q - 1.3574 & 2.3477\,r - 0.0147\,p & 0.3371\,p + 0.3379\,r + 2.3477\,v \\ -0.4028\,r & -0.1266\,w & A_{23} \\ -0.1107\,r & -0.1363\,w & A_{33} \end{bmatrix} \tag{13}$$

Where the dimensions in heave, roll and pitch has been removed and

$$A_{23} = 0.1167\,q + 0.1301\,r - 0.4028\,u + 1.3012\,|r| \tag{14}$$

$$A_{33} = 0.1257\,q - 2.0752\,r - 0.1107\,u - 20.7524\,|r| \tag{15}$$

$$\boldsymbol{B} = \begin{bmatrix} 0.0175 & 0 & 0 \\ 0 & 0.0078 & -0.0014 \\ 0 & -0.0014 & 0.0229 \end{bmatrix} \tag{16}$$

2.1 Reduction

Assiming that

$$w = 0$$
$$p = 0$$
$$q = 0$$

The system equations simplify to

$$\boldsymbol{A}(u,v,r) = \begin{bmatrix} -1.3574 & 2.3477\,r & 0.3379\,r + 2.3477\,v \\ -0.4028\,r & 0 & 0.0651 - 0.4028\,u + 1.3012\,|r| \\ -0.1107\,r & 0 & -1.0376 - 0.1107\,u - 20.7524\,|r| \end{bmatrix} \tag{17}$$

$$\boldsymbol{B} = \begin{bmatrix} 0.0175 & 0 & 0 \\ 0 & 0.0078 & -0.0014 \\ 0 & -0.0014 & 0.0229 \end{bmatrix} \tag{18}$$

Choosing the linearization point $(u,v,r) = (3,0,0)$ yields the following linearization

$$\boldsymbol{A} = \begin{bmatrix} -1.3574 & 0 & 0 \\ 0 & 0 & -1.1433 \\ 0 & 0 & -1.3696 \end{bmatrix}$$

$$\boldsymbol{B} = \begin{bmatrix} 0.0175 & 0 & 0 \\ 0 & 0.0078 & -0.0014 \\ 0 & -0.0014 & 0.0229 \end{bmatrix} \tag{19}$$

3 Simple state truncation

$$\boldsymbol{A} = -(M_{RB} + M_A)^{-1}(C_{RB} + C_A + D) = \begin{bmatrix} 0.2944\,w - 0.0294\,q - 1.3574 & 0.0294\,p + 0.9037\,r & 0.1690\,r - 0.0742\,p + 1.4439\,v \\ -0.3601\,r & 0.2532\,w & 0.6506\,|r| - 0.0427\,u - 0.1504\,q + 0.0651 \\ -0.1186\,r & 0.2726\,w & 0.0079\,u - 0.1620\,q - 10.3762\,|r| - 1.0376 \end{bmatrix} \tag{20}$$

$$\boldsymbol{B} = (M_{RB} + M_A)^{-1} = \begin{bmatrix} 0.0175 & 0 & 0 \\ 0 & 0.0078 & -0.0014 \\ 0 & -0.0014 & 0.0229 \end{bmatrix} \tag{21}$$

where the heave, roll and pitch dimensions has been removed from A and B

3.1 Reduction

Assiming that

$$w = 0$$
$$p = 0$$
$$q = 0$$

The system equations simplify to

$$\boldsymbol{A}(u,v,r) = \begin{bmatrix} -1.3574 & 0.9037\,r & 0.1690\,r + 1.4439\,v \\ -0.3601\,r & 0 & 0.6506\,|r| - 0.0427\,u + 0.0651 \\ -0.1186\,r & 0 & 0.0079\,u - 10.3762\,|r| - 1.0376 \end{bmatrix}$$

$$\boldsymbol{B} = \begin{bmatrix} 0.0175 & 0 & 0 \\ 0 & 0.0078 & -0.0014 \\ 0 & -0.0014 & 0.0229 \end{bmatrix}$$

4 Selection matrix approch

$$C^*_{RB} = UM_{RB}L \qquad\qquad C^*_A = UM_AL \tag{22}$$

Where U is the surge of the vessel at the linarization point and L is a selection matrix seen in HMCHMC (3.63). For this lineariztion $U = 3$

$$M_{RB} = \begin{bmatrix} 55 & 0 & 0 \\ 0 & 55 & 11 \\ 0 & 11 & 18.1500 \end{bmatrix} \qquad M_A = \begin{bmatrix} 5.5000 & 0 & 0 \\ 0 & 82.5000 & 0 \\ 0 & 0 & 27.1150 \end{bmatrix} \tag{23}$$

$$C^*_{RB} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 165 \\ 0 & 0 & 33 \end{bmatrix} \qquad C^*_A = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 247.5000 \\ 0 & 0 & 0 \end{bmatrix} \tag{24}$$

$$D = \begin{bmatrix} 77.5544 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 452.6500\,|r| + 45.2650 \end{bmatrix} \tag{25}$$

$$\dot{\nu} = A\nu + B\tau \tag{26}$$

$$M = M_{RB} + M_A \tag{27}$$
$$N = C^*_{RB} + C^*_A + D* \tag{28}$$
$$A = -M^{-1}N \tag{29}$$
$$B = M^{-1} \tag{30}$$

$$A = \begin{bmatrix} -1.2819 & 0 & 0 \\ 0 & 0 & 0.8159\,|r| - 2.9184 \\ 0 & 0 & -10.1983\,|r| - 1.0198 \end{bmatrix}$$
$$B = \begin{bmatrix} 0.0165 & 0 & 0 \\ 0 & 0.0074 & -0.0018 \\ 0 & -0.0018 & 0.0225 \end{bmatrix}$$

5 Waterfixed coordinates

In the common system equations ν is defined relative to the seabed

$$\boldsymbol{M}_{RB}\dot{\boldsymbol{\nu}} + \boldsymbol{M}_A\dot{\boldsymbol{\nu}}_{\boldsymbol{r}} + \boldsymbol{C}_A(\boldsymbol{\nu})\boldsymbol{\nu} + \boldsymbol{C}_{RB}(\boldsymbol{\nu}_r)\boldsymbol{\nu}_r + \boldsymbol{D}(\boldsymbol{\nu}_r)\boldsymbol{\nu}_r = \boldsymbol{\tau} + \boldsymbol{w}(t) \tag{31}$$
$$\dot{\boldsymbol{\eta}} = \boldsymbol{J}(\eta)\boldsymbol{\nu} \tag{32}$$

If the velocities ν is defined relative to the water $\nu = \nu_r$. Since it is still preferred to have the position in land relative coordinates the velocities from the current needs to be included in the second equation.

$$(\boldsymbol{M}_{RB} + \boldsymbol{M}_A)\,\dot{\boldsymbol{\nu}} + (\boldsymbol{C}_A(\boldsymbol{\nu}) + \boldsymbol{C}_{RB}(\boldsymbol{\nu}) + \boldsymbol{D}(\boldsymbol{\nu}))\,\boldsymbol{\nu} = \boldsymbol{\tau} + \boldsymbol{w}(t) \tag{33}$$
$$\dot{\boldsymbol{\eta}} = \boldsymbol{J}(\eta)\boldsymbol{\nu} + \boldsymbol{\nu}_c \tag{34}$$

Where $\boldsymbol{\nu}_c$ is the velocity vector of the current

$$\boldsymbol{\nu}_c = \begin{bmatrix} u_c & v_c & 0 & 0 & 0 & 0 \end{bmatrix}^T \tag{35}$$

6 Crossflow damping

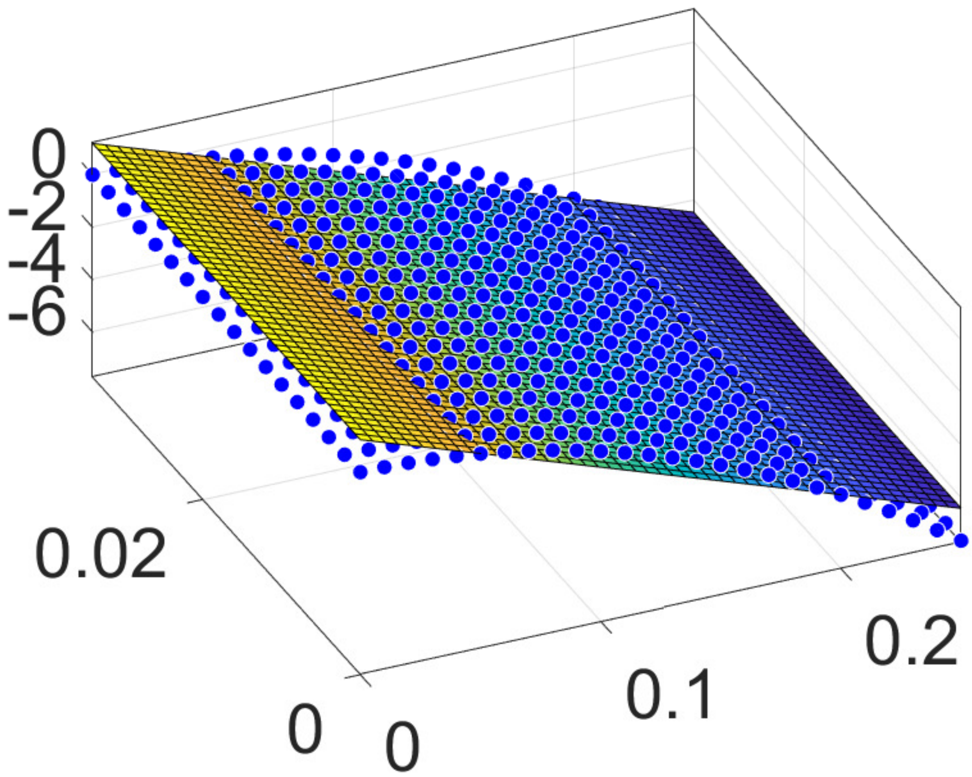

Figure 1: Linearized crossflow damping in sway

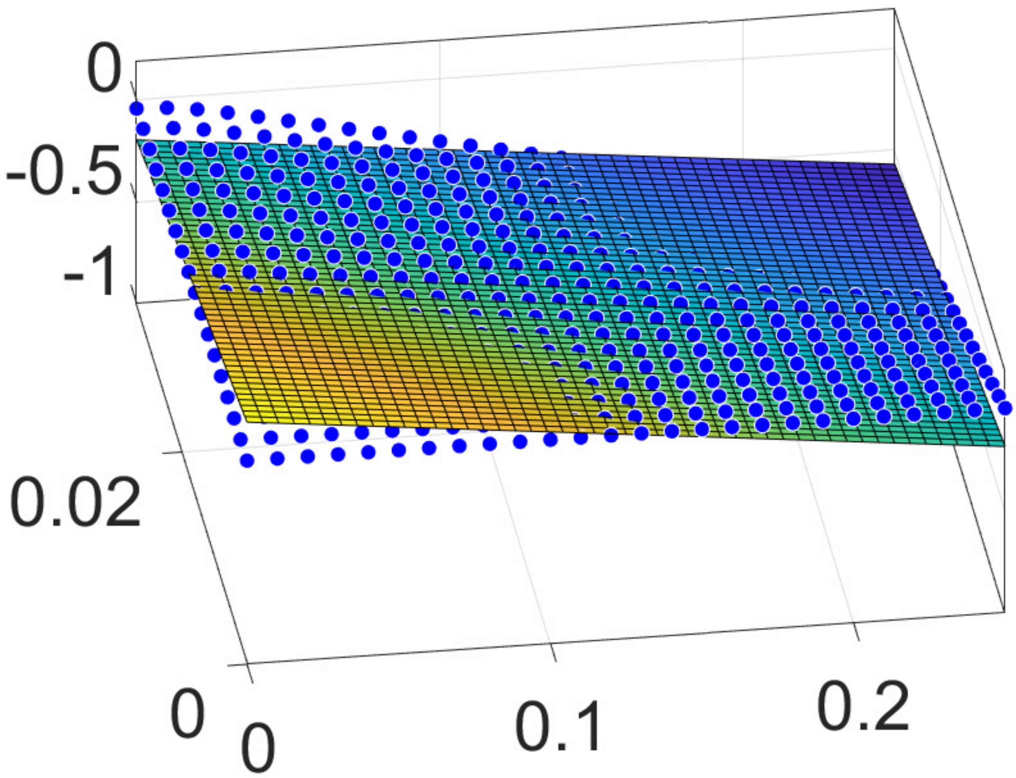

Figure 2: Linearized crossflow damping in yaw

$$\boldsymbol{\tau}_{cf} = \begin{bmatrix} 0 \\ 1.2336 - 30.6073\,v - 1.4439\,r \\ 0 \\ 0 \\ 0 \\ 0.1891 - 1.5099\,v - 11.2817\,r \end{bmatrix}$$

$$\boldsymbol{D} = \frac{d}{d\boldsymbol{\nu}}(-\boldsymbol{\tau}_d - \boldsymbol{\tau}_{cf}) =$$

$$= \begin{bmatrix} 77.5544 & 0 & 0 & 0 & 0 & 0 \\ 0 & 30.6073 & 0 & 0 & 0 & 1.4439 \\ 0 & 0 & 546.4805 & 0 & 0 & 0 \\ 0 & 0 & 0 & 54.3823 & 0 & 0 \\ 0 & 0 & 0 & 0 & 246.0496 & 0 \\ 0 & 1.5099 & 0 & 0 & 0 & 905.3000\,|r| + 56.5467 \end{bmatrix}$$