

# 1 System matrices of the 6 DOF model

$$M\dot{\nu} + C(\nu)\nu + D(\nu)\nu = \tau \quad (1)$$

$$M = M_{RB} + M_A \quad (2)$$

$$C = C_{RB} + C_A \quad (3)$$

$$\dot{\nu} = A\nu + B\tau$$

In the case of no payload, no current and no external force  $\tau$  we have

$$M_{RB} = \begin{bmatrix} 55 & 0 & 0 & 0 & -11 & 0 \\ 0 & 55 & 0 & 11 & 0 & 11 \\ 0 & 0 & 55 & 0 & -11 & 0 \\ 0 & 11 & 0 & 14.6643 & 0 & 4.4000 \\ -11 & 0 & -11 & 0 & 22.5500 & 0 \\ 0 & 11 & 0 & 4.4000 & 0 & 18.1500 \end{bmatrix}$$

$$C_{RB} = \begin{bmatrix} 0 & -55r & 55q & -11r & -11q & -11r \\ 55r & 0 & -55p & 0 & 11p - 11r & 0 \\ -55q & 55p & 0 & 11p & 11q & 11p \\ 11r & 0 & -11p & 0 & 4.4000p + 13.7500r & -18.1500q \\ 11q & 11r - 11p & -11q & -4.4000p - 13.7500r & 0 & 10.2643p + 4.4000r \\ 11r & 0 & -11p & 18.1500q & -10.2643p - 4.4000r & 0 \end{bmatrix}$$

$$M_A = \begin{bmatrix} 5.5000 & 0 & 0 & 0 & 0 & 0 \\ 0 & 82.5000 & 0 & 0 & 0 & 0 \\ 0 & 0 & 55 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2.4929 & 0 & 0 \\ 0 & 0 & 0 & 0 & 14.5200 & 0 \\ 0 & 0 & 0 & 0 & 0 & 27.1150 \end{bmatrix}$$

$$C_A = \begin{bmatrix} 0 & 0 & 0 & 0 & 55w & -82.5000v \\ 0 & 0 & 0 & -55w & 0 & 5.5000u \\ 0 & 0 & 0 & 82.5000v & -5.5000u & 0 \\ 0 & 55w & -82.5000v & 0 & 27.1150r & -14.5200q \\ -55w & 0 & 5.5000u & -27.1150r & 0 & 2.4929p \\ 0 & 0 & 0 & 14.5200q & -2.4929p & 0 \end{bmatrix}$$

$$D = \begin{bmatrix} 77.5544 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 546.4805 & 0 & 0 & 0 \\ 0 & 0 & 0 & 54.3823 & 0 & 0 \\ 0 & 0 & 0 & 0 & 246.0496 & 0 \\ 0 & 0 & 0 & 0 & 0 & 452.6500|r| + 45.2650 \end{bmatrix}$$

# 2 Reduction from system equations

$$\dot{\nu} = (M_{RB} + M_A)^{-1}(-C_{RB} - C_A - D)\nu \quad (4)$$

$$\dot{\nu} = f(\nu) = f(u, v, w, p, q, r) \quad (5)$$

$$\dot{\boldsymbol{\nu}} = [\dot{u} \quad \dot{v} \quad \dot{w} \quad \dot{p} \quad \dot{q} \quad \dot{r}]^T \quad (6)$$

$$\begin{aligned} \dot{u} = & 0.3371 p r - 1.3574 u - 0.2925 w - 1.3169 q - 0.0147 p v - 0.0265 q u \\ & - 1.8664 q w + 2.3477 r v + 0.2649 u w + 0.0177 p^2 + 0.1866 q^2 + 0.1690 r^2 \end{aligned} \quad (7)$$

$$\begin{aligned} \dot{v} = & 0.2504 p + 0.0651 r - 0.0364 p q + 0.1167 q r + 0.7867 p w - 0.4028 r u \\ & - 0.1266 v w + 0.6506 r |r| \end{aligned} \quad (8)$$

$$\begin{aligned} \dot{w} = & 0.5354 q u - 0.0415 u - 5.1289 w - 0.0146 p r - 1.2581 p v - 0.7243 q \\ & - 0.0265 q w + 0.0412 r v + 0.1457 u w - 0.0903 p^2 - 0.0974 q^2 - 0.0071 r^2 \end{aligned} \quad (9)$$

$$\begin{aligned} \dot{p} = & 0.2244 r - 3.3993 p - 0.1257 p q - 0.5847 q r + 0.1266 p w - 0.3545 r u \\ & + 1.7190 v w + 2.2439 r |r| \end{aligned} \quad (10)$$

$$\begin{aligned} \dot{q} = & 0.8539 p r - 0.4151 u - 1.6087 w - 7.2431 q - 0.0810 p v - 0.1457 q u \\ & - 0.2649 q w + 0.4121 r v + 1.4572 u w + 0.0971 p^2 + 0.0265 q^2 - 0.0707 r^2 \end{aligned} \quad (11)$$

$$\begin{aligned} \dot{r} = & 0.2696 p - 1.0376 r - 0.4188 p q + 0.1257 q r + 0.0395 p w - 0.1107 r u \\ & - 0.1363 v w - 10.3762 r |r| \end{aligned} \quad (12)$$

$$\begin{aligned} \mathbf{A} = \frac{d\mathbf{f}}{d\boldsymbol{\nu}} = & \begin{bmatrix} 0.2649 w - 0.0265 q - 1.3574 & 2.3477 r - 0.0147 p & 0.3371 p + 0.3379 r + 2.3477 v \\ -0.4028 r & -0.1266 w & A_{23} \\ -0.1107 r & -0.1363 w & A_{33} \end{bmatrix} \end{aligned} \quad (13)$$

Where the dimensions in heave, roll and pitch has been removed and

$$A_{23} = 0.1167 q + 0.1301 r - 0.4028 u + 1.3012 |r| \quad (14)$$

$$A_{33} = 0.1257 q - 2.0752 r - 0.1107 u - 20.7524 |r| \quad (15)$$

$$\mathbf{B} = \begin{bmatrix} 0.0175 & 0 & 0 \\ 0 & 0.0078 & -0.0014 \\ 0 & -0.0014 & 0.0229 \end{bmatrix} \quad (16)$$

## 2.1 Reduction

Assuming that

$$w = 0$$

$$p = 0$$

$$q = 0$$

The system equations simplify to

$$\mathbf{A}(u, v, r) = \begin{bmatrix} -1.3574 & 2.3477 r & 0.3379 r + 2.3477 v \\ -0.4028 r & 0 & 0.0651 - 0.4028 u + 1.3012 |r| \\ -0.1107 r & 0 & -1.0376 - 0.1107 u - 20.7524 |r| \end{bmatrix} \quad (17)$$

$$\mathbf{B} = \begin{bmatrix} 0.0175 & 0 & 0 \\ 0 & 0.0078 & -0.0014 \\ 0 & -0.0014 & 0.0229 \end{bmatrix} \quad (18)$$

Choosing the linearization point  $(u, v, r) = (3, 0, 0)$  yields the following linearization

$$\mathbf{A} = \begin{bmatrix} -1.3574 & 0 & 0 \\ 0 & 0 & -1.1433 \\ 0 & 0 & -1.3696 \end{bmatrix}$$

$$\mathbf{B} = \begin{bmatrix} 0.0175 & 0 & 0 \\ 0 & 0.0078 & -0.0014 \\ 0 & -0.0014 & 0.0229 \end{bmatrix} \quad (19)$$

### 3 Simple state truncation

$$\mathbf{A} = -(M_{RB} + M_A)^{-1}(C_{RB} + C_A + D) =$$

$$\begin{bmatrix} 0.2944 w - 0.0294 q - 1.3574 & 0.0294 p + 0.9037 r & 0.1690 r - 0.0742 p + 1.4439 v \\ -0.3601 r & 0.2532 w & 0.6506 |r| - 0.0427 u - 0.1504 q + 0.0651 \\ -0.1186 r & 0.2726 w & 0.0079 u - 0.1620 q - 10.3762 |r| - 1.0376 \end{bmatrix} \quad (20)$$

$$\mathbf{B} = (M_{RB} + M_A)^{-1} = \begin{bmatrix} 0.0175 & 0 & 0 \\ 0 & 0.0078 & -0.0014 \\ 0 & -0.0014 & 0.0229 \end{bmatrix} \quad (21)$$

where the heave, roll and pitch dimensions has been removed from  $A$  and  $B$

#### 3.1 Reduction

Assuming that

$$w = 0$$

$$p = 0$$

$$q = 0$$

The system equations simplify to

$$\mathbf{A}(u, v, r) = \begin{bmatrix} -1.3574 & 0.9037 r & 0.1690 r + 1.4439 v \\ -0.3601 r & 0 & 0.6506 |r| - 0.0427 u + 0.0651 \\ -0.1186 r & 0 & 0.0079 u - 10.3762 |r| - 1.0376 \end{bmatrix}$$

$$\mathbf{B} = \begin{bmatrix} 0.0175 & 0 & 0 \\ 0 & 0.0078 & -0.0014 \\ 0 & -0.0014 & 0.0229 \end{bmatrix}$$

## 4 Selection matrix approach

$$C_{RB}^* = U M_{RB} L \quad C_A^* = U M_A L \quad (22)$$

Where  $U$  is the surge of the vessel at the linearization point and  $L$  is a selection matrix seen in HMCHMC (3.63). For this linearization  $U = 3$

$$M_{RB} = \begin{bmatrix} 55 & 0 & 0 \\ 0 & 55 & 11 \\ 0 & 11 & 18.1500 \end{bmatrix} \quad M_A = \begin{bmatrix} 5.5000 & 0 & 0 \\ 0 & 82.5000 & 0 \\ 0 & 0 & 27.1150 \end{bmatrix} \quad (23)$$

$$C_{RB}^* = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 165 \\ 0 & 0 & 33 \end{bmatrix} \quad C_A^* = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 247.5000 \\ 0 & 0 & 0 \end{bmatrix} \quad (24)$$

$$D = \begin{bmatrix} 77.5544 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 452.6500 |r| + 45.2650 \end{bmatrix} \quad (25)$$

$$\dot{\nu} = A\nu + B\tau \quad (26)$$

$$M = M_{RB} + M_A \quad (27)$$

$$N = C_{RB}^* + C_A^* + D^* \quad (28)$$

$$A = -M^{-1}N \quad (29)$$

$$B = M^{-1} \quad (30)$$

$$A = \begin{bmatrix} -1.2819 & 0 & 0 \\ 0 & 0 & 0.8159 |r| - 2.9184 \\ 0 & 0 & -10.1983 |r| - 1.0198 \end{bmatrix}$$

$$B = \begin{bmatrix} 0.0165 & 0 & 0 \\ 0 & 0.0074 & -0.0018 \\ 0 & -0.0018 & 0.0225 \end{bmatrix}$$

## 5 Waterfixed coordinates

$$M_{RB}\dot{\nu} + M_A\dot{\nu}_r + C_{RB}(\nu)\nu + C_{RB}(\nu_r)\nu_r + D(\nu_r)\nu_r = \tau + w(t) \quad (31)$$

$$\dot{\eta} = J(\eta)\nu \quad (32)$$

$$(M_{RB} + M_A)\dot{\nu} + (C_{RB}(\nu) + C_{RB}(\nu_r) + D(\nu_r))\nu = \tau + w(t) \quad (33)$$

$$\dot{\eta} = J(\eta)\nu + \nu_c \quad (34)$$

Where  $\nu_c$  is the velocity vector of the current

$$\nu_c = [u_c \ v_c \ 0 \ 0 \ 0 \ 0]^T \quad (35)$$