1 Modelling

1.1 Kinematics

The dynamic of a vessel can be described as follows, using a 6 DOF model.

$$\boldsymbol{M}_{RB}\dot{\boldsymbol{\nu}} + \boldsymbol{M}_{A}\dot{\boldsymbol{\nu}}_{\boldsymbol{r}} + \boldsymbol{C}_{A}(\boldsymbol{\nu})\boldsymbol{\nu} + \boldsymbol{C}_{RB}(\boldsymbol{\nu}_r)\boldsymbol{\nu}_r + \boldsymbol{D}(\boldsymbol{\nu}_r)\boldsymbol{\nu}_r + \boldsymbol{G}\boldsymbol{\eta} = \boldsymbol{\tau} + \boldsymbol{w}(t) \tag{1}$$

$$\dot{\boldsymbol{\eta}} = \boldsymbol{J}(\eta)\boldsymbol{\nu} \tag{2}$$

where ν is the velocities and η is the positions. This description includes a velocity and a position in each of the 6 modes. The modes are translation in x, y and z and rotation around x, y and z. ν_r is the relative speed of the vessel to the water

1.2 State space

Assuming $\nu = \nu_r$ (no movement of the water relative to the seabed), we can define

$$\boldsymbol{M} = \boldsymbol{M}_{RB} + \boldsymbol{M}_{A} \tag{3}$$

$$\boldsymbol{C}(\boldsymbol{\nu}) = \boldsymbol{C}_{RB}(\boldsymbol{\nu}) + \boldsymbol{C}_{A}(\boldsymbol{\nu}) \tag{4}$$

And write the system as

$$\boldsymbol{M}\dot{\boldsymbol{\nu}} + \boldsymbol{C}(\boldsymbol{\nu})\boldsymbol{\nu} + \boldsymbol{D}(\boldsymbol{\nu})\boldsymbol{\nu} + \boldsymbol{G}\boldsymbol{\eta} = \boldsymbol{\tau} + \boldsymbol{w}(t) \tag{5}$$

$$\dot{\boldsymbol{\eta}} = \boldsymbol{J}(\eta)\boldsymbol{\nu} \tag{6}$$

Solving for $\dot{\boldsymbol{\nu}}$ we get

$$\dot{\boldsymbol{\nu}} = \boldsymbol{A}(\boldsymbol{\nu})\boldsymbol{\nu} + \boldsymbol{G}\boldsymbol{\eta} + \boldsymbol{B}\boldsymbol{\tau} \tag{7}$$

$$\dot{\boldsymbol{\eta}} = \boldsymbol{J}(\eta)\boldsymbol{\nu} \tag{8}$$

Where

$$\boldsymbol{A}(\boldsymbol{\nu}) = -\boldsymbol{M}^{-1}(\boldsymbol{C}(\boldsymbol{\nu}) + \boldsymbol{D}(\boldsymbol{\nu})) \tag{9}$$

$$\boldsymbol{B} = \boldsymbol{M}^{-1} \tag{10}$$

Writing (7) and (8) in matrix form gives us a state space description of the whole system

$$\dot{\boldsymbol{x}} = \begin{bmatrix} \boldsymbol{A} & \boldsymbol{G} \\ \boldsymbol{J}(\eta) & \boldsymbol{0} \end{bmatrix} \boldsymbol{x} + \begin{bmatrix} \boldsymbol{B} \\ \boldsymbol{0} \end{bmatrix} \boldsymbol{\tau} \tag{11}$$

where

$$\boldsymbol{x} = \begin{bmatrix} \boldsymbol{\nu} \\ \boldsymbol{\eta} \end{bmatrix} \tag{12}$$

1.3 Model Reduction

2 Measurements

The vessel is fitted with a satellite compass and GPS like the one seen in figure 1. This device can provide the following data

SoG	Speed over Ground also know as track
CoG	Course over Ground
RoT	Rate of Turn (in yaw)
Pos	Position of vessel in x and y
HDT	Heading of vessel

Figure 1: Furuno SC70. Differential GPS (DGPS) and compass

Assuming normal noise distribution the measurement equations can be written as

$$HDT = \psi + w \tag{13}$$

$$Pos = \begin{bmatrix} x \\ y \end{bmatrix} + w \tag{14}$$

$$SoG = \left| \begin{bmatrix} u \\ v \end{bmatrix} \right| + w \tag{15}$$

$$CoG = atan2(v, u) + w \tag{16}$$

$$RoT = r + w \tag{17}$$

$$\boldsymbol{y} = \boldsymbol{C}_m \boldsymbol{x} \tag{18}$$

$$\boldsymbol{C}_m = \boldsymbol{I} \tag{19}$$

3 Control

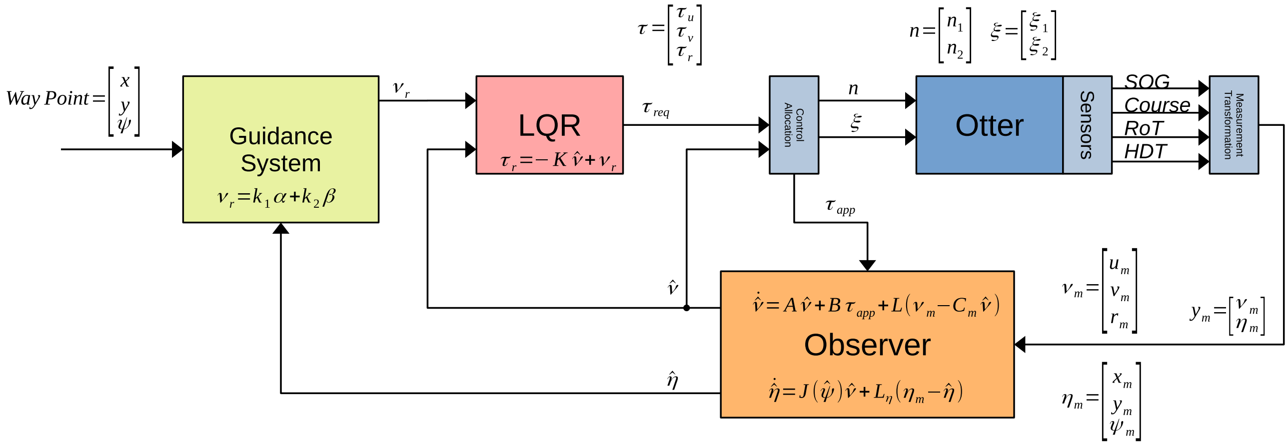

Figure 2: