

1 Kinematics

$$M_{RB}\dot{\boldsymbol{\nu}} + M_A\dot{\boldsymbol{\nu}}_r + \mathbf{C}_A(\boldsymbol{\nu})\boldsymbol{\nu} + \mathbf{C}_{RB}(\boldsymbol{\nu}_r)\boldsymbol{\nu}_r + \mathbf{D}(\boldsymbol{\nu}_r)\boldsymbol{\nu}_r + \mathbf{G}\boldsymbol{\eta} = \boldsymbol{\tau} + \mathbf{w}(t) \quad (1)$$

$$\dot{\boldsymbol{\eta}} = \mathbf{J}(\boldsymbol{\eta})\boldsymbol{\nu} \quad (2)$$

2 State space

$$\dot{\boldsymbol{\nu}} = \mathbf{A}\boldsymbol{\nu} + \mathbf{G}\boldsymbol{\eta} + \mathbf{B}\boldsymbol{\tau} \quad (3)$$

$$\dot{\boldsymbol{\eta}} = \mathbf{J}(\boldsymbol{\eta})\boldsymbol{\nu} \quad (4)$$

Where

$$\mathbf{A} = -\mathbf{M}^{-1}(\mathbf{C} + \mathbf{D}) \quad (5)$$

$$\mathbf{B} = \mathbf{M}^{-1} \quad (6)$$

$$\mathbf{M} = \mathbf{M}_{RB} + \mathbf{M}_A \quad (7)$$

$$\mathbf{C} = \mathbf{C}_{RB} + \mathbf{C}_A \quad (8)$$

Notice that \mathbf{A} still depends on the $\boldsymbol{\nu}$ and $\boldsymbol{\nu}_r$ even though it is not shown. Combining (3) and (4) gives us a state space description of the whole system

$$\dot{\mathbf{x}} = \begin{bmatrix} \mathbf{A} & \mathbf{G} \\ \mathbf{J}(\boldsymbol{\eta}) & \mathbf{0} \end{bmatrix} \mathbf{x} + \begin{bmatrix} \mathbf{B} \\ \mathbf{0} \end{bmatrix} \boldsymbol{\tau} \quad (9)$$

where

$$\mathbf{x} = \begin{bmatrix} \boldsymbol{\nu} \\ \boldsymbol{\eta} \end{bmatrix} \quad (10)$$

2.1 Measurements

Assuming that the vessel is fitted with a compass, a GPS and a speedometer in surge.

$$\mathbf{y} = \mathbf{C}_m \mathbf{x} \quad (11)$$

$$\mathbf{C}_m = \begin{bmatrix} 1 & 0 & 0 & \dots \\ 0 & 0 & 0 & \dots \\ 0 & 0 & 0 & \dots \\ \vdots & \vdots & \vdots & I \end{bmatrix} \quad (12)$$