1 System matrices of the 6 DOF model

$$\boldsymbol{M}\dot{\boldsymbol{\nu}} + \boldsymbol{C}(\boldsymbol{\nu})\boldsymbol{\nu} + \boldsymbol{D}(\boldsymbol{\nu})\boldsymbol{\nu} = \boldsymbol{\tau} \tag{1}$$

$$\boldsymbol{M} = \boldsymbol{M}_{RB} + \boldsymbol{M}_A \tag{2}$$

$$\boldsymbol{C} = \boldsymbol{C}_{RB} + \boldsymbol{C}_A \tag{3}$$

$$\dot{\boldsymbol{\nu}} = \boldsymbol{A}\boldsymbol{\nu} + \boldsymbol{B}\boldsymbol{\tau}$$

In the case of no payload, no current and no external force τ we have

$$\boldsymbol{M}_{RB} = \begin{bmatrix} 55 & 0 & 0 & 0 & -11 & 0 \\ 0 & 55 & 0 & 11 & 0 & 11 \\ 0 & 0 & 55 & 0 & -11 & 0 \\ 0 & 11 & 0 & 14.6643 & 0 & 4.4000 \\ -11 & 0 & -11 & 0 & 22.5500 & 0 \\ 0 & 11 & 0 & 4.4000 & 0 & 18.1500 \end{bmatrix}$$

$$\boldsymbol{C}_{RB} =$$

$$\begin{bmatrix} 0 & -55\,r & 55\,q & -11\,r & -11\,q & -11\,r \\ 55\,r & 0 & -55\,p & 0 & 11\,p - 11\,r & 0 \\ -55\,q & 55\,p & 0 & 11\,p & 11\,q & 11\,p \\ 11\,r & 0 & -11\,p & 0 & 4.4000\,p + 13.7500\,r & -18.1500\,q \\ 11\,q & 11\,r - 11\,p & -11\,q & -4.4000\,p - 13.7500\,r & 0 & 10.2643\,p + 4.4000\,r \\ 11\,r & 0 & -11\,p & 18.1500\,q & -10.2643\,p - 4.4000\,r & 0 \end{bmatrix}$$

$$\boldsymbol{M}_A = \begin{bmatrix} 5.5000 & 0 & 0 & 0 & 0 & 0 \\ 0 & 82.5000 & 0 & 0 & 0 & 0 \\ 0 & 0 & 55 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2.4929 & 0 & 0 \\ 0 & 0 & 0 & 0 & 14.5200 & 0 \\ 0 & 0 & 0 & 0 & 0 & 27.1150 \end{bmatrix}$$

$$\boldsymbol{C}_A = \begin{bmatrix} 0 & 0 & 0 & 0 & 55\,w & -82.5000\,v \\ 0 & 0 & 0 & -55\,w & 0 & 5.5000\,u \\ 0 & 0 & 0 & 82.5000\,v & -5.5000\,u & 0 \\ 0 & 55\,w & -82.5000\,v & 0 & 27.1150\,r & -14.5200\,q \\ -55\,w & 0 & 5.5000\,u & -27.1150\,r & 0 & 2.4929\,p \\ 0 & 0 & 0 & 14.5200\,q & -2.4929\,p & 0 \end{bmatrix}$$

$$\boldsymbol{D} = \begin{bmatrix} 77.5544\,u & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 546.4805\,w & 0 & 0 & 0 \\ 0 & 0 & 0 & 54.3823\,p & 0 & 0 \\ 0 & 0 & 0 & 0 & 246.0496\,q & 0 \\ 0 & 0 & 0 & 0 & 0 & r\,\left(452.6500\,|r| + 45.2650\right) \end{bmatrix}$$

2 Reduction from system equations

$$\dot{\boldsymbol{\nu}} = (\boldsymbol{M}_{RB} + \boldsymbol{M}_A)^{-1}(-\boldsymbol{C}_{RB} - \boldsymbol{C}_A - \boldsymbol{D})\boldsymbol{\nu} \tag{4}$$

$$\dot{\boldsymbol{\nu}} = f(\boldsymbol{\nu}) = f(u, v, w, p, q, r) \tag{5}$$

$$\dot{\boldsymbol{\nu}} = \begin{bmatrix}\dot{u} & \dot{v} & \dot{w} & \dot{p} & \dot{q} & \dot{r}\end{bmatrix}^T \tag{6}$$

$$\begin{aligned}\dot{u} = 0.0177\,p^2 + 0.3371\,p\,r - 0.0147\,v\,p - 1.1303\,q^2 - 0.0265\,q\,u - 1.8664\,q\,w \\ + 0.1690\,r^2 + 2.3477\,v\,r - 1.3574\,u^2 + 0.2649\,u\,w - 0.2925\,w^2\end{aligned} \tag{7}$$

$$\begin{aligned}\dot{v} = 0.6506\,r^2\,|r| - 0.0364\,p\,q + 0.1167\,q\,r + 0.7867\,p\,w \\ - 0.4028\,r\,u - 0.1266\,v\,w + 0.2504\,p^2 + 0.0651\,r^2\end{aligned} \tag{8}$$

$$\begin{aligned}\dot{w} = -0.0903\,p^2 - 0.0146\,p\,r - 1.2581\,v\,p - 0.8217\,q^2 + 0.5354\,q\,u - 0.0265\,q\,w \\ - 0.0071\,r^2 + 0.0412\,v\,r - 0.0415\,u^2 + 0.1457\,u\,w - 5.1289\,w^2\end{aligned} \tag{9}$$

$$\begin{aligned}\dot{p} = 2.2439\,r^2\,|r| - 0.1257\,p\,q - 0.5847\,q\,r + 0.1266\,p\,w \\ - 0.3545\,r\,u + 1.7190\,v\,w - 3.3993\,p^2 + 0.2244\,r^2\end{aligned} \tag{10}$$

$$\begin{aligned}\dot{q} = 0.0971\,p^2 + 0.8539\,p\,r - 0.0810\,v\,p - 7.2166\,q^2 - 0.1457\,q\,u - 0.2649\,q\,w \\ - 0.0707\,r^2 + 0.4121\,v\,r - 0.4151\,u^2 + 1.4572\,u\,w - 1.6087\,w^2\end{aligned} \tag{11}$$

$$\begin{aligned}\dot{r} = 0.1257\,q\,r - 0.4188\,p\,q - 10.3762\,r^2\,|r| + 0.0395\,p\,w \\ - 0.1107\,r\,u - 0.1363\,v\,w + 0.2696\,p^2 - 1.0376\,r^2\end{aligned} \tag{12}$$

$$\boldsymbol{A} = \frac{df}{d\boldsymbol{\nu}} = \begin{bmatrix} 0.2649\,w - 2.7147\,u - 0.0265\,q & 2.3477\,r - 0.0147\,p & 0.3371\,p + 0.3379\,r + 2.3477\,v \\ -0.4028\,r & -0.1266\,w & A_{23} \\ -0.1107\,r & -0.1363\,w & A_{33} \end{bmatrix} \tag{13}$$

Where the dimensions in heave, roll and pitch has been removed and

$$A_{23} = 0.1167\,q + 0.1301\,r - 0.4028\,u + 0.6506\,r^2\,\mathrm{sign}\,(r) + 1.3012\,r\,|r| \tag{14}$$

$$A_{33} = 0.1257\,q - 2.0752\,r - 0.1107\,u - 10.3762\,r^2\,\mathrm{sign}\,(r) - 20.7524\,r\,|r| \tag{15}$$

This can be simplified further as $r^2\,\mathrm{sign}(r) = r\,|r|$

$$A_{23} = 0.1167\,q + 0.1301\,r - 0.4028\,u + 1.9518\,r\,|r| \tag{16}$$

$$A_{33} = 0.1257\,q - 2.0752\,r - 0.1107\,u - 31.1286\,r\,|r| \tag{17}$$

$$\boldsymbol{B} = M^{-1} = \begin{bmatrix} 0.0175 & 0 & 0 \\ 0 & 0.0078 & -0.0014 \\ 0 & -0.0014 & 0.0229 \end{bmatrix} \tag{18}$$

2.1 Reduction

Assiming that

$$
\begin{aligned}
w &= 0 \\
p &= 0 \\
q &= 0
\end{aligned}
$$

The system equations simplify to

$$
\boldsymbol{A}(u,v,r) = \begin{bmatrix} -2.7147\,u & 2.3477\,r & 0.3379\,r + 2.3477\,v \\ -0.4028\,r & 0 & 0.1301\,r - 0.4028\,u + 1.9518\,r\ |r| \\ -0.1107\,r & 0 & -2.0752\,r - 0.1107\,u - 31.1286\,r\ |r| \end{bmatrix} \tag{19}
$$

$$
\boldsymbol{B} = \begin{bmatrix} 0.0175 & 0 & 0 \\ 0 & 0.0078 & -0.0014 \\ 0 & -0.0014 & 0.0229 \end{bmatrix} \tag{20}
$$

3 Simple state truncation

$$
\boldsymbol{A} = -(M_{RB} + M_A)^{-1}(C_{RB} + C_A + D) =
$$
$$
\begin{bmatrix} 0.2944\,w - 1.3574\,u - 0.0294\,q & 0.0294\,p + 0.9037\,r & 0.1690\,r - 0.0742\,p + 1.4439\,v \\ -0.3601\,r & 0.2532\,w & 0.0651\,r - 0.1504\,q - 0.0427\,u + 0.6506\,r\ |r| \\ -0.1186\,r & 0.2726\,w & 0.0079\,u - 1.0376\,r - 0.1620\,q - 10.3762\,r\ |r| \end{bmatrix}
$$

$$
\boldsymbol{B} = (M_{RB} + M_A)^{-1} =
$$
$$
\begin{bmatrix} 0.0175 & 0 & 0 \\ 0 & 0.0078 & -0.0014 \\ 0 & -0.0014 & 0.0229 \end{bmatrix}
$$

where the heave, roll and pitch dimensions has been removed from A and B

3.1 Reduction

Assiming that

$$
\begin{aligned}
w &= 0 \\
p &= 0 \\
q &= 0
\end{aligned}
$$

The system equations simplify to

$$
\boldsymbol{A}(u,v,r) = \begin{bmatrix} -1.3574\,u & 0.9037\,r & 0.1690\,r + 1.4439\,v \\ -0.3601\,r & 0 & 0.0651\,r - 0.0427\,u + 0.6506\,r\ |r| \\ -0.1186\,r & 0 & 0.0079\,u - 1.0376\,r - 10.3762\,r\ |r| \end{bmatrix} \tag{21}
$$

$$
\boldsymbol{B} = \begin{bmatrix} 0.0175 & 0 & 0 \\ 0 & 0.0078 & -0.0014 \\ 0 & -0.0014 & 0.0229 \end{bmatrix} \tag{22}
$$

4 Selection matrix approch

$$C_{RB}^* = UM_{RB}L \qquad\qquad C_A^* = UM_AL \tag{23}$$

Where U is the surge of the vessel at the linarization point and L is a selection matrix seen in HMCHMC (3.63). For this lineariztion $U = 3$

$$M_{RB} = \begin{bmatrix} 55 & 0 & 0 \\ 0 & 55 & 11 \\ 0 & 11 & 18.1500 \end{bmatrix} \qquad M_A = \begin{bmatrix} 5.5000 & 0 & 0 \\ 0 & 82.5000 & 0 \\ 0 & 0 & 27.1150 \end{bmatrix} \tag{24}$$

$$C_{RB}^* = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 165 \\ 0 & 0 & 33 \end{bmatrix} \qquad C_A^* = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 247.5000 \\ 0 & 0 & 0 \end{bmatrix} \tag{25}$$

$$D = \begin{bmatrix} 232.6633 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & r\,(452.6500\,|r| + 45.2650) \end{bmatrix} \tag{26}$$

$$\dot{\nu} = A\nu + B\tau \tag{27}$$

$$M = M_{RB} + M_A \tag{28}$$
$$N = C_{RB}^* + C_A^* + D* \tag{29}$$
$$A = -M^{-1}N \tag{30}$$
$$B = M^{-1} \tag{31}$$

$$A = \begin{bmatrix} -3.8457 & 0 & 0 \\ 0 & 0 & 0.0816\,r + 0.8159\,r\,|r| - 3 \\ 0 & 0 & -1.0198\,r - 10.1983\,r\,|r| \end{bmatrix} \tag{32}$$

$$B = \begin{bmatrix} 0.0165 & 0 & 0 \\ 0 & 0.0074 & -0.0018 \\ 0 & -0.0018 & 0.0225 \end{bmatrix} \tag{33}$$

5 Waterfixed coordinates

$$\boldsymbol{M}_{RB}\dot{\boldsymbol{\nu}} + \boldsymbol{M}_A\dot{\boldsymbol{\nu}}_{\boldsymbol{r}} + \boldsymbol{C}_{RB}(\boldsymbol{\nu})\boldsymbol{\nu} + \boldsymbol{C}_{RB}(\boldsymbol{\nu}_r)\boldsymbol{\nu}_r + \boldsymbol{D}(\boldsymbol{\nu}_r)\boldsymbol{\nu}_r = \boldsymbol{\tau} + \boldsymbol{w}(t) \tag{34}$$
$$\dot{\boldsymbol{\eta}} = \boldsymbol{J}(\eta)\boldsymbol{\nu} \tag{35}$$

$$(\boldsymbol{M}_{RB} + \boldsymbol{M}_A)\,\dot{\boldsymbol{\nu}} + (\boldsymbol{C}_{RB}(\boldsymbol{\nu}) + \boldsymbol{C}_{RB}(\boldsymbol{\nu}) + \boldsymbol{D}(\boldsymbol{\nu}))\,\boldsymbol{\nu} = \boldsymbol{\tau} + \boldsymbol{w}(t) \tag{36}$$
$$\dot{\boldsymbol{\eta}} = \boldsymbol{J}(\eta)\boldsymbol{\nu} + \boldsymbol{\nu}_c \tag{37}$$

Where $\boldsymbol{\nu}_c$ is the velocity vector of the current

$$\boldsymbol{\nu}_c = \begin{bmatrix} u_c & v_c & 0 & 0 & 0 & 0 \end{bmatrix}^T \tag{38}$$