1 Constants and Dimensions

Symbol	Value	Unit	Description
g	9.81	$\frac{m}{s^2}$	acceleration of gravity
ρ	1025	$\frac{kg}{m^3}$	density of water
L	2.0	m	length of hull
B	1.08	m	beam of hull
m	55.0	kg	mass of hull
rg	$\begin{bmatrix}0.2 & 0 & -0.2\end{bmatrix}^T$	m	CG of hull
R_{44}	$0.4 \cdot B$	m	radii of gyrations
R_{55}	$0.25 \cdot L$	m	radii of gyrations
R_{66}	$0.25 \cdot L$	m	radii of gyrations
T_{yaw}	1	s	time constant in yaw
U_{max}	6	$knot$	max forward speed
B_{pont}	0.25	m	beam of one pontoon
y_{pont}	0.395	m	distance from centerline to waterline area center
Cw_{pont}	0.75	$-$	waterline area coefficient
Cb_{pont}	0.4	$-$	block coefficient

Waterline area of one pontoon

$$Aw_{pont} = Cw_{pont} L B_{pont} \tag{1}$$

2 Skew symetric matrix

$$S\left(\begin{bmatrix}a_1\\a_2\\a_3\end{bmatrix}\right) = \begin{bmatrix}0 & -a_3 & a_2\\ a_3 & 0 & -a_1\\ -a_2 & a_1 & 0\end{bmatrix} \tag{2}$$

3 Kinetics

$$\nu_1 = \begin{bmatrix}u & v & w\end{bmatrix}^T \qquad \nu_2 = \begin{bmatrix}p & q & r\end{bmatrix}^T \tag{3}$$

$$M_{RB}^{CG} = \begin{bmatrix}(m+m_p)I & 0\\ 0 & I_g\end{bmatrix} \qquad C_{RB}^{CG} = \begin{bmatrix}(m+m_p)S(\nu_2) & 0\\ 0 & -S(I_g\nu_2)\end{bmatrix} \tag{4}$$

Transform M_{RB} and C_{RB} from the C_G to the C_O

$$H = \begin{bmatrix}I & S(rg)^T\\ 0 & I\end{bmatrix} \tag{5}$$

$$M_{RB} = H^T M_{RB}^{CG} H \tag{6}$$

$$C_{RB} = H^T C_{RB}^{CG} H \tag{7}$$

$$\tag{8}$$

4 Hydrodynamics

Hydrodynamic added mass

$$M_A = \begin{bmatrix} 0.1*m & 0 & 0 & 0 & 0 & 0 \\ 0 & 1.5*m & 0 & 0 & 0 & 0 \\ 0 & 0 & 1.0*m & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.2*Ig(1,1) & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.8*Ig(2,2) & 0 \\ 0 & 0 & 0 & 0 & 0 & 1.7*Ig(3,3) \end{bmatrix} \tag{9}$$

$$C_A = \begin{bmatrix} 0 & -S(M_{A,11}\nu_{r,1} + M_{A,12}\nu_{r,2}) \\ -S(M_{A,11}\nu_{r,1} + M_{A,12}\nu_{r,2}) & -S(M_{A,21}\nu_{r,1} + M_{A,22}\nu_{r,2}) \end{bmatrix} \tag{10}$$

System mass and Coriolis-centripetal matrices

$$M = M_{RB} + M_A \tag{11}$$

$$C = C_{RB} + C_A \tag{12}$$

5 Hydro statics

Water volume displacement

$$\nabla = \frac{m + m_p}{\rho} \tag{13}$$

Draft

$$T = \frac{\nabla}{2Cb_{pont}B_{pont}L} \tag{14}$$

$$KB = \frac{1}{3}(5\frac{T}{2} - \frac{\nabla}{2LB_{pont}}); \tag{15}$$

$$I_T = \frac{2}{12}LB_{pont}^3\frac{6 \cdot Cw_{pont}^3}{(1 + Cw_{pont})(1 + 2Cw_{pont})} + 2 * Aw_{pont}y_{pont}^2 \tag{16}$$

$$I_L = \frac{0.8 \cdot 2}{12}B_{pont}L^3 \tag{17}$$

$$GM_T = KB + \frac{I_T}{\nabla} - T + rg_z \tag{18}$$

$$GM_L = KB + \frac{I_L}{\nabla} - T + rg_z \tag{19}$$

$$\tag{20}$$

$$G_{CF} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \rho g(2Aw_{pont}) & 0 & 0 & 0 \\ 0 & 0 & 0 & \rho g\nabla GM_T & 0 & 0 \\ 0 & 0 & 0 & 0 & \rho g\nabla GM_L & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{21}$$

$$G = H^T G_{CF} H; \tag{22}$$

$$\omega_3 = \sqrt{G_{33}/M_{33}} \tag{23}$$
$$\omega_4 = \sqrt{G_{44}/M_{44}} \tag{24}$$
$$\omega_5 = \sqrt{G_{55}/M_{55}} \tag{25}$$

6 Linear Damping

$$h = \begin{bmatrix} -24.4\frac{g}{U_{max}} \\ 0 \\ -2 \cdot 0.3 \cdot \omega_3 M_{33} \\ -2 \cdot 0.2 \cdot \omega_4 M_{44} \\ -2 \cdot 0.4 \cdot \omega_5 M_{55} \\ \frac{-M_{66}}{T_{yaw}} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \frac{-M_{66}}{T_{yaw}} 10abs(r) \end{bmatrix} \tag{26}$$

$$\tau_{damp} = h \cdot \nu_r \tag{27}$$

7 Transformation

$$R = R_z R_y R_x = \begin{bmatrix} \cos(\phi) & -\sin(\phi) & 0 \\ \sin(\phi) & \cos(\phi) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos(\theta) & 0 & \sin(\theta) \\ 0 & 1 & 0 \\ -\sin(\theta) & & \cos(\theta) \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(\phi) & -\sin(\phi) \\ 0 & \sin(\phi) & \cos(\phi) \end{bmatrix} \tag{28}$$

$$T = \begin{bmatrix} 1 & \sin(\phi)\tan(\theta) & \cos(\phi)\tan(\theta) \\ 0 & \cos(\phi) & -\sin(\phi) \\ 0 & \dfrac{\sin(\phi)}{\cos(\theta)} & \dfrac{\cos(\phi)}{\cos(\theta)} \end{bmatrix} \tag{29}$$

$$J = \begin{bmatrix} R & 0 \\ 0 & T \end{bmatrix} \tag{30}$$

8 State derivative

$$\dot{x} = \begin{bmatrix} \dfrac{M}{\tau + \tau_{damp} + \tau_{cf} - C\nu_r - G\eta - g_0} \\ J\nu \end{bmatrix} \tag{31}$$