1 Kinematics

$$\boldsymbol{M}_{RB}\dot{\boldsymbol{\nu}} + \boldsymbol{M}_A\dot{\boldsymbol{\nu}}_{\boldsymbol{r}} + \boldsymbol{C}_A(\boldsymbol{\nu})\boldsymbol{\nu} + \boldsymbol{C}_{RB}(\boldsymbol{\nu}_r)\boldsymbol{\nu}_r + \boldsymbol{D}(\boldsymbol{\nu}_r)\boldsymbol{\nu}_r = \boldsymbol{\tau} + \boldsymbol{w}(t) \tag{1}$$

$$\dot{\boldsymbol{\eta}} = \boldsymbol{J}(\eta)\boldsymbol{\nu} \tag{2}$$

$$\boldsymbol{\tau} = \boldsymbol{\tau}_{control} \tag{3}$$

2 Thruster model

$$\boldsymbol{\tau}_{control} = \begin{bmatrix} \boldsymbol{\tau}_{control,linear} \\ \boldsymbol{\tau}_{control,torque} \end{bmatrix} \tag{4}$$

Each propeller can rotate with an angle ξ_p. The thrust of each propeller can be describe as a vector by spilting it into components that align with the ship coordinate frame

$$t_p = T_{nn}n_p^2 + T_{nv}V_A n_p \tag{5}$$

$$\boldsymbol{T}_p = \begin{bmatrix} \cos(\xi_p) \\ \sin(\xi_p) \\ 0 \end{bmatrix} t_p \tag{6}$$

where t_p is the magnitude of the thrust and $\boldsymbol{T}_p$ is the thrust vector. The linear control force is found as the sum of forces from each propeller p.

$$\boldsymbol{\tau}_{control,linear} = \sum^{P} \boldsymbol{T}_p \tag{7}$$

$$\boldsymbol{\tau}_{control,torque} = \sum^{P} \boldsymbol{r}_p \times \boldsymbol{T}_p = \sum^{P} S(\boldsymbol{r}_p)\boldsymbol{T}_p \tag{8}$$

where $\boldsymbol{r}_p$ is the position vector of thruster p and $S(\boldsymbol{r}_p)$ is the skew-symetric matrix of vector $\boldsymbol{r}_p$.

In the case of two propellers we have

$$\boldsymbol{\tau}_{control} = \begin{bmatrix} I & I \\ S(\boldsymbol{r}_1) & S(\boldsymbol{r}_2) \end{bmatrix} \begin{bmatrix} \boldsymbol{T}_1 \\ \boldsymbol{T}_2 \end{bmatrix} \tag{9}$$