Exercise 1

Give the asymptotic tight bounds (Θ) on the following functions of n. Here, $k \geq 1$ and $c > 1$ are constants. Hint: you might want to take a look at some of the classic functions and their properties in Section 3.3 CLRS.

1. $0.001n^2 + 70000n$
2. $2^n + n^{1000}$
3. $n^k + c^n$
4. $20 \log n + n^k$
5. $2^n + 2^{n/2}$
6. $n^{\log c} + c^{\log n}$
7. $13n^4 + 7n^3 - 9n^2 + 127n + \frac{n!}{7}$

Exercise 2

Consider the following function $f(n) = 3n \log n + 15n + 1800$ and prove that it is in $O(n \log n)$ (hint: find constants c and n_0 such that $f(n) \leq c(n \log n)$ for all $n \geq n_0$.

Exercise 3

Consider the pseudo-code for the following mysterious sorting function SORT(A):

```
SORT(A)
n = A.length
for i = 1 to n
    for j = i + 1 to n
        if A[i] > A[j]
            key = A[i]
            A[i] = A[j]
            A[j] = key
return sum
```

1. Explain informally how the algorithm works.
2. Prove that SORT correctly solves the solving problem (hint: determine suitable invariants for both loops)
3. Determine the asymptotic worst-case running time in terms of upper bound O and lower bound Ω.