Notes on minimizing synthesized controller strategies

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1 Introduction

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2 Overview

The process involves several steps.

The first step is to synthesize a strategy for a controller in UPPAAL Stratego and output it as a `json` file. UPPAAL stores the strategy as a set of Q-trees (one for each possible action). A Q-tree is a binary tree whose (internal) branching nodes each split the state space according to some bound on a given variable and whose leaf nodes are labelled with the Q-value of one particular action (the one that the given tree represents) in the state determined by the constraints on the path from the root node to that leaf. As such, the set of Q-trees together represents a complete lookup-table as known from classical RL, except that the discretization (partitioning) of the state space is unique for each action and is something that is learned during training instead of being pre-dertimined by experts or engineers.

The second step is to convert the set of Q-trees into an actual decision tree, where the leaf nodes are labelled not with Q-values but with actions. The decision tree should be constructed so that for each state S, the action label on the leaf node corresponding to S is always the same as the action with largest Q-value when evaluating S in all the original Q-trees. An algorithm for performing this conversion is given in Algorithm **??**. In short, it works by obtaining all the state, action and Q-value tuples of the Q-trees and sorting them according to Q-value. The first tuple is then used to create a root node of the new DT and for each constraint needed to specify the state, a branching node is created. At the end of this path, a leaf node with the action is inserted. Now, the rest of the tuples can be inserted in order, creating branch nodes when the state needs to be further specified or discarding the insertion otherwise (since every new insertion will have a 'worse' Q-value than every previous one).

Thirdly, the resulting DT, that represents a single, complete partitioning of the state space with individual optimal actions assigned to every possible state, can then be scanned for 'neighbouring' leaves with identical action labels and aligned constraints. 'Neighbouring' in this sense refers to the situation where the states of two leaves are spatially adjacent and therefore could be represented by a single leaf instead of two. This will happen quite often, since UPPAAL will create a lot of arbitrary or experimental partitions especially during the early stages of training, which is then carried over into our new DT. This can —

potentially — give larger areas of the state space, where the same action is optimal but where multiple leaves (partitions) are used to represent it in the tree. Algorithm **??** describes a procedure for minimizing a given complete partitioning in the number of partitions used.

The fourth and final part is to recreate a DT from the newly obtained minimal partitioning. This has the challenge that the partitions most likely wont be able to be represented excactly by a tree structure. For example, the partitioning in Figure **??** cannot be represented by a binary tree, since no matter what (axis aligned) constraint we would choose for a root node, it would 'cut' one of the partitions in two and thus create a larger set of leaves than the set of partitions in the minimal partitioning. Instead, we then need a heuristic for how to best represent the minimal partitioning, which we give a suggestion for in Algorithm **??**.

After this process, we can export the newly created DT to a format readable by UPPAAL Stratego and verify, that it performs just as well on the original task as the original strategy did.

3 Q-trees

In Reinforcement Learning (RL) the goal is to learn a policy π for which action to take in any given state of the environment. This policy is thus essentially a mapping from a state $S \in \mathcal{S}$ to an action $A \in \mathcal{A}$ where $\mathcal{S}$ is the state space and $\mathcal{A}$ is the set of all possible actions. Most often, the policy will be to choose the optimal action in any given state according to some metric, for example expected cumulative reward or cost. This is called a *greedy* policy.

The classical approaches either considers S to be a vector of continuous values and the task of learning π as one of approximating the function $f : S, A \mapsto Q^{\pi}_{S,A}$ where $Q^{\pi}_{S,A}$ is the expected value of taking action A in state S when following π (this can for example be done using a Neural Network **??**), or considers S to describe a *discrete* state for which the true value of $Q^{\pi}_{S,A}$ can be learned for any $S \in \mathcal{S}$ and any $A \in \mathcal{A}$. In the latter case, if the state space originally is continuous, some kind of artificial discretization is required and then the strategy will typically be represented by a look-up table with dimensions $|\mathcal{S}| \times |\mathcal{A}|$. This is called a Q-table.

What is the correct and concise way to write 'the dimensionality is the number of discrete states times the number of actions'?

In UPPAAL Stratego **??** the gap between a continuous and discretized state space is gapped by having the discretization happen as part of the training process and representing the Q-values of state-action pairs as values in the leaf nodes of binary decision trees. During training, UPPAAL Stratego will build such trees by creating branch nodes that splits the state space in a particular dimension on a bound that it finds (or guesses) separates diverging Q-values of an action. These predicates take the form of $x \leq b$ where x is a dimension (variable) in the state space and b is a real valued constant.

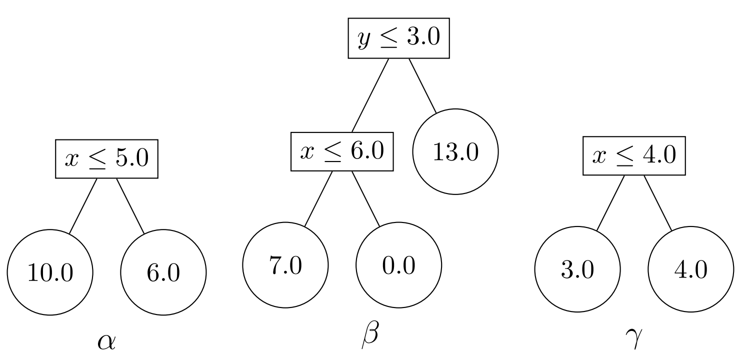

Figure 1: A simple example of Q-trees for three actions, α, β and γ, and predicates on two variables, x and y. Evaluating a state with $x = 3$ and $y = 5$, we would get Q-values 10.0, 13.0 and 3.0 for α, β and γ respectively, meaning γ would be the optimal action.

The end result is a set of trees $\mathcal{T}$, one for each action $A \in \mathcal{A}$, where any continuous valued state $S \in \mathcal{S}$

can be evaluated to $Q^{\pi}_{S,A}$ in any tree $T_A \in \mathcal{T}$ by following the path from the root node to a leaf node as specified by the predicates in the branching nodes. That is, at each branch node we follow the path to the 'left' if the predicate is true for S and we follow the path to the 'right' otherwise, untill we end up at a leaf node which will then carry the value of $Q^{\pi}_{S,A}$. The subscript A in T_A indicates that it is the tree specifying the Q-values of action A and we can now define $T_A(S) : S, A \mapsto Q^{\pi}_{S,A}$. The greedy policy then just have to choose a an action A where $A = \operatorname{argmin}_{A \in \mathcal{A}} T_A(S)$[1].

Should I ditch the π superscript now?

We propose to call these trees *Q-trees* and provide a small example in Figure 1.

4 Converging Q-trees to decision tree

A strategy represented by a set of Q-trees has a couple of disadvantages: first of all, having to iterate all the trees and evaluate S in each one of them creates an amount of overhead, and second, the structure of the representation is not very intuitive to humans in the sense that the unique evaluation of S in each tree and the Q-values that they then return are very hard to interpret.

I don't know

Maybe just cut this paragraph?

We can convert the Q-trees into a single decision tree with actions — not Q-values — in the leaf nodes so that the evaluation of S in the tree yields the optimal action for that state.

Let $\mathcal{V} = \{V_1, V_2, \ldots, V_m\}$ be the set of all variables (dimensions) in the state space. We then have that a state $S = [v_1, v_2, \ldots, v_m]^{\intercal}$ is a vector of instantiated values of each $V \in \mathcal{V}$, ie. $V_i = v_i$. Following the path from the root node of any Q-tree T_A to a leaf node L_i, we can collect the constraints entailed by each predicate in the branching nodes along the path to get a symbolic state S_i that represents an m-dimensional area (also called partition) of the state space. The symbolic state is not given by concrete values of the dimensions but rather by lower and upper bounds, that is $S_i = \{(l_{1,i}, u_{1,i}), (l_{2,i}, u_{2,i}), \ldots, (l_{m,i}, u_{m,i})\}$ where $l_{j,i}$ and $u_{j,i}$ are the lower and upper bounds respectively of V_j in the symbolic state S_i that the leaf L_i entails.

With this, we can define a leaf in a Q-tree as $L_i = (S_i, q_a, A_i)$ where S_i is the symbolic state obtained from applying all the constraints on the path to L_i, A_i is the action pertaining to the tree that the leaf belongs to and q_{A_i} is the Q-value of action A_i in all $S \in S_i$. The algorithm for converting a set of Q-trees to a decision tree then starts by gathering all these leaf triplets and sorting them in ascending order[2] according to their Q-value, and then pops the first leaf to create a root node and a path of branching nodes until S_1 is fully specified. Then a decision leaf can be inserted with label A_1, knowing that this is the optimal action for this state. From here, we can continually pop and insert L_2, $L_2, \ldots, L_n$ in that order, creating new branching nodes when needed and ignoring insertions into areas that have already been covered by previous leafs (since the actions of these will necessarily be more optimal than the later one because of inital sorting).

The algorithm for this operation is given in Algorithm **??**.

5 Minimizing the decision tree

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[1] Note that UPPAAL Stratego uses *cost* rather than *reward*, therefore we minimize instead of maximizing

[2] Again, UPPAAL Stratego computes for Q-values the expected *cost*, so the action with the lowest Q-value is the most preferable.

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5.1 Empirical pruning

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5.2 Maximizing partition sizes

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6 Rebuilding the tree

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