Spread of Infectious Diseases with Finite Infectious Period on Temporal Networks

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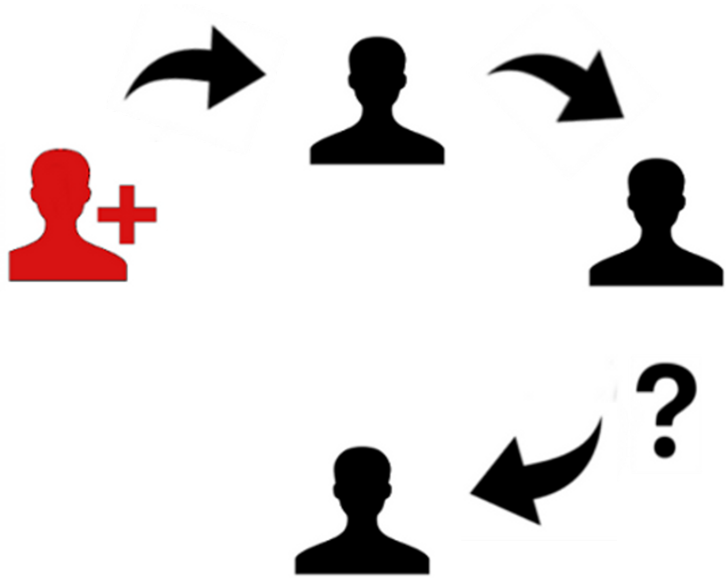
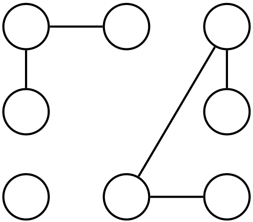
9. Dezember 2014



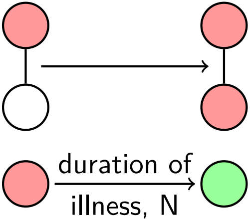
What is a Temporal Network?

$$\{A_0, A_1, ..., A_T\}$$



An Epidemiological Toy-Model

Rules for an SIR-Model:



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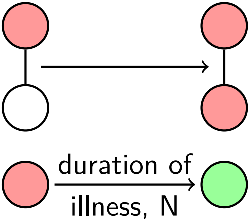
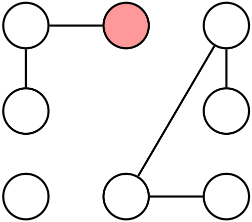


One possible Realisation



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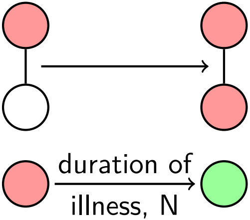

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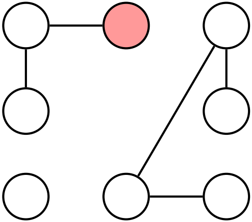
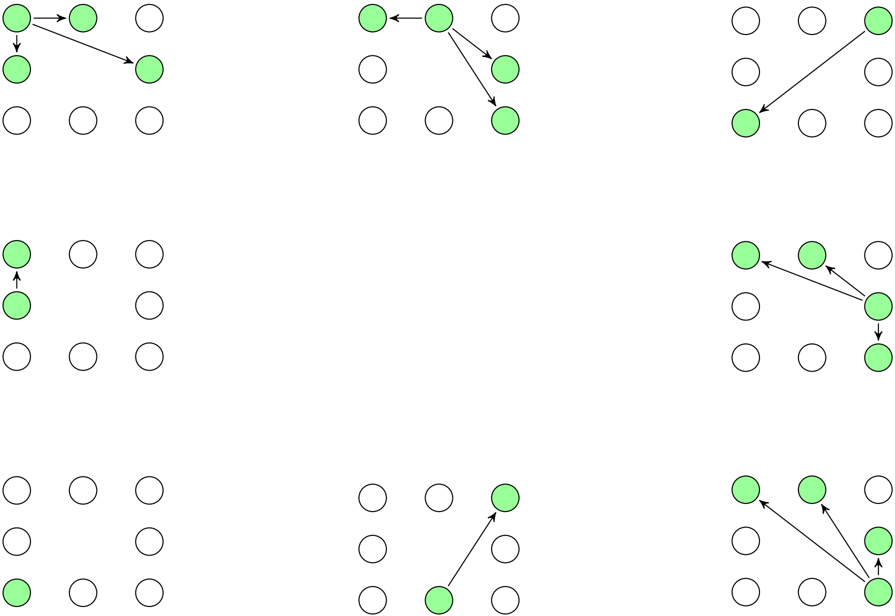


The reachability matrix



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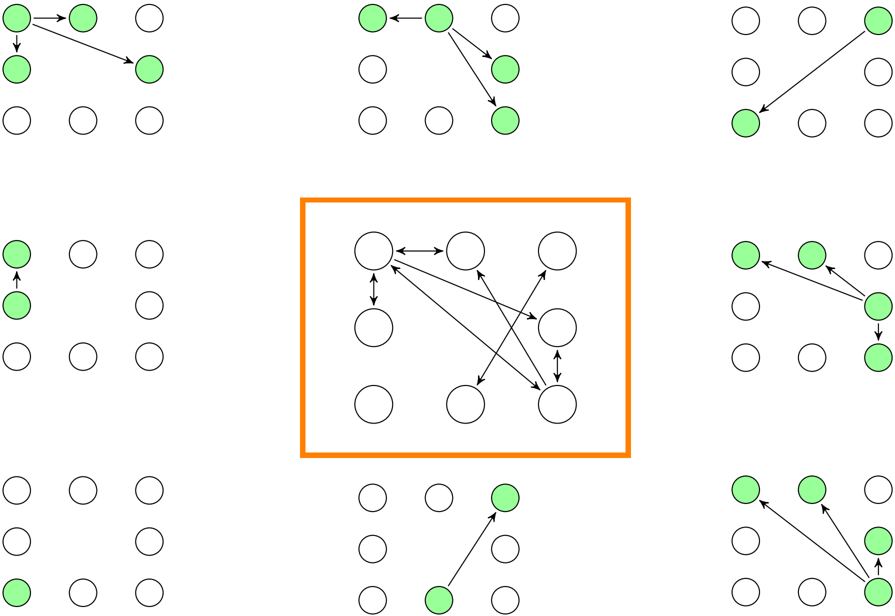
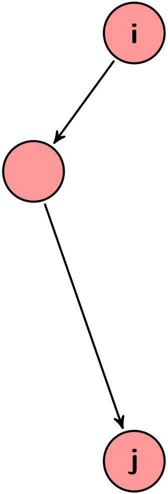
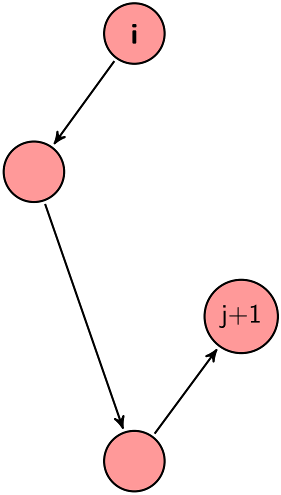


Infection Matrix



Infection Matrix $I(T)$

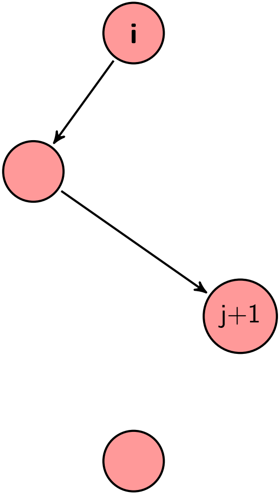
$$\begin{array}{cc} \begin{array}{ccc} & \mathbf{i} & \end{array} & \\ \left[\begin{array}{ccc} & \vdots & \\ & 1 & \cdots \\ & & \end{array}\right] & \mathbf{j} \end{array}$$



$$A_{T+1} \cdot I(T)$$

$$\begin{array}{cc} \mathbf{i} & \\ \left[\begin{array}{cc} \vdots & \\ 1 & \cdots \\ & \\ & \end{array}\right] & \mathbf{j+1} \end{array}$$

Infection Matrix



$$A_{T+1} \cdot I(T-1)$$

$$\begin{array}{cc} \begin{array}{c} \mathbf{i} \end{array} & \\ \begin{bmatrix} & \vdots & & \\ & 1 & \cdots\cdots & \\ & & & \\ & & & \end{bmatrix} & \mathbf{j+1} \end{array}$$

Rule #1

Finte Waiting Time

$$I^*(T+1) = \sum_{t=0}^{N} A_{T+1} \cdot I(T-t)$$

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Rule #2

No Loops

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$$\boxed{I(T+1) = I^*(T+1) \wedge \bigwedge_{t=0}^{T} \overline{I(T-t)}}$$

Infection Matrix

$$I(T+1) = \bigvee_{t=0}^{N} A_{T+1} \cdot I(T-t) \wedge \bigwedge_{t=0}^{T} \overline{I(T-t)}$$

$$I(0) = \mathbb{1}$$

Visualization

$$\rho(I_T) = \frac{\mathrm{nnz}(I_T)}{\dim^2(I_T)}$$

$\langle \#$ of newly infected individuals $\rangle$

Incidence

newly infected individuals

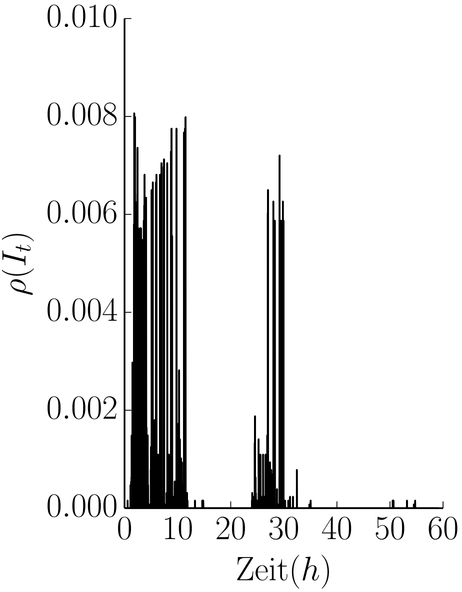
$I(T)$

Numerical Results - sociopatterns.org

Incidence

\# newly infected individuals

$I(T)$



Numerical Results - sociopatterns.org

Accumulated Incidence

$\#$ of infected individuals
up to time T

$$C(T) = \bigvee_{t=0}^{T} I(t)$$

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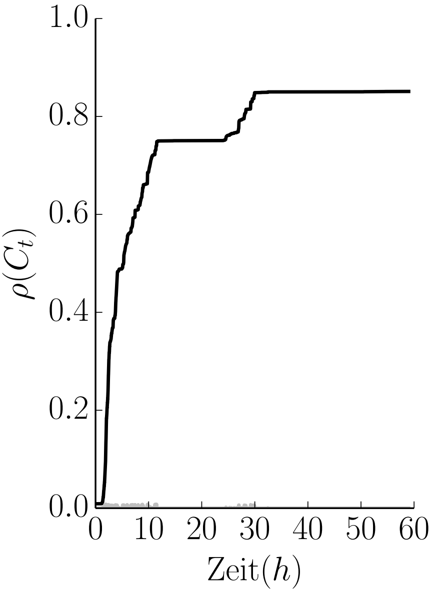

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# of infected individuals up to time T

$$C(T) = \bigvee_{t=0}^{T} I(t)$$

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Prevalence

# of infected individuals at time T

$$P(T) = \bigvee_{t=0}^{N} I(T - t)$$

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# of infected individuals at time T

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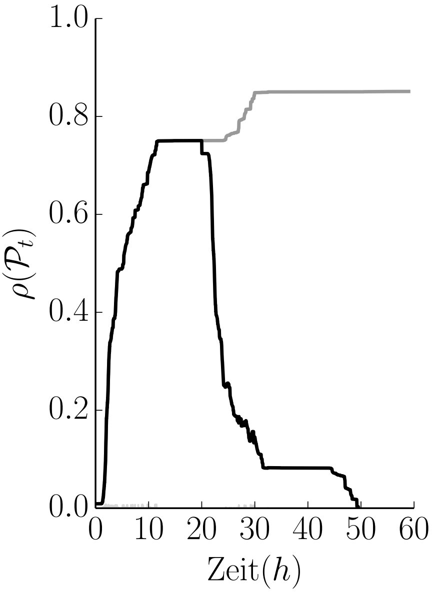
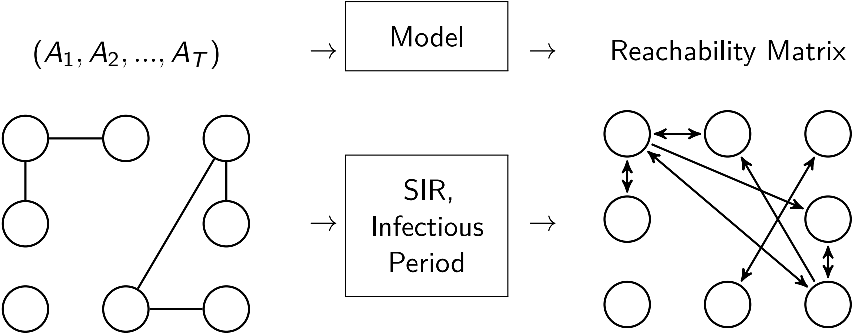


Summary

We present a compact matrix formalism:



Thank you!