FRAUD DETECTION BY A MULTINOMIAL MODEL: SEPARATING HONESTY FROM UNOBSERVED FRAUD

ANDREAS OLDEN*

JOINT WITH
JONAS ANDERSSON*
AND AIJA POLAKOVA*

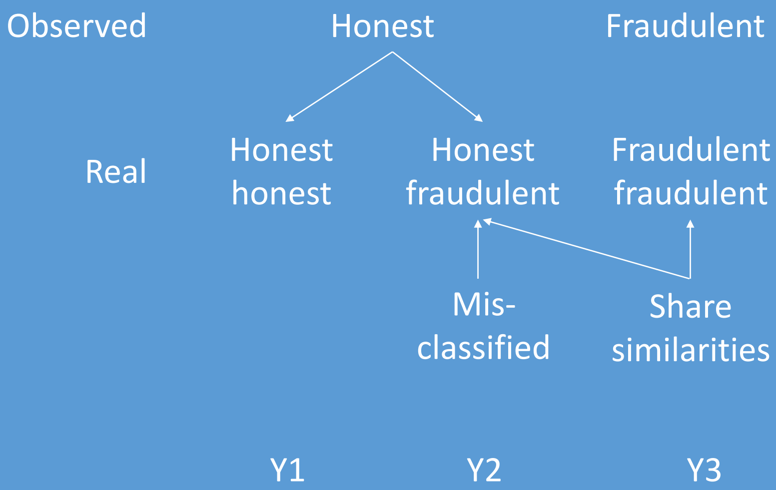
*NORWEGIAN SCHOOL OF ECONOMICS

Overview

Fraud is often detected through audits. We then use this information to predict which new claims, tax returns etc. are fraudulent.

However, an audit process (almost) never has a perfect detection rate. This implies that a group of cheaters are (mis)classified as honest.

Hausman, Abrevaya, and Scott-Morton (1998) analyses misclassification in a survey setting and show that with misclassification of only 2 percent, parameters in a probit model are biased by 15 percent to 25 percent, which reduces predictive power + cheaters get away with it!


Observed
Honest
Fraudulent
Real
Honest honest
Honest fraudulent
Fraudulent fraudulent
Mis-classified
Share similarities
Y1
Y2
Y3

Literature

Much work on fraud, but not that much on misclassification

Hausman, Abrevaya and Scott-Morton (1998) runs a simulation study similar to ours based on a known issue of misclassification in surveys

Artís, Ayuso and Guillén (2002) shows that the method of HAS-M can be used in an insurance fraud setting, but no evaluation of performance

Caudill, Ayuso and Guillén (2005) introduces a new model based on the EM-algorithm, again without evaluation

The two latter articles shows that these methods can be used in setting close to tax fraud, but has nothing to say on whether we should do so.

The Trinomial Logit

If we observe all three categories, the estimation is straight forward

$$p_k = \frac{e^{\alpha_k + \beta_k x}}{1 + e^{\alpha_2 + \beta_2 x} + e^{\alpha_3 + \beta_3 x}},$$

for $k = 1, 2, 3$ and $\alpha_1 = \beta_1 = 0$.

And we max the log-likelihood

$$\ln L(\alpha_2, \alpha_3, \beta_2, \beta_3) = \sum_{i=1}^{n} (Y_{1i} \ln p_1 + Y_{2i} \ln p_2 + Y_{3i} \ln p_3)$$

However, since we do not observe Y_1 and Y_2, only $(Y_1 + Y_2)$, we have to consider them as latent variables. We now call

$$\ln L(\alpha_2, \alpha_3, \beta_2, \beta_3) = \sum_{i=1}^{n} (Y_{1i} \ln p_1 + Y_{2i} \ln p_2 + Y_{3i} \ln p_3)$$

the log-likelihood function for the *full data* (which is not completely observed).

The identifying assumption for the model is $\beta_2 = \beta_3$, meaning that the "HF" and the FF" have similar characteristics.

But we do not get any further with the standard multinomial model without full data.

The EM-algorithm

1. Select starting values for $\alpha_2, \alpha_3, \beta_2$
2. E-step: Compute the expectation of $\ln L(\alpha_2, \alpha_3, \beta_2)$ given the *observed data*. $Q(\alpha_2, \alpha_3, \beta_2) = E(\ln L(\alpha_2, \alpha_3, \beta_2 | Y, X)$
3. M-step: Maximize Q to obtain new parameters
4. Use new parameters as new starting values, repeat until convergence.

In step 2 of the algorithm above we need to compute the following conditional expectations and use instead of Y_1 and Y_2

$$Y_1^* = E(Y_1 | Y_3 = 0) = \frac{1}{1 + e^{\alpha_2 + \beta_2 x}}$$

and

$$Y_2^* = E(Y_2 | Y_3 = 0) = \frac{e^{\alpha_2 + \beta_2 x}}{1 + e^{\alpha_2 + \beta_2 x}}$$

What we do

We do a **Monte Carlo simulation study**: 1000 random draws (N) of X-variables 1000 times (nrepl) with standard deviations from CAG (2005)

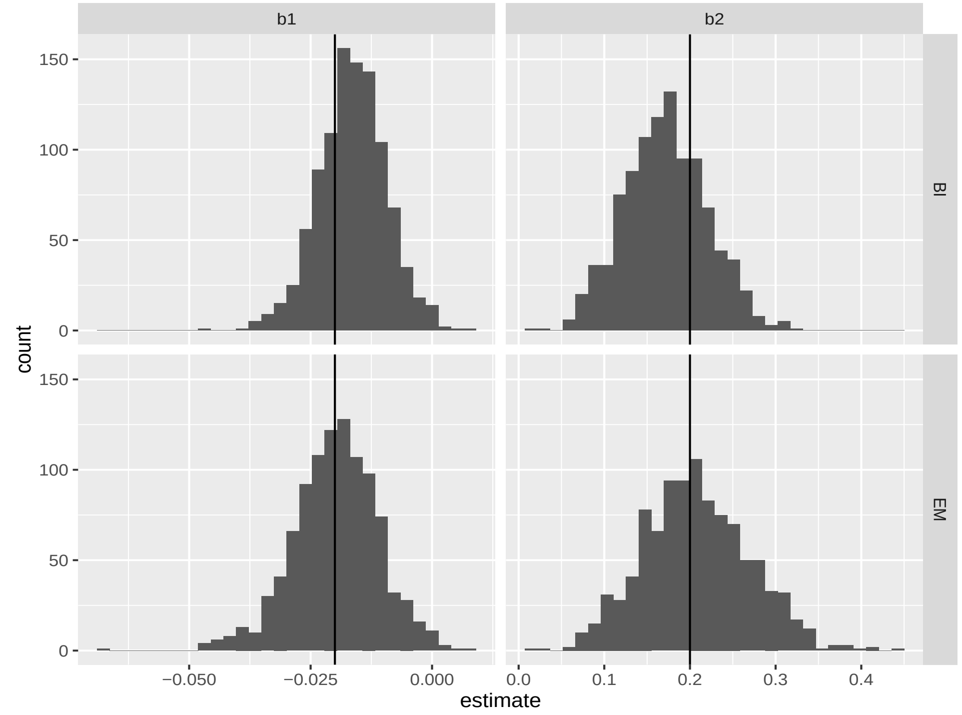
We then apply the full trinomial model together with parameters from CAG (2005) to create Y-variables for the three categories, HH, HF and FF.

We pretend not to observe all three categories and use the EM-algorithm for missing data to see how close to the true values we come

We manipulate several conditions and see how the EM-algorithm performs and compare it to a naive binomial model which assumes no misclassification

Variable	Mean	Std dev	Coeff
CONSTANT	-	-	-1.440
AGE	38.02	12.32	-0.021
LICENSE	14.23	9.09	0.003
RECORDS	1.42	1.80	0.177
COVERAGE	0.91	0.29	0.795
DEDUCTIBLE	0.03	0.16	-0.303
ACCESORI	0.07	0.25	-0.350
VEHUSE	0.88	0.32	-0.507
VEHAGE	6.17	4.48	0.012
FAULT	0.32	0.47	1.388
NONURBAN	0.07	0.26	0.559
NIGHT	0.13	0.34	1.488
WEEKEND	0.27	0.44	0.274
WITNESS	0.01	0.08	1.140
POLICE	0.11	0.31	-1.805
ZONE1	0.14	0.34	0.320
ZONE3	0.49	0.50	0.642
REPORT	0.59	0.49	0.562
NAMES	0.06	0.24	1.172
PROXIM	0.02	0.13	1.716
DELAY	0.24	0.43	1.212

Statistic	N	Mean	St. Dev.	Min	Max
n	1,000	1,000.000	0.000	1,000	1,000
px	1,000	2.000	0.000	2	2
iter	1,000	13.918	25.704	3	100
nrepl	1,000	1,000.000	0.000	1,000	1,000
ad	1,000	0.167	0.000	0.167	0.167
y1	1,000	712.710	14.123	674	758
y2	1,000	122.105	10.528	81	161
y3	1,000	165.185	12.045	129	200
ta1	1,000	−1.800	0.000	−1.800	−1.800
ta2	1,000	−1.500	0.000	−1.500	−1.500
tb1	1,000	−0.020	0.000	−0.020	−0.020
tb2	1,000	0.200	0.000	0.200	0.200
EMa1	1,000	−1.694	0.173	−2.551	−0.951
EMa2	1,000	−1.488	0.109	−1.800	−1.110
EMb1	1,000	−0.020	0.009	−0.068	0.008
<u>EMb2</u>	1,000	0.207	0.062	0.019	0.446
BIa	1,000	−1.672	0.091	−1.941	−1.412
BIb1	1,000	−0.016	0.007	−0.047	0.007
<u>BIb2</u>	1,000	0.172	0.048	0.018	0.331
PIa1	1,000	−1.810	0.102	−2.259	−1.508
PIa2	1,000	−1.507	0.090	−1.799	−1.258
PIb1	1,000	−0.020	0.006	−0.042	0.003
PIb2	1,000	0.203	0.040	0.053	0.334


b1
b2
BI
EM
count
estimate
150
100
50
0
−0.050
−0.025
0.000
0.0
0.1
0.2
0.3
0.4

Some more results

	EM a1	EM a2	EM b1	EM b2	BI a	BI b1	BI b2	BI $b1/\beta1$	BI $b2/\beta2$	EM $b1/\beta1$	EM $b2/\beta2$	y1	y2	y3
2 var														
0 Corr	-1.694	-1.488	-0.02	0.207	-1.672	-0.016	0.172	0.8	0.86	1	1.035	713	122	165
0.5 corr	-1.673	-1.48	-0.02	0.203	-1.661	-0.017	0.17	0.85	0.85	1	1.015	715	121	164
0.9 corr	-1.668	-1.484	-0.021	0.205	-1.661	-0.018	0.174	0.9	0.87	1.05	1.025	719	120	162
0 corr x-sd*10	-1.851	-1.527	-0.02	0.204	-1.63	-0.007	0.072	0.35	0.36	1	1.02	579	179	241
0.9 corr x-sd*10	-1.904	-1.517	-0.02	0.203	-1.72	-0.013	0.13	0.65	0.65	1	1.015	651	149	201
1 var														
Base	-1.665	-1.477	-0.02		-1.657	-0.017		0.85		1		717	121	163
a-dev	-1.567	-1.356	-0.022		-1.554	-0.019		0.95		1.1		783	40	178

Some more results

2 variables	BI $b1/\beta1$	BI $b2/\beta2$	EM $b1/\beta1$	EM $b2/\beta2$	y1	y2	y3
0 Corr	0.8	0.86	1	1.035	713	122	165
0.5 corr	0.85	0.85	1	1.015	715	121	164
0.9 corr	0.9	0.87	1.05	1.025	719	120	162
0 corr x-sd*10	0.35	0.36	1	1.02	579	179	241
0.9 corr x-sd*10	0.65	0.65	1	1.015	651	149	201
1 variable							
Base	0.85		1		717	121	163
a-dev 200% y2 small	0.95		1.1		783	40	178

Two variables with true values: $\beta1 = -0.02$, $\beta2 = 0.2$

Summary

EM does better than a naive binomial model in most cases

Particularly in cases with large misclassification and lots of variation in the data

with low misclassification the naive binomial case is slightly better

The EM-algorithm has higher standard deviation and more uncertainty (partly because more parameters are estimated).

Starting-values matter, but our suggested solution of using the numbers from the binomial logit works well

The EM-algorithm does not always converge to our criteria

The EM-algorithm is slow

Next steps

Add variables

Move to predictions- here the additional uncertainty with the EM-algorithm will matter

Try other estimation techniques- omission errors that do not explicitly model HF

Apply to data

Possible real-life testing

The ideal would be an RCT with tax audits

One alternative would be to look at data for known evaders

If we for instance have data on foreign evaded income in Denmark and Sweden, we could pretend not to observe both and use the one to estimate the other.

References

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