Basic Mathematics

- Add, subtract, multiply and divide numbers
- Combine operations and use brackets
- Rounding to decimal places or significant figures
- scientific notation
- Use symbols to represent relationships
- constants and variables
- Evaluate expressions
- Understand the concept of a model
- Use a spreadsheet
- Simplify expressions containing symbols

We assume that:

- ⋆ you know how to add, subtract, multiply and divide positive numbers
- ⋆ you can cope with numbers like 5.123 in which the fractional part is given using decimal places

- ... -6 -5 -4 -3 -2 -1 0 1 2 3 4 5 6...
- 0
- 2, 4, 5
- 2.14, 3.76, 21.9351
- -2, -4, -5
- -2.43, -12.54, -17.9136

⋆ OPPOSITE SIGNS

- + (- number) or - (+ number) gives a -
- $3 + (-5) = 3 - 5 = -2$

⋆ SAME SIGNS

- - (- number) or + (+ number) gives a +
- $(-3) - 7 = -10$
- $12.42 - (-3.1) = 12.42 + 3.1 = 15.52$

- ⋆ the same signs gives a $+$
- ⋆ different signs gives a $-$

- ⋆ $+$ x $+$ $=$ $+$ 2 x 3 $=$ 6
- ⋆ $+$ x - $=$ $+$ 2 x (-3) $=$ -6
- ⋆ - x $+$ $=$ - (-2) x 3 $=$ -6
- ⋆ - x - $=$ $+$ (-2) x (-3) $=$ -6

- the same signs gives a $+$
- different signs gives a $-$

- $+ \div + = +\ 6 \div 3 = 2$
- $+ \div - = -\ (-6) \div 3 = -2$
- $- \div + = -\ 6 \div (-3) = -2$
- $- \div - = +\ (-6) \div (-3) = 2$

In order:

- ⋆ Brackets
- ⋆ Multiply and Divide (from left to right)
- ⋆ Add and Subtract (from left to right)

- ⋆ $2 \times 2 \times (27 \div 3) + (1 - 20)$
- ⋆ $= 2 \times 2 \times (9) + (-19)$
- ⋆ $= 36 - 20$
- ⋆ $= 17$

- To decimal places – the number of digits after a decimal point
- 1234.56789
  - is 1234.568 to 3 decimal places
  - is 1234.6 to 1 decimal place

- $a \times 10^b$
  - Where $1 \leq a < 10$
  - B is an integer

- $12000 = 1.2 \times 10^4$
- $0.0012 = 1.2 \times 10^{-3}$

- Letters are used to give a general representation of a constant or variable

- $S =$ the speed of a car: variable
- W $=$ the weight of a book - constant

- $C =$ the cost of hiring a boat for a trip
- $F =$ the fixed costs of the hire
- $P =$ the cost of petrol an hour (variable)
- $H =$ the time spent on a trip (variable)
- $C = F + HP$

- If n people hire the boat each pays:
- $\frac{C}{n} = \frac{F+NP}{n}$

- ⋆ The relationships form models of problems
- ⋆ Finding a solution to a problem means solving the equations
- ⋆ We have to find previously unknown from the known ones
- ⋆ For this we have to manipulate the equations into the required form

⋆ The rules for addition, subtraction, multiplication and division are exactly the same as for arithmetic with numbers

- $-(-a) = a \;\; -(+a) = -a \; etc...$
- $a \times (-b) = -ab \; (-a) \times (-b) = ab \; etc...$
- $\frac{a}{-b} = \frac{-a}{b} \; \frac{-a}{-b} \frac{a}{b} \; etc...$

- Remember explicit multiplication $2a = 2 \times a$

⋆ Often we have to collect like terms together

- $2pq + pq - 5pq = -2pq$
- $\frac{s}{2r} + \frac{4s}{2r} = \frac{5s}{2r}$

- A fraction is:
- $\frac{numerator}{denominator}$
- A fraction keeps the same value when you do the same thing to both the numerator and denominator.
- Dividing top and bottom means that
- $\frac{84}{162} = \frac{42}{81} = \frac{14}{27}$
- When no further cancelling can be done, a fraction is in its simplest form.

Multiplying fractions

- Multiply the numerators and the denominators
- $\frac{2}{3} \times \frac{4}{5} = \frac{8}{15}$
- $\frac{a}{b} \times \frac{c}{b} = \frac{ab}{cd}$

Dividing fractions

- Turn the second fraction upside down and multiply
- $\frac{1}{2} \div \frac{2}{3} = \frac{1}{2} \times \frac{3}{2} = \frac{3}{4}$
- $\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \times \frac{d}{c} = \frac{ad}{bc}$

- Percentages are the number of hundredths
- $5\% = 5 \div 100$
- $6\% of 300 = 300 \times \frac{6}{100}$
- $\frac{25}{400} = \frac{6.25}{100} = 6.25\%$

Expanding Brackets

- ⋆ The value before a bracket is multiplied by everything inside the brackets
- ⋆ $a(b+c) = ab + ac$
- ⋆ $x(y+z) - xy = xy + xz - xy = xz$
- ⋆ $\frac{b+c}{a} = \frac{b}{c} + \frac{c}{a}$

Product of Brackets

- ⋆ Everything inside the first bracket is multiplied by everything inside the second bracket
- ⋆ $(a+b)(c+d) = ac + ad + bc + bd$
- ⋆ $(x-2)(y+1) = xy - 2y + x - 2$

The opposite of expanding brackets; means taking an equation and finding the factors

- $5ma + 15m = 5m(a + 3)$
- $y2 + 4y - 5 = (y + 5)(y - 1)$
- $a^2 + 2ac + c2 = (a + c)^2$

Factorising is useful in solving equations

- $a^2 + 2ab + b^2 = (a + b)^2$
- $a^2 - 2ab + b^2 = (a - b)^2$
- $a^2 - b^2 = (a + b)(a - b)$
- $a^2 + b^2 =$

Multiplying a value by itself a number of times

- a squared $= a \times a = a^2$
- 2 cubed $= 2 \times 2 \times 2 = 2^3 = 8$
- -2 to the power $4 = (-2)^4 = 16$
- $\frac{2}{3}$ to the power $5 = (\frac{2}{3})^5 = 32/243$
- $a^0 = 1$ for any value of a

Take the reciprocal of the positive power

- ⋆ $b^{-m} = (\frac{1}{b})^m$
- ⋆ $2^{-2} = (\frac{1}{2})^2 = \frac{1}{4}$
- ⋆ $3^{-4} = (\frac{1}{3})^4 = \frac{1}{81}$
- ⋆ $(1+a)^{-2} = \frac{1}{(1+a)^2}$

To multiply powers of the same number, add the power

- ⋆ $b^m b^n = b^{m+n}$
- ⋆ $2^4 \times 2^{-5} = 2^{4-5} = 2^{-1} = \frac{1}{2}$

To raise a power to a power, multiply the powers together

- ⋆ $(b^m)^n = b^{m\times n}$
- ⋆ $(3^2)^3 = 3^6 = 729$

To divide powers of the same number, subtract the power

- ⋆ $\frac{b^m}{b^n} = b^{m-n}$
- ⋆ $\frac{2^4}{2^{-5}} = 2^{4+5} = 2^9 = 512$

- $(a \times b)^n = a^n \times b^n$
- $(3 \times 4)^3 = 3^3 \times 4^3 = 27 \times 64 = 1728$
- $(2ab)^m = 2^m \times a^m \times b^m$
- $(\frac{a}{b})^n = \frac{a^n}{b^n}$
- $(\frac{5}{2})^3 = \frac{5^3}{2^3} = \frac{5}{8}$

- $(a \times b)^n = a^n \times b^n$
- $(3 \times 4)^3 = 3^3 \times 4^3 = 27 \times 64 = 1728$
- $(2ab)^m = 2^m \times a^m \times b^m$
- $(\frac{a}{b})^n = \frac{a^n}{b^n}$
- $(\frac{5}{2})^3 = \frac{5^3}{2^3} = \frac{125}{8}$

- The square root of $a = \sqrt{a} = a^{\frac{1}{2}}$
- The cube root of $64 = a^{\frac{1}{3}} = 4$
- The q^{th} root of a is $a^{\frac{1}{q}}$
- $b^{\frac{n}{q}} = (b^{\frac{1}{q}})^n$

- ⋆ Understand how equations arise
- ⋆ Find solutions to equations
- ⋆ Solve equations where the unknown is to the power of 1
- ⋆ Rearrange an equation and substitute an expression
- ⋆ Formulate and rearrange inequalities

- ⋆ Show the relationship between constants and variables
- ⋆ They form quantitative models of problems
- ⋆ When there is a single unknown value, rearrange the equation so that this value is on one side of the equals sign, and all the known values are on the other side.
- ⋆ Then doing the calculations gives the unknown value or solves the equation

- $C = \frac{60+5h}{n}$
- When $h = 10$ and $n = 5$ the solution is $C = \frac{60+50}{5} = 22$

- $V = 1000(1+i)^n$
- When $I = 0.05$ and $n = 10$
- $V = 1000(1.05)^10 = 1629$

- When you do the same thing to both sides of an equation, it remains true
- $3x + 1000 = x + 2 + 1000 \Rightarrow 3x - 500 = x + 2 - 500$
- $6x = 2x + 4 \Rightarrow \frac{3x}{10} = \frac{x+2}{10}$

- ⋆ Get x on top $\Rightarrow$ by multiplying by any expressions containing x that are in the denominators of fractions
- ⋆ Get x outside any brackets $\Rightarrow$ Multiply out any brackets
- ⋆ Get all the xs on one side $\Rightarrow$ Collect together on one side of the equation all the terms involving x
- ⋆ Get x alone on one side $\Rightarrow$ Remove all other terms from that side of the equation

- $3x - x = x + 2 - x \Rightarrow 2x = 2 \Rightarrow x = 1$
- $4x + 3 = 11 \Rightarrow 4x = 8 \Rightarrow x = 2$
- $\frac{y}{y-2} = 3 \Rightarrow y = 3(y-2) \Rightarrow y = 3y - 6 \Rightarrow 2y = 6 \Rightarrow y = 3$
- $\frac{2x}{x-2} = 1 + \frac{4}{x-2} \Rightarrow \frac{2x}{x-2} = \frac{(x-2}{x-2} + \frac{4}{x-2} \Rightarrow 2x = x - 2 + 4 \Rightarrow x = 2$

- $3x - x = x + 2 - x \Rightarrow 2x = 2 \Rightarrow x = 1$
- $4x + 3 = 11 \Rightarrow 4x = 8 \Rightarrow x = 2$
- $\frac{y}{y-2} = 3 \Rightarrow y = 3(y-2) \Rightarrow y = 3y - 6 \Rightarrow 2y = 6 \Rightarrow y = 3$

- A break-even analysis shows that
- $100 - 0.5P = 80 + 0.2P \Longrightarrow 20 = 0.3P P = 67$

- A concert organiser anticipates selling 2000 tickets, a quarter of them at a 40% reduction. He needs to make \$18,000.
- 2000×0.75 at price p and 2000×0.25 at $0.6p \Rightarrow$
- $1500p + 500(0.6p) = 18000 \Rightarrow 1800p = 18000 \Rightarrow p = 10$

- $=$ is equal to
- $\neq$ is not equal to
- $<$ is less than
- $\leq$ is less than or equal to
- $>$ is greater than
- $\geq$ is greater than or equal to

- $\star$ $5x + 2 > 3x - 1 \Rightarrow 2x > -3 \Rightarrow x > -\frac{3}{2}$
- $\star$ $\frac{3}{x} > 2 \Rightarrow x < \frac{3}{2}$
- $\star$ $3 - x < 1 < 5 - x \Rightarrow 2 < x < 4$

- $\star$ $-2x > 6 \Rightarrow -x > 3 \Rightarrow x < -3$
- $\star$ $-\frac{p}{2} > 3 \Rightarrow \frac{3}{2} < -3 \Rightarrow p < -6$

- ⋆ Plot a straight line on a graph;
- ⋆ Calculate the slope or gradient of a line and the intercept with the y axis;
- ⋆ Model a problems using a linear equation;
- ⋆ Recognise a linear equation in more than two variables;
- ⋆ Solve a pair of simultaneous equations with two variables.

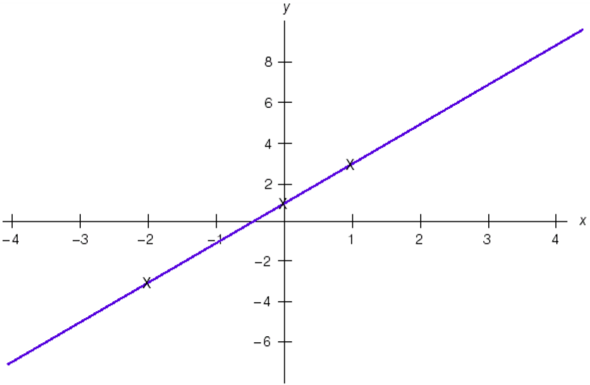

Figure 4.1

$$y = 2x + 1$$

x -5 -4 -3 -2 -1 0 1 2 3 4 5

y 17 14 11 8 5 2 -1 -4 -7 -10 -13

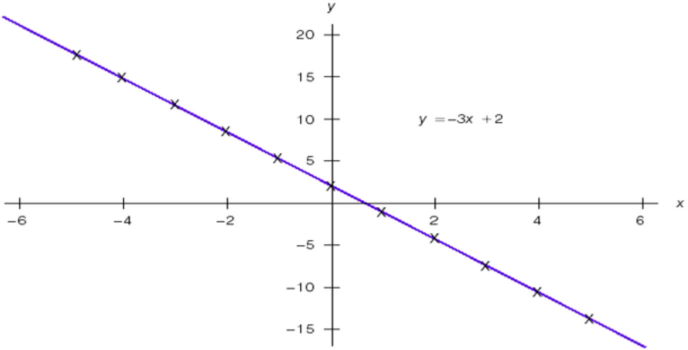

Figure 4.2

$$y = 2x + 1$$

- $y = ax + b$
- $y = 2x + 1$
- $3y - 3 = x$
- $2x - 4y = 5$

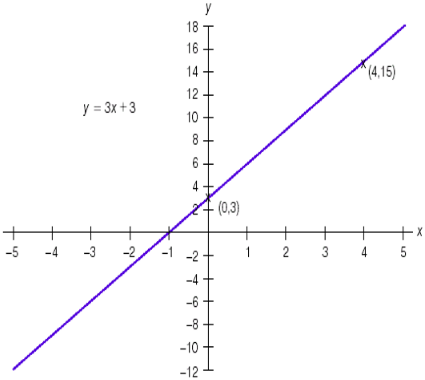

Figure 4.3

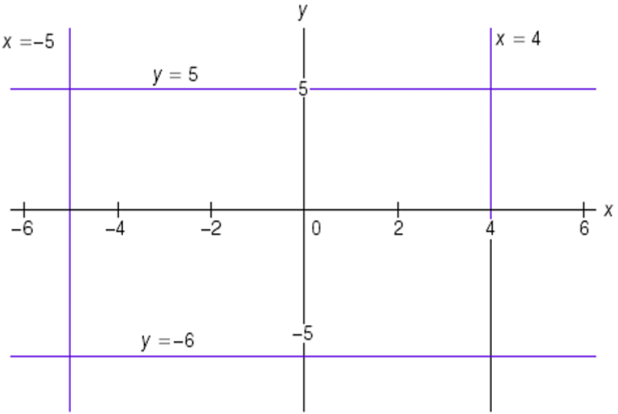

Figure 4.5

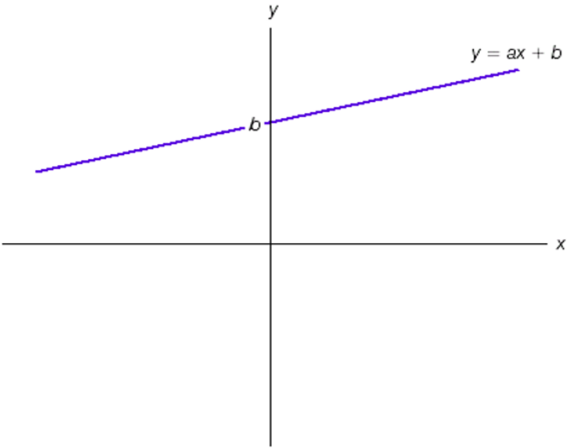

Figure 4.6

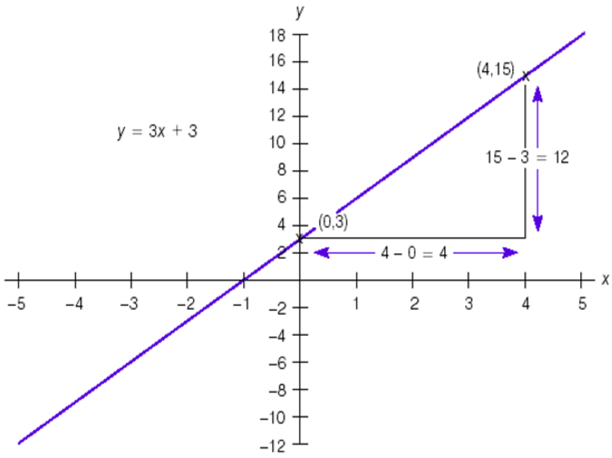
y
x
18
16
14
12
10
8
6
4
2
−2
−4
−6
−8
−10
−12
−5
−4
−3
−2
−1
1
2
3
4
5
y = 3x + 3
(4,15)
(0,3)
15 − 3 = 12
4 − 0 = 4

Figure 4.7

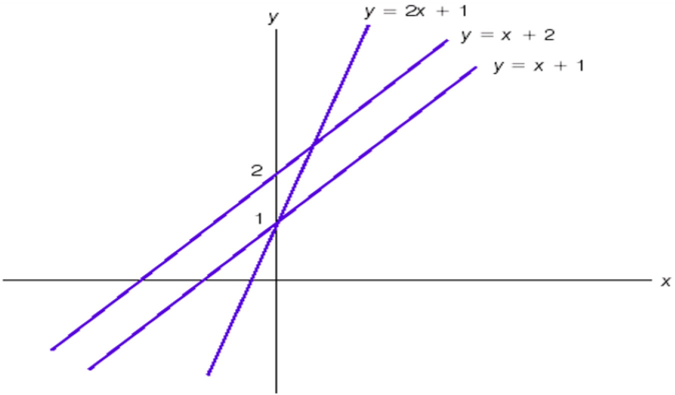

Figure 4.10

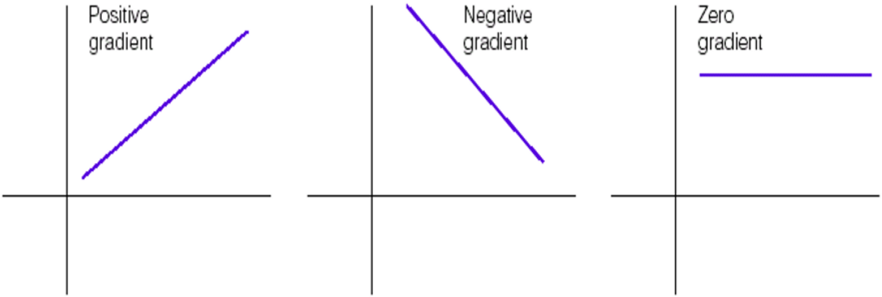

Figure 4.9

- You only need two points to draw a straight line
- Find the equation of the line through the points $(-1, 3)$ and $(2, 9)$
- Gradient is: $\frac{y2-y1}{x2-x1}$
- To find the value of b, just use one of the points. For instance $(2, 9)$ tells us that $9 = 2x2 + b$, so b is 5
- the equation is $y = 2x + 5$

- ⋆ Company has \$2000 per week to spend making radios and televisions. It costs \$5 to make a radio, \$40 to make a television
- ⋆ Draw a graph to show the possible numbers of radios (x) and televisions (y) they could manufacture

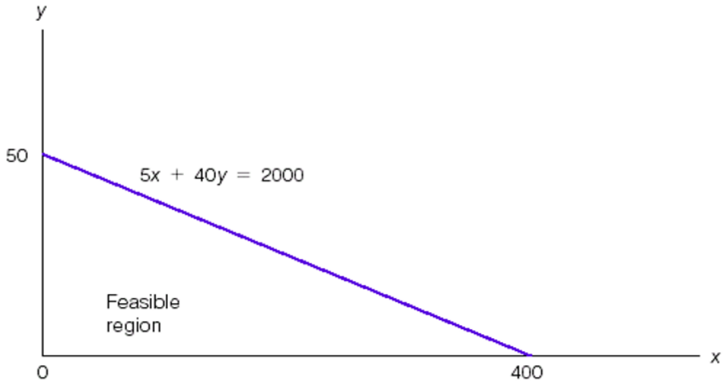

Figure 4.11

- $Q = -\frac{1}{2}P + 100$ Demand equation
- $Q = 2P - 20$ Supply equation
- Notice negative and positive slopes
- Market is in equilibrium at the values of Q and P, which makes both equations true

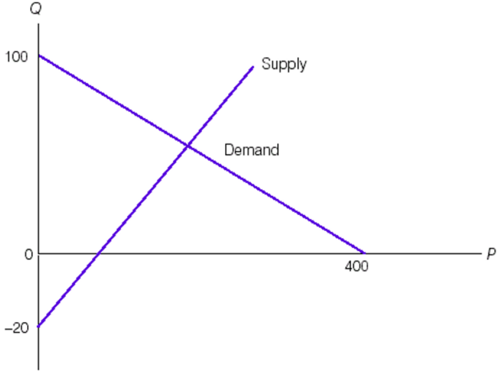

Figure 4.13

Example: we want the x and y which makes both equations true

$$\begin{cases} 2x - y = -1 \\ 14x + 3y = 43 \end{cases}$$

Multiplying equation 1 by 3 gives

$$\begin{cases} 6x - 3y = -3 \\ 14x + 3y = 43 \end{cases}$$

If we put x =2 into equation 1 we can solve for y

$$2x - y = -1 \Rightarrow 4 - y = -1, \Rightarrow y = 5$$

Therefore: $x = 2; y = 5$

- Multiply both sides of one equation by a number that makes the same term (or its negative) appear in both equations
- Add or subtract the two equations so that these two identical terms become 0.
- This new equation only contains one variable so solve for this
- Substitute your solution for that variable into one of the original equations to solve.

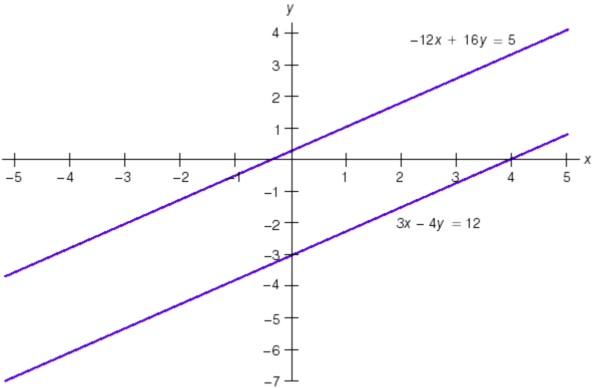

Figure 4.14

$$\begin{cases} Q = -\frac{1}{2}P + 100 \\ Q = 2P - 20 \end{cases}$$

Subtracting equation 1 from equation 2:

$$0 = \frac{5P}{2} - 120 \Rightarrow \frac{5P}{2} = 120 \Rightarrow P = 48$$

Substituting in equation 2:

$$Q = 96 - 20 \Rightarrow Q = 76$$

The solution is $P = 48$ and $Q = 76$