September 28, 2022

Herleitung: Analythische Lösung für konvektiven Wärmetransport bei stationären Zuständen

$$
\begin{aligned}
Q_{cd} &= c_{pa}\dot{M}_a \cdot T(x) \\
Q_{cd} &= c_{pa}\dot{M}_a \frac{Te^{L/\ell} - T_2}{e^{L/\ell} - 1} \\
\frac{Q_{cd}}{c_{pa}\dot{M}_a} &= T(x) - \ell \cdot \frac{dT}{dx} \\
-\ell e^{x/\ell} \frac{d}{dx}\left(e^{-x/\ell} T(x)\right) &= T(x) - \ell \cdot \frac{dT}{dx} \\
\frac{d}{dx}(e^{-x/\ell} T(x)) = -\frac{e^{-x/\ell}}{\ell} \cdot \left(T(x) - \ell \cdot \frac{dT}{dx}\right) &= -\frac{e^{-x/\ell}}{\ell} \cdot \frac{Q_{cd}}{c_{pa}\dot{M}_a} \\
\int_0^L \frac{d}{dx}(e^{-x/\ell} T(x))\, dx \ &= -\int_0^L \frac{e^{-x/\ell}}{\ell} \cdot \frac{Q_{cd}}{c_{pa}\dot{M}_a}\, dx \\
\left[e^{-x/\ell} T(x)\right]_0^L &= -\frac{1}{\ell} \cdot \frac{Q_{cd}}{c_{pa}\dot{M}_a} \int_0^L e^{-x/\ell}\, dx \\
\left[e^{-x/\ell} T(x)\right]_0^L &= -\frac{1}{\ell} \cdot \frac{Q_{cd}}{c_{pa}\dot{M}_a} \left[-\ell e^{-x/\ell}\right]_0^L \\
e^{-L/\ell} T(L) - 1 \cdot T(0) &= -\frac{1}{\ell} \cdot \frac{Q_{cd}}{c_{pa}\dot{M}_a} (-\ell) \left(e^{-L/\ell} - 1\right) \quad | \cdot e^{L/\ell} \\
T(L) - T(0) \cdot e^{L/\ell} &= -\frac{1}{\ell} \cdot \frac{Q_{cd}}{c_{pa}\dot{M}_a} (-\ell) \left(1 - e^{L/\ell}\right) \quad | \cdot (-1) \\
T(0) \cdot e^{L/\ell} - T(L) &= \frac{Q_{cd}}{c_{pa}\dot{M}_a} \left(e^{L/\ell} - 1\right) \\
Q_{cd} &= c_{pa}\dot{M}_a \cdot \frac{T(0) \cdot e^{L/\ell} - T(L)}{e^{L/\ell} - 1}
\end{aligned} \tag{1}
$$