ATS 421/521

Climate Modeling

Spring 2013

Lecture 6

- Meridional Energy Transport
- 1D EBM

April 17

No Lecture on Monday.

How about watching a movie instead?

http://thiniceclimate.org/

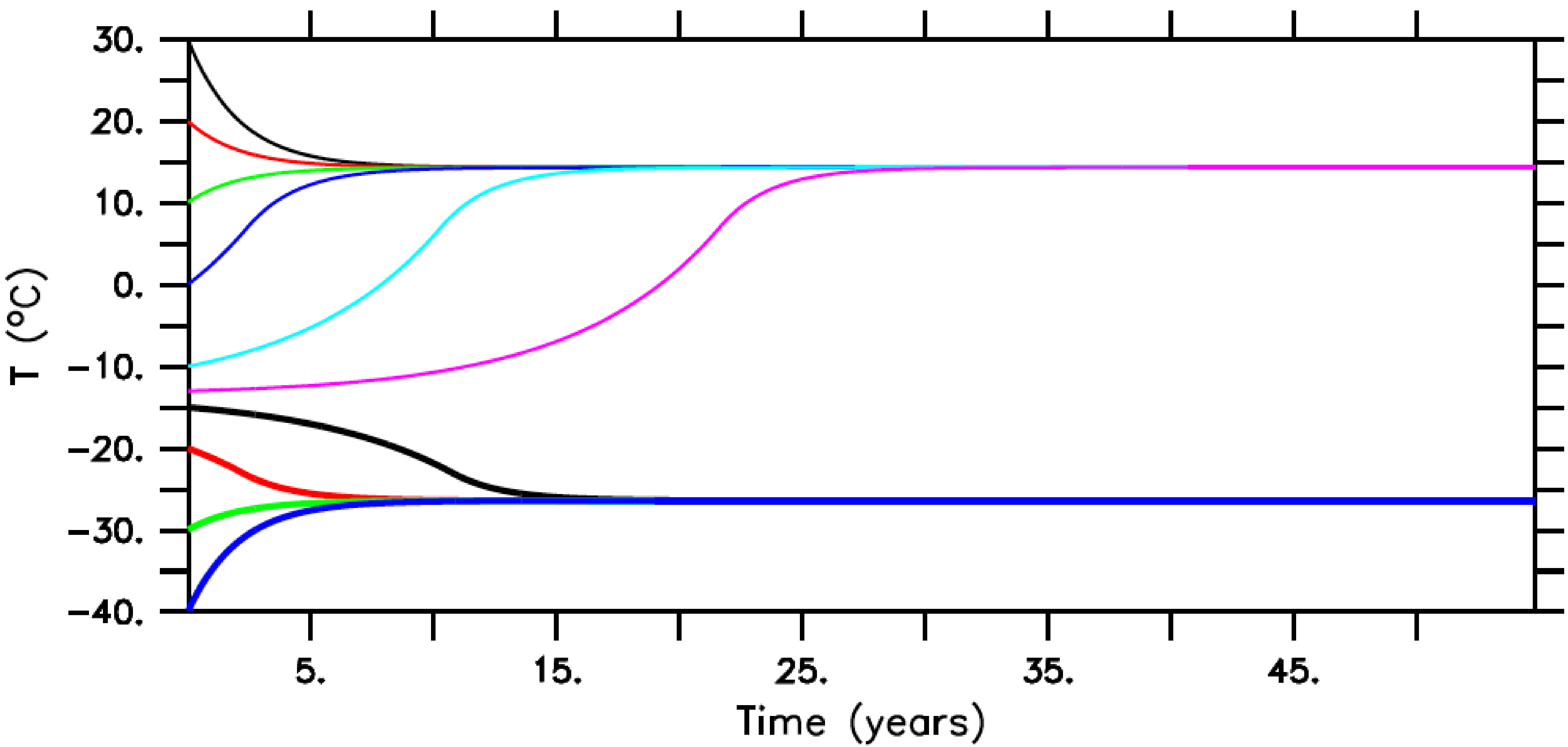
Runs for free Monday and Tuesday.

Reading

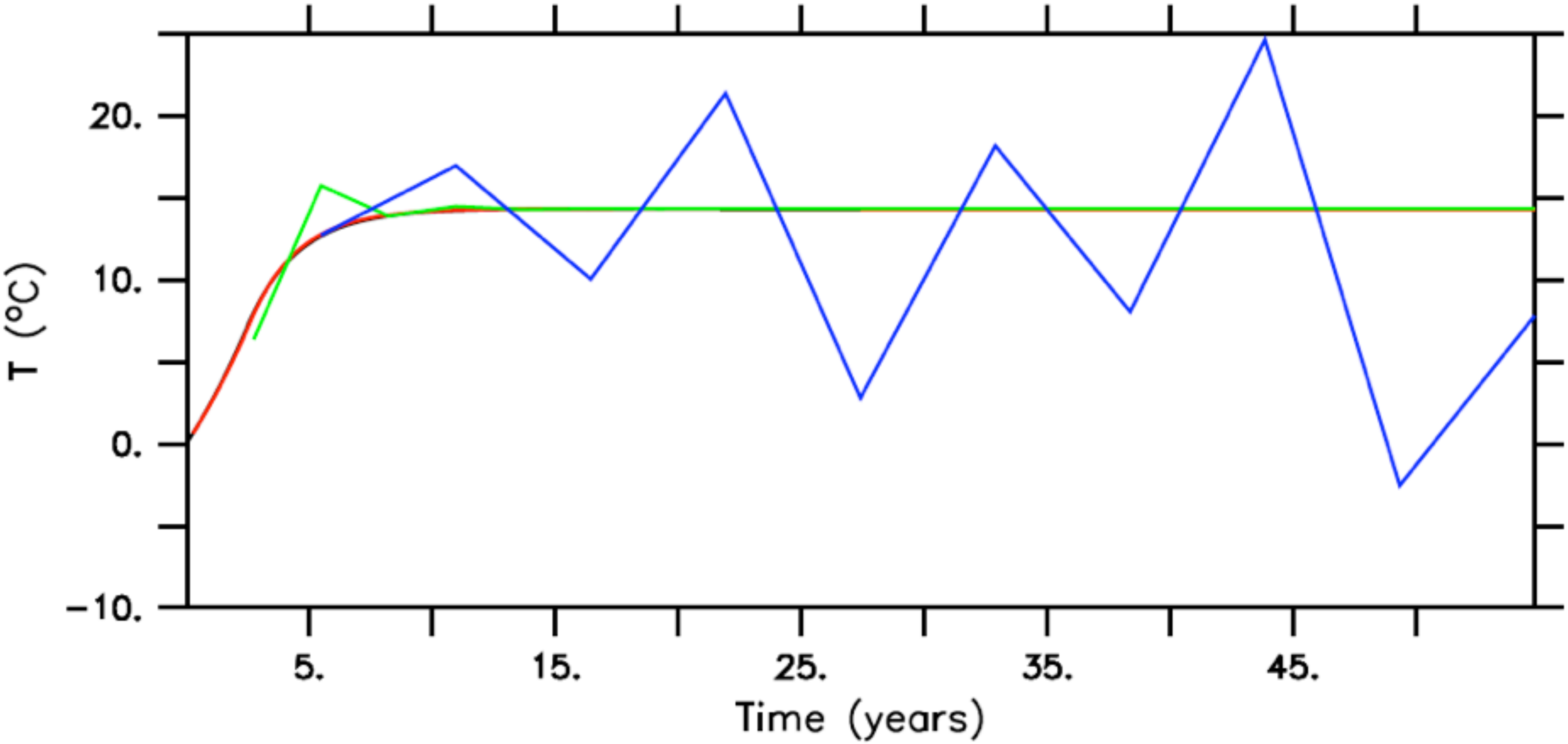
- For Friday: Huybers & Curry (2006)
- For Tuesday:
  - Script chapter 2.6
  - Textbook chapter 3.4

Previous Lecture

HW1



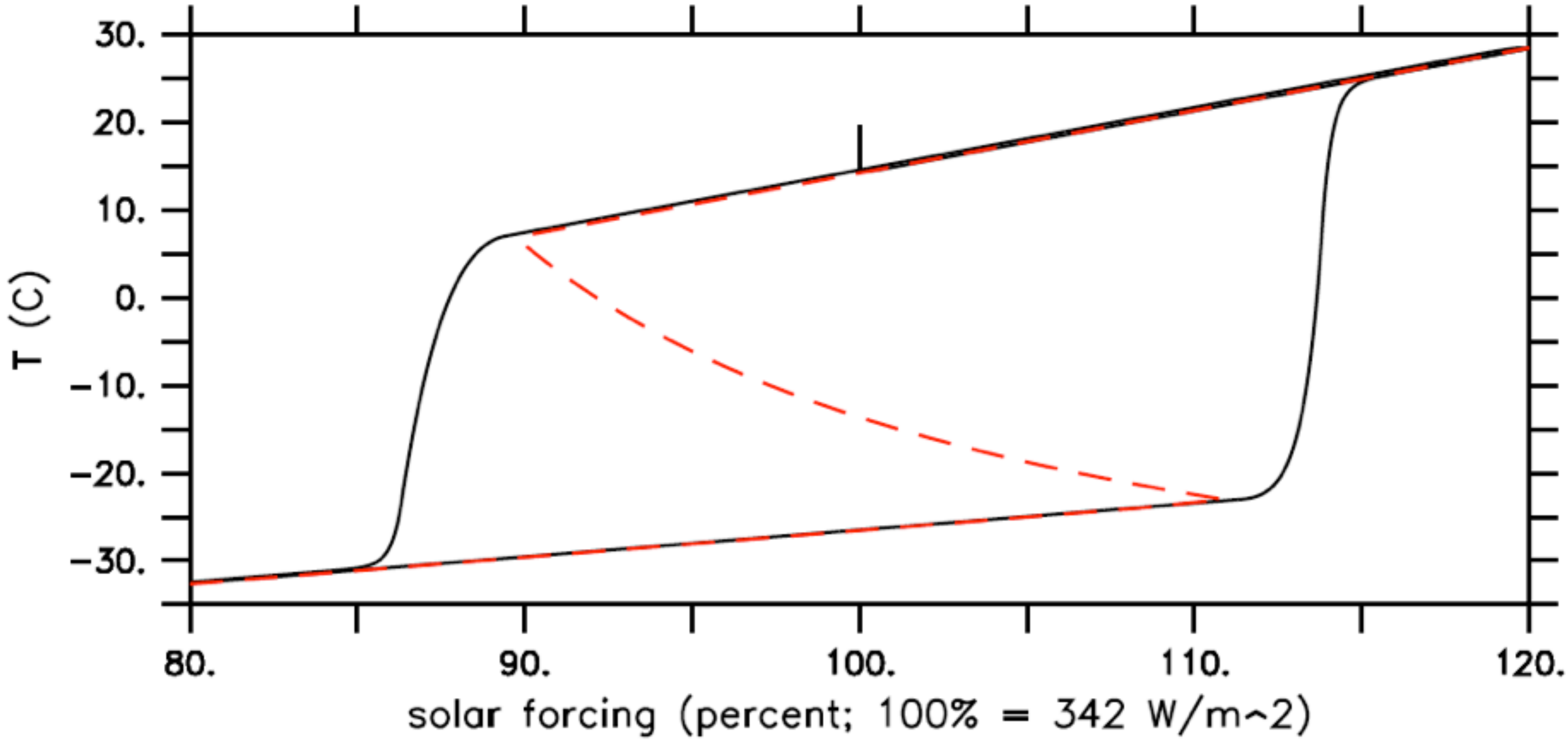
HW1



***3.** Model spin-up from T_0=0°C for different time steps . Red: Δt=100 days, green: Δt=1000 days, blue: Δt=2000 days. Oscillations occur (Δt=2000 days) and for Δt >2000 the model crashes.*

HW1

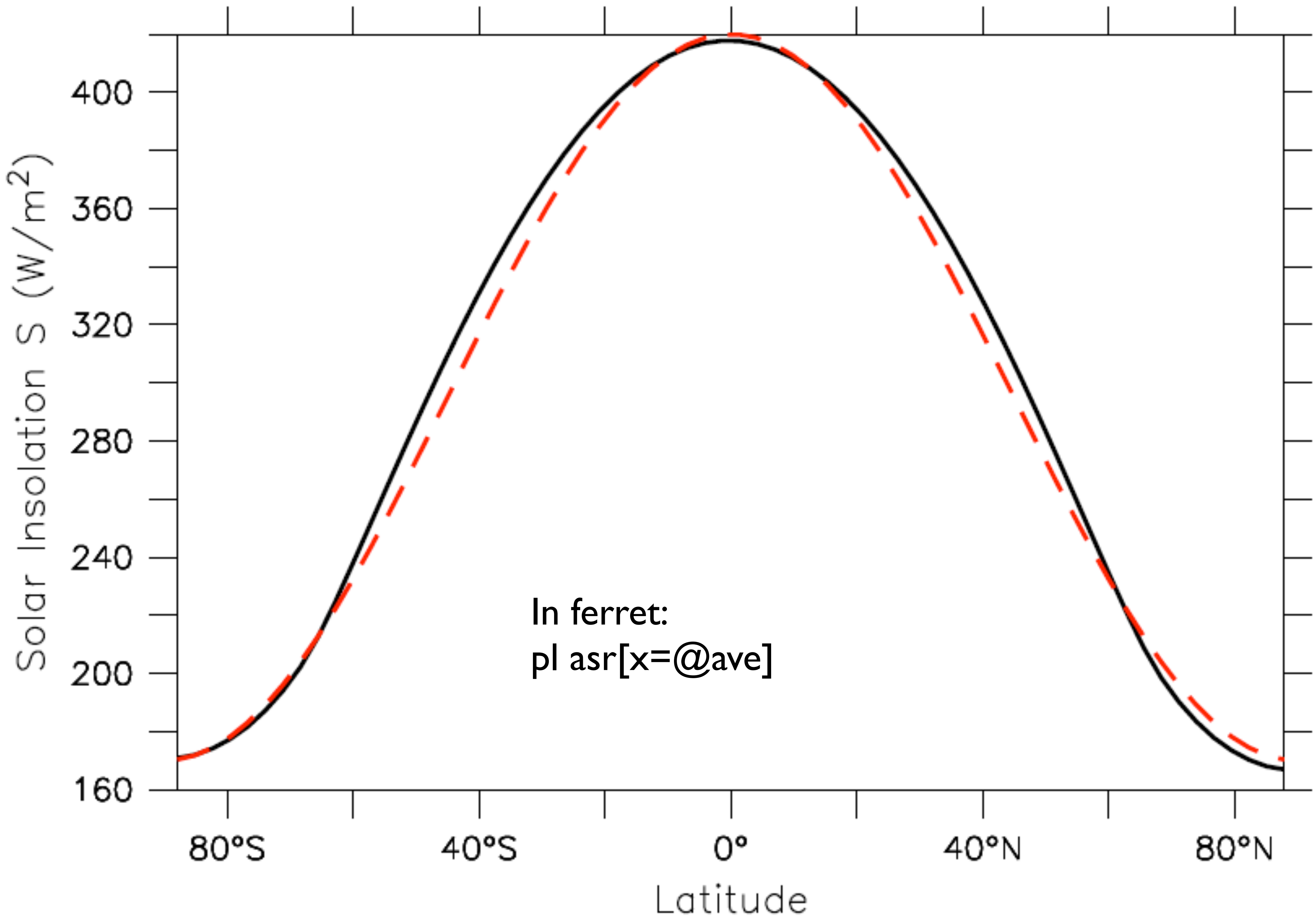
$$S = \frac{A + BT}{1 - a(T)}$$



Summary 0D EBM

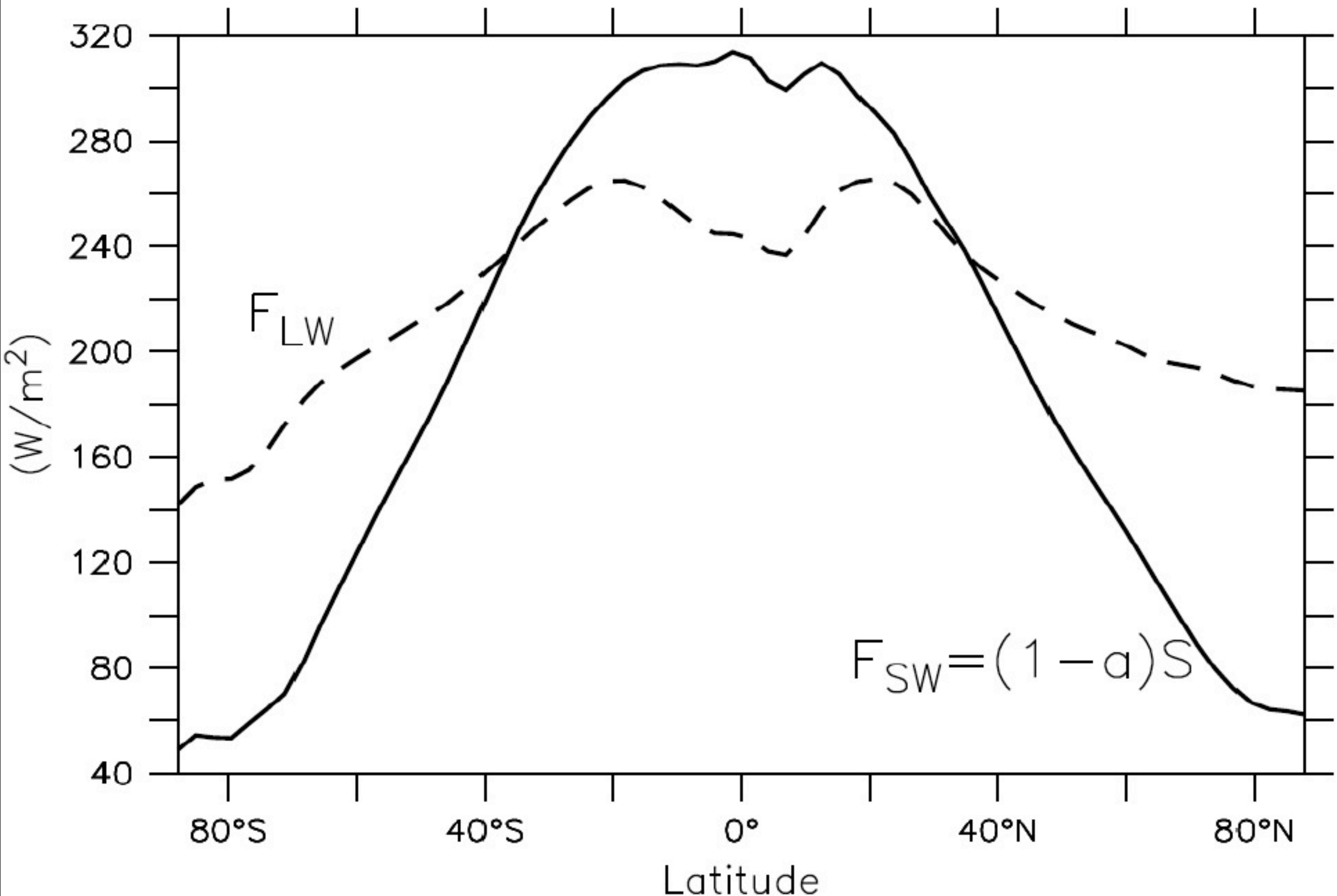
- Quantification of greenhouse effect
- Ice-Albedo Feedback gives interesting properties of hysteresis and multiple steady states
- We'll continue to explore effects of stochastic forcing (HW2)
- What are the limitations?

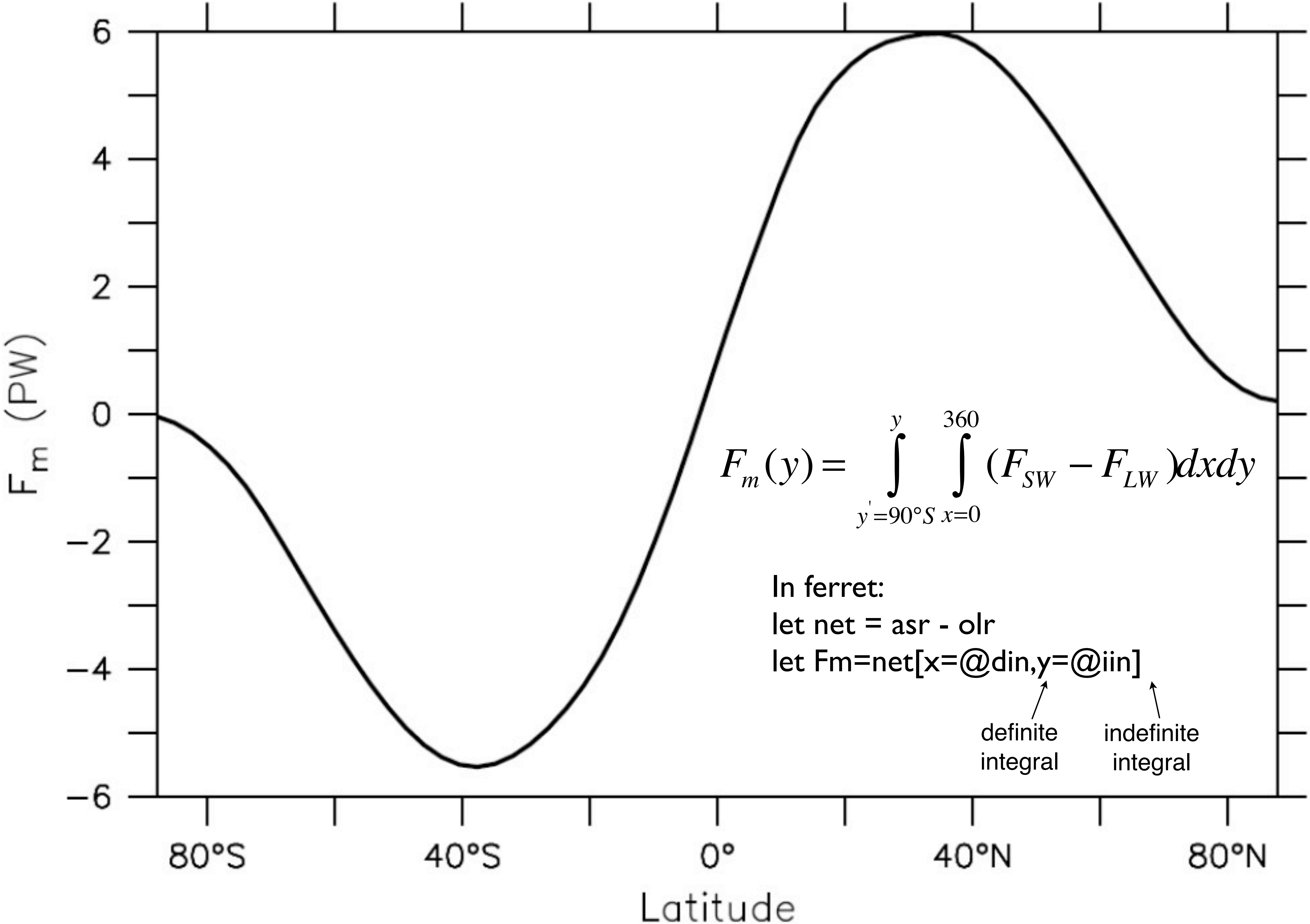
Zonally Averaged Incident Solar Radiation



red: $S(\phi) = 195 + 125\cos(2\phi)$
use in 1D EBM

Zonally averaged absorbed solar and outgoing longwave radiation





$$F_m(y) = \int_{y'=90°S}^{y} \int_{x=0}^{360} (F_{SW} - F_{LW}) dx dy$$

In ferret:

```
let net = asr - olr
let Fm=net[x=@din,y=@iin]
```

definite integral

indefinite integral

Diffusive parameterization of meridional heat transport:

Lorenz, E. N. (1979) Forced and free variations of weather and climate, J. Atmos. Sci. 36, 1367-1376.

$$\vec{F}_m = -CK\vec{\nabla}T = -CK\frac{\partial T}{\partial y} \qquad (2.18)$$

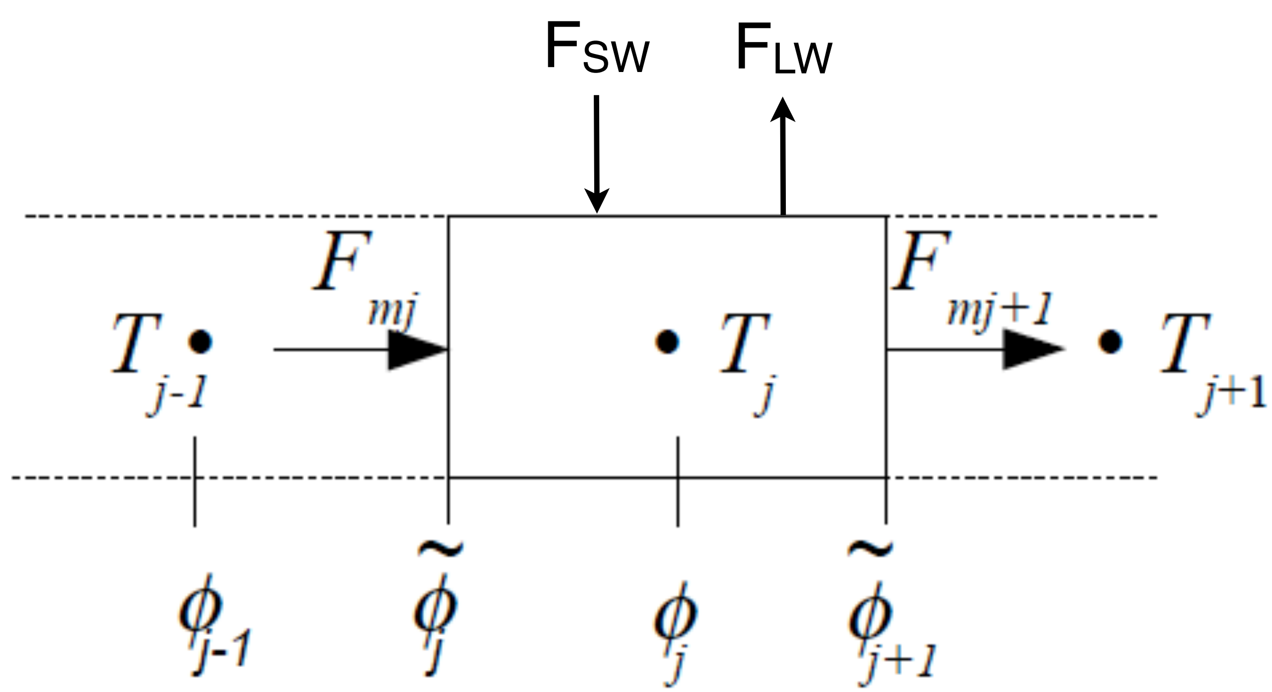
Heat Capacity

Diffusivity

Temperature Gradient

$$C\frac{\partial T}{\partial t} = -\vec{\nabla}\vec{F}_m + F_{SW} - F_{LW}$$

Meridional Heat Flux Convergence



in spherical coordinates

Meridional Heat Flux Divergence:

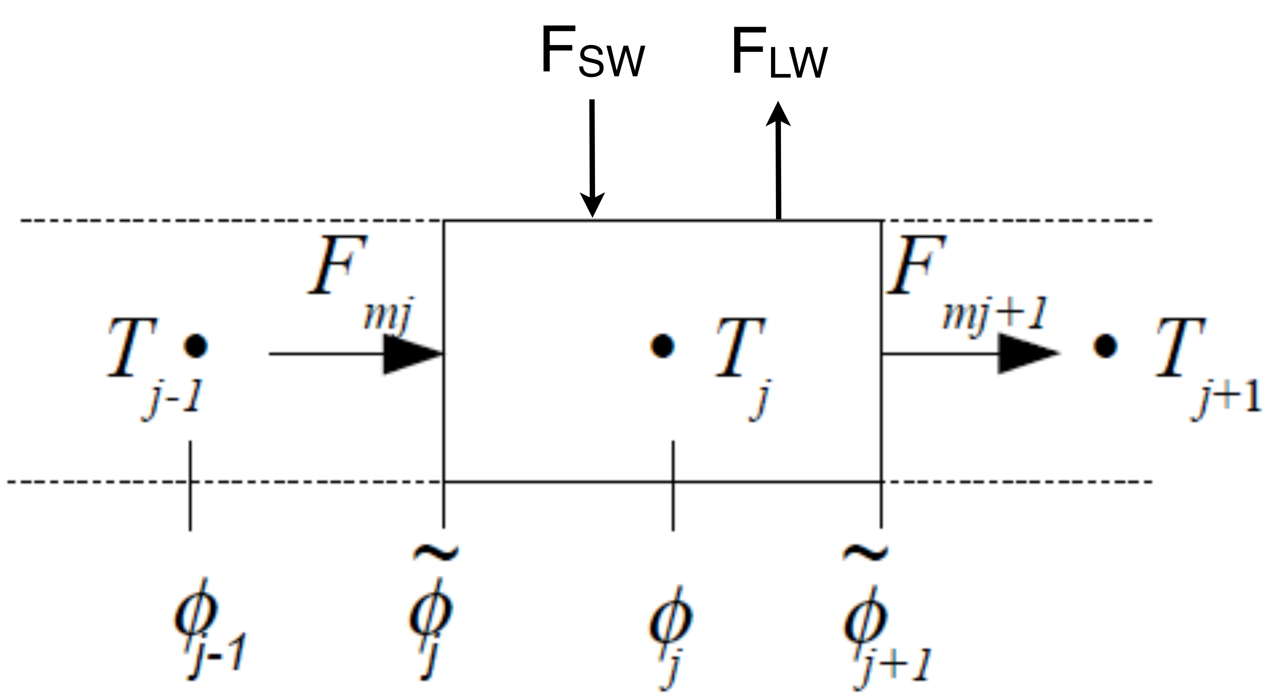
$$\vec{\nabla}\vec{F}_m = -\vec{\nabla}\left(CK\,\vec{\nabla}T\right) = \frac{-1}{R^2\cos\phi}\frac{\partial}{\partial\phi}\left(CK\cos\phi\frac{\partial T}{\partial\phi}\right) \quad (2.20)$$

latitude

Discretized:

$$-\vec{\nabla}\vec{F}_m = \frac{-1}{R\cos\phi}\frac{\Delta F_m}{\Delta\phi} = \frac{-1}{R\cos\phi}\frac{F_{mj+1}-F_{mj}}{\tilde{\phi}_{j+1}-\tilde{\phi}_j}$$

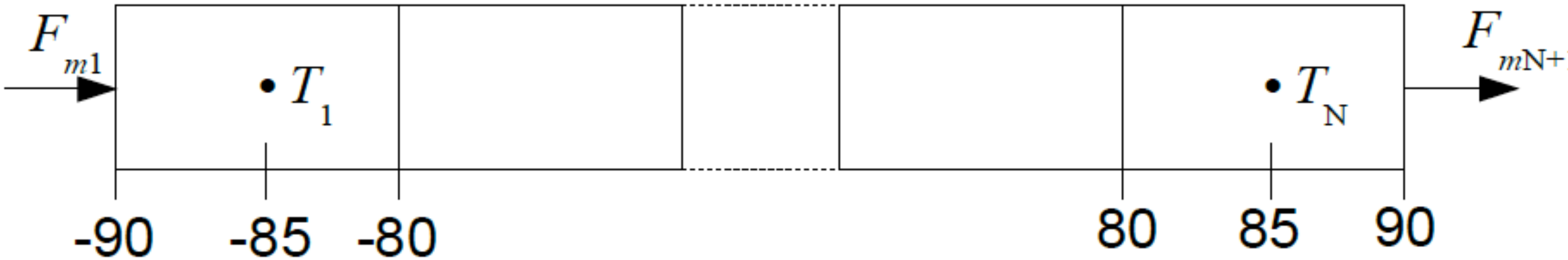
$$F_{mj} = -CK_j\frac{\cos\tilde{\phi}_j}{R}\frac{T_j - T_{j-1}}{\phi_j - \phi_{j-1}}$$



Set up 10º grid from pole to pole.

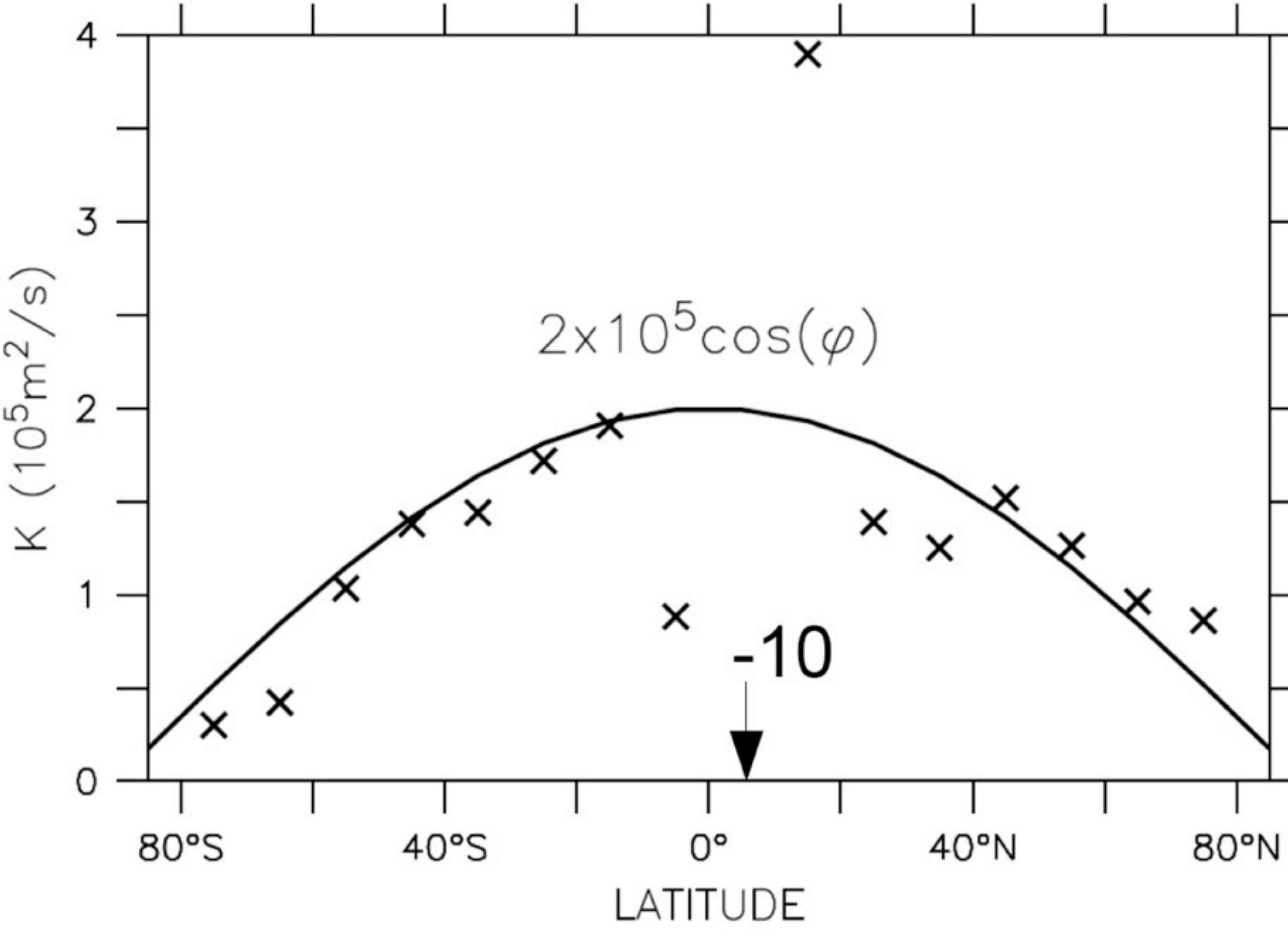
Boundary Conditions:

$$F_{m1} = F_{mN+1} = 0$$

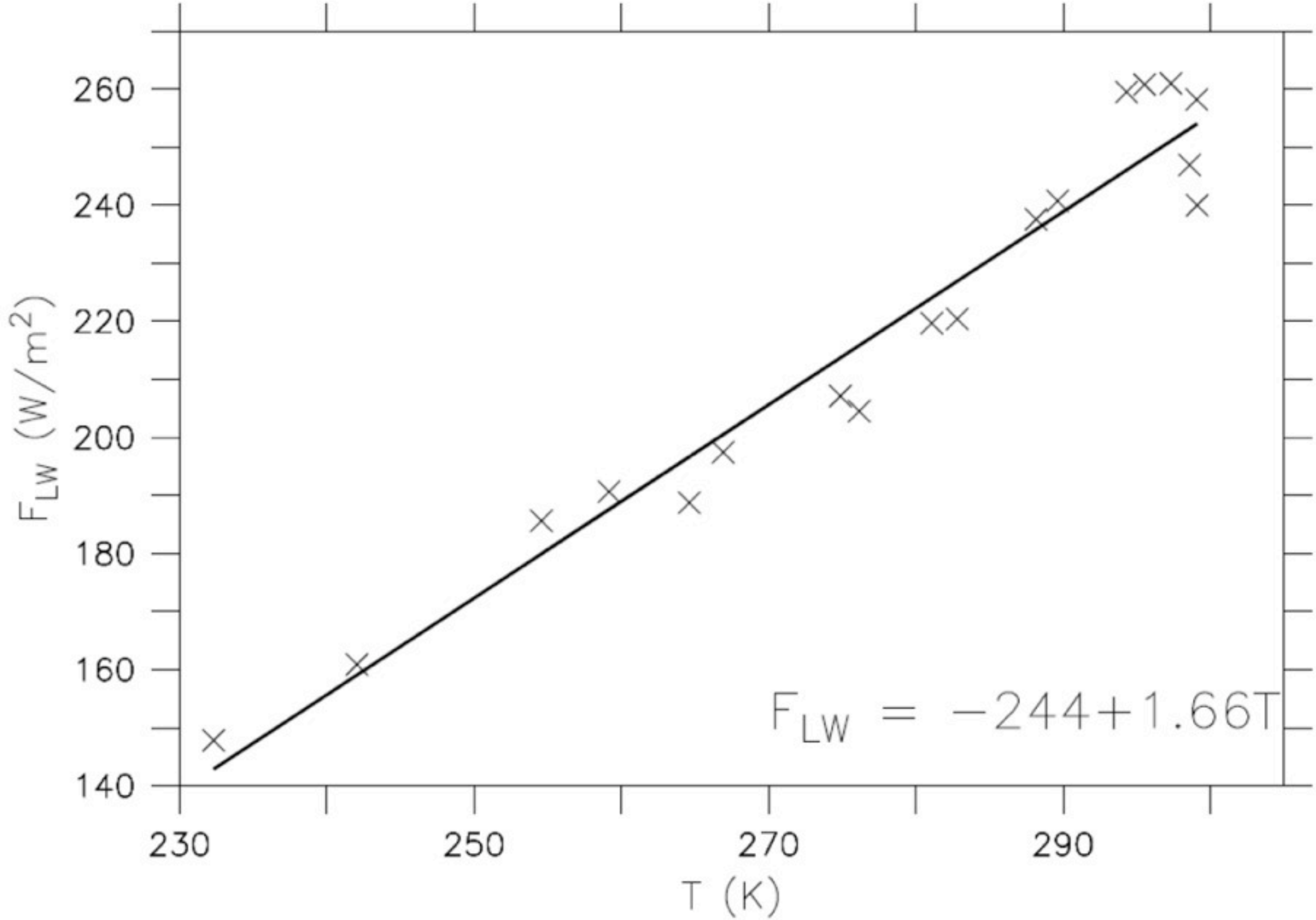


In FORTRAN use vectors:

```
parameter (jmax = 18) ! number of grid boxes
real temp(1:jmax), fm(1:jmax+1), phi(1:jmax), phim(1:jmax+1)
...
do i=1:imax ! time loop
   do j=1:jmax ! loop over latitudes
       ...
      temp(j) = temp(j) - divFm + FSW(j) - FLW(j)
```



4
3
2
1
0
K (10⁵m²/s)
2x10⁵cos(φ)
-10
80°S
40°S
0°
40°N
80°N
LATITUDE


260
240
220
200
180
160
140
230
250
270
290
F_{LW} (W/m^2)
T (K)
$F_{LW} = -244+1.66T$

Update Albedo Parameters:

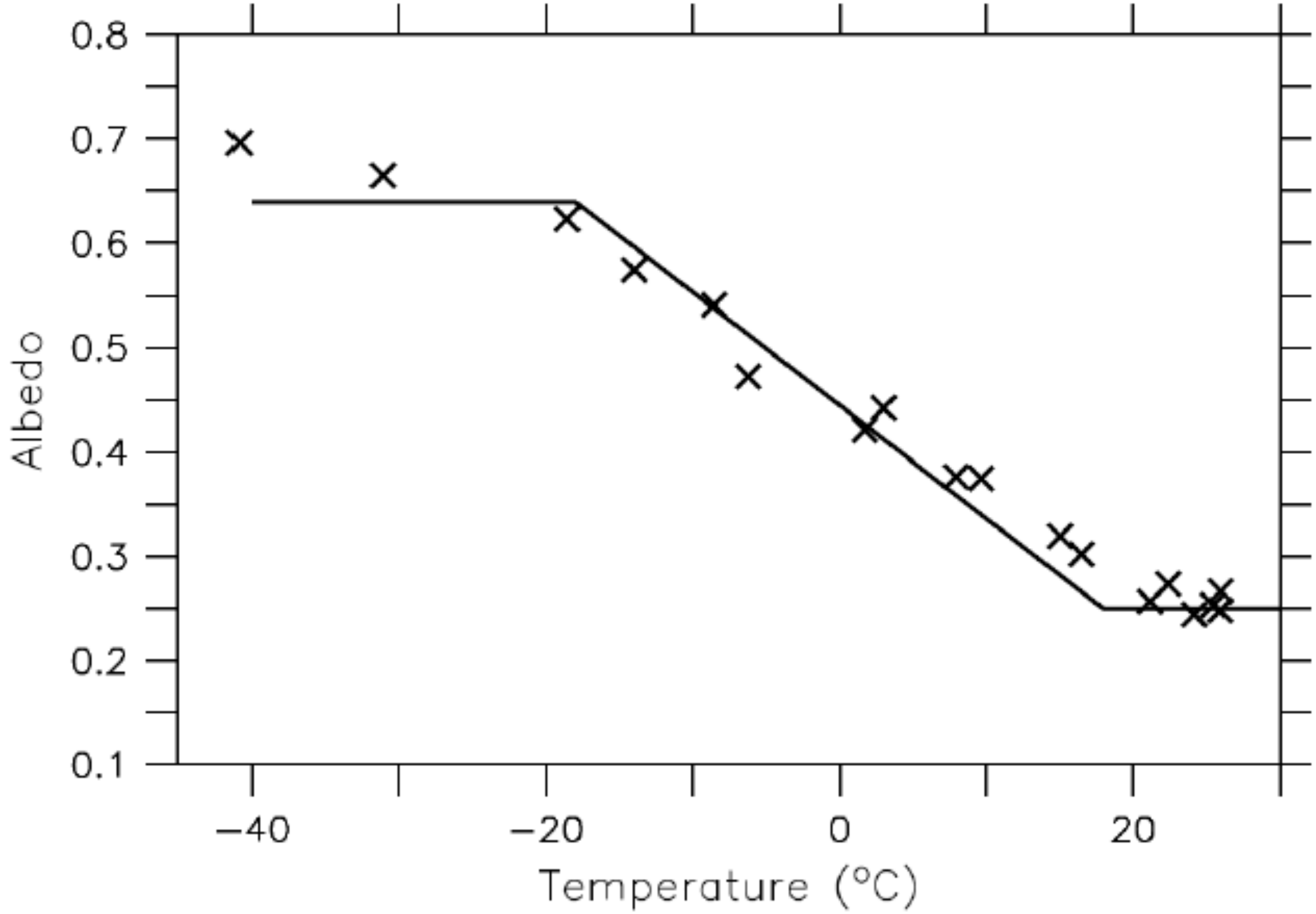


Figure 2.16: Albedo (from ERBE) as a function of surface air temperature (from NCEP) calculated from zonally averaged (on a 10° grid) data. The solid line shows a simple ramp function approximation (eq. 2.5) with T_L=-18°C, T_U =18°C, a_1=0.64 and a_2=0.25.

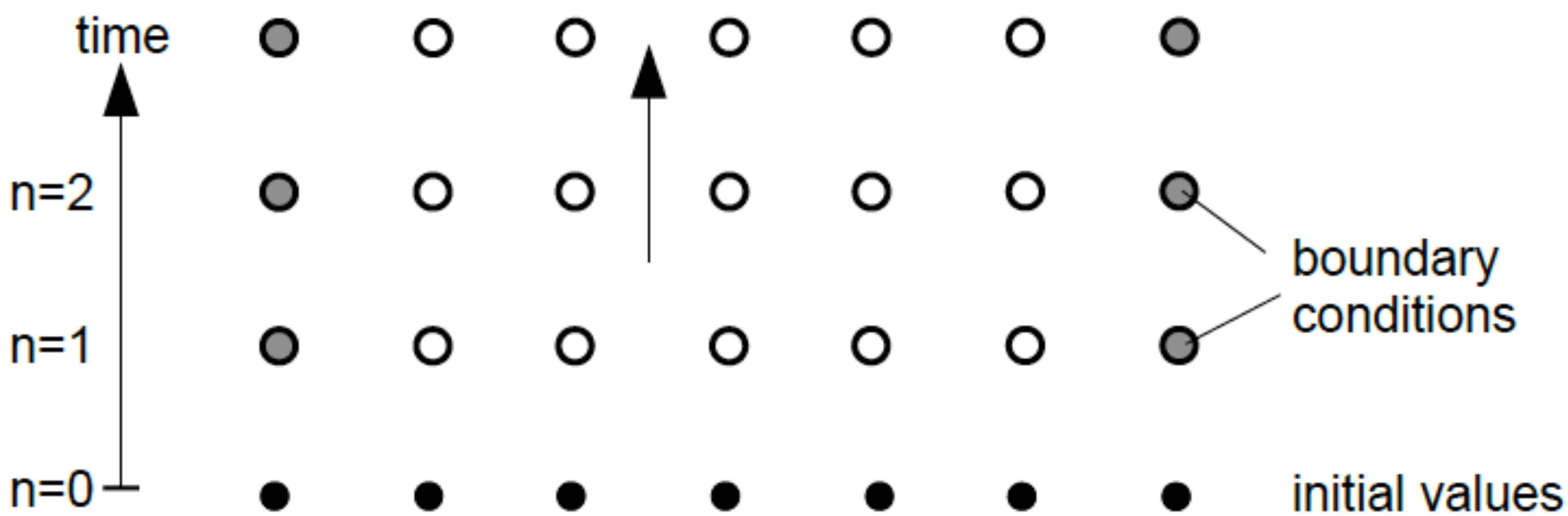
Numerics

Script chapter 2.6

Important criteria for numerical schemes:

1) Convergence for $\Delta x, \Delta t \to 0$
2) Stability
3) Accuracy
4) Conservation
5) Behavior of Amplitudes and Phases
6) Positive definite
7) No (or Small) Numerical Artifacts

Boundary Conditions



Two types of boundary conditions:

- Dirichlet: specify values
- Neuman: specify normal gradients

Of which type are our 1D EBM boundary conditions?

Develop *T* in Taylor series around *t*:

$$T(t+\Delta t) = T(t) + \frac{dT}{dt}\Big|_t \Delta t + \frac{1}{2!}\frac{d^2 T}{dt^2}\Big|_t (\Delta t)^2 + \ldots \qquad (2.23)$$

neglecting these terms gives the "Centered Differences" scheme

more accurate than Euler Forward since errors scale with

$$(\Delta t)^2$$

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$$\longrightarrow \quad \frac{dT}{dt}\Big|_t = \frac{T(t+\Delta t) - T(t)}{\Delta t} \underbrace{- \frac{1}{2!}\frac{d^2T}{dt^2}\Big|_t \Delta t - \frac{1}{3!}\frac{d^3T}{dt^3}\Big|_t (\Delta t)^2 - \ldots}_{\text{correction of order } \Delta t} \qquad (2.24)$$

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as $\Delta t \rightarrow 0$ the Euler scheme converges to the true solution

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Now replace Δt with $-\Delta t$ in eq. (2.24) and add this new equation to (2.24)

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$$\longrightarrow \quad \boxed{\frac{dT}{dt}\Big|_t=\frac{T(t+\Delta t)-T(t-\Delta t)}{2\cdot\Delta t}}\underbrace{-\frac{1}{3!}\frac{d^3T}{dt^3}\Big|_t(\Delta t)^2-\ldots}_{\text{correction of order }(\Delta t)^2}$$

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