ATS 421/521

Climate Modeling

Spring 2015

Lecture 14

- Ice Sheets

Reading

- Today:
- Textbook chapter 3.3.6 (Ice Sheets)
- Course Notes chapter 7.2 (Ice Sheets)
- Pollard and DeConto (2009) let by Felicio

Homeworks vs Project

- Instead of two remaining HW you can do a project of your own choice
- E.g. compare different CMIP5 models with each other and with observations; write short (5 pages) assessment report
- Think about it. If you have a project in mind, let me know. We'll to decide by Wednesday.

Ice Sheets

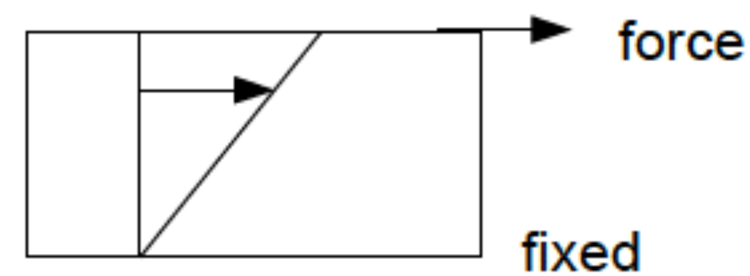
Oerlemans (1981)

J. Oerlemans (1981) developed the following simple vertically-integrated model of a continental ice sheet.

7.2.1 Perfectly plastic solution for an ice sheet on a flat base

Assumptions:

- flow is quasi-two dimensional
- normal stress deviations are small
- the surface slope (s<0.1) is small



Balance of forces: $$\frac{\partial \tau_{xz}}{\partial z} = \rho g s \quad \Rightarrow \tau_{xz} = \rho g (H - z) s$$

vertical gradient of the shear stress — pressure gradient

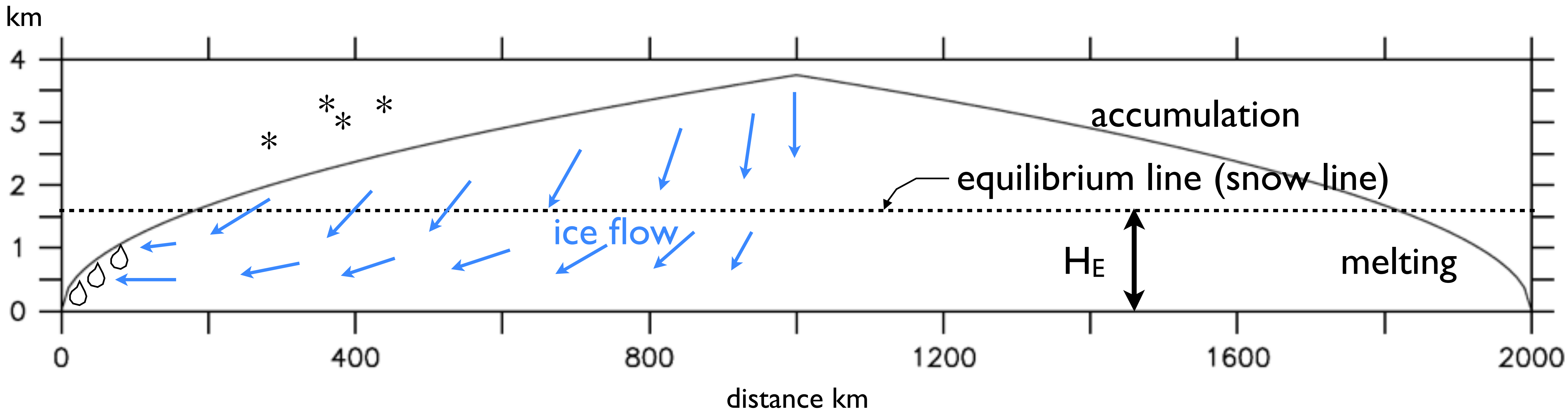
Stress at base: $$\Rightarrow \tau_b = \rho g H s = \rho g H \frac{\partial H}{\partial x} = const. = \tau_0$$

=> $$\frac{1}{2} \frac{\partial H^2}{\partial x} = \frac{\tau_0}{\rho g} \quad \Rightarrow H = \sqrt{\frac{2\tau_0}{\rho g} x} = \Lambda \sqrt{x} \qquad (7.3)$$

$$[3.5\,\mathrm{m}^{1/2} < \Lambda < 4\,\mathrm{m}^{1/2}]$$

$$\frac{1}{2}\frac{\partial H^2}{\partial x}=\frac{\tau_0}{\rho g} \quad \Rightarrow H=\sqrt{\frac{2\tau_0}{\rho g}x}=\Lambda\sqrt{x}$$

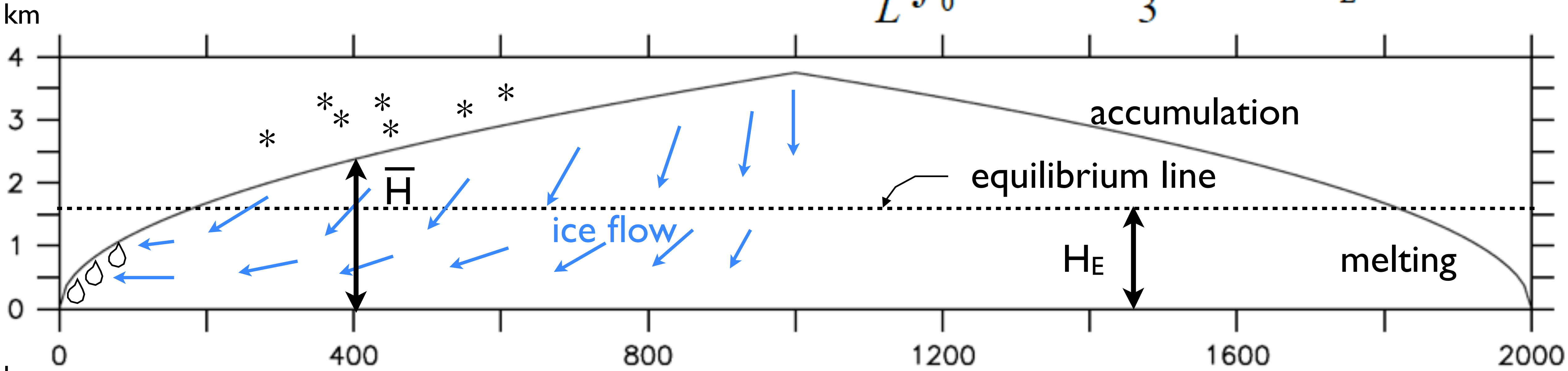
parabolic ice sheet profile

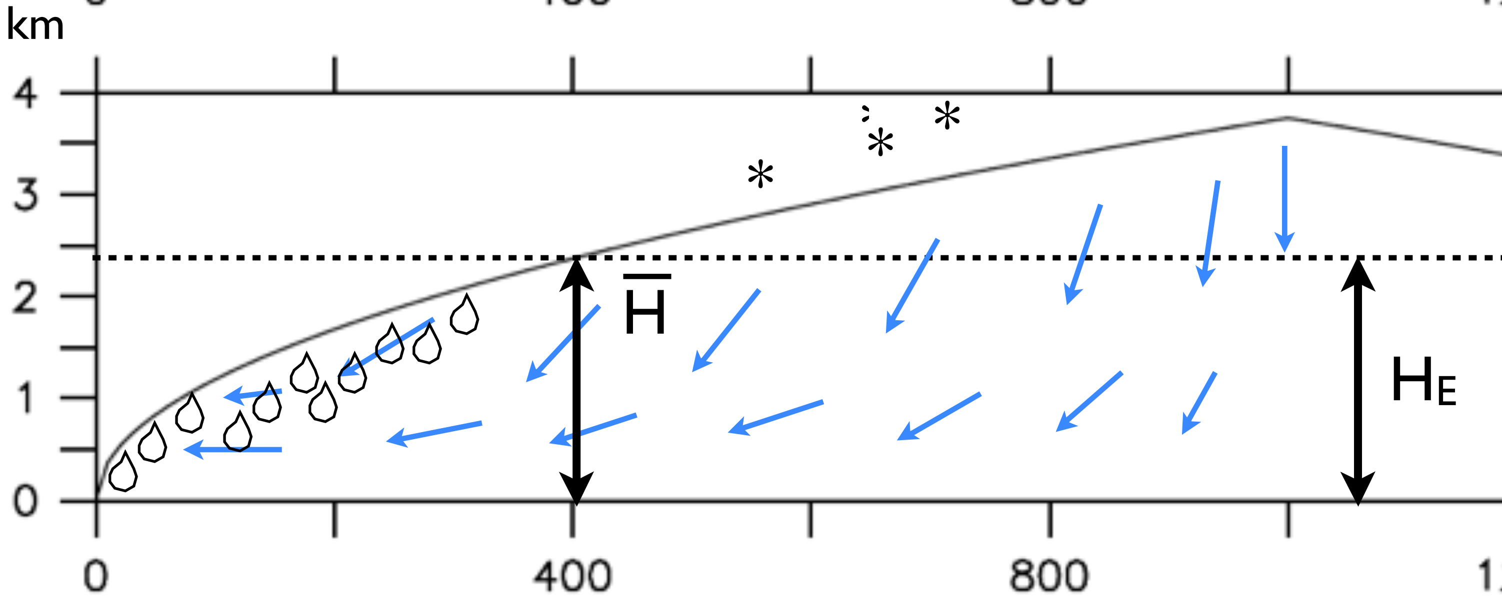


Ice sheet is in steady state if total accumulation (snow fall) equals total melting.
Area above equilibrium line experiences accumulation. (cold because at high elevation)
Area below equilibrium line experiences melting. (warm because at low elevation)

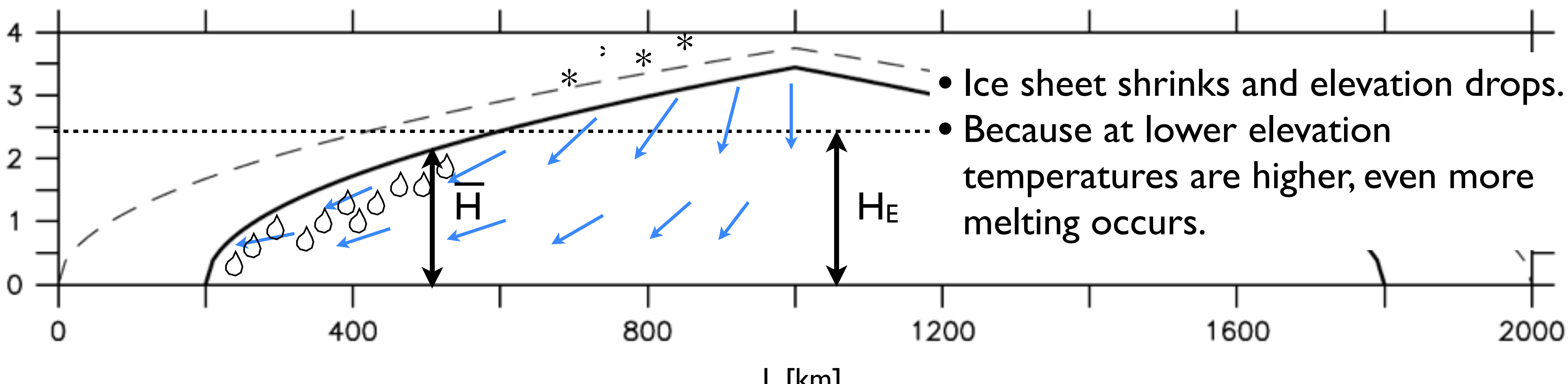
Under the overlaying weight ice flows slowly from accumulation region to melt region.
What happens to the equilibrium line if climate warms?

$\overline{H}$ = average ice sheet height $$\overline{H} = \frac{1}{L}\int_0^L H\,dx = \frac{\sqrt{2\Lambda}}{3}\sqrt{L} > H_E \quad (7.5)$$



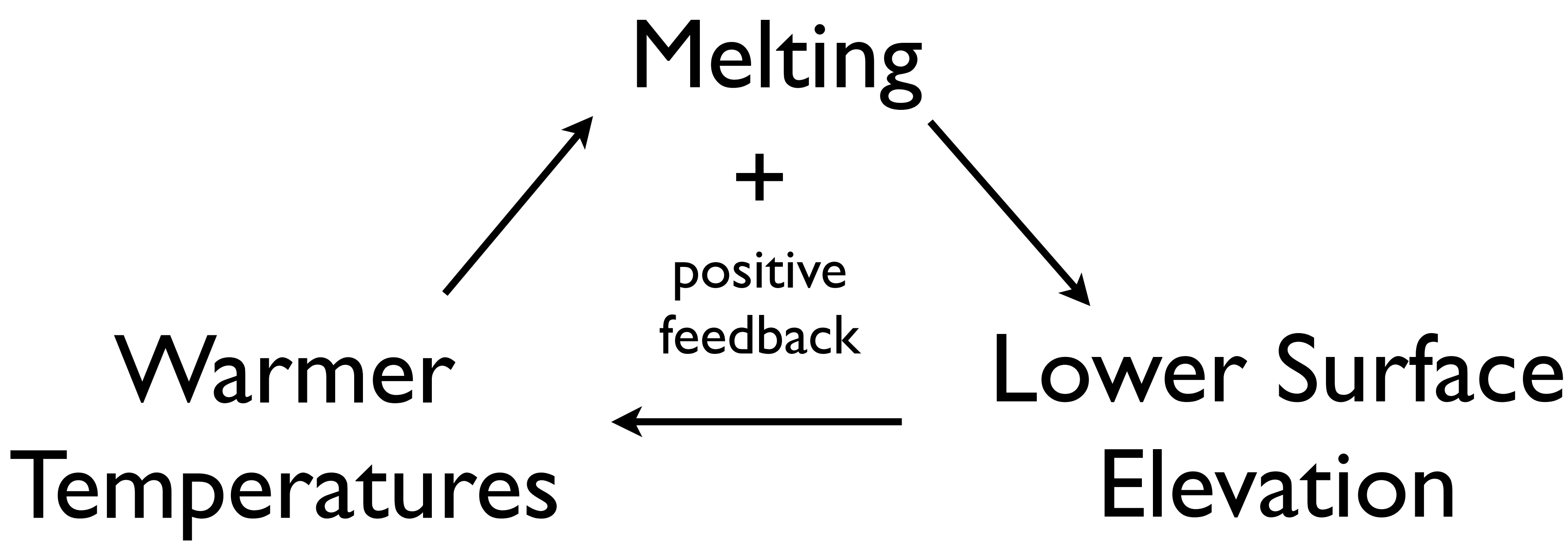


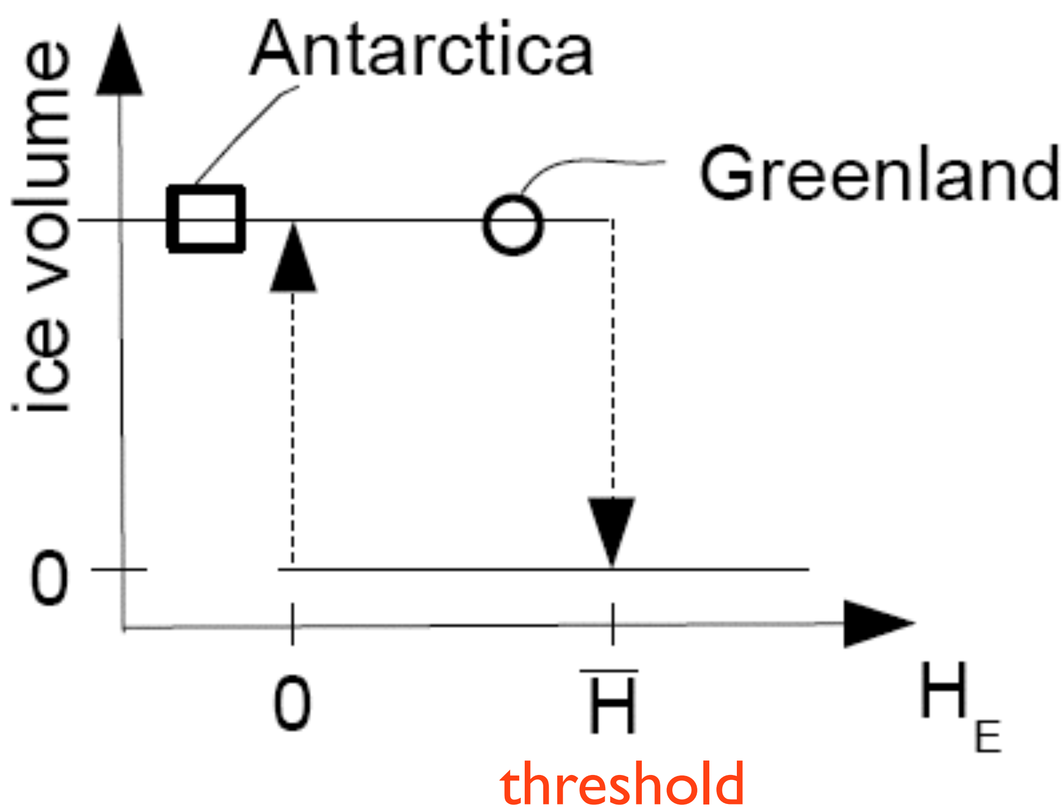
- Equilibrium line rises.
- More surface are is below equilibrium line in melt zone.
- Increased melting.



- Ice sheet shrinks and elevation drops.
- Because at lower elevation temperatures are higher, even more melting occurs.

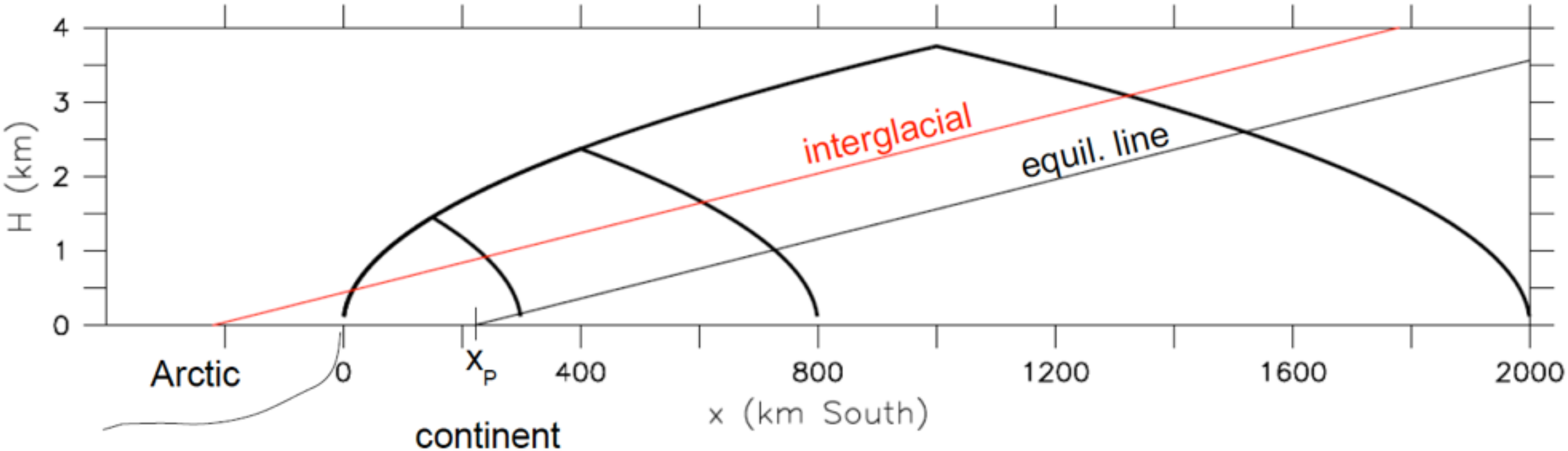
Ice Sheet Elevation-Mass Balance Feedback





- Positive feedback leads to a threshold for the equilibrium line.
- If higher than average ice sheet height, the ice sheet will disappear.
- It will only grow back if equilibrium line is decreased below zero (ground), that is below its current elevation. Irreversibility.
- **Greenland** is close to threshold. Vulnerable to warming. 2-3°C global mean warming will lead to its irreversible demise with 7 m sea level rise.
- **East Antarctic** ice sheet is not close to the threshold. May even grow due to increased snowfall in warmer climate.

Northern Hemisphere Ice Sheets



Mass Balance $$B = \alpha(x - x_P) + \beta H \qquad \alpha < 0 \qquad (7.6)$$

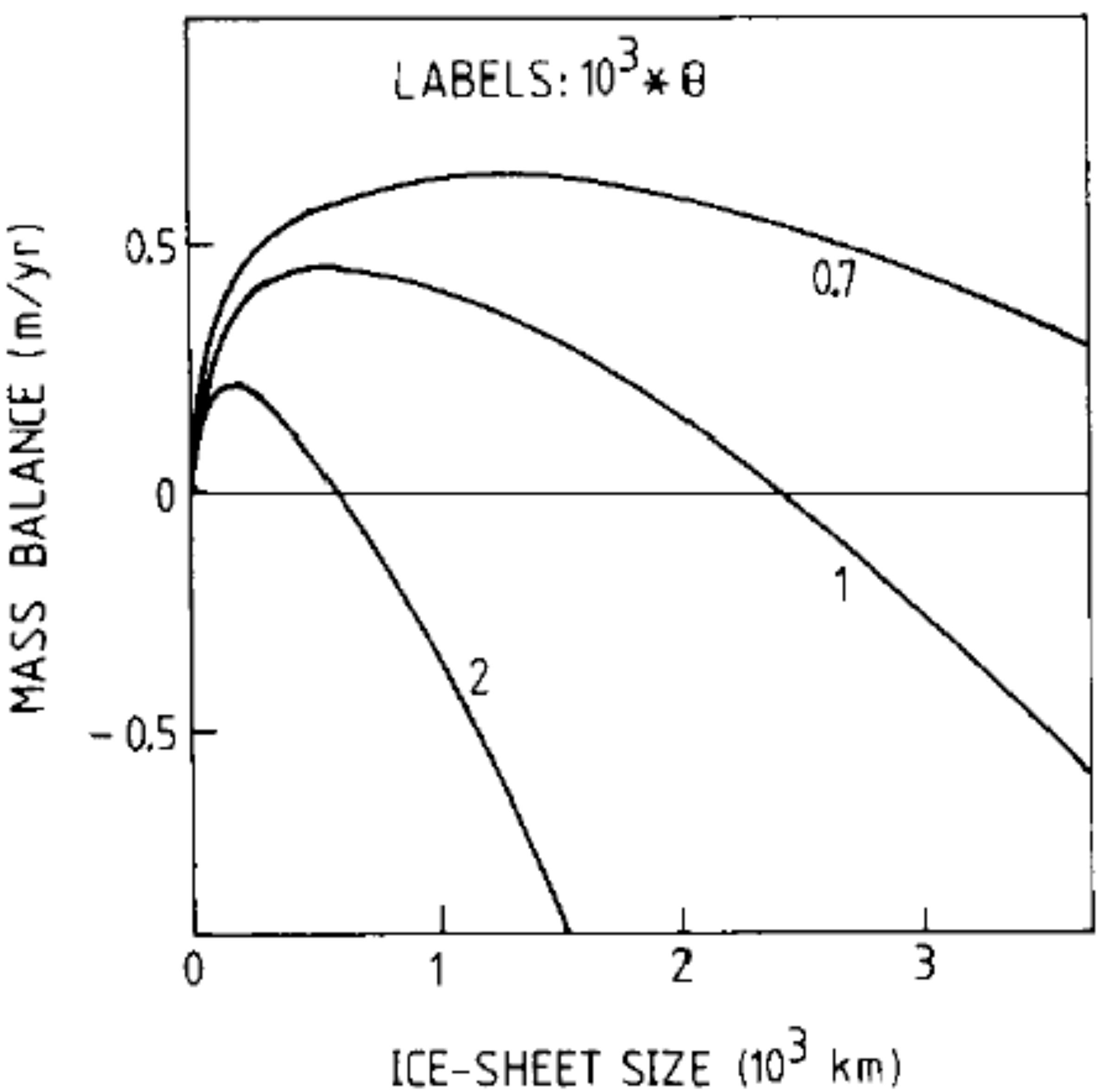
Northern half will loose mass by calving if ice bergs into Arctic.
Equilibrium: mass balance integrated over southern half = zero.

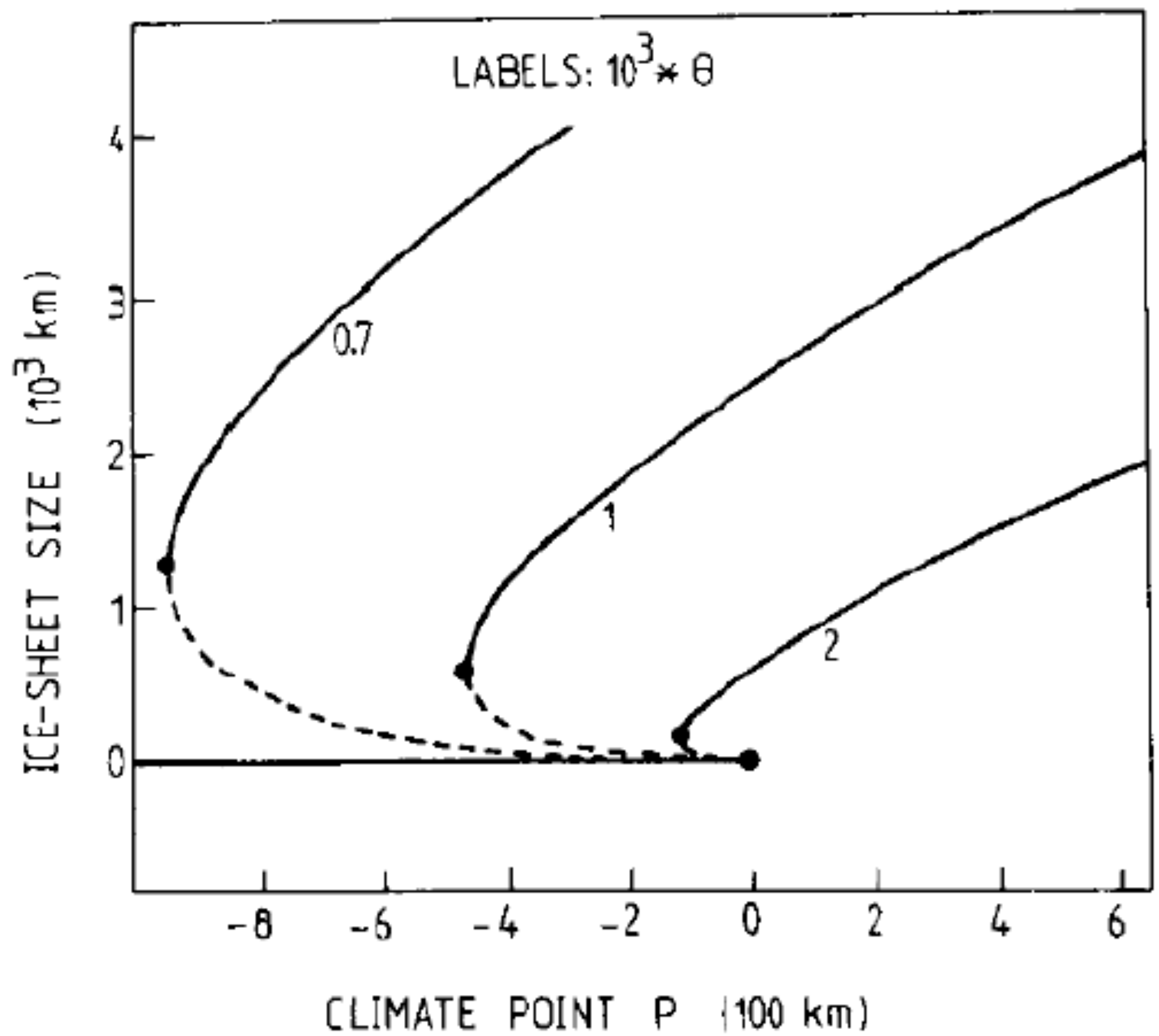
Equi. line slope: $\Theta = -\alpha / \beta$

$$\bar{B}(L) = \frac{2}{L} \int_{L/2}^{L} B dx = B_1 + B_2 L^{1/2} + B_3 L = 0$$

$$B_1 = -\alpha x_P \quad , \quad B_2 = \sqrt{2} \beta \Lambda / 3 \quad , \text{ and } \quad B_3 = 3 \alpha / 4$$

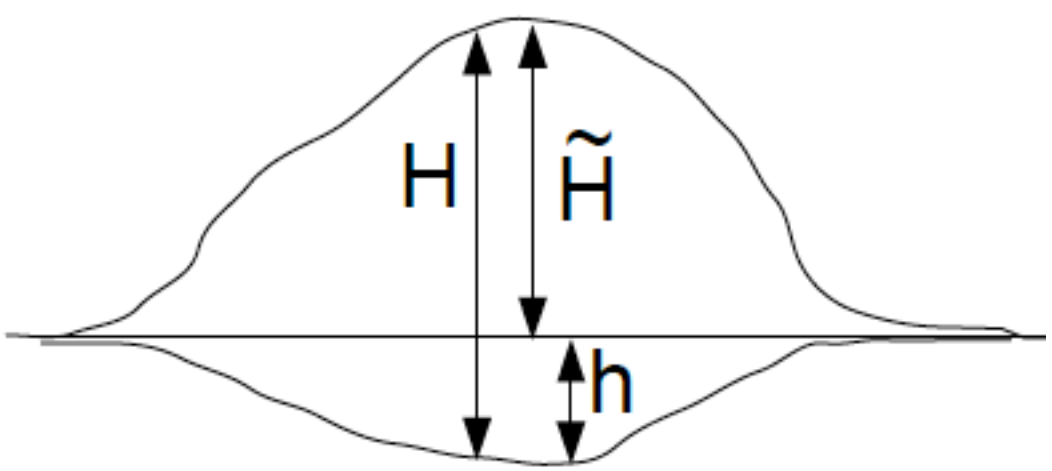
The equilibrium is stable if $\partial \bar{B} / \partial L < 0$ and unstable if $\partial \bar{B} / \partial L > 0$


LABELS: $10^3 * \theta$
0.5
0
- 0.5
MASS BALANCE (m/yr)
0.7
1
2
0
1
2
3
ICE-SHEET SIZE (10^3 km)


LABELS: $10^3 * \theta$
4
3
2
1
0
ICE-SHEET SIZE (10^3 km)
0.7
1
2
-8
-6
-4
-2
0
2
4
6
CLIMATE POINT P (100 km)

Hysteresis

Bedrock adjustment



$$\frac{\partial h}{\partial t} = \frac{(\rho_i/\rho_B)H - h}{\tau_B} \qquad (7.8)$$

$\tau_B \sim$ 3-5 ka

$\rho_i/\rho_B \simeq 1/4 - 1/3$

A numerical model using Glen's law

$$\frac{\partial H}{\partial t} = \vec{\nabla}\vec{M} + B \qquad (7.9)$$

Vertically integrated mass flux: *M*=*Hu* $\quad u = C\tau_b^{\,m} \quad \tau_b = \rho gHs = \rho gH\frac{\partial \tilde{H}}{\partial x} \qquad (7.2)$

m=3

$$\vec{M} = \underbrace{A H^{m+1}\left|\vec{\nabla}\tilde{H}^{m-1}\right|}_{D}\vec{\nabla}\tilde{H} \qquad (7.10)$$

$$\frac{\partial H}{\partial t} = \vec{\nabla}(D\vec{\nabla}\tilde{H}) + B \qquad (7.11)$$

$$D = A H^{m+1}\left[\left(\frac{\partial \tilde{H}}{\partial x}\right)^2 + \left(\frac{\partial \tilde{H}}{\partial y}\right)^2\right]^{(m-1)/2} \qquad (7.12)$$

Your 2D ice sheet model !

State-of-the-Science Ice Sheet Models

- Calculate three-dimensional velocities and temperature distributions within the ice
- Some include ice shelves
- Efforts are currently underway to coupled to ocean-atmosphere models (Earth System Models)
- Problem: different time scales (GCMs 100 yr; ice sheets 1,000+ years)