ATS 421/521

Climate Modeling

Spring 2015

Lecture 16

- Ice Sheets (Oerlemans, 1981)
- Ocean Biogeochemistry

May 27, 2015

Reading

- Wednesday: textbook chapter 3.3.4 (Land Surface)
- Friday: Friedlingstein et al. (2006)

Ice Sheets

Oerlemans (1981)

J. Oerlemans (1981) developed the following simple vertically-integrated model of a continental ice sheet.

7.2.1 Perfectly plastic solution for an ice sheet on a flat base

Assumptions:

- flow is quasi-two dimensional
- normal stress deviations are small
- the surface slope (s<0.1) is small

Balance of forces: $$\frac{\partial \tau_{xz}}{\partial z} = \rho g s \qquad \Rightarrow \tau_{xz} = \rho g (H - z) s$$

vertical gradient of the shear stress

pressure gradient

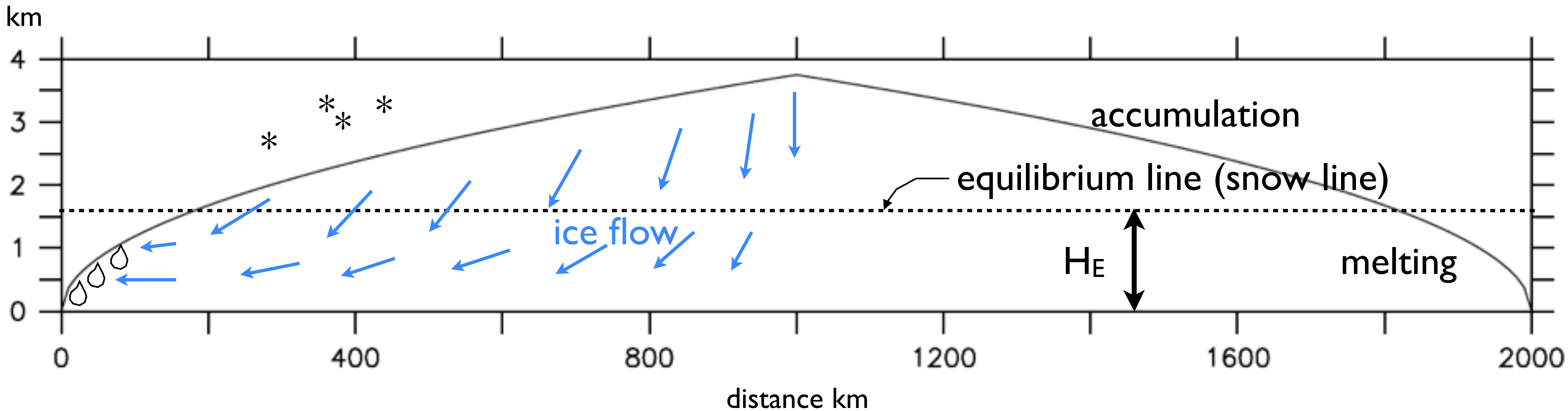
Stress at base: $$\Rightarrow \tau_b = \rho g H s = \rho g H \frac{\partial H}{\partial x} = const. = \tau_0$$

=> $$\frac{1}{2} \frac{\partial H^2}{\partial x} = \frac{\tau_0}{\rho g} \qquad \Rightarrow H = \sqrt{\frac{2 \tau_0}{\rho g} x} = \Lambda \sqrt{x} \tag{7.3}$$

$$[3.5\,\mathrm{m}^{1/2} < \Lambda < 4\,\mathrm{m}^{1/2}]$$

$$\frac{1}{2}\frac{\partial H^2}{\partial x}=\frac{\tau_0}{\rho g} \qquad \Rightarrow H=\sqrt{\frac{2\tau_0}{\rho g}x}=\Lambda\sqrt{x}$$

parabolic ice sheet profile



Assume surface mass balance linear with height.

$\overline{H}$ = average ice sheet height $\qquad \overline{H}=\frac{1}{L}\int_0^L H\,dx=\frac{\sqrt{2}\Lambda}{3}\sqrt{L}>H_E \quad (7.5)$

Ice Sheet Elevation-Mass Balance Feedback

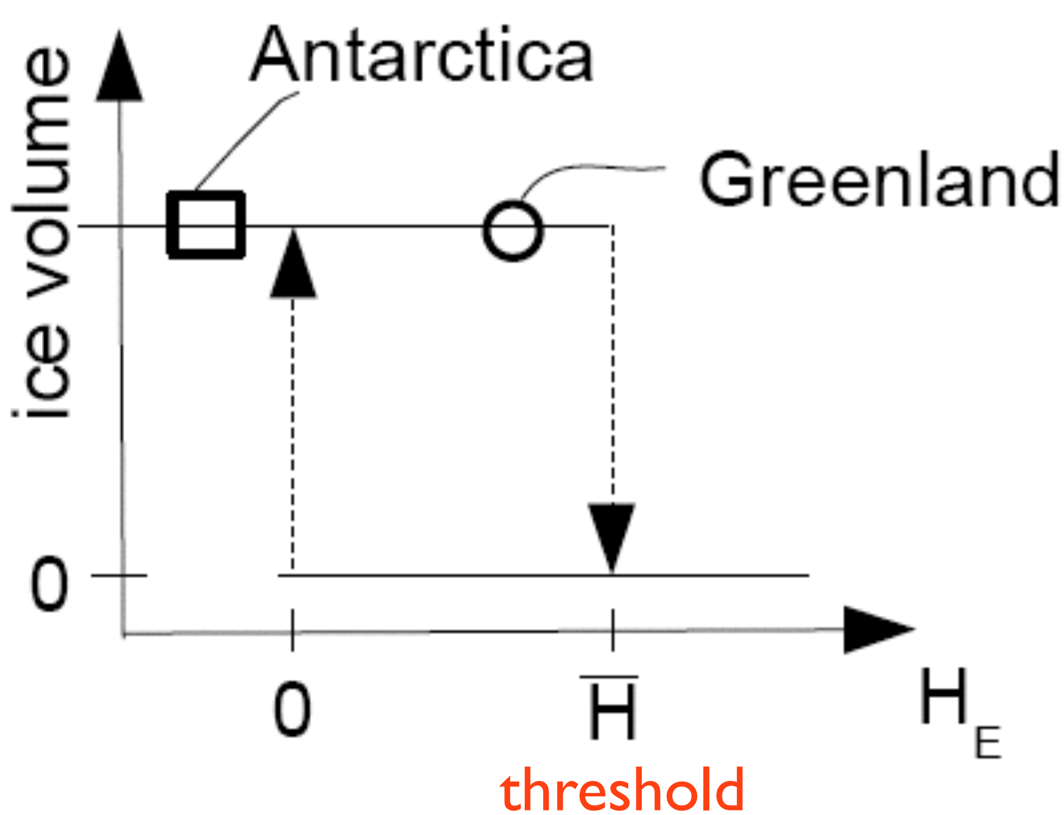
Melting

+

positive
feedback

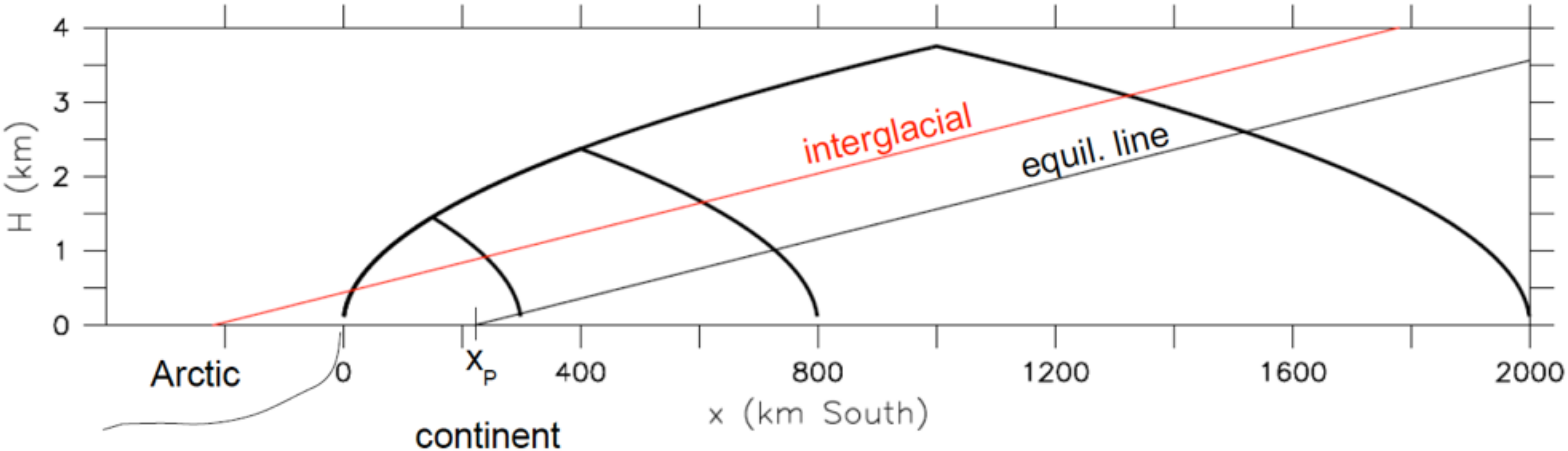
Warmer
Temperatures

Lower Surface
Elevation



- Positive feedback leads to a threshold for the equilibrium line.
- If higher than average ice sheet height, the ice sheet will disappear.
- It will only grow back if equilibrium line is decreased below zero (ground), that is below its current elevation. Irreversibility.
- **Greenland** is close to threshold. Vulnerable to warming. 2-3°C global mean warming will lead to its irreversible demise with 7 m sea level rise.
- **East Antarctic** ice sheet is not close to the threshold. May even grow due to increased snowfall in warmer climate.

Northern Hemisphere Ice Sheets



Mass Balance $$B = \alpha(x - x_P) + \beta H \qquad \alpha < 0 \qquad (7.6)$$

Northern half will loose mass by calving if ice bergs into Arctic.
Equilibrium: mass balance integrated over southern half = zero.

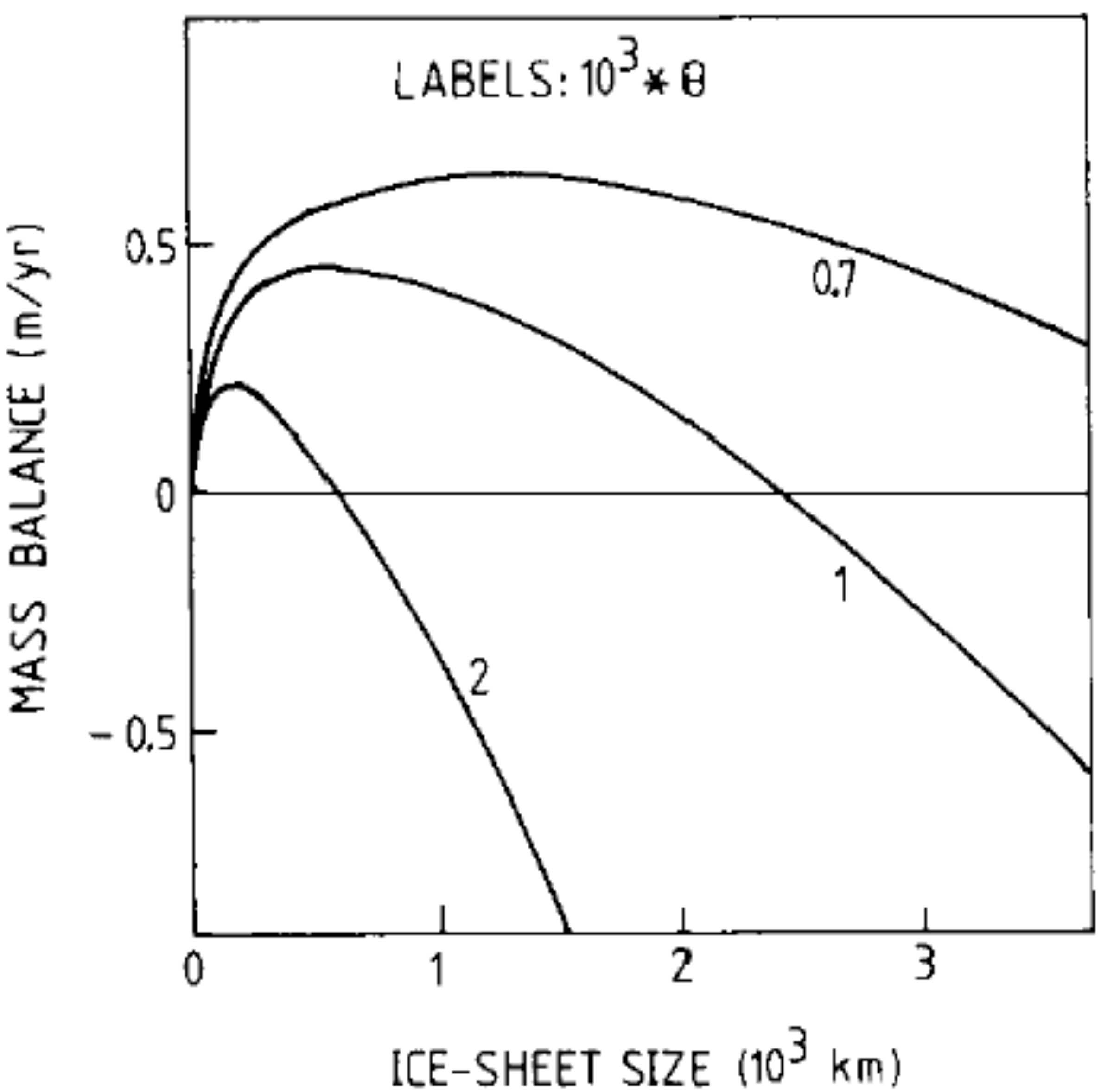
Equil. line slope: $\Theta = -\alpha/\beta$

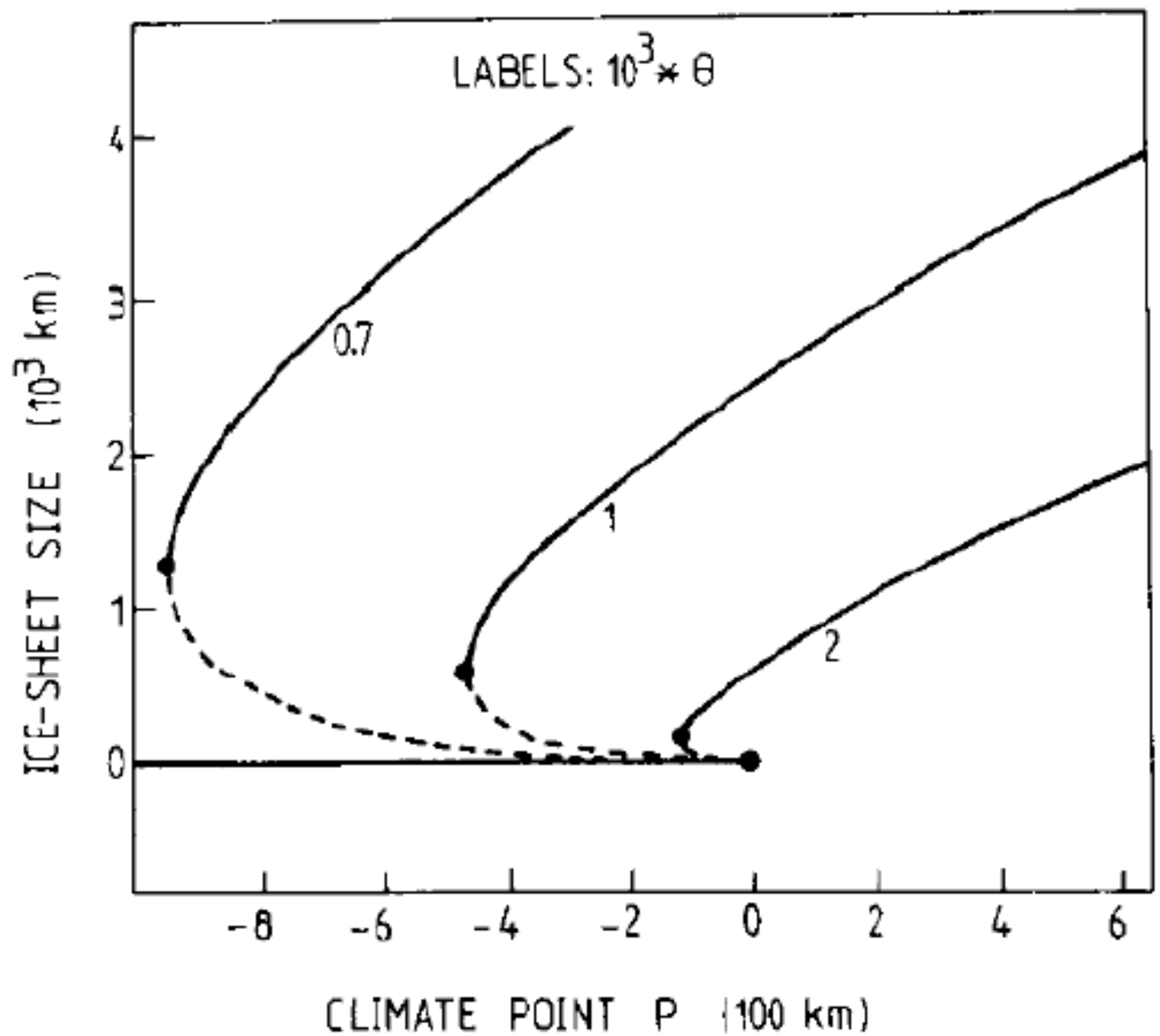
$$\bar{B}(L) = \frac{2}{L}\int_{L/2}^{L} B\,dx = B_1 + B_2 L^{1/2} + B_3 L = 0$$

$$B_1 = -\alpha x_P \quad , \quad B_2 = \sqrt{2}\,\beta\Lambda/3 \quad , \text{ and } \quad B_3 = 3\alpha/4$$

The equilibrium is stable if $\partial \bar{B}/\partial L < 0$ and unstable if $\partial \bar{B}/\partial L > 0$

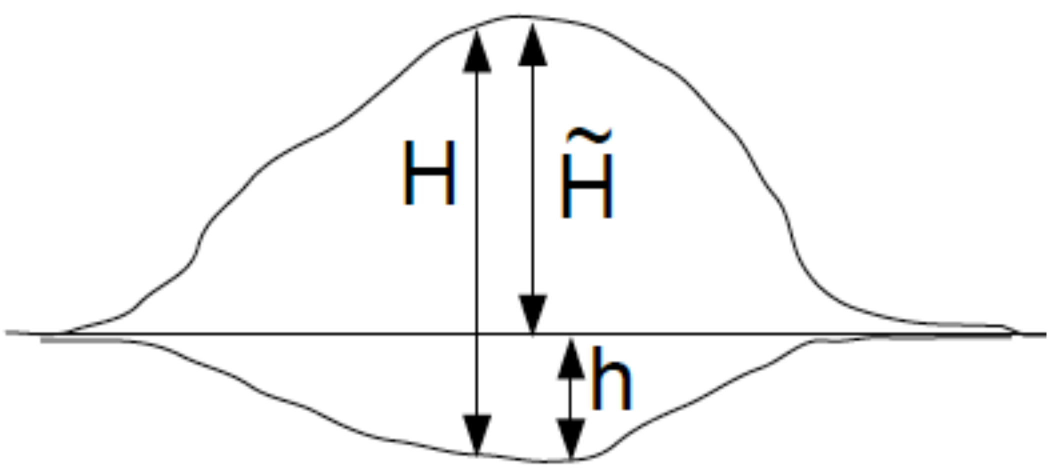
$x_P=0$





Hysteresis

Bedrock adjustment



$$\frac{\partial h}{\partial t} = \frac{(\rho_i / \rho_B) H - h}{\tau_B} \qquad (7.8)$$

$$\tau_B \sim 3\text{-}5 \text{ ka}$$

$$\rho_i / \rho_B \simeq 1/4 - 1/3$$

A numerical model using Glen's law

$$\frac{\partial H}{\partial t} = \vec{\nabla} \vec{M} + B \qquad (7.9)$$

Vertically integrated mass flux: *M*=*Hu* $\quad u = C\,\tau_b^{\,m} \quad \tau_b = \rho\, gHs = \rho\, gH \frac{\partial \tilde{H}}{\partial x} \qquad (7.2)$

m=3

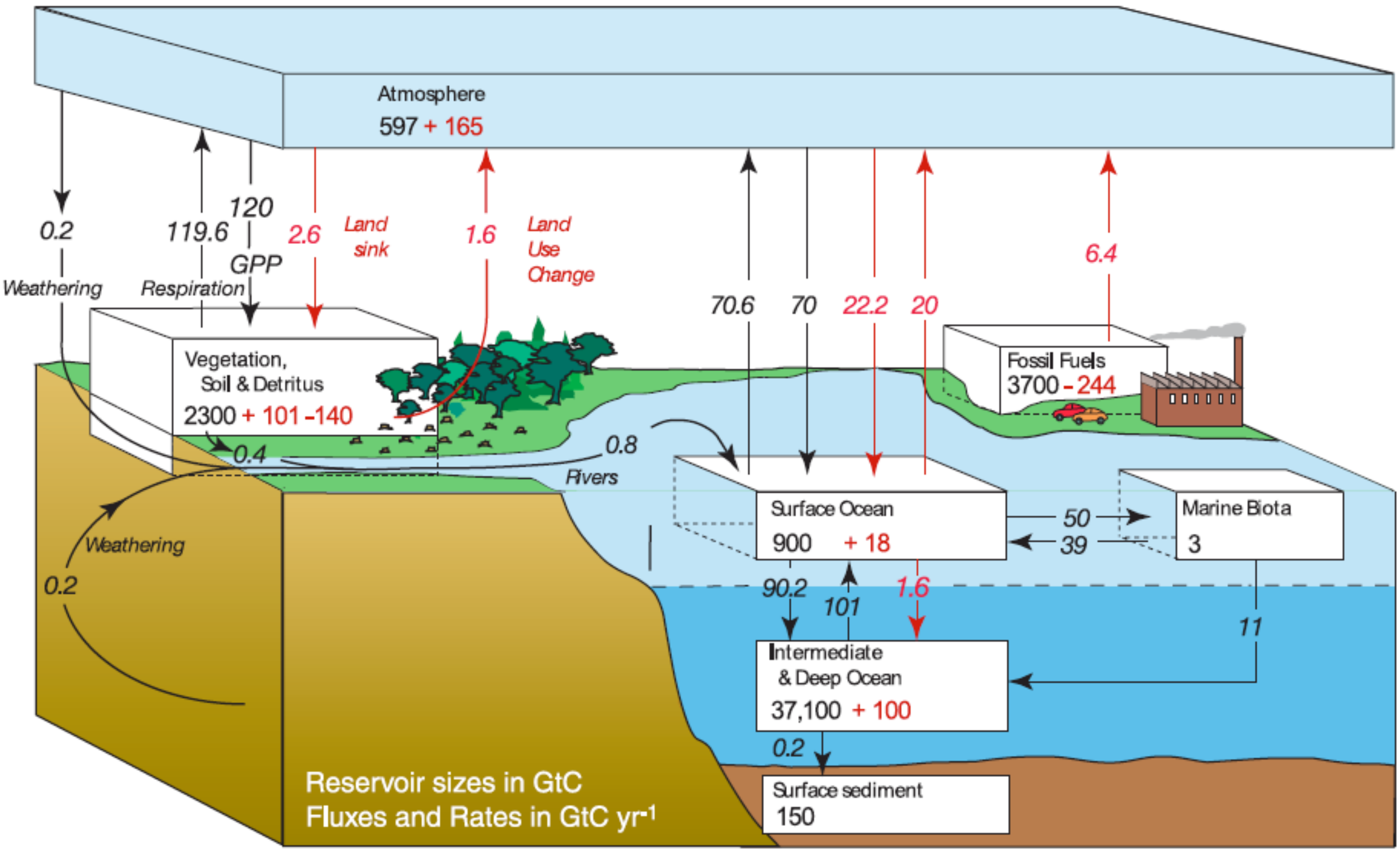
$$\vec{M} = \underbrace{A\,H^{m+1}\left|\vec{\nabla}\,\tilde{H}^{m-1}\right|}_{D}\vec{\nabla}\,\tilde{H} \qquad (7.10)$$

$$\frac{\partial H}{\partial t} = \vec{\nabla}\left(D\vec{\nabla}\,\tilde{H}\right) + B \qquad (7.11)$$

$$D = A\,H^{m+1}\left[\left(\frac{\partial \tilde{H}}{\partial x}\right)^2 + \left(\frac{\partial \tilde{H}}{\partial y}\right)^2\right]^{(m-1)/2} \qquad (7.12)$$

Your 2D ice sheet model !

The Global Carbon Cycle

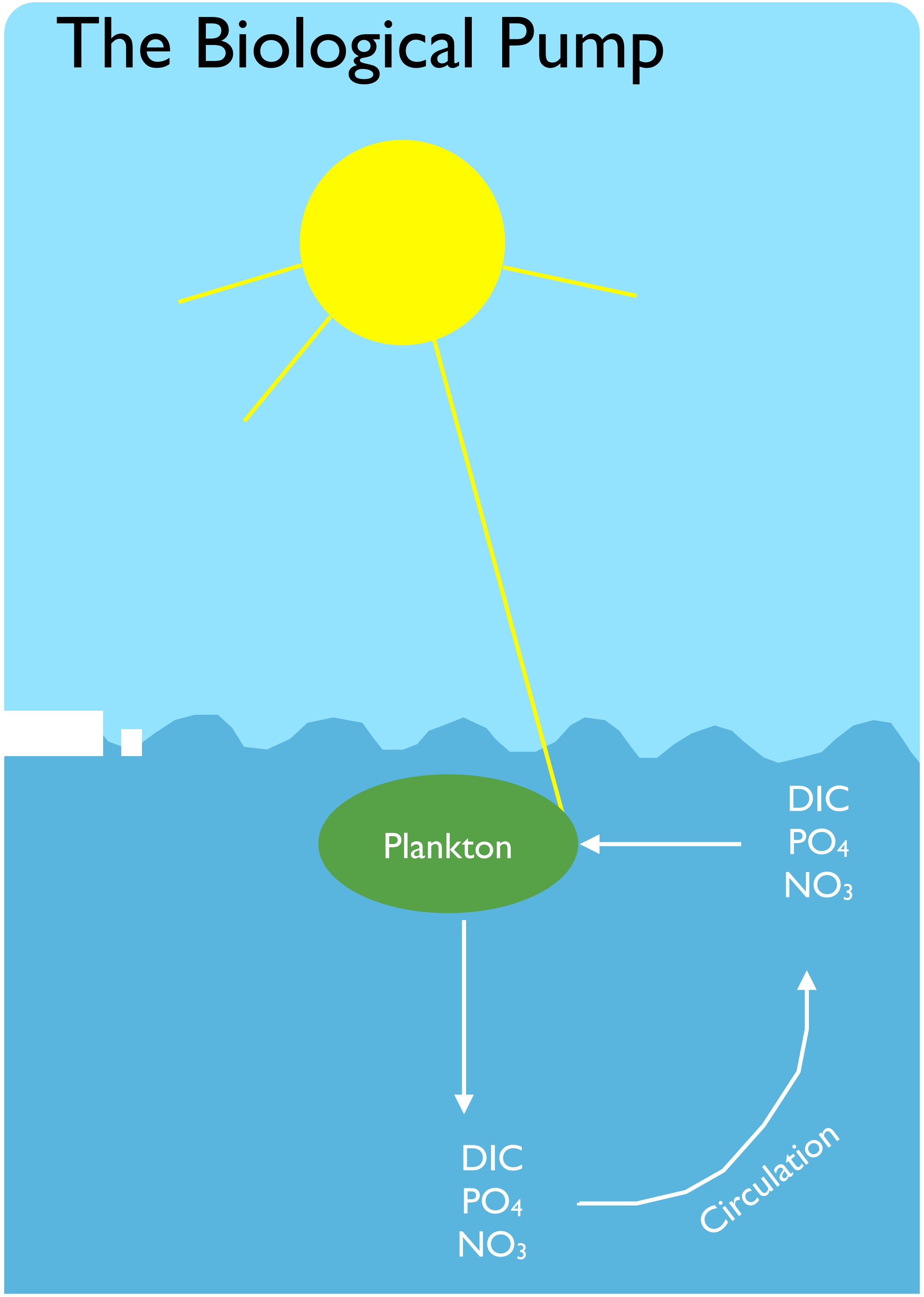


2 GtC = 1 ppmv

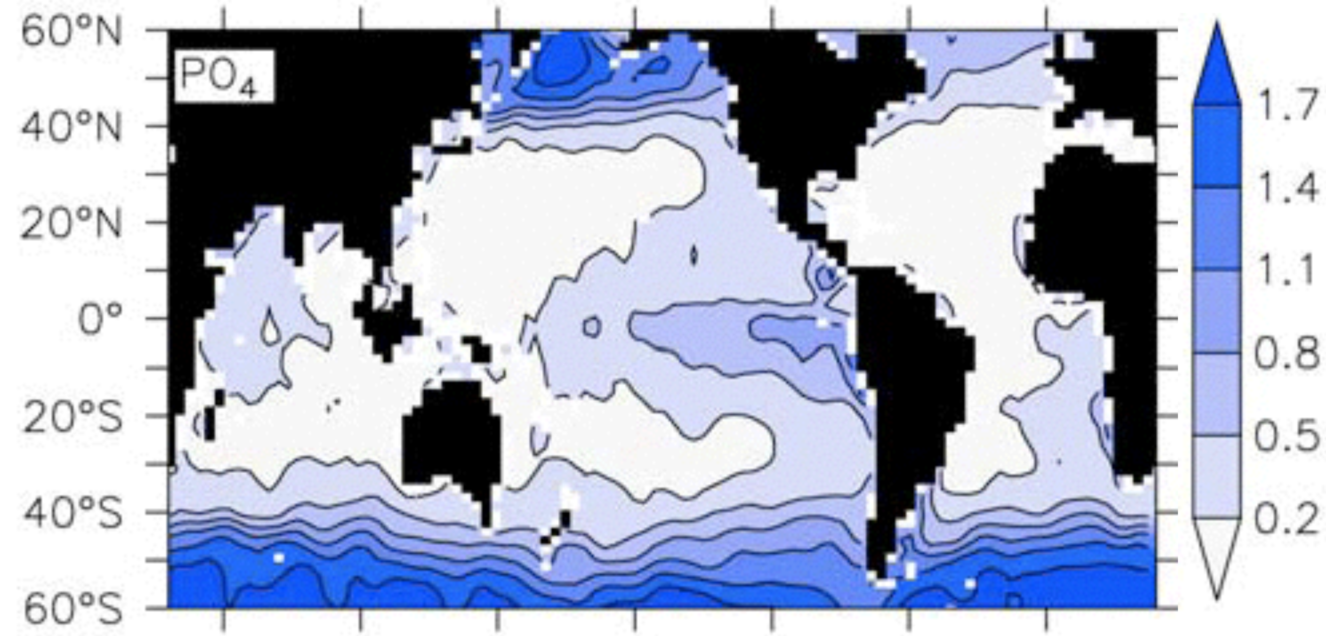
IPCC (2007)

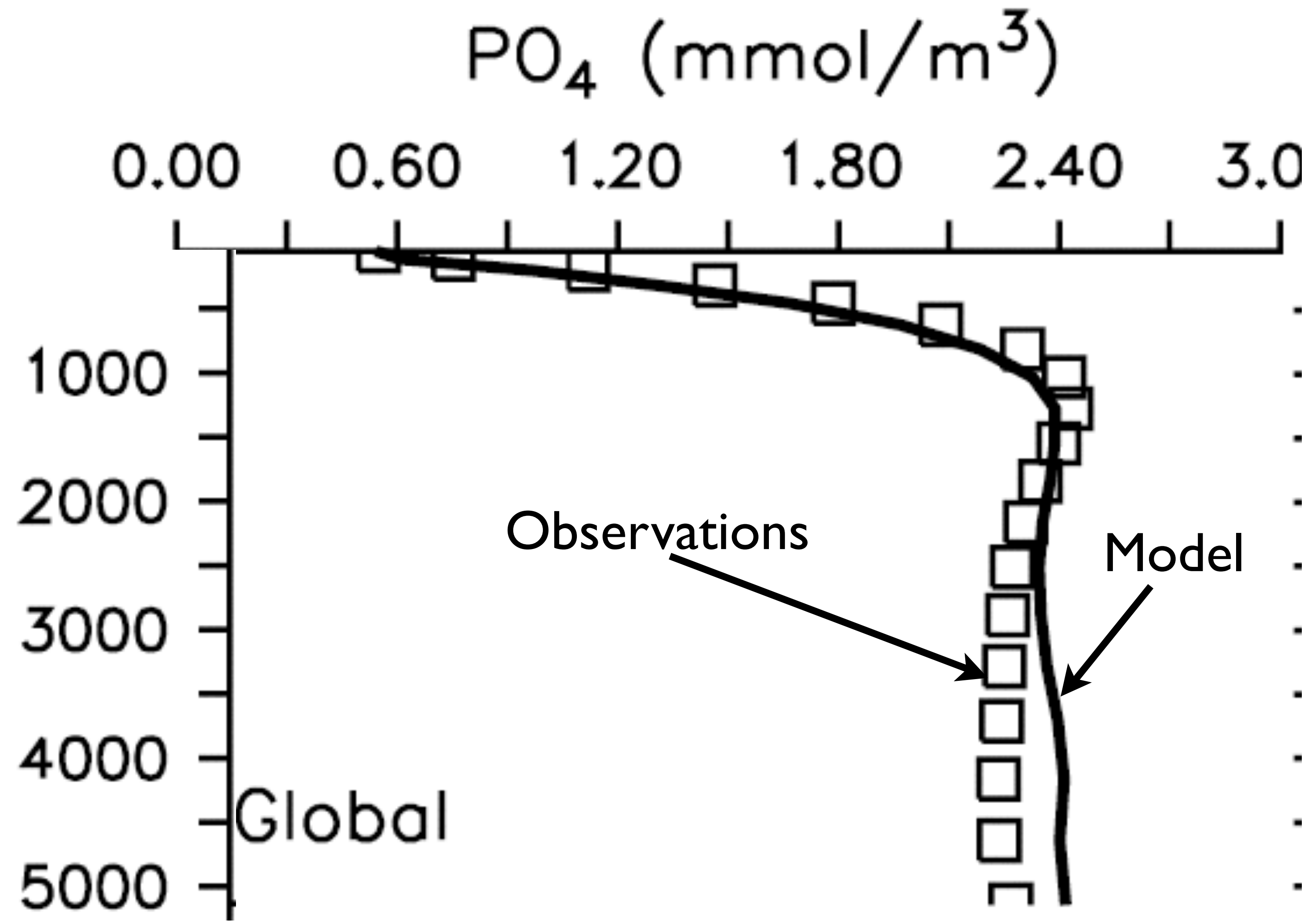
The Biological Pump

1. The Soft Tissue (Organic Matter) Pump

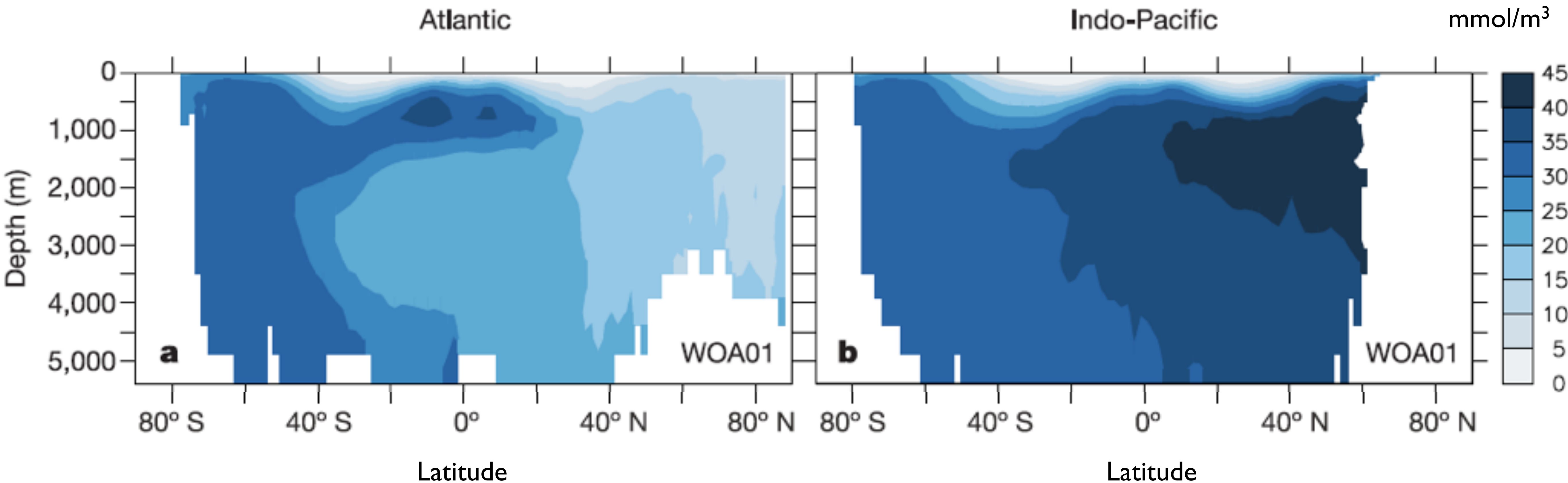


Surface Nutrients

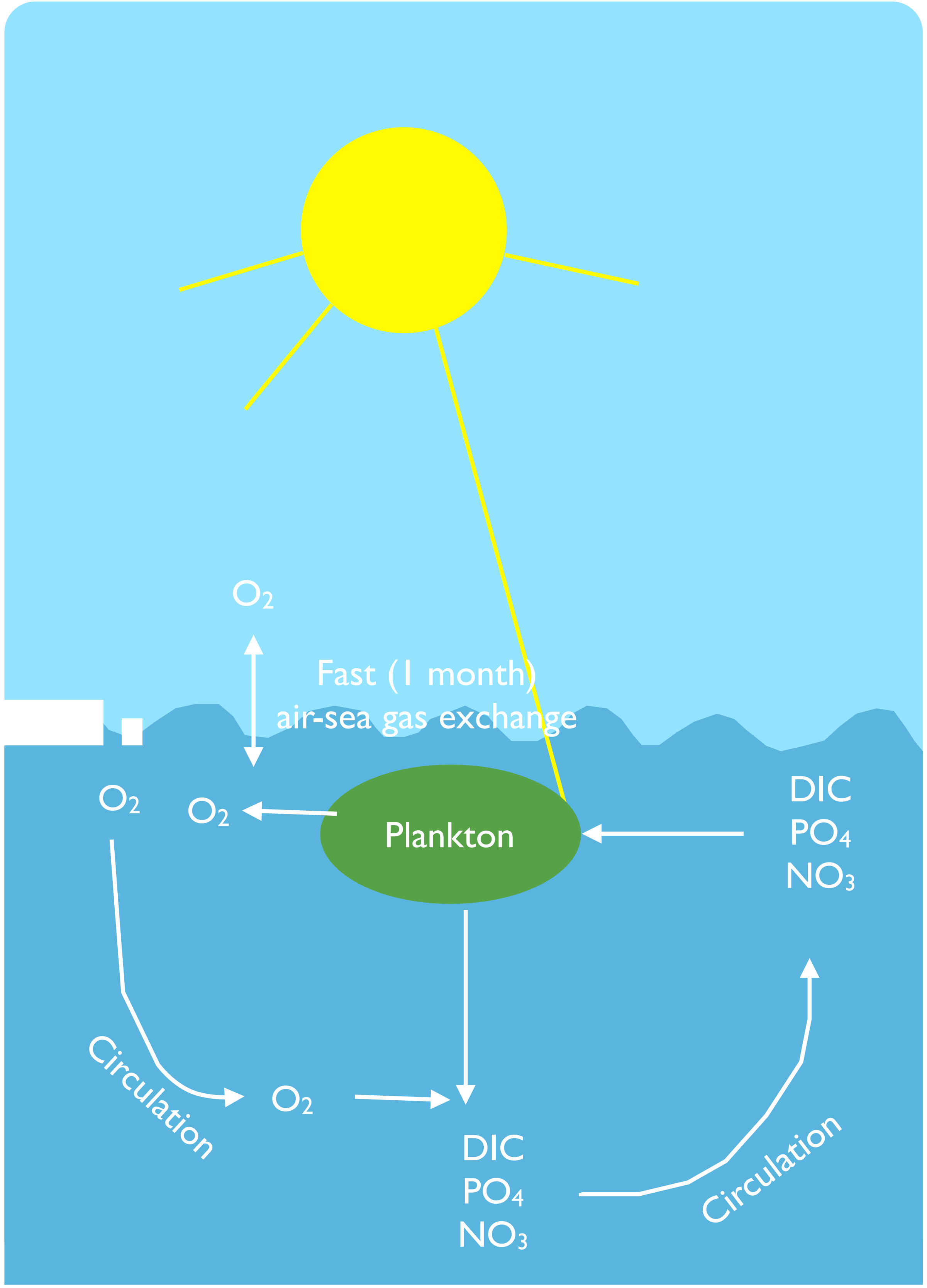




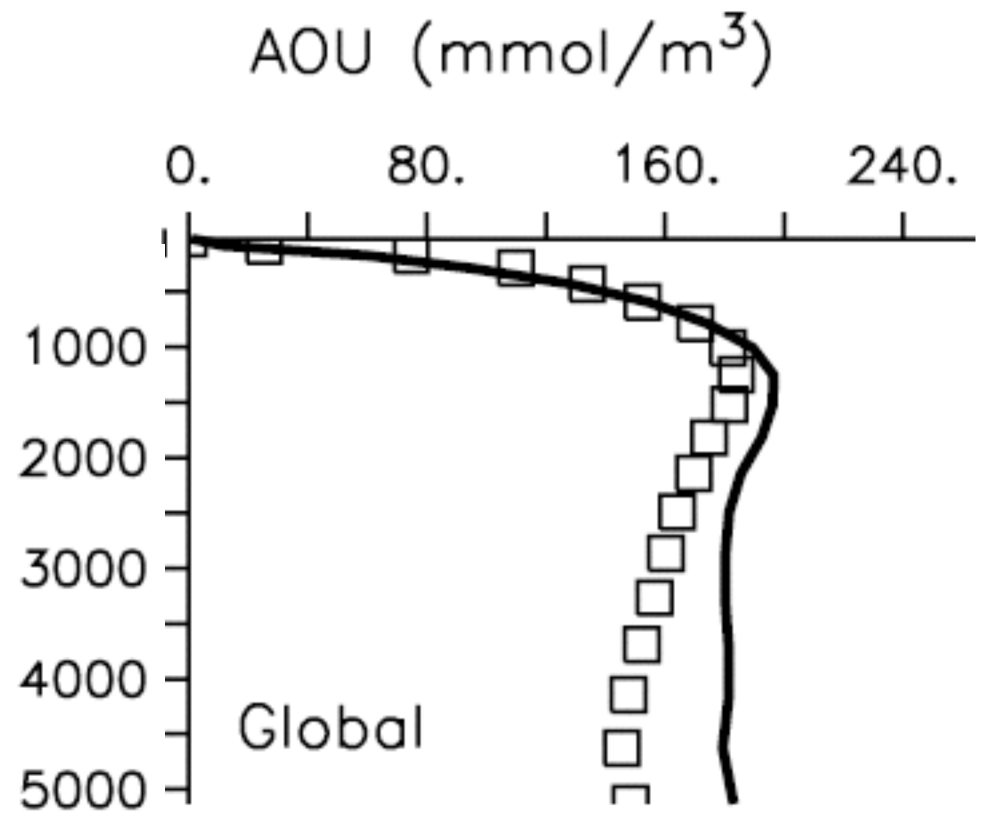
NO_3 in the deep ocean

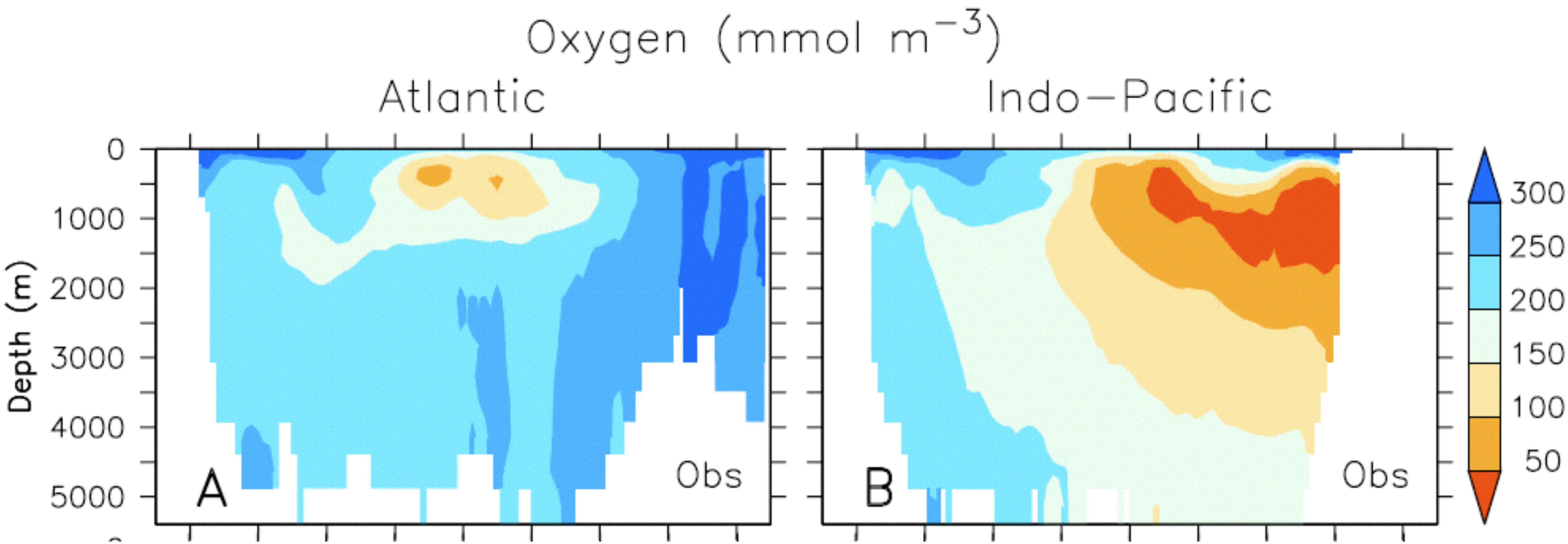


Oxygen and Apparent Oxygen Utilization (AOU)

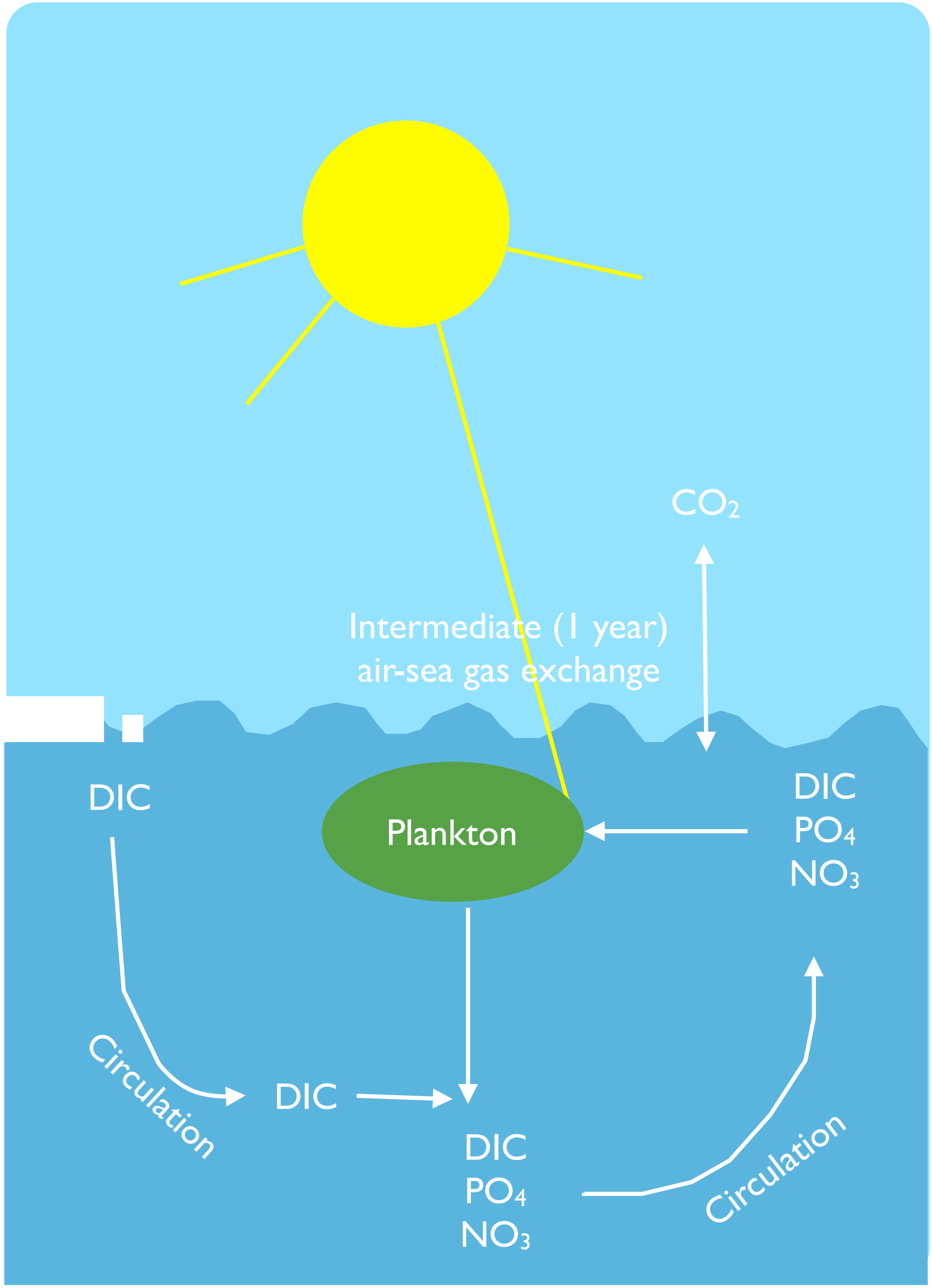


$$AOU = {}^{sat}O_2(T) - O_2$$

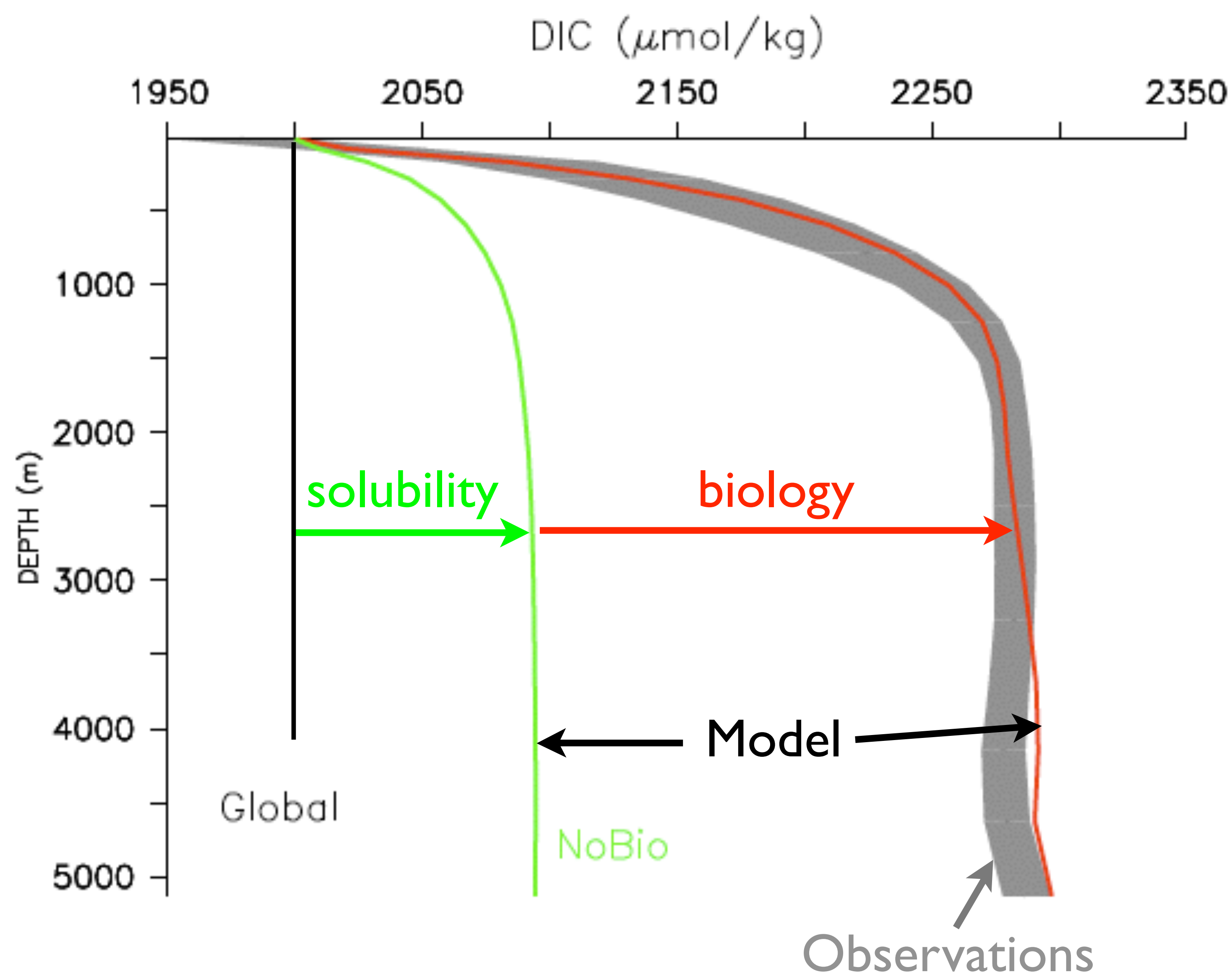




Oxygen (mmol m^{-3})
Atlantic
Indo−Pacific
Depth (m)
0
1000
2000
3000
4000
5000
A
Obs
B
Obs
80°S
40°S
0°
40°N
80°N
300
250
200
150
100
50



solubility pump biological pump



Air sea gas exchange:

$$q=-K(|v|,T,S)(pCO_2^{atm} - pCO_2^{ml})$$

$$pCO_2^{ml}=[CO_2]^{ml}/\alpha(T,S)$$

Solubility

Chemistry

$$CO_2 + H_2O \Leftrightarrow HCO_3^- + H^+$$

$$HCO_3^- \Leftrightarrow CO_3^{2-} + H^+$$

Total Carbon

Dissolved Inorganic Carbon

$$DIC = \sum CO_2 = [HCO_3^-]+[CO_3^{2-}]+[CO_2]$$

bicarbonate carbonate 1%

The Biological Pump

2. The Hard Tissue (Inorganic Matter/Alkalinity) Pump

$$[H^+][HCO_3^-]=K_1[CO_2]^{ml}$$
$$[H^+][CO_3^{2-}]=K_2[HCO_3^-]$$

$$\Rightarrow [CO_2]^{ml}=K_2[HCO_3^-]^2/(K_1[CO_3^{2-}])$$

$CaCO_3$ production increases $[CO_2]$ because $[CO_3^{2-}]$ is taken up by organisms:

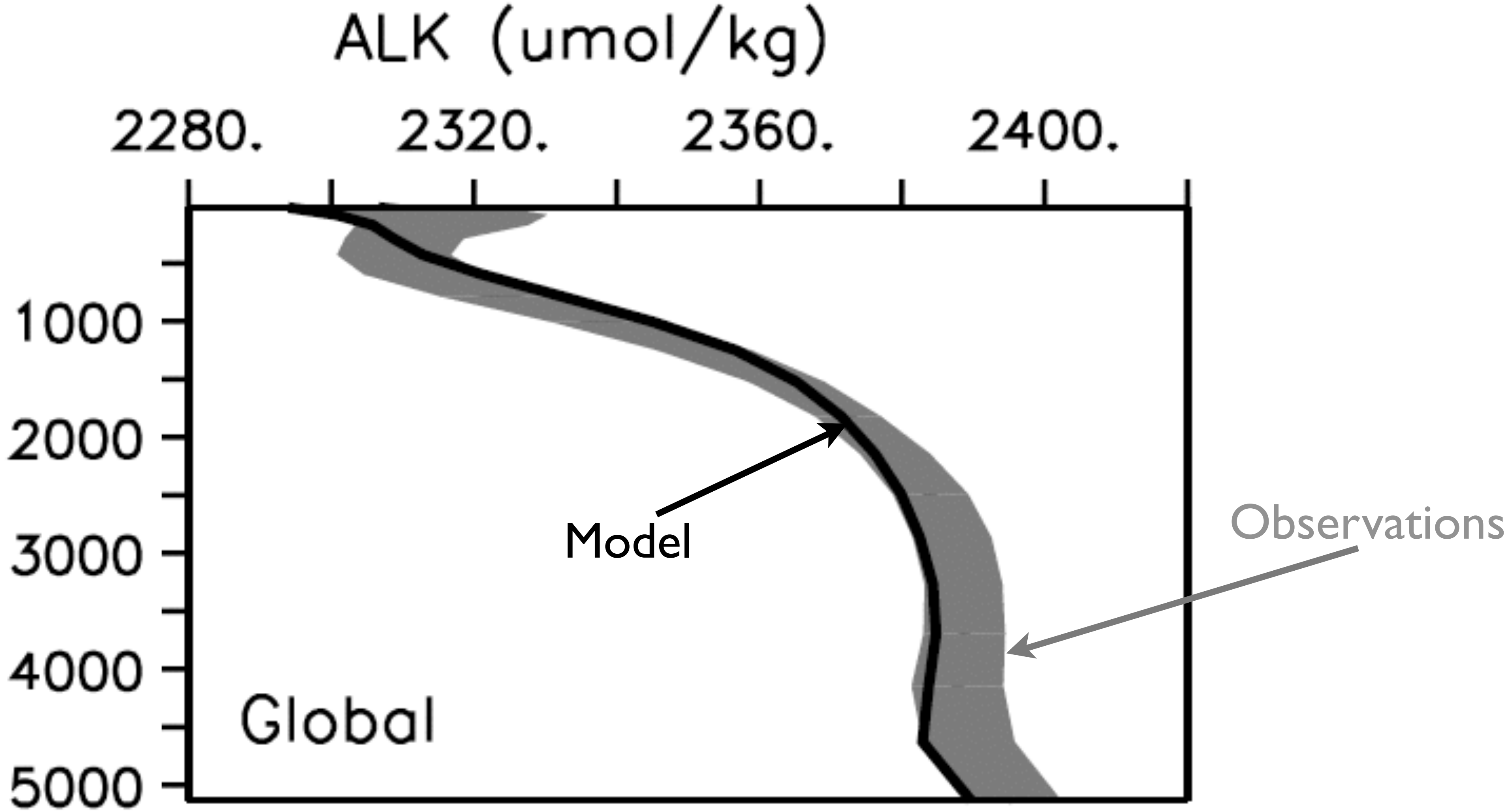
- Coccolithophorids (phytoplankton)
- Foraminifera (zooplankton)

$\Big\}$ Calcite

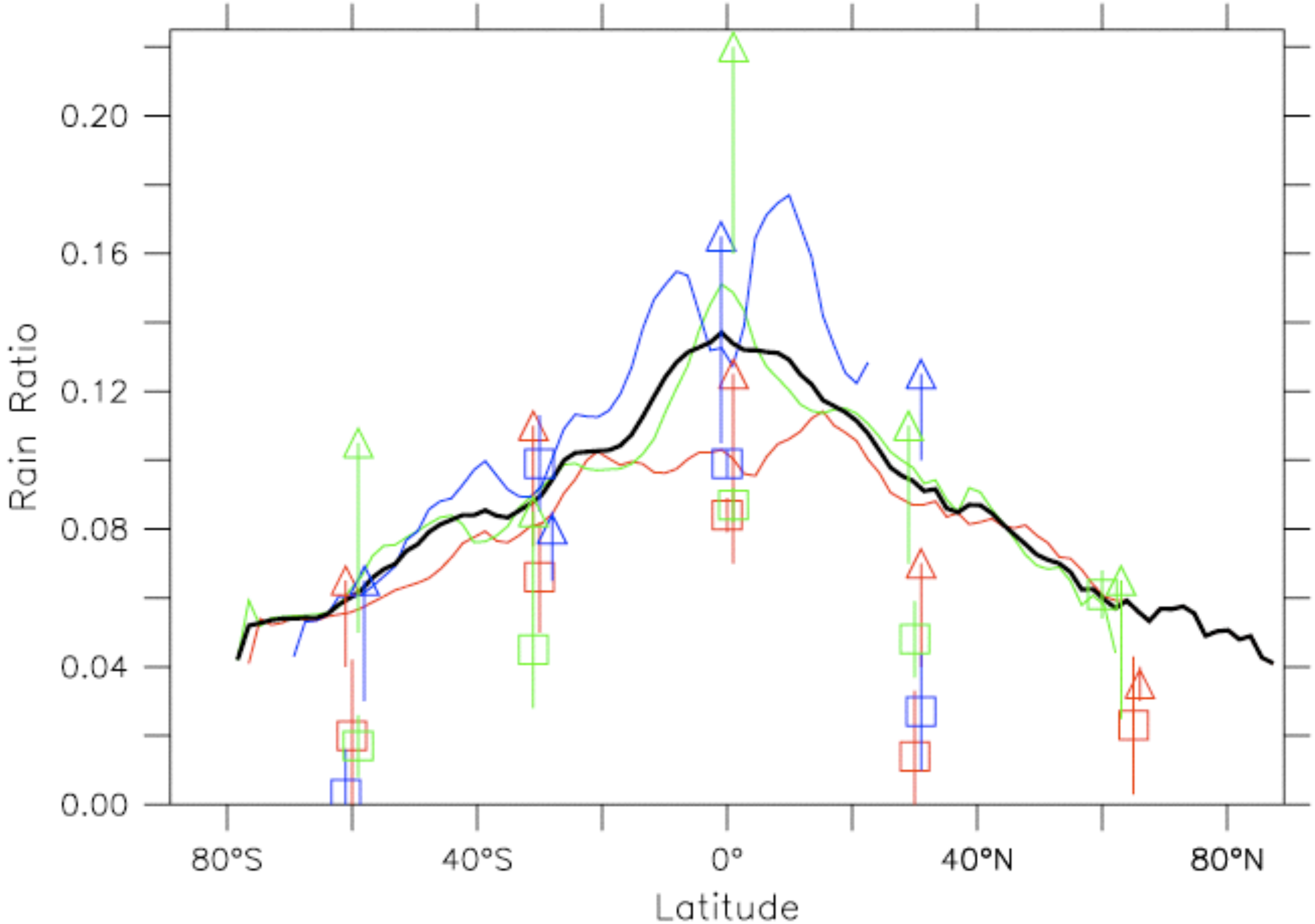
- Pteropods (zooplankton) Aragonite

Carbonate Alkalinity

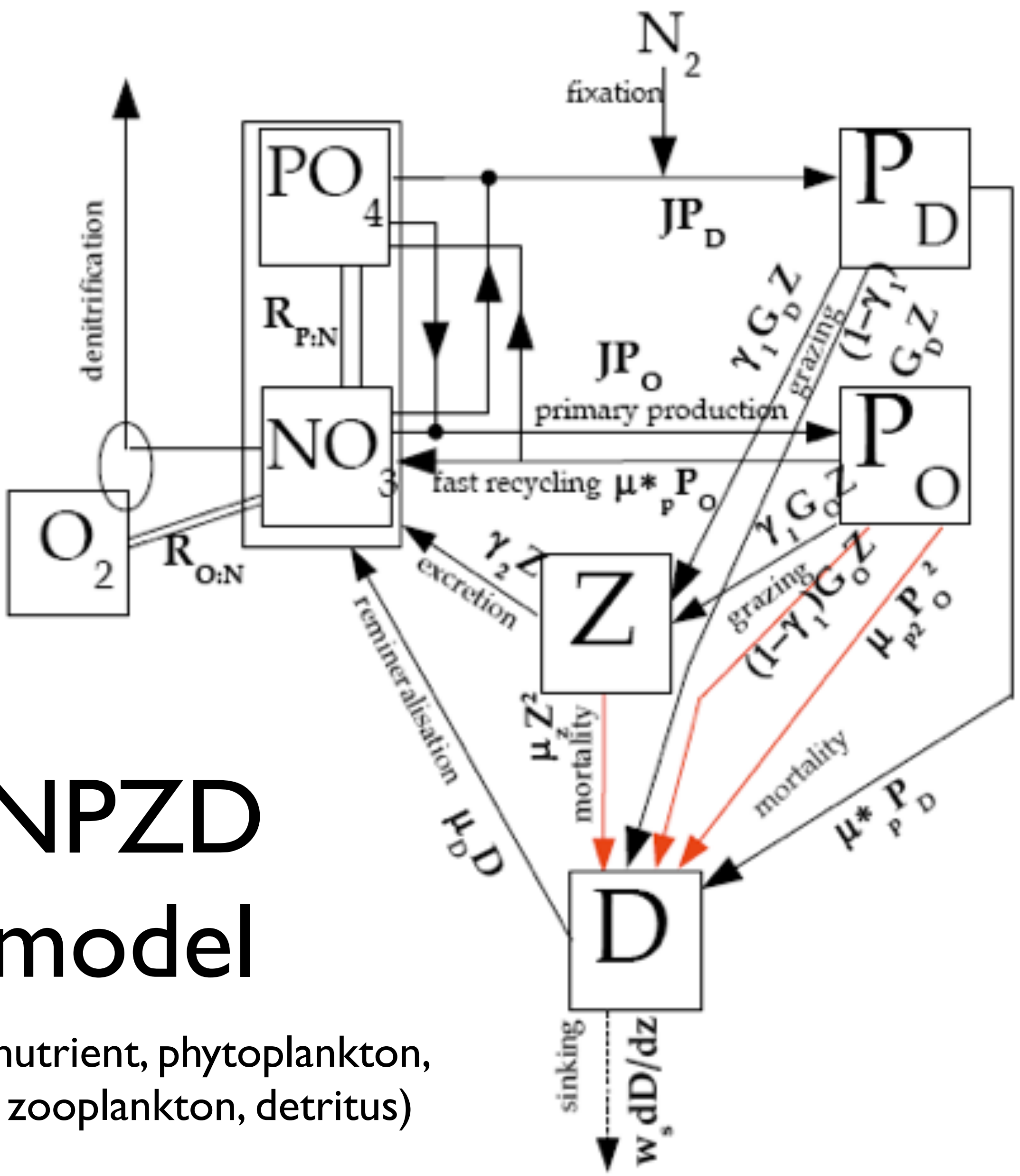
$$ALK=[HCO_3^-]+2[CO_3^{2-}]=2DIC-[HCO_3^-]$$



Rain Ratio = Export of $CaCO_3$ / Export of POC



Details of Ecosystem and Carbon Cycle Model



NPZD model

(nutrient, phytoplankton, zooplankton, detritus)

$$\frac{\partial C}{\partial t} = T + S,$$

Transport — Biological Sources/Sinks

$$S(\mathrm{DIC}) = S(\mathrm{PO_4})R_{C:P} - S(\mathrm{CaCO_3})$$

$$S(\mathrm{ALK}) = -S(\mathrm{NO_3}) \times 10^{-3} - 2S(\mathrm{CaCO_3}).$$

$$S(\mathrm{PO_4}) = (\mu_D \mathrm{D} + \mu_P^* \mathrm{P}_O + \gamma_2 \mathrm{Z} - J_O \mathrm{P}_O - J_D \mathrm{P}_D) R_{P:N}$$

$$S(\mathrm{NO_3}) = (\mu_D \mathrm{D} + \mu_P^* \mathrm{P}_O + \gamma_2 \mathrm{Z} - J_O \mathrm{P}_O - u_{\mathrm{N}} J_D \mathrm{P}_D) \cdot (1 - 0.8 R_{O:N}\ r_{sox}^{NO3})$$

$$S(\mathrm{P}_O) = J_O \mathrm{P}_O - \mu_P^* \mathrm{P}_O - G(\mathrm{P}_O)\mathrm{Z} - \mu_{P2} \mathrm{P}_O^2$$

$$S(\mathrm{P}_D) = J_D \mathrm{P}_D - G(\mathrm{P}_D)\mathrm{Z} - \mu_P \mathrm{P}_D$$

$$S(\mathrm{Z}) = \gamma_1 [G(\mathrm{P}_O) + G(\mathrm{P}_D)]\mathrm{Z} - \gamma_2 \mathrm{Z} - \mu_Z \mathrm{Z}^2$$

$$S(\mathrm{D}) = (1 - \gamma_1)[G(\mathrm{P}_O) + G(\mathrm{P}_D)]\mathrm{Z} + \mu_P \mathrm{P}_D + \mu_{P2} \mathrm{P}_O^2 + \mu_Z \mathrm{Z}^2 - \mu_D \mathrm{D} - w_{\mathrm{D}}\ \partial \mathrm{D} \partial z$$

$$J(I, \mathrm{NO_3}, \mathrm{PO_4}) = \min(J_{OI}, J_{O\max} u_N, J_{O\max} u_P), \qquad u_{\mathrm{P}} = \mathrm{PO_4}/(k_{\mathrm{P}} + \mathrm{PO_4}).$$

$$J_{OI} = \frac{J_{O\max} \alpha I}{\left[J_{O\max}^2 + (\alpha I)^2\right]^{1/2}} \qquad J_{O\max} = a \times \exp(T/T_b)$$

$$w_D = \left\{ \begin{array}{c} w_{D0} + m_w z, z \leq 1000m \\ w_{D0} + m_w 1000m, z > 1000m \end{array} \right\},$$

$$\mu_D = \mu_{D0} \exp(T/T_b)[0.65 + 0.35 \tanh(O_2 - 6)]$$

$$S(\mathrm{O_2}) = F_{sfc} - S(\mathrm{PO_4}) R_{O:P}\ r_{sox}^{O2}$$

$$\Pr(\mathrm{CaCO_3}) = ((1 - \gamma_1) G(\mathrm{P}_O)\mathrm{Z} + \mu_{P2} \mathrm{P}_O^2 + \mu_Z \mathrm{Z}^2) R_{\mathrm{CaCO3/POC}} R_{C:P},$$

$$Di(\mathrm{CaCO_3}) = \int Pr(\mathrm{CaCO_3}) dz \cdot \frac{d}{dz}\left(e^{-z/D_{\mathrm{CaCO_3}}}\right)$$

Schmittner et al. 2008 GBC