ATS 421/521

Climate Modeling

Spring 2015

Lecture 7

- Numerics (cont'd)
- Radiative Convective Models

April 22

Reading

- Today: Course Notes Chapter 5.1
- For Friday: Pierrehumbert (2011)

$$\Delta t \leqslant \frac{\Delta x}{|u|} \quad . \tag{2.41}$$

CFL criterion

(Courant-Friedrichs-Lewy, 1928)

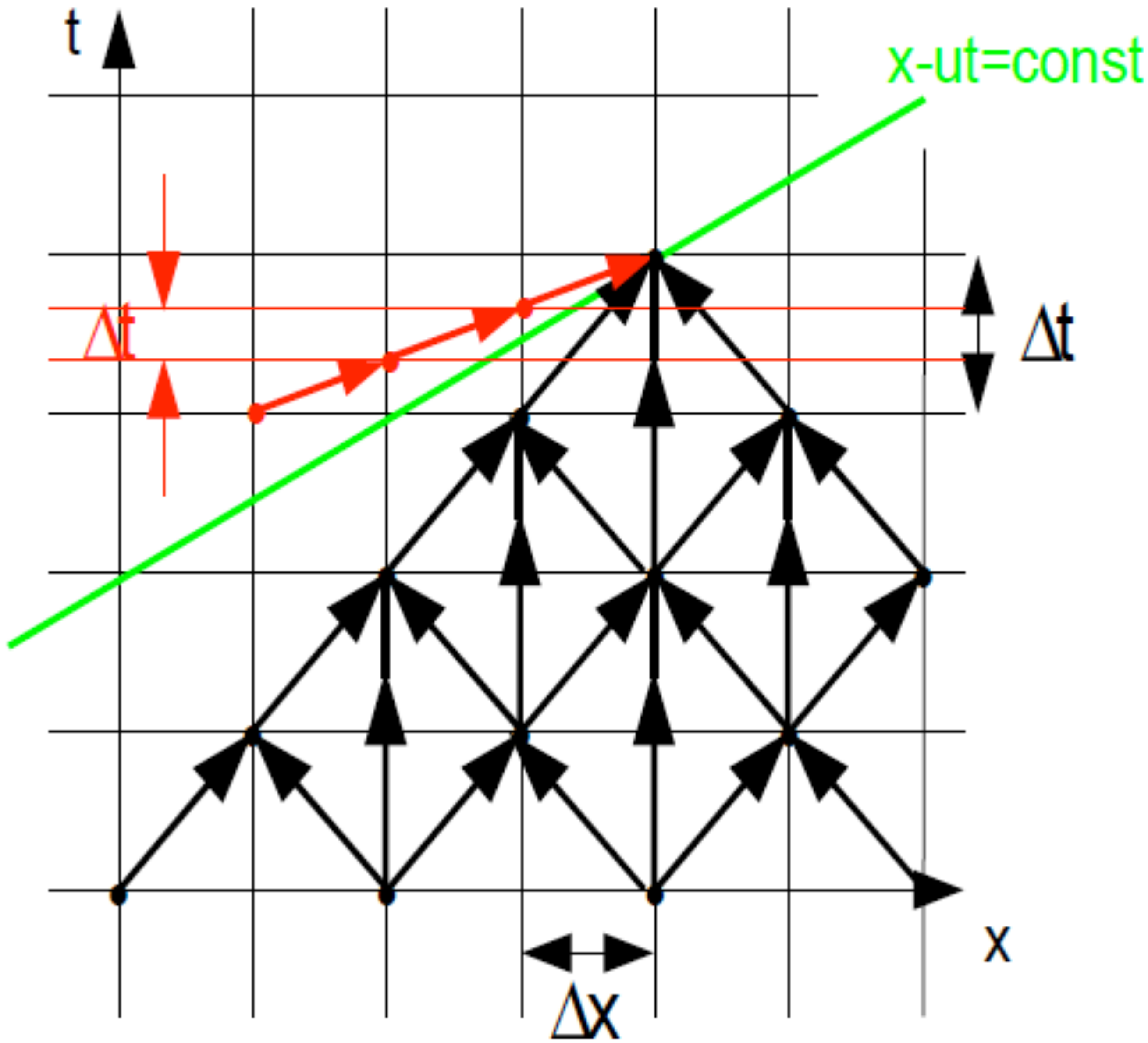
The CFL criterion limits the maximum possible time step.

For Δx = 300 km

ocean: max(u) = 1 m/s => $\Delta t < 3$ days

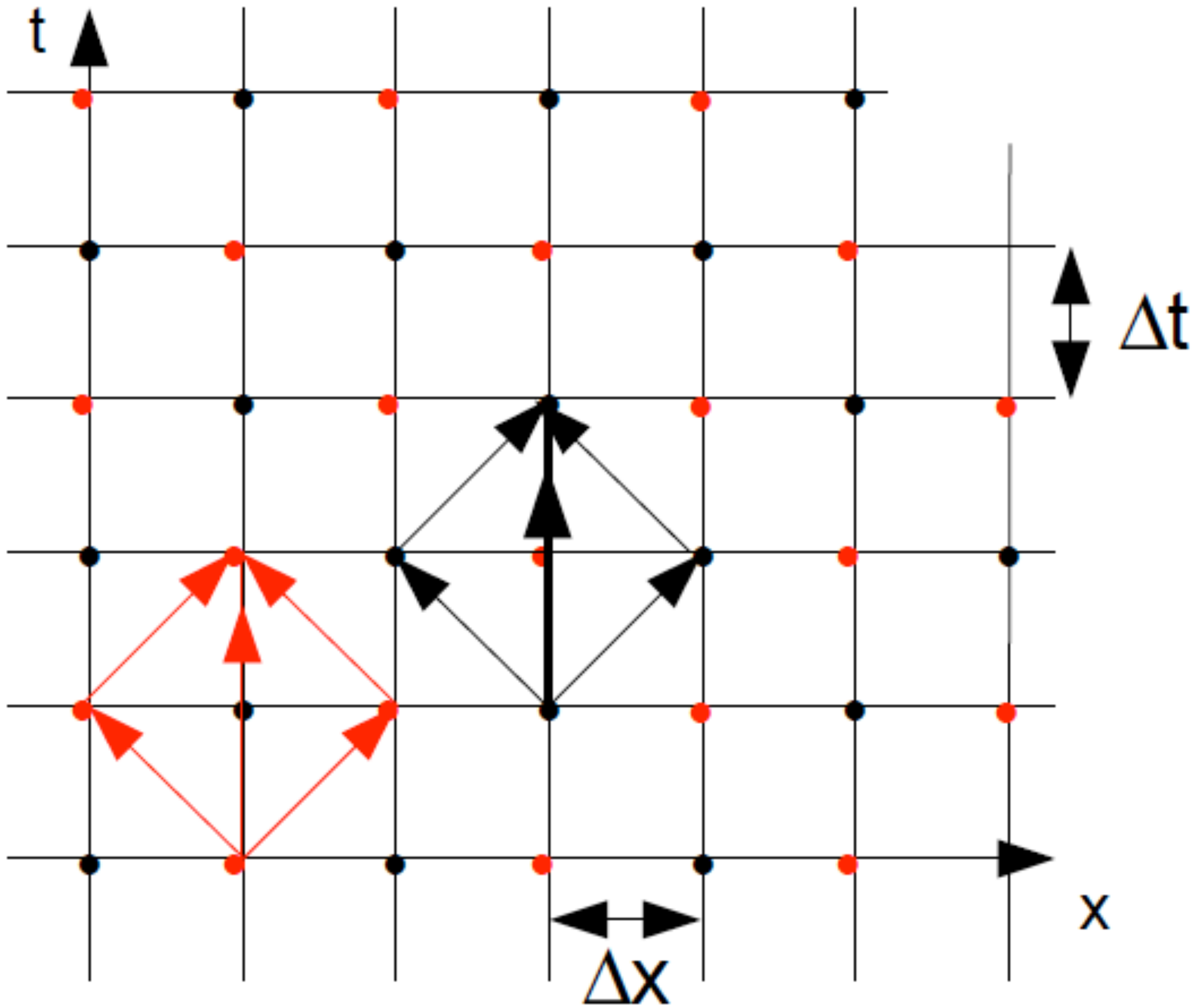
atmosphere: max(u) = 80 m/s => $\Delta t < 1$ hour

CFL criterion



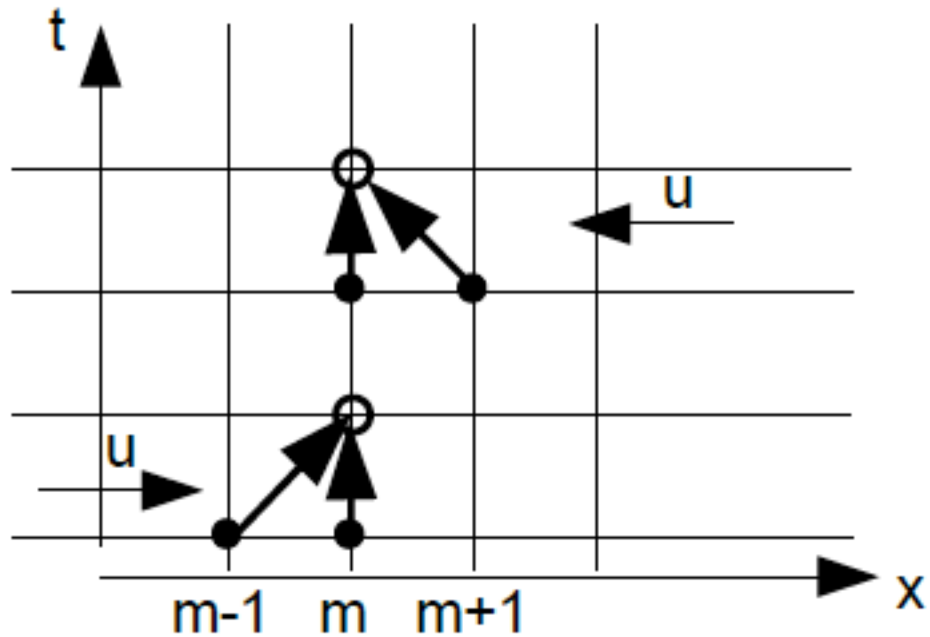
Signal propagates faster than the cone of influence for large time step **Δt**.
Signal propagates slower than the cone of influence for small time step **Δt**.

Numerical Mode (artifact)



Decoupling of red and black grid points.
Can be removed by using an Euler (FTCS) time step.

The Upwind Scheme



$$\frac{C_{m,n+1}-C_{m,n}}{\Delta t} = -u \begin{cases} \dfrac{C_{m,n}-C_{m-1,n}}{\Delta x}, u>0 \\ \dfrac{C_{m+1,n}-C_{m,n}}{\Delta x}, u\leqslant 0 \end{cases}$$

$$\xi = 1 - \left|\frac{u\Delta t}{\Delta x}\right|(1-\cos(k\Delta x)) - i\frac{u\Delta t}{\Delta x}\sin(k\Delta x)$$

$$|\xi^2| = 1 - 2\left|\frac{u\Delta t}{\Delta x}\right|\left(1-\left|\frac{u\Delta t}{\Delta x}\right|\right)(1-\cos(k\Delta x))$$

Again CFL criterion for stability.

<u>Advantage:</u> Positive definite

<u>Disadvantage:</u> only first order accurate (numerical diffusion)

Other Schemes

- Prather: higher order terms are calculated and stored (positive definite, very accurate, no numerical diffusion but requires more memory and computations)
- FCT (Flux corrected transport)

Consider **diffusion equation**: $$\frac{\partial C}{\partial t} = K \frac{\partial^2 C}{\partial x^2}$$

FTCS: $$\frac{C_{m,n+1} - C_{m,n}}{\Delta t} = K \frac{C_{m+1,n} - 2C_{m,n} + C_{m-1,n}}{\Delta x^2} \quad , \tag{2.42}$$

$$C_{m,n+1} = C_{m,n} + \frac{K \Delta t}{\Delta x^2} (C_{m+1,n} - 2C_{m,n} + C_{m-1,n})$$

$$\xi = 1 - \frac{4 K \Delta t}{(\Delta x)^2} \sin^2 (\frac{k \Delta x}{2})$$

$$\xi^2 = 1 - 2 \frac{4 K \Delta t}{(\Delta x)^2} \sin^2 (\frac{k \Delta x}{2}) + \left(\frac{4 K \Delta t}{(\Delta x)^2} \right)^2 \sin^2 (\frac{k \Delta x}{2})$$

$$|\xi| \leqslant 1 \longrightarrow \Delta t \leqslant \frac{(\Delta x)^2}{2K}$$

Analogous to CFL criterion.

FTCS **stable** for diffusion equation.

FTCS: $$\frac{C_{m,n+1}-C_{m,n}}{\Delta t}=K\frac{C_{m+1,n}-2C_{m,n}+C_{m-1,n}}{\Delta x^2}\quad, \tag{2.42}$$

Replace n with n+1:

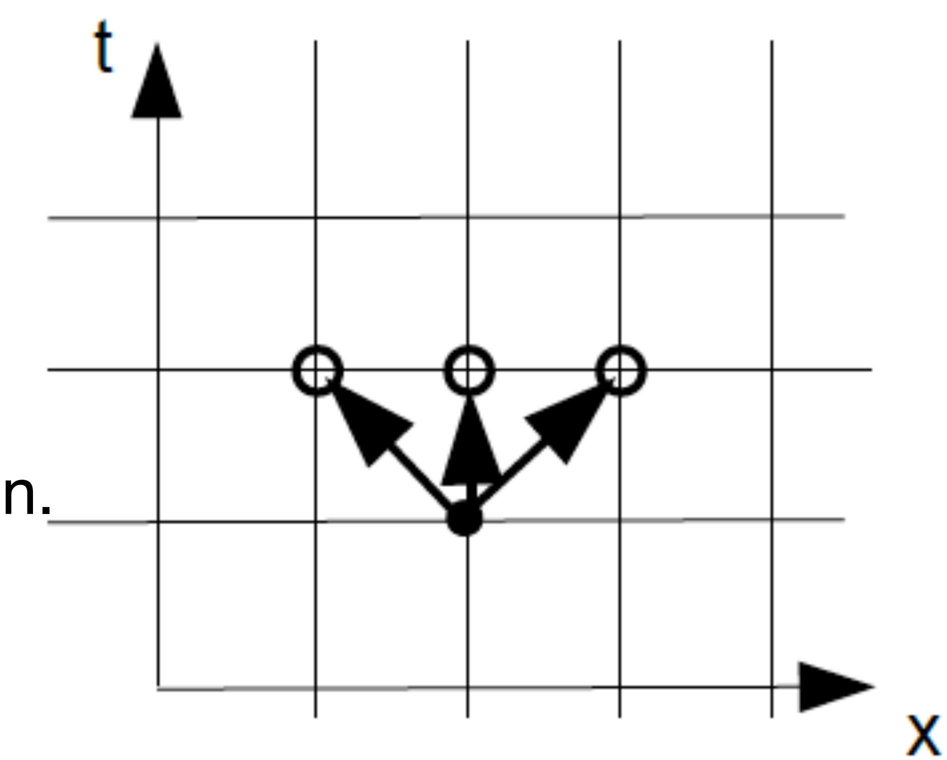
$$\frac{C_{m,n+1}-C_{m,n}}{\Delta t}=K\frac{C_{m+1,n+1}-2C_{m,n+1}+C_{m-1,n+1}}{\Delta x^2}$$

fully implicit (or backward in time) scheme

Can be solved by solving set of linear equations:

$$-\alpha C_{m-1,n+1}+(1+2\alpha)C_{m,n+1}-\alpha C_{m+1,n+1}=C_{m,n}$$

with $\alpha=K\Delta t/(\Delta x)^2$



Tridiagonal system can be solved by matrix inversion.
Unconditionally stable for any Δt !
Only first order accurate: numerical diffusion (not a big problem here since we're solving a diffusion equation, but for advection equation it is an issue).

Numerics Summary

- Schemes for advection equation:
  - FTCS: unstable
  - CTCS (leap-frog): stable if CFL criterion is met, second order accurate (low numerical diffusion), not positive definite, numerical mode
  - Upwind: stable if CFL criterion is met, only first order accurate (numerical diffusion), positive definite
- Diffusion equation:
  - FTCS: stable if criterion analogous to CFL is met
  - Fully implicit scheme (backward in time): can be solved by matrix inversion; unconditionally stable

Radiative-Convective Models

Manabe and Strickler (1964)

Radiative Properties of Atmospheric Gases

- Transmission
- Scattering (change in direction):
  - e.g. water droplets in clouds
- Absorption (photon is absorbed raising the internal energy of a molecule)
  - e.g. H_2O, CO_2
- Emission (photon is emitted lowering the internal energy of a molecule)

Absorption and Emission of Atmospheric Gases

- Absorption and emission of photons can only occur at those discrete frequencies that correspond to the quantized energy levels of a molecule => atmospheric gases are not blackbodies
- Rotational Energy (dipole needed)
- Vibrational Energy

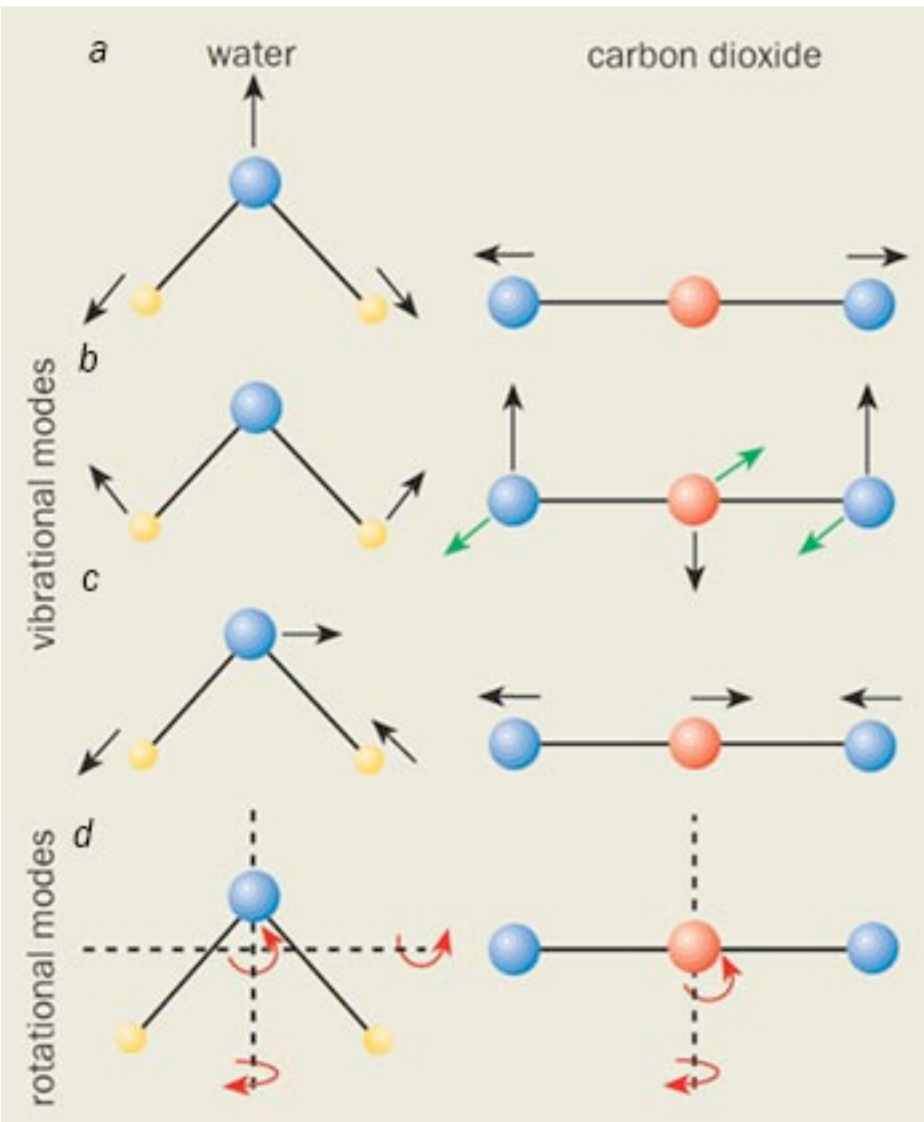
CO_2

No Permanent Dipole Moment

but

Vibrational Modes

(b) and (c) below can induce temporary dipole leading to vibration-rotation absorption bands



Hartmann, 1994

No Dipole Moment

Dipole Moment: 15 um (important because near peak of terrestrial emission spectrum)

Dipole Moment: 4.3 um

Broadening of sharp spectral lines due to

- natural broadening (finite time of absorption = energy uncertainty)
- pressure broadening (due to collisions with other molecules during absorption/emission)
- doppler broadening (due to movement of molecule relative to photon)

- Lifetime of high energy states are long (10^{-1} - 10^{-3} s) compared to time between collisions (10^{-7} s)

- Energy is redistributed increasing temperature

Pierrehumbert (2011) Physics Today

Planck's Law

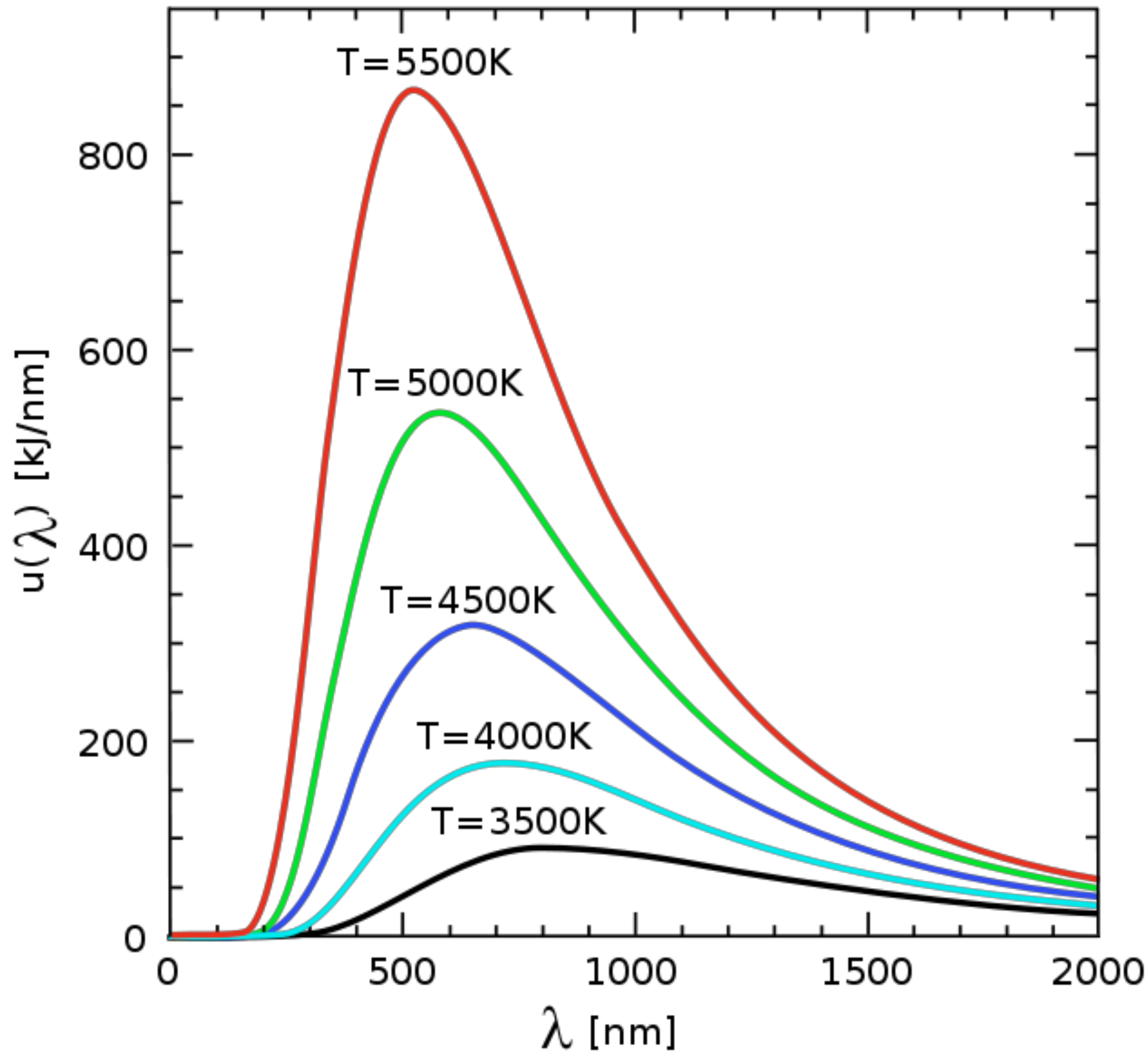
$$I(\nu,T) = \frac{2h\nu^3}{c^2}\frac{1}{e^{\frac{h\nu}{kT}}-1}.$$

frequency

$$I'(\lambda,T) = \frac{2hc^2}{\lambda^5}\frac{1}{e^{\frac{hc}{\lambda kT}}-1}.$$

wavelength

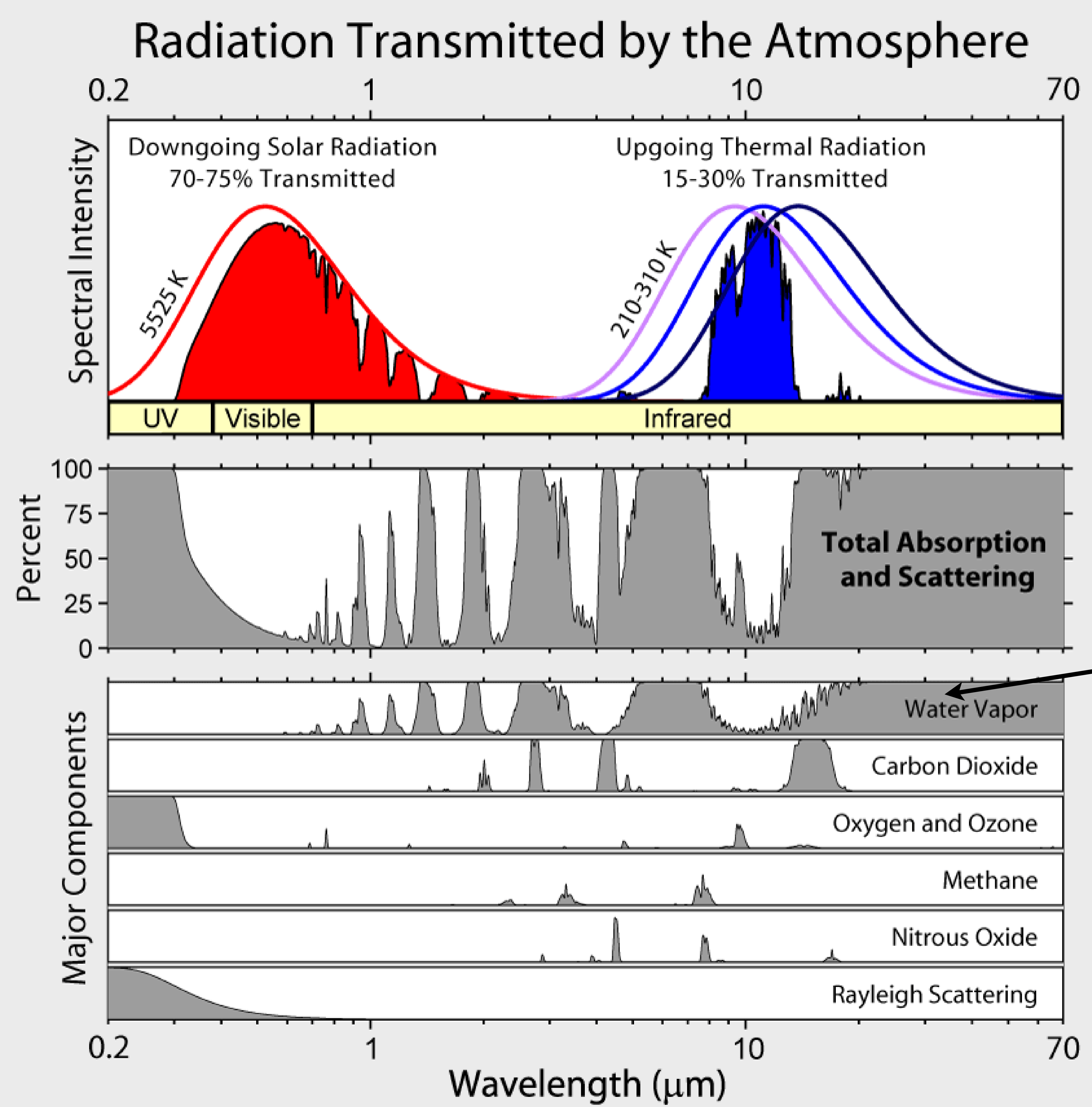
Black body spectrum (spectral energy density **inside** a blackbody cavity). Indicated units are correctly kJ/m^4, or nJ/cm^3/µm. Scale by c/4π to achieve $I'(\lambda,T)$.



Integration over all frequencies gives Stephan Boltzmann law ~T^4

http://en.wikipedia.org/wiki/Planck%27s_law

Line-by-Line Model



Rotation

5.1 Radiative-Convective Models

The simple energy balance model of Figure (2.4) can be modified and extended to include more layers as shown in Figure (5.1). Now we assume the atmosphere is transparent to shortwave radiation and atmospheric layers 1 and 2 are completely opaque for longwave radiation. Further assuming that the atmospheric layers are perfect black bodies, the energy balance at the top of the atmosphere becomes

$$S(1-\alpha)=\sigma T_1^4 \quad . \tag{5.1}$$

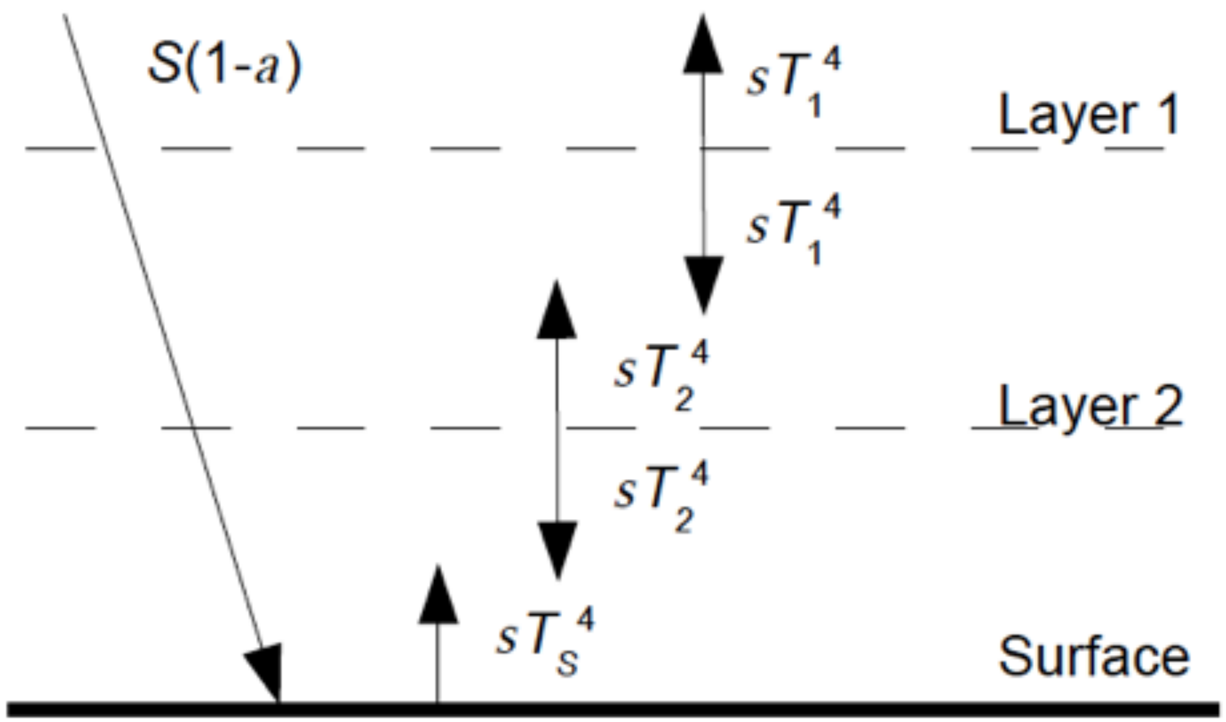


Figure 5.1: Simple two-layer radiative equilibrium model.

For layer 1 the energy balance is

$$\sigma T_2^4=2\sigma T_1^4=2S(1-\alpha) \quad , \tag{5.2}$$

for layer 2 we have

$$\sigma T_1^4+\sigma T_s^4=2\sigma T_2^4=4S(1-\alpha) \quad ,$$

and at the surface

$$S(1-\alpha)+\sigma T_2^4=\sigma T_s^4 \quad . \tag{5.3}$$

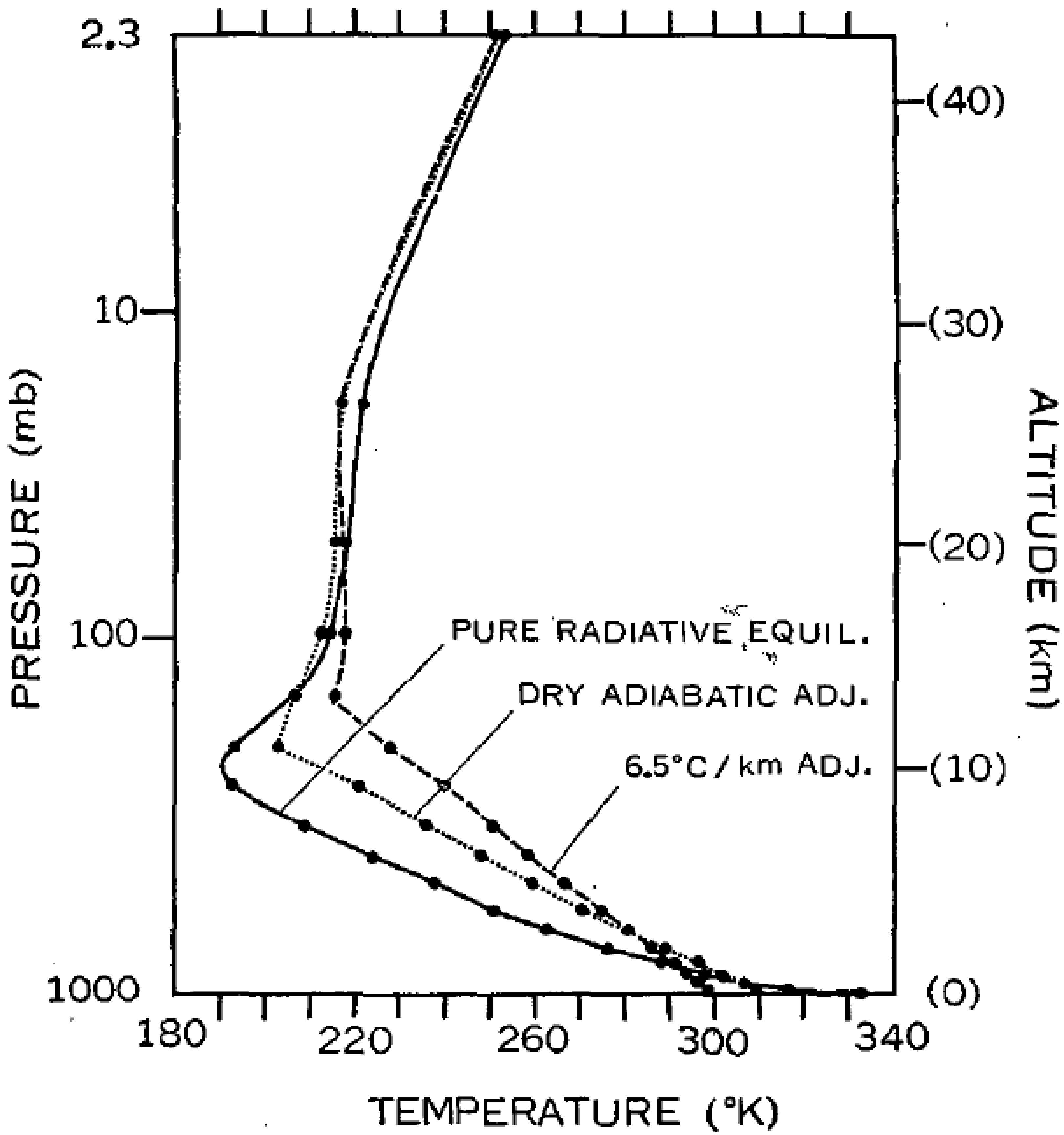
We notice that the temperatures increase downward. Solving these equations for the surface temperature we get

$$T_s^4=3S\frac{1-\alpha}{\sigma}=3T_1^4 \quad . \tag{5.4}$$

Extending the model to n layers we see that the surface temperature is equilibrium will be always be larger than the temperature of the upper layer.

$$T_s=\sqrt[4]{n+1}\,T_1 \quad . \tag{5.5}$$

For 2 layers the surface temperature is T_s = 335 K and the atmospheric temperatures are T_2 = 303 K and T_1 = 255 K. We see that the surface temperature is much too warm compared to the observed surface temperature of the Earth of about 288 K. What could be the reason for this discrepancy? First, we know that the real



Radiative transfer models resolve frequency bands.

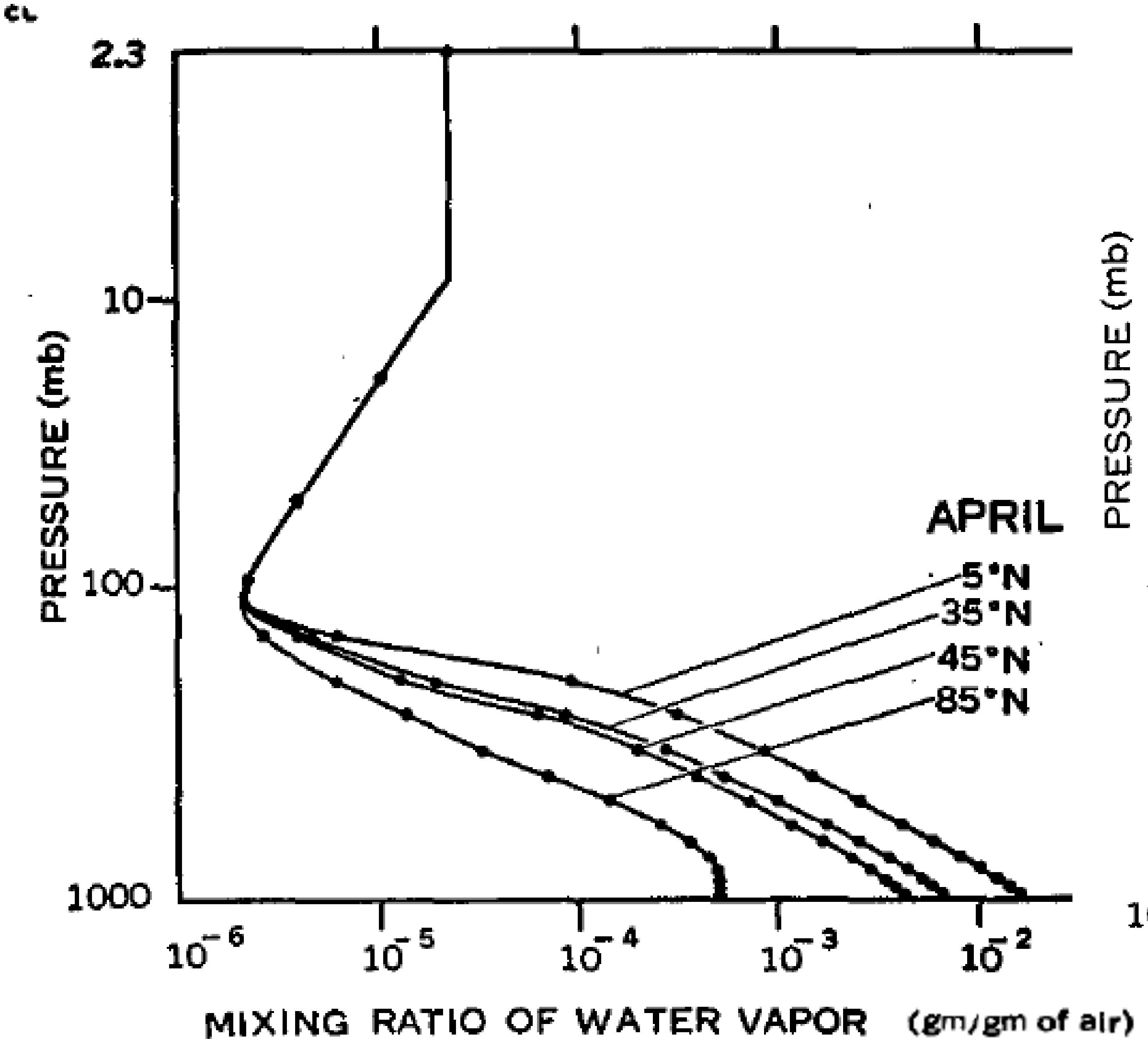
Radiative fluxes only (pure radiative equil.) gives very high surface temps.

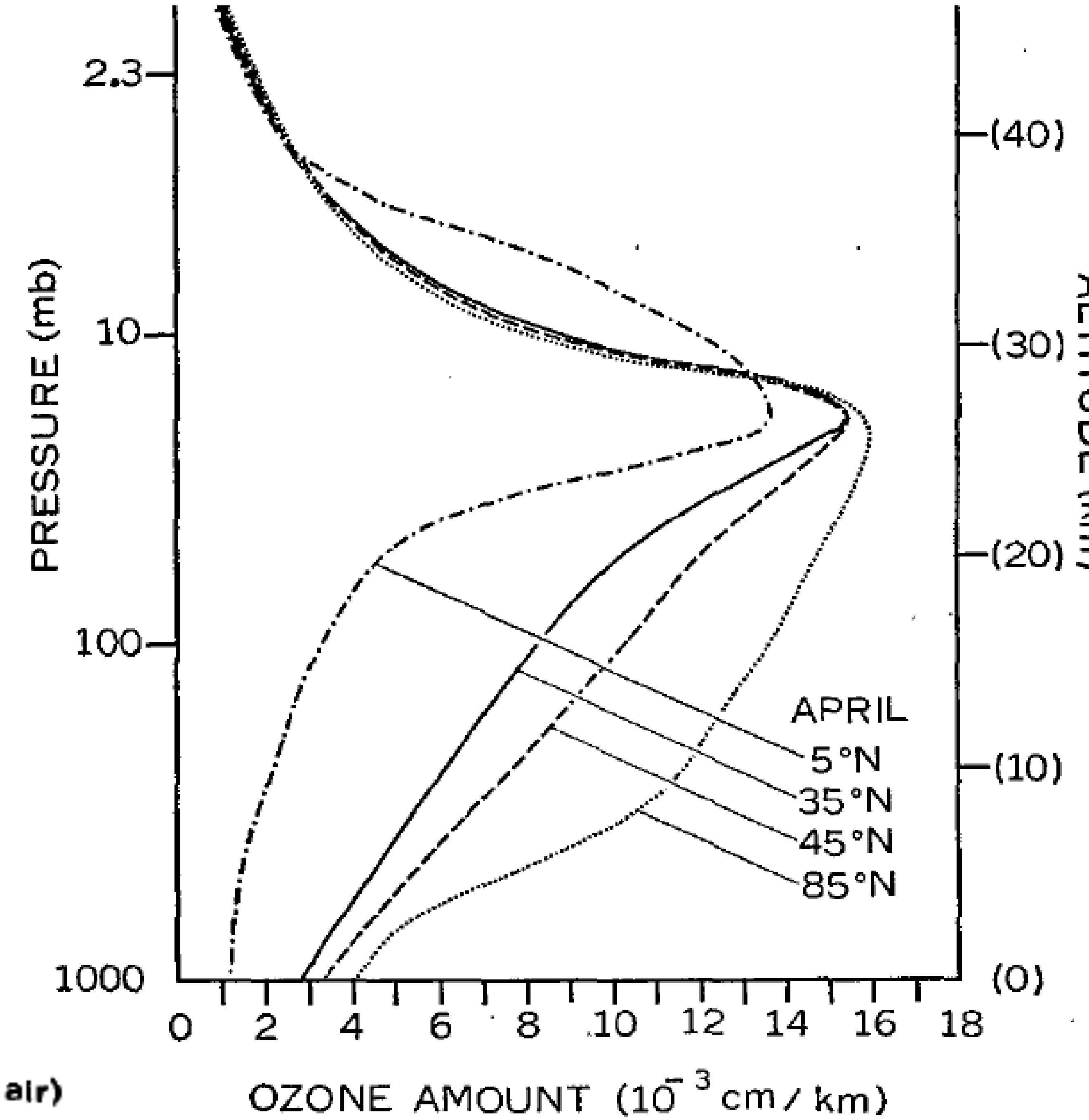
This leads to low densities and instability, which will cause convection.

Convective overturning, in the presence of liquid water at the surface (ocean), will lead to moist adiabatic lapse rate.

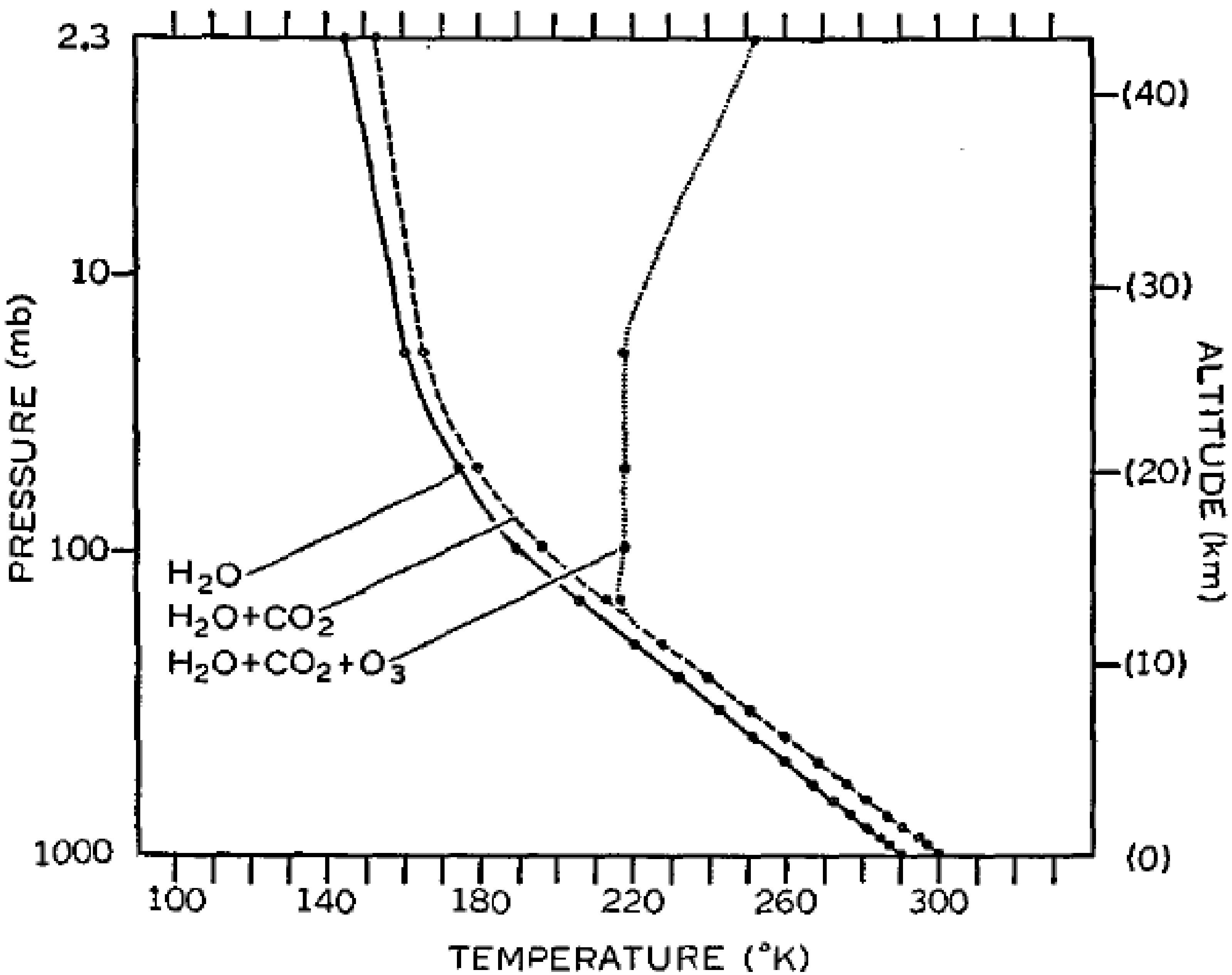
Manabe and Strickler (1964)

CO_2=290 ppmv constant with height





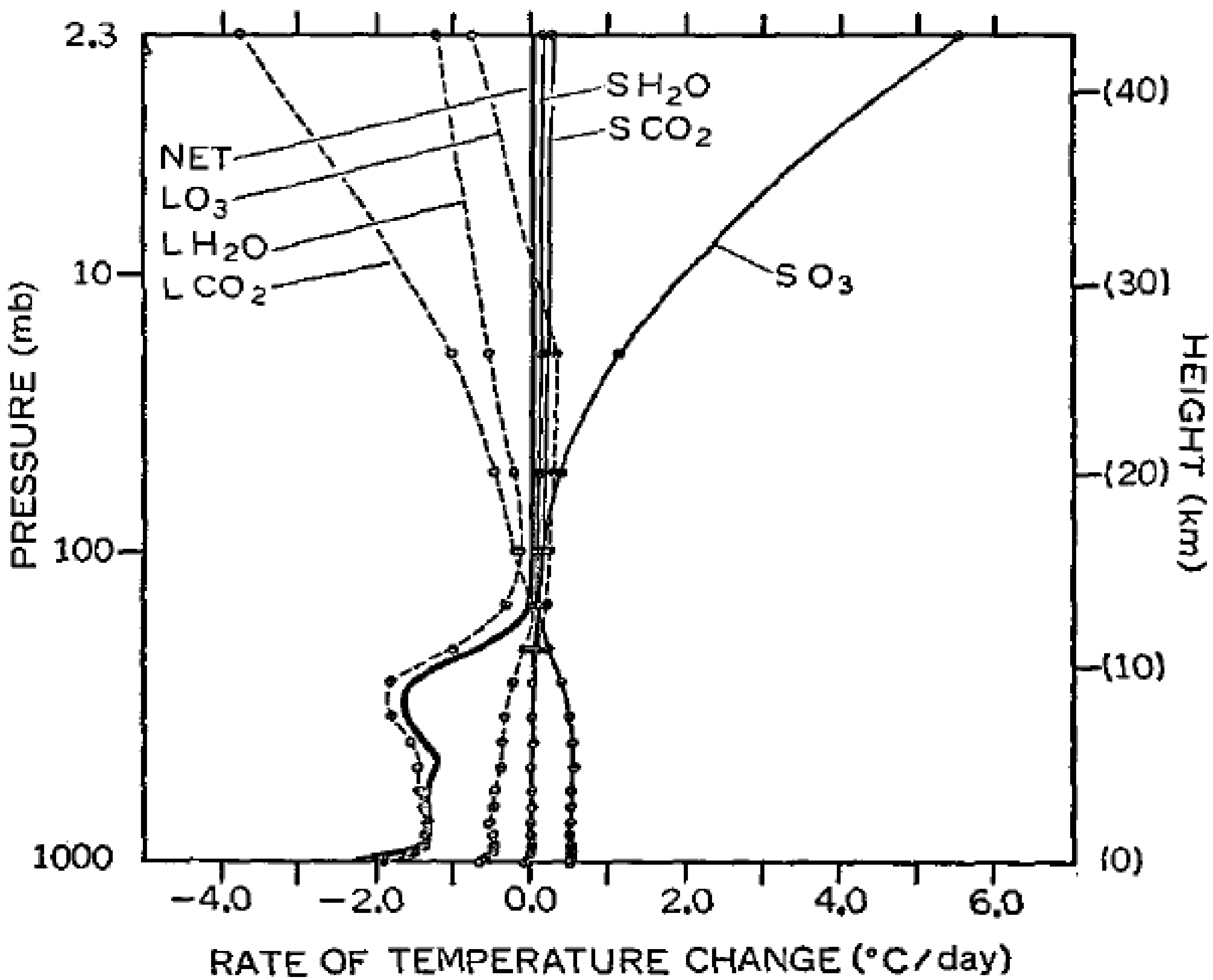
Manabe and Strickler (1964)



Ozone leads to warming in stratosphere, but not at surface.

CO_2 leads to surface warming of ~10 K.

Manabe and Strickler (1964)



Ozone absorbs sunlight in stratosphere, which leads to warming.

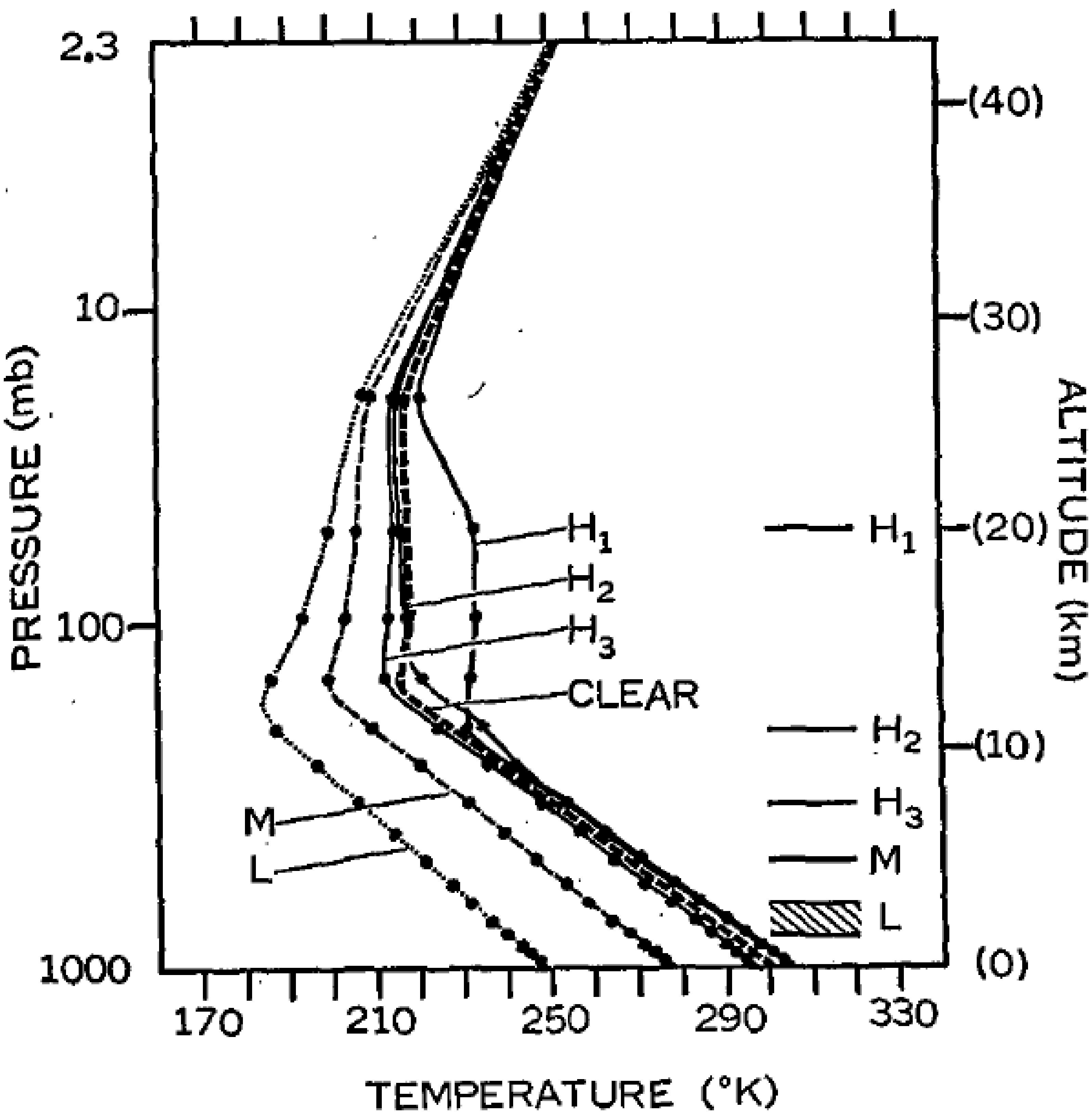
Stratosphere is cooled mainly by long wave radiation due to CO_2.

Long wave radiation by H_2O and CO_2 cool the troposphere.

Convective fluxes heat the troposphere by transporting heat from the ground upwards.

Manabe and Strickler (1964)

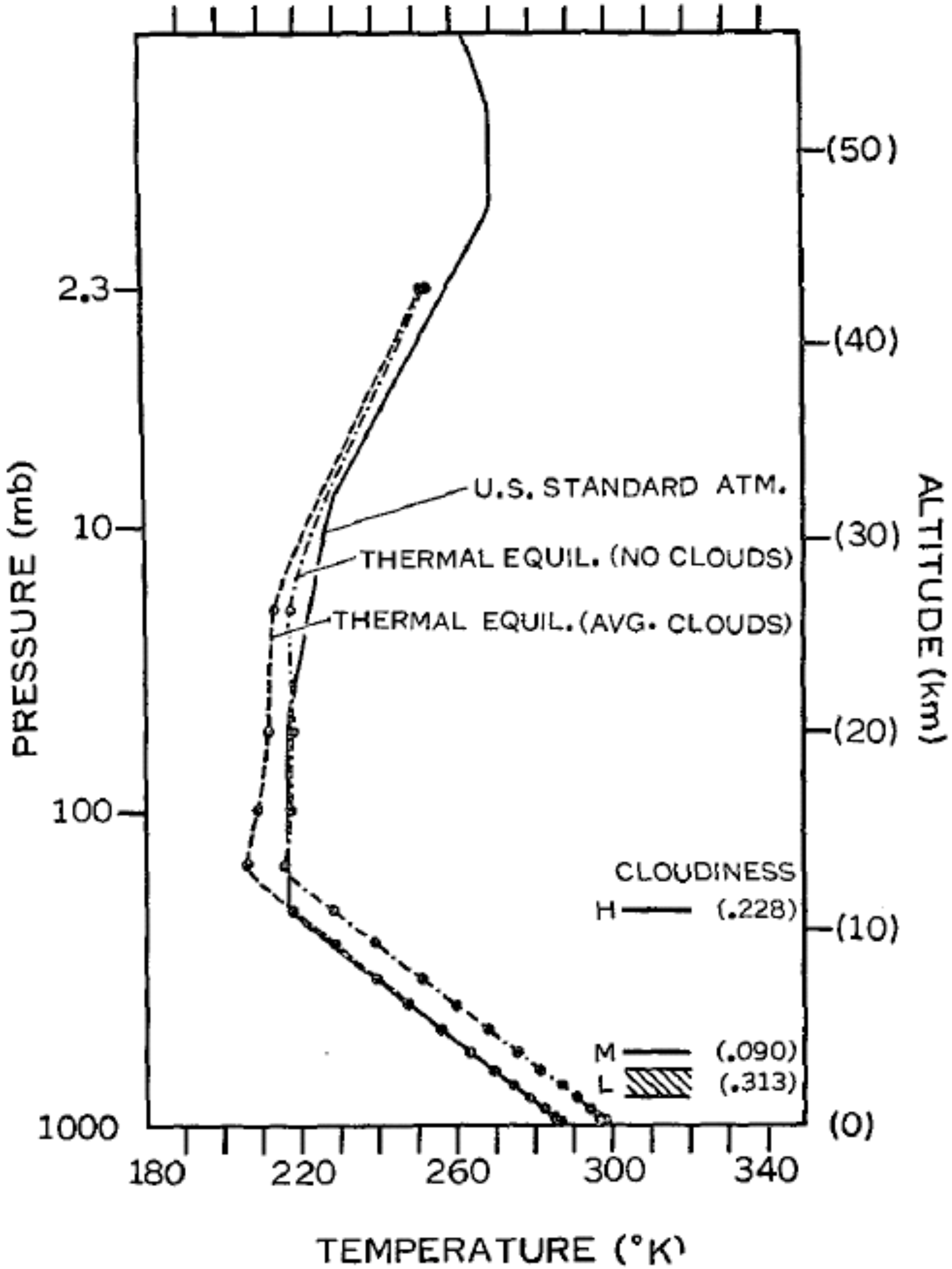
Effect of Clouds



Low and mid level clouds cool the surface and troposphere.

High clouds can heat the surface.

Manabe and Strickler (1964)



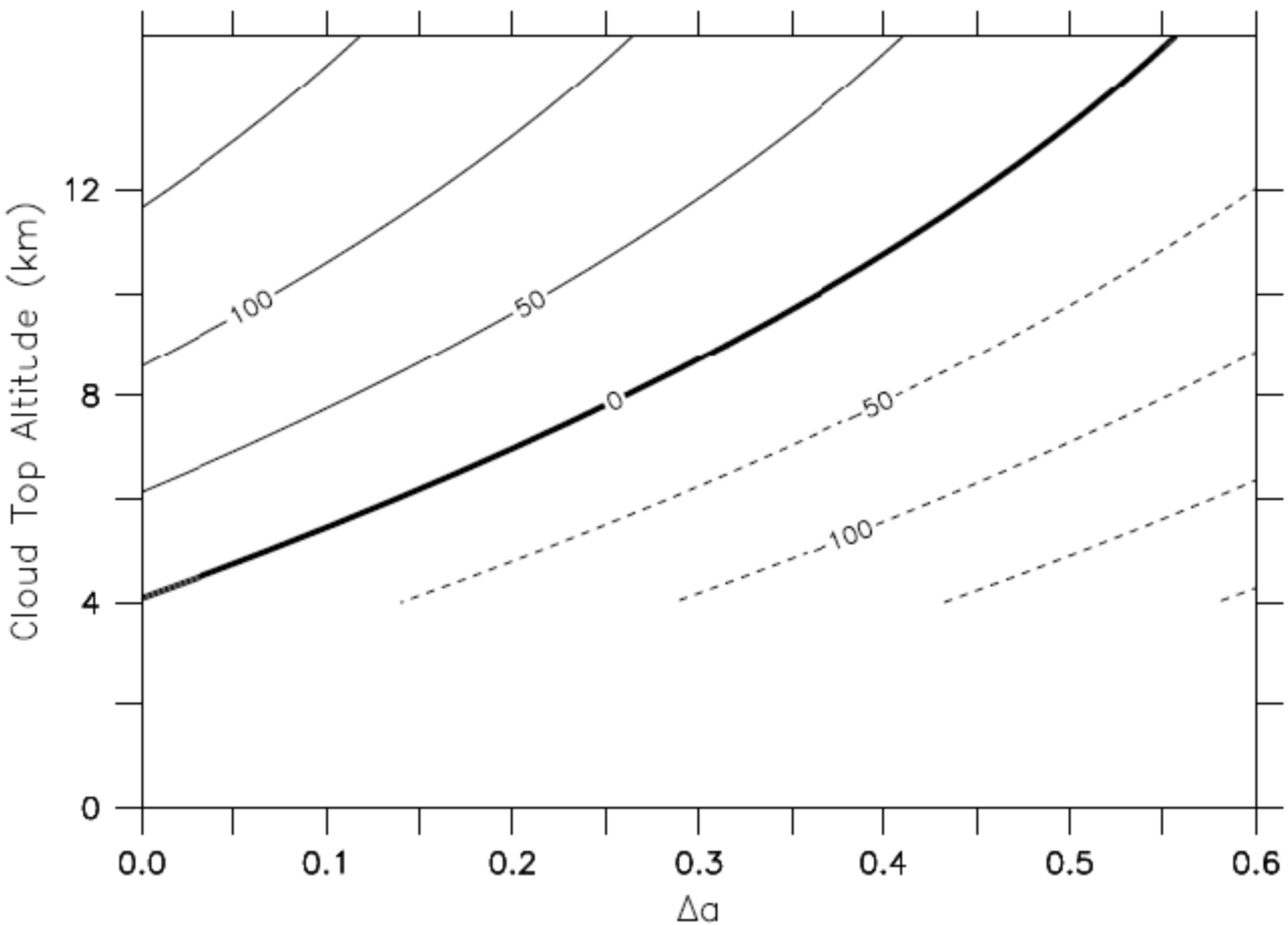
Average effect of clouds is to cool the surface and troposphere.

$$\Delta F_{SW} = S(1-a_{cloud}) - S(1-a_{clear}) = -S(a_{cloud}-a_{clear}) = -S\Delta a \leqslant 0$$

$$\Delta F_{LW} = \sigma T_{ct}^4 - F_{LWclear} \qquad\qquad F_{LW} = \sigma T_{ct}^4$$

$$\Delta R_{TOA} = \Delta F_{SW} - \Delta F_{LW} = -S\Delta\alpha + F_{LWclear} - \sigma T_{ct}^4$$

$$T_{ct} = T_s - \Gamma z_{ct}$$



Cloud radiative forcing ΔR_{TOA} as a function of change in albedo and cloud top altitude. Negative values are show as dashed lines. S = 342 Wm^{-2}, $F_{LWclear}$ = 265 Wm^{-2}, T_s = 288 K, Γ = 6.5 K/km. From Hartmann (1994).

RCNs Conclusions

- Radiative transfer heats the surface
- Convection leads to upward heat transport causing temperature in the troposphere to follow the moist adiabatic lapse rate
- Absorption of shortwave radiation by ozone in the upper atmosphere leads to the temperature increase in the stratosphere