ATS 421/521

Climate Modeling

Spring 2013

Lecture 11

- General Circulation Models
    - The Primitive Equations
    - Surface Processes
    - Parameterizations
    - Grids and Resolution

Primitive Equations

Momentum Conservation:

Newton's second law

Velocity

$$\frac{d\vec{u}}{dt} = \sum \vec{F}$$

Total (Lagrangian) derivative

Force per mass

Navier-Stokes Equations:

$$\frac{d\vec{u}}{dt} = \underbrace{\frac{\partial \vec{u}}{\partial t} + \vec{u}\vec{\nabla}\vec{u}}_{inertia} = \underbrace{\frac{-1}{\rho}\vec{\nabla} p}_{pressure\ gradient} + \underbrace{\mu \nabla^2 \vec{u}}_{viscosity} + \underbrace{f}_{other\ body\ forces}$$

Total derivative: $$\frac{d}{dt} = \frac{\partial}{\partial t} + \vec{u}\vec{\nabla}$$

On rotating sphere Coriolis and centripetal forces:

$$\frac{du}{dt} - \left(f + u\frac{\tan\phi}{R}\right)v = -\frac{1}{\rho R \cos\phi}\frac{\partial p}{\partial \lambda} + F_u \qquad (5.17)$$

zonal velocity

$$\frac{dv}{dt} + \left(f + u\frac{\tan\phi}{R}\right)u = -\frac{1}{\rho R}\frac{\partial p}{\partial \phi} + F_v \qquad (5.18)$$

meridional velocity

$$g = -\frac{1}{\rho}\frac{\partial p}{\partial z} \quad , \qquad (5.19)$$

vertical velocity

$$f = 2\Omega \sin\phi \qquad \vec{u} = (u, v, w)$$

Mass Conservation:
Continuity Equation

$$\frac{\partial \rho}{\partial t} = -\vec{\nabla}(\rho \vec{u}) = 0 \rightarrow \vec{\nabla} \vec{u} = 0 \quad . \qquad (5.20)$$

Incompressible Fluid

Energy Conservation:
First Law of Thermodynamics

$$c_v \frac{dT}{dt} = -p \frac{d}{dt}\left(\frac{1}{\rho}\right) + F_T \qquad (5.21)$$

Adiabatic Expansion/ Compression

Diabatic Processes (e.g. radiation, latent heat release)

Potential Temperature = Temperature of fluid parcel if adiabatically brought to surface

$$\Theta = T\left(\frac{p_0}{p}\right)^{\kappa} \rightarrow \frac{d\Theta}{dt} = F_{\Theta}$$

$$\kappa = R'/c_p$$

gas constant dry air

heat capacity const. pressure

Conservation of Water Vapor:

(In Ocean analogous equation for salinity)

$$\frac{dq}{dt} = F_q \qquad (5.22)$$

If RH > 80% then precipitation

Equation of State:

Air: $$p = \rho R' T \qquad (5.23)$$

Sea Water: $$\rho = \rho(p, T, S) \qquad (5.24)$$

Primitive Equations = 5.17 - 5.23 (5.24)

7 equations with 7 unknowns (u,v,w,ρ,p,T,q)

Surface Processes

Empirical Relations (Bulk Formulae)

Momentum (wind stress):

$$F_u = \rho C_m |\vec{u}| u$$

$$F_v = \rho C_m |\vec{u}| v$$

Sensible heat flux:

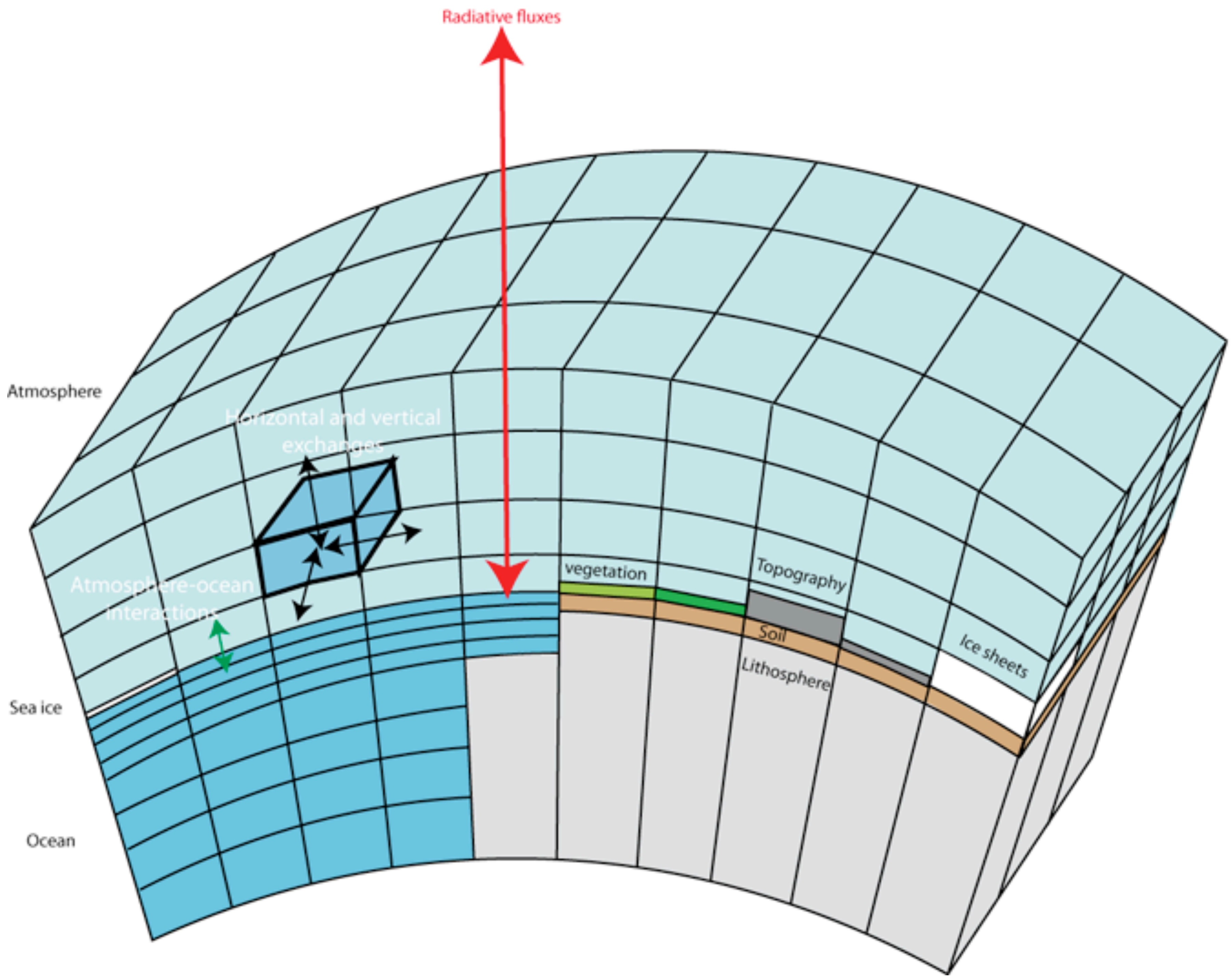
$$F_\Theta = \rho c_p C_\Theta |\vec{u}| (\Theta_s - \Theta_a)$$

Moisture flux:

$$F_q = \rho C_q |\vec{u}| (q_s - q_a)$$

Cs are transfer/drag coefficients

Equations are solved on a three-dimensional grid covering the Earth. In each grid box the primitive equations as well as other equations are solved. Fluxes between neighboring boxes are calculated and used to update the tendencies for the next time step.



Typically ocean and atmosphere models have different grid box sizes (resolution), which requires a “coupler” to interpolate/average between the two grids.

Parameterizations

required due to large grid box sizes

Reynolds Decomposition:

$$u = \overline{u} + u' \quad \text{and} \quad C = \overline{C} + C' \quad \text{, with} \quad \overline{C'} = 0$$

grid box mean

deviation from mean

$$\overline{uC} = \overline{(\overline{u} + u')(\overline{C} + C')} = \overline{\overline{u}\,\overline{C}} + \underbrace{\overline{\overline{u}C'}}_{0} + \underbrace{\overline{u'\overline{C}}}_{0} + \overline{u'C'} = \overline{u}\,\overline{C} + \overline{u'C'}$$

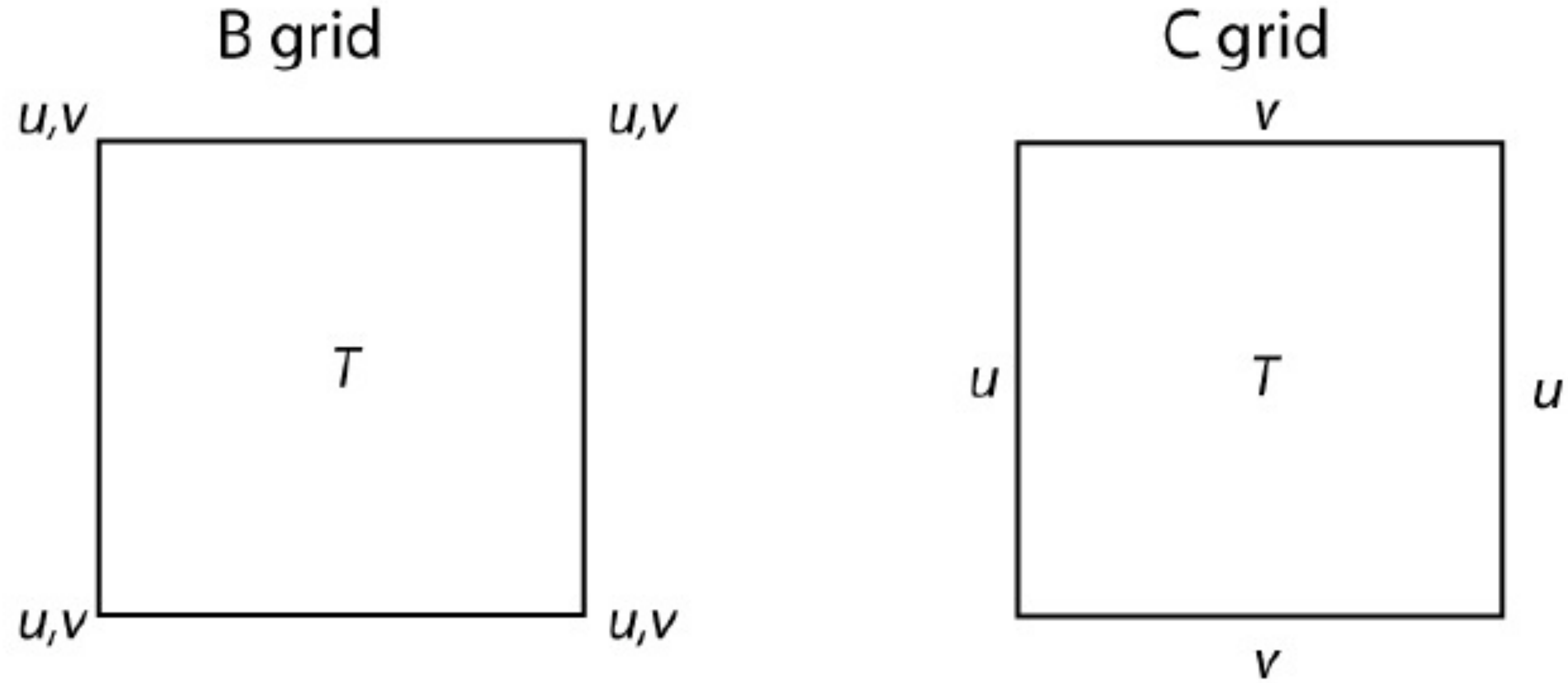
mean flow

eddy fluxes

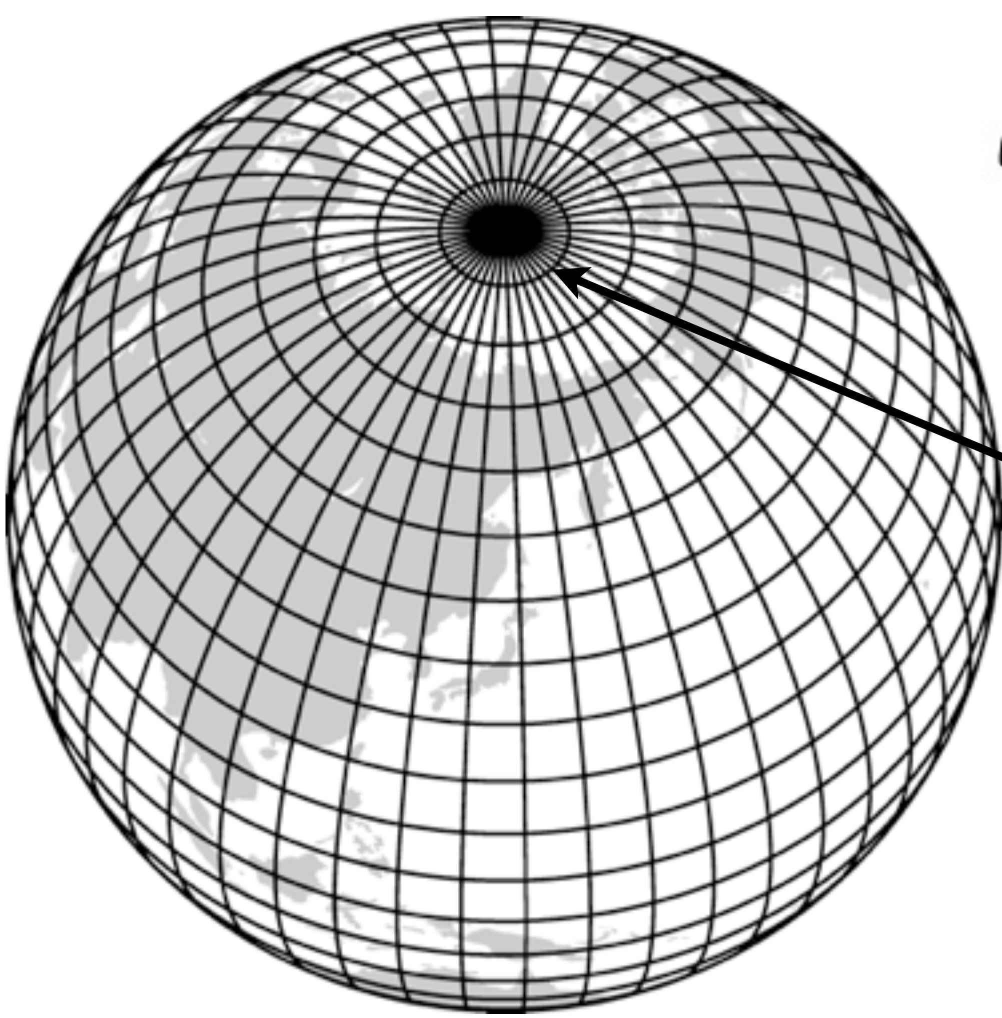
Sub-grid scale eddy fluxes have to be parameterized. E.g. in ocean models they are often treated as diffusion:

$$\overline{u'C'} = -K_C \frac{\partial \overline{C}}{\partial x}$$

Finite Differences
Horizontal Staggered Grids

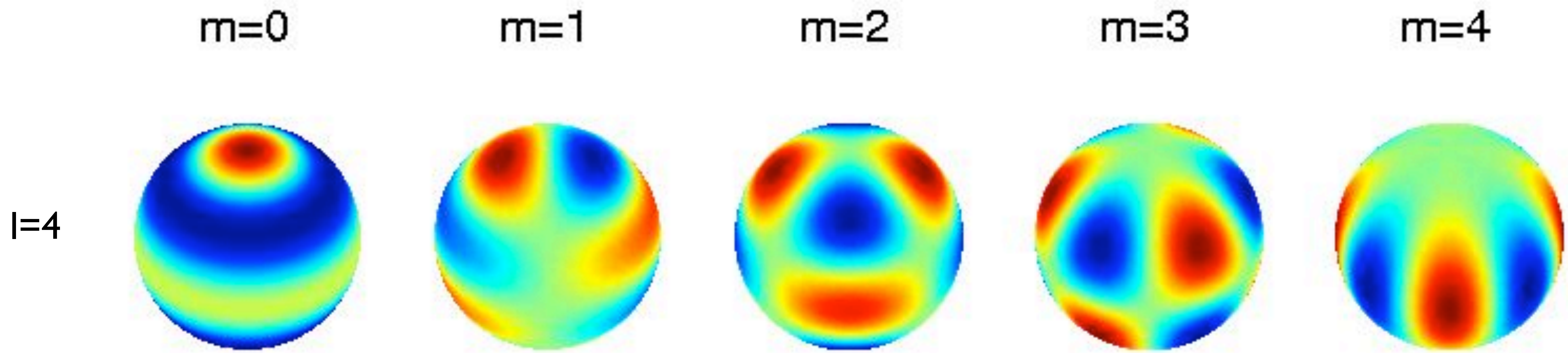


Arakawa and Lamb (1977)



Convergence of meridians at poles leads to small grid spacings Δx, which restricts the time step.

Spectral Models
Spherical Harmonics

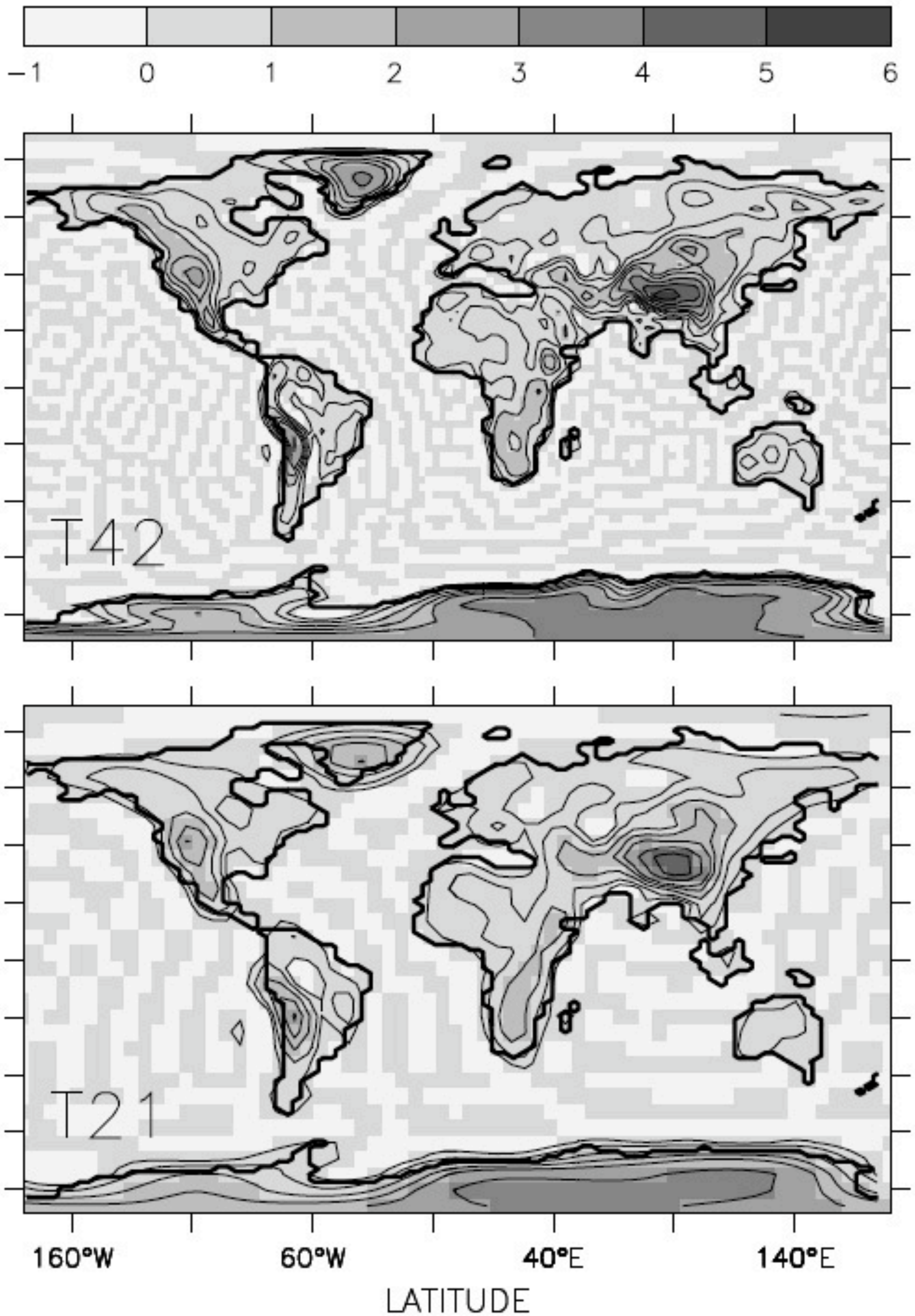


- Frequently used for atmospheric models
- No problem due to convergence of meridians at pole
- Advection is accurate
- But: not positive definite

Not used for ocean models due to zonal boundaries at continents

http://www.maths.nottingham.ac.uk/personal/pcm/sphere/sphere.html

Orography at different spectral resolutions

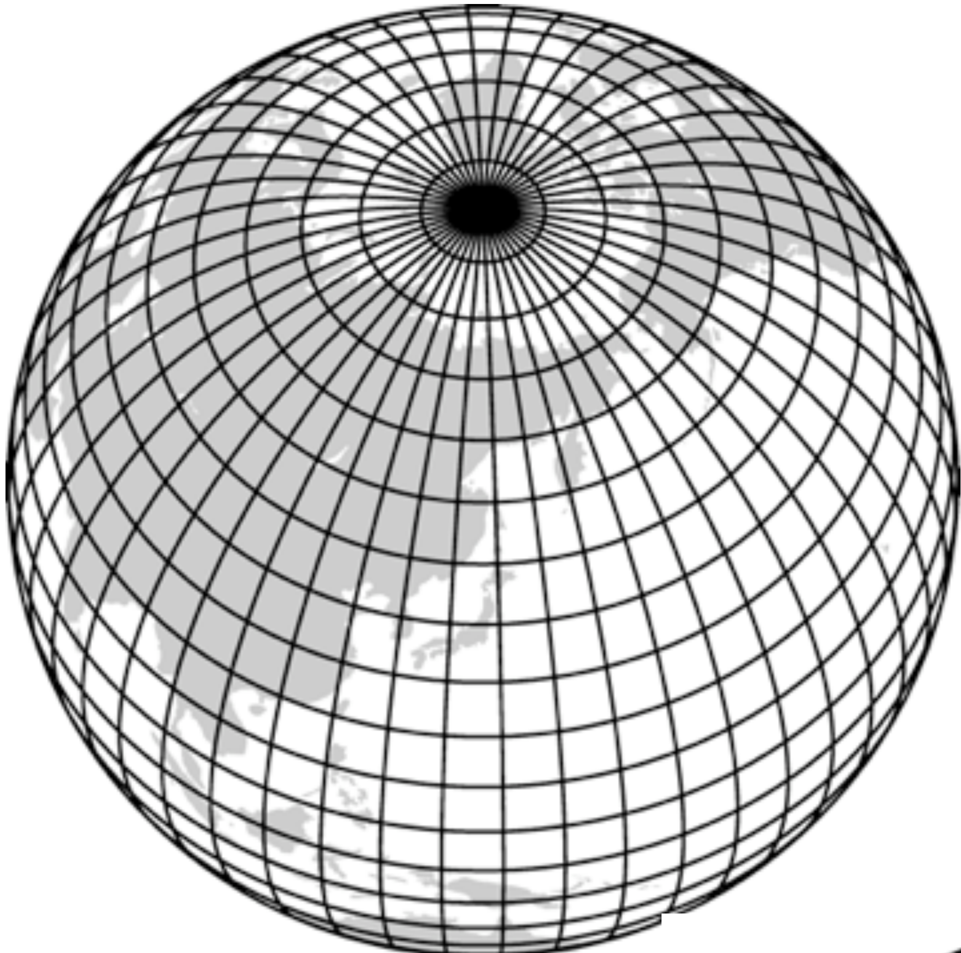


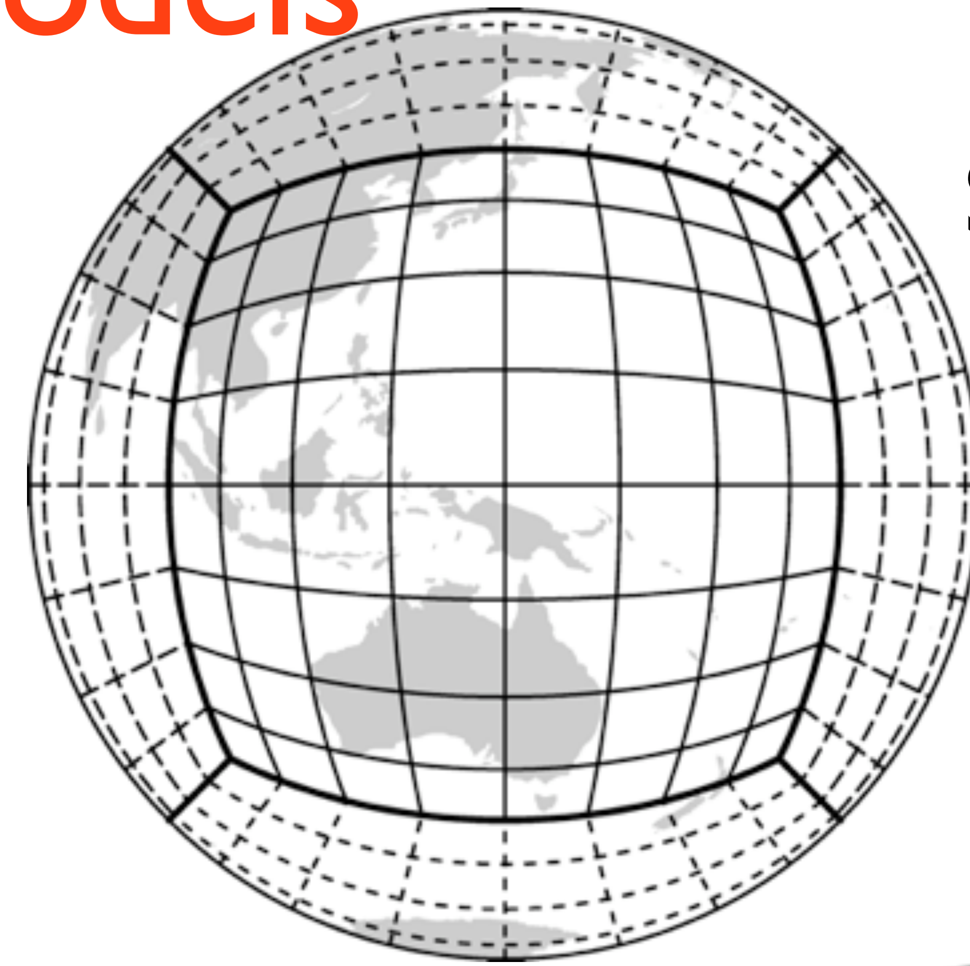
T42 ~ 250 km

T21 ~ 500 km

Schmittner et al. in press J. Climate

Finite Element Models

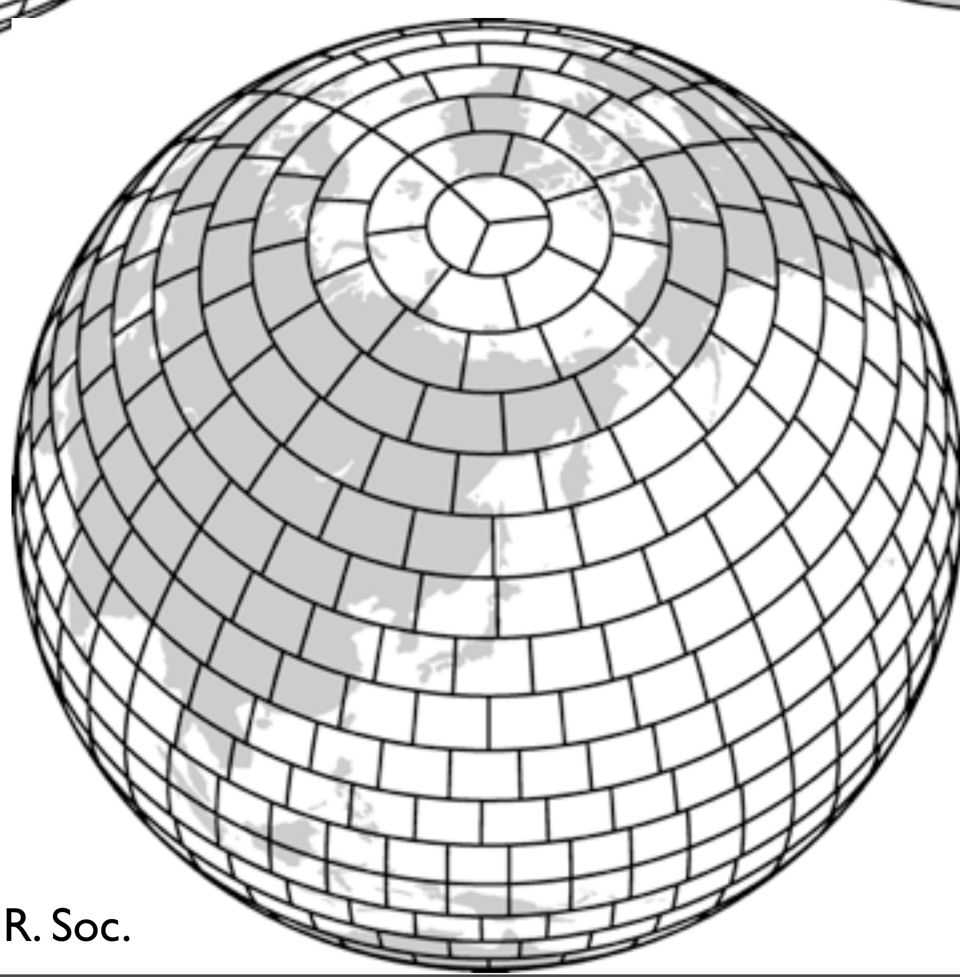


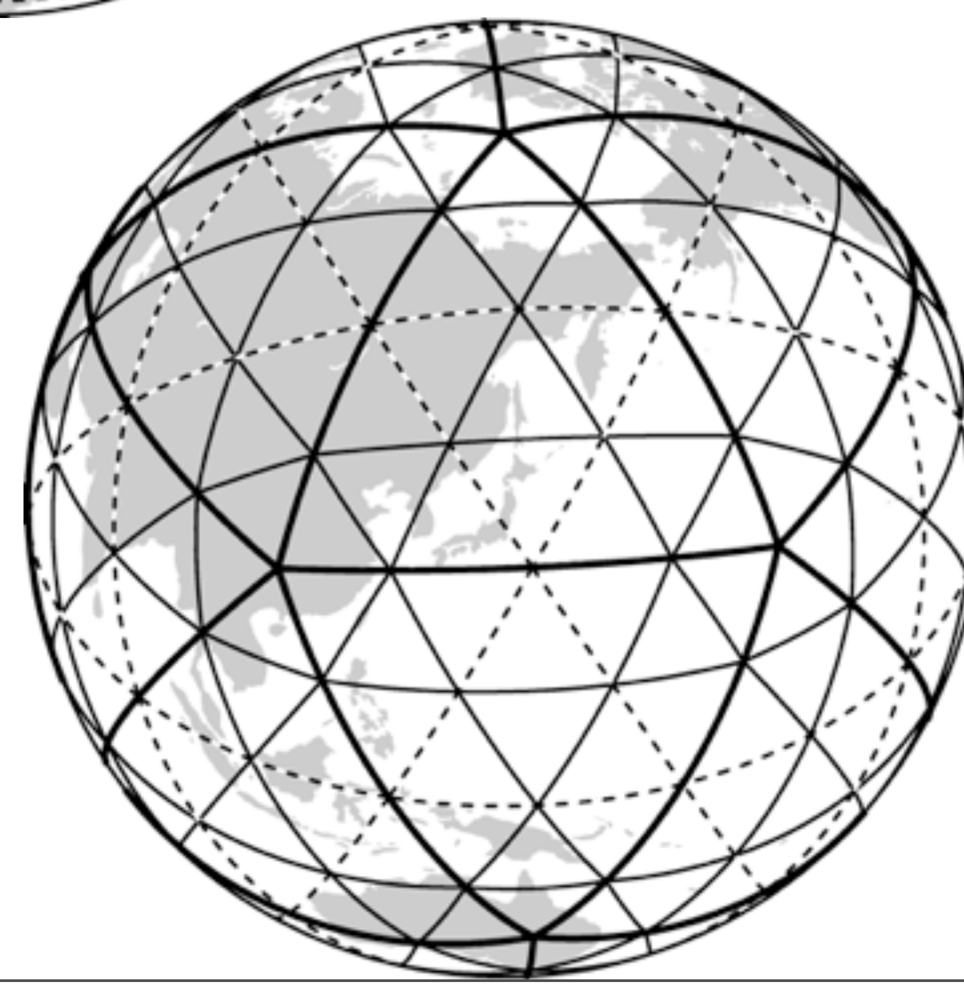


Cubed Sphere

Geodesic

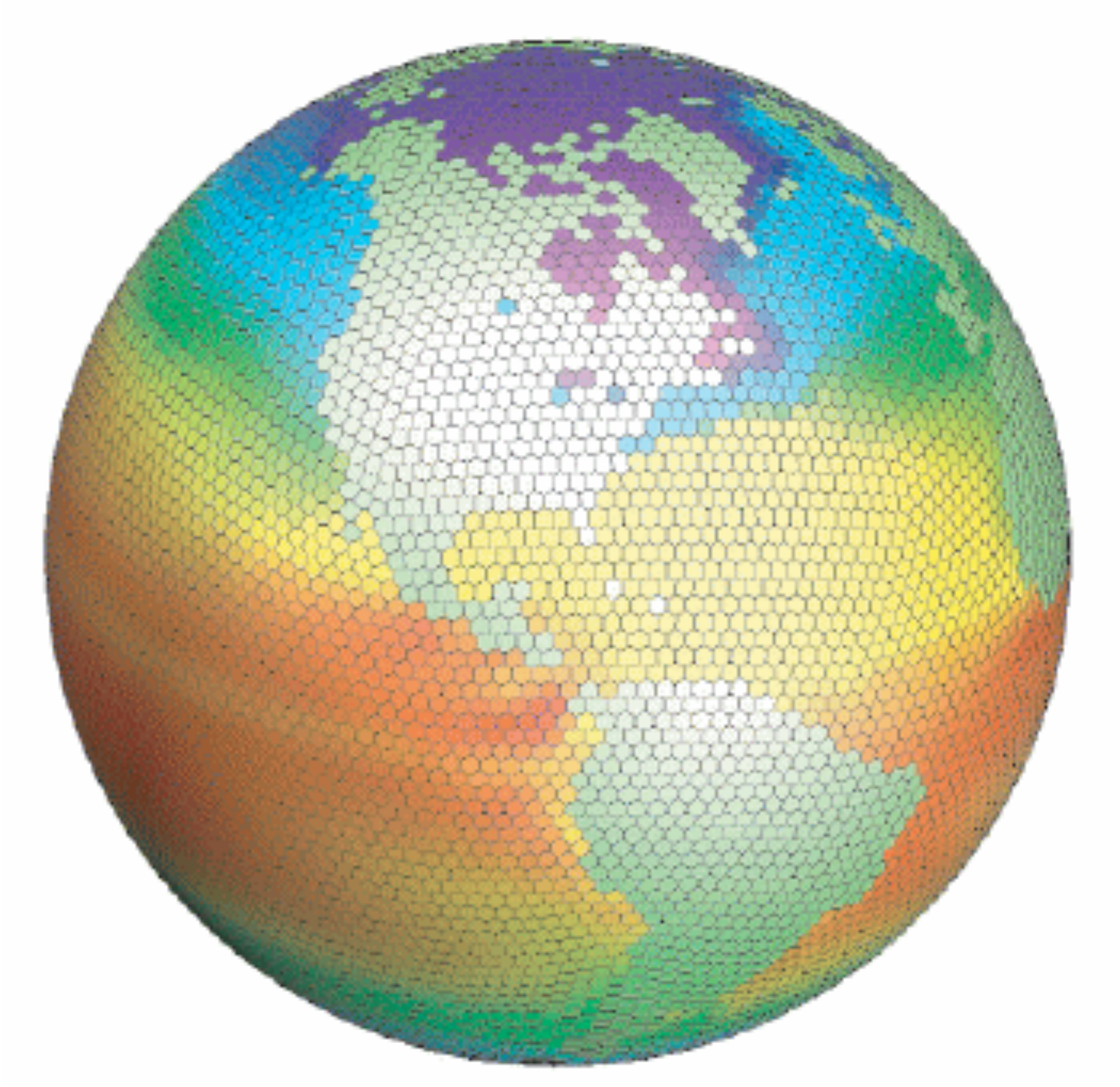
Kurihata





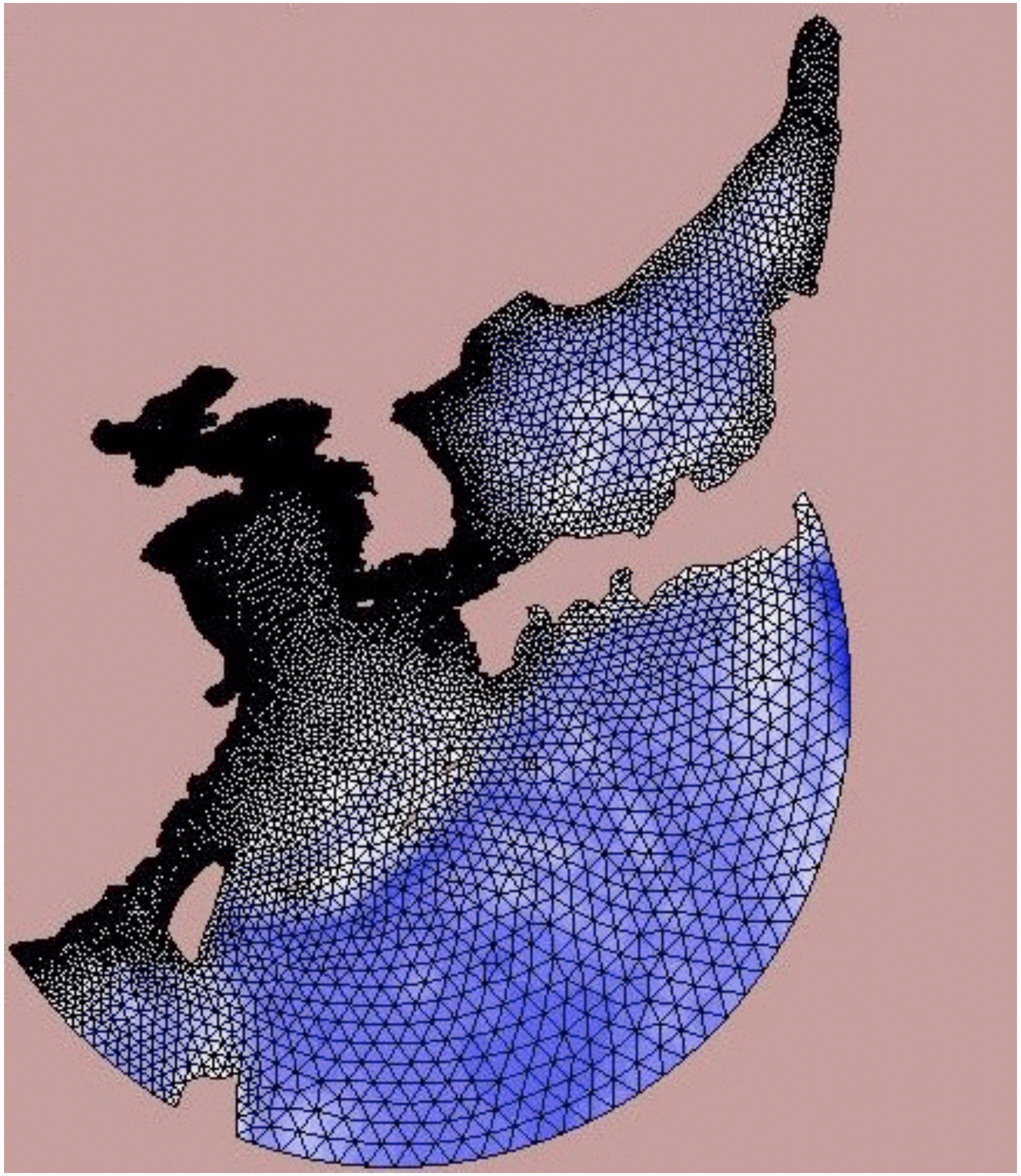
Washington et al. 2009 Phil. Trans. R. Soc.

Finite Element Models



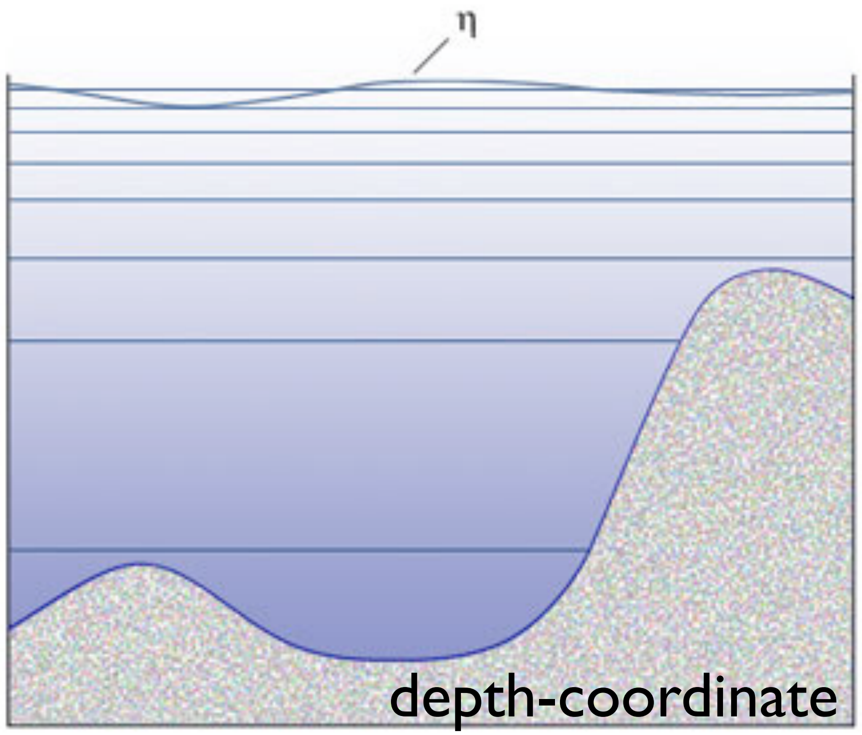
An example of a global climate model geodesic grid with a color-coded plot of the observed sea-surface temperature distribution. The continents are depicted in white. This grid has 10,242 cells, each of which is roughly 240 km across. Twelve of the cells are pentagons; the rest are hexagons.

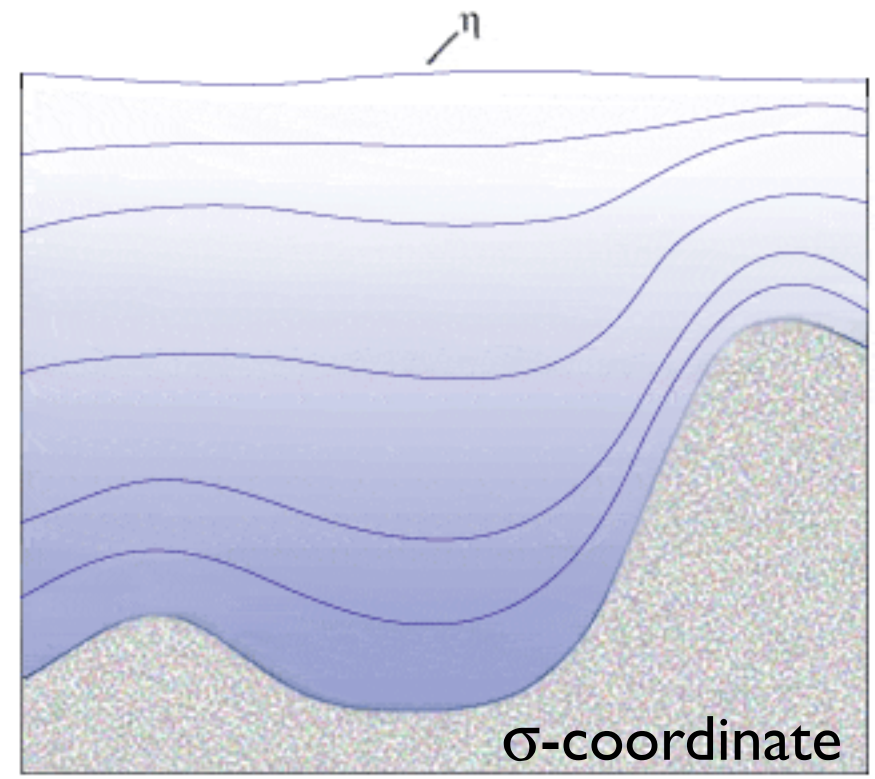
Source: Randall, D. A. et. al., *Climate modeling with spherical geodesic grids*, Computing in Science and Engineering, **4**, 5, 32-41.

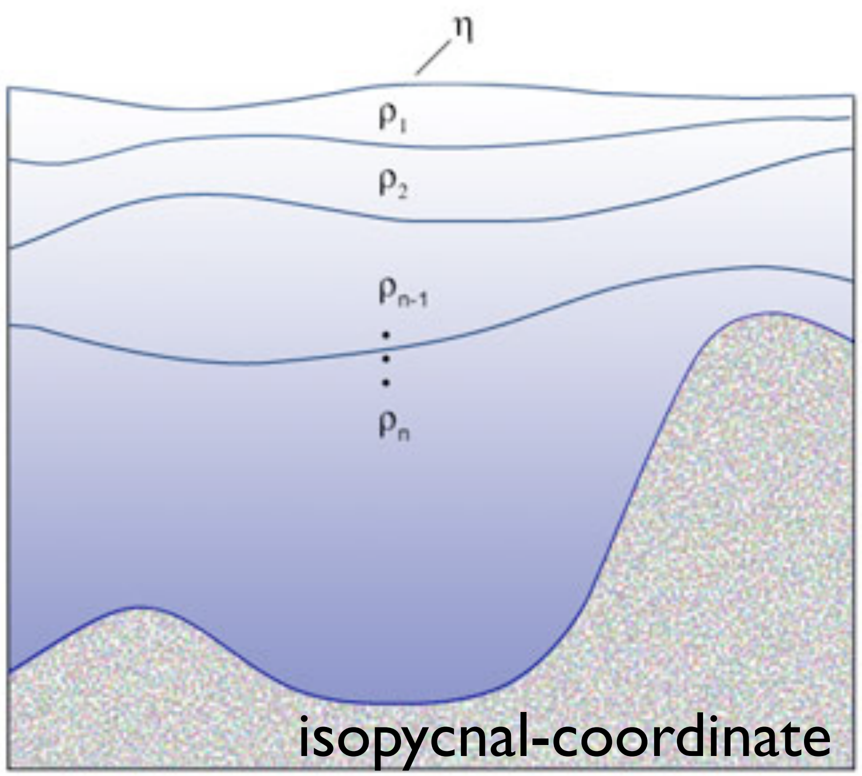


http://cnls.lanl.gov/~petersen/

Vertical Grids





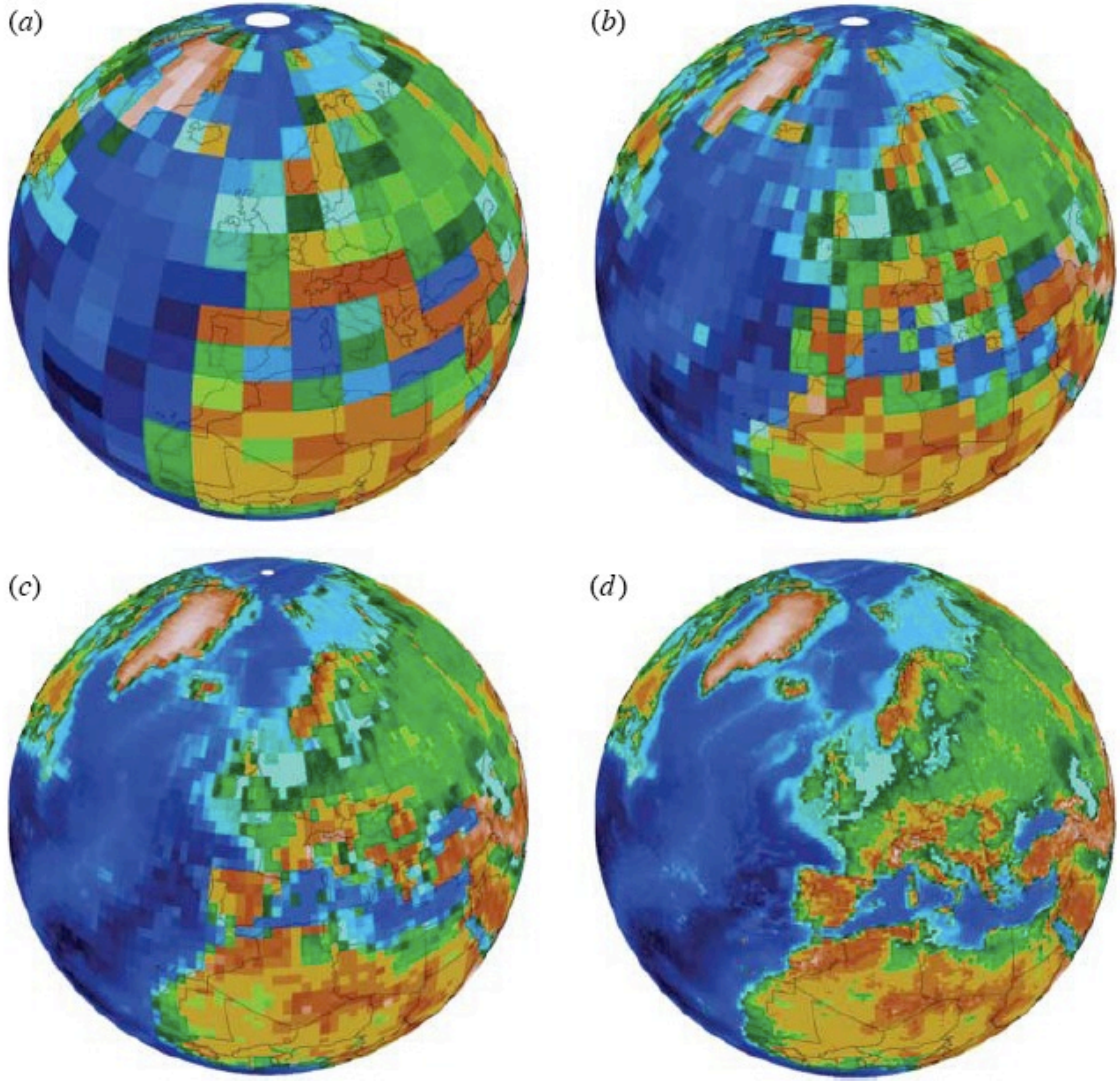


http://www.oc.nps.edu/nom/modeling/vertical_grids.html

Resolution

500 km
(T21)

300 km
(T42)



150 km
(T85)

75 km

Washington et al. 2009 Phil. Trans. R. Soc.