ATS 421/521

Climate Modeling

Spring 2015

Lecture 12

Non-Linear Dynamics, Chaos and the Lorenz (1963) Model

May 15, 2015

Comments on HW5

- Comparing data and models
- Use the same units
- It helps if you use the same color scale / contour interval
- Plot two lines on the same graph

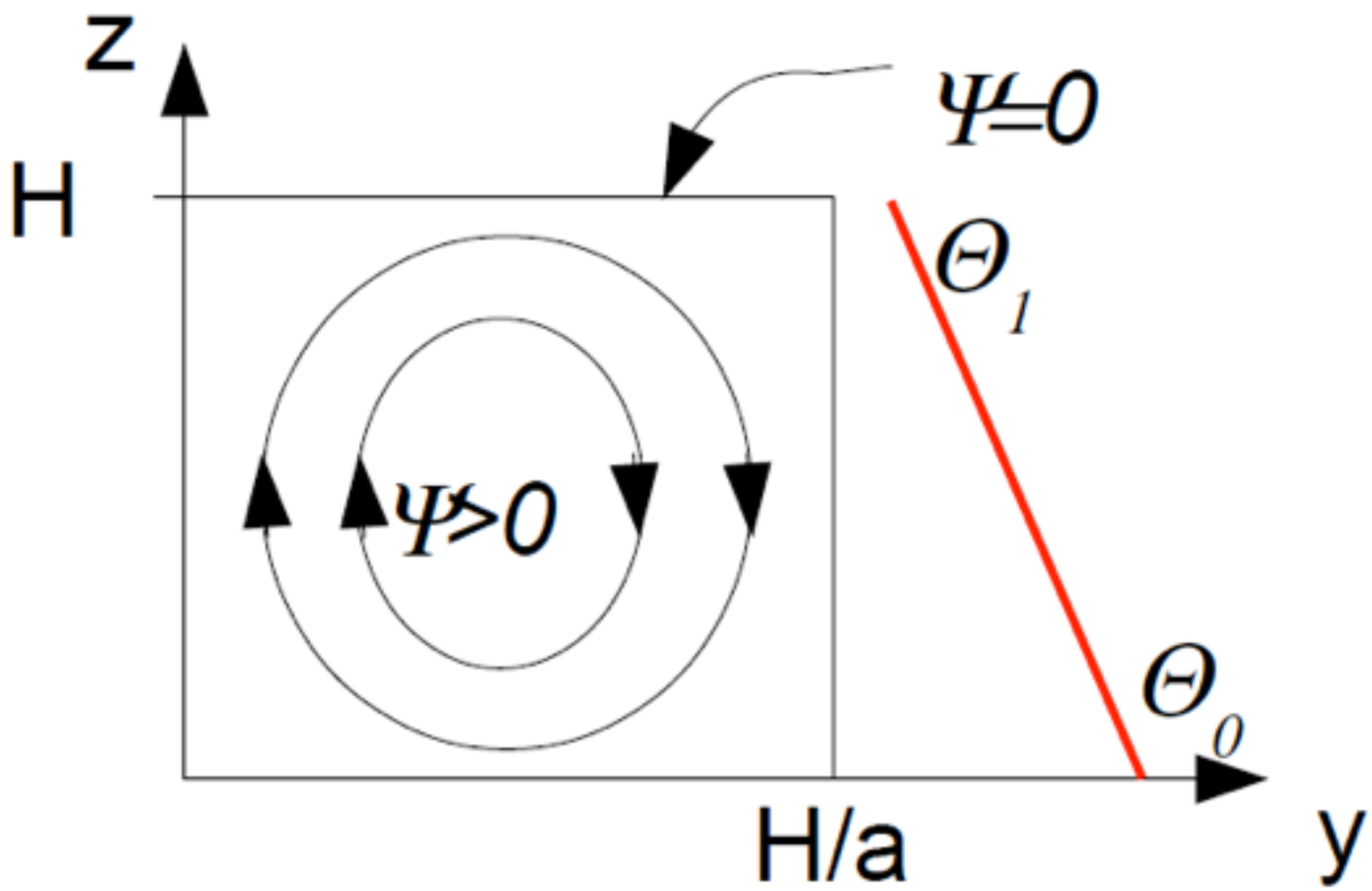
Reading

- Textbook chapter 3.3.4 (Land surface)
- Course Notes chapter 5.4 (Lorenz 1963)

Non-linear Dynamics and Deterministic Chaos

- The primitive equations are a system of non-linear coupled differential equations
- Why non-linear?
- Why coupled?
- Lorenz (1963) showed that a simpler set of non-linear coupled differential equations is not predictable for long time scales

Lorenz (1963)
Two-dimensional convection, incompressible, viscous fluid without rotation



Constant vertical temperature gradient

Mass conservation: $$\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0 \tag{5.34}$$

viscosity

$$\frac{Dv}{Dt}=-\frac{1}{\rho_0}\frac{\partial p}{\partial y}+\mu\nabla^2 v \tag{5.35}$$

Momentum conservation:

small density variation from constant

$$\frac{Dw}{Dt}=-\frac{1}{\rho_0}\frac{\partial p}{\partial z}+\mu\nabla^2 w-\frac{g}{\rho_0}\tilde{\rho} \tag{5.36}$$

$$\frac{D}{Dt}=\frac{\partial}{\partial t}+v\frac{\partial}{\partial y}+w\frac{\partial}{\partial z}$$

Streamfunction: $$v=-\frac{\partial\psi}{\partial z}\quad,\qquad w=\frac{\partial\psi}{\partial y}\tag{5.37}$$

Vorticity: $$\zeta=\frac{\partial w}{\partial y}-\frac{\partial v}{\partial z}=\nabla^2\psi\tag{5.38}$$

$\frac{\partial}{\partial z}(5.36)-\frac{\partial}{\partial z}(5.35)$ $$\frac{D\zeta}{Dt}=\mu\nabla^2\zeta-\frac{g}{\rho_0}\frac{\partial\tilde{\rho}}{\partial y}$$

Expansion Coefficient: $$\alpha=-\frac{1}{\rho_0}\frac{\partial\tilde{\rho}}{\partial\Theta}$$

$$\frac{D\zeta}{Dt}=\mu\nabla^2\zeta+g\,\alpha\frac{\partial\Theta}{\partial y}$$

Vorticity generated by buoyancy forcing and destroyed by viscosity small deviation from constant vertical gradient

Assume temperature distrib.

$$\Theta = \Theta_0 - \frac{\Delta T}{H} z + \tilde{\Theta}(y, z, t) \tag{5.41}$$

Energy conservation:

$$\frac{D\Theta}{Dt} = \kappa \nabla^2 \Theta \tag{5.42}$$

$$\frac{D}{Dt} = \frac{\partial}{\partial t} + v\frac{\partial}{\partial y} + w\frac{\partial}{\partial z}$$

with (5.37)

$$\frac{\partial \tilde{\Theta}}{\partial t} - \frac{\partial \psi}{\partial z}\frac{\partial \tilde{\Theta}}{\partial y} + \frac{\partial \psi}{\partial y}\frac{\partial \tilde{\Theta}}{\partial z} = \kappa \nabla^2 \tilde{\Theta} + \frac{\Delta T}{H}\frac{\partial \psi}{\partial y} \tag{5.43}$$

From the vorticity equation (5.40) together with (5.37) and (5.38) we get

$$\frac{\partial}{\partial t}\nabla^2\psi - \frac{\partial \psi}{\partial z}\frac{\partial}{\partial y}\nabla^2\psi + \frac{\partial \psi}{\partial y}\frac{\partial}{\partial z}\nabla^2\psi = \mu \nabla^4 \psi + g\alpha \frac{\partial \Theta}{\partial y} \tag{5.44}$$

Equations (5.43) and (5.44) are a coupled, non-linear system of partial differential equations. With appropriate boundary conditions those can be solved. The following Fourier expansion of the streamfunction and temperature deviation satisfies the boundary conditions of zero streamfunction and no horizontal gradient at y=0 and y=H/a and $\tilde{\Theta}=0$ at z=0 and z=H:

$$\psi(y, z, t) = X(t)\sin\left(\frac{\pi a y}{H}\right)\sin\left(\frac{\pi z}{H}\right) + \ldots \tag{5.45}$$

$$\tilde{\Theta}(y, z, t) = Y(t)\cos\left(\frac{\pi a y}{H}\right)\sin\left(\frac{\pi z}{H}\right) - Z(t)\sin\left(\frac{2\pi z}{H}\right) + \ldots \tag{5.46}$$

$\tau = (\pi / H)^2 (1 + a^2) \kappa t$ dimensionless time

$$\frac{dX}{dt} = -\sigma X + \sigma Y \tag{5.47}$$

$$\frac{dY}{dt} = -XZ + rX - Y \tag{5.48}$$

$$\frac{dZ}{dt} = XY - bZ \quad , \tag{5.49}$$

$\sigma = \mu / \kappa$ is the Prandtl number,

$r = R_c^{-1} R_a = g \alpha H^3 \Delta T a^2 \mu^{-1} \kappa^{-1} \pi^{-4} (1 + a^2)^{-3}$

$b = 4(1 + a^2)^{-1}$

HW6 implement eqs.
(5.47-5.49) in fortran code !

Homeworks vs Project

- Instead of two remaining HW you can do a project of your own choice
- E.g. compare different CMIP5 models with each other and with observations; write short (5 pages) assessment report
- Think about it. If you have a project in mind, let me know. We'll decide next week.