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Hubble parameter – Conformal time – Distance measures – Free electron fraction – Optical depth – Visibility function

Milestone 1 Introduction In this project we are going to look at the uniform background in the Universe and investigate the exp

$\Omega_{\nu 0} = N_{eff} \cdot \frac{7}{8}\left(\frac{4}{11}\right)^{\frac{4}{3}} \Omega_{\gamma 0}$ where $T_{CDM0} = 2.7255$K is today's temperature of the CMB, and $N_{eff} = 3.0466$ is the effective number of

All these components and equation eq:Friedmann$_{e}qisknownasthestandardmodelofcosmology.$

$WecanalsowritetheFriedmannequationonthefollowingformH = H_0 = \sqrt{\frac{\rho_x}{\rho_c}}where\rho_{x\,\overline{\rho_c=\Omega_{m0}+\Omega_{r0}+\Omega_{k0}+\Omega_{\Lambda 0}}}$ and $\rho_c = 3H^2/(8\pi G)$.

The Friedmann equation can show us how each density components changes with time, this is given by the following equation $\rho +$

$\omega = \frac{P}{\rho}$ and ω will be constant. Hence we have that $\rho \propto a^{-3(1+\omega)}$ Where

When we look at the cosmic microwave background, CMB, it is important to look at the *comoving horizon*. Comoving horizon tells

$\frac{d\eta}{dx} = \frac{da}{dx}\frac{d\eta}{da} = \frac{c}{\mathcal{H}}$

$\eta(x) = \int_{-\infty}^{x} \frac{cdx'}{\mathcal{H}(x')} where$H=aH$isascaledversionoftheHubbleparameter.\mathcal{H} = aH_0\sqrt{\Omega_{m0}a^{-3} + \Omega_{r0}a^{-4}\Omega_{k0}a^{-2} + \Omega_{\Lambda 0}}Therearesevaralw$

$Anotherwaytocalculatetimeistousetheredshift, whichisdonebymeasuringthetimebyobservinghowfarthewavelengthofaphotonhasb$

$ThereareotherwaystocalculatethedistancesintheUniverseaswell.Thecalculationsonthedistancesdependsonthegeometry.Weareon$

$= \eta - \eta_0 where\chi = \int_{t}^{t_{today}} \frac{cdt}{a} = randcdt_{\overline{a=dr}}$ if a photon moves radially to us in a flat universe.

There is also distance measure that is called *angular distance measure*. This is a distance that is defined at the size, Δs, of an object

Another distance measure is the luminosity distance, this distance can be calculated by finding the flux, F, and the luminosity, L, an

$= \frac{d_A}{a^2}$

Method By calculating expansion history of the universe numerical we us a model which contains the cosmological parameter

We will compute and make plots of $H(x)$, $\mathcal{H}(x)$, $\frac{1}{\mathcal{H}(x)}\frac{d\mathcal{H}(x)}{dx}$, $\frac{1}{\mathcal{H}(x)}\frac{d^2\mathcal{H}(x)}{dx^2}$, $\eta(x)$, $\frac{\eta(x)is\mathcal{H}(x)}{c}$, $t(x)$, Ω_m, Ω_r and Ω_Λ, we use different functio

$\Omega_{CDM}(a) = \frac{\Omega_{CDM0}}{a^2H(a)^2/H_0^2}$

$\Omega_b(a) = \frac{\Omega_{n0}}{a^3H(a)^2/H_0^2}$

$\Omega_\gamma(a) = \frac{\Omega_{\gamma 0}}{a^4H(a)^2/H_0^2}$

$\Omega_\nu(a) = \frac{\Omega_{\nu 0}}{a^4H(a)^2/H_0^2}$

$\Omega_\Lambda(a) = \frac{\Omega_{\Lambda 0}}{a^4H(a)^2/H_0^2}$ Position, redshift and time First of all we want calculate the times - x, z, t for radiation-matter equality, matter-d

We start to compute the time for radiation-matter equality, $\Omega_m = \Omega_r$. We now that $\Omega_{m0} \propto a^{-3}$ and $\Omega_{r0} \propto a^{-4}$ we then get the equatio

$a_{MR} = \frac{\Omega_{r0}}{\Omega_{m0}}$ Further we compute the time for matter-dark energy equality, $\Omega_m = \Omega_\Lambda$, and we have that $\Omega_\Lambda \propto 1$, which gives us a$^{-3}\Omega$

$a_{MDE} = \left(\frac{\Omega_{m0}}{\Omega_{\Lambda 0}}\right)^{\frac{1}{3}}$ Finally we compute when the Universe starts to accelerate. We have that a=aH= $\sqrt{\Omega_{m0}a^{-3} + \Omega_{\Lambda 0}}$

$= H_0\sqrt{\Omega_{m0}a^{-1} + \Omega_{\Lambda 0}a^2}$ This gives us now a=H$_0\frac{1}{2}\frac{1}{\sqrt{\Omega_{m0}a^{-1}+\Omega_{\Lambda 0}a^2}}\left(\Omega_{m0}\left(-\frac{1}{a^2}\right)a + 2\Omega_{\Lambda 0}aa\right) = 0$ We simplify this expression by cance

$-\Omega_{m0}\frac{1}{a^2} + 2\Omega_\Lambda a = 0$

$2\Omega_{\Lambda 0}a = \Omega_{m0}\frac{1}{a^2}$

$a = \left(\frac{\Omega_{m0}}{\Omega_{\Lambda 0}}\right)^{\frac{1}{3}}$

We now insert equation eq:a$_{M}R, eq : a_mdeandeq : a_accintoequationeq : x - eq : twhichwillthengiveusthedifferentvaluesforposit$

$z = \frac{1}{a} - 1$

$t = t(\ln a)$HandH$Wewillnowcompute$H$, soweuseequationeq : Friedmann_eqwhichiswrittenasH(x) = H_0\sqrt{\Omega_{m0}e^{-3x} + \Omega_{r0}e^{-4x}\Omega_{k0}e^{-2}}$

$Weinsertequationeq : Omega_k - eq : Omega_lambdaintoequationeq : H_x.$

$Tocompute$H$weuseequationeq : Hp_eq\mathcal{H}(x) = e^xH_0\sqrt{\Omega_{m0}e^{-3x} + \Omega_{r0}e^{-4x}\Omega_{k0}e^{-2x} + \Omega_{\Lambda 0}}$

$= H_0\sqrt{\Omega_{m0}e^{-x} + \Omega_{r0}e^{-2x} + \Omega_{k0} + \Omega_{\Lambda 0}e^{2x}}andwewillsaythat\Omega_{\nu 1} = \Omega_{m0}e^{-x} + \Omega_{r0}e^{-2x} + \Omega_{k0} + \Omega_{\Lambda 0}e^{2x}whichgivesus\mathcal{H}(x) = H_0\sqrt{\Omega_{\nu 1}}1$

$= H_0\frac{1}{\sqrt{\Omega_{\nu 1}}}(-\Omega_{m0}e^{-x} - 2\Omega_{r0}e^{-2x} + \Omega_{k0} + 2\Omega_{\Lambda 0}e^{2x})$

$= H_0\frac{1}{2}\frac{1}{\sqrt{\Omega_{\nu 1}}}\Omega_{\nu 2}where\Omega_{\nu 2} = (-\Omega_{m0}e^{-x} - 2\Omega_{r0}e^{-2x} + \Omega_{k0} + 2\Omega_{\Lambda 0}e^{2x})$

$Wearenowabletocalculate1\frac{}{\mathcal{H}(x)\frac{d^2\mathcal{H}(x)}{dx^2}}$ by finding the derivative of equation eq:dHp$_{d}x.\frac{d^2\mathcal{H}(x)}{dx^2} = \frac{1}{2}H_0(\Omega'_{\nu 2}\Omega_{\nu 1}^{-1/2} + \Omega_{\nu 2}(\Omega_{\nu 2}^{-1/2})')$

$= \frac{1}{2}H_0\left(\frac{\Omega_{m0}e^{-x}+4\Omega_{r0}e^{-2x}+\Omega_{k0}+4\Omega_{\Lambda 0}e^{2x}}{\Omega_{\nu 1}}\right.$

$\left.+ \Omega_{\nu 2}\left(-\frac{1}{2}\frac{1}{\Omega_{\nu 1}^{3/2}}\Omega'_{\nu 1}\right)\right)wenowsaythat\Omega_{\nu 3} = \Omega_{m0}e^{-x} + 4\Omega_{r0}e^{-2x} + \Omega_{k0} + 4\Omega_{\Lambda 0}e^{2x}Thiswillthengiveus\frac{d^2\mathcal{H}(x)}{dx^2} = \frac{1}{2}H_0\left(\frac{\Omega_{\nu 3}}{\sqrt{\Omega_{\nu 1}}} - \frac{1}{2}\frac{\Omega_{\nu 2}}{\Omega_{\nu 1}^{3/2}}\Omega_{\nu 2}\right)$

$= \frac{1}{2}H_0\frac{1}{\sqrt{\Omega_{\nu 1}}}\left(\Omega_{\nu 3} - \frac{1}{2}\frac{\Omega_{\nu 2}^2}{\Omega_{\nu 1}}\right)Wehavenowfoundvaluesfor$dH(x)$_{\overline{dx}}$ and $\frac{d^2\mathcal{H}(x)}{dx^2}$ analytically and we now insert these values into two functi

$\eta(x)$ and $t(x)$ To compute $\eta(x)$ and $t(x)$ we use the 4th-Order Runge Kutta for solving our ODEs. After we solved the ODEs we mac

Now that we have computed $\eta(x)$ and $t(x)$ we will find the age of the Universe today, $t(0)$, and the conformal time today, $\eta(0)/c$. Ω-

Milestone 2 Introduction In this project we are going to look at the recombination history of the universe. More detailed, we w

If we consider a smooth universe we can write the Boltzmann equation on the following form $1\frac{}{a^3\frac{d(n_1a^3)}{dt}=-\alpha n_1n_2+\beta n_3n_4}$ where a is the s

$\frac{1}{a^3}\frac{d(n_1a^3)}{dt} = -\langle\sigma v\rangle\left(n_1n_2 - n_3n_4\left(\frac{n_1n_2}{n_3n_4}\right)_{eq}\right)$ which we rewrite as $1\frac{}{n_1a^3\frac{d(n_1a^3)}{dx}=-\frac{\Gamma_1}{H}\left(1-\frac{n_3n_4}{n_1n_2}\left(\frac{n_1n_2}{n_3n_4}\right)_{eq}\right)}$ where H is the expansion rate and $\Gamma_1 \equiv n_2$

$Toreachchemicalequilibriumwemusthavethat\Gamma_1 \gg H$, which means that the interaction rate is greater than the expansion rate of the

$\frac{n_1n_2}{n_3n_4} \approx \left(\frac{n_1n_2}{n_3n_4}\right)_{eq} whentheefficiencyofinteractionishighenoughtogetclosetoequilibrium.ThisequationisalsoknownastheSahaapproxi$

$Wecanrewriteequationeq : Saha_approx.Weknowthattheuniverseiselectricallyneutral, hence$n$_e = n_p$. Further we have that $n_\gamma = n_\gamma^{eq}$

By using the definition of electron fraction, $n_\gamma = n_\gamma^{eq}$ and $n_e = n_p$ as we used for the Saha approximation and insert those into equati

The downside with the Peebles equation is that the electron in a hydrogen atom can have different energy and spin states. To recomb

So from these two possible ways there is introduced a factor which looks at the probability of an atom to reach it ground state from

$\Lambda_{2s\rightarrow1s} = 8.227\text{s}^{-1}$,
$\Lambda_\alpha = H\frac{(3\epsilon_0)^3}{(8\pi)^2c^3\hbar^3n_{1s}}$
$n_{1s} = (1 - X_e)n_H$
$n_H = (1 - Y_p)n_b$
$n_b = (1 - Y_p)\frac{3H_0^2\Omega_{b0}}{8\pi Gm_Ha^3}$
$\beta^{(2)}(T_b) = \beta(T_b)e^{\frac{3\epsilon_0}{4k_bT_b}}$
$\beta(T_b) = \alpha^{(2)}(T_b)\left(\frac{m_ek_bT_b}{2\pi\hbar^2}\right)^{3/2}e^{-\frac{\epsilon_0}{k_bT_b}}$
$\alpha^{(2)}(T_b) = \frac{8}{\sqrt{3\pi}}c\sigma_T\sqrt{\frac{\epsilon_0}{k_bT_b}}\phi_2(T_b)$
$\phi_2(T_b) = 0.448\ln\left(\frac{\epsilon_0}{k_bT_b}\right)$ Optical Depth Optical depth, τ, describes the level of transparency through a medium. An appropriate way
It can also be looked at the number of scatterings of photons by electrons per unit time.
At the recombination time where the free protons and the free electrons ties together and create hydrogen is also the time when the
$\int_\infty^0\tilde{g}(x)dx = 1$ Method We will solve the recombination history of the universe numerically. To do so we first start to solve the Saha
With n_e we are able to calculate the optical depth. We use the Runge-Kutta 4 method and spline the solution. We do the same thing
For the visibility function $\tilde{g}$ we use the Runge-Kutta 4 ODE solver and spline the solution and use a '$deriv_x$'-function on $\frac{d\tilde{g}}{dx}$ and $\frac{d^2\tilde{g}}{dx^2}$
The calculation on the decoupling and recombination on x is to use 'binary search for value'-function and set $\tau = 1$ for decoupling
$z_{decoupling} = 1081.15$ Further we found the time when the recombination was half-way at $X_e = 0.5$ x$_{rec} = -7.16$

Electron_fraction.png

$z_{rec} = 1291.25$ Then we found the freeze-out abudance of free electrons X$_e(x = 0) = 0.000197$ [H]

Optical depth τ, $\frac{d\tau}{dx}$, $\frac{d^2\tau}{dx^2}$ as a function of time x. Figure fig:Otical$_{d}epthshowsusthebehaviouroftheopticaldepth.Weseethattheoptic$

Visibility function g, $\frac{dg}{dx}$ $\frac{d^2}{dx^2}$ as a function of time x. Where g is scaled with $(g)_{max} = 4.85$, $\left(\frac{dg}{dx}\right)_{max} = 50.40$, $\left(\frac{d^2g}{dx^2}\right)_{max} = 724.94$ With

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Figure taken from Baumann. The evolution of the free electron from the early Universe till today. As mentioned earlier we said tha
I would like to add a comment that the recombination should be later than the value we got. It should have been close to $x_{rec} \sim -7.2$

Milestone 3 Introduction In the previous milestones we have considered a smooth Universe(?). We will now take it a step furth
We will use a Newtonian gauge for our metric which is defined as $\mathrm{d}\,s^2 = -dt^2(1+2\Psi) + a^2(1+2\Phi)(dx^2+dy^2+dz^2)$ where Ψ and
The perturbations to the distribution function is defined as $\mathrm{T} = \overline{T}(1+\Theta(t,\vec{x},p,\hat{p}))$ and it is also known as the perturbation in the lo
$p^+ + \gamma \leftrightharpoons p^+ + \gamma$ And the collision term of this process for first order in perturbation theory is given as $\mathrm{C} = \text{-p}^2\frac{\partial f}{\partial p} n_e \sigma_T[\Theta_0 - \Theta + \hat{p}$
The Thompson cross-section, $p^+ + \gamma \rightleftharpoons p^+ + \gamma$, has the proportionality $\propto 1/m^2$. Since the protons has a much larger mass than the

Finally we are able to find all of the Boltzmann equations we need as well as the potential Ψ and Φ from the Einstein equations. We

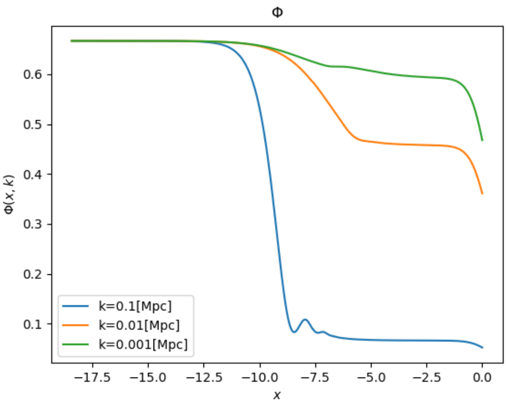
We can use a Fourier transform on equation eq:final$_{photon}$*toreducethenumberoffreevariables.Westartbyintroducingthe*$\mu \equiv \frac{\hat{p}\cdot\vec{k}}{k}$-va

We can now write the Einstein-Boltzmann equations which describes the velocity and energy contents as well as the the metric pert
$\Psi = -\Phi - \frac{12H_0^2}{c^2k^2a^2}\left[\Omega_{\gamma 0}\Theta_2\right]$ The initial conditions photon temperature multipoles $\Theta_0' = -\frac{ck}{\mathcal{H}}\Theta_1 - \Phi'$,
$\Theta_1' = \frac{ck}{3\mathcal{H}}\Theta_0 - \frac{2ck}{3\mathcal{H}}\Theta_2 + \frac{ck}{3\mathcal{H}}\Psi + \tau'\left[\Theta_1 + \frac{1}{3}v_b\right],$
$\Theta_\ell' = \frac{\ell ck}{(2\ell+1)\mathcal{H}}\Theta_{\ell-1} - \frac{(\ell+1)ck}{(2\ell+1)\mathcal{H}}\Theta_{\ell+1}$
$+\tau'\left[\Theta_\ell - \frac{1}{10}\Pi\delta_{\ell,2}\right], \qquad 2 \leq \ell\ell\text{max}$
$\Theta_\ell' = \frac{ck}{\mathcal{H}}\Theta_{\ell-1} - c\frac{\ell+1}{\mathcal{H}\eta(x)}\Theta_\ell + \tau'\Theta_\ell, \qquad \ell = \ell\text{max}$ The initial condition for CDM and baryons $\delta_{CDM}' = \frac{ck}{\mathcal{H}}v_{CDM} - 3\Phi'$
$v_{CDM}' = -v_{CDM} - \frac{ck}{\mathcal{H}}\Psi$
$\delta_b' = \frac{ck}{\mathcal{H}}v_b - 3\Phi'$
$v_b' = -v_b - \frac{ck}{\mathcal{H}}\Psi + \tau' R(3\Theta_1 + v_b)$ HVA DE ER, SE W

And the initial conditions are given as following $\Psi = -\frac{1}{\frac{3}{2}+\frac{2f_\nu}{5}}$
$\Phi = -(1+\frac{2f_\nu}{5})\Psi$
$\delta_{CDM} = \delta_b = -\frac{3}{2}\Psi$
$v_{CDM} = v_b = -\frac{ck}{2\mathcal{H}}\Psi$
Photons :
$\Theta_0 = -\frac{1}{2}\Psi$
$\Theta_1 = +\frac{ck}{6\mathcal{H}}\Psi$
$\Theta_2 = \begin{cases} -\frac{8ck}{15\mathcal{H}\tau'}\Theta_1, & \text{(with polarization)} \\ -\frac{20ck}{45\mathcal{H}\tau'}\Theta_1, & \text{(without polarization)} \end{cases}$
$\Theta_\ell = -\frac{\ell}{2\ell+1}\frac{ck}{\mathcal{H}\tau'}\Theta_{\ell-1}$ We want to expand the μ-dependence of Θ in Legendre multipoles. The reason why we do this is that we want

The optical depth, τ, is very high at early times. Hence electrons is only affected by temperature fluctions of electrons that are close

As we solve the Einstein-Boltzmann equations we have to us the approximation given in equation eq:that$_{a}$*pproxandignoringthemu*



Results and Discussion [H]

Evolution of the Newtonian potential Φ. [H]

We see the same behaviour for the velocity perturbation in figure fig:v.*Thelowmodevelocityperturbationforbaryonsstarttooscillat*
Figurefig : *phishowsusthespatialcurvature.Weseethatithasaconstantmovementuntilitreachesthehorizon.Thereasonforwhythisp*
Infigurefig : *delta$_g$ammaandfig* : *v$_g$ammaweseethephotondensitycontrastandphotonvelocityperturbation.Weseethatthereareos*

Appendix A $\frac{d\mathcal{H}(x)}{dx}/H$ and $\frac{d^2\mathcal{H}(x)}{dx^2}/H$ H=e$^x H$
$= e^x H_0 \sqrt{\frac{\rho_x}{\rho_c}}$
$= H_0 \frac{1}{\sqrt{\rho_c}} e^{-\frac{3}{2}(1+\omega)+x}$
$= \frac{H_0}{\sqrt{\rho_c}} e^{-\frac{1}{2}x-\frac{3}{2}(x\omega}$
$= H_0 \frac{1}{\sqrt{\rho_c}} e^{-\frac{1}{2}x(1+\omega)}$
We now use the $\mathcal{H}$ we got to find $\frac{d\mathcal{H}}{dx}$ dH$\overline{dx=\frac{H_0}{\rho_c}\left(-\frac{1}{2}(1+3\omega)\right)e^{-\frac{1}{2}x}}$ Which finally gives us dH(x)$\overline{dx/H=\frac{\frac{H_0}{\rho_c}\left(-\frac{1}{2}(1+3\omega)\right)e^{-\frac{1}{2}x}}{\frac{H_0}{\rho_c}e^{-\frac{1}{2}x}}=-\frac{1}{2}(1+3\omega)}$ We will now

Appendix B

$$\frac{\mathrm{df}}{d\lambda = C(f)\,\frac{df}{d\lambda} = -P^0 p\,\frac{\partial\bar{f}}{\partial p}\left[\frac{\partial\Theta}{\partial t}+\frac{\partial\Theta}{\partial x^i}\,\frac{\hat{p}^i}{a}+\left(\frac{\partial\Phi}{\partial t}+\frac{\partial\Psi}{\partial x^i}\,\frac{\hat{p}^i}{a}\right)\right]\frac{\partial\Theta}{\partial t}+\frac{\partial\Theta}{\partial x^i}\,\frac{\hat{p}^i}{a}+\left(\frac{\partial\Phi}{\partial t}+\frac{\partial\Psi}{\partial x^i}\,\frac{\hat{p}^i}{a}\right)=n_e\sigma_T\left[\Theta_0-\Theta+\hat{p}\cdot\vec{v}_b\right]}$$

https://cmb.wintherscoming.no/pdfs/baumann.pdf

https://cmb.wintherscoming.no/milestone2.phpOverview

https://cmb.wintherscoming.no/theory$_t$hermodynamics.php
https://arxiv.org/pdf/astro-ph/0606683.pdf
https://cmb.wintherscoming.no/milestone3.php