Chapter 1 A Tour of Computer Systems

Problem 1.1

A. We use the formula with $\alpha = \frac{3}{5}$ and $k = \frac{150}{100} = \frac{3}{2}$.

$$\begin{aligned} S &= \frac{1}{\left(1 - \frac{3}{5}\right) + \frac{3}{5} \cdot \frac{2}{3}} \\ &= \frac{1}{\frac{2}{5} + \frac{2}{5}} \\ &= \frac{5}{4} \\ &= 1.25\times \end{aligned}$$

B. We use the formula and work our way back:

$$\begin{aligned} \frac{5}{3} &= \frac{1}{\left(1 - \frac{3}{5}\right) + \frac{3}{5k}} \\ \frac{3}{5} &= \frac{2}{5} + \frac{3}{5k} \\ \frac{1}{5} &= \frac{3}{5k} \\ 1 &= \frac{3}{k} \\ k &= 3 \end{aligned}$$

So the drive through Montana needs a speedup of $3\times$ which is 300 km/hr.

Problem 1.2

Use the formula with $\alpha = \frac{4}{5}$ and $S = 2$ and solve for k.

$$\begin{aligned} 2 &= \frac{1}{\left(1 - \frac{4}{5}\right) + \frac{4}{5k}} \\ \frac{2}{5} + \frac{8}{5k} &= 1 \\ \frac{8}{5k} &= \frac{3}{5} \\ \frac{1}{k} &= \frac{3}{8} \\ k &= \frac{8}{3} \end{aligned}$$

Chapter 2 Representing and Manipulating Information

Problem 2.1

A. `0x39A7F8` to binary: `0011 1001 1010 0111 1111 1000`

B. `1100100101111011` to hexadecimal: `0xC97B`

C. `0xD5E4C` to binary: `1101 0101 1110 0100 1100`

D. `1001101110011110110101` to hexadecimal: `0x26E7B5`

Problem 2.2

n	2^n (decimal)	2^n (hexadecimal)
`9`	`512`	`0x200`
`19`	`524288`	`0x80000`
`14`	`16384`	`0x4000`
`16`	`65536`	`0x10000`
`17`	`131072`	`0x20000`
`5`	`32`	`0x20`
`7`	`128`	`0x80`

Problem 2.3

Decimal	Binary	Hexadecimal
`0`	`0000 0000`	`0x00`
`167`	`1010 0111`	`0xA7`
`62`	`0011 1110`	`0x3E`
`188`	`1011 1100`	`0xBC`
`55`	`0011 0111`	`0x37`
`136`	`1000 1000`	`0x88`
`243`	`1111 0011`	`0xF3`
`82`	`0101 0010`	`0x52`
`172`	`1010 1100`	`0xAC`

Decimal	Binary	Hexadecimal
`231`	`1110 0111`	`0xE7`

Problem 2.4

A. `0x503c` + `0x8` = `0x5044`

B. `0x503c` − `0x40` = `0x4ffc`

C. `0x503c` + `64` = `0x507c`

D. `0x50ea` − `0x503c` = `0xae`

Problem 2.5

	Little endian	Big endian
A.	`21`	`87`
B.	`21 43`	`87 65`
C.	`21 43 65`	`87 65 43`

Problem 2.6

A.

`0x00359141` in binary: `0000 0000 0011 0101 1001 0001 0100 0001`

`0x4A564504` in binary: `0100 1010 0101 0110 0100 0101 0000 0100`

B.

```
00000000001101011001000101000001
  01001010010101100100010100000100
           *********************
```

There are 21 matching bits.

C.

The whole integer occurs in the float representation, except for the most-significant bit which is a 1. Similarly, some of the most-significant bits of the float representation do not occur in the int representation.

Problem 2.7

It prints `61 62 63 64 65 66` (it does not print the terminating null character because the `strlen` function does not count it).

Problem 2.8

Operation	Result
`a`	`[01101001]`
`b`	`[01010101]`
`~a`	`[10010110]`
`~b`	`[10101010]`
`a & b`	`[01000001]`
`a | b`	`[01111101]`
`a ^ b`	`[00111100]`

Problem 2.9

A. The following colors completent each other:

Black ↔ White
Blue ↔ Yellow
Green ↔ Magenta
Cyan ↔ Red

B.

Blue | Green = Cyan
Yellow & Cyan = Green
Red ^ Magenta = Blue

Problem 2.10

Step	$*x$	$*y$
Initially	`a`	`b`
Step 1	`a`	`a ^ b`
Step 2	`a ^ (a ^ b) = b`	`a ^ b`
Step 3	`b`	`b ^ (a ^ b) = a`

Problem 2.11

A. In the final iteration we have `first = k` and `last = k` (swap the middle element with itself).

B. In this case `*x` and `*y` point to the same address and the steps become:

Step	*x	*y
Initially	`a`	`a`
Step 1	`a ^ a = 0`	`a ^ a = 0`
Step 2	`0 ^ 0 = 0`	`0 ^ 0 = 0`
Step 3	`0 ^ 0 = 0`	`0 ^ 0 = 0`

C. We can fix it by changing the condition to `first < last` since the middle element does not need to be swapped anyway.

Problem 2.12

A. `x & 0xFF` leaves the least significant byte and sets everything else to zero.

B. `x ^ ~0xFF` inverts everything except the least significant byte.

C. `x | 0xFF` sets the least significant byte to ones and leaves everything else.

Problem 2.13

`x | y` is equivalent to `bis(x, y)`.

`x ^ y` is equivalent to `bis(bic(x, y), bic(y, x))`.

Problem 2.14

We have `x = 0110 0110` and `y = 0011 1001`.

Expression	Value	Expression	Value
`x & y`	`0010 0000`	`x && y`	`1`
`x | y`	`0111 1111`	`x || y`	`1`
`~x | ~y`	`1111 1111 1111 1111 1111 1111 1101 1111` (assuming 32-bit int)	`!x || !y`	`0`
`x & !y`	`0`	`x && ~y`	`1`

Problem 2.15

`!(x ^ y)` is equivalent to `x == y` because `x ^ y` will be `0` only if all the bits match.

Problem 2.16

x	x << 3	x >> 2 (logical)	x >> 2 (arithmetic)
`0xC3 = 1100 0011`	`0001 1000 = 0x18`	`0011 0000 = 0x30`	`1111 0000 = 0xF0`
`0x75 = 0111 0101`	`1010 1000 = 0xA8`	`0001 1101 = 0x1D`	`0001 1101 = 0x1D`
`0x87 = 1000 0111`	`0011 1000 = 0x38`	`0010 0001 = 0x21`	`1110 0001 = 0xE1`
`0x66 = 0110 0110`	`0011 0000 = 0x30`	`0001 1001 = 0x19`	`0001 1001 = 0x19`

Problem 2.17

Hexadecimal	Binary	$B2U_4(x)$	$B2T_4(x)$
`0xE`	`[1110]`	$2^3 + 2^2 + 2^1 = 14$	$-2^3 + 2^2 + 2^1 = -2$
`0x0`	`[0000]`	0	0
`0x5`	`[0101]`	$2^2 + 2^0 = 5$	$2^2 + 2^0 = 5$
`0x8`	`[1000]`	$2^3 = 8$	$-2^3 = -8$
`0xD`	`[1101]`	$2^3 + 2^2 + 2^0 = 13$	$-2^3 + 2^2 + 2^0 = -3$
`0xF`	`[1111]`	$2^3 + 2^2 + 2^1 + 2^0 = 15$	$-2^3 + 2^2 + 2^1 + 2^0 = -1$

Problem 2.18

A. `0x2e0 = 736`

B. `-0x58 = -88`

C. `0x28 = 40`

D. `-0x30 = -48`

E. `0x78 = 120`

F. `0x88 = 136`

G. `0x1f8 = 504`

H. `0xc0 = 192`

I. `-0x48 = -72`

Problem 2.19

x	$T2U_4(x)$
-8	8
-3	$2^3 + 2^2 + 2^0 = 13$
-2	$2^3 + 2^2 + 2^1 = 14$
-1	$2^3 + 2^2 + 2^1 + 2^0 = 15$
0	0
5	5

Problem 2.20

Equation 2.5 can be used to solve the previous problem. Since $\omega = 4$, we need to add $2^4 = 16$ to all negative numbers in Two's Complement. For example, $-8 + 16 = 8$ and $-1 + 16 = 15$. Positive numbers (and zero) stay the same.

Problem 2.21

Expression	**Type**	**Evaluation**
$-2147483647 - 1 == 2147483648\text{U}$	Unsigned	1
$-2147483647 - 1 < 2147483647$	Signed	1
$-2147483647 - 1\text{U} < 2147483647$	Unsigned	0
$-2147483647 - 1 < -2147483647$	Signed	1
$-2147483647 - 1\text{U} < -2147483647$	Unsigned	1

Problem 2.22

A. $[1011] = -2^3 + 2^1 + 2^0 = -5$
B. $[11011] = -2^4 + 2^3 + 2^1 + 2^0 = -5$
C. $[111011] = -2^5 + 2^4 + 2^3 + 2^1 + 2^0 = -5$

Problem 2.23

`w`	`fun1(w)`	`fun2(w)`
`0x00000076`	`0x00000076`	`0x00000076`
`0x87654321`	`0x00000021`	`0x00000021`
`0x000000C9`	`0x000000C9`	`0xFFFFFFC9`
`0xEDCBA987`	`0x00000087`	`0xFFFFFF87`

`fun1` keeps only the least significant byte and sets the other three to all zeroes, resulting in a value between 0 and 255. `fun2` also extracts the least significant byte, but it performs sign extension instead of zero extension, which results in a value between −128 and 127.

Problem 2.24

Hex		**Unsigned**		**Two's complement**	
Original	Truncated	Original	Truncated	Original	Truncated
0	0	0	0	0	0
2	2	2	2	2	2
9	1	9	1	−7	1
B	3	11	3	−5	3
F	7	15	7	−1	−1

We can use the equations to verify these results. For example, in hex F truncates to 7, in unsigned $B2U_4(1111) \bmod 2^3 = 7$ and in two's complement $U2T_3(B2U_4(1111) \bmod 2^3) = -1$.

Problem 2.25

Because `length` is unsigned the expression $0 - 1$ evaluates to UMax. The comparison has an unsigned integer on one side, which means the other side will also be treated as unsigned. Of course every unsigned number is $\leq$ UMax and so we try to access invalid array elements.

We can fix it by changing the condition to `i < length` or changing `length` to a signed integer.

Problem 2.26

A. The function returns wrong results in case `t` is longer than `s`.

B. The problem is that `strlen` returns a `size_t` which is unsigned. When calculating `strlen(s) - strlen(t)` where `t` is longer than `s` unsigned arithmetic is used, resulting in a number close to UMax instead of a negative number. This is obviously greater than 0 so the function incorrectly says that `s` is longer.

C. We can fix it by changing the condition to `strlen(s) > strlen(t)`.

Problem 2.27

```
/* Determine whether arguments can be added without overflow */
int uadd_ok(unsigned x, unsigned y) {
  return x + y >= x;
}
```

Problem 2.28

x		$-^u_w x$	
Hex	**Decimal**	**Decimal**	**Hex**
0	0	0	0
5	5	11	B
8	8	8	8
D	13	3	3
F	15	1	1

Problem 2.29

x	y	$x + y$	$x +^t_5 y$	Case
−12 [10100]	−15 [10001]	−27 [100101]	5 [00101]	1
−8 [11000]	−8 [11000]	−16 [110000]	−16 [10000]	2
−9 [10111]	8 [01000]	−1 [111111]	−1 [11111]	2
2 [00010]	5 [00101]	7 [000111]	7 [00111]	3
12 [01100]	4 [00100]	16 [010000]	−16 [10000]	4

Problem 2.30

```c
/* Determine whether arguments can be added without overflow */
int tadd_ok(int x, int y) {
  int sum = x + y;

  if (x > 0 && y > 0) {
    return sum > 0;
  }

  if (x < 0 && y < 0) {
    return sum < 0;
  }

  return 1;
}
```

Problem 2.31

sum $-$ x can overflow again, since it's another two's complement addition. For example, if x and y are large positive numbers whose sum overflows to a negative number, then sum $-$ x will cause a negative overflow "wrapping back around" to y. So this check will not detect the overflow.

Problem 2.32

The function will be incorrect for $y = \text{TMin}_\omega$. This is because the two's complement representation is not symmetric. $-y = -\text{TMin}_\omega = \text{TMin}_\omega$ causes an overflow possibly resulting in an incorrect return value.

Problem 2.33

x		$-^t_4 x$	
Hex	Decimal	Decimal	Hex
0	0	0	0
5	5	−5	B
8	−8	−8	8
D	−3	3	3
F	−1	1	1

The bit patterns for two's complement and unsigned negation are the same.

Problem 2.34

Mode	x	y	$x \cdot y$	Truncated $x \cdot y$
Unsigned	$4 = [100]$	$5 = [101]$	$20 = [010100]$	$4 = [100]$
Two's complement	$-4 = [100]$	$-3 = [101]$	$12 = [001100]$	$-4 = [100]$
Unsigned	$2 = [010]$	$7 = [111]$	$14 = [001110]$	$6 = [110]$
Two's complement	$2 = [010]$	$-1 = [111]$	$-2 = [111110]$	$-2 = [110]$
Unsigned	$6 = [110]$	$6 = [110]$	$36 = [100100]$	$4 = [100]$
Two's complement	$-2 = [110]$	$-2 = [110]$	$4 = [000100]$	$-4 = [100]$

Problem 2.35

1. Let $t = u + p_{\omega-1}$ where u is the two's complement number represented by the ω upper bits of the 2ω-bit representation of $x \cdot y$. Since $p_{\omega-1}$ is either 0 or 1, there are two possibilities for t to equal 0.

1. If $p_{\omega-1} = 0$ then it must be that $u = 0$ (upper ω bits are all 0s).
2. If $p_{\omega-1} = 1$ then it must be that $u = -1$ (upper ω bits are all 1s).

So $t = 0$ if the upper $\omega + 1$ bits are all 0s or all 1s. These are exactly the cases where the multiplication does not overflow. All other cases do overflow.

This means we can write $x \cdot y = p + t2^\omega$ which overflows iff $t \neq 0$.

2. To show that p can be written in the form $p = x \cdot q + r$, where $|r| < |q|$ we consider integer division. Dividing p by nonzero x gives a quotient q and remainder r, such that $|r| < |q|$.

3. By plugging in we get $x \cdot y = x \cdot q + r + t2^\omega$. If $r + t2^\omega = 0$ then $q = y$. Since $|r| < |q| < 2^\omega$ this can only hold if $r = t = 0$.

Problem 2.36

```
/* Determine whether arguments can be multiplied without overflow */
int tmult_ok(int x, int y) {
  int64_t prod = ((int64_t)x) * y;
  int64_t upper = prod >> 31;
  // if the upper 33 bits are all 1s or 0s the number fits into 32 bits
  return upper == 0 || upper == -1;
}
```

Problem 2.37

A. The new code does not improve the situation since the 64-bit number will be truncated to 32 bits when passed to `malloc`. This truncation is the same that also happens when the multiplication overflows.

B. Check the multiplication for overflow (by one of the previous methods) and if it overflows immediately abort and don't allocate any memory.

Problem 2.38

A single LEA instruction can compute the following multiples:

k	b	$(a \ll k) + b$
0	0	$(2^0 + 0)a = 1a$
0	a	$(2^0 + 1)a = 2a$
1	0	$(2^1 + 0)a = 2a$
1	a	$(2^1 + 1)a = 3a$
2	0	$(2^2 + 0)a = 4a$
2	a	$(2^2 + 1)a = 5a$
3	0	$(2^3 + 0)a = 8a$
3	a	$(2^3 + 1)a = 9a$

Problem 2.39

In this case the expression simplifies to $-(x \ll m)$. This is because shifting by $n + 1 = \omega$ to the left results in 0 so we can ignore the first term.

Problem 2.40

K	Shifts	Add/Subs	Expression
6	2	1	$(x \ll 2) + (x \ll 1)$
31	1	1	$(x \ll 5) - x$
-6	2	1	$(x \ll 1) - (x \ll 3)$
55	2	2	$(x \ll 6) - (x \ll 3) - x$

Problem 2.41

Consider two cases:

- $m > 0$: In this case form A requires $n - m + 1$ shifts and $n - m$ additions while form B requires 2 shifts and 1 subtraction. So if $n = m$, form A is favorable since it requires only 1 shift and 0 additions which is less than the constant numbers required for form B. If $n = m + 1$, form A takes 2 shifts and 1 addition, so either form is equally efficient in this case. If $n > m + 1$ form A takes $n - m + 1 > 2$ shifts and $n - m > 1$ additions so form B is favorable in this case.
- $m = 0$: In this case the last bit does not cause a shift so form A requires n shifts and n additions, while form B takes 1 shift and 1 subtraction. The same three cases apply in the same way here, so the above analysis extends to this case as well.