CPSC 322: Introduction to Artificial Intelligence

CSPs: Stochastic Local Search Variants

Textbook reference: [4.7.3, 4.8]

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Credit: These slides are adapted from the slides of the previous offerings of the course. Thanks to all instructors for creating and improving the teaching material and making it available!

Announcements

- Final exam scheduled: **Dec 9 at 7:00pm**
- Reminder: Assignment 2 is due on **Oct 21 at 11:59pm**.
- Reminder: Midterm next Friday, **Oct 25 from 6pm to 7pm** in Woodward 2
- Midterm **practice questions** are available on Piazza.
- I will be holding **extra office hours** next week on Thursday during class time (5 to 6:30pm) in ICCS 185.

Midterm

- Included in midterm: Everything till today's lecture
- Planning won't be included in midterm
- Tips on studying for midterm
  - Make sure you understand the purpose and basic ideas of the different algorithms we have learned so far.
  - Go through lecture notes and the lecture learning outcomes and make sure you can do things that are expected of you
  - Go through the iclicker questions to test your understanding
  - Make use of extra office hours next week

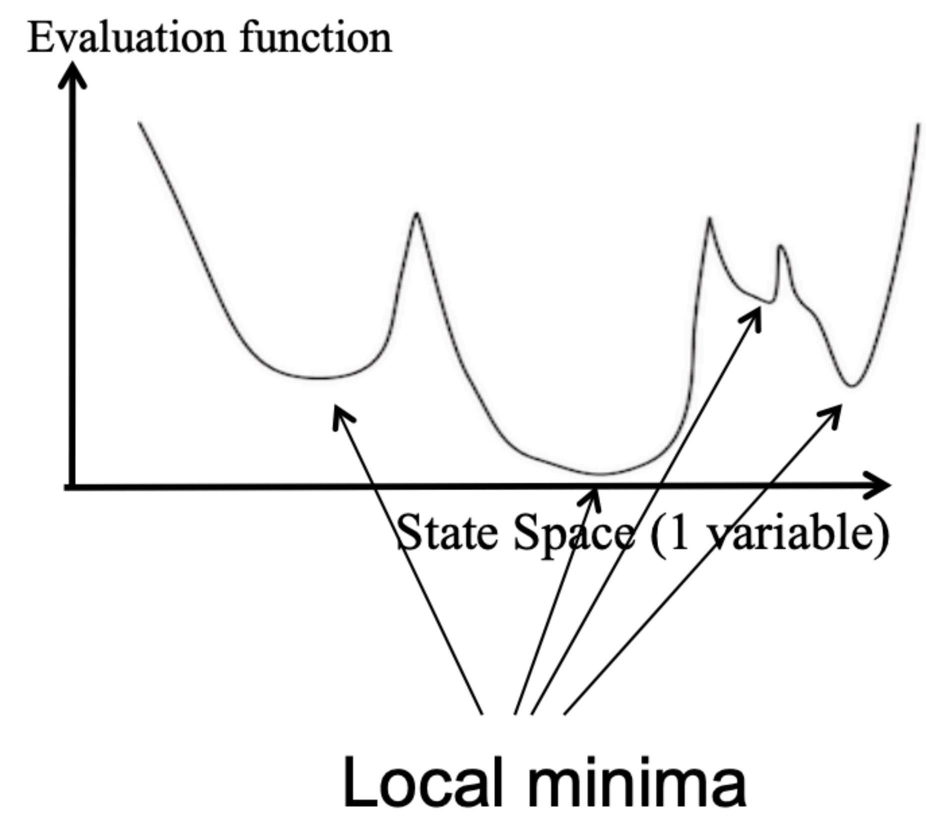
Lecture outline

- Recap stochastic local search (~5 mins) 👈
- SLS variants (~50 mins)
  - Tabu lists
  - Simulated Annealing
  - Beam search
  - Genetic algorithms
- Summary and wrap up (~5 mins)

Local search problem: Local minima

The primary problem associated with local search methods (i.e., greedy descent and hill climbing) is getting stuck in local minima/maxima.

The greedy methods always choose a "better" scoring neighbour and there is no such neighbour at local minima/maxima.



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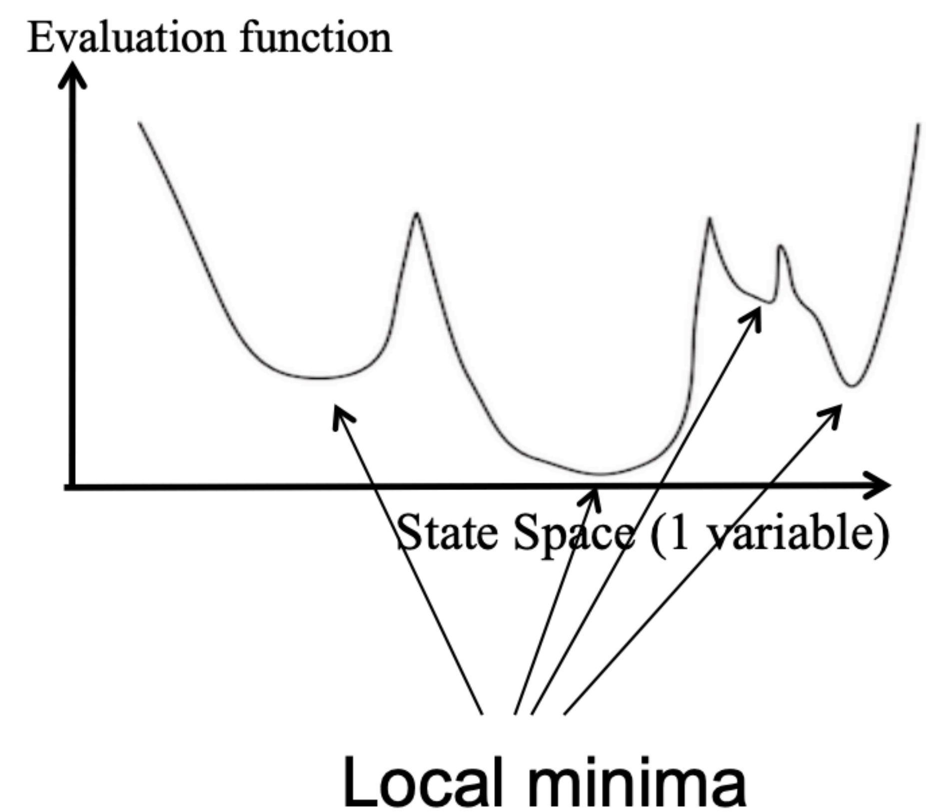


The greedy methods always choose a "better" scoring neighbour and there is no such neighbour at local minima/ maxima.

Solution: Stochastic local search!! Add randomness to avoid getting trapped in local minima! 💡

Greedy descent vs. Random sampling

- **Greedy descent** is good for finding local minima – bad for exploring new parts of the search space

- **Random sampling** is good for exploring new parts of the search space – bad for finding local minima

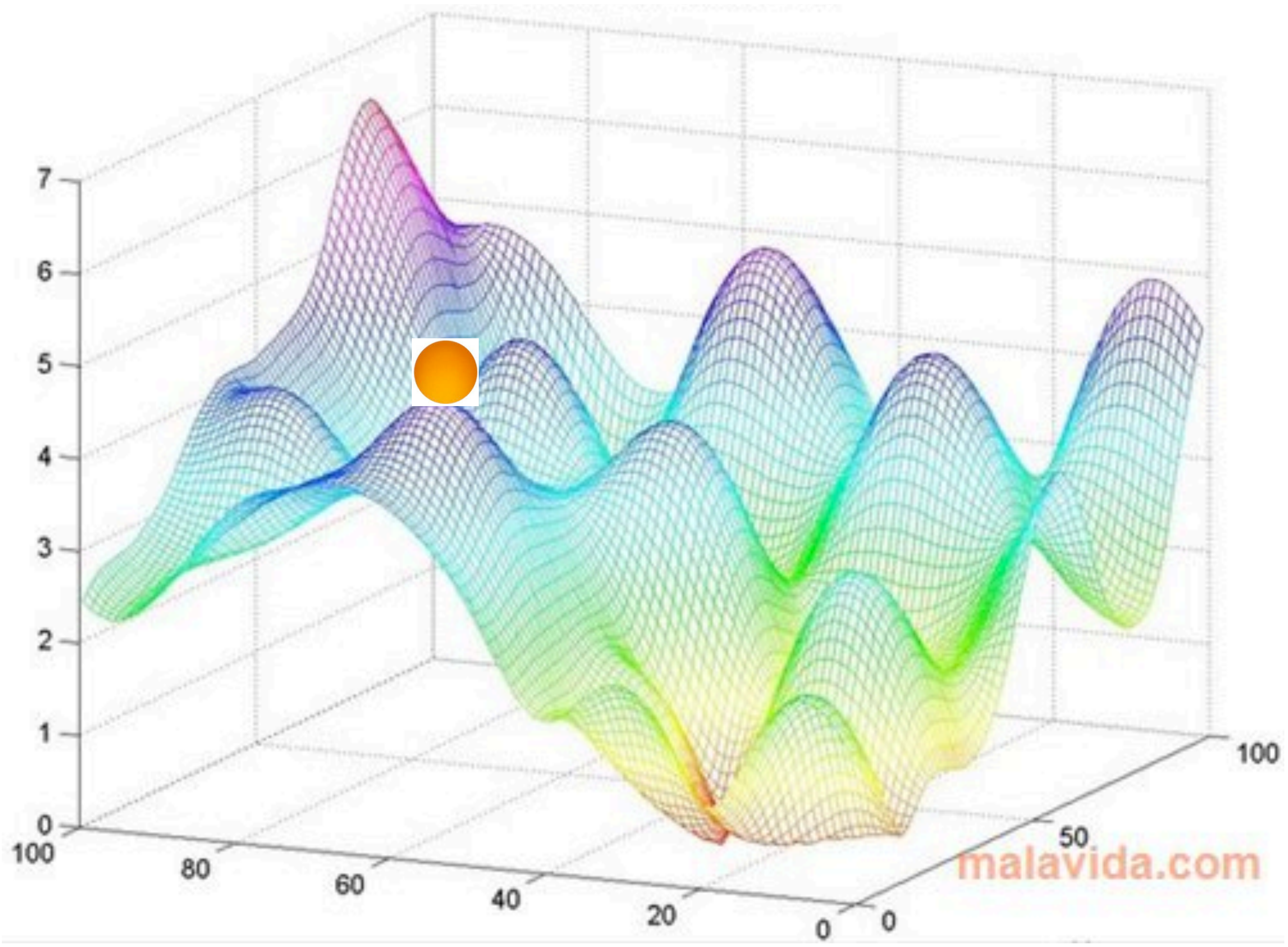
A mix of the two can work very well.

Adding randomness to greedy descent

- **Random restart**: reassign random values to all variables (i.e. start fresh)

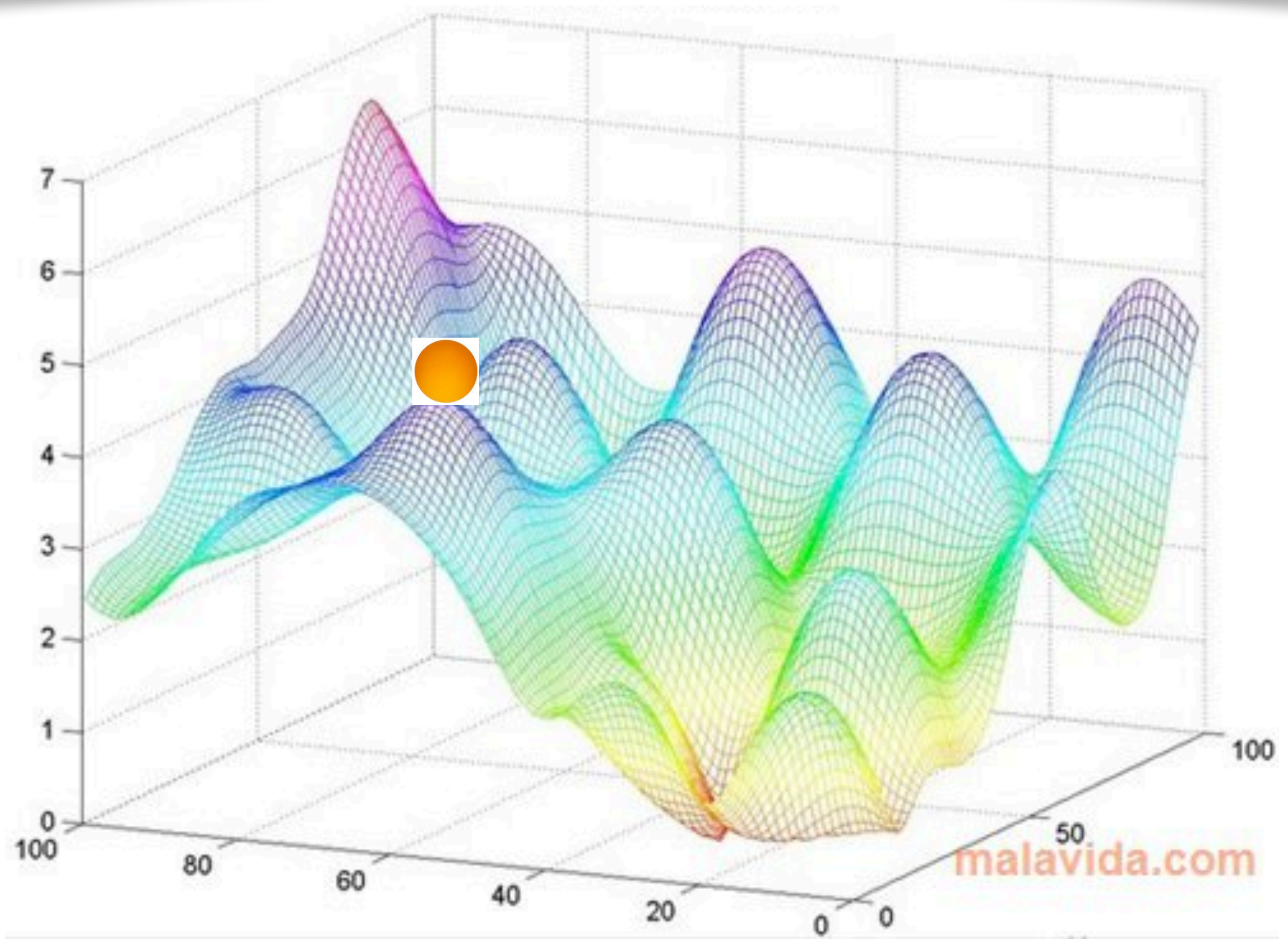
- **Random step**: move to a random neighbour

Adding randomness



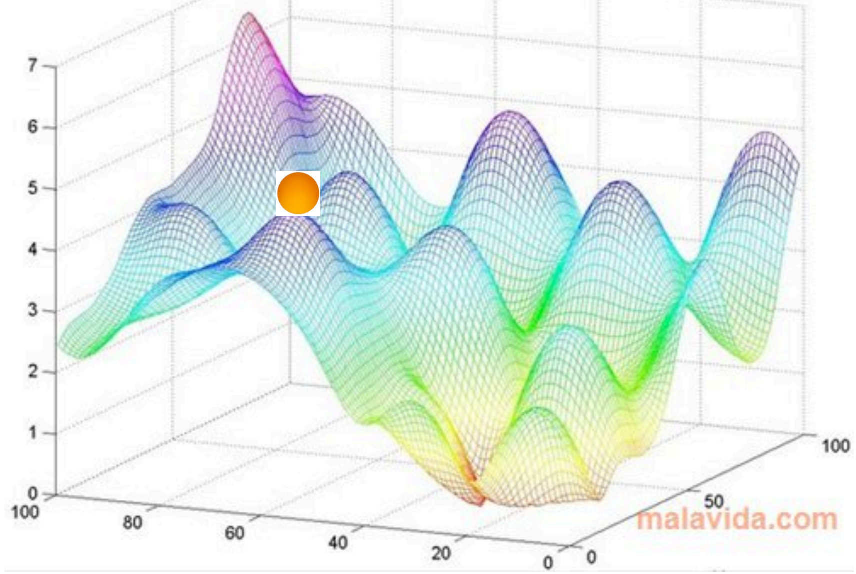
Adding randomness

Consider the task of getting a ping-pong ball into the deepest crevice



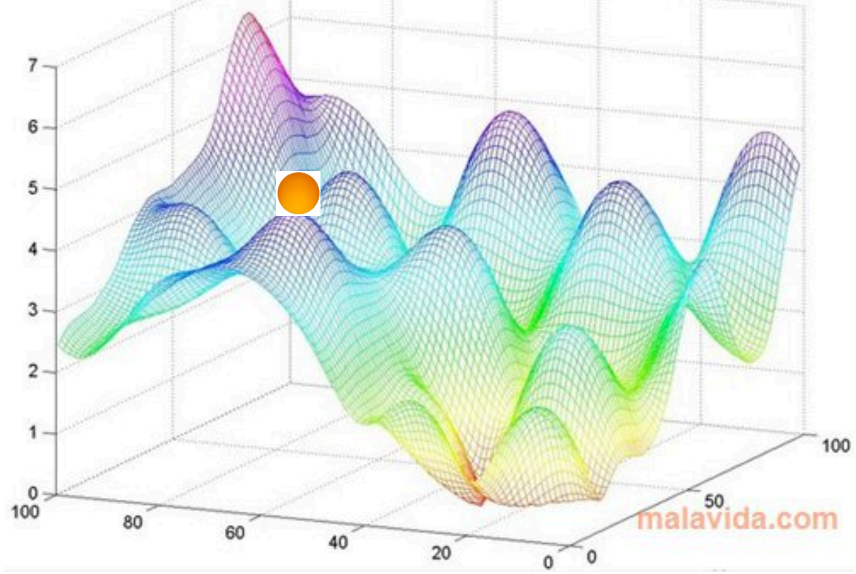
Adding randomness

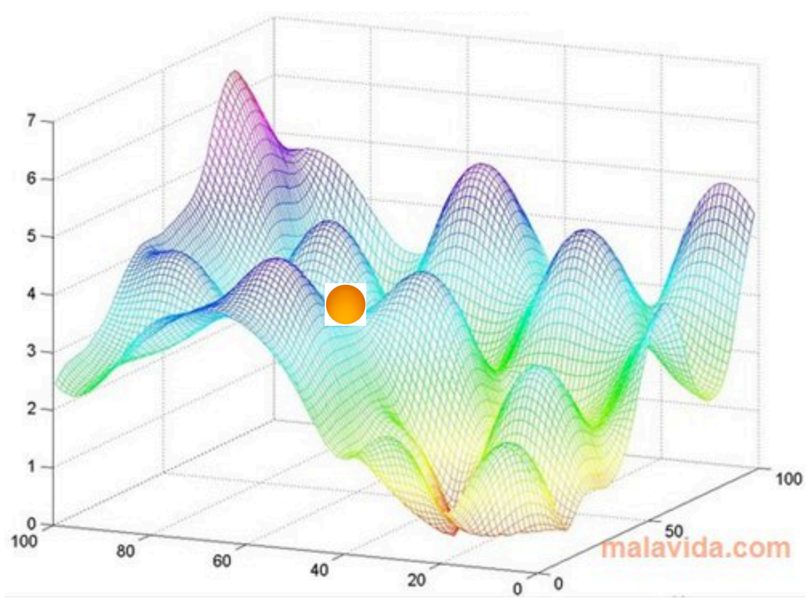
Current state



Adding randomness

Current state

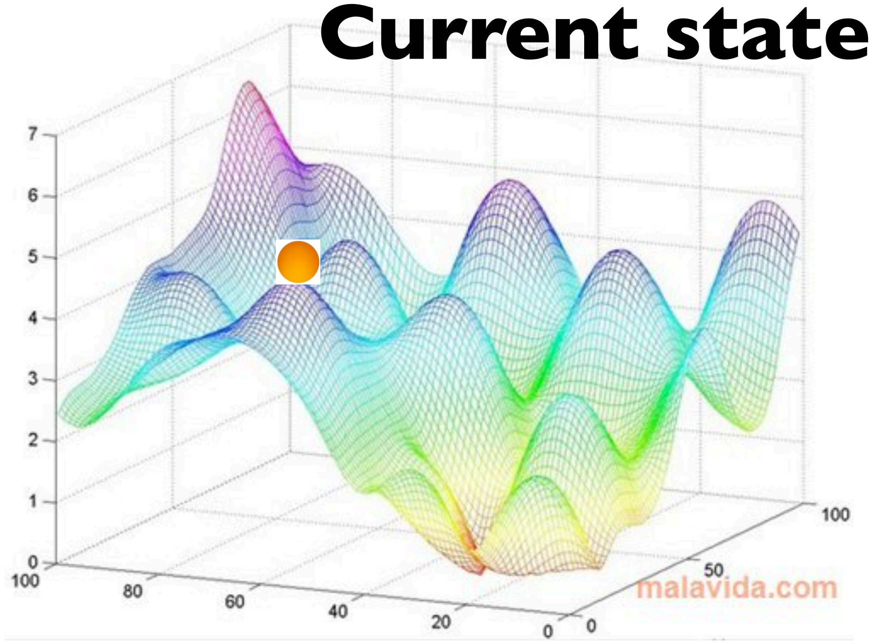


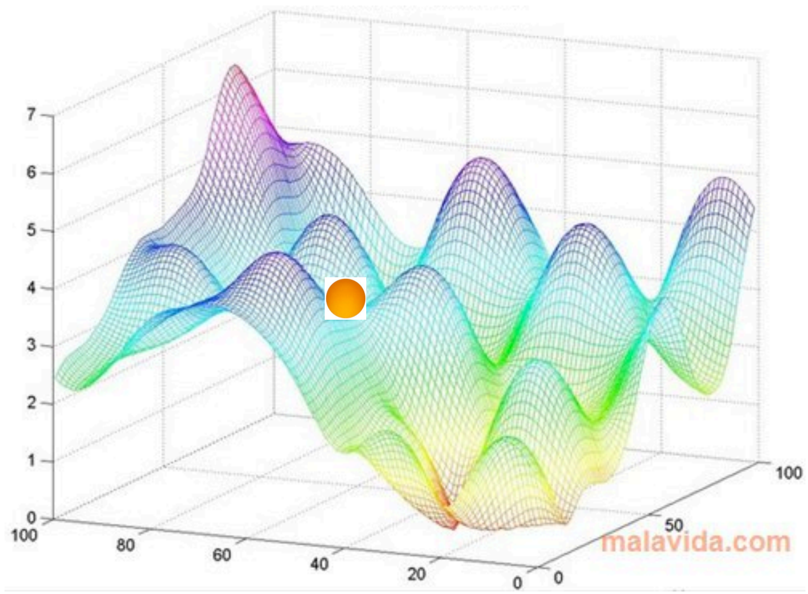


Greedy descent: Let it roll.
Move to the “best” neighbour.

Adding randomness

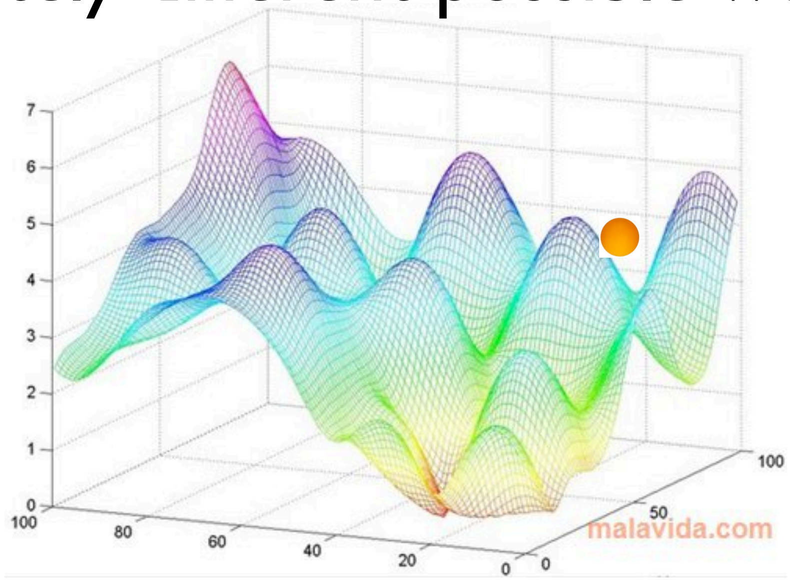
Current state





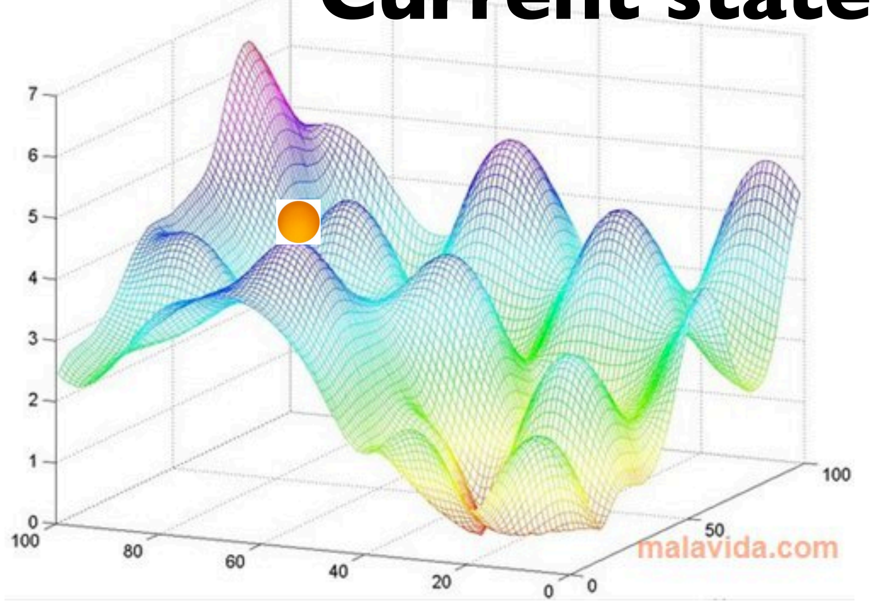
Greedy descent: Let it roll.
Move to the "best" neighbour.

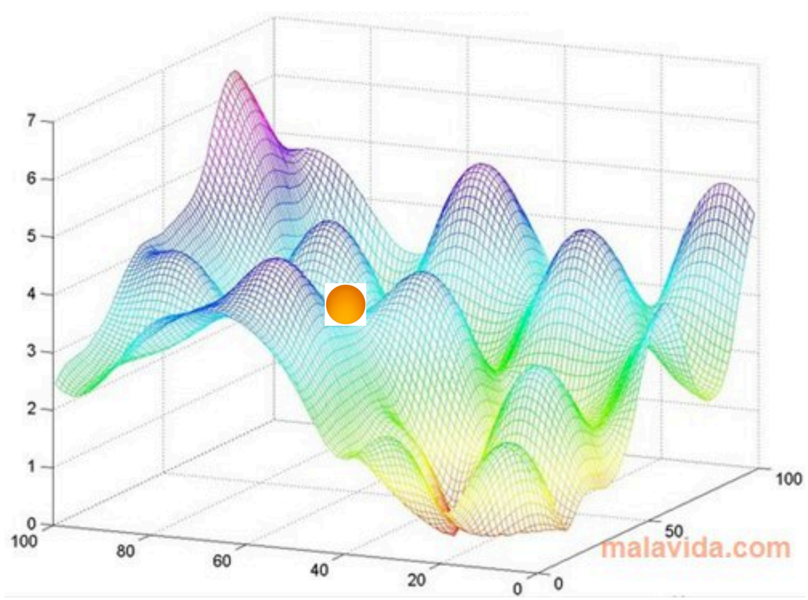
Random restart: Move to a completely different possible world



Adding randomness

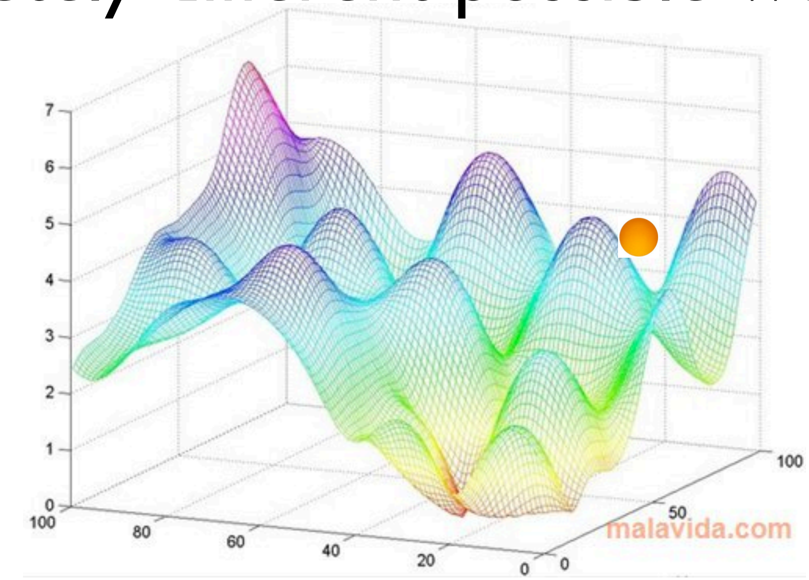
Current state



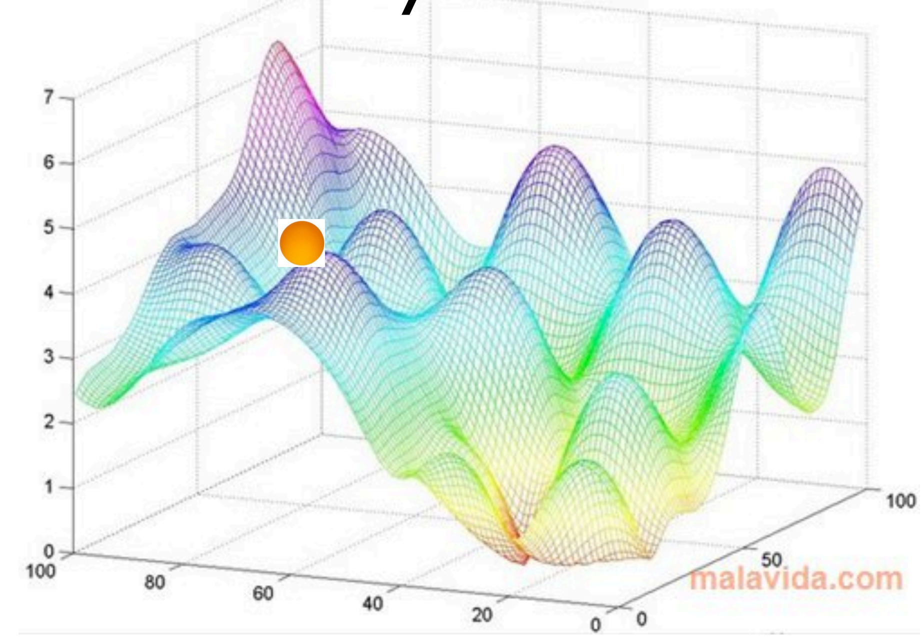


Greedy descent: Let it roll. Move to the “best” neighbour.

Random restart: Move to a completely different possible world



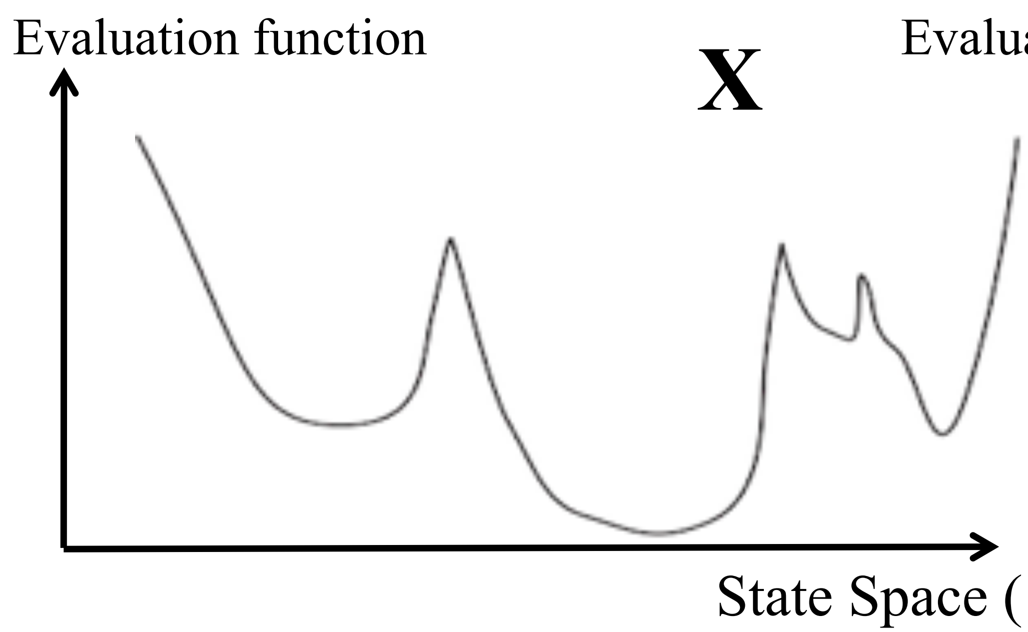
Random step: Move to a **random** neighbour, not necessarily the “best” one

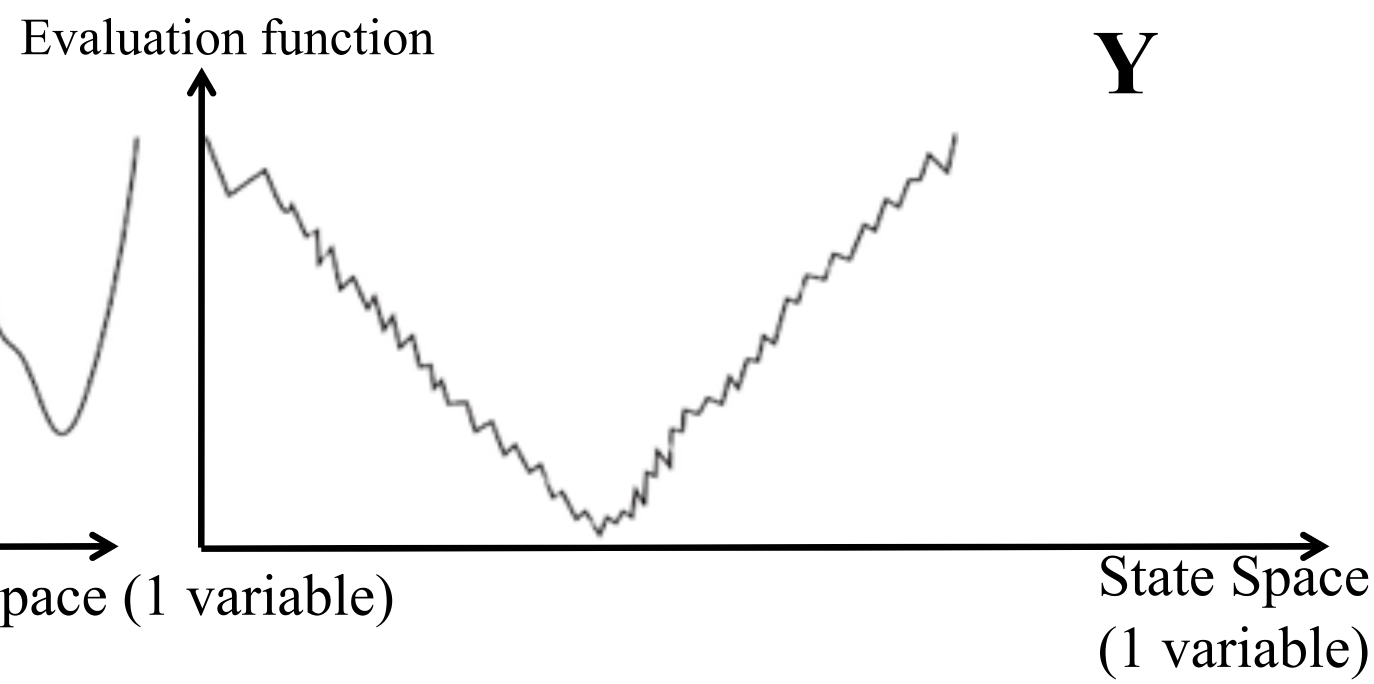


Greedy descent + randomness

Greedy descent with random restart for X

Greedy descent with random steps for Y





Today: Learning outcomes

From this lecture, students are expected to be able to:

- Explain the principles and ideas behind the following SLS variants
- Tabu list
- Simulated annealing
- Beam search
- Genetic algorithms

SLS variants

- There are many different SLS algorithms
- Each could easily be a lecture by itself
- We will only touch on each of them very briefly
- If you want to know more:
  - See this book "Stochastic Local Search: Foundations and Applications" by Holger Hoos & Thomas Stützle, 2004 (available in reading room)

Lecture outline

- Recap stochastic local search (~5 mins)
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  - Tabu lists 👈
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- Summary and wrap up (~5 mins)

Tabu list idea

Greedy descent + randomness + memory

- **Randomness:** accepts a worse neighbour if no better neighbour is available
- **Memory:** Avoid potentially problematic behaviour by maintaining a tabu list.
  - E.g., Returning to recently visited nodes or cycling

Tabu list

- Mark partial assignments as tabu ('taboo'= forbidden)
- Prevents repeatedly visiting the same (or similar) local minima
- Maintain a queue of k variable = value assignments that are tabu
  - E.g., when changing V7 's value from 2 to 4, we cannot change V7 back to 2 for the next k steps
  - k is a parameter that needs to be optimized empirically

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Simulated annealing

Key idea: Change the degree of randomness. Escape local minima by initially allowing some “bad” moves. Then gradually decrease the frequency of such moves.

Annealing: a metallurgical process where metals are heated and then slowly cooled.

- Analogy: start with a high “temperature”: a high tendency to take random steps

- Over time, cool down: more likely to follow the scoring function (only take random steps that are not too bad)

Temperature reduces over time, according to an annealing schedule

Simulated annealing: algorithm

Here's how it works (for maximizing $h(n)$):

- You are in node n. Pick a variable at random and a new value for it at random. You generate a neighbour node n'.

- If it **is** an "improvement" i.e., $h(n') >= h(n)$, adopt it.

- If it **isn't** an improvement, adopt it **probabilistically** depending on the difference and the temperature T.

  - We move to n' with probability: $e^{\frac{h(n') - h(n)}{T}}$

Pair-share (2 mins)

If it **isn't** an improvement, adopt it probabilistically depending on the difference and the temperature T.

- We move to n' with probability: $e^{\frac{h(n') - h(n)}{T}}$

Discuss with your neighbour the effect of higher and lower values of T on the probability of moving to n'.

Simulated annealing: "temperature" parameter

iclicker.

If it **isn't** an improvement, we move to n' with probability: $e^{\frac{h(n') - h(n)}{T}}$

Having a higher temperature T means that the algorithm is

A. more likely to move to n' if it is "better" than n

B. less likely to move to n' if it is "better" than n

C. more likely to move to n' if it is "worse" than n

D. less likely to move to n' if it is "worse" than n

Simulated annealing: "temperature" parameter

i▸clicker

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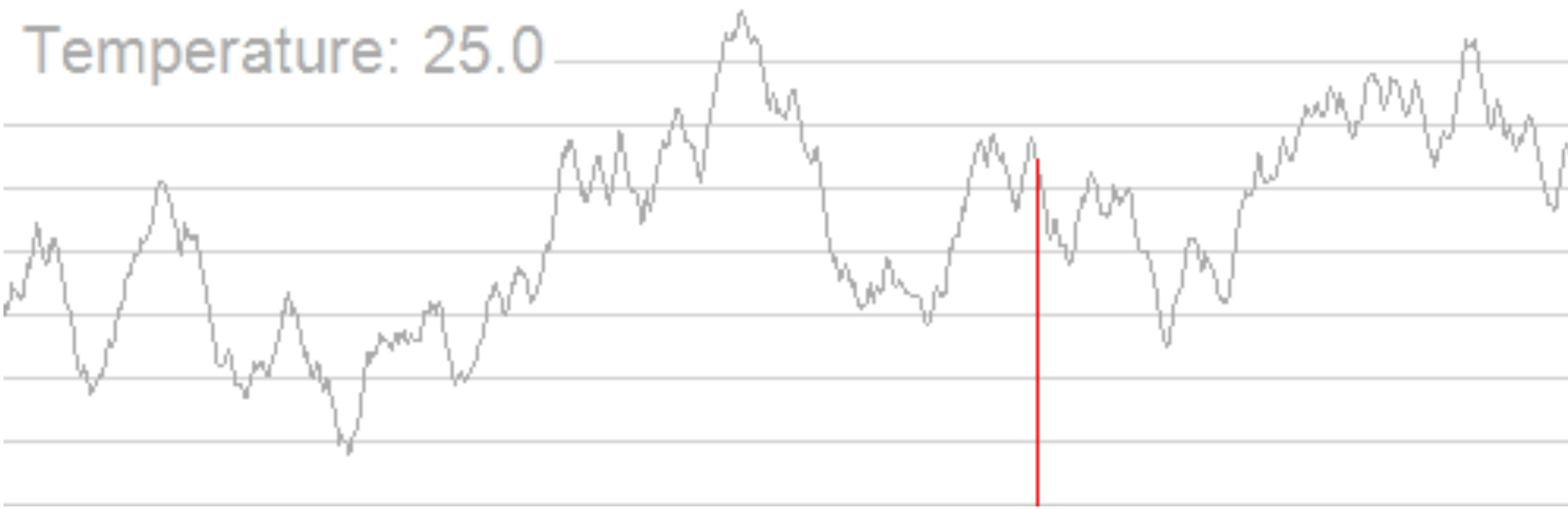
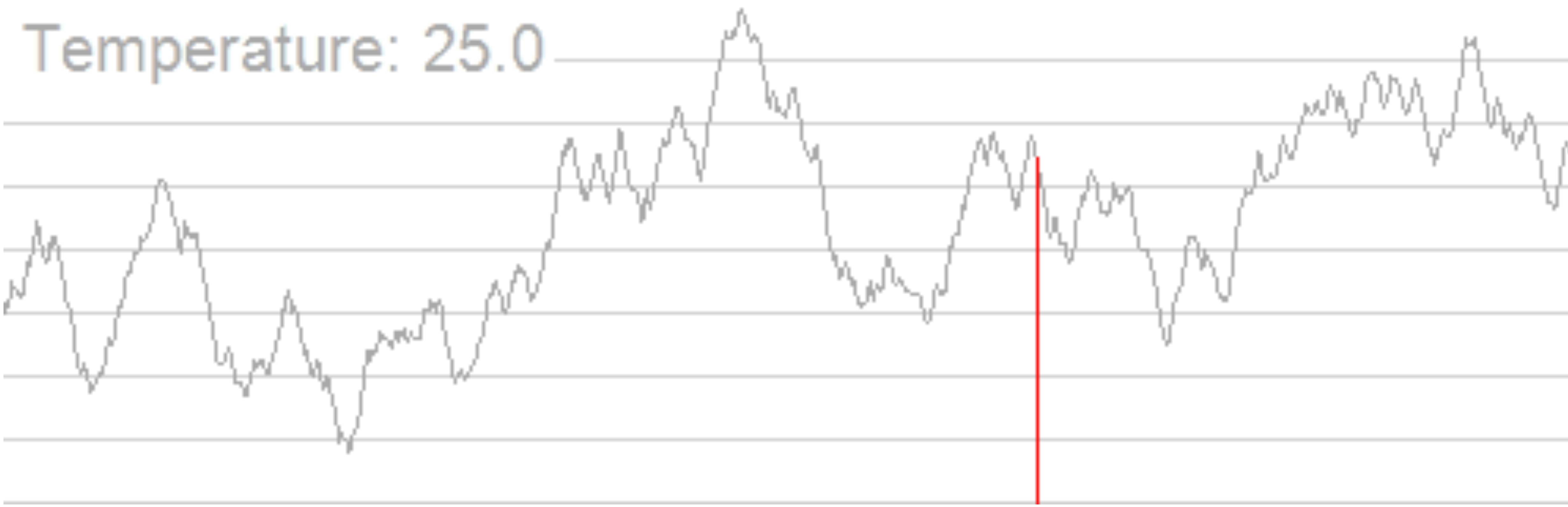
Simulated annealing: T parameter



Source: https://en.wikipedia.org/wiki/Simulated_annealing

Simulated annealing: T parameter



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Simulated annealing: temperature parameter

If it **isn't** an improvement, we move to n' with probability: $e^{\frac{h(n') - h(n)}{T}}$

Higher T → higher probability for given $h(n') - h(n)$

Higher $|h(n') - h(n)|$ → smaller probability for given T

Properties of simulated annealing

- One can prove: If T decreases slowly enough, then simulated annealing search will find a global optimum with probability approaching 1.
- Widely used in VLSI layout, airline scheduling, etc.
- Finding the ideal cooling schedule is unique to each class of problems
  - Want to move off of local maxima but not global maxima
  - Want to minimize computation time while taking enough time to ensure we reach a good minimum

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Population based SLS

Key Idea: maintain a population of k individuals

- Often we have more memory than the one required for current node (+ best so far + tabu list)
- Maintain a population of k individuals
- At every stage, update your population.
- Whenever one individual is a solution, report it.

Beam search

- Keep not just 1 assignment, but k assignments at once
- A 'beam' with k different assignments (k is the 'beam width')
- Useful information is passed among the k parallel search threads
- The neighbourhood is the union of the k neighbourhoods
- At each step, keep only the k best neighbours
- Never backtrack

The value of k lets us limit space and parallelism.

Beam search

Keep not just 1 assignment,
but k assignments at once

When k = 1, beam search is identical to

A. Best-first search

B. Greedy descent

C. Breadth-first search

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Keep not just 1 assignment,
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At step m, the beam contains all nodes m steps away from the start node, like breadth-first search, but expanding a whole level of the search tree at once

Beam search: Problems

- Troublesome case: If one individual generates several good neighbours and the other $k-1$ all generate bad successors…
  - Lack of diversity
  - The next generation will comprise very similar individuals

Stochastic beam search

Non-stochastic Beam Search may suffer from lack of diversity among the k individual (just a more expensive hill climbing)

Stochastic version alleviates this problem

- Selects k individuals at random
- But probability of selection proportional to their value (according to scoring function)

Stochastic beam search

Stochastic beam search selects the k individual at random but probability of selection is proportional to their value (according to scoring function).

Suppose node n has m neighbours: $\{n_1, n_2, \ldots, n_m\}$
Let the scoring function be h. Then the probability of selecting n_j is

A. $\dfrac{n_j}{\sum_i n_i}$

B. $\dfrac{h(n_j)}{\sum_i h(n_i)}$

C. $\dfrac{\sum_i h(n_i)}{h(n_j)}$

Stochastic beam search

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Stochastic beam search: advantages

- It maintains diversity in the population.
- Biological metapor (asexual reproduction):
  - Each individual generates "mutated" copies of itself (its neighbours)
  - The scoring function value reflects the fitness of the individual. The higher the fitness the more likely the individual will survive (i.e., the neighbour will be in the next generation)

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Genetic algorithms

Key Idea: Like stochastic beam search, but pairs of nodes are combined to create the offspring.

Genetic algorithms is a large research field

- Motivated by theory of evolution
- Appealing biological metaphor
- Several conferences are devoted to the topic

Genetic algorithms

- Start with k randomly generated individuals (population)
- An individual is represented as a string over a finite alphabet (often a string of 0s and 1s)
- A successor is generated by combining two parent individuals (loosely analogous to how DNA is spliced in sexual reproduction)
- Evaluation/Scoring function (fitness function): Higher values for better individuals.
- Produce the next generation of individuals by **selection**, **crossover**, and **mutation**

Genetic algorithms

Like stochastic beam search, but pairs of nodes are combined to create the offspring. For each generation:

- Choose pairs of nodes n_1 and n_2 ('parents'), where nodes with low $h(n)$ are more likely to be chosen from the population
- For each pair (n_1, n_2), perform a cross-over: create offspring combining parts of their parents
- Mutate some values for each offspring.
- Select from previous population and all offspring which nodes to keep in the population

Genetic algorithms: Crossover example

- Given two nodes with 8-digit representation:
  $n_1 = 13573333$ and $n_2 = 24682222$

- Select i at random and form two offspring:

- For example, for $i = 4$, what would be the offspring?

Genetic algorithms: Crossover example

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  $n_1 = 13573333$ and $n_2 = 24682222$

- Select i at random and form two offspring:

- For example, for $i = 4$, the two offsprings would be
  $n_3 =$ 13572222 and $n_4 =$ 24683333

Genetic algorithms: Crossover

Given two nodes:

$X_1 = a_1, X_2 = a_2, \ldots, X_m = a_m$ and

$X_1 = b_1, X_2 = b_2, \ldots, X_m = b_m$

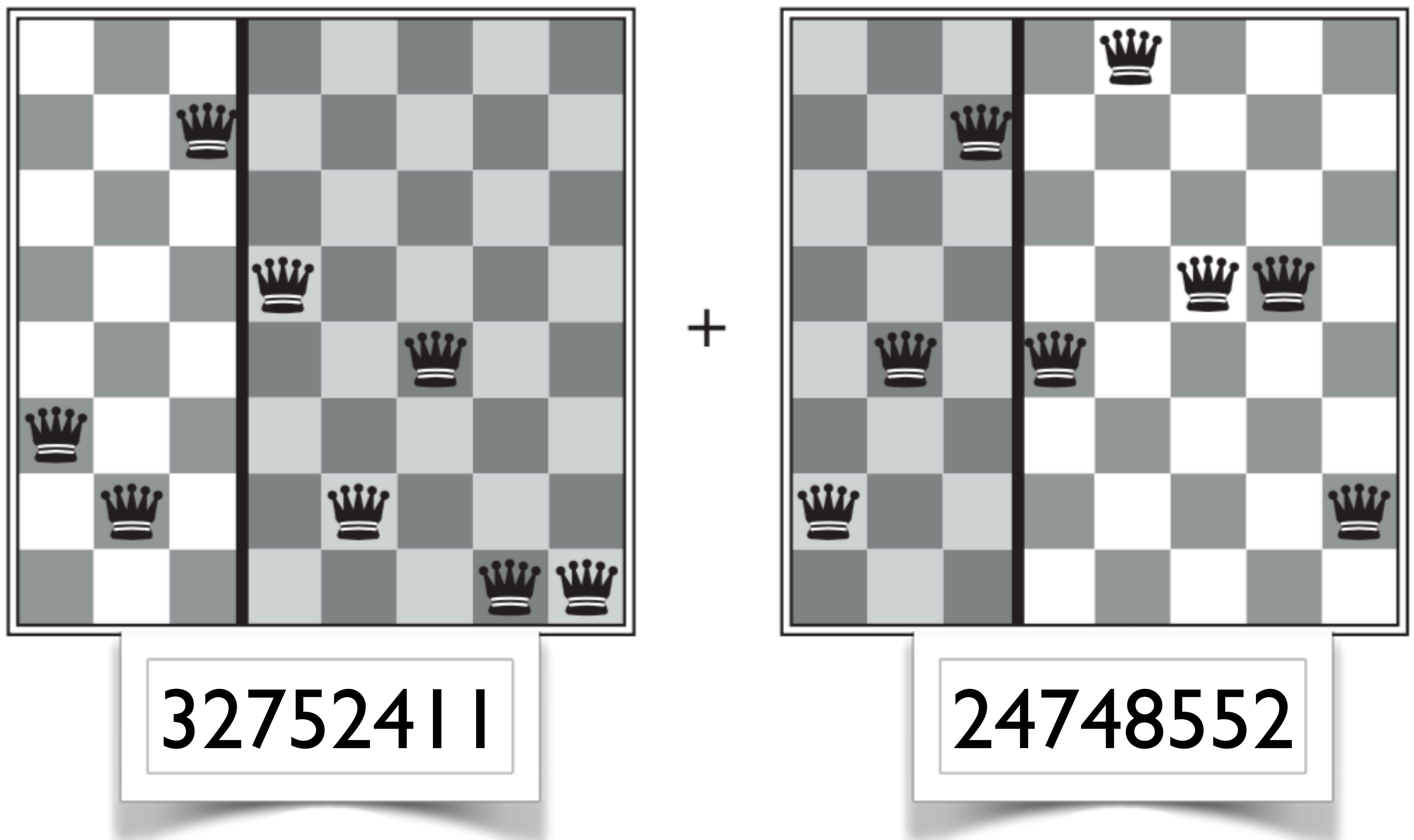
Select i at random and form two offspring:

$X_1 = a_1, \ldots, X_i = a_i, X_{i+1} = b_{i+1}, \ldots, X_m = b_m$ and

$X_1 = b_1, \ldots, X_i = b_i, X_{i+1} = a_{i+1}, \ldots, X_m = a_m$

Many different crossover operators are possible.

Genetic algorithms: Example 8-Queen



State: String over finite alphabet (8-digit representation)

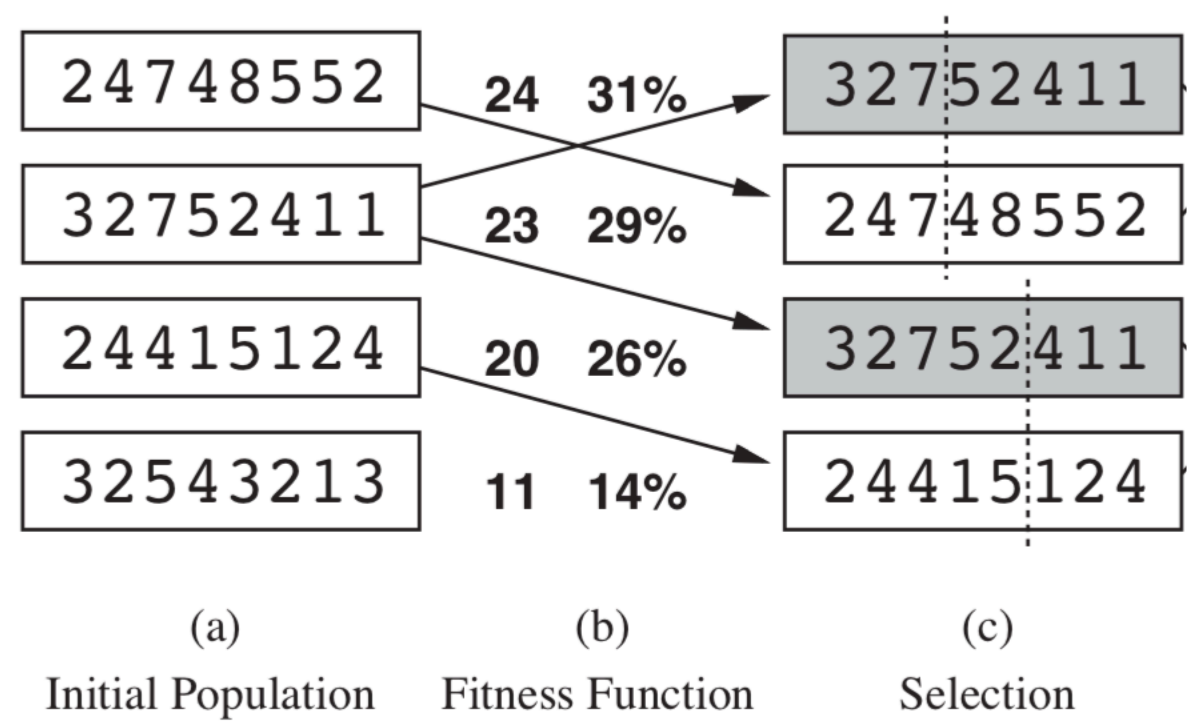
Fitness function: higher values means better states

Genetic algorithms: Example 8-Queen

Selection: common strategy, probability of being chosen for reproduction is directly proportional to fitness score (as with beam search)

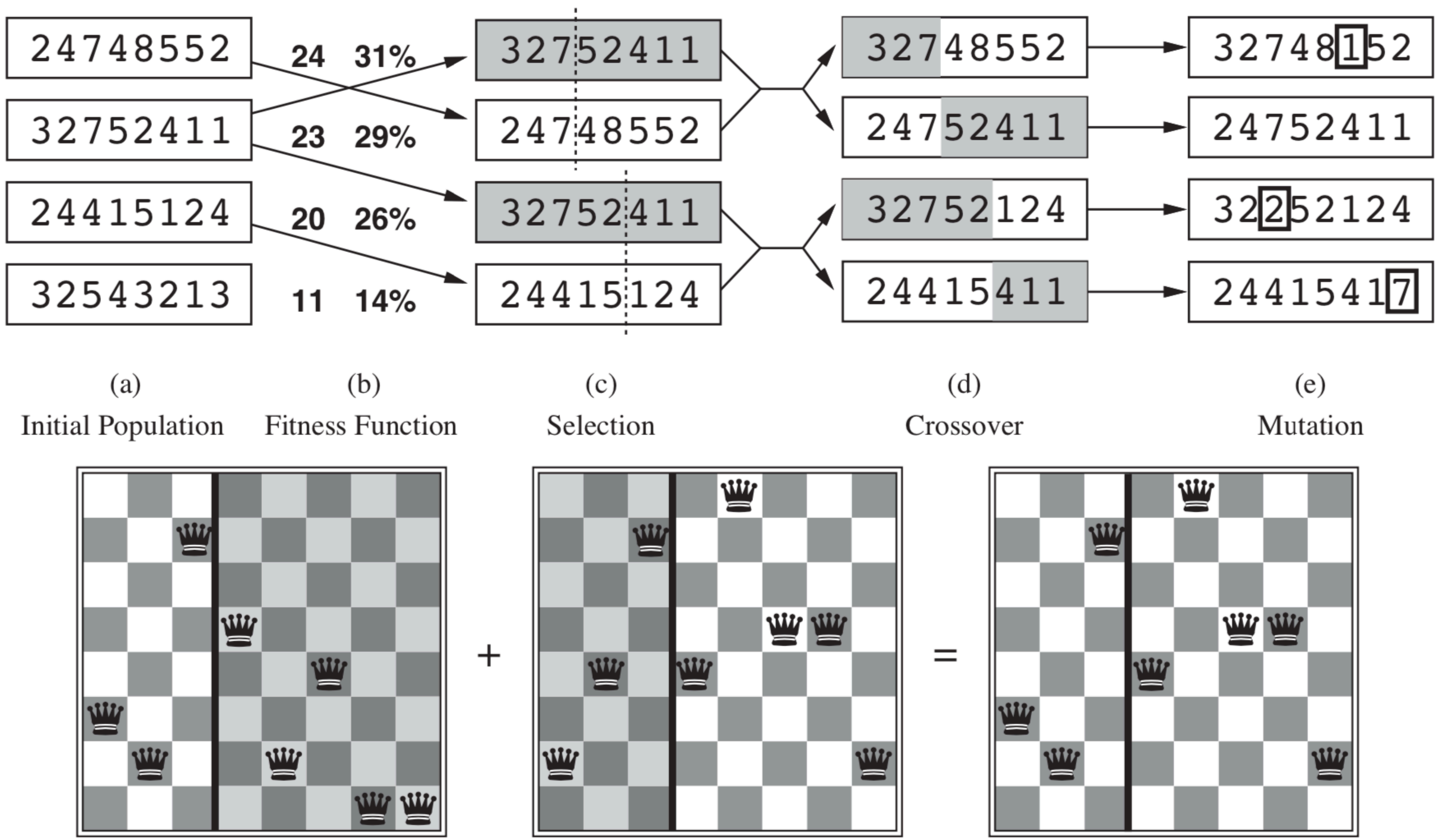
Fitness function: Number of non-attacking queens. Higher value means better state.

$$24/(24 + 23 + 20 + 11) = 31\%$$
$$23/(24 + 23 + 20 + 11) = 29\%$$



Genetic algorithms: Example 8-Queen

Reproduction: Cross-over and mutation



Genetic algorithms: Conclusion

- Their performance is very sensitive to the choice of state representation and fitness function
- Extremely slow (not surprising as they are inspired by evolution!)
- Ongoing work to make them practical for real-world problems

Parameters in SLS

Algorithm	**Parameters**
Simple SLS	Neighbourhoods, variable and value selection heuristics, percentages of random steps, restart probability
Tabu search	Tabu length (or interval for randomized tabu length)
Simulated annealing	Temperature
Beam search	*k* (beam width)
Genetic algorithms	Population size, mating scheme, cross-over operator, mutation rate

Revisit: learning outcomes for local search

- Implement local search for a CSP.
- Implement different ways to generate neighbours
- Implement scoring functions to solve a CSP by local search through either greedy descent or hill-climbing.
- Implement SLS with
    - random steps (1-step, 2-step versions) and random restart
- Compare SLS algorithms with runtime distributions
- Understand principles of types of SLS algorithms

A rough CPSC 322 overview

Representation and reasoning

Environment

Problem		Deterministic	Stochastic
Static	Constraint satisfaction	Arc consistency Variables + constraints Search	
	Query	Logics Search	Belief networks Variable elimination
Sequential	Planning	<u>STRIPS</u> Search	Decision networks Variable elimination Markov decision processes Value iteration

Coming up

Planning: How to select and organize a sequence of actions to achieve a given goal…

Readings for next class

- 6.1 Representing States, Actions, and Goals
- 6.2 Forward Planning
- 6.4 Planning as a CSP