CPSC 322: Introduction to Artificial Intelligence

Decision Theory: Introduction to Decision Networks

Textbook reference: [9.2]

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University of British Columbia

Credit: These slides are adapted from the slides of the previous offerings of the course. Thanks to all instructors for creating and improving the teaching material and making it available!

TA evaluations (~10 mins)

Pick up a form and a pencil and fill out TA evaluations.
Evaluate the TAs you interacted with the most
(one form per TA)

Kyle Clarkson **Beriwan** Ravandi **Federico** Arribas **Ganesh** Jawahar

Raymond Li **Siddhesh** Khandelwal Tzu-yun (**Ariel**) Shann

Announcements

- Teaching evaluations are open. You should have received an email.
  - I am teaching an undergrad course for the first time and I will very much appreciate constructive feedback.
- Final exam
  - **Time:** Dec 9 at 7:00pm and **Location:** SRC A
  - Final is cumulative. Refer to midterm and final practice exam question on Piazza.
- Assignment 4 is due on **Friday, Nov 29th, 11:59 PM**

A rough CPSC 322 overview

Representation and reasoning

Problem		Environment: Deterministic	Environment: Stochastic
Static	Constraint satisfaction	Variables + constraints Arc consistency Search	
Static	Query	Logics Search	Belief networks Variable elimination
Sequential	Planning	<u>STRIPS</u> Search	Decision networks Variable elimination Markov decision processes Value iteration

Lecture outline

- Assignment 4 related 👈🏻
- One-off decision example
- Utilities / Preferences and optimal decisions
- Single-stage decision network
- Optimal decisions using variable elimination

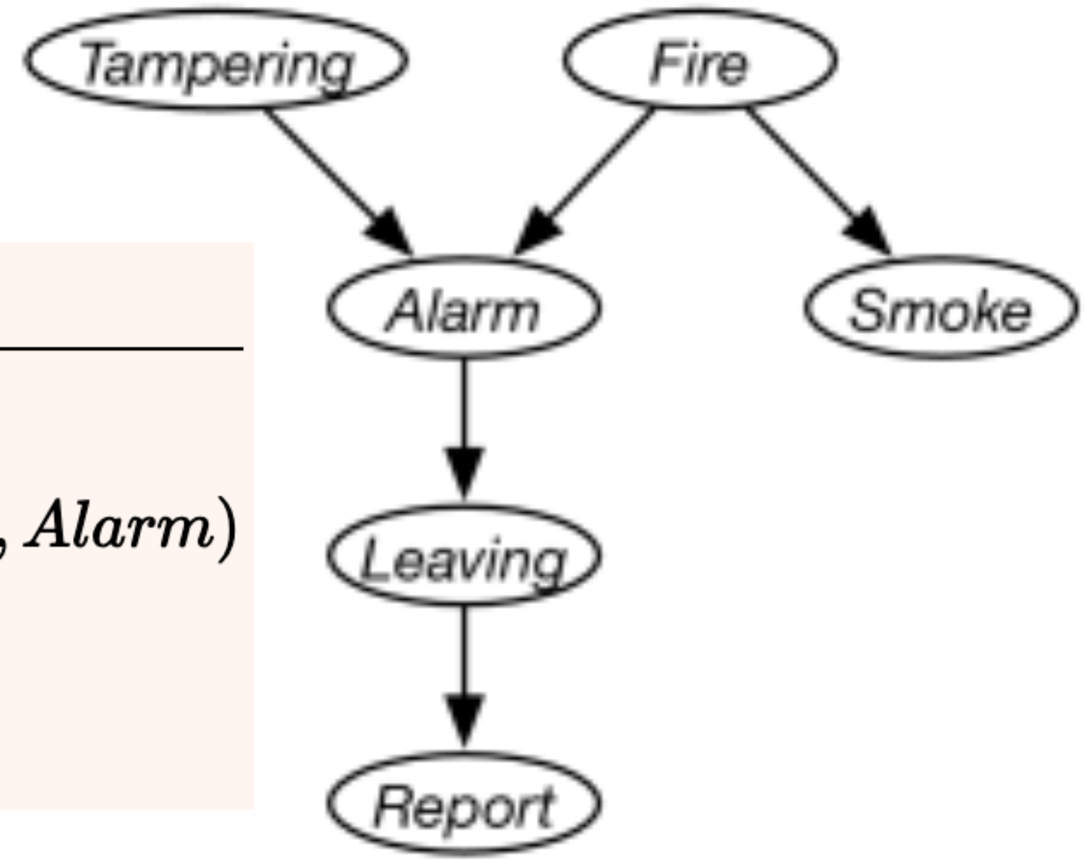
Hint for A4 question 4

You may use AIspace for this question.

- Create a belief network similar to the one from the assignment
- Make observation
- Query on the variable of interest and apply VE
  - Prune irrelevant variables
  - Project observations
  - Sum out variables
  - Multiply
  - Normalize

Hint for A4 question 4

$P(Tampering \mid Smoke = true \wedge Report = true)$



$Conditional Probability$	$Factor$
$P(Tampering)$	$f_0(Tampering)$
$P(Fire)$	$f_1(Fire)$
$P(Alarm \mid Tampering, Fire)$	$f_2(Tampering, Fire, Alarm)$
$P(Smoke = yes \mid Fire)$	$f_3(Fire)$
$P(Leaving \mid Alarm)$	$f_4(Alarm, Leaving)$
$P(Report = yes \mid Leaving)$	$f_5(Leaving)$

$$\sum_L f_5(L) \sum_A f_4(A, L) \sum_F f_0(F) f_2(T, F, A) f_3(F)$$

Today: Learning outcomes

From this lecture, students are expected to be able to:

- Compare and contrast stochastic single-stage (one-off) decisions vs. multistage (sequential) decisions
- Define a utility function on possible worlds
- Define and compute optimal one-off decisions
- Represent one-off decisions as single stage decision networks
- Compute optimal decisions by variable elimination

Decisions under uncertainty: Intro

Earlier in the course, we focused on decision making in deterministic domains

- Search/CSPs: single-stage decisions
- Planning: sequential decisions

Now we face stochastic domains

- So far we've considered how to represent and update beliefs
- What if an agent has to make decisions under uncertainty?
- Making decisions under uncertainty is important
- We mainly represent the world probabilistically so we can use our beliefs as the basis for making decisions

Decisions under uncertainty: Intro

An agent's decision will depend on

What **actions** are available

What **beliefs** the agent has

Which **goals** the agent has

Planning under uncertainty: Intro

Planning how to select and organize a sequence of actions/ decisions to achieve a given goal.

Deterministic goal: A possible world in which some propositions are true

Planning under uncertainty: how to select and organize a sequence of actions/decisions to "maximize the probability" of "achieving a given goal"

Goal under uncertainty: we'll move from all-or-nothing goals to a richer notion: rating how happy the agent is in different possible worlds.

Decisions under uncertainty: Intro

Obvious difference in representation: need to represent our uncertain **beliefs**

How to represent **actions** and **goals**?

- Actions will be pretty straightforward: **decision variables**
- Goals will be interesting: we'll move from all-or-nothing goals to a richer notion: rating **how happy the agent is** in different situations.
- Putting these together, we'll extend Bayesian Networks to make a new representation called **Decision Networks**

Single action vs. sequential action

- "Single" or "One-off": Set of one or more primitive decisions that can be treated as a single macro decision to be made before acting
- "Sequential": Set of one or more decisions, each of which depends on observations
    - Agents makes observations
    - Decides on an action
    - Carries out the action
    - Repeats with future decisions

Lecture outline

- Assignment 4 related
- One-off decision example 👈
- Utilities / Preferences and optimal Decision
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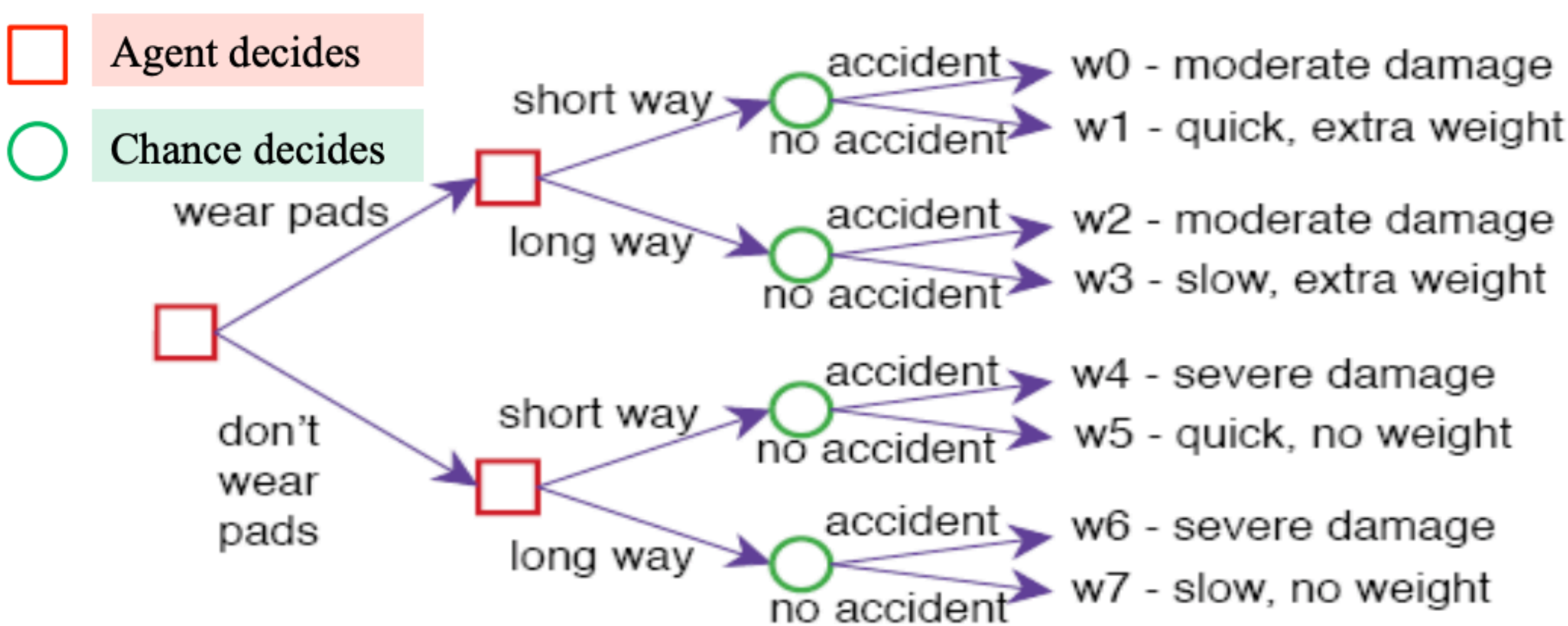
One-off decision example: Delivery robot

- Robot needs to reach a certain room.
- Going through stairs may cause an accident.
- It can go the short way down the stairs, or the long way down a long ramp (that reduces the chance of an accident but takes more time).
- The Robot can choose to wear pads to protect itself (in case of an accident) or not but pads slow it down.
- If there is an accident the Robot does not get to the room.

Delivery robot example

This scenario can be represented as the following decision tree

- The agent has a set of decisions to make (a macro-action it can perform)
- Decisions can influence random variables
- Decisions have probability distributions over outcomes



Delivery robot example

Decision variable 1: the robot can choose to wear pads

Yes: protection against accidents, but extra weight

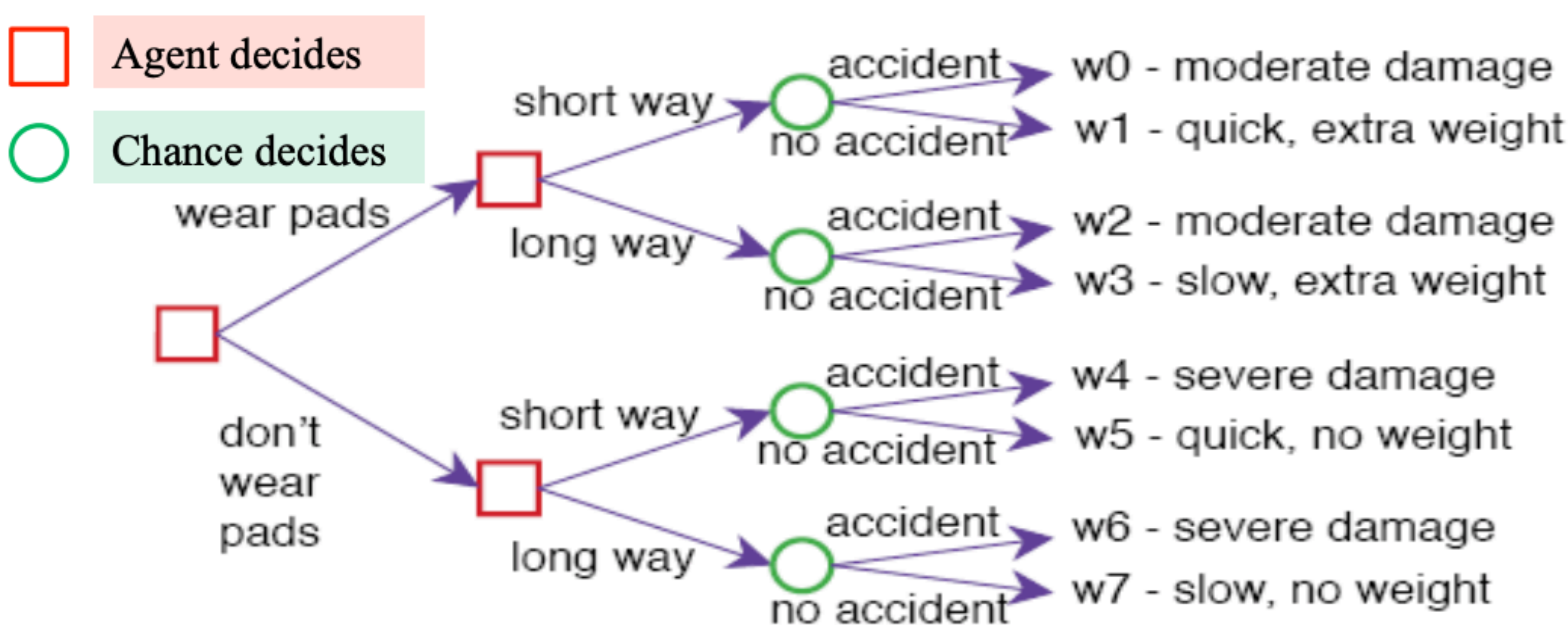
No: fast but no protection

Decision variable 2: the robot can choose the way

Short way: quick, but higher chance of accident

Long way: safe, but slow

Random variable: is there an accident?



Single action vs. sequence of actions

Single Action (aka One-Off Decisions)

One or more primitive decisions that can be treated as a single macro decision to be made before acting

E.g., "WearPads" and "WhichWay" can be combined into macro decision (WearPads, WhichWay) with domain {yes,no} × {long, short}

Sequence of Actions (Sequential Decisions)

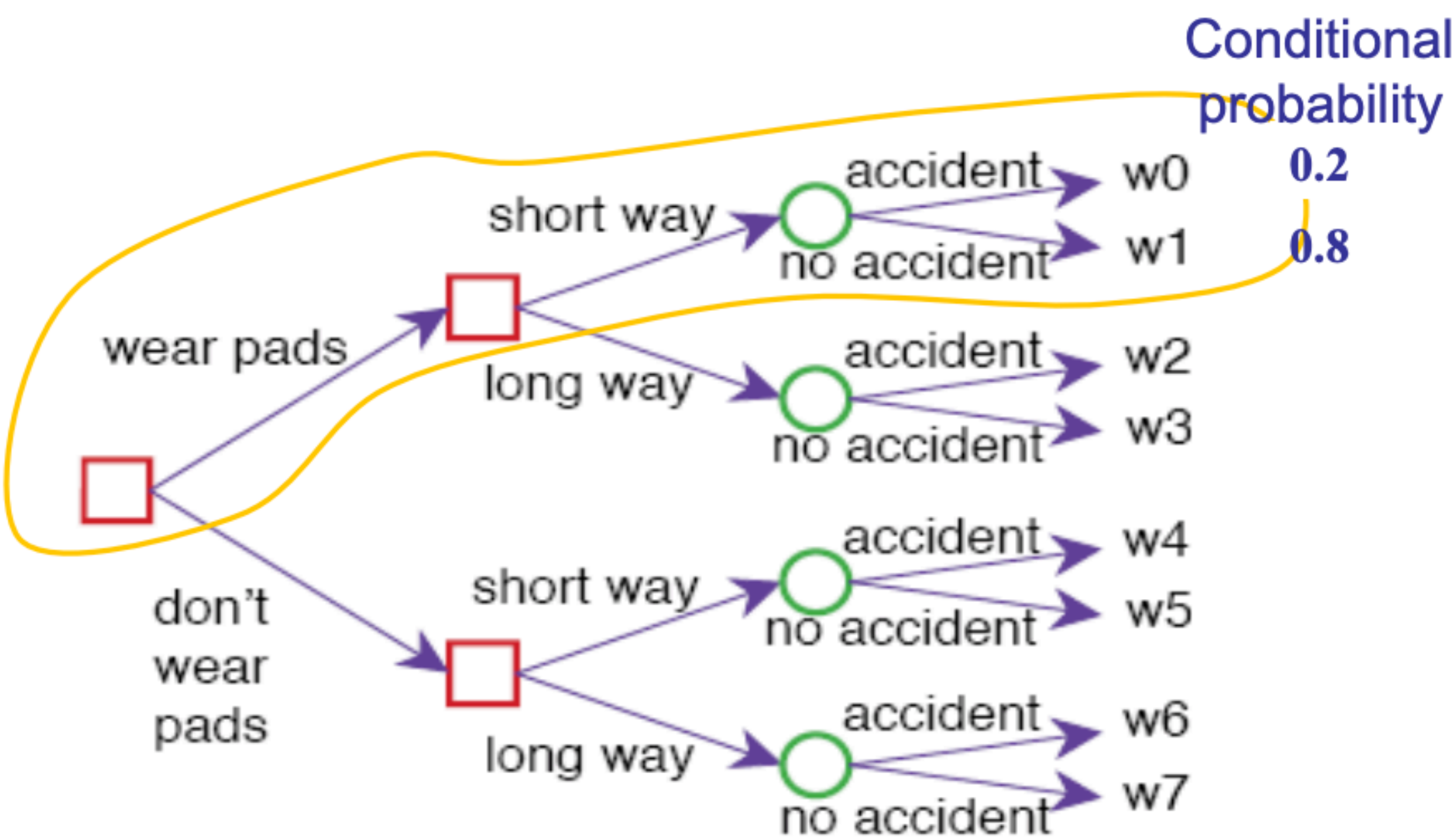
Repeat: make observations, decide on an action, carry out the action

Agent has to take actions not knowing what the future brings

This is fundamentally different from everything we've seen so far. Planning was sequential, but we still could still think first and then act

Possible worlds and decision variables

- A possible world specifies a value for each random variable and each decision variable
- For each assignment of values to all decision variables
  - the probabilities of the worlds satisfying that assignment sum to 1.



Possible worlds and decision variables

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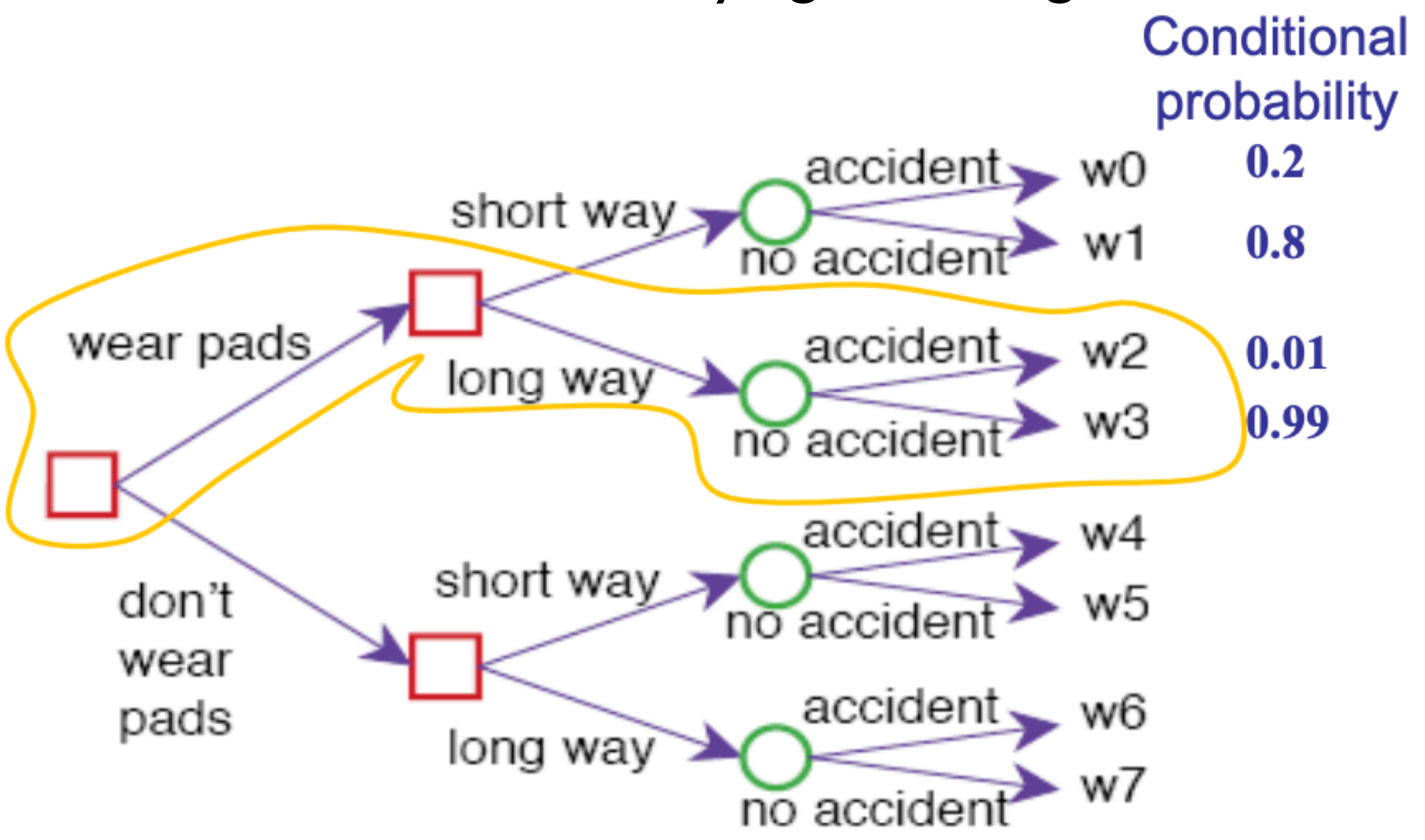

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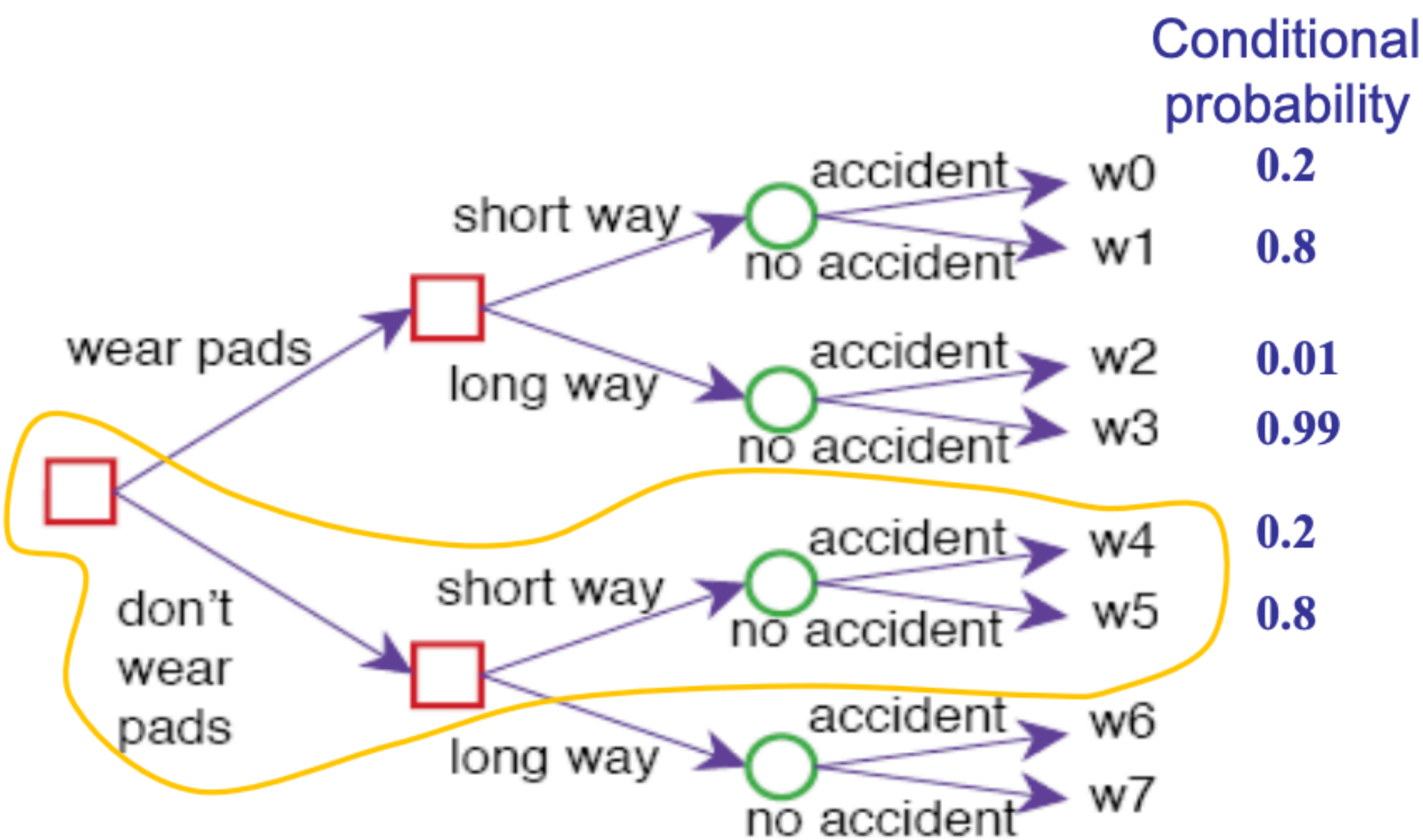

Possible worlds and decision variables

- A possible world specifies a value for each random variable and each decision variable
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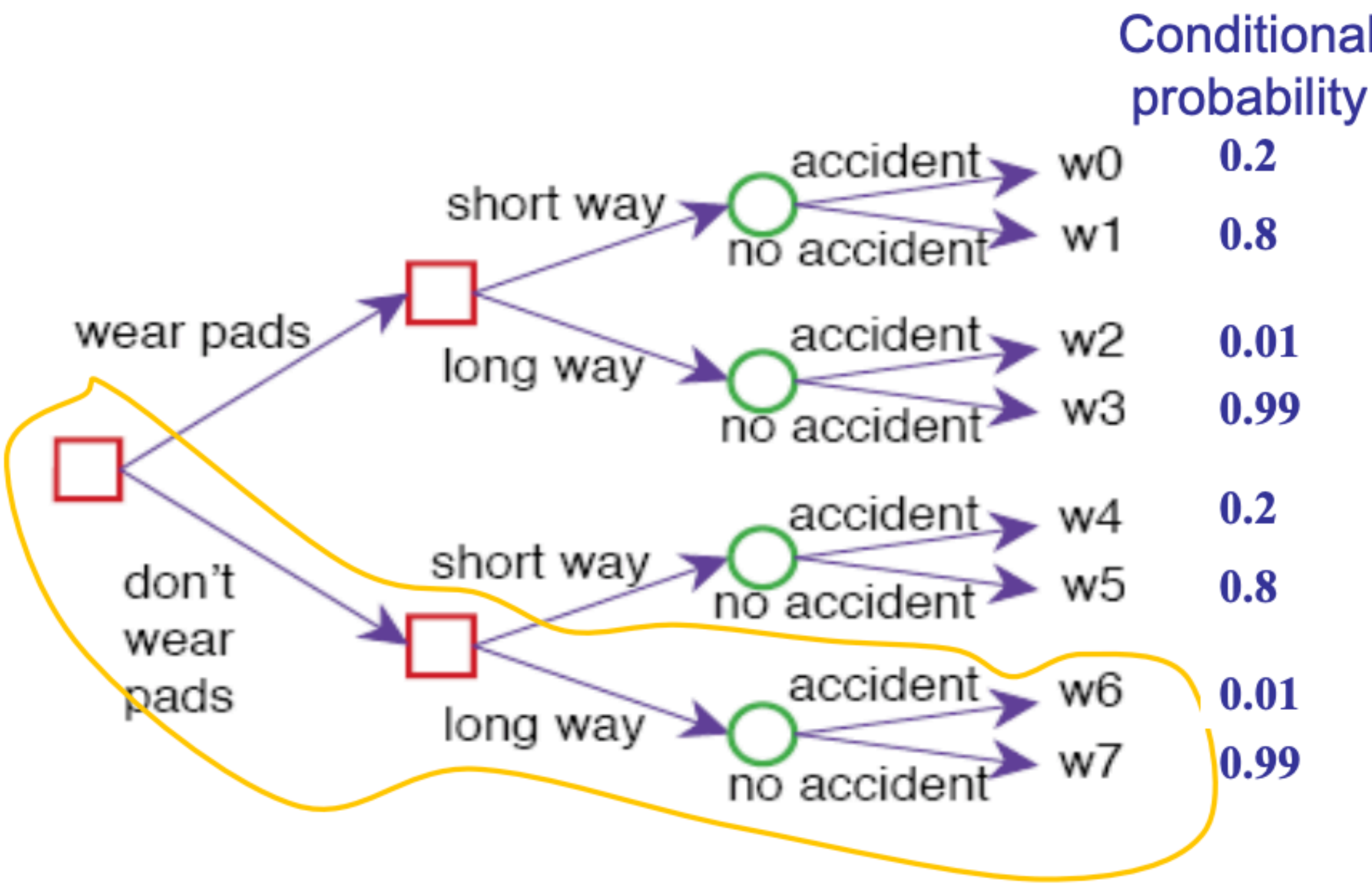


Lecture outline

- Assignment 4 related
- One-off decision example
- Utilities / Preferences and optimal decisions 👈
- Single-stage decision networks
- Optimal decisions using variable elimination

Utility

Utility is a measure of desirability of possible worlds to an agent

Utility U is a real-valued function such that $U(w)$ represents an agent's degree of preference for world w

Usually expressed by a number in $[0,100]$

Simple goals can still be specified

- Worlds that satisfy the goal have utility 100
- Other worlds have utility 0

Utility

Utilities can be more complicated

For example, in the robot delivery domains, they could involve

- Amount of damage
- Reached the target room?
- Energy left
- Time taken

Utility function

Utility is a measure of desirability of possible worlds to an agent. Utility U is a real-valued function such that $U(w)$ represents an **agent's degree of preference** for world w

Would this be a reasonable utility function for our Robot, who wants to reach the room?

A. Yes ✅
B. No

Which way	Accident	Wear Pads	Utility	World
short	true	true	18	w0, moderate damage
short	false	true	95	w1, reaches room, quick, extra weight
long	true	true	11	w2, moderate damage, low energy
long	false	true	75	w3, reaches room, slow, extra weight
short	true	false	3	w4, severe damage
short	false	false	100	w5, reaches room, quick
long	true	false	0	w6, severe damage, low energy
long	false	false	80	w7, reaches room, slow

Utility simple goals

i▸clicker.

How can the simple boolean goal "reach the room" be specified?

A.

Which way	Accident	Wear Pads	Utility
long	true	true	0
long	true	false	0
long	false	true	0
long	false	false	0
short	true	true	0
short	true	false	0
short	false	true	100
short	false	false	90

B.

Which way	Accident	Wear Pads	Utility
long	true	true	0
long	true	false	0
long	false	true	0
long	false	false	100
short	true	true	0
short	true	false	0
short	false	true	0
short	false	false	0

C. ✅

Which way	Accident	Wear Pads	Utility
long	true	true	0
long	true	false	0
long	false	true	100
long	false	false	100
short	true	true	0
short	true	false	0
short	false	true	100
short	false	false	100

Hint: If there is an accident the Robot does not get to the room.

Expected utility and conditional expected utility

Suppose $U(w)$ represents an agent's degree of preference for world w. How can we combine probability with utility?

Definition: The expected utility is

$$E[U] = \sum_w P(w)U(w)$$

Definition: The conditional expected utility given e is

$$E[U \mid e] = \sum_w P(w \mid e)U(w)$$

Combining probabilities and utilities

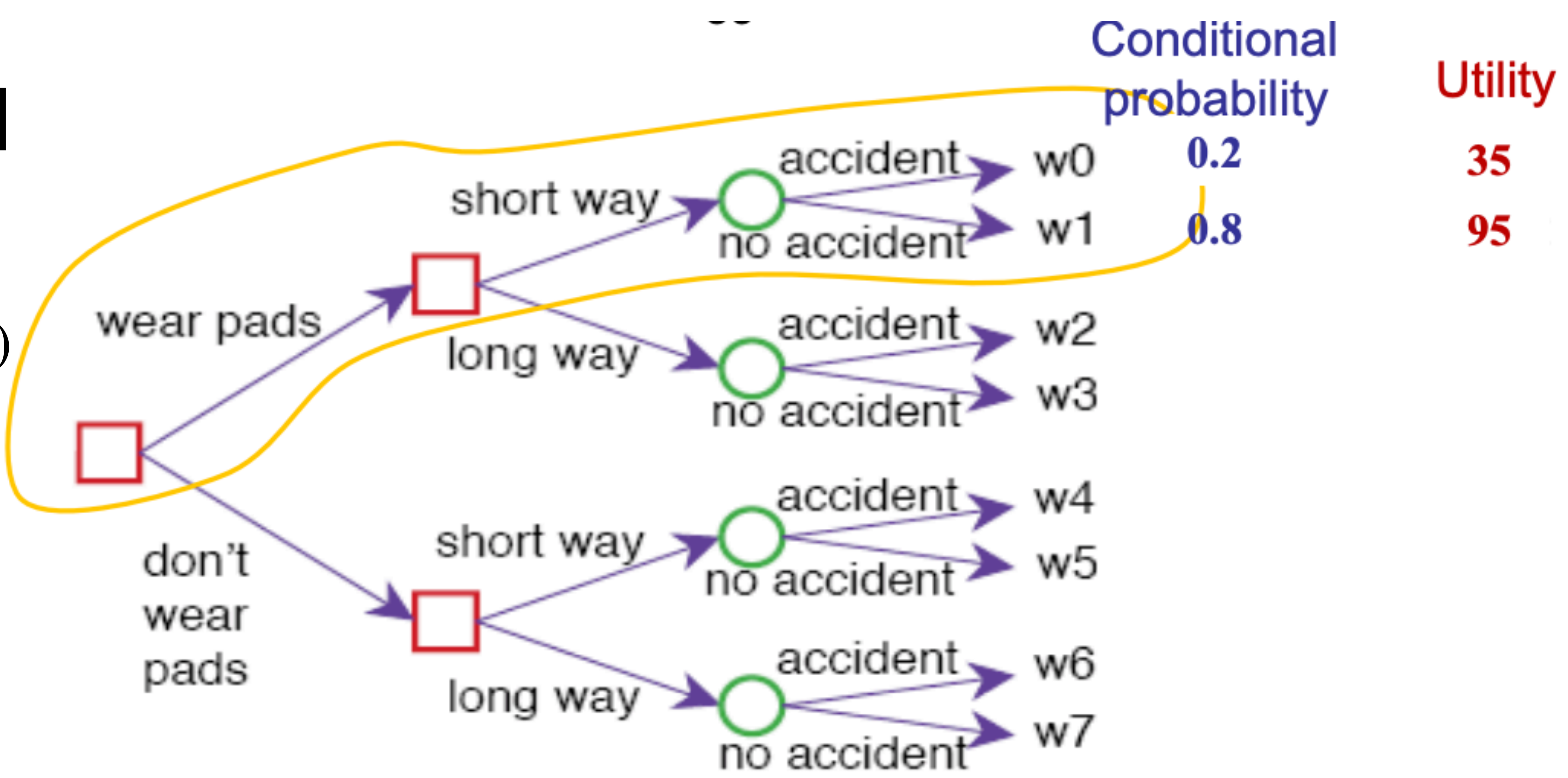
The **expected utility** of a probability distribution over possible worlds average utility, weighted by probabilities of possible worlds

What is the **expected** utility of Wearpads=yes, Way=short ?

It is $0.2 \times 35 + 0.8 \times 95 = 83$

We write the expected utility of a decision as

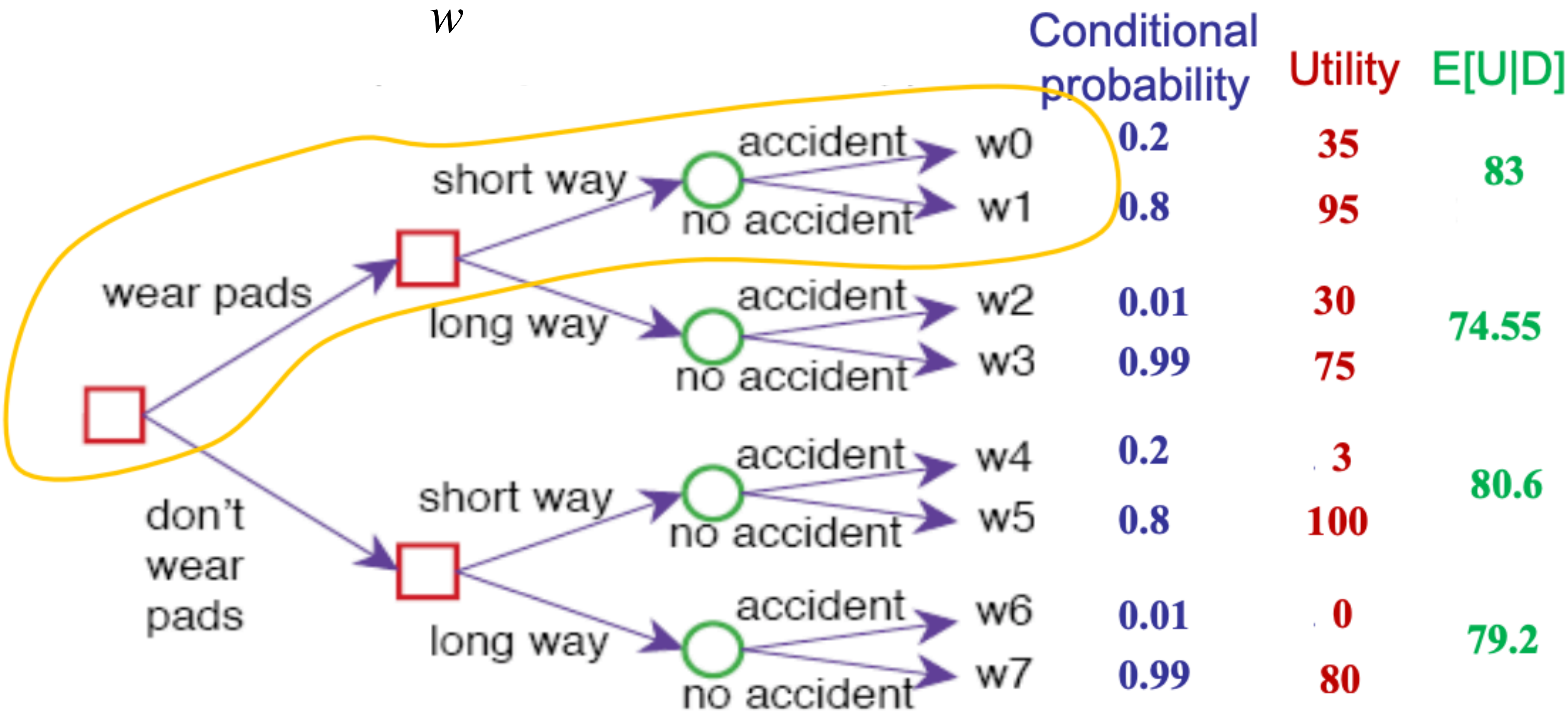
$$E[U \mid D = d] = \sum_w P(w \mid D = d) U(w)$$



Expected utility of a decision

We write the expected utility of a decision as

$$E[U\,|\,D=d]=\sum_{w} P(w\,|\,D=d)\,U(w)$$



Optimal single-stage decision

Given a single decision variable D, the agent can choose $D = d_i$ for any value $d_i \in dom(D)$

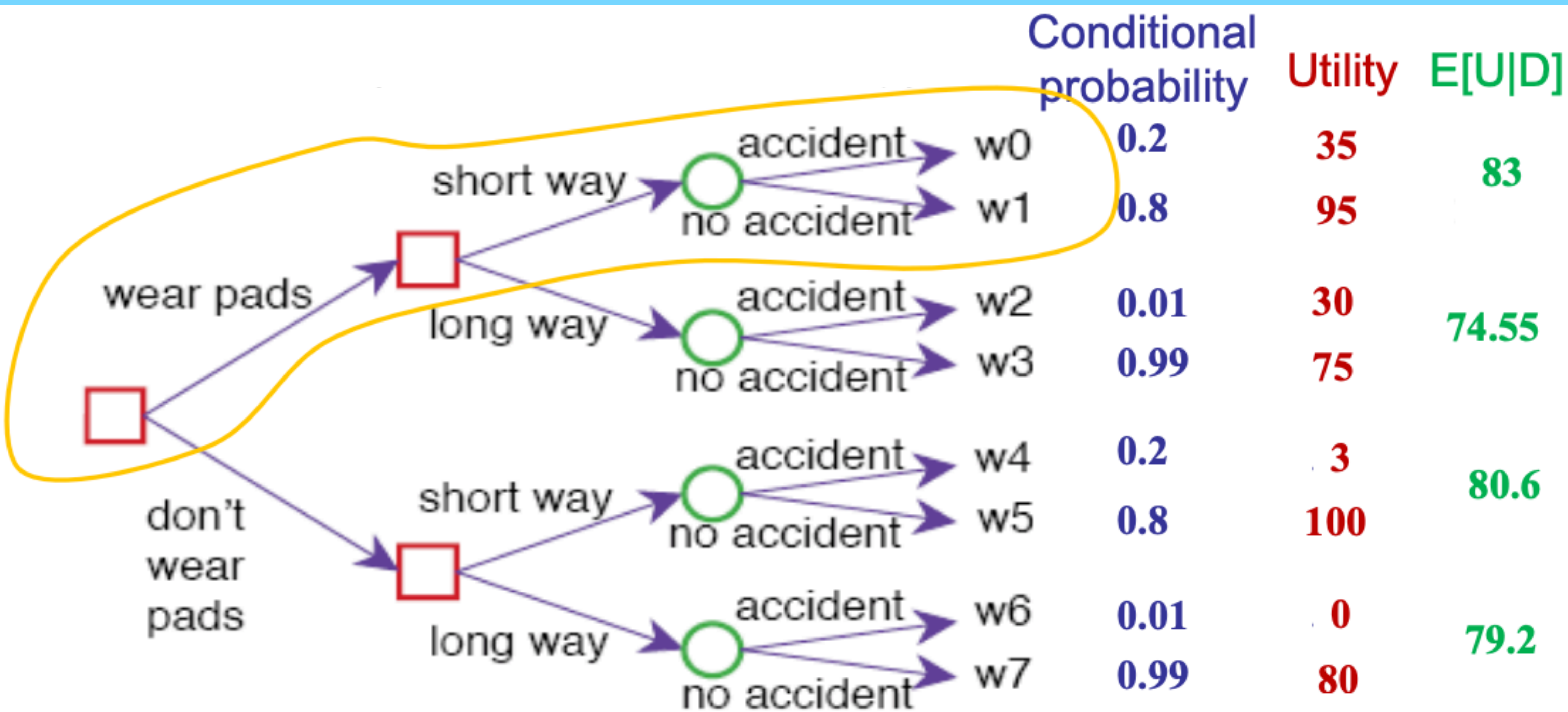
Definition: An **optimal single-stage decision** is the decision $D = d_{max}$ whose expected value is maximal:

$$d_{max} \in argmax_{d_i \in dom(D)} E[U \mid D = d_i]$$

What's the optimal decision in the example?

An **optimal single-stage decision** is the decision $D = d_{max}$ whose expected value is maximal:

$$d_{max} \in argmax_{d_i \in dom(D)} E[U \mid D = d_i]$$

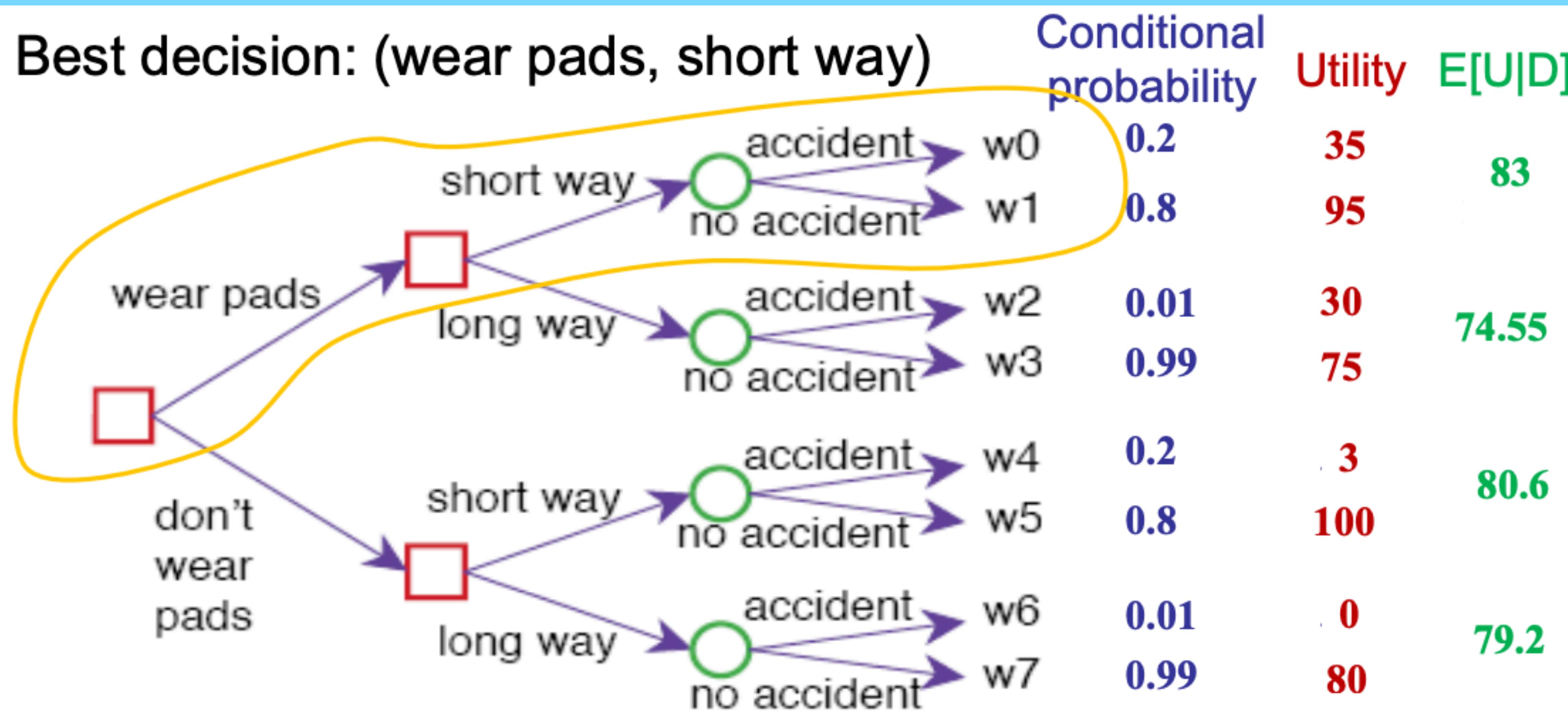


What's the optimal decision in the example?

An **optimal single-stage decision** is the decision $D = d_{max}$ whose expected value is maximal:

$$d_{max} \in argmax_{d_i \in dom(D)} E[U \mid D = d_i]$$

Best decision: (wear pads, short way)



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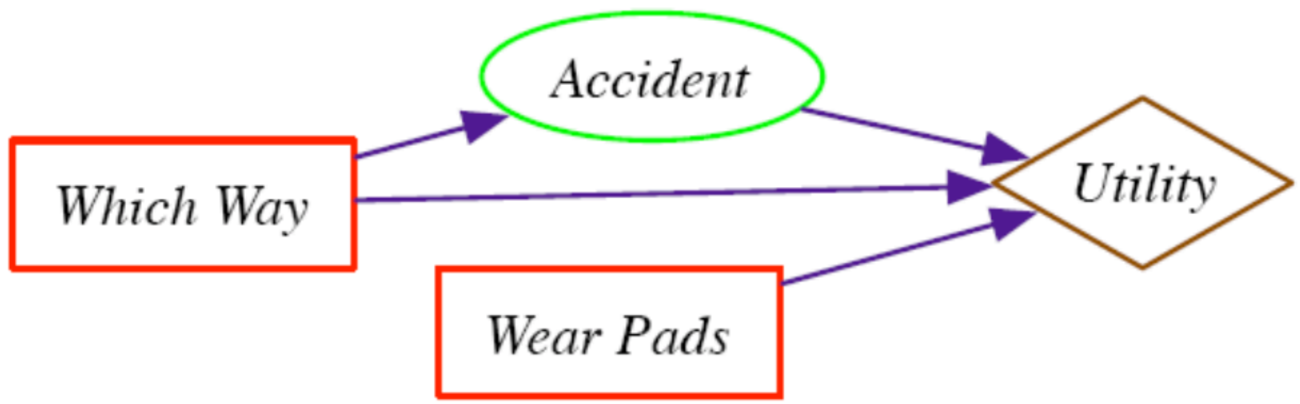
Single-stage decision networks



Extend belief networks

Decision nodes, that the agent chooses the value for Parents:

only other decision nodes allowed as parents

Domain is the set of **possible actions**

Drawn as a **rectangle**

Exactly one utility node

Parents: all random & decision variables on which the utility depends

Does **not have a domain**

Drawn as a **diamond**

Types of nodes in decision networks

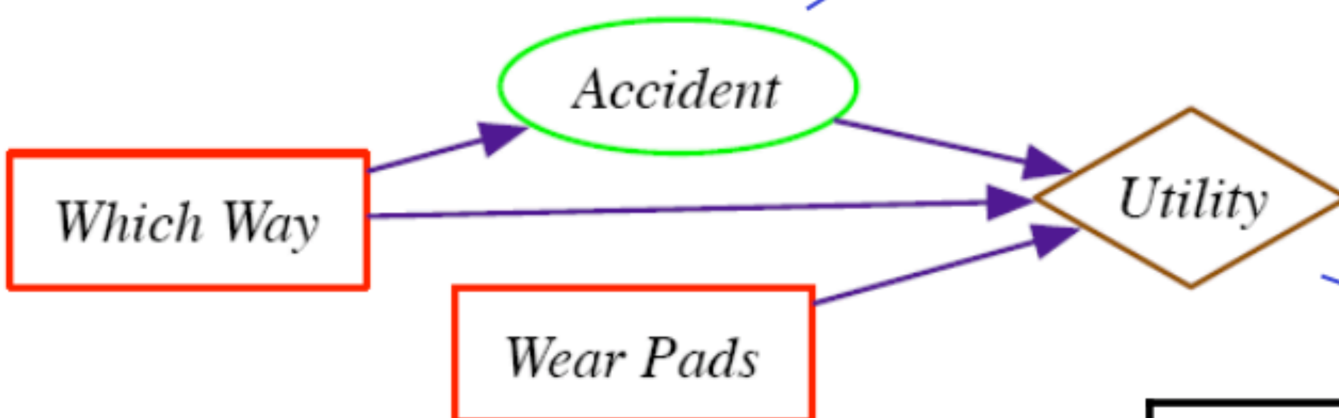
A **random variable** is drawn as an ellipse.
Arcs into the node represent probabilistic dependence.
As in Bayesian networks: a random variable is conditionally independent of its non-descendants given its parents

A **decision variable** is drawn as an rectangle.
Arcs into the node represent information available when the decision is made

A **utility node** is drawn as a diamond.
Arcs into the node represent variables that the utility depends on.
Specifies a utility for each instantiation of its parents

Example decision network

Which Way W	Accident A	P(A|W)
long	true	0.01
long	false	0.99
short	true	0.2
short	false	0.8



Decision nodes do not have an associated table.

The utility node does not have a domain.

Which way	Accident	Wear Pads	Utility
long	true	true	30
long	true	false	0
long	false	true	75
long	false	false	80
short	true	true	35
short	true	false	3
short	false	true	95
short	false	false	100

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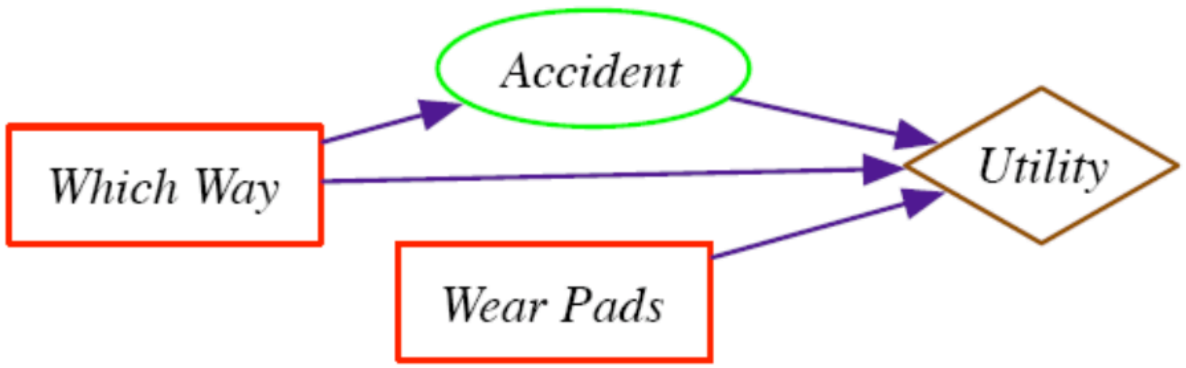
Computing the optimal decision: Use VE



Denote
the random variables as $X_1, \ldots, X_n$
the decision variable as D
the parents of node N as $pa(N)$

$$E(U) = \sum_{X_1,\ldots,X_n} P(X_1, \ldots, X_n \mid D) \cup (pa(U)) \qquad = \sum_{X_1,\ldots,X_n} \prod_{i=1}^{n} P(X_i \mid pa(X_i)) \cup (pa(U))$$

To find the most optimal decision we can use VE
Create a factor for each conditional probability and for the utility

Sum out all random variables, one at a time. This creates a factor on D that gives the expected utility for each d_i

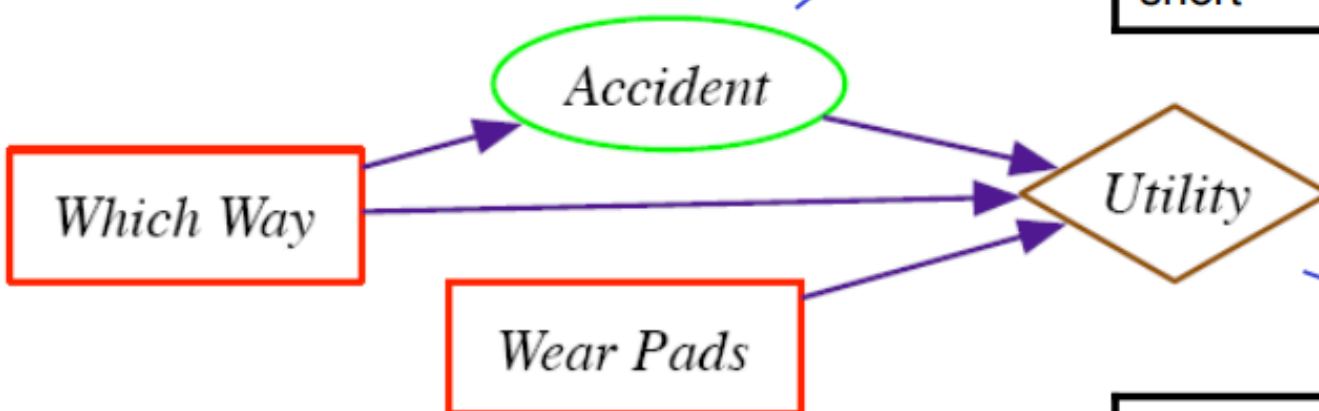
Choose the d_i with the maximum value in the factor

VE example: Step 1 Create initial factors

Abbreviations:
W = Which Way
P = Wear Pads
A = Accident

$f_1(A,W)$

Which Way W	Accident A	P(A|W)
long	true	0.01
long	false	0.99
short	true	0.2
short	false	0.8

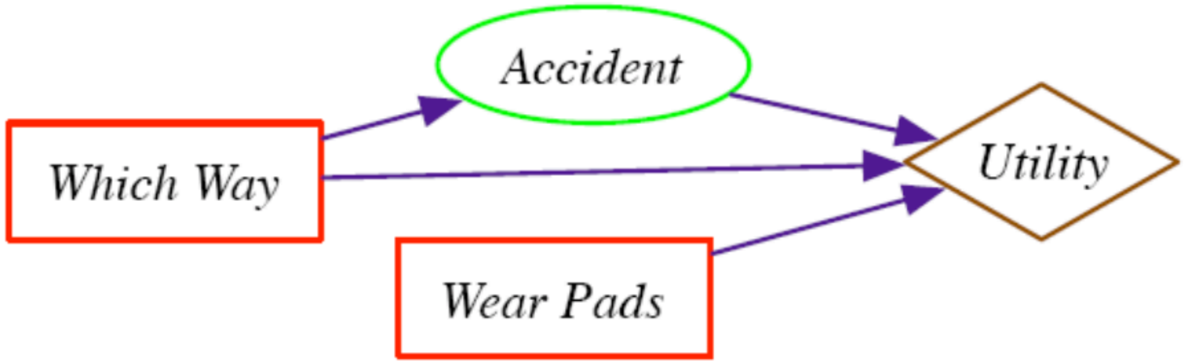


$f_2(A,W,P)$

Which way W	Accident A	Pads P	Utility
long	true	true	30
long	true	false	0
long	false	true	75
long	false	false	80
short	true	true	35
short	true	false	3
short	false	true	95
short	false	false	100

$$E(U) = \sum_A P(A \mid W)\, U(A, W, P)$$
$$= \sum_A f_1(A, W)\, f_2(A, W, P)$$

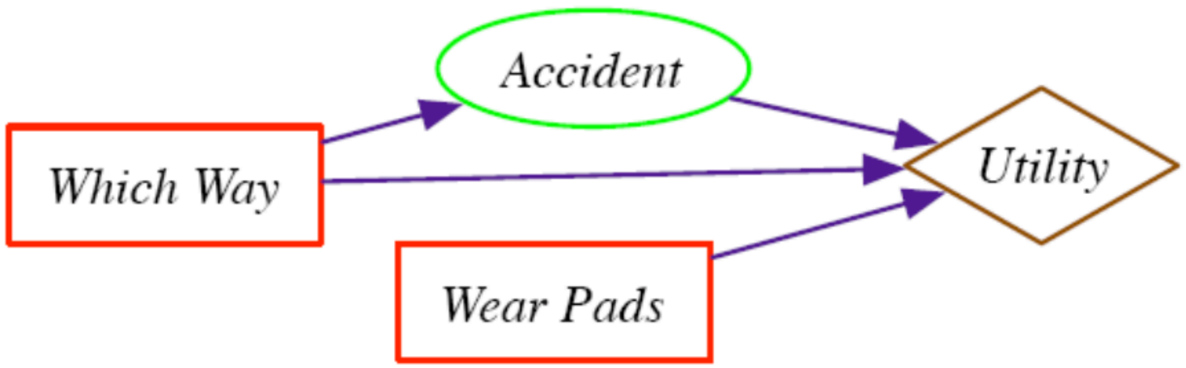
VE example: Step 2 Sum out A



Step 2a: compute product
$f_1(A, W) \times f_2(A, W, P)$

What is the right form for the product
$f_1(A, W) \times f_2(A, W, P)$?

VE example: Step 2 Sum out A



Step 2a: compute product
$f_1(A, W) \times f_2(A, W, P)$

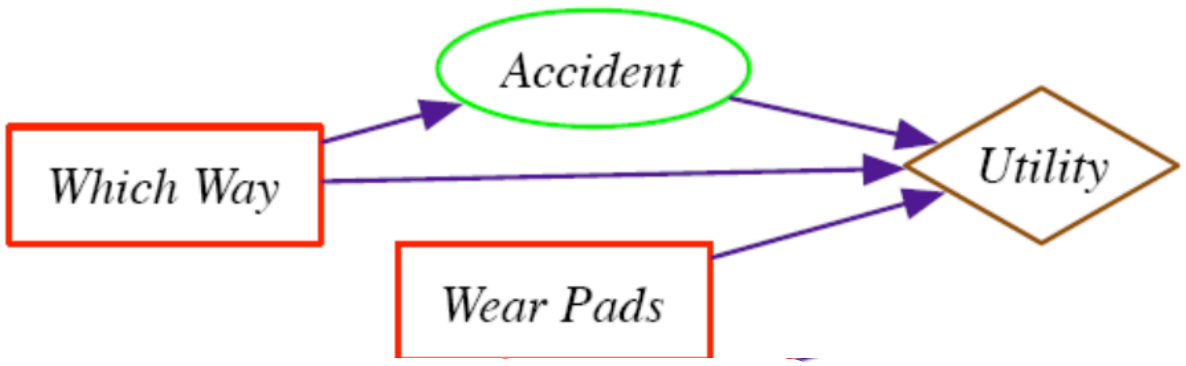
What is the right form for the product $f_1(A, W) \times f_2(A, W, P)$?

It is $f(A, P, W)$
the domain of the product is the union of the multiplicands' domains

$$f(A, P, W) = f_1(A, W) \times f_2(A, W, P)$$

$$f(A = a, P = p, W = w) = f_1(A = a, W = w) \times f_2(A = a, W = w, P = p)$$

VE example: Step 2 Sum out A



Step 2a: compute product

$$f_1(A, W) \times f_2(A, W, P)$$

f (A=a,P=p,W=w) = f_1(A=a,W=w) × f_2(A=a,W=w,P=p)

Which way W	Accident A	f_1(A,W)
long	true	0.01
long	false	0.99
short	true	0.2
short	false	0.8

Which way W	Accident A	Pads P	f_2(A,W,P)
long	true	true	30
long	true	false	0
long	false	true	75
long	false	false	80
short	true	true	35
short	true	false	3
short	false	true	95
short	false	false	100

Which way W	Accident A	Pads P	f(A,W,P)
long	true	true	0.01 * 30
long	true	false	
long	false	true	
long	false	false	???
short	true	true	
short	true	false	
short	false	true	
short	false	false	

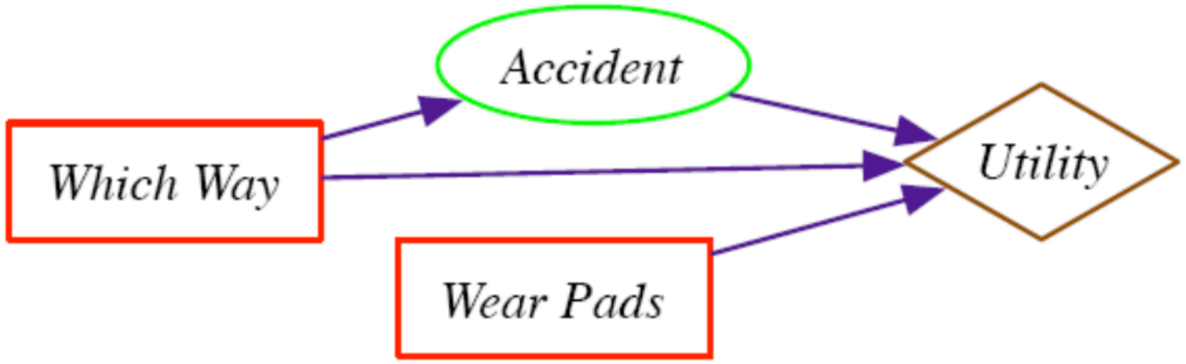
0.99 * 30

0.01 * 80

0.99 * 80

0.8 * 30

VE example: Step 2 Sum out A



Step 2a: compute product

$$f_1(A, W) \times f_2(A, W, P)$$

f (A=a,P=p,W=w) = f_1(A=a,W=w) × f_2(A=a,W=w,P=p)

Which way W	Accident A	f_1(A,W)
long	true	0.01
long	false	0.99
short	true	0.2
short	false	0.8

Which way W	Accident A	Pads P	f_2(A,W,P)
long	true	true	30
long	true	false	0
long	false	true	75
long	false	false	80
short	true	true	35
short	true	false	3
short	false	true	95
short	false	false	100

Which way W	Accident A	Pads P	f(A,W,P)
long	true	true	0.01 * 30
long	true	false	0.01*0
long	false	true	0.99*75
long	false	false	0.99*80
short	true	true	0.2*35
short	true	false	0.2*3
short	false	true	0.8*95
short	false	false	0.8*100

VE example: Step 2 Sum out A

Step 2b: sum A out of the product f(A,W,P):

$$f_3(W,P) = \sum_A f(A,W,P)$$

Which way W	Pads P	f_3(W,P)
long	true	0.01*30+0.99*75=74.55
long	false	
short	true	??
short	false	

Which way W	Accident A	Pads P	f(A,W,P)
long	true	true	0.01 * 30
long	true	false	0.01*0
long	false	true	0.99*75
long	false	false	0.99*80
short	true	true	0.2*35
short	true	false	0.2*3
short	false	true	0.8*95
short	false	false	0.8*100

0.2*35 + 0.2*0.3

0.2*35 + 0.8*95

0.99*80 + 0.8*95

0.8 * 95 + 0.8*100

VE example: Step 2 Sum out A

Step 2b: sum A out of the product f(A,W,P):

$$\boxed{f_3(W,P) = \sum_A f(A,W,P)}$$

Which way W	Accident A	Pads P	f(A,W,P)
long	true	true	0.01 * 30
long	true	false	0.01*0
long	false	true	0.99*75
long	false	false	0.99*80
short	true	true	0.2*35
short	true	false	0.2*3
short	false	true	0.8*95
short	false	false	0.8*100

Which way W	Pads P	f_3(W,P)
long	true	0.01*30+0.99*75=74.55
long	false	0.01*0+0.99*80=79.2
short	true	0.2*35+0.8*95=83
short	false	0.2*3+0.8*100=80.6

VE example: Step 2 Sum out A

Step 2b: sum A out of the product f(A,W,P):

$$f_3(W,P) = \sum_A f(A, W, P)$$

Which way W	Pads P	f_3(W,P)
long	true	0.01*30+0.99*75=74.55
long	false	0.01*0+0.99*80=79.2
short	true	0.2*35+0.8*95=83
short	false	0.2*3+0.8*100=80.6

Which way W	Accident A	Pads P	f(A,W,P)
long	true	true	0.01 * 30
long	true	false	0.01*0
long	false	true	0.99*75
long	false	false	0.99*80
short	true	true	0.2*35
short	true	false	0.2*3
short	false	true	0.8*95
short	false	false	0.8*100

The final factor encodes the expected utility of each decision.

Thus, taking the short way but wearing pads is the best choice, with an expected utility of **83**.

VE for single-stage decision networks: Summary

- Create a factor for each conditional probability and for the utility

- Sum out all random variables, one at a time
  This creates a factor on D that gives the expected utility for each d_i

- Choose the d_i with the maximum value in the factor

Final

Included in Final:

- Everything till (and including) today's lesson
- Please read the corresponding sections from the textbook

Structure of the exam will be similar to midterm

Final exam studying tips

- Try to **understand** the material rather than memorizing it
- Go through lecture notes and the lecture learning outcomes and make sure you can do things that are expected of you
- Alspace exercises are a great way to check whether you have understood corresponding material or not
- Make use of extra office hours

We like answers that are: **correct**, **easy to read**, and **as short as possible while including important information**. We may penalize answers containing irrelevant information.

AI grad courses at UBC

522: Artificial Intelligence II : Reasoning and Acting Under Uncertainty

503: Computational Linguistics I / Natural Language Processing

532: Topics in Artificial Intelligence

540: Machine Learning

505: Image Understanding I: Image Analysis

525: Image Understanding II: Scene Analysis

515: Computational Robotics

Reading groups

https://www.cs.ubc.ca/cs-research/lci/reading-groups

- Meet regularly to discuss papers and/or topics of interest
- Have email lists to keep people informed
- Designed for faculty/graduate students, but undergrads are also welcome.
- Excellent way to get to know faculty/grad students in areas of interest

Final remarks

I hope you learned something from the course that will stay with you. This was my first time teaching an undergraduate course, and I am aware that many things can be improved. But **thank you** for your support, your engagement, great questions, and your feedback. You made it an enjoyable experience for me!

I wish you every success in future! Stay in touch!

Coming up

Final exam!!