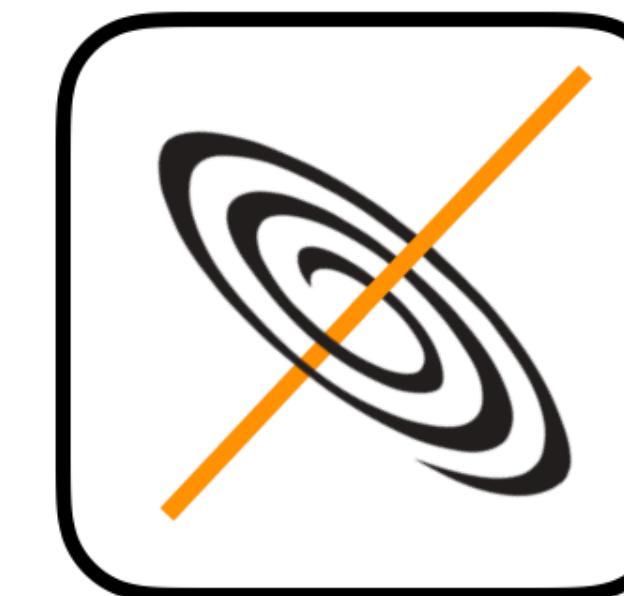


Numerical modelling of radiative processes with the JetSeT code: hands-on session for the high-energy astrophysics course



JetSeT

Jets SED modeler and fitting Tool

Andrea Tramacere

<https://jetset.readthedocs.io/en/latest/>

<https://github.com/andreatramacere/jetset>

<https://www.facebook.com/jetsetastro/>

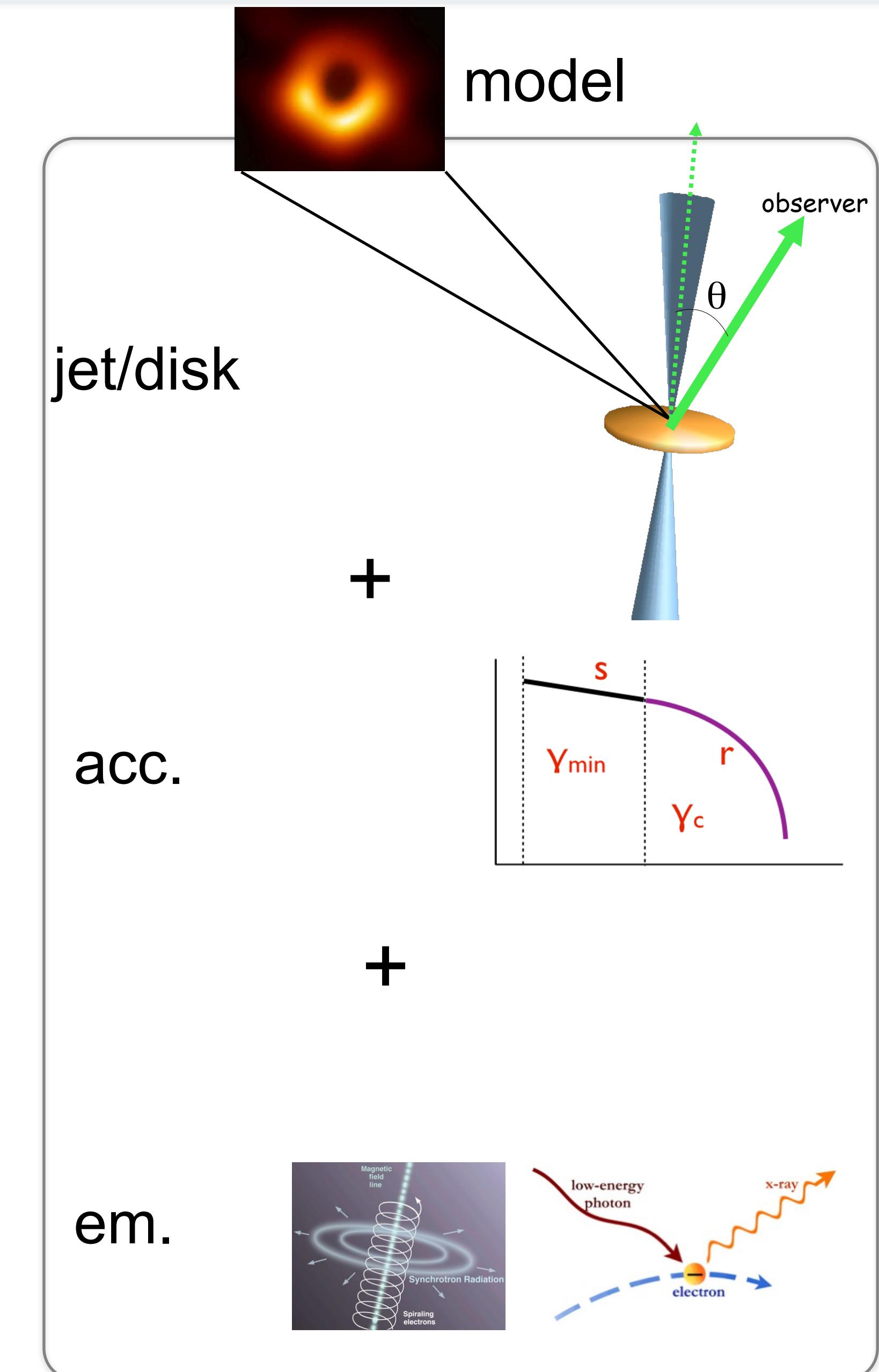
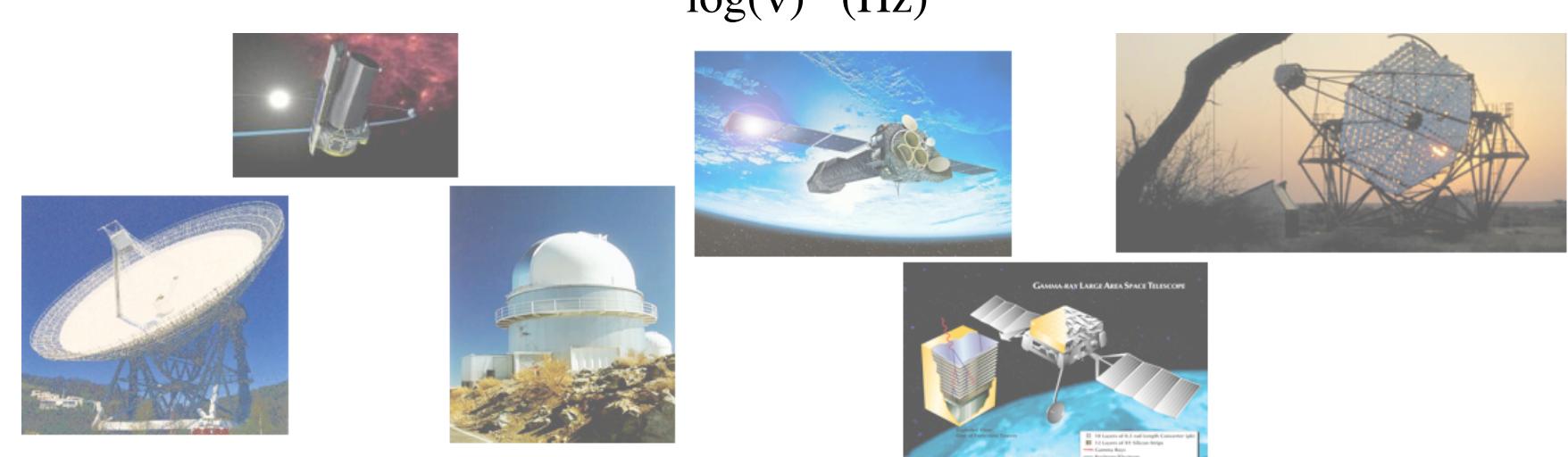
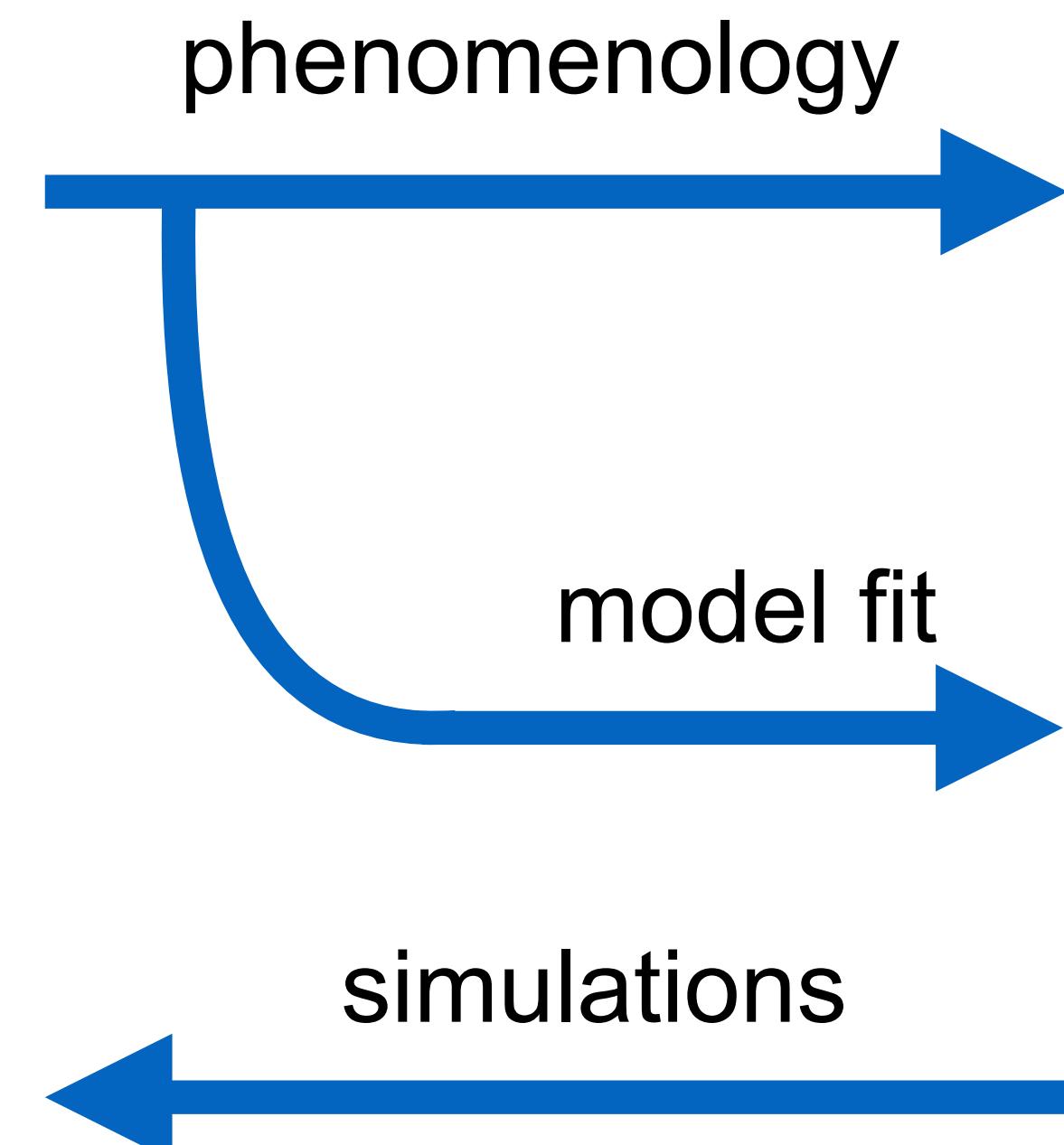
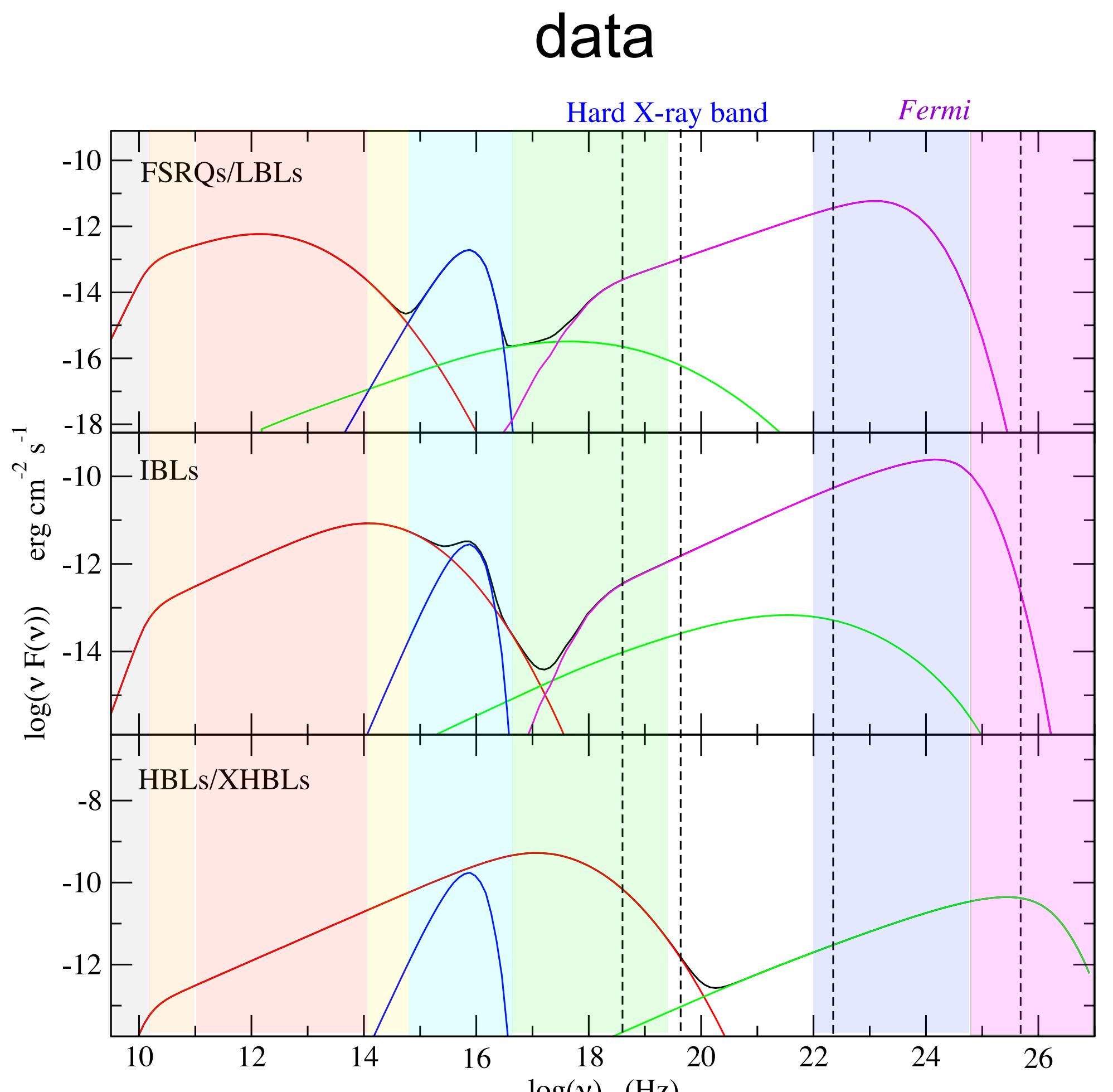
- Theoretical background
- definition of complex radiative **models** SSC/EC IC against CMB/BLR/DT, plus analytical and template models
- handling observed **data** (grouping, definition of data sets, etc...)
- **constraining** of the model in the pre-fitting stage, based on accurate and already published **phenomenological trends**
- **fitting of multiwavelength SEDs** using both **frequentist** approach (iminuit/scipy) and Bayesian **MCMC** sampling (emcee)
- Textbooks
 - Radiative Processes in Astrophysics, Ribicky & Lightman, John Wiley & Sons, 1991
 - High Energy Radiation from Black Holes: Gamma Rays, Cosmic Rays, and Neutrinos, Dermer & Menon, Princeton University Press 2009
 - Bayesian Reasoning in Data Analysis: A Critical Introduction, D'Agostini G., World Scientific, 2003

https://github.com/andreatramacere/Geneva_HighEnergy_Course_2021

- Tutorial 1: basic operation with jet models
- Tutorial 2: phenomenological trends for synchrotron and SSC emission
- Tutorial 3: phenomenological trends for EC emission
- Tutorial 4: composite models and application to EBL (not covered in these slides)
- Tutorial 5: constraining of the model in the pre-fitting stage, based on accurate and already published phenomenological trends and fitting of multiwavelength SEDs using both frequentist approach ([iminuit/scipy](#)) and Bayesian MCMC sampling ([emcee](#))

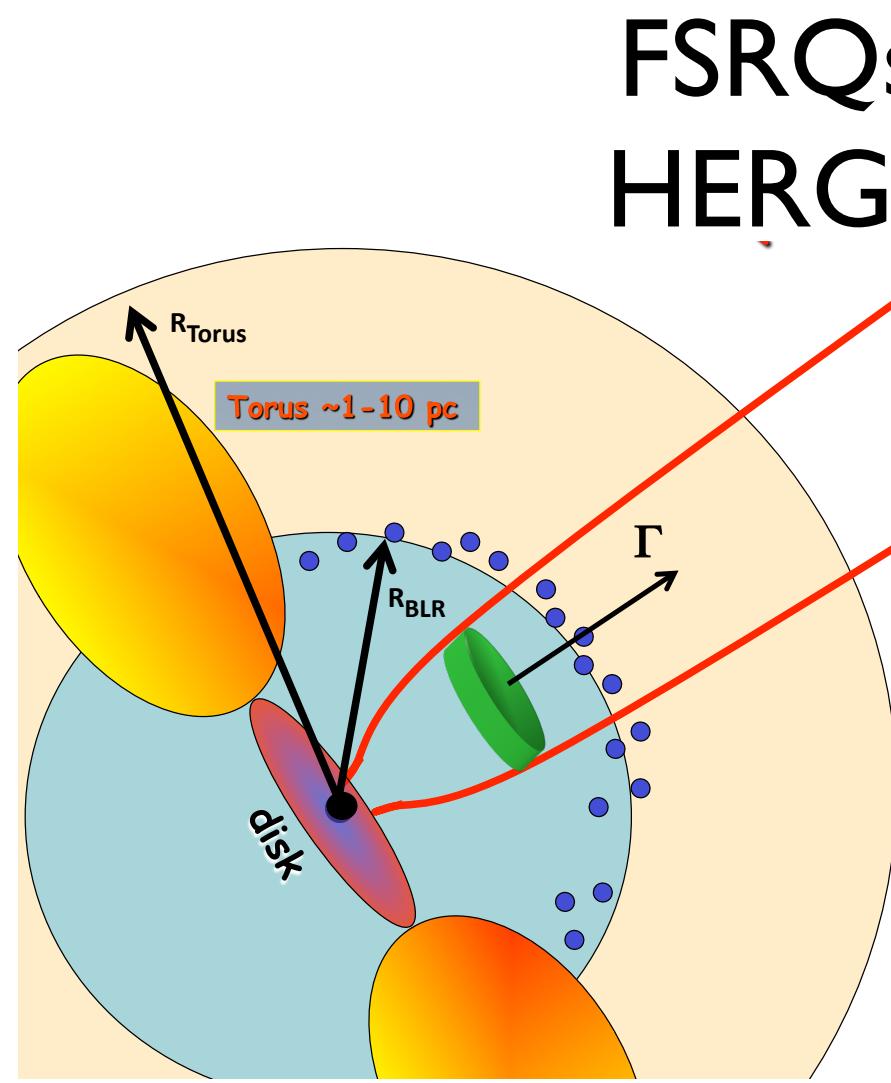
Today will focus only on the first two tutorials

Blazars in a nutshell



standard picture: acceleration/cooling balance

(Ghisellini,Fossati,Celotti 99-16)



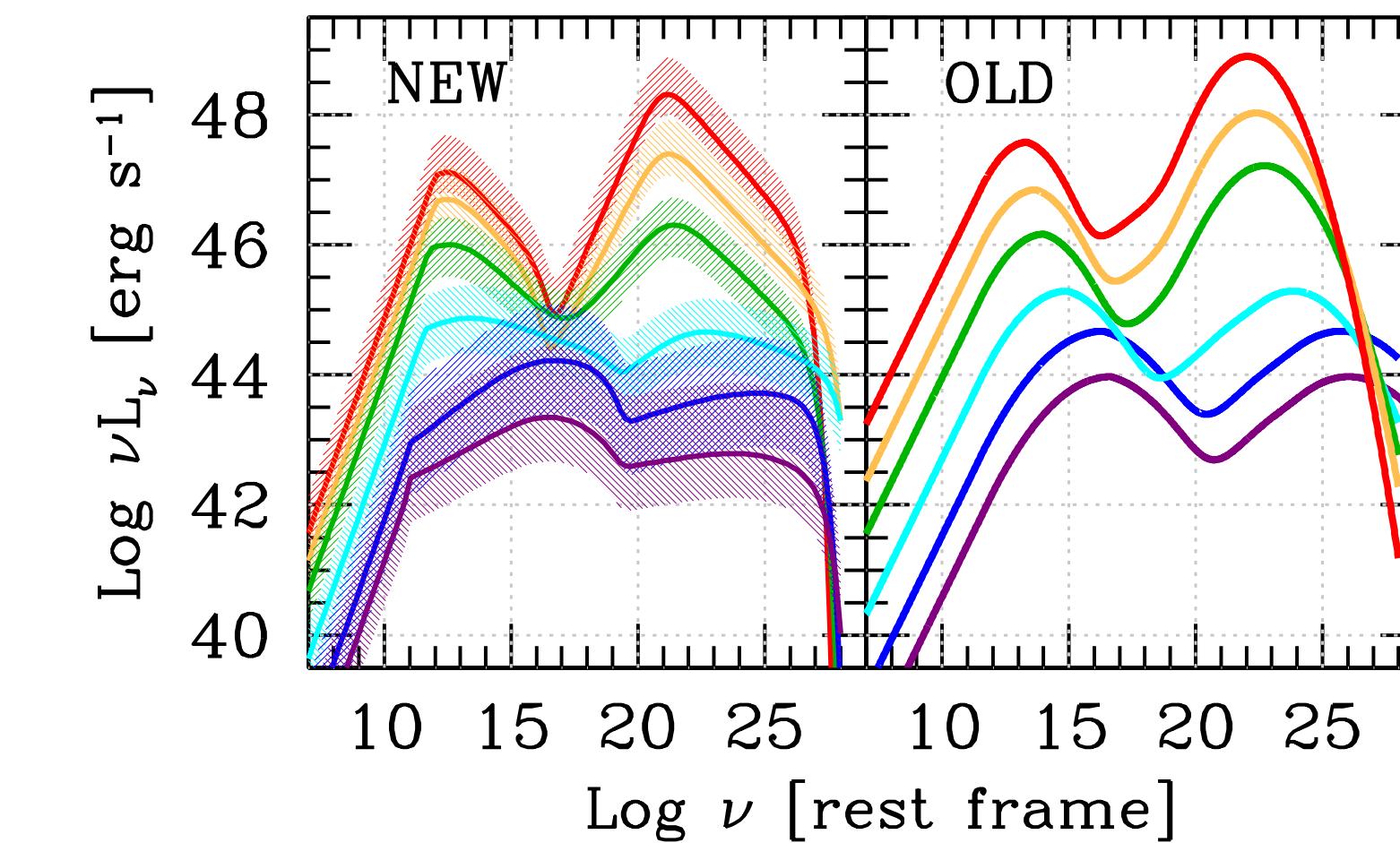
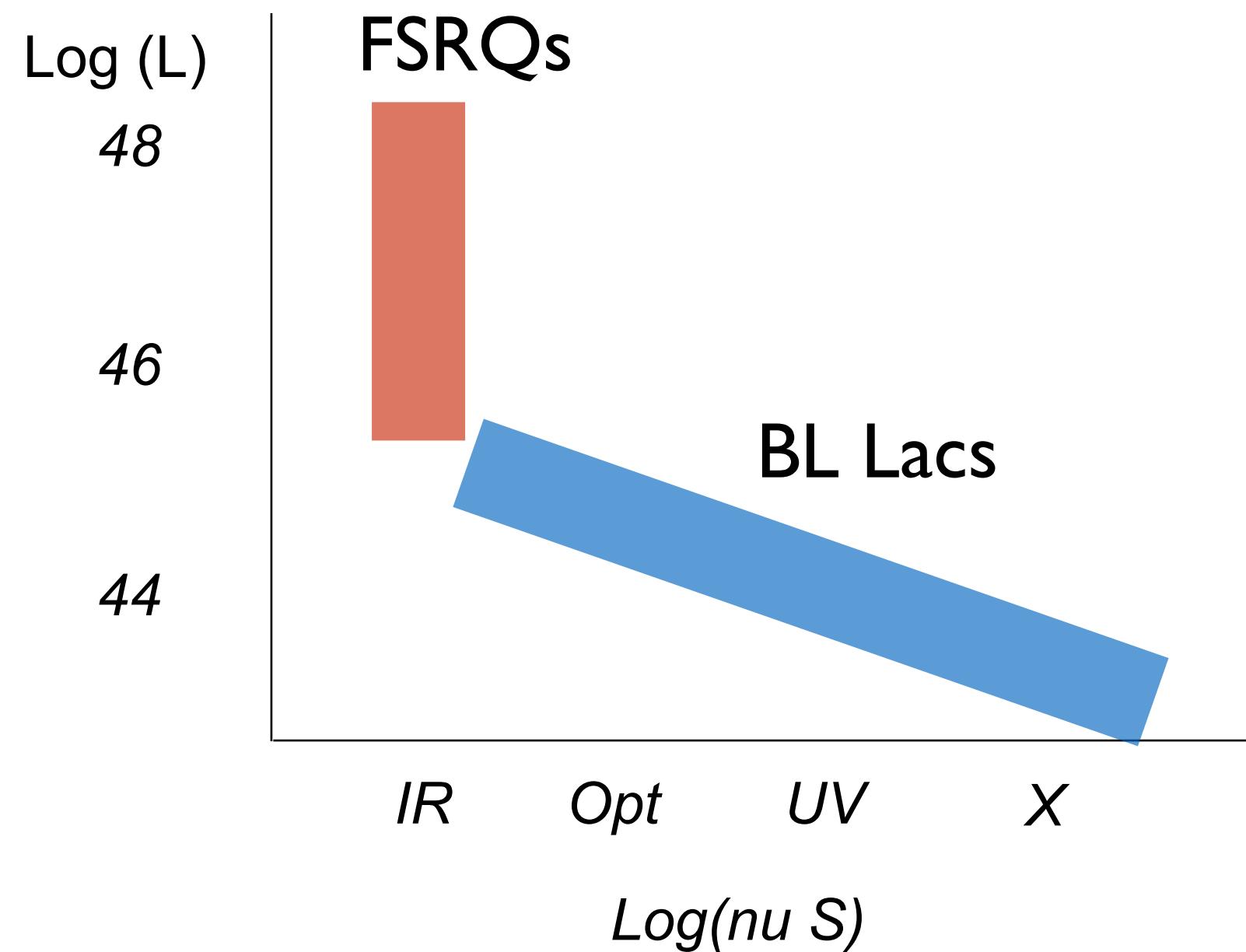
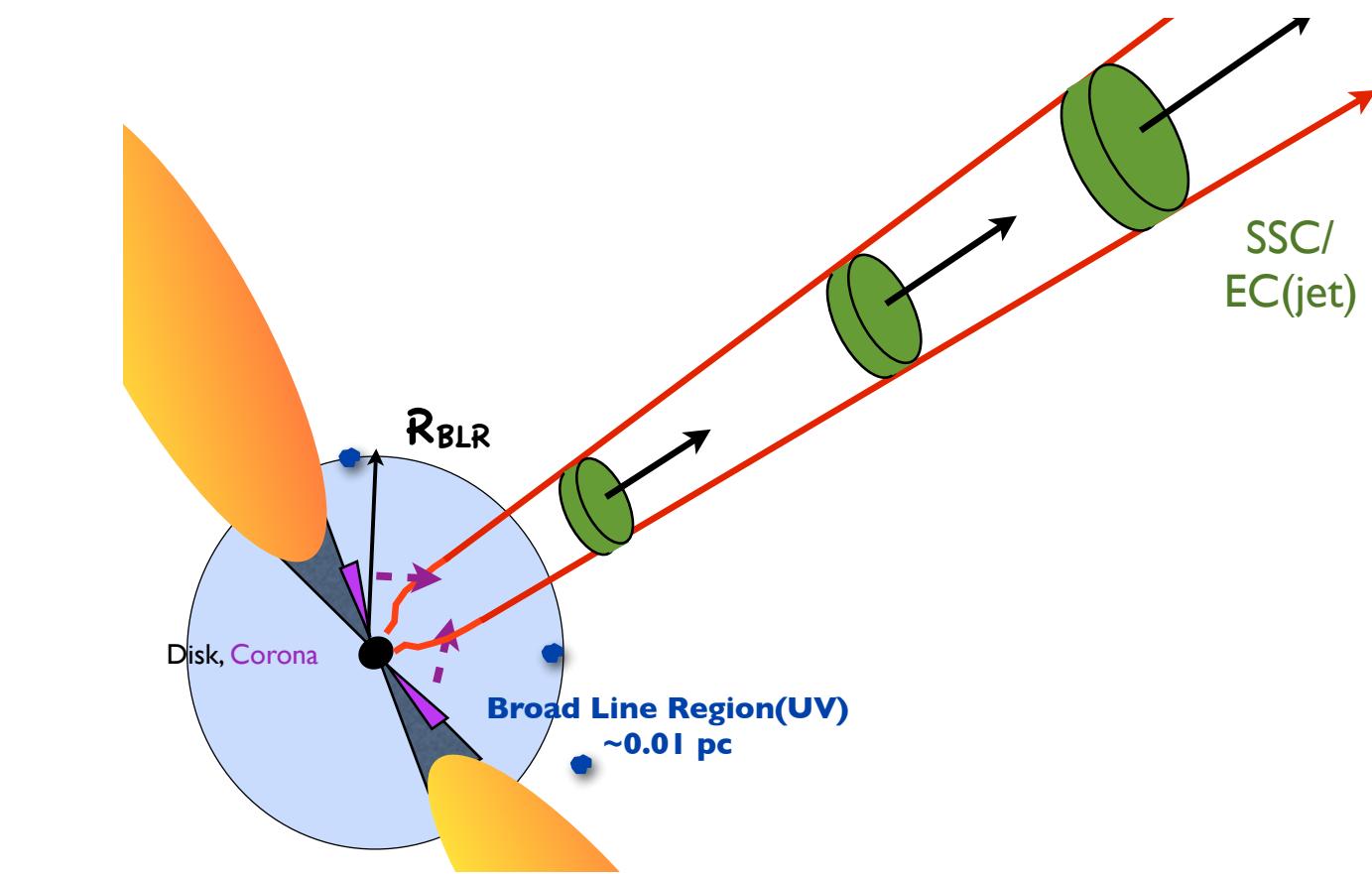
rad eff.
 $L_d > 10^{-2} L_{EDD}$

$$L_{Edd} \approx 1.4 \times 10^{44} \frac{M_{BH}}{10^6 M_\odot} \text{ erg s}^{-1}$$

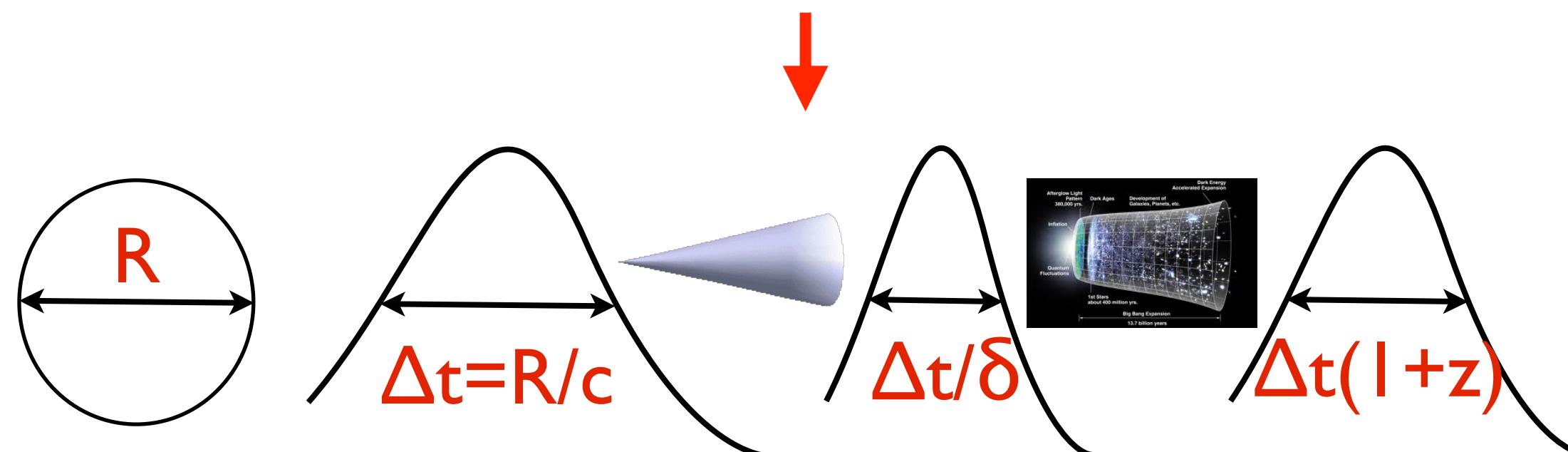
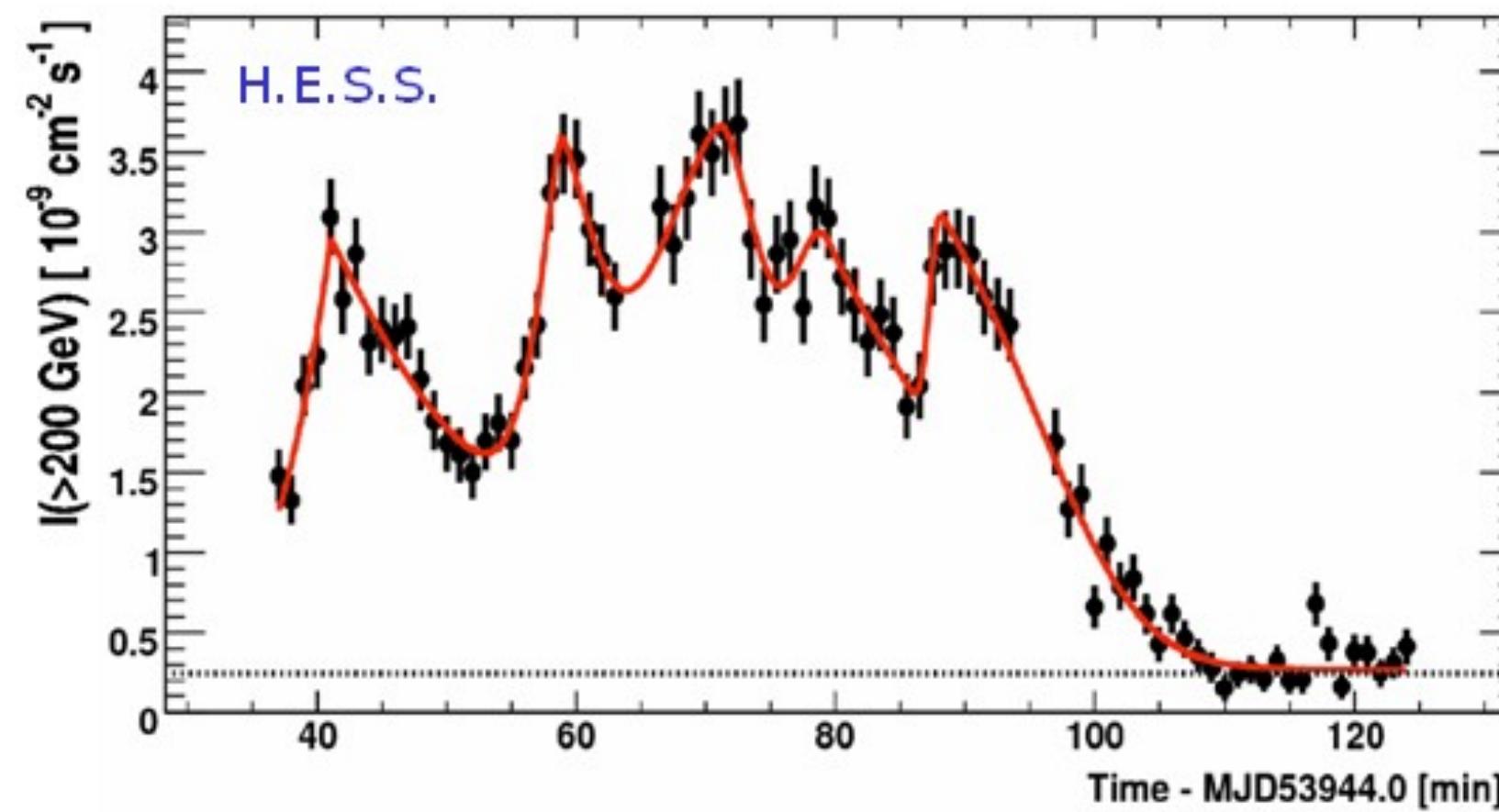
$$\left. \begin{aligned} U_{ext} &\simeq \frac{L_d}{R_{ext}^2 c} \\ R_{ext} &\simeq L_d^{1/2} \end{aligned} \right\} \rightarrow \begin{aligned} &\sim 0.1 \text{ erg/cm}^3 \text{ BLR} \\ &\sim 0.01 \text{ erg/cm}^3 \text{ DT} \end{aligned}$$

BL Lacs
LERGs

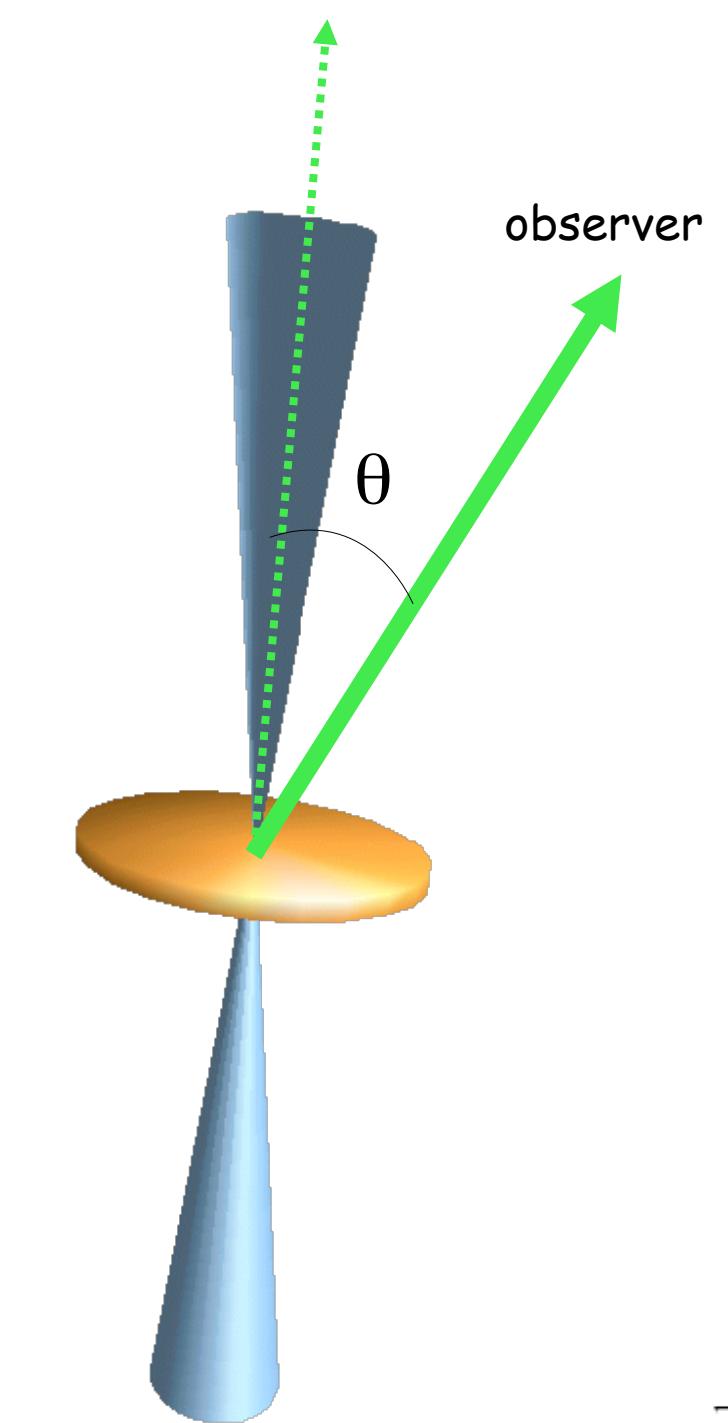
rad ineff.
 $L_d < 10^{-2} L_{EDD}$



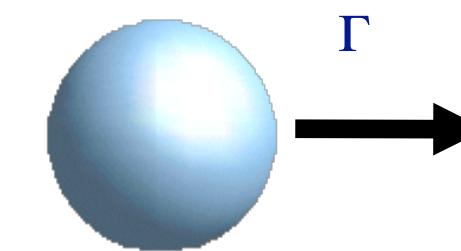
Beamed Emission



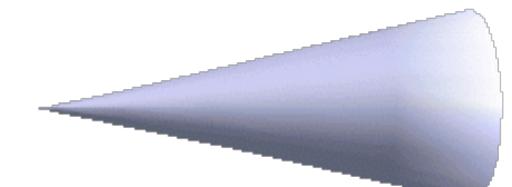
$$R \leq c \Delta t \delta / (1+z)$$



rest frame :
isotropic emission

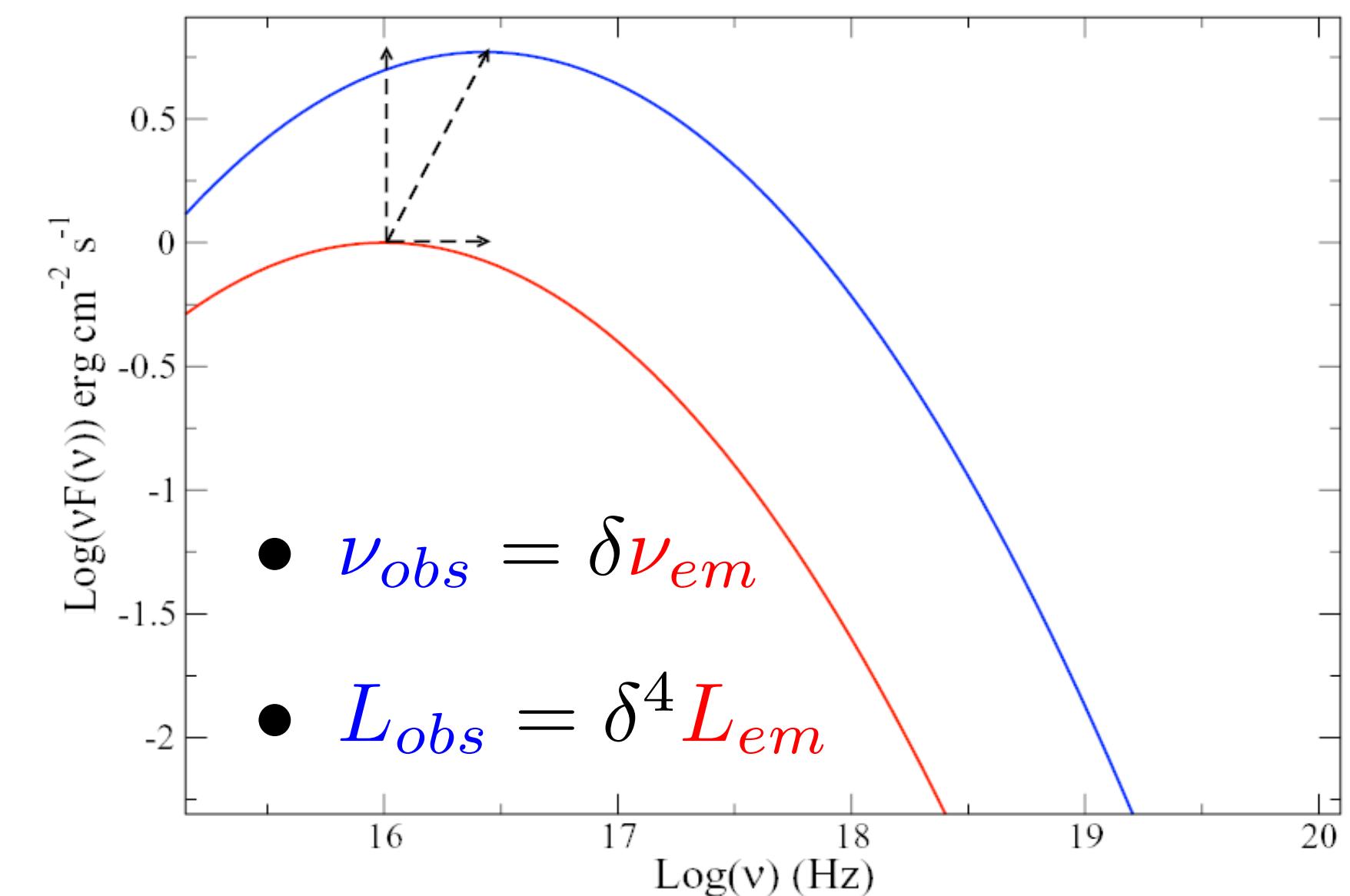


Observer frame: beamed

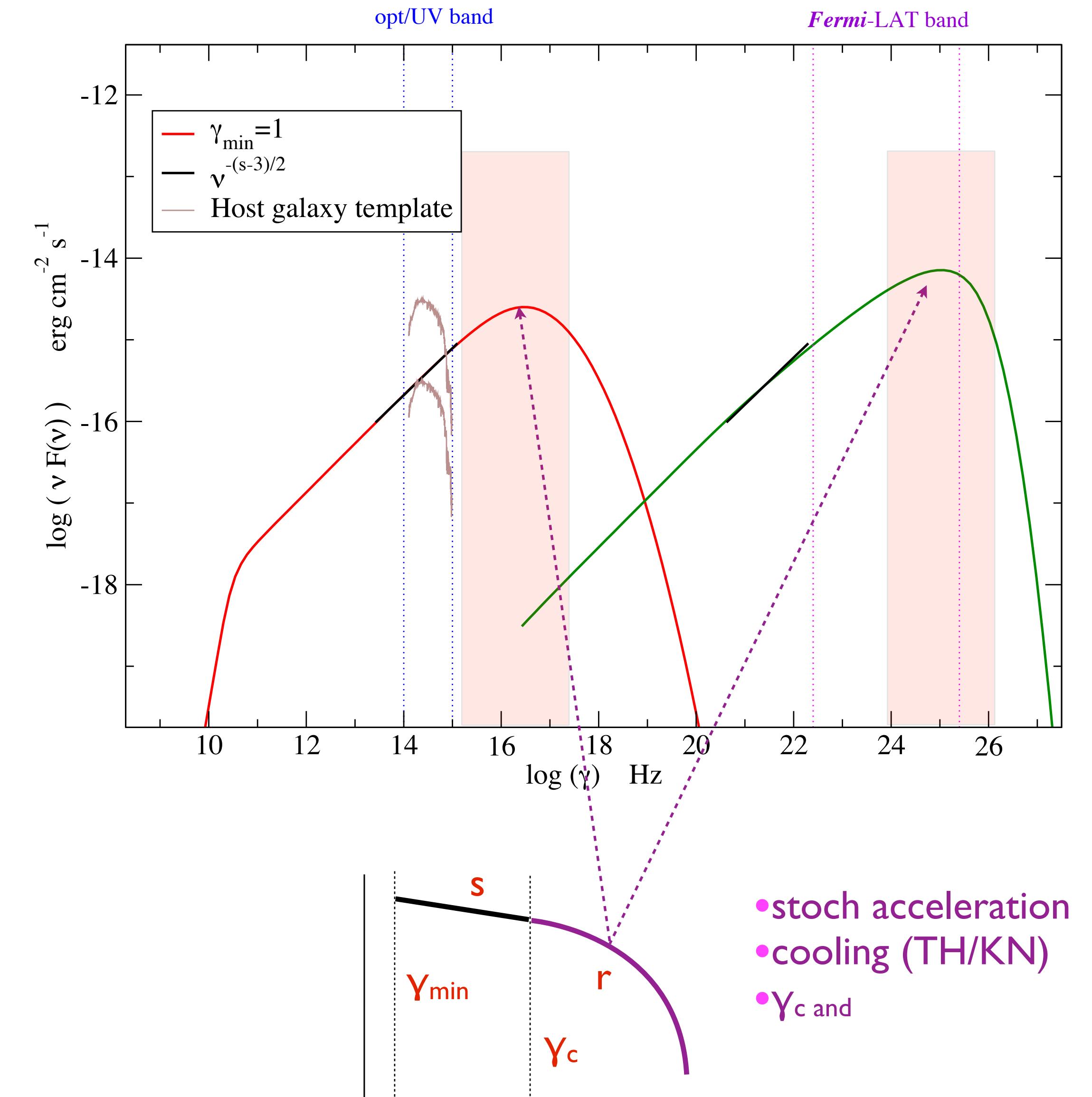
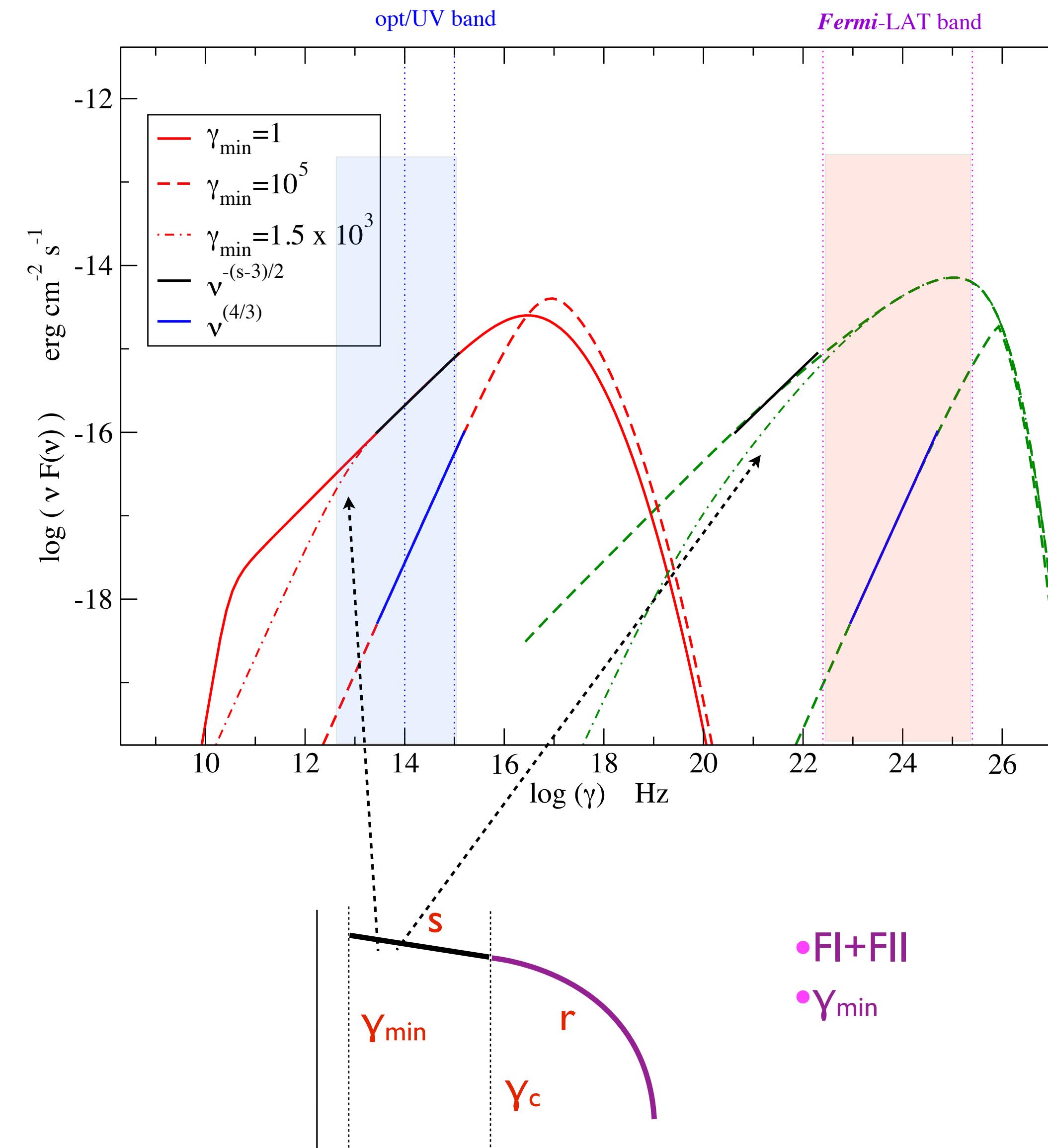


Beaming factor:

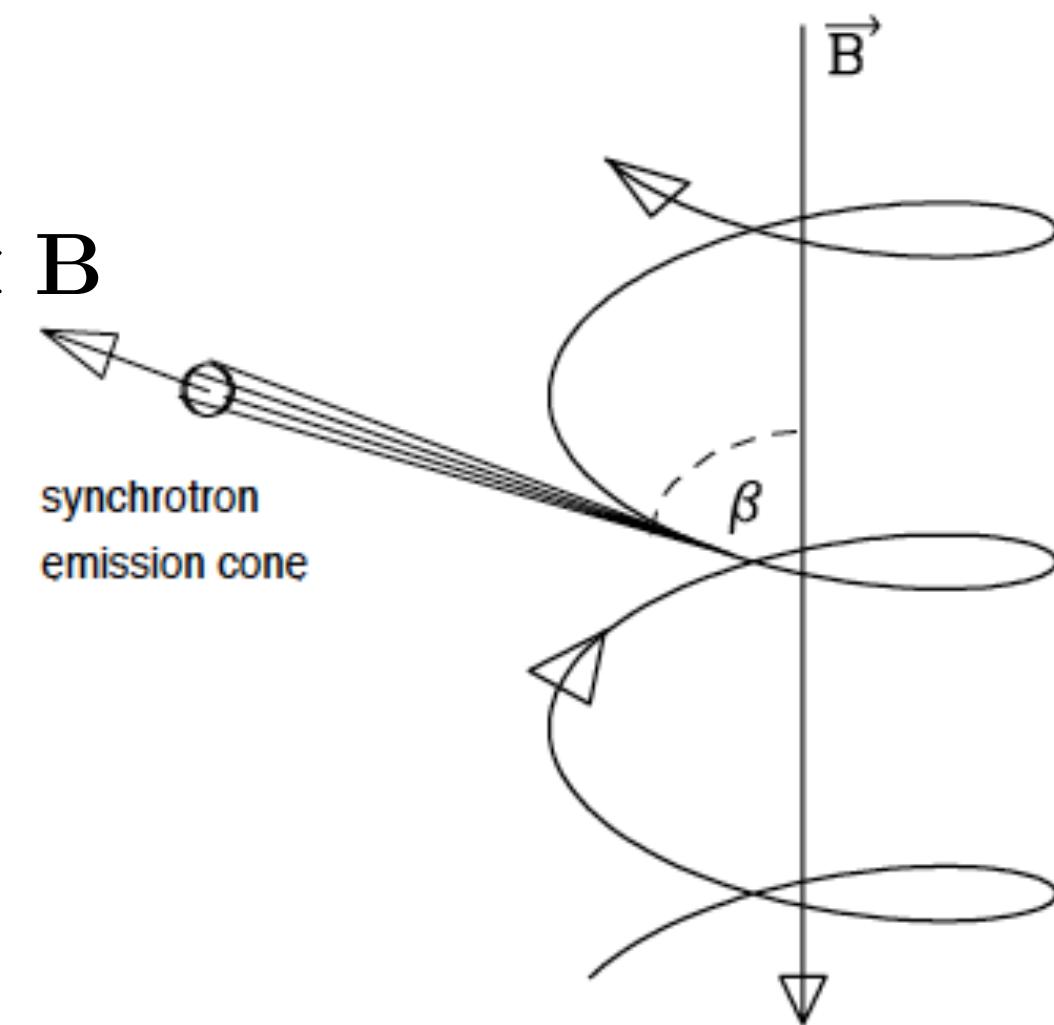
- $\delta = \frac{1}{\Gamma(1-\beta \cos(\theta))}$
- $\theta = 1/\Gamma$



SED shaping and constraining the electron distribution



$$m\gamma \frac{d(\mathbf{v})}{dt} = \frac{q}{c} \mathbf{v} \times \mathbf{B}$$



$$a_{\parallel} = 0$$

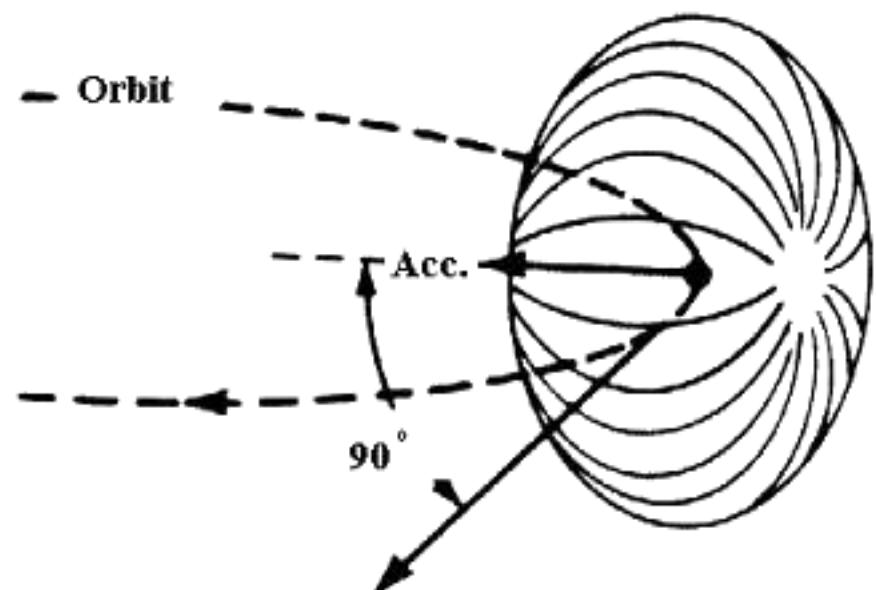
$$a_{\perp} = \frac{evB\sin\alpha}{\gamma m_e c}$$

$$\nu_B = \frac{eB}{2\pi\gamma mc} = \frac{\nu_L}{\gamma}$$

total emitted power

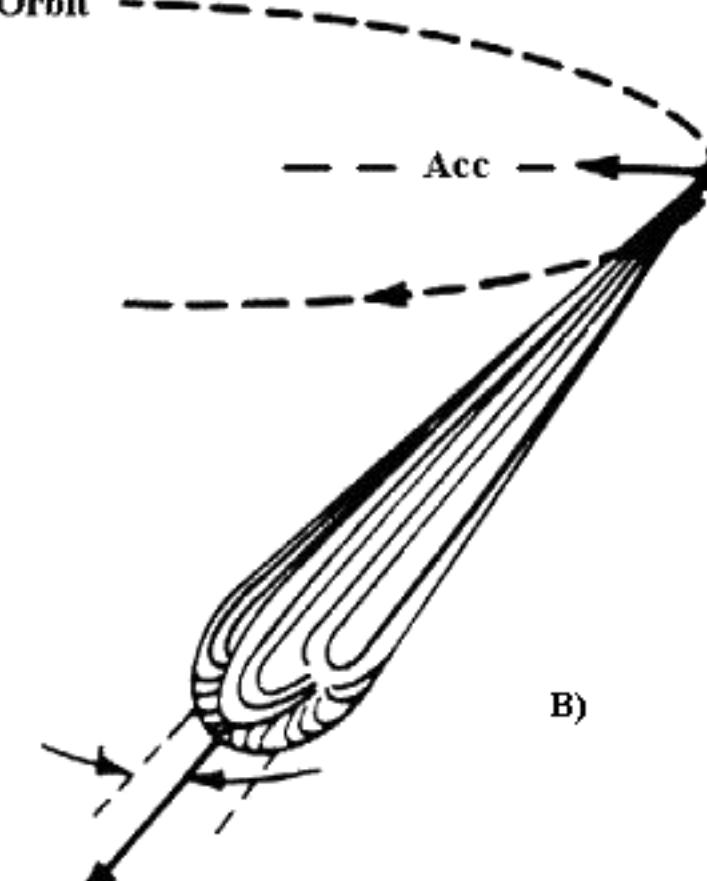
$$P_e = P'_e = \frac{2e^2}{3c^3} a'^2 = \frac{2e^2}{3c^3} [a'^2_{\parallel} + a'^2_{\perp}]$$

$$P_S = \frac{2e^4}{3m_e c^3} B^2 \gamma^2 \beta^2 \sin^2 \alpha$$



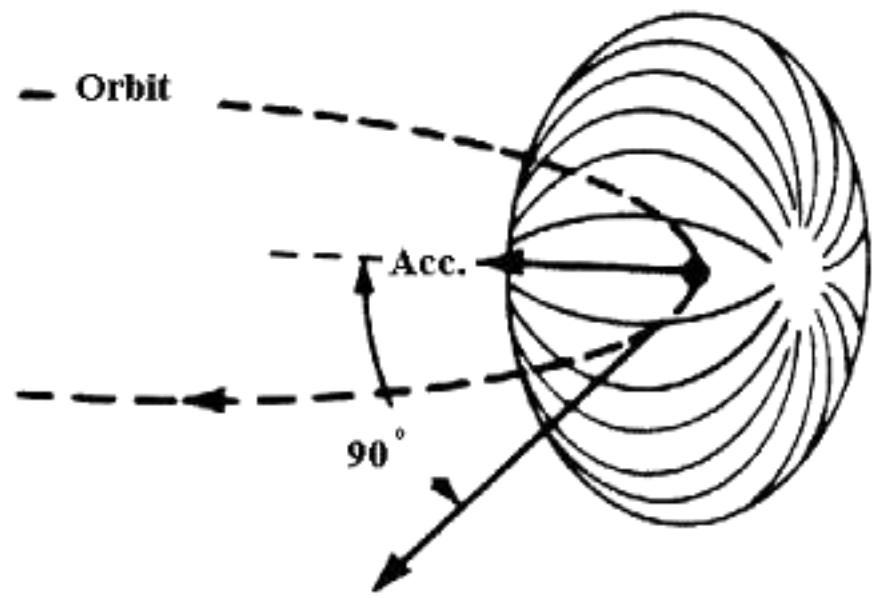
A)

$$\Delta \phi = \frac{1}{\gamma}$$

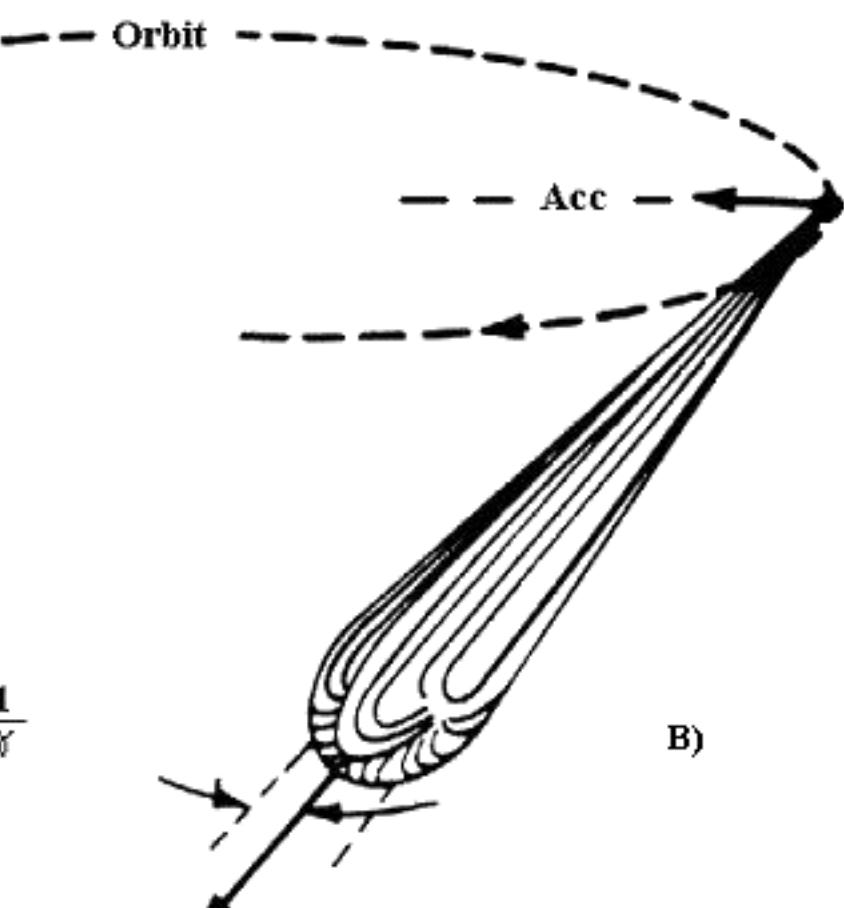
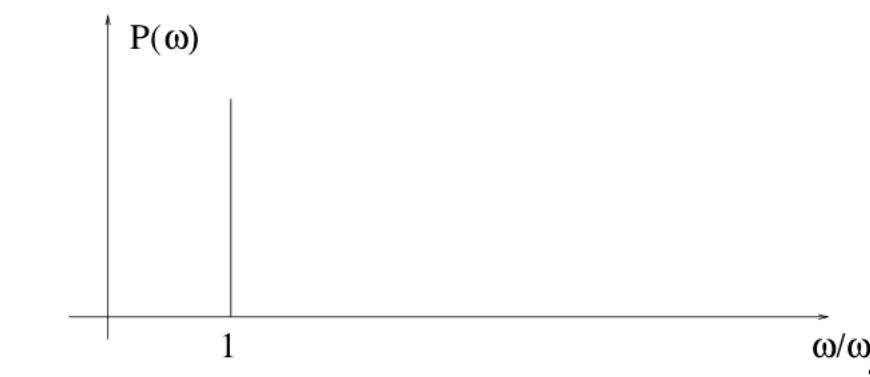
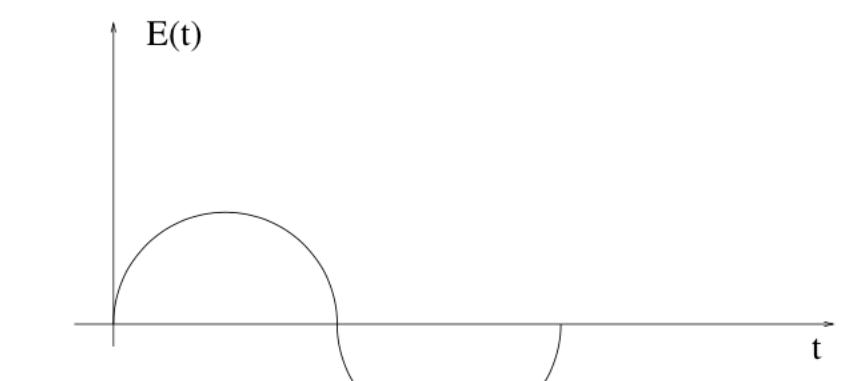


B)

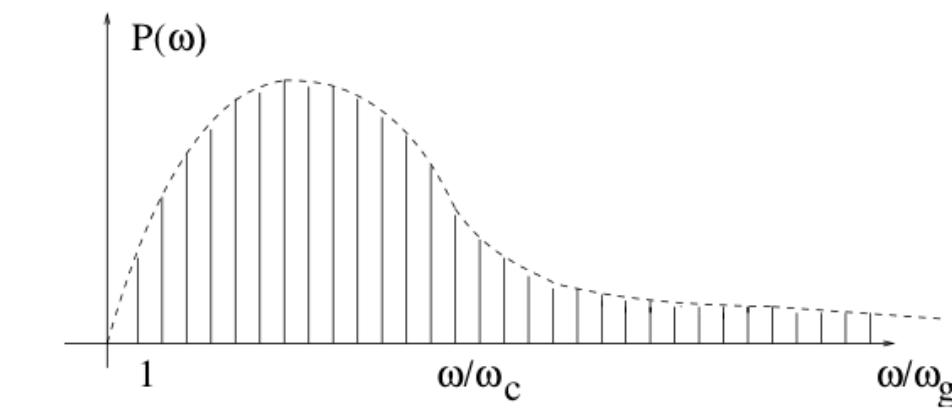
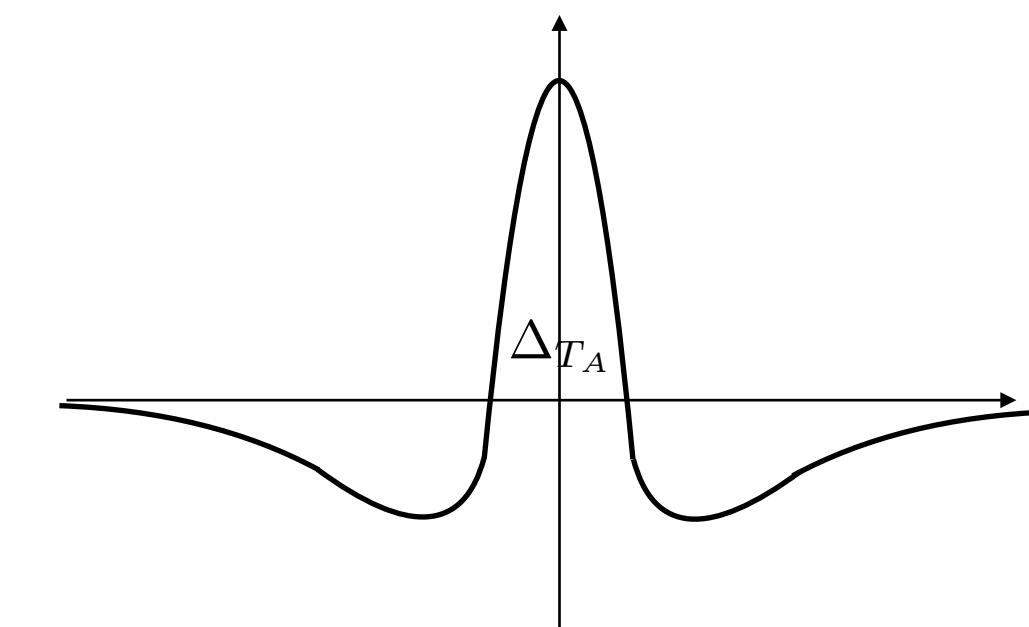
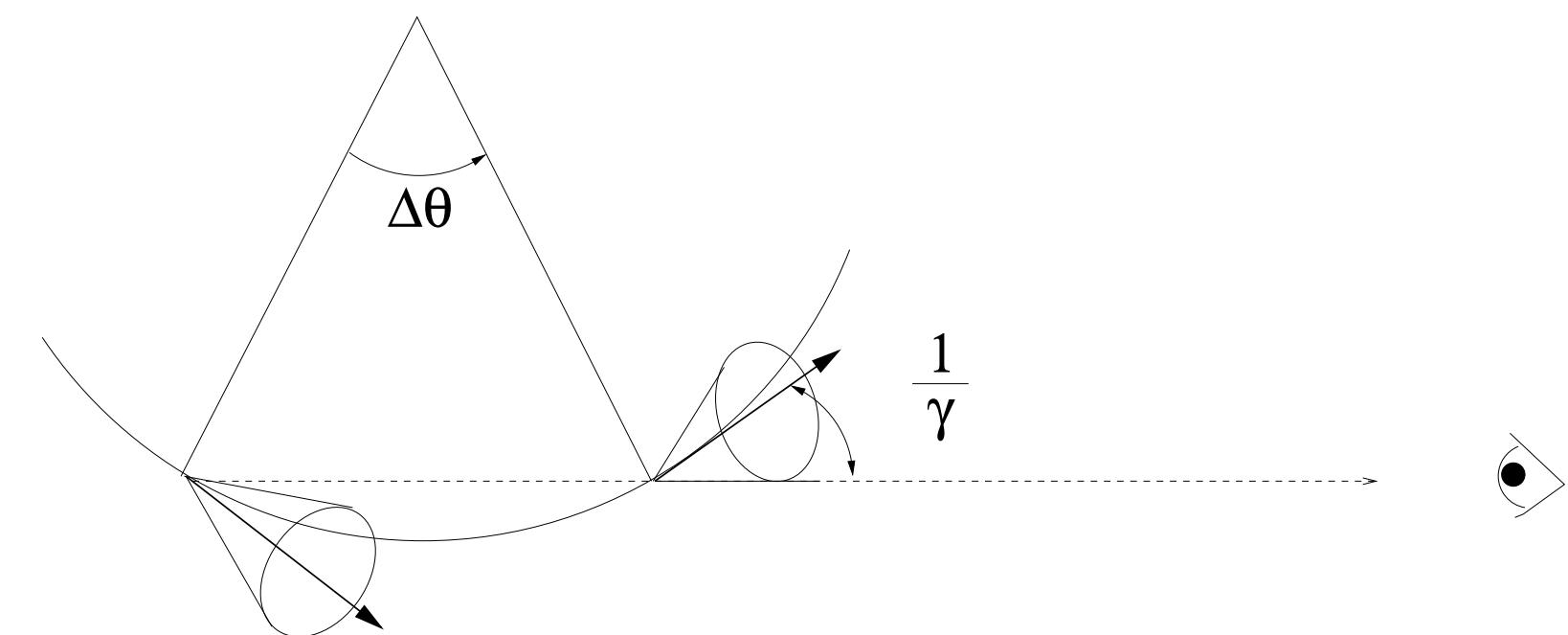
synchrotron basics



A)

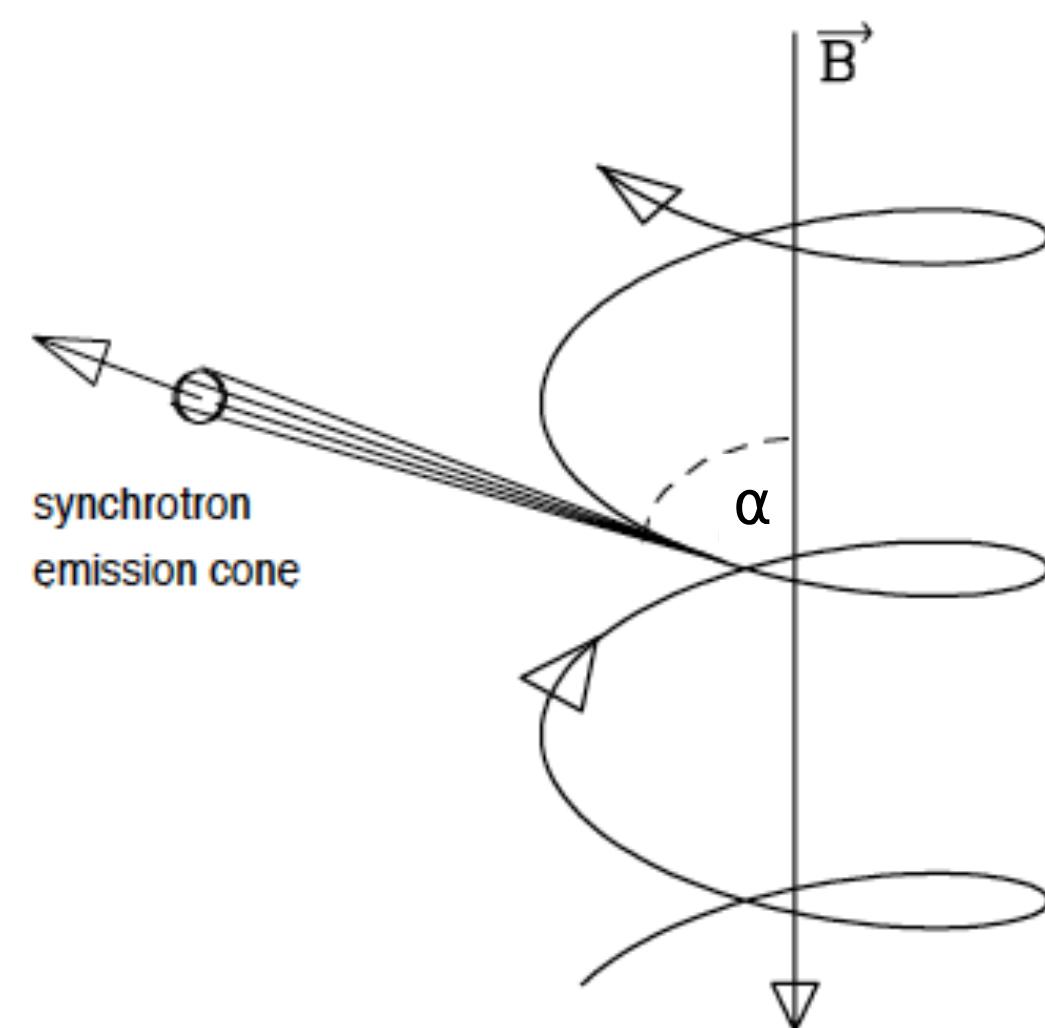


$$\Delta\phi = \frac{1}{\gamma}$$



synchrotron basics

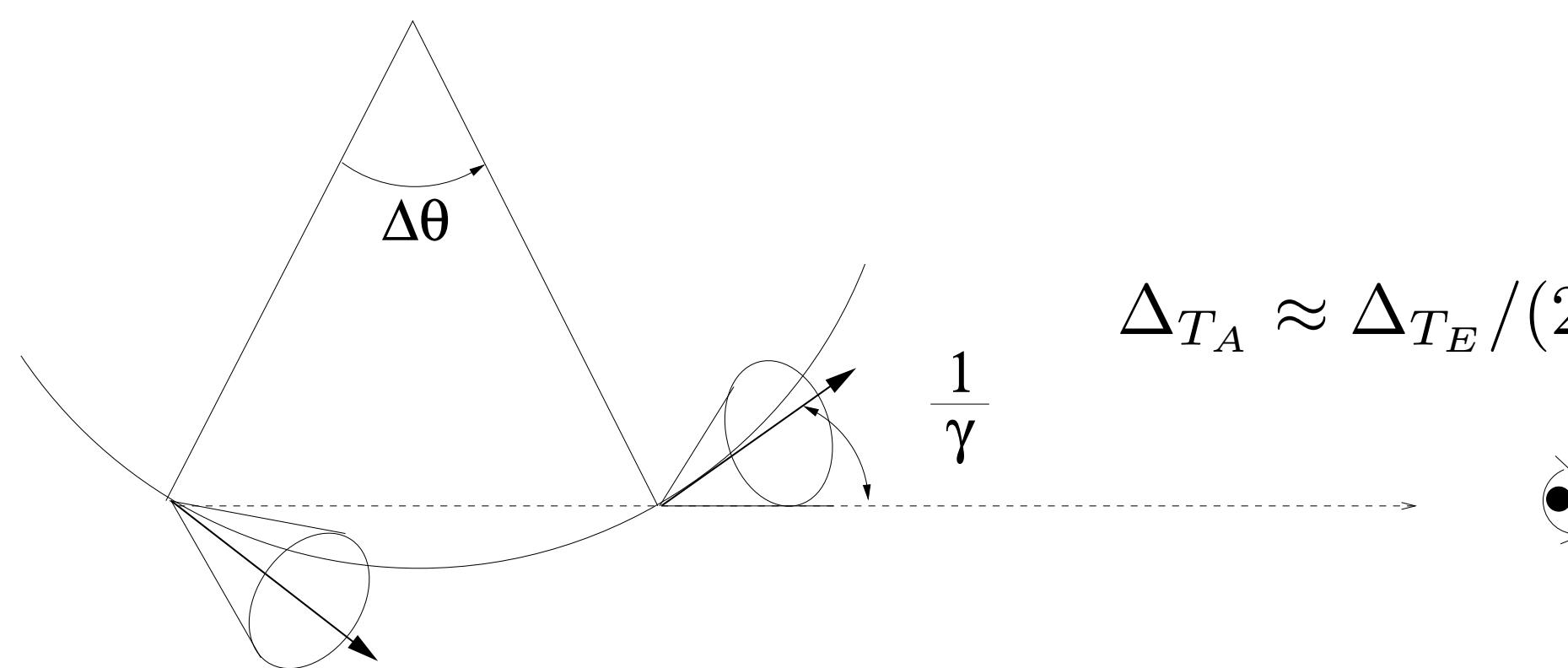
single particle



emitted spectrum

$$P_e(\nu, \gamma) = \frac{\sqrt{3}e^3 B \sin \alpha}{2m_e c^2} F(x)$$

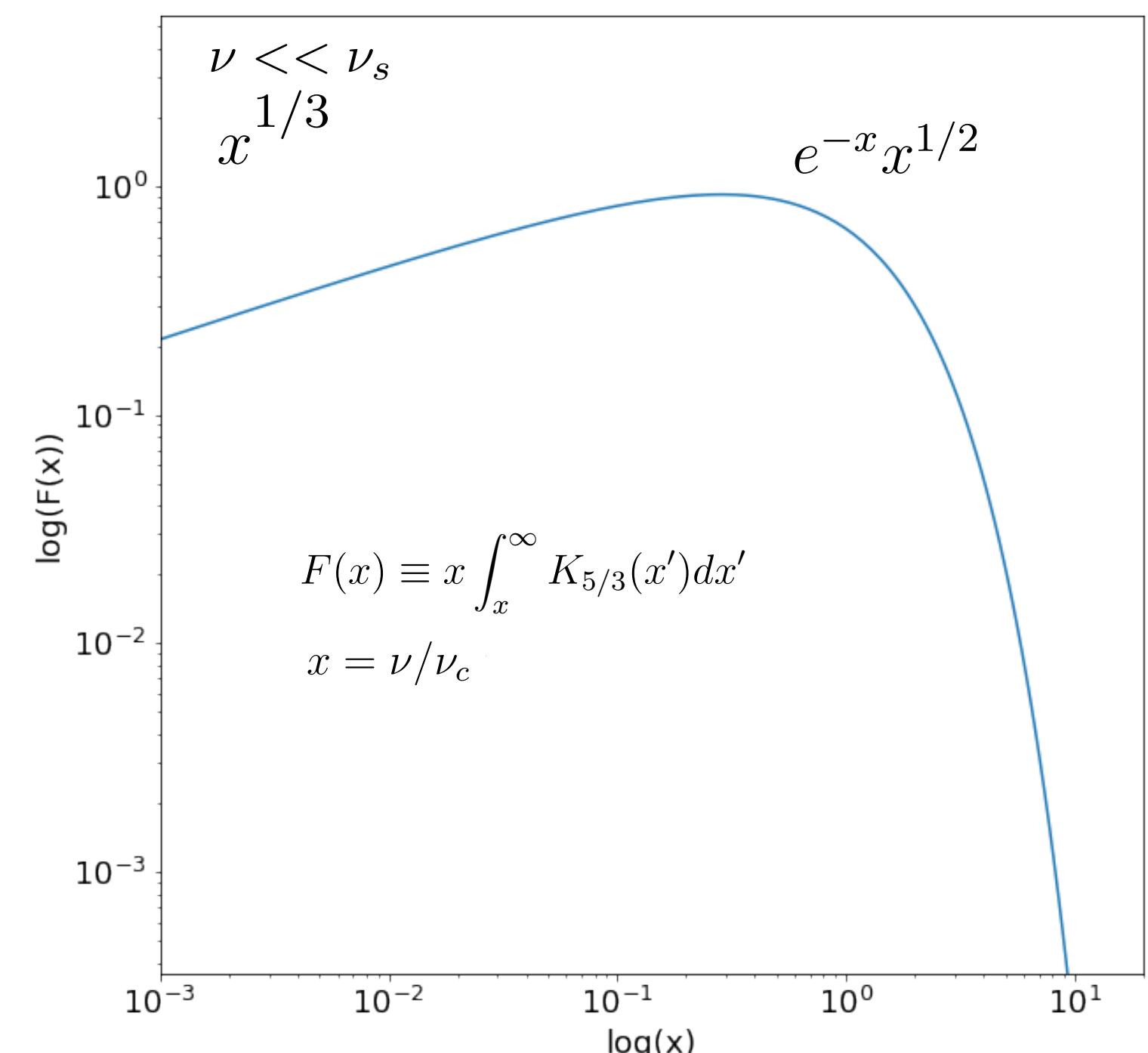
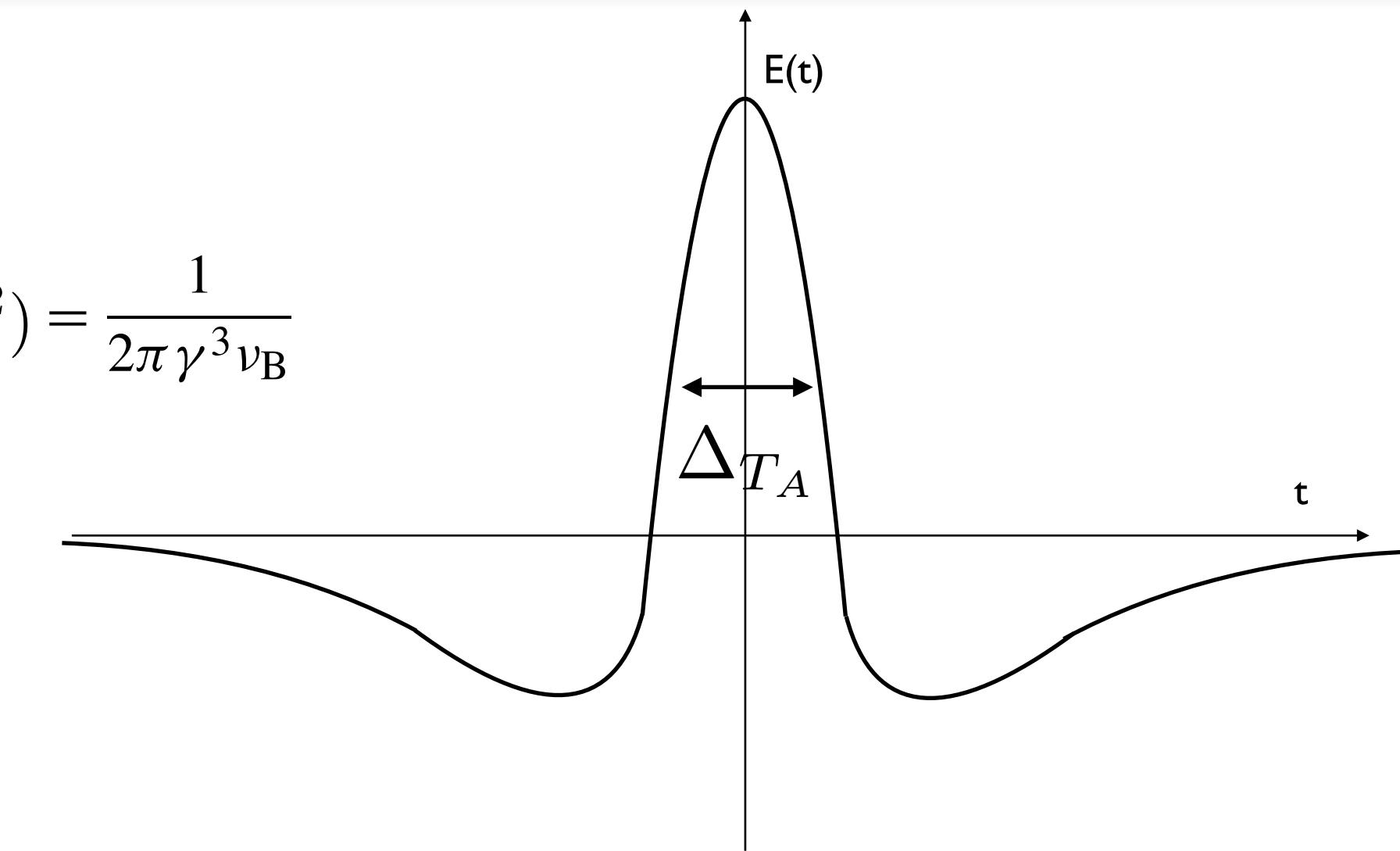
$$\nu_c = \frac{3}{2}\nu_B \gamma^3 \sin \alpha = \frac{3\gamma^2 e B \sin \alpha}{4\pi m_e c}$$



power spectrum

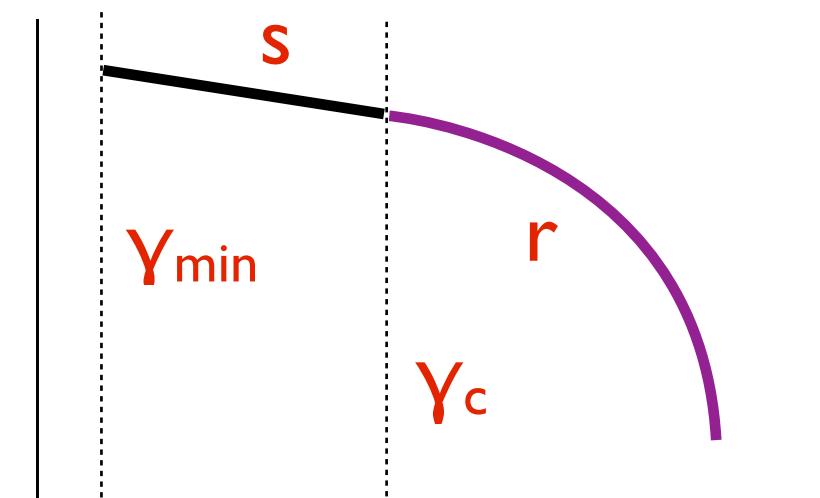
$$\Delta_{T_A} \nu_T = \frac{1}{2\pi} \rightarrow \nu_T = \gamma^3 \nu_B$$

$$\nu_s \propto 10^6 B \gamma^2 \sin(\alpha) \text{ Hz}$$



Tutorial 2

$$N(\gamma) \propto \gamma^{-s}$$



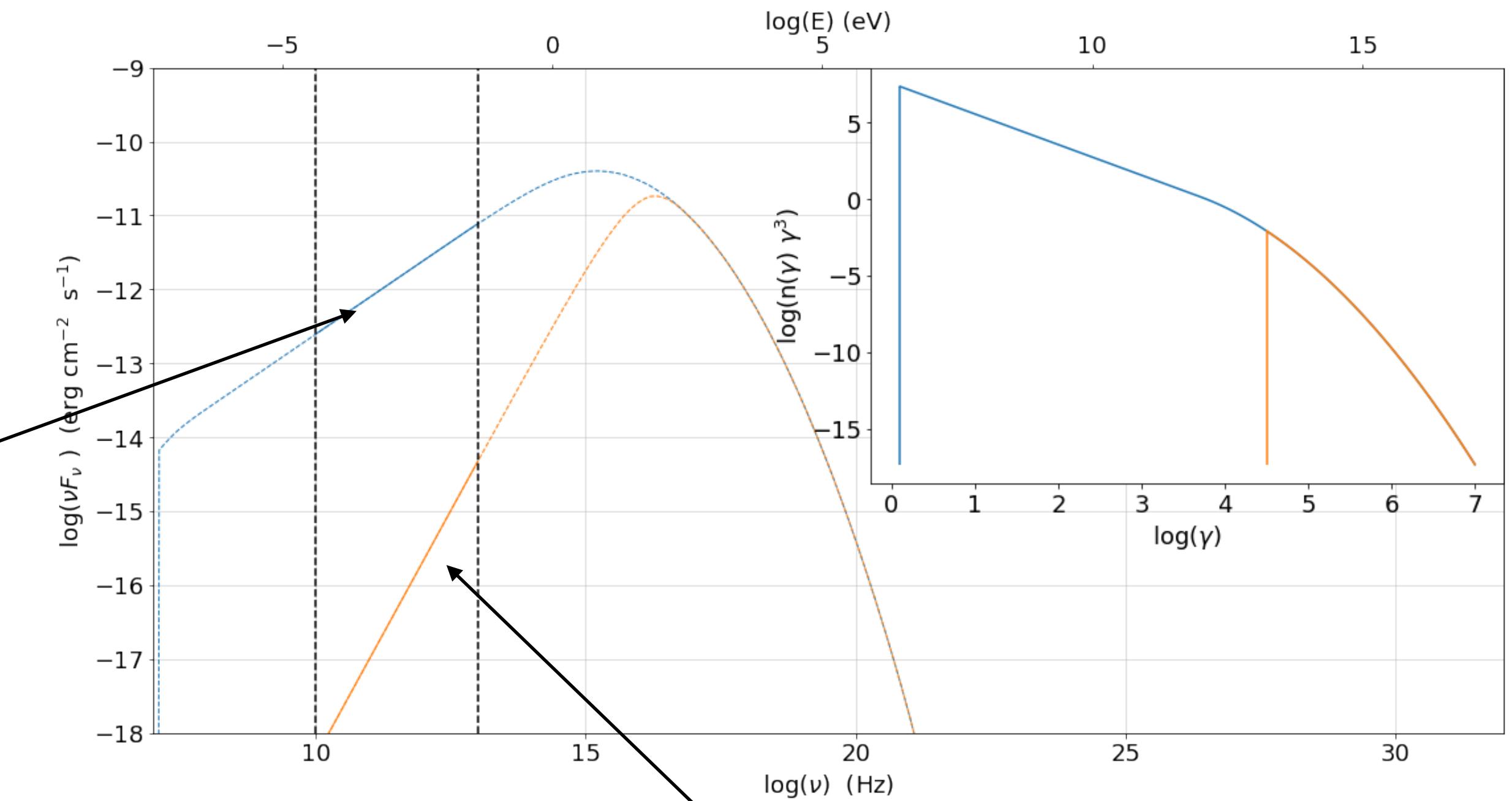
$$j_\nu^S(\nu) = \frac{1}{4\pi} \int_{\gamma_{min}}^{\gamma_{max}} P(\nu, \gamma) N(\gamma) d\gamma \propto \nu^{-\frac{s-1}{2}}$$

F_ν	refers to spectral index => SED =	$\frac{\nu F_\nu}{\nu I_\nu}$
I_ν		$\frac{\nu F_\nu}{\nu L_\nu}$
L_ν		

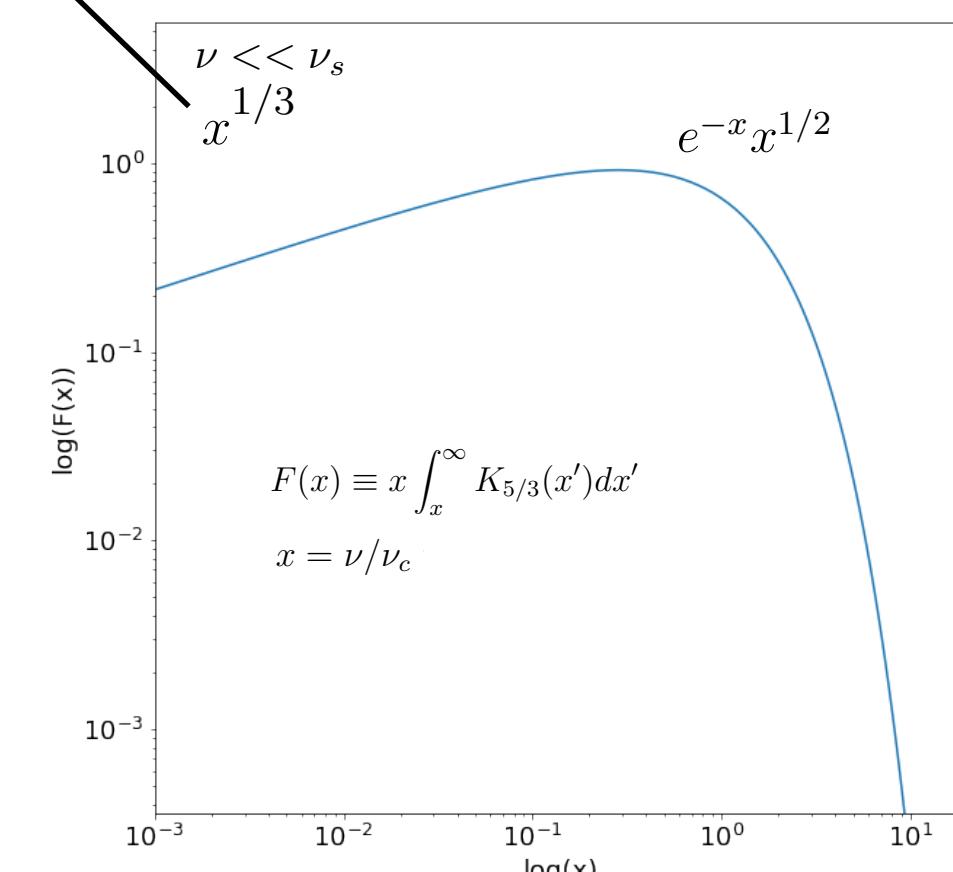
δ -approx relations

$$S_p^{Sync} \sim \frac{dN(\gamma)}{d\gamma} \gamma_{3p}^3 B^2 \delta^4 \quad \nu_p^{Sync} \sim 3.2 \times 10^6 (\gamma_{3p})^2 B \delta$$

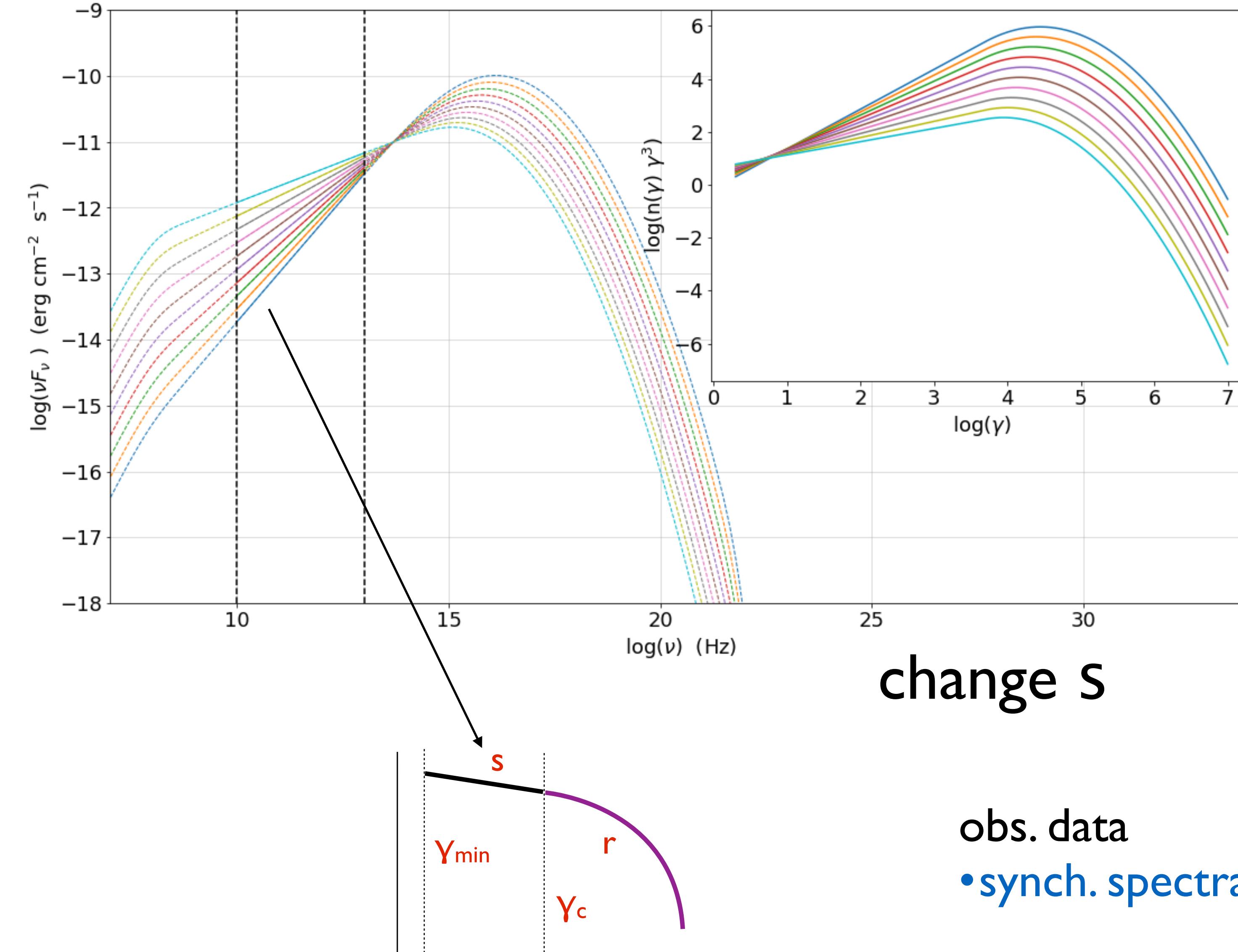
$$\text{SED} \propto N(\gamma) \gamma^3$$



4/3 in SED corresponds
to 1/3 in spectral index

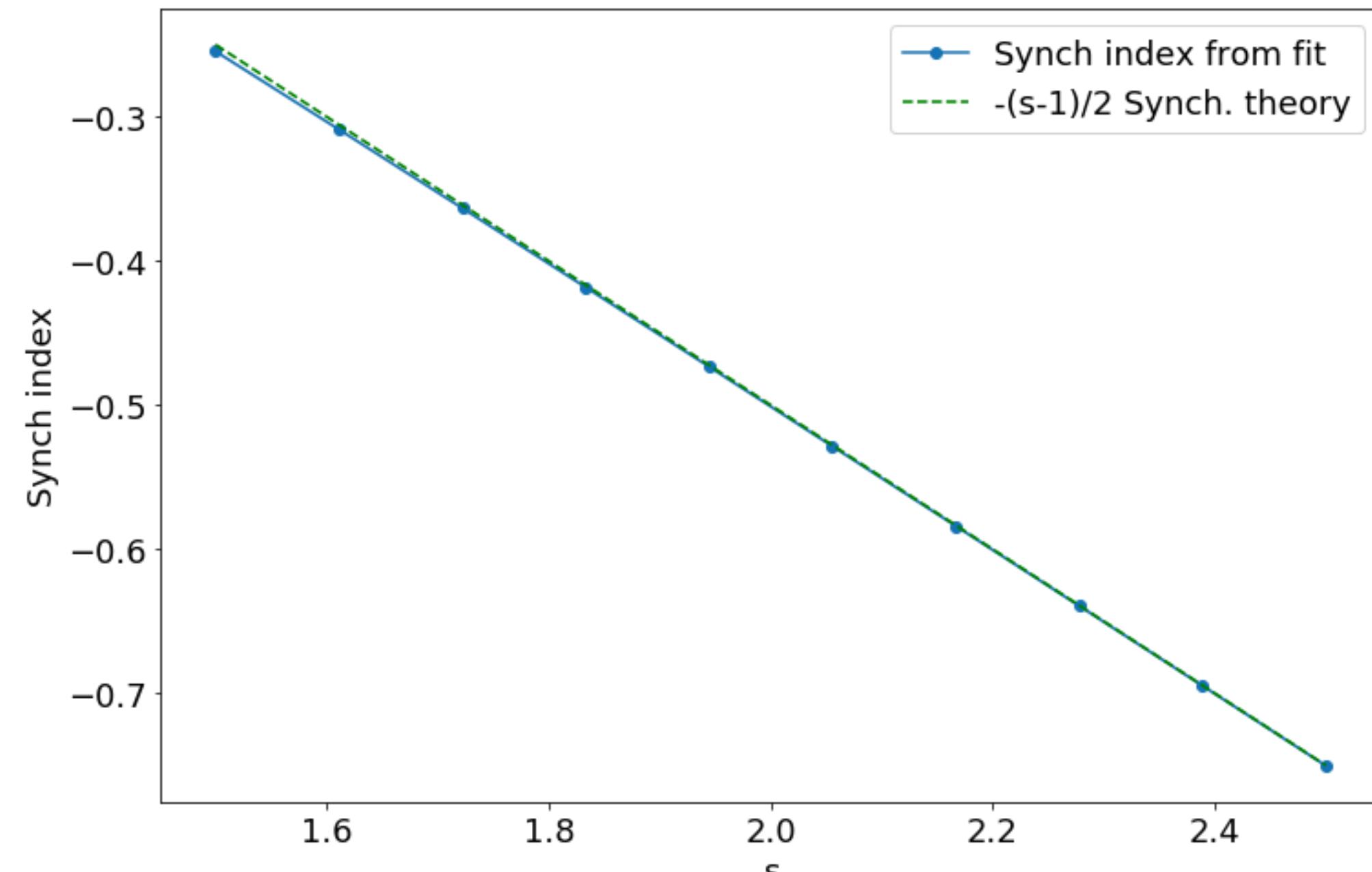


Tutorial 2

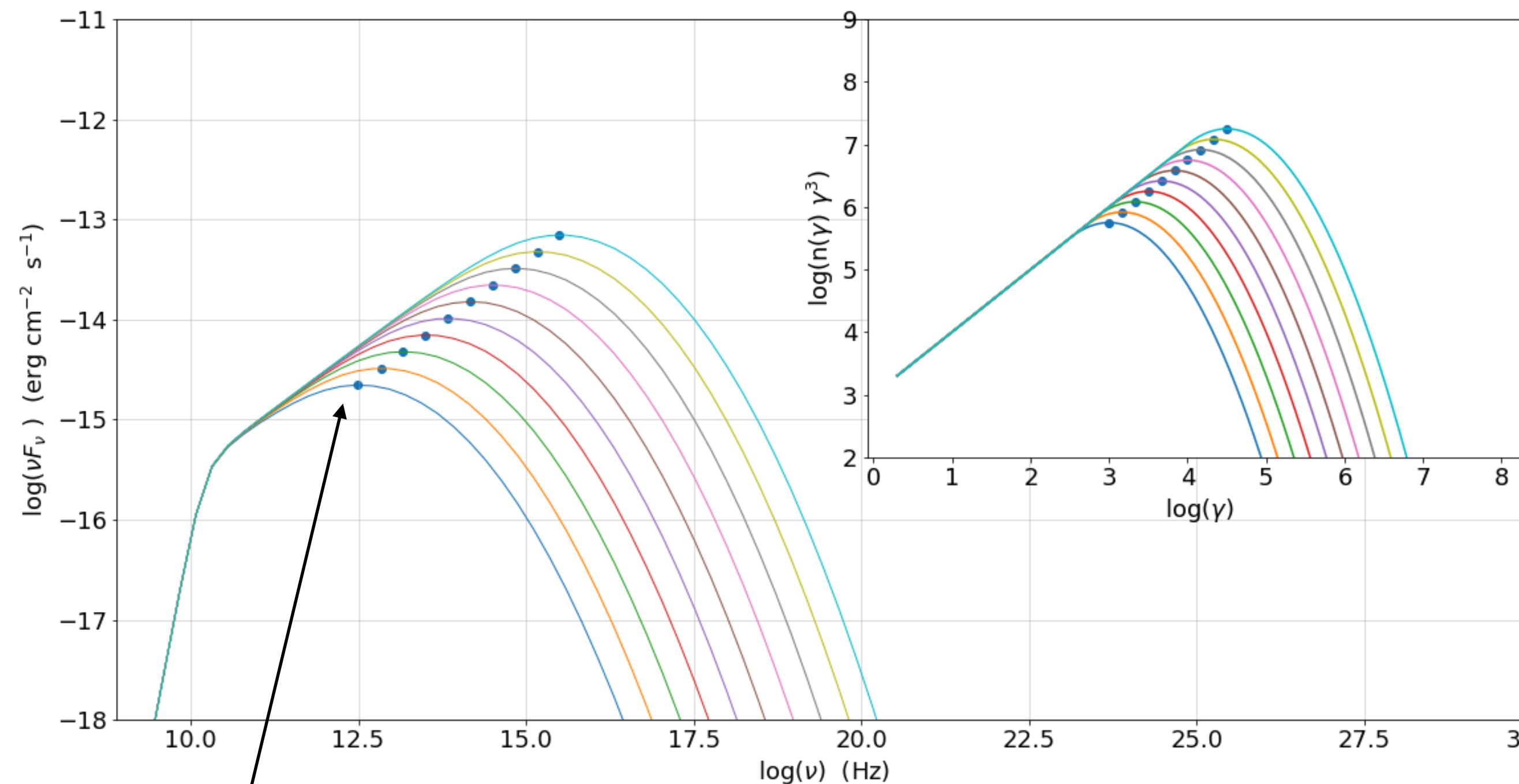
change S

obs. data
•synch. spectral index

model
• s el. distr.



$$j_\nu^S(\nu) = \frac{1}{4\pi} \int_{\gamma_{min}}^{\gamma_{max}} P(\nu, \gamma) N(\gamma) d\gamma \propto \nu^{-\frac{s-1}{2}}$$



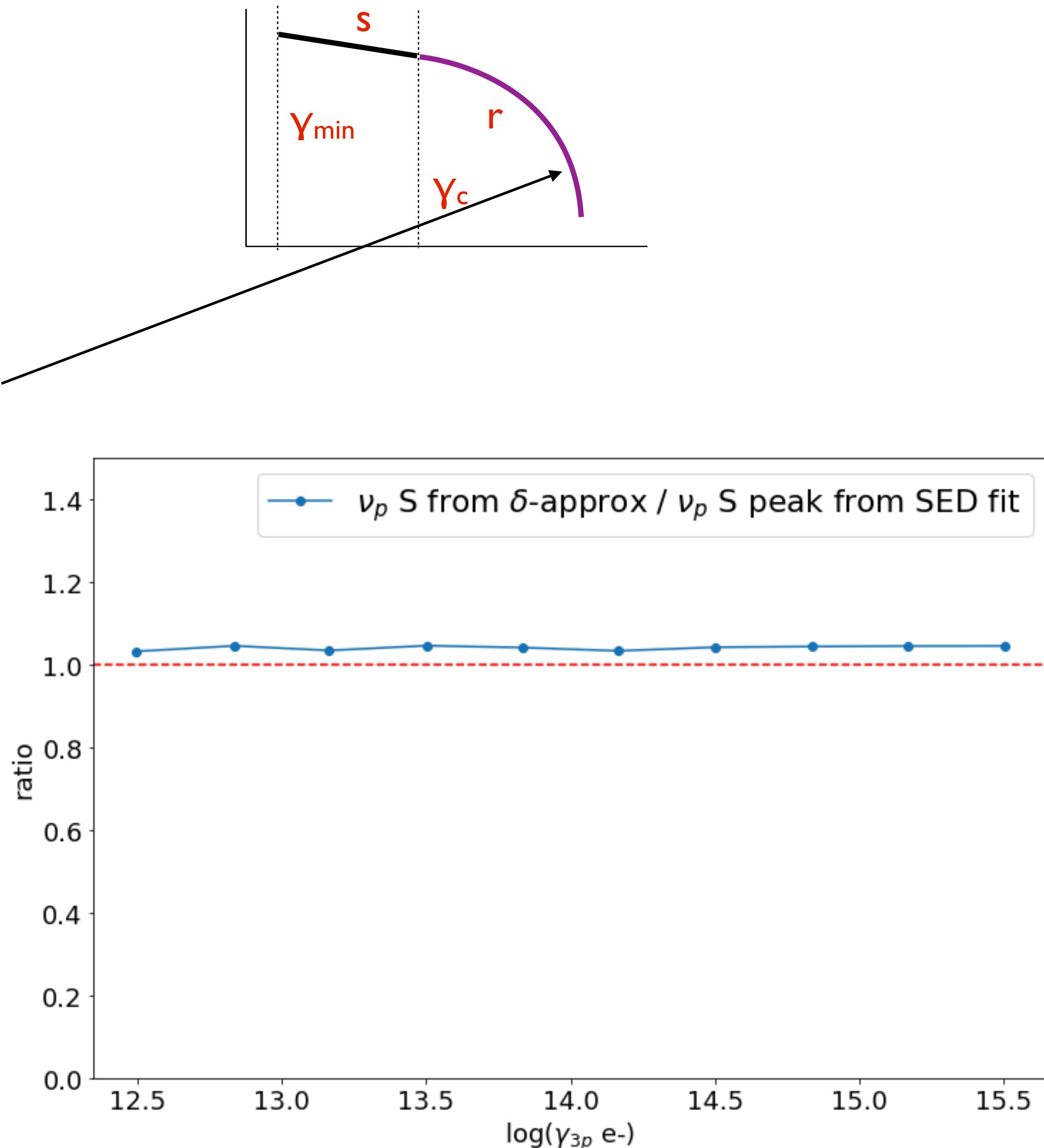
$$\nu_p^{Sync} \sim 3.2 \times 10^6 (\gamma_{3p})^2 B \delta$$

$$S_p^{Sync} \sim \frac{dN(\gamma)}{d\gamma} \gamma_{3p}^3 B^2 \delta^4$$

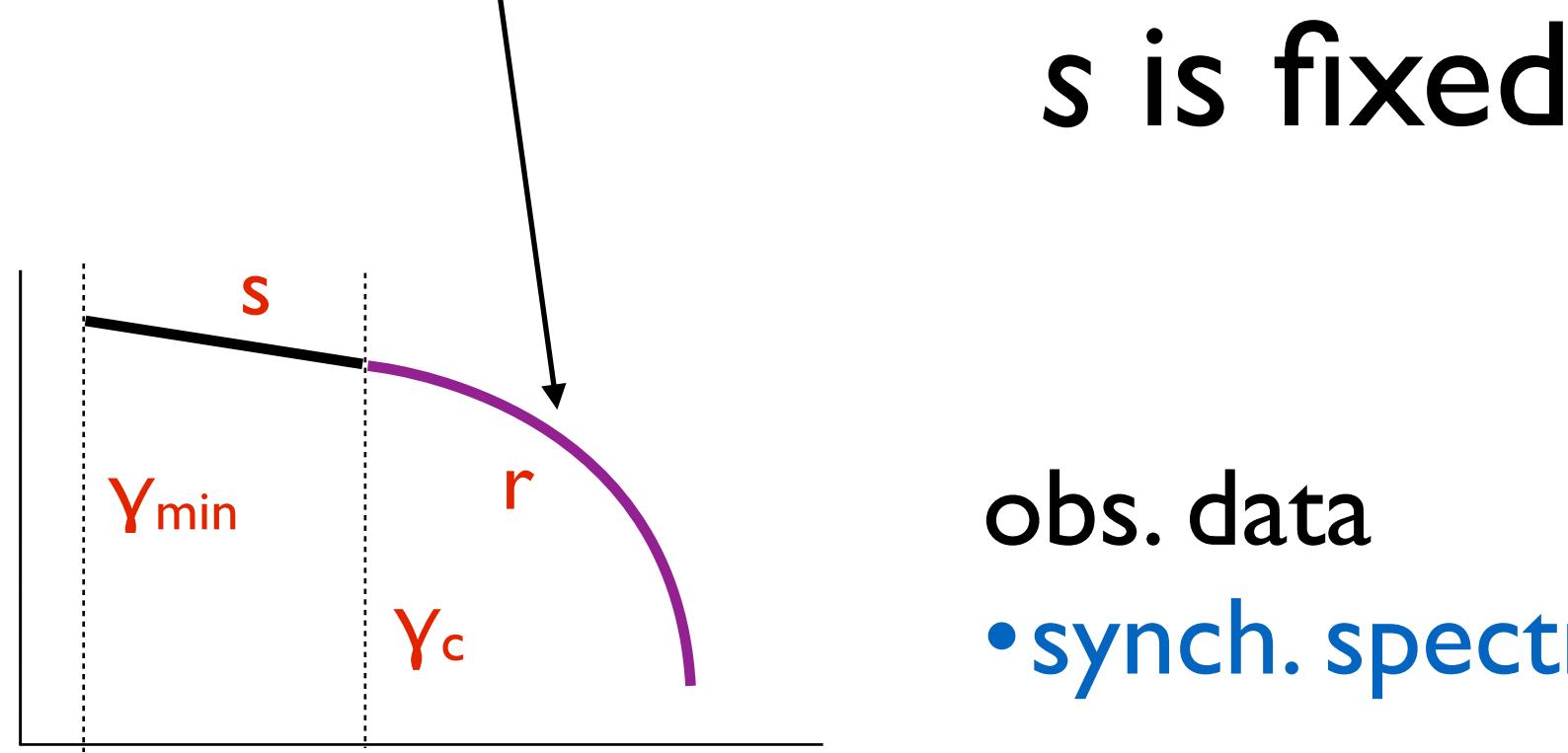
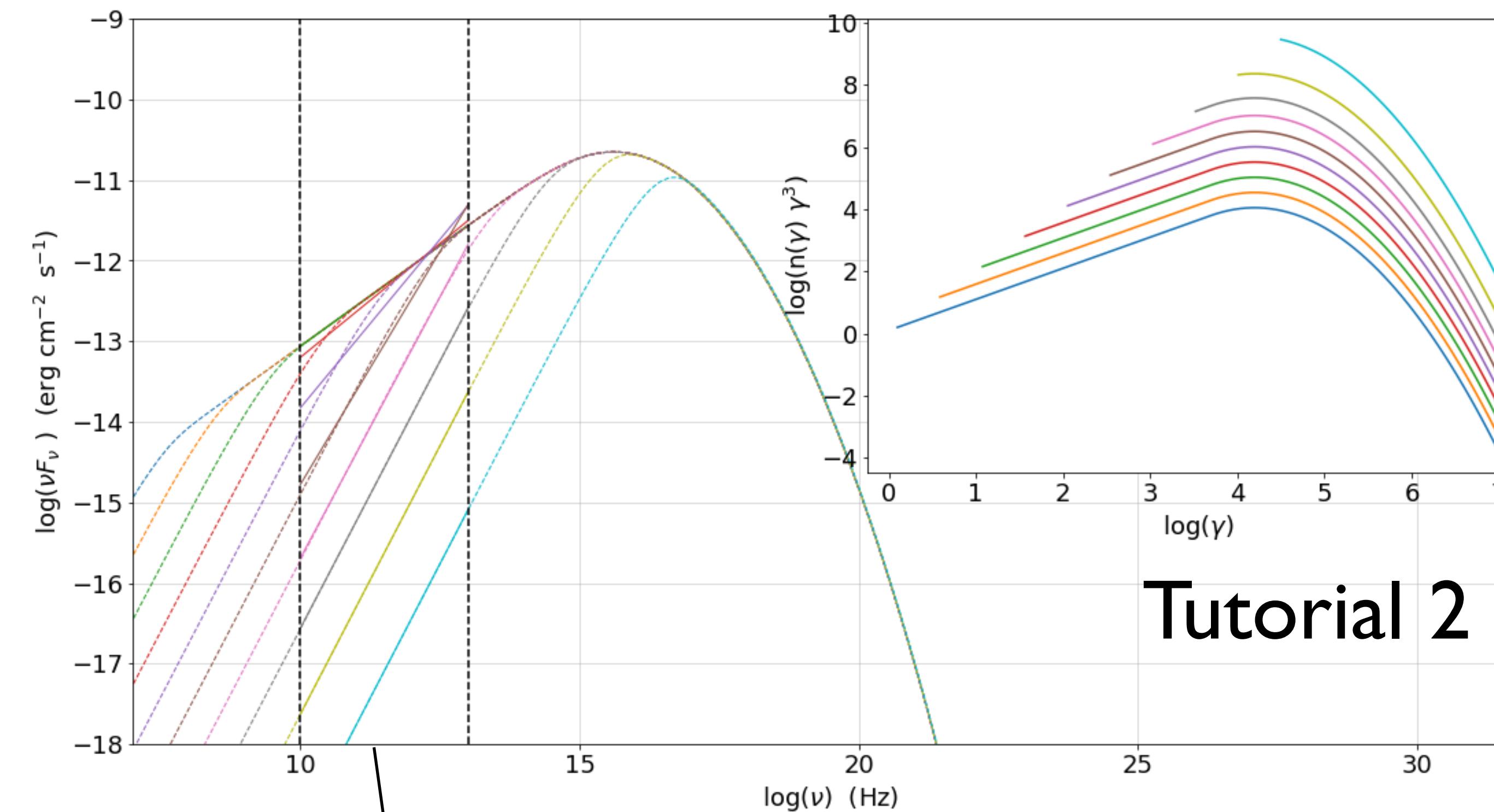
change γ_c

obs. data \longrightarrow model

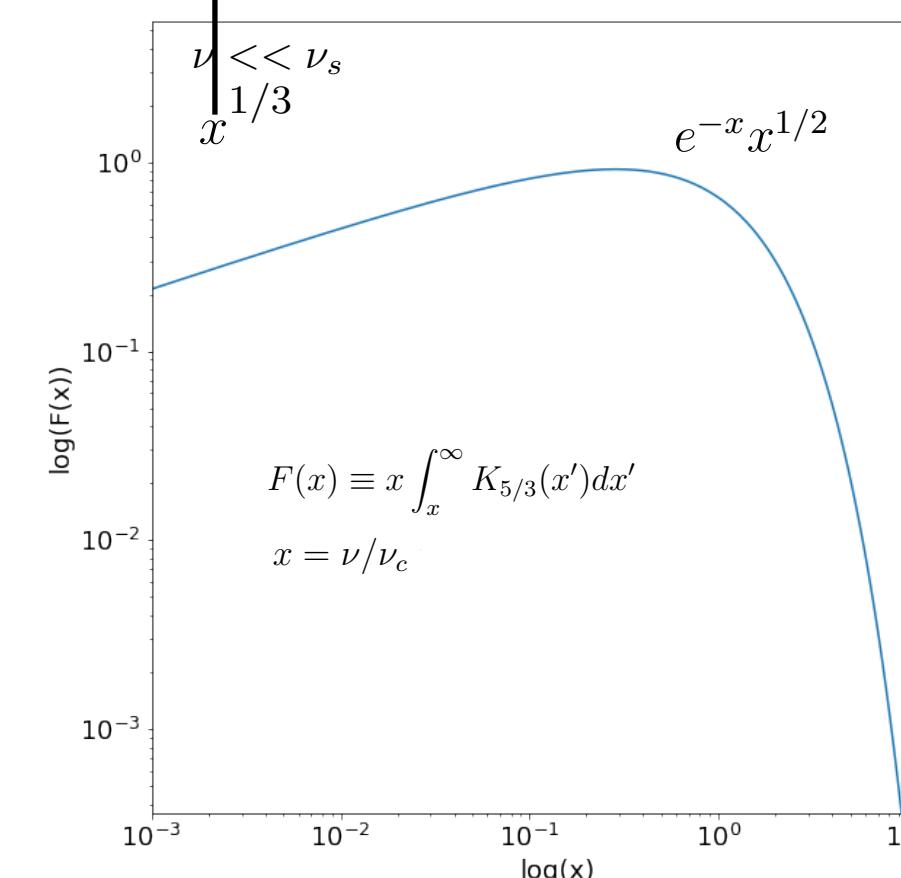
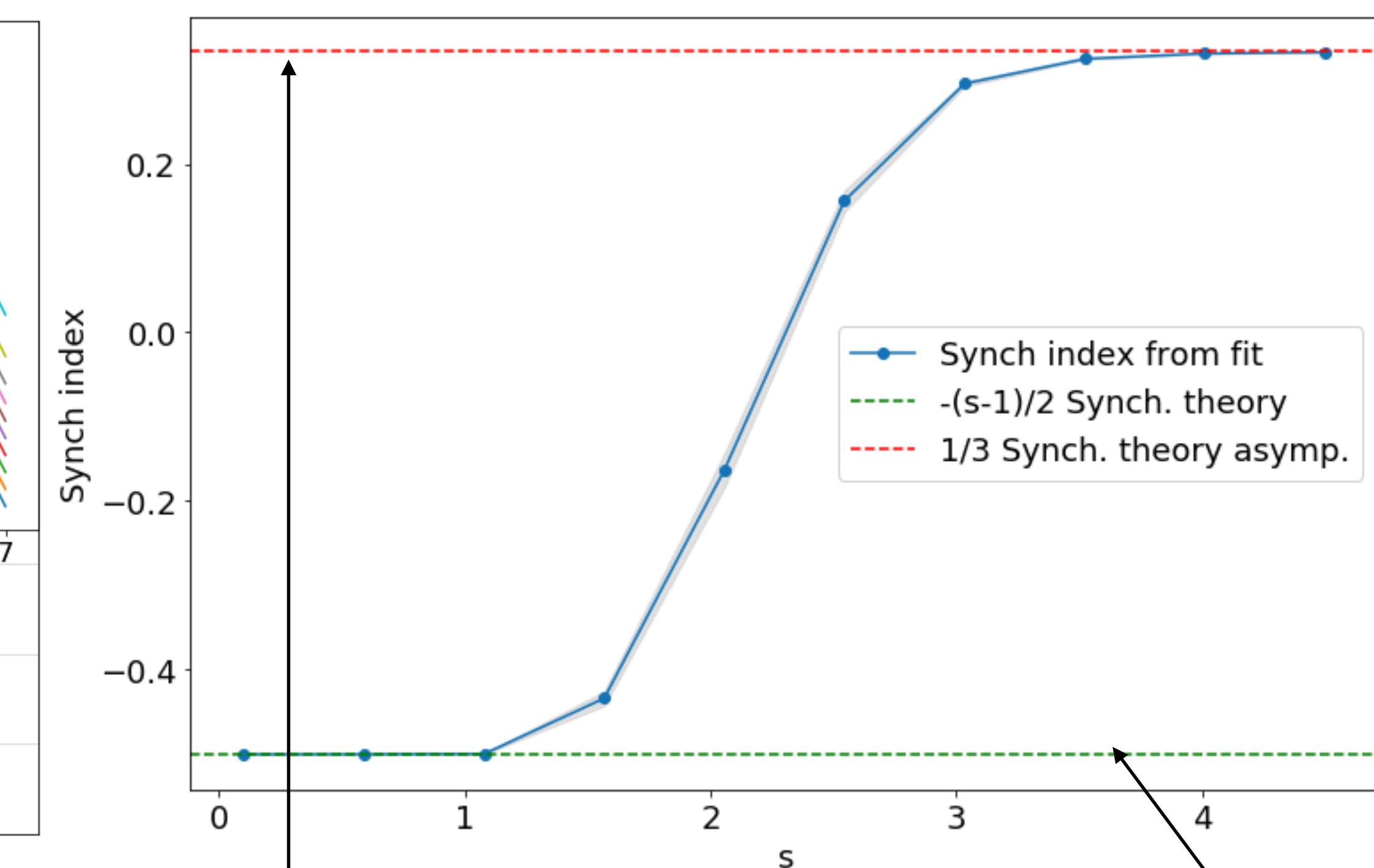
- ν_p^s
- γ_p^s



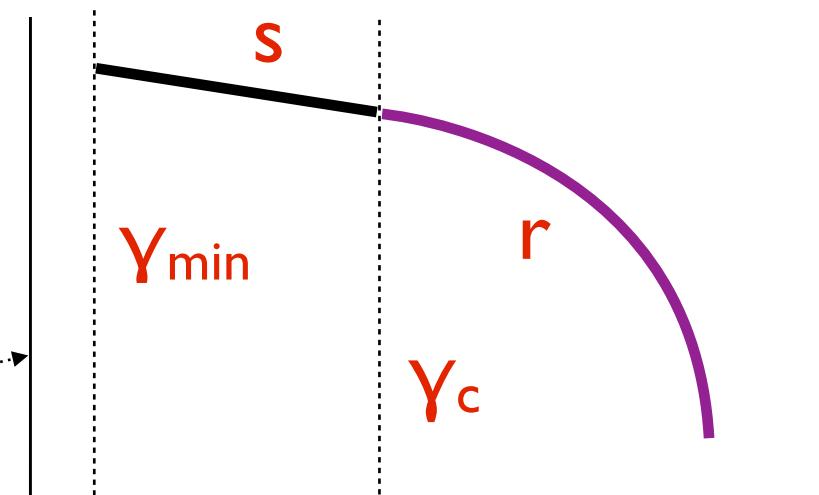
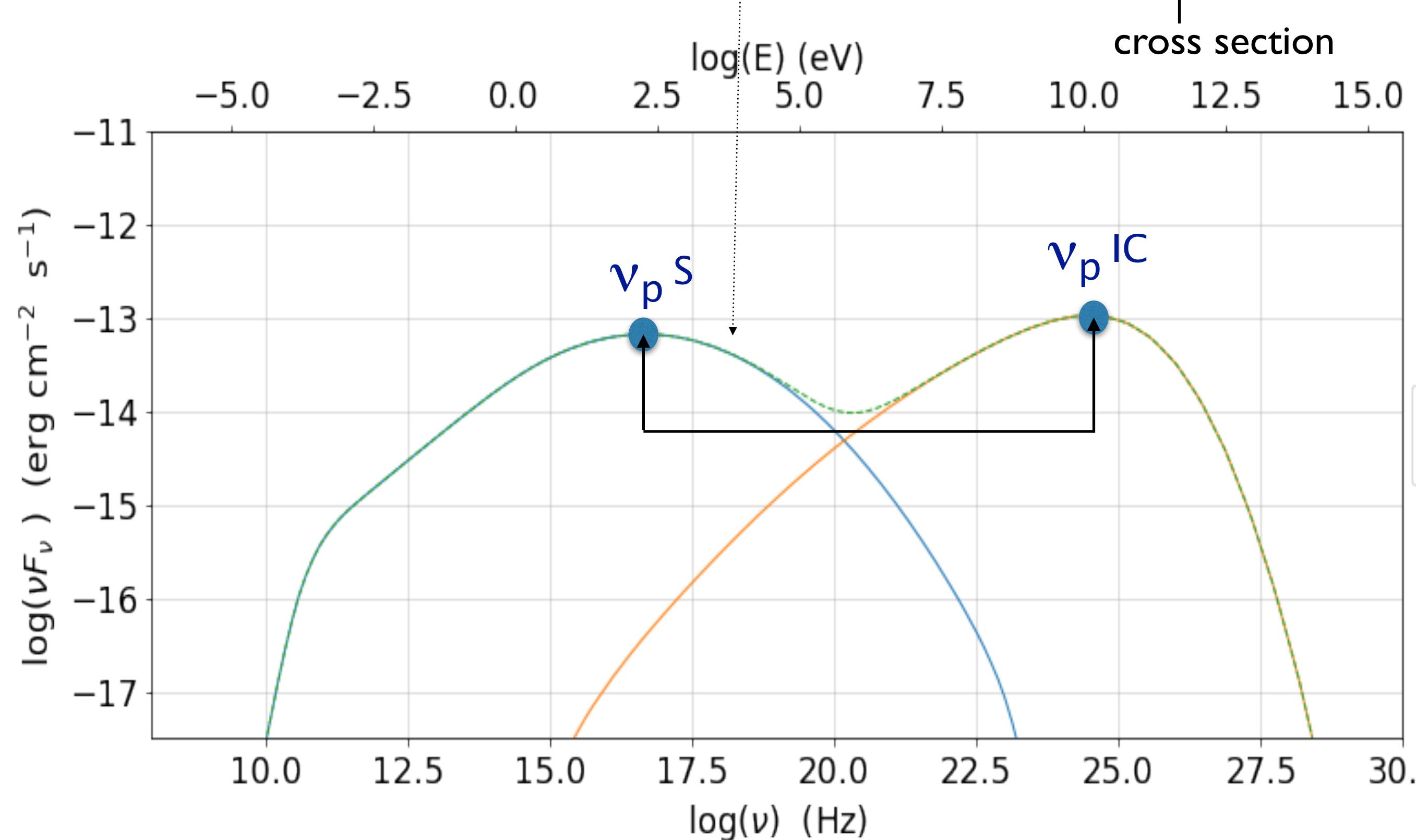
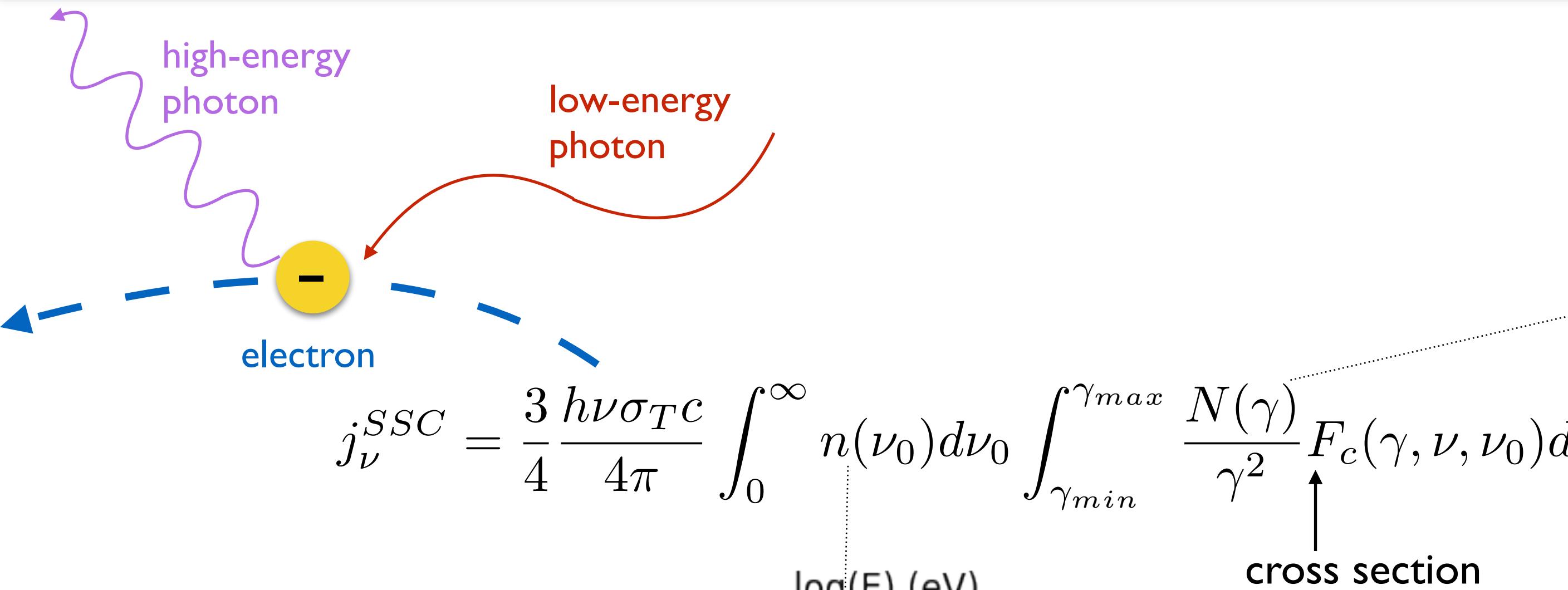
Synchrotron emission Estimate of γ_{min} from spectral index



obs. data → model
 • synch. spectral index
 • γ_{min}



IC emission basics



Tutorial 2

soft photon energy
in the e- rest frame

$$\epsilon' = \frac{h\nu'}{m_e c^2}$$

$$\epsilon' \ll m_e c^2$$

TH regime

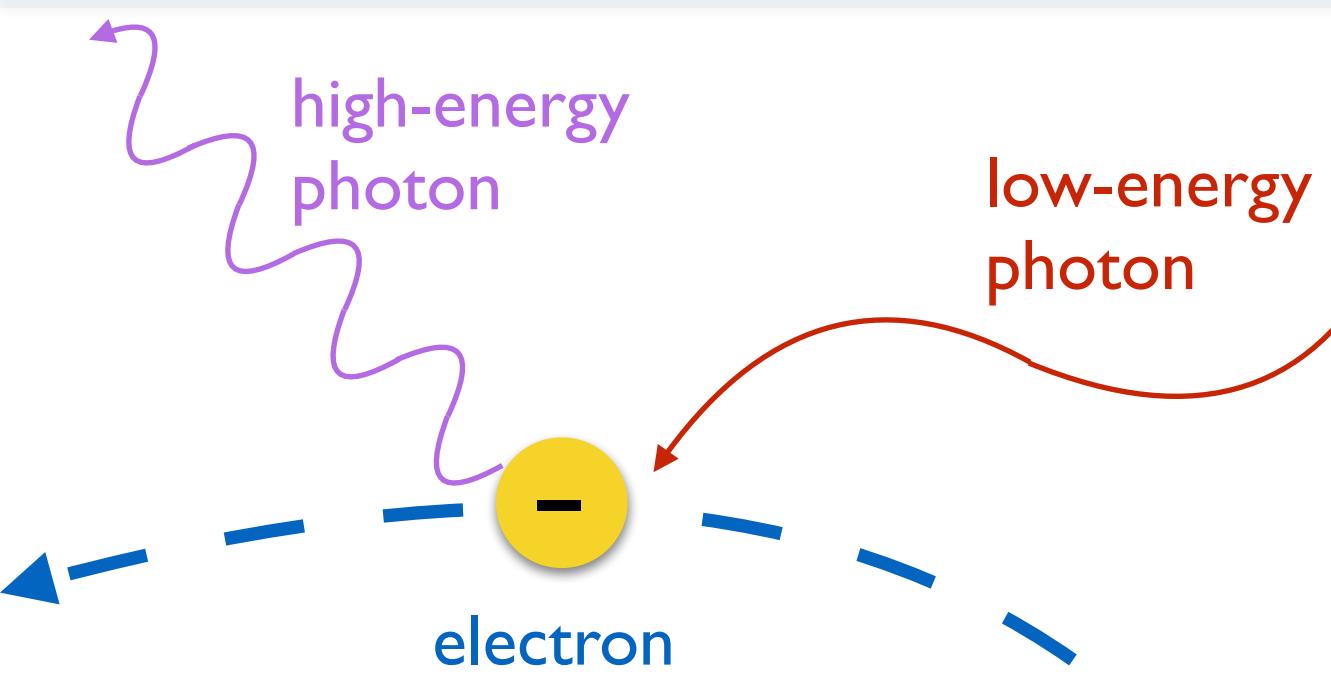
$$\nu_p^{IC} / \nu_p^{SSC} \sim (4/3) \gamma_p^2$$

$$\epsilon' \geq m_e c^2$$

KN regime

$$\nu_p^{IC} / \nu_p^{SSC} \sim \gamma_p$$

$$h\nu_p^{IC} \sim m_e c^2 \gamma_p$$

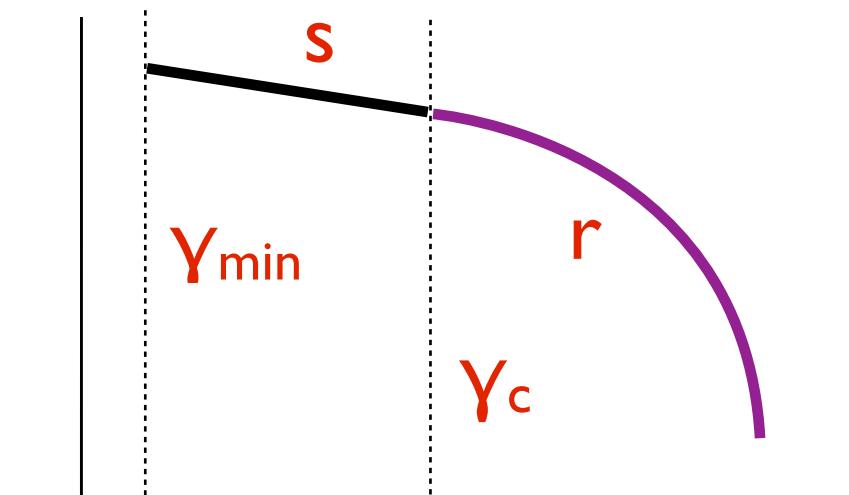
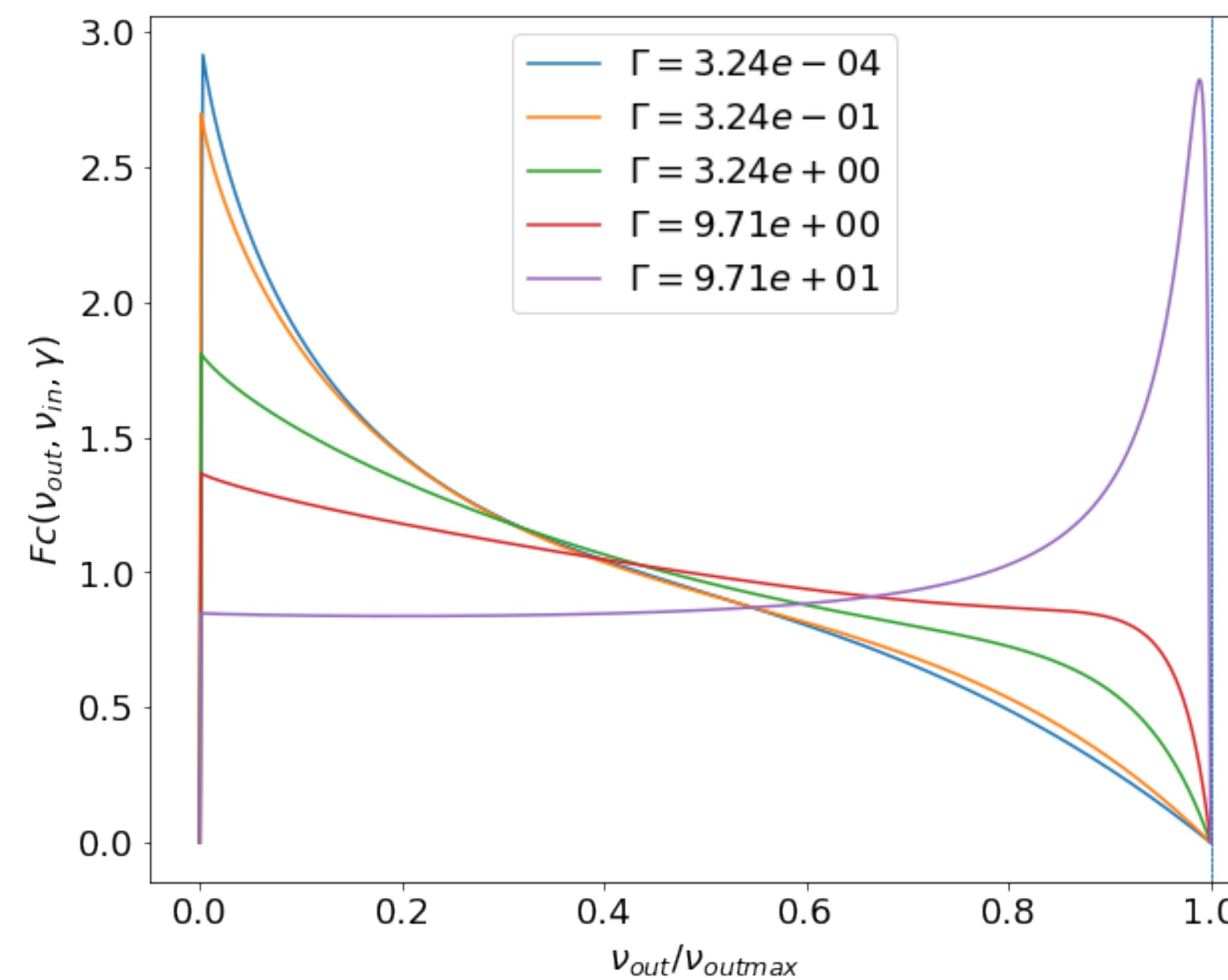


soft photon energy
in the e- rest frame

$$\epsilon' = \frac{h\nu'}{m_e c^2}$$

$$\Gamma = 4\epsilon' \gamma_e$$

$$\nu_{\text{out max}} = \frac{4\nu_{\text{in}} \gamma_e^2}{1+4\gamma_e \epsilon'}$$

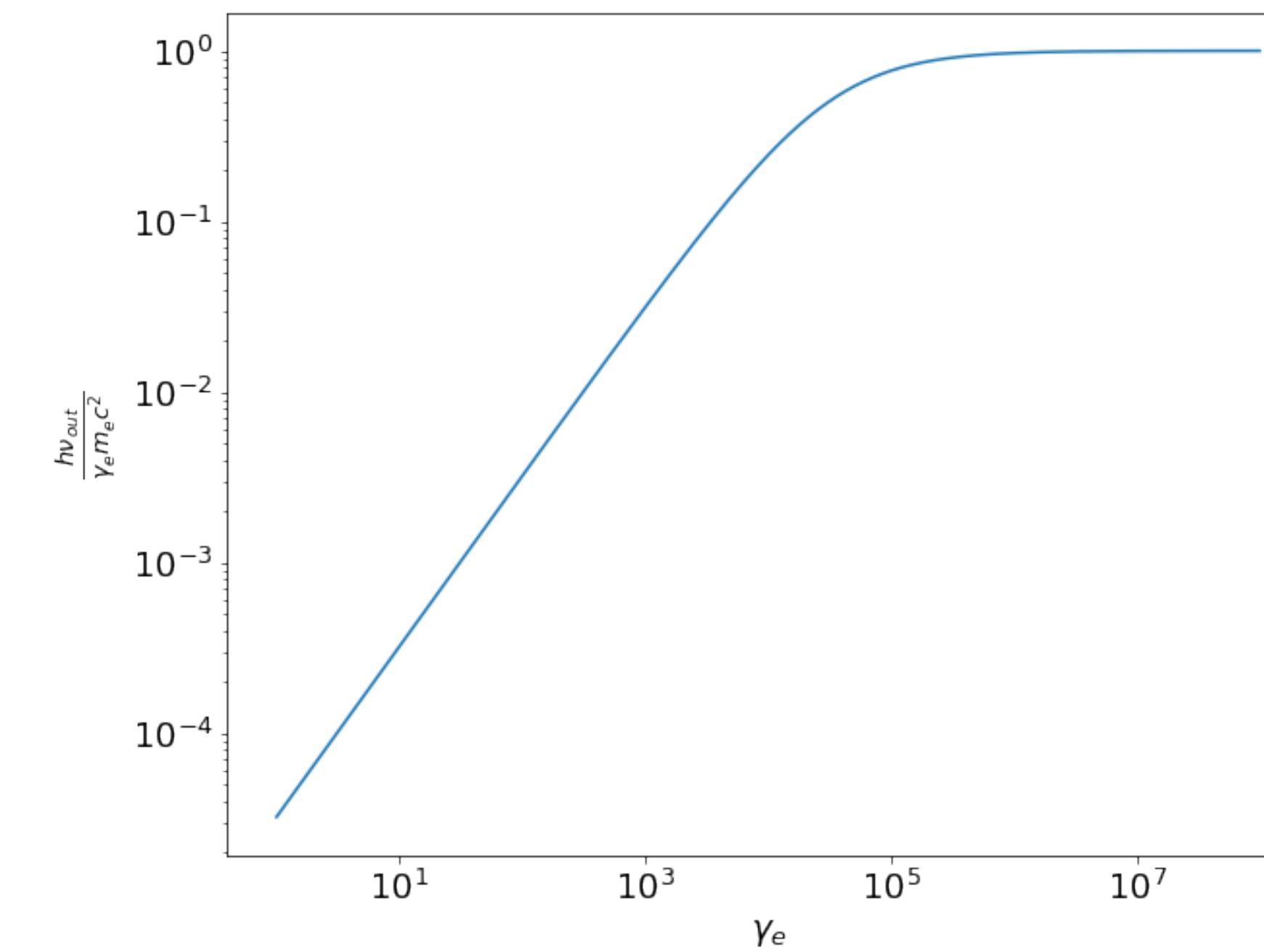


Tutorial 2

TH regime

$$\epsilon' \ll m_e c^2$$

$$\nu_p^{\text{IC}} / \nu_p^S \sim (4/3) \gamma_p^2$$



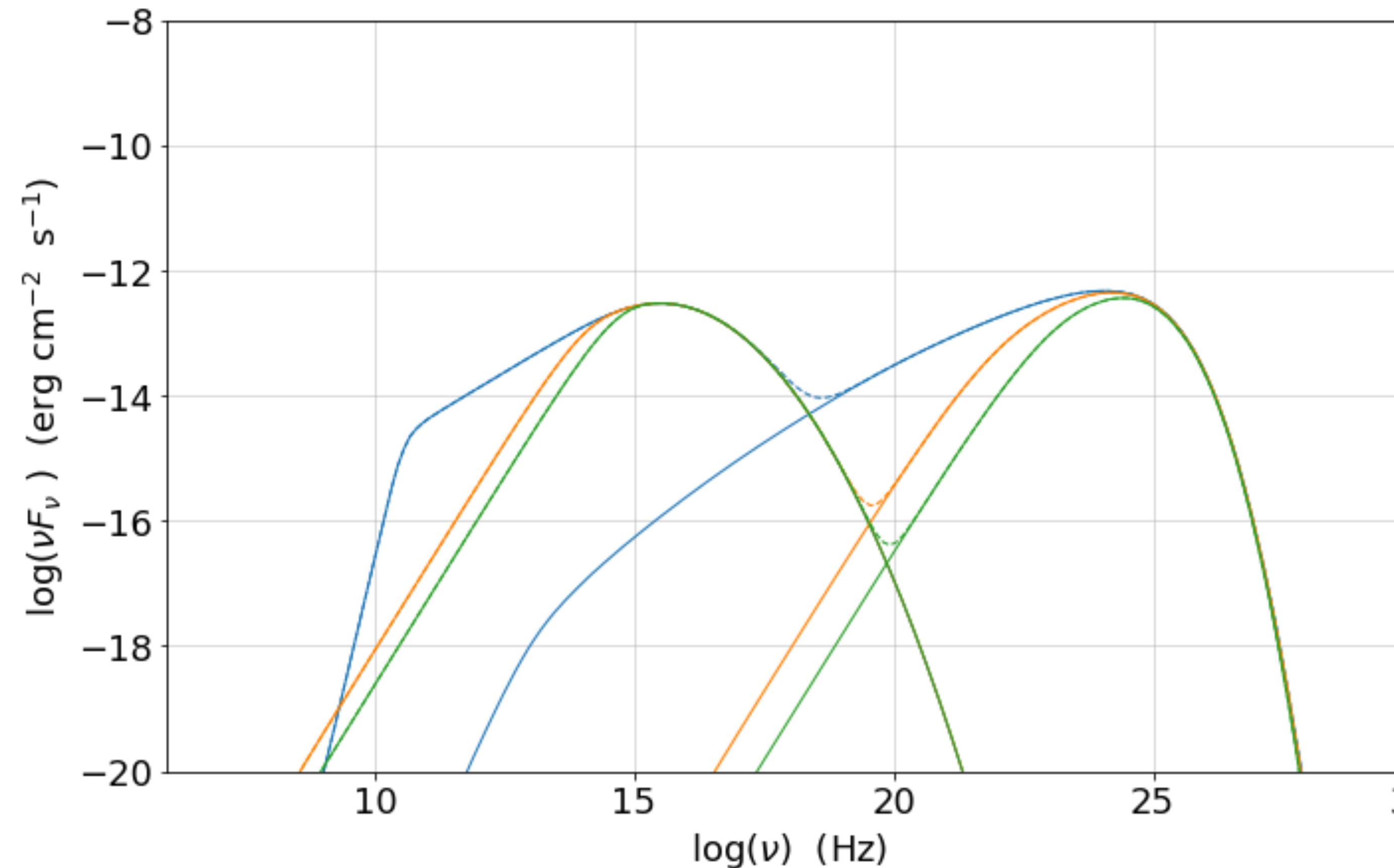
KN regime

$$\epsilon' \geq m_e c^2$$

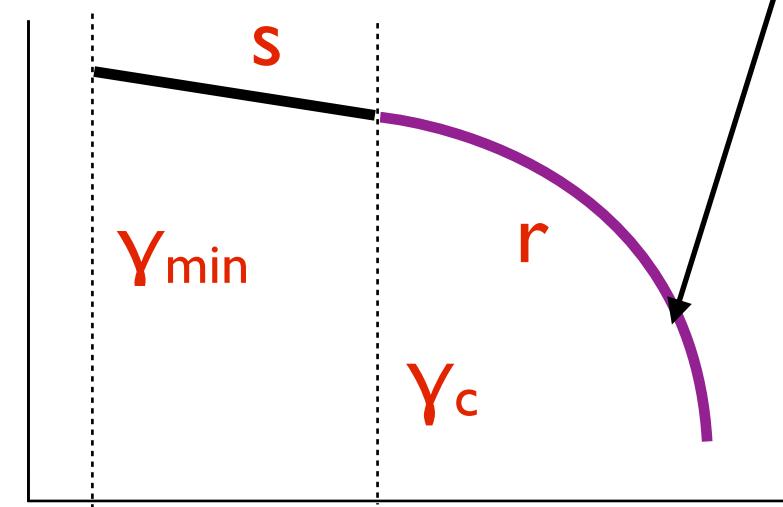
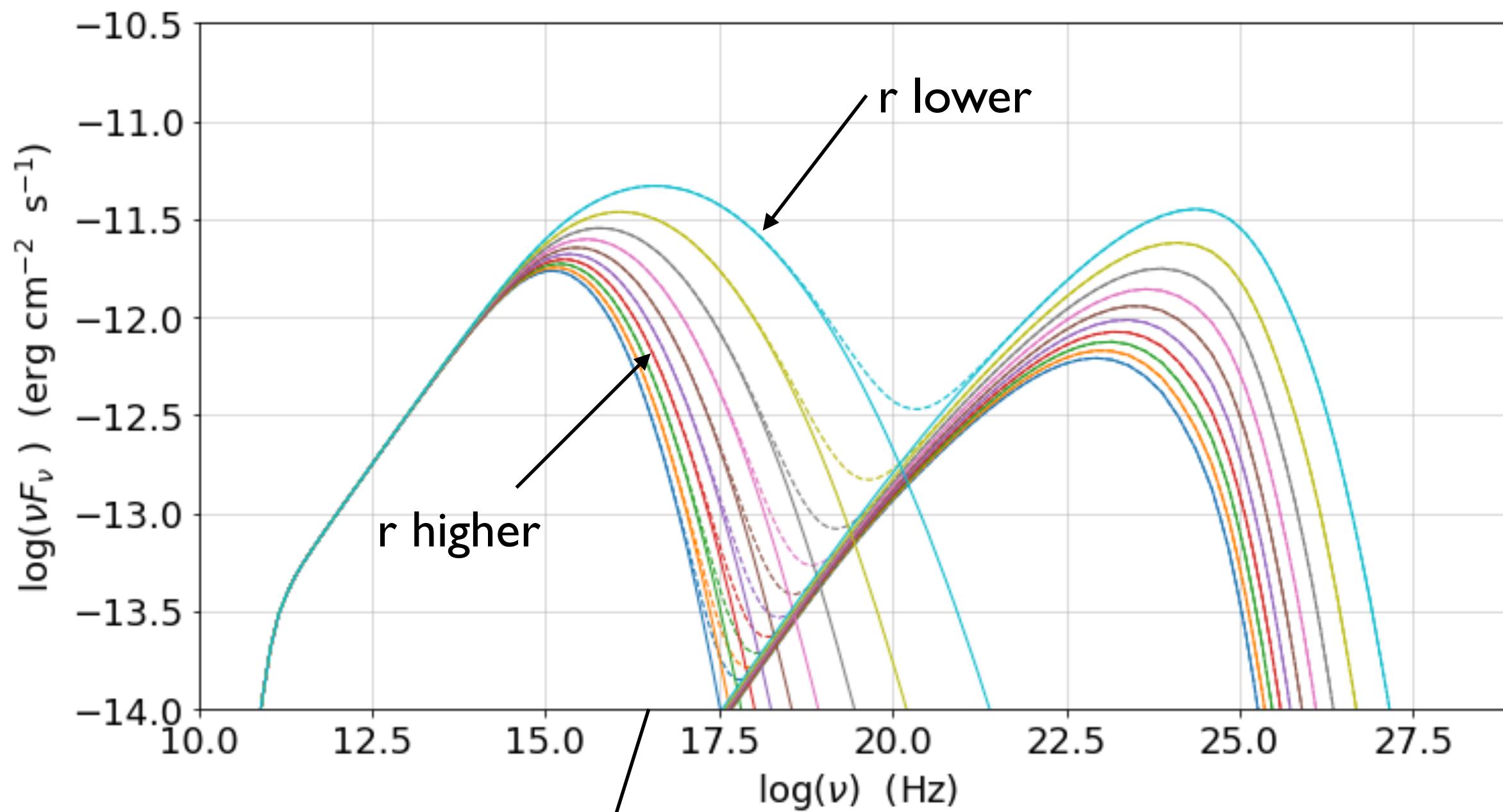
$$\nu_p^{\text{IC}} / \nu_p^S \sim \gamma_p$$

$$h\nu_p^{\text{IC}} \sim m_e c^2 \gamma_p$$

Adding the IC emission SSC case



IC emission TH/KN regime and peak curvature



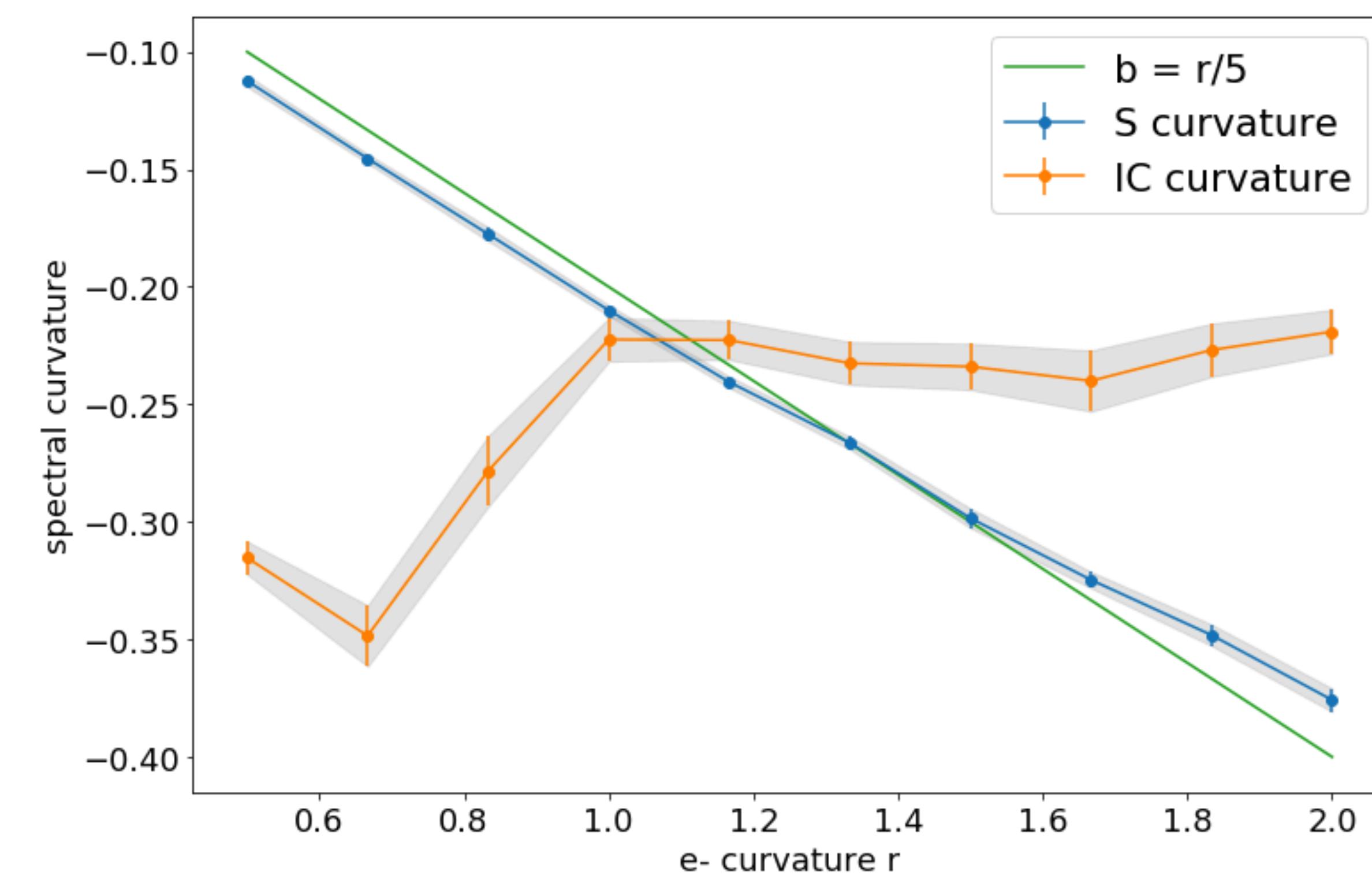
obs. data
• spectral curvature



model
• KN regime
• r

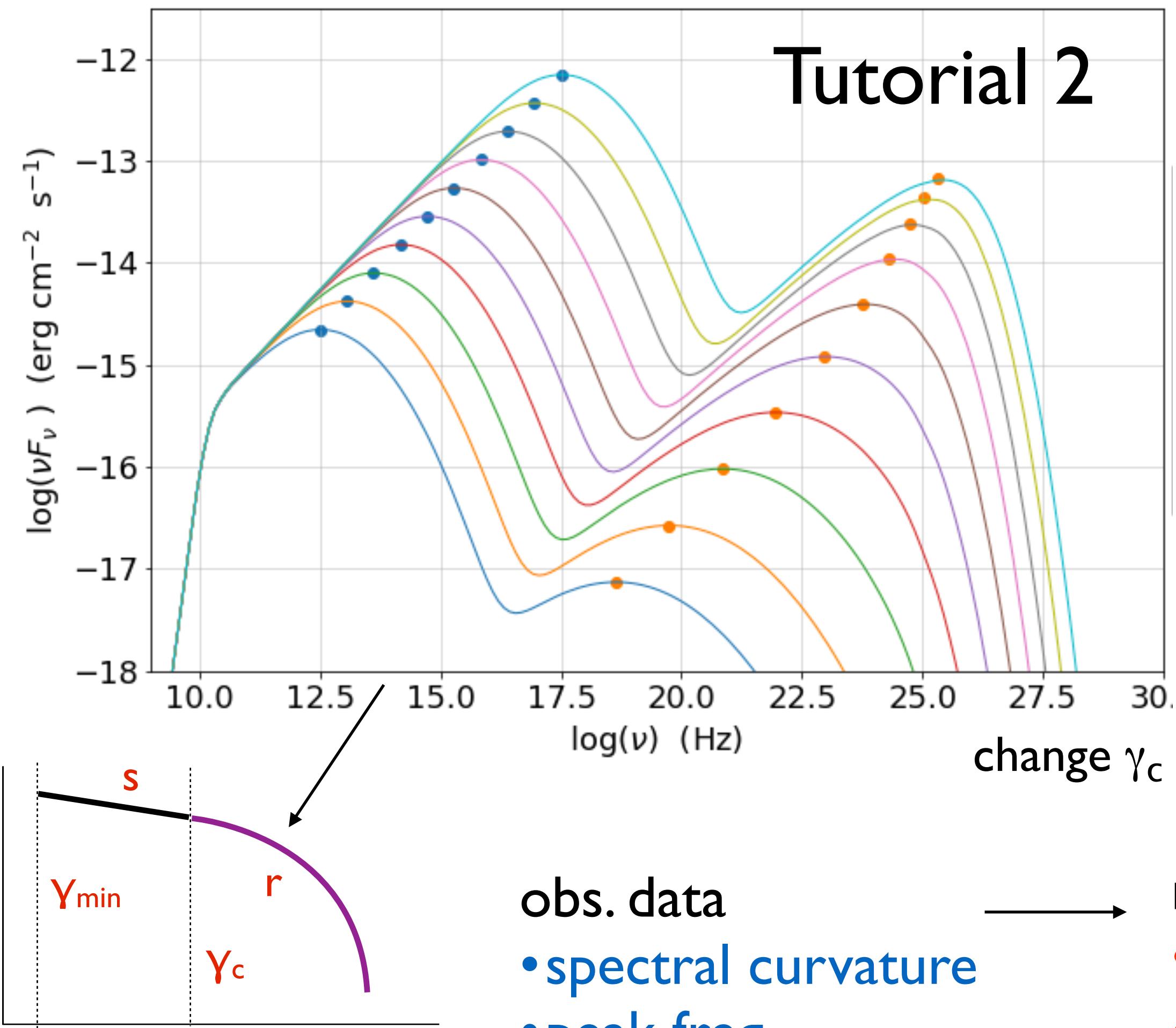
change r

Tutorial 2



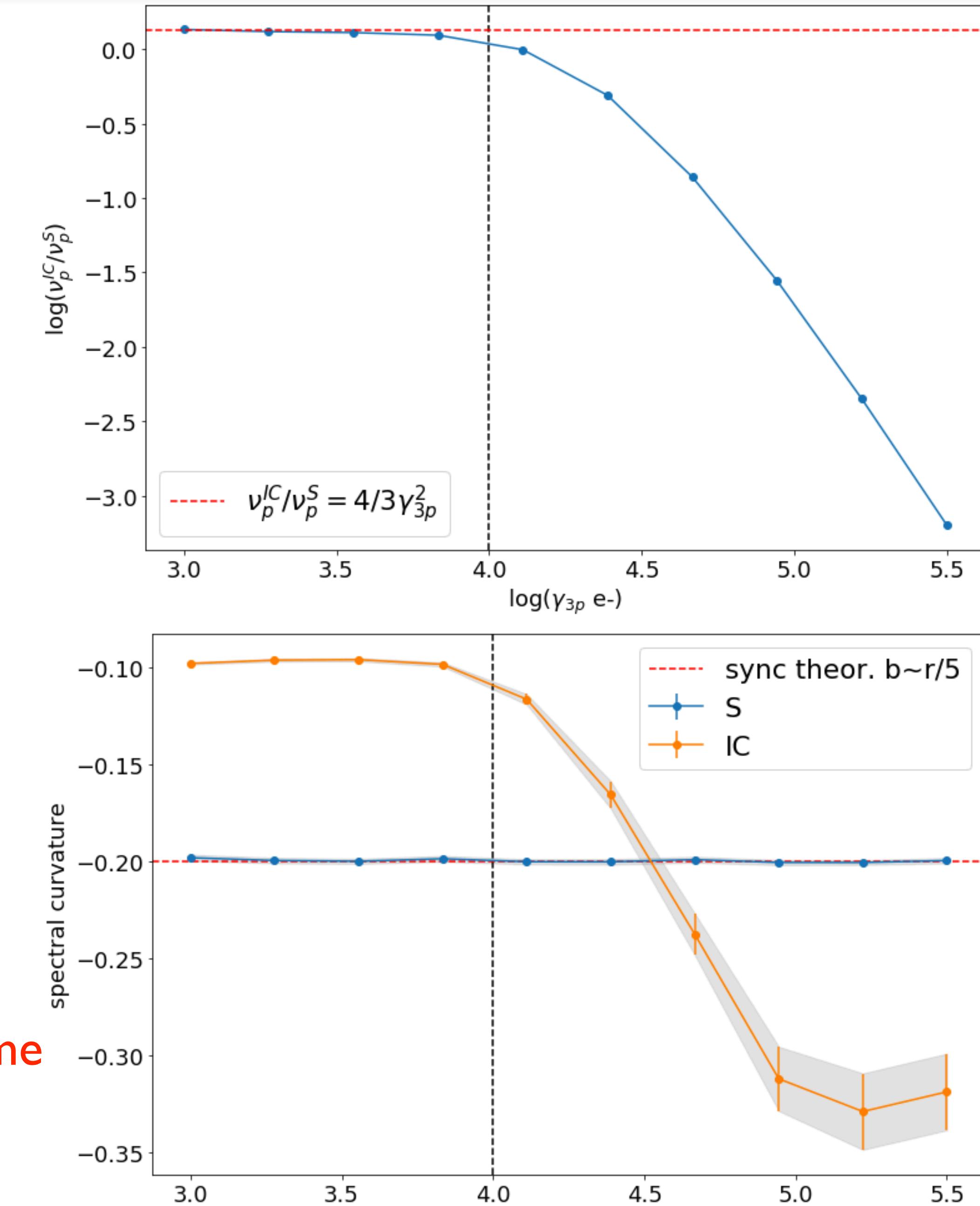
IC emission TH/KN regime and peak freq.

- $v_p^{IC} / v_p^S \sim (4/3) \gamma_p^2 \equiv \gamma_{3p}^2$ is true only in TH regime

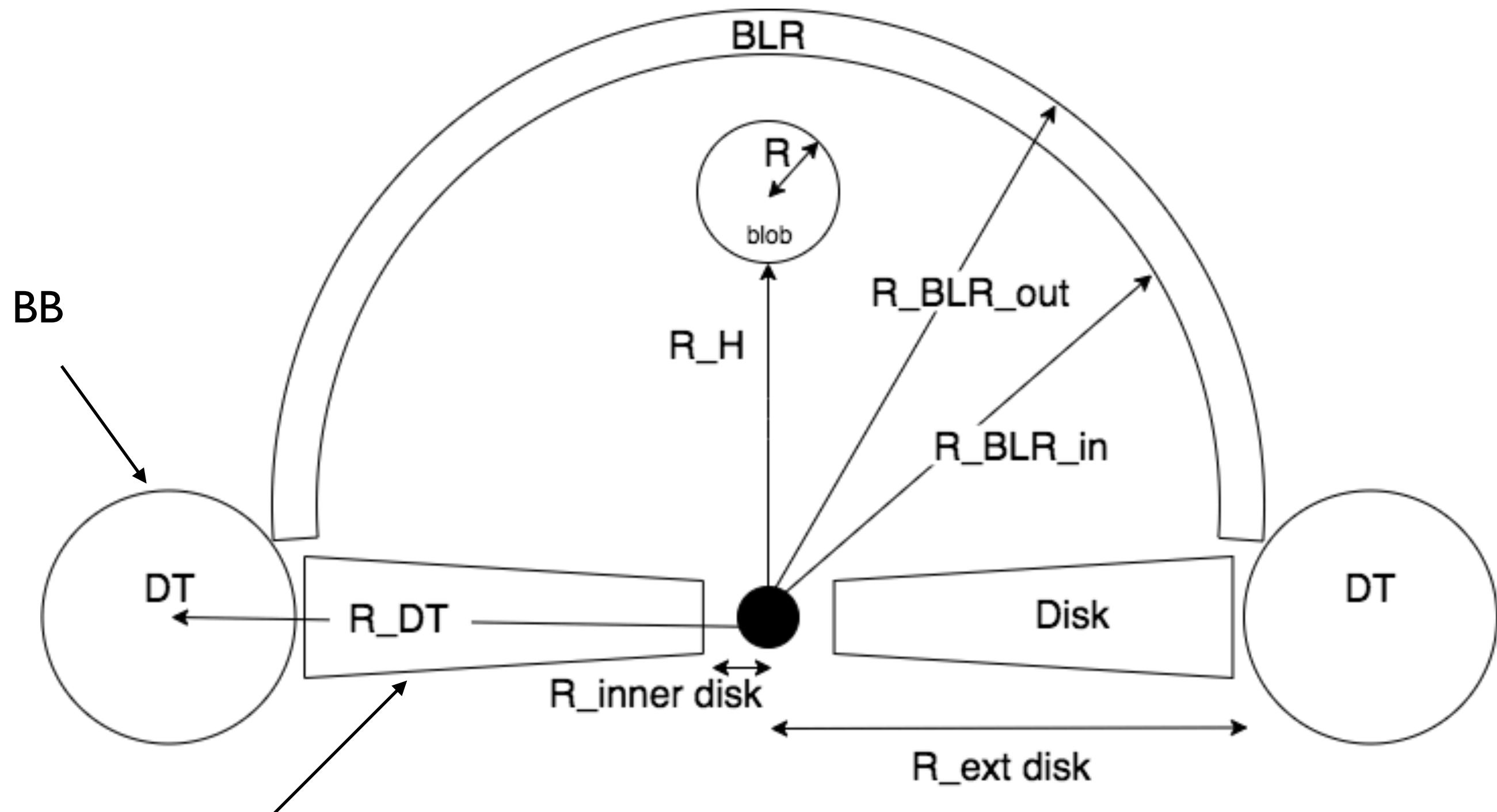


- obs. data
- spectral curvature
 - peak freq.

- model
- KN regime
 - γ_p, γ_c



Extra slides

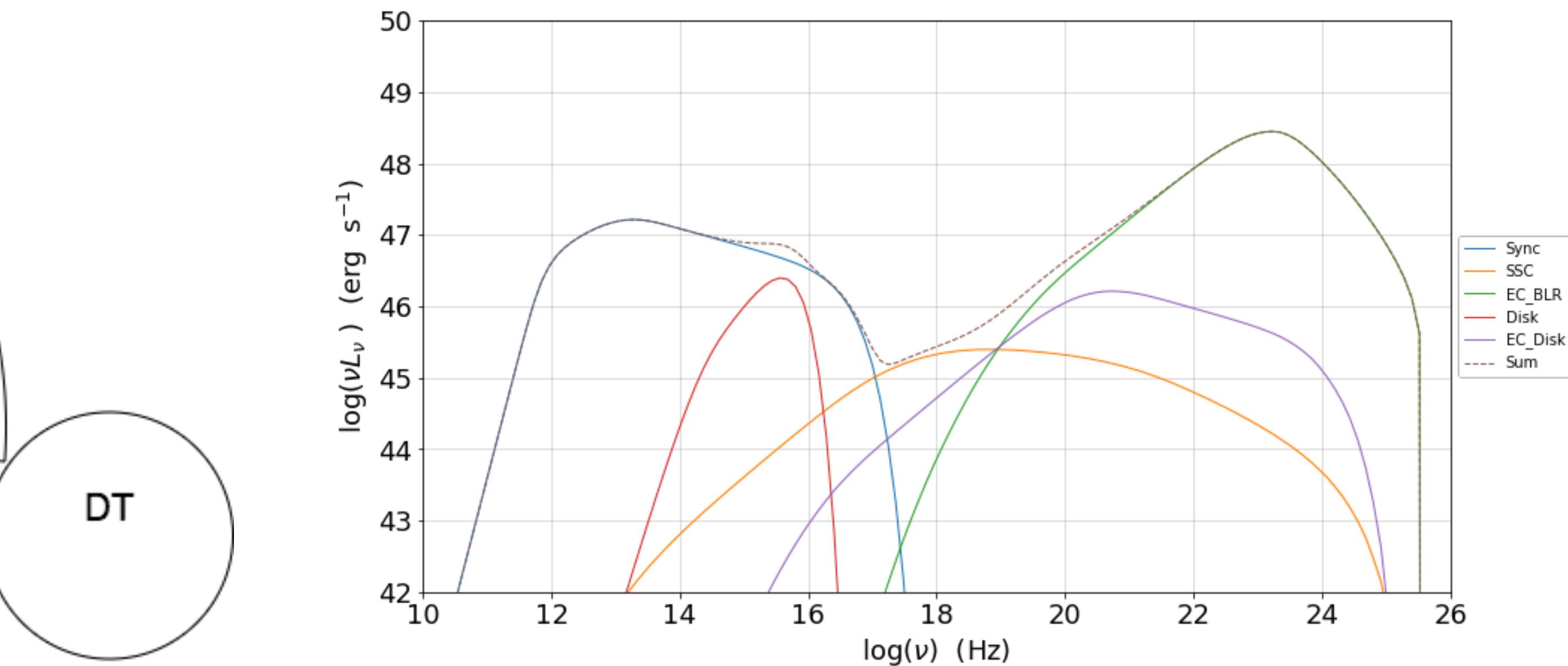


multi T BB $T(R) \approx T_*(R/R_*)^{-3/4}$

Transformation of the radiative fields

$$\epsilon^{-3} I_\epsilon \text{ and } \epsilon^{-2} j(\epsilon, \Omega)$$

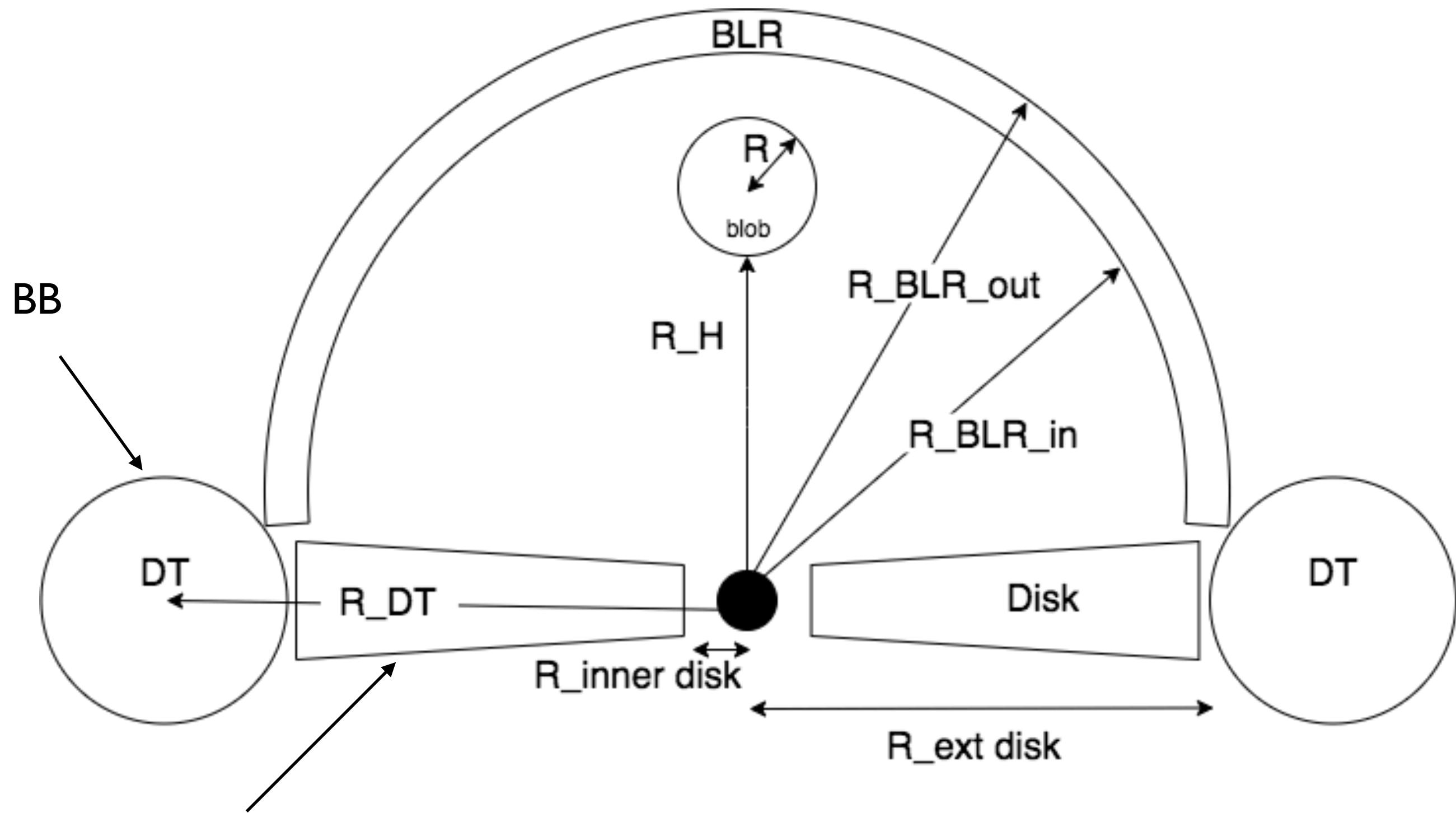
$$\frac{u(\epsilon, \Omega)}{\epsilon^3} = \frac{u'(\epsilon', \Omega')}{\epsilon'^3} = \text{inv.}$$



$$d\Omega' = \frac{d\Omega}{\Gamma^2(1 - \beta_\Gamma \cos \theta)^2} = \frac{2\pi d\mu}{\Gamma^2(1 - \beta_\Gamma \mu)^2} \quad u_\nu(\nu, \Omega) = \frac{I_\nu}{c} \text{ (erg cm}^{-3} \text{ Hz}^{-1} \text{ sterad}^{-1}\text{)}$$

$$\begin{aligned} I_{v'} &= \frac{1}{4\pi} \int d\Omega' \delta^3 I_{v=(v'/\Gamma)} \\ &= \Gamma \tau \frac{L_{\text{nuc}}}{4\pi R^2} f_{v=(v'/\Gamma)}(T_{\text{ext}}) \end{aligned} \quad \begin{aligned} u'_{ext} &\simeq \Gamma^2 u_{ext} \\ L_{ERC} &\simeq \Gamma^6 U_{ext} \end{aligned}$$

$$\eta = \frac{\dot{\gamma}_{IC}}{\dot{\gamma}_{sync}} = \frac{U_{ph}}{U_B}$$



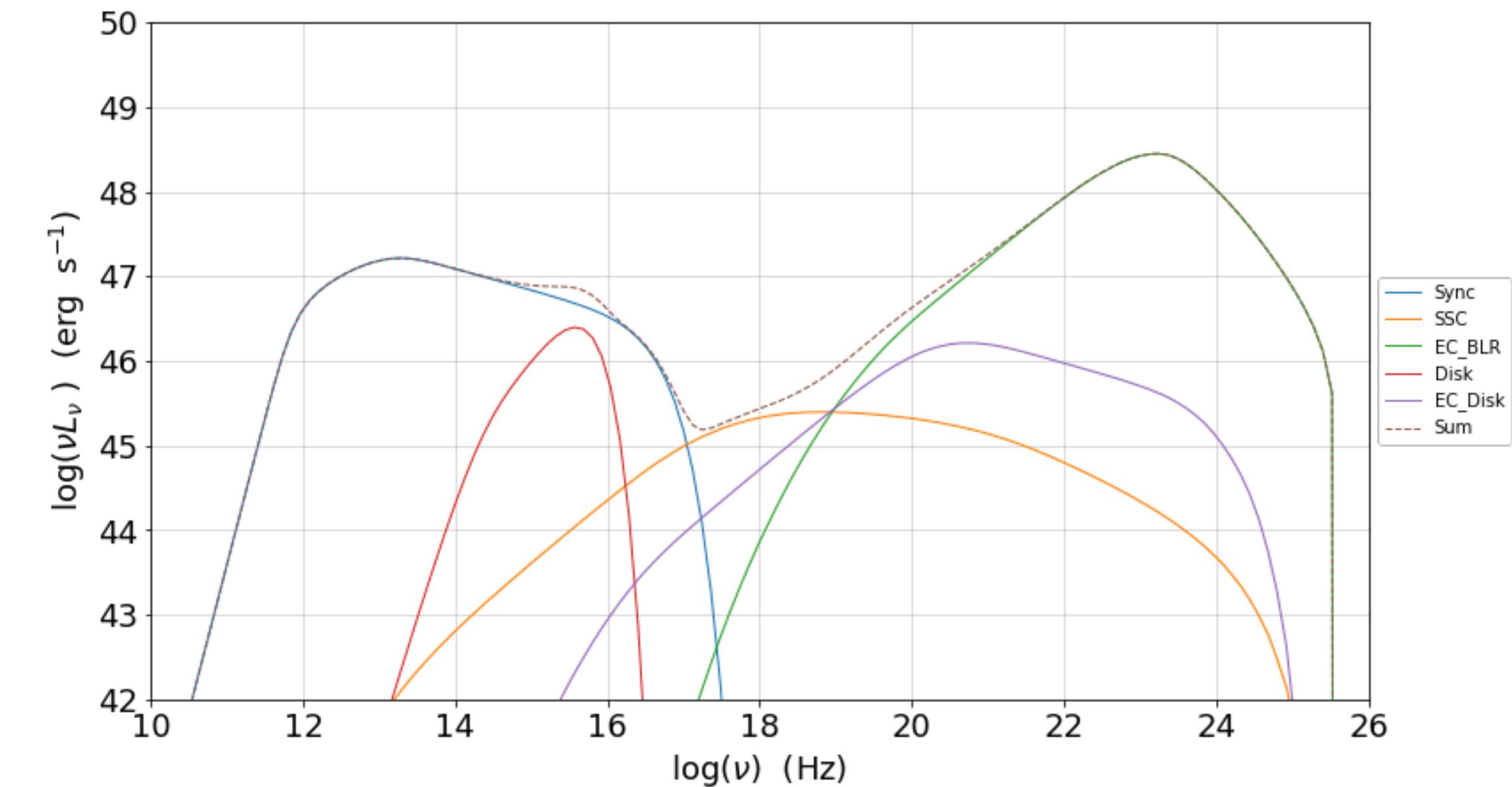
multi T BB $T(R) \approx T_*(R/R_*)^{-3/4}$

$$n'(\gamma', \Omega') = n'(\gamma')/4\pi$$

$$n/\gamma^2$$

Isotropic emitters distr.

Invariant.



Transformation of the emitters distribution

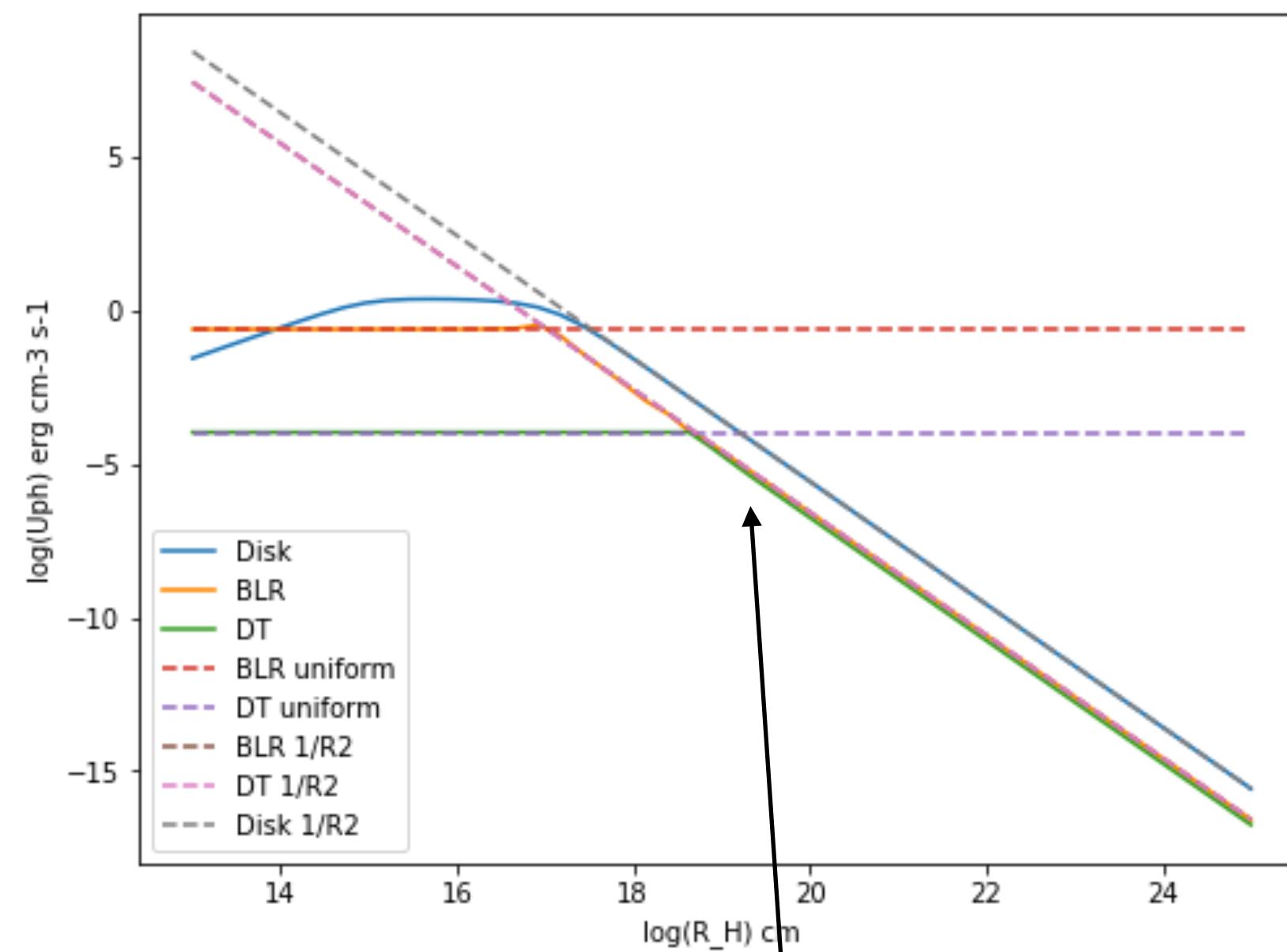
$$n(\gamma, \Omega) = \delta^2 n'(\gamma', \Omega) = \delta^2 n'(\gamma')/4\pi$$

$$\gamma = \delta\gamma'$$

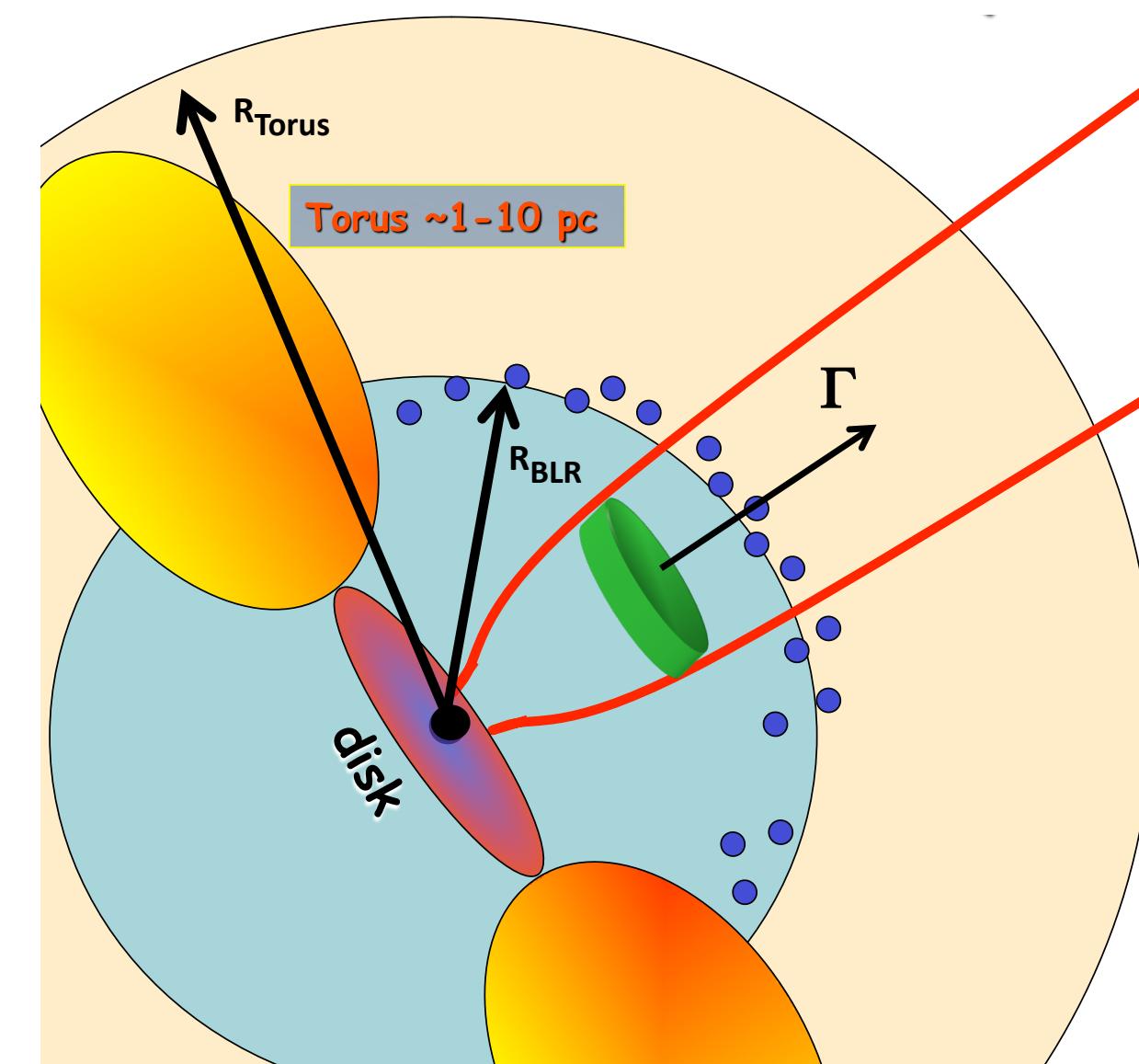
$$V = V'\delta$$

External Compton Scenario

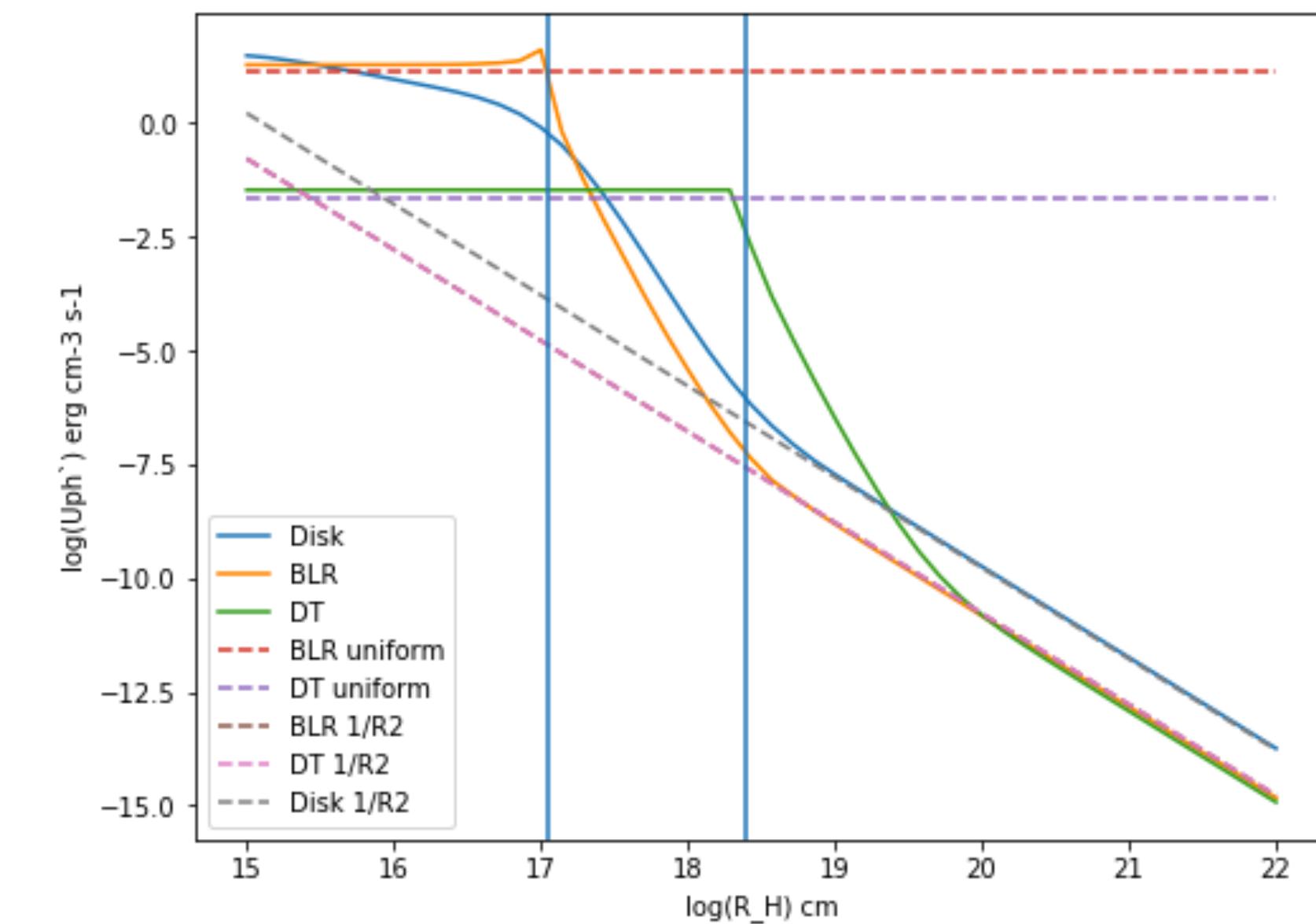
external photon fields in the disk rest frame



constant within
the sphere



external photon fields in the blob rest frame

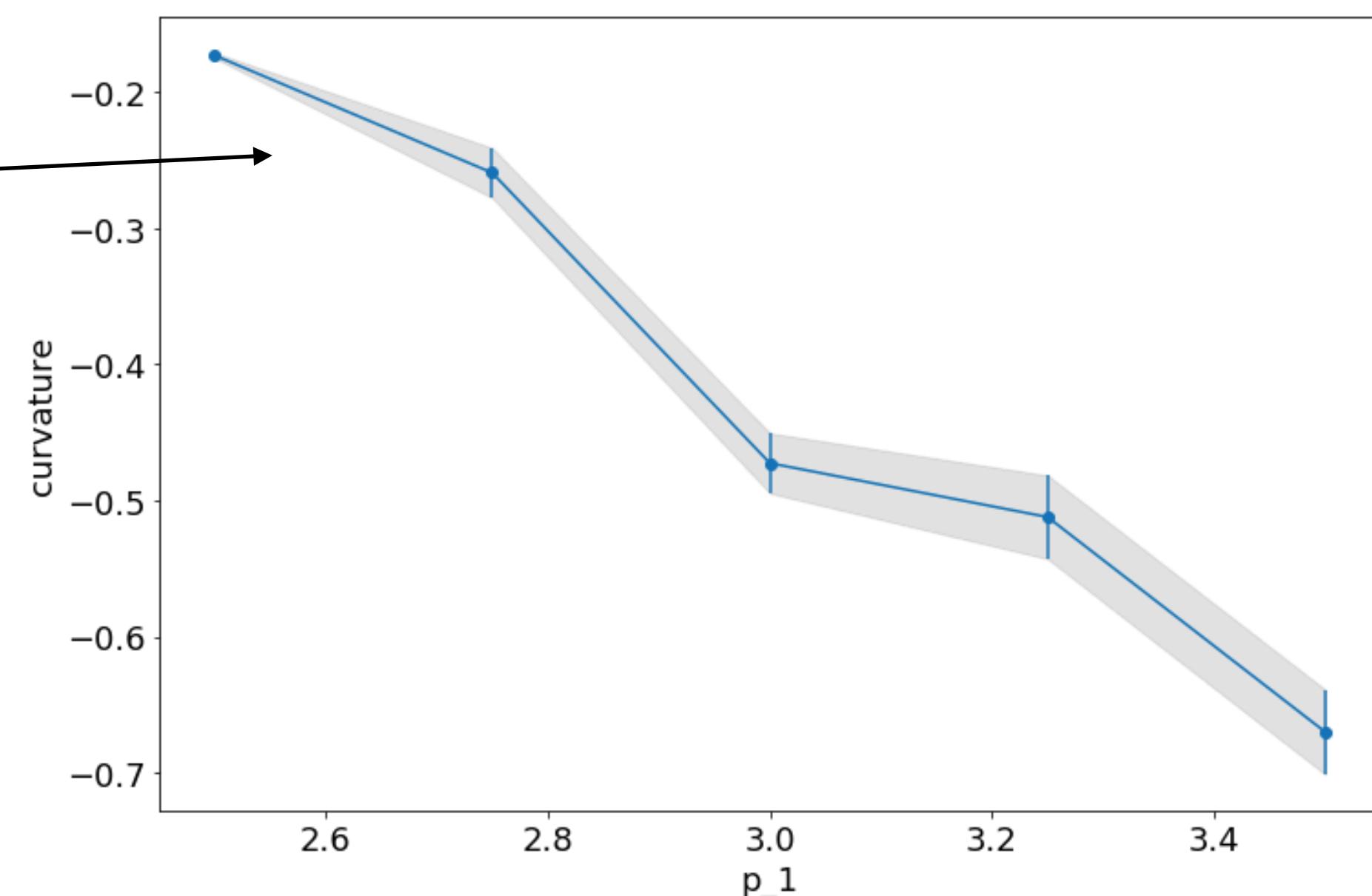
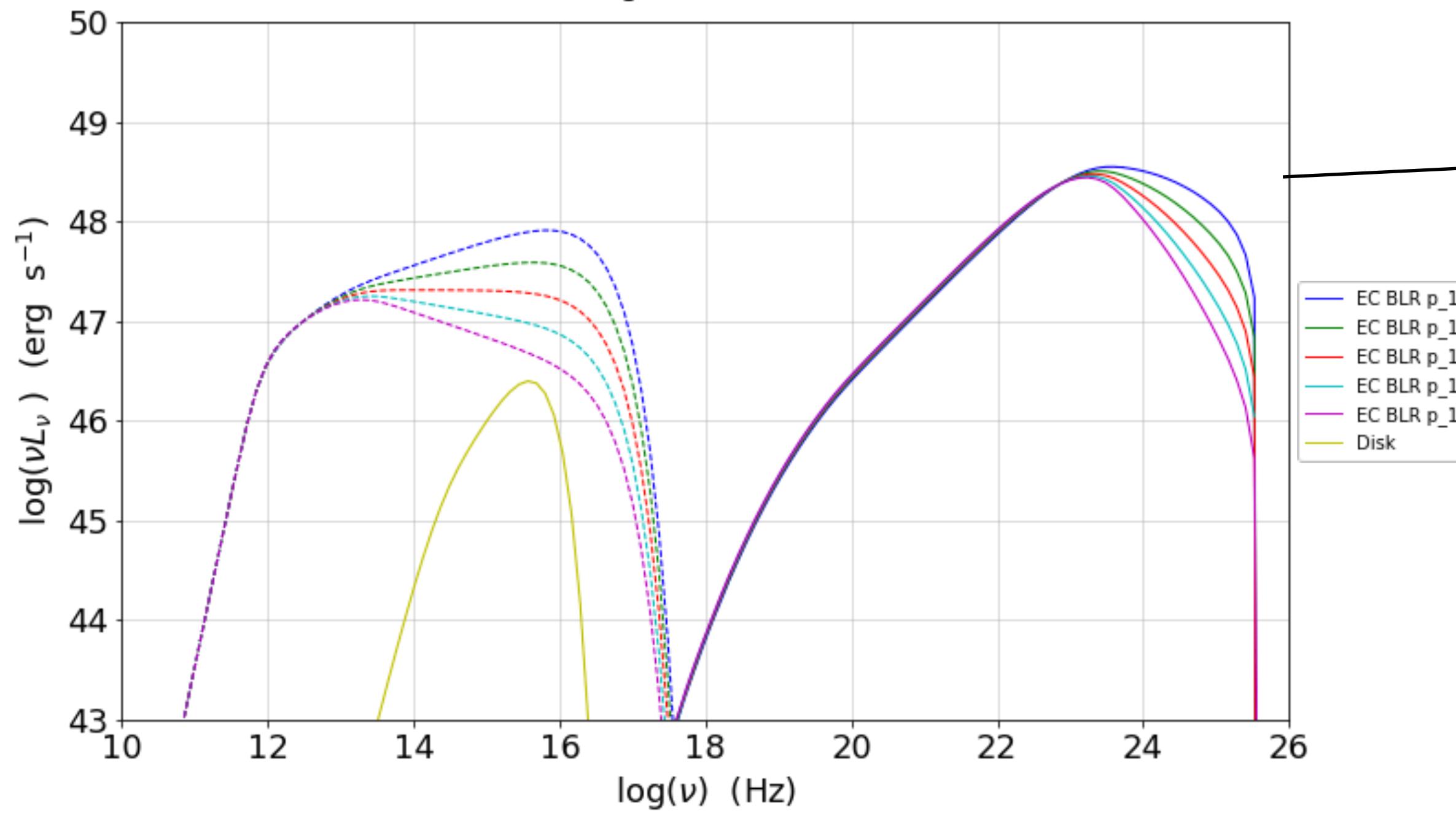
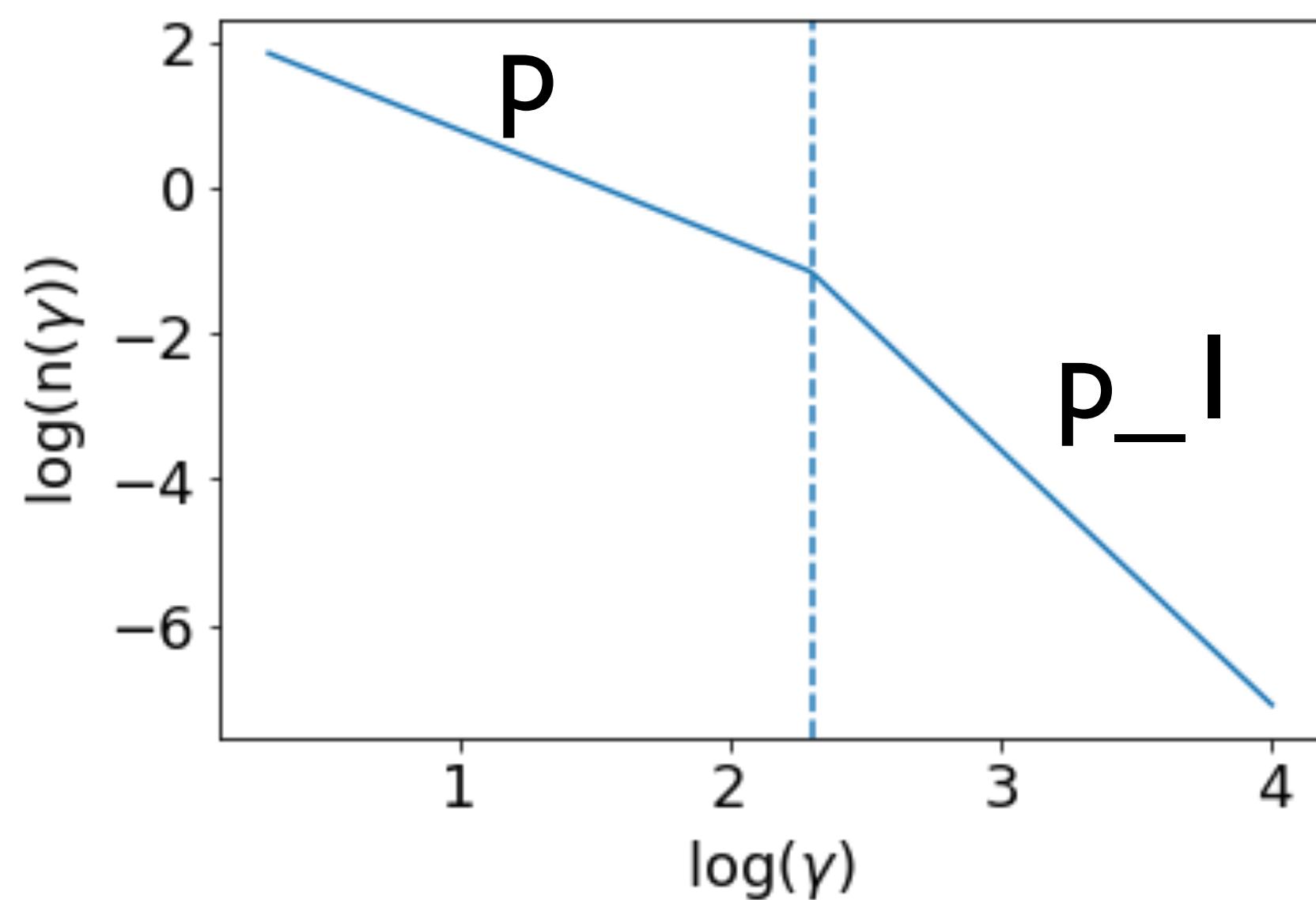
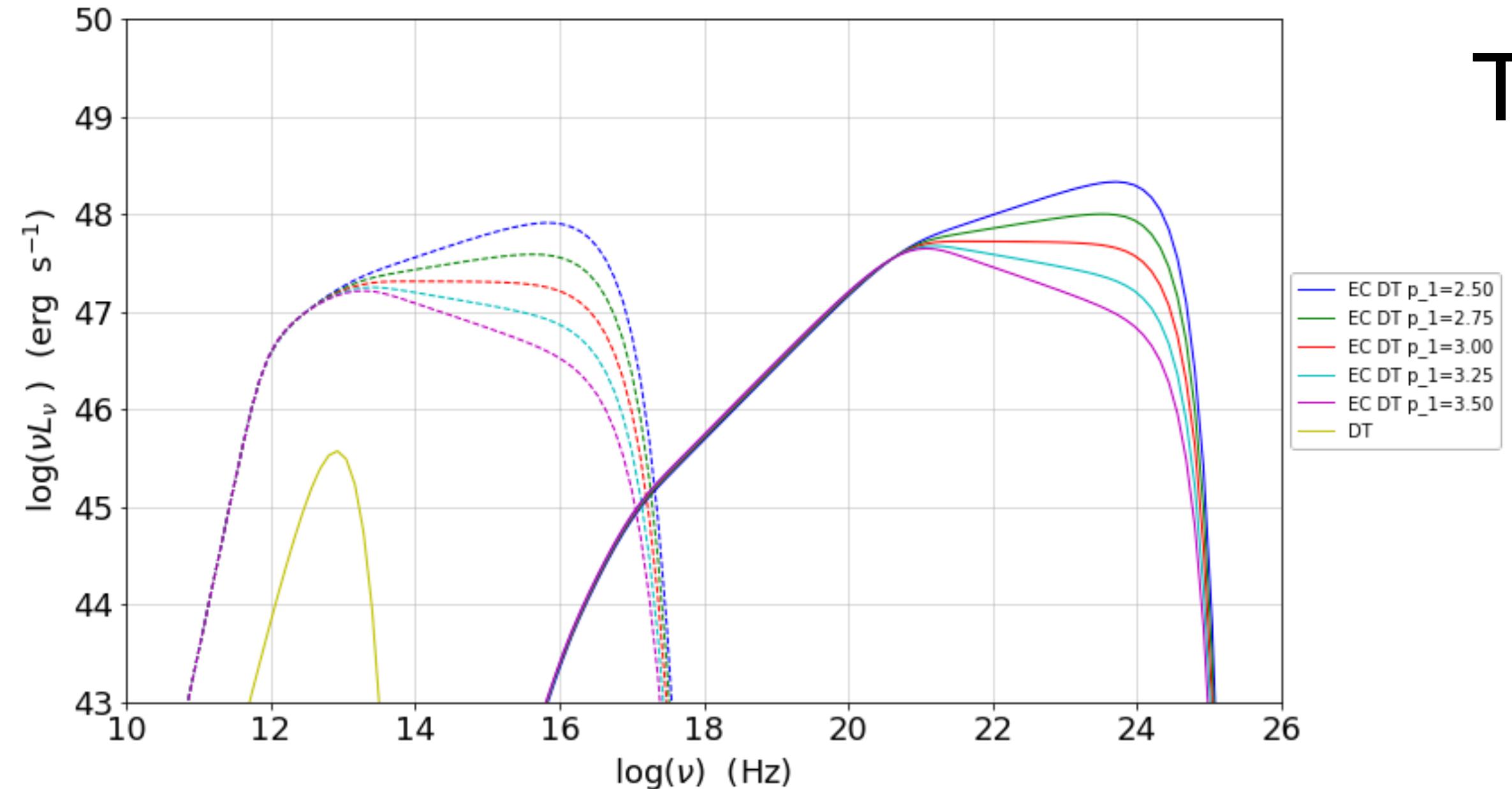


$$\left. \begin{aligned} U_{ext} &\simeq \frac{L_d}{R_{ext}^2 c} \\ R_{ext} &\simeq L_d^{1/2} \end{aligned} \right\} \rightarrow \begin{aligned} &\sim 0.1 \text{ erg/cm}^3 \text{ BLR} \\ &\sim 0.01 \text{ erg/cm}^3 \text{ DT} \end{aligned}$$

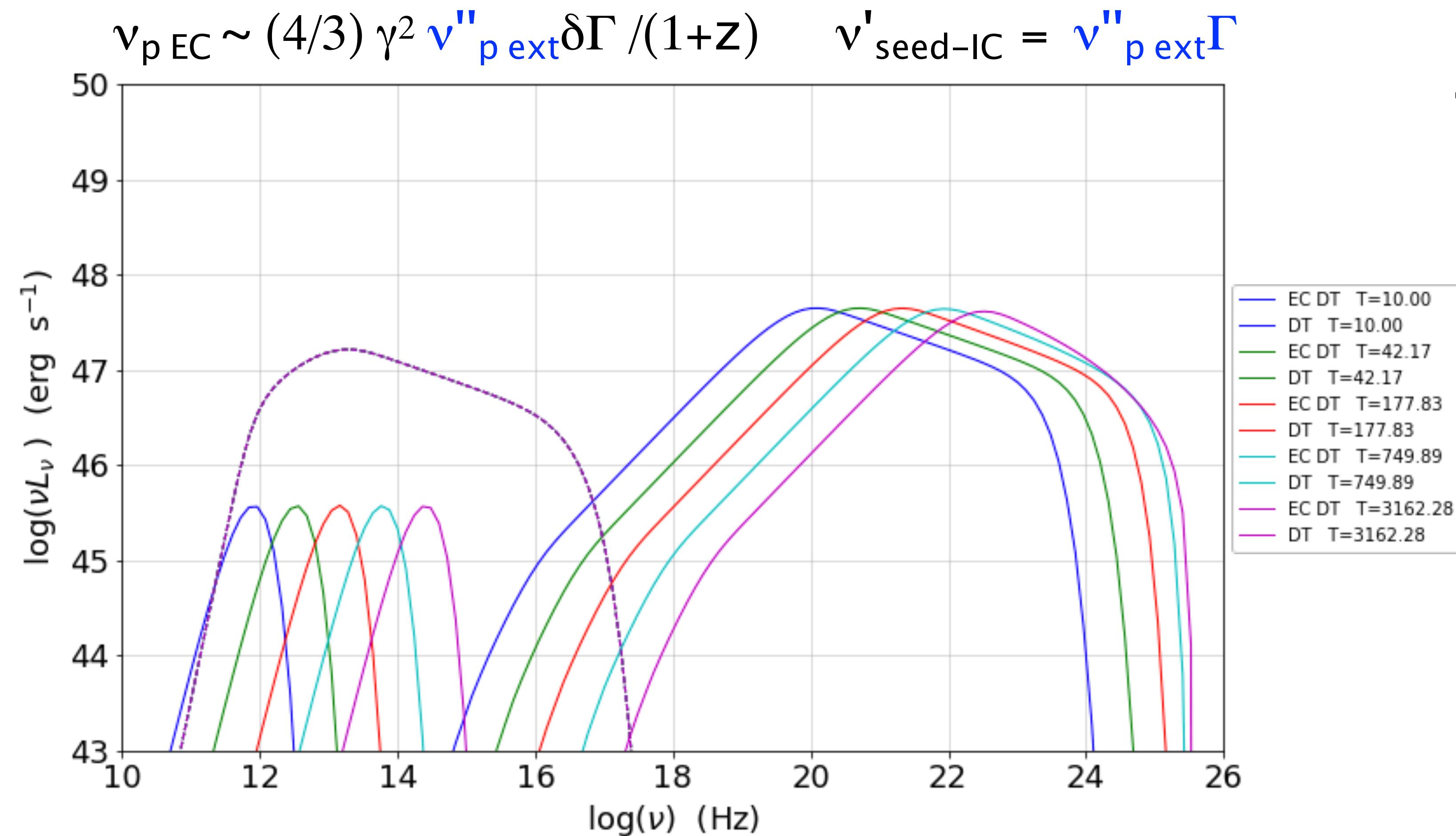
$$u'_{ext} \simeq \Gamma^2 u_{ext}$$

External Compton Scenario and TH/KN

Tutorial 3



External Compton Scenario and external seed photons energy



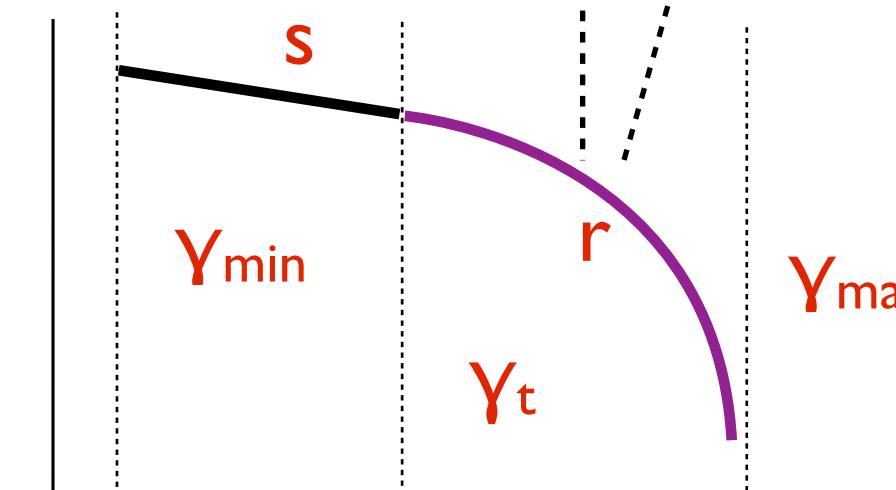
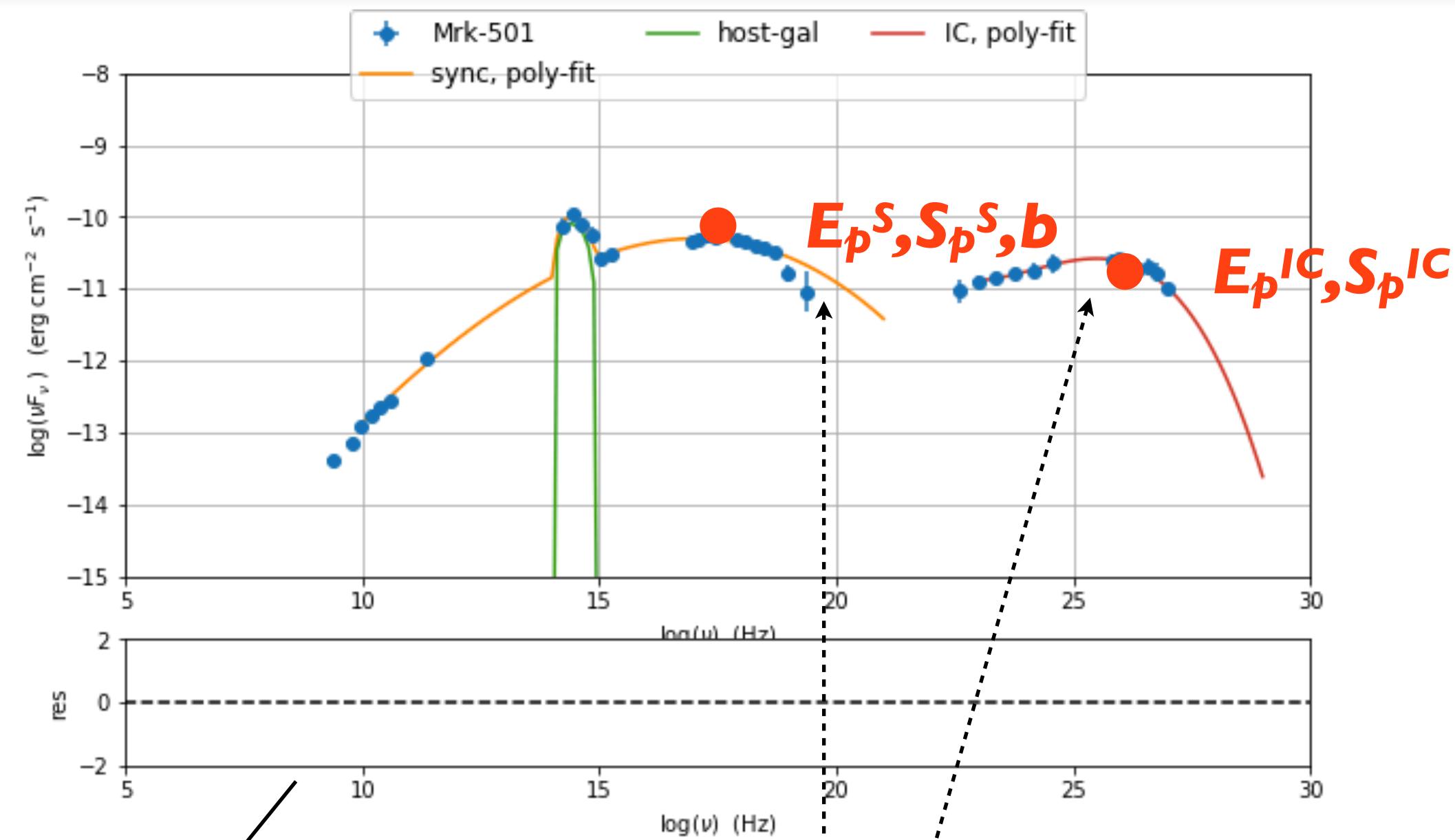
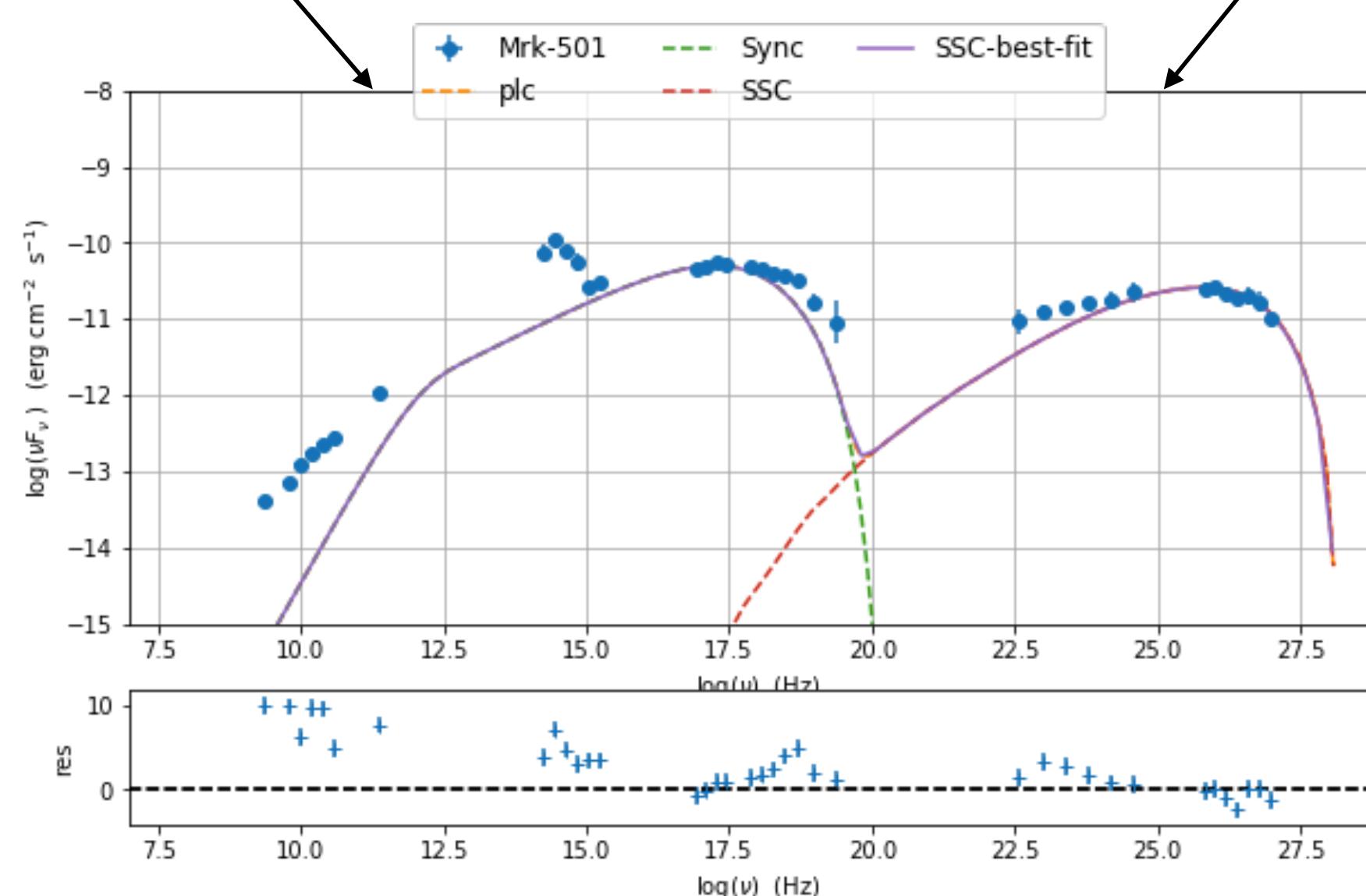
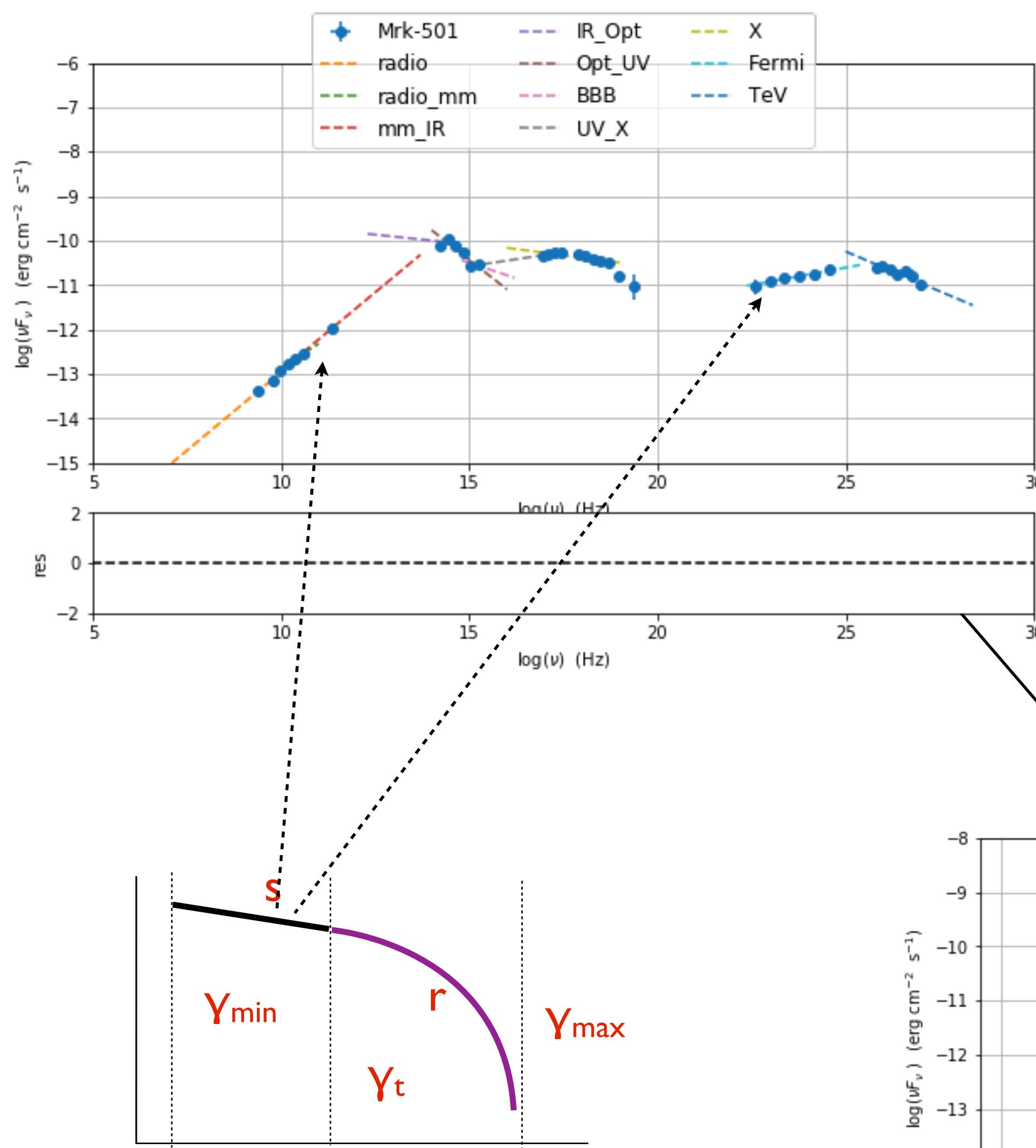
obs. data

→ model

•peak freq.

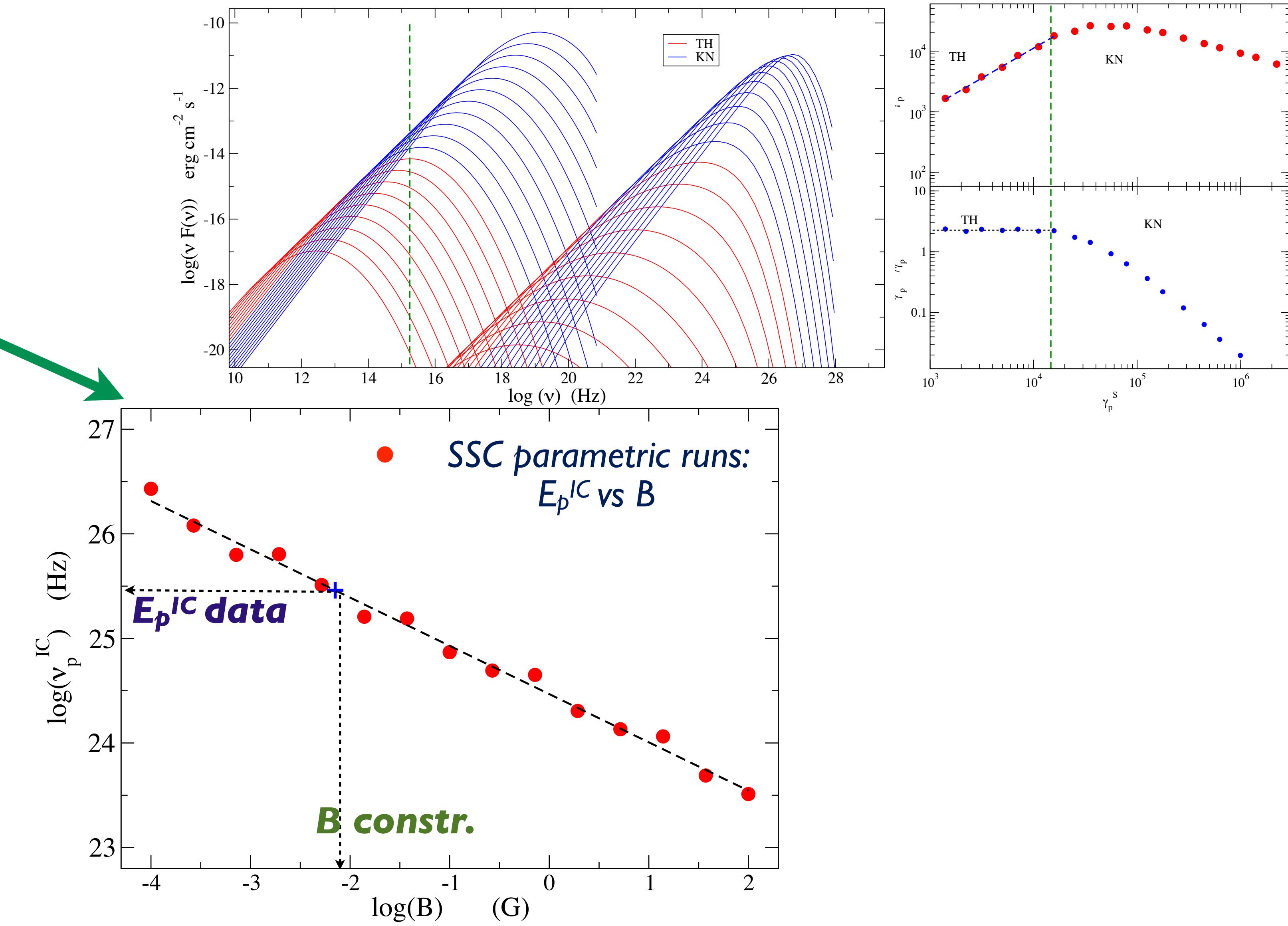
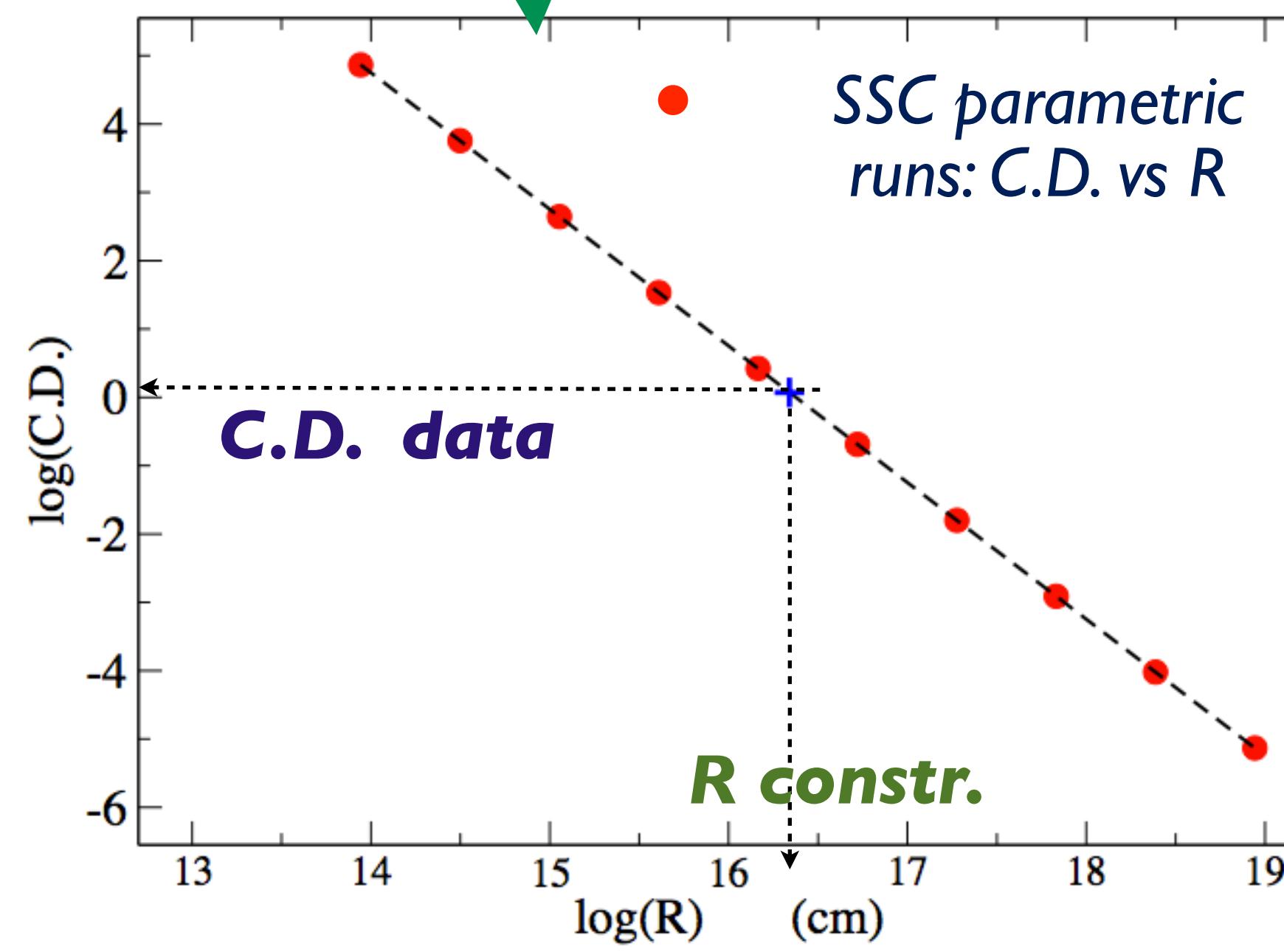
•T seed ext.

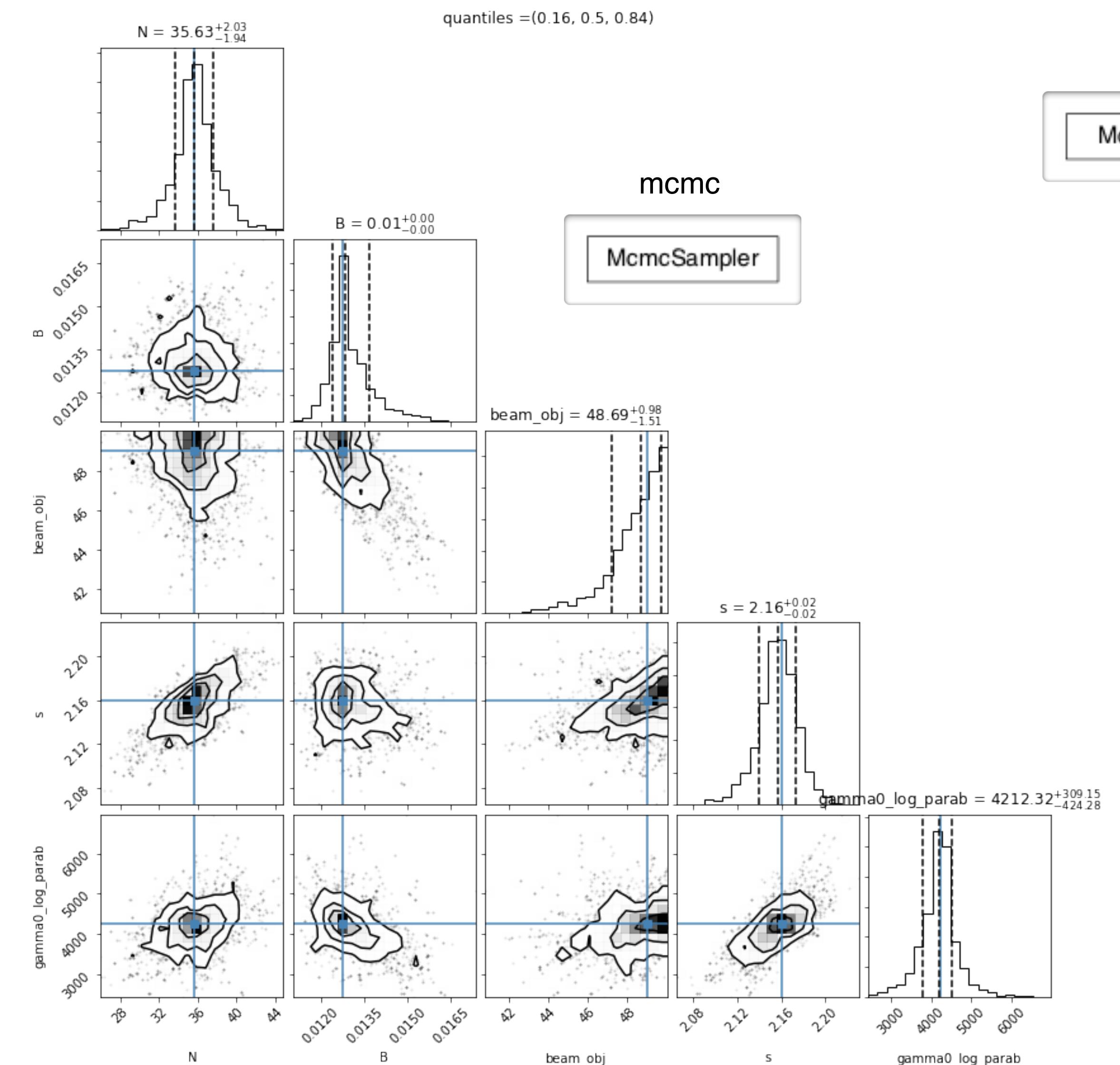
Tutorial 5



SED shaping and model constraining

- ✓ $\Gamma \Rightarrow s$
- ✓ $b \Rightarrow r$
- ✓ $E_p^s, r, s \Rightarrow \gamma_0$
- ✓ $t_{var}, \delta \Rightarrow R$ u.l.
- ✓ $N \Rightarrow$ best S_p^s match
- $B \Rightarrow$ best E_p^{IC}, S_p^{IC} match
- $R \Rightarrow CD$





$$p(\text{parameters given data}) \propto p(\text{data given parameters}) \times (\text{parameters})$$

Posterior = Likelihood \times Prior

