Numerical modelling of radiative processes with the JetSeT code: hands-on session for the high-energy astrophysics course

JetSeT

Jets SED modeler and fitting Tool

Andrea Tramacere

https://jetset.readthedocs.io/en/latest/
https://github.com/andreatramacere/jetset
https://www.facebook.com/jetsetastro/

Outline

- Theoretical background
- definition of complex radiative **models** SSC/EC IC against CMB/BLR/DT, plus analytical and template models
- handling observed **data** (grouping, definition of data sets, etc…)
- **constraining** of the model in the pre-fitting stage, based on accurate and already published **phenomenological trends**
- **fitting of multiwavelength SEDs** using both **frequentist** approach (iminuit/scipy) and Bayesian **MCMC** sampling (emcee)
- Textbooks
  - Radiative Processes in Astrophysics, Ribicky & Lightman, John Wiley & Sons, 1991
  - High Energy Radiation from Black Holes: Gamma Rays, Cosmic Rays, and Neutrinos, Dermer & Menon, Princeton University Press 2009
  - Bayesian Reasoning in Data Analysis: A Critical Introduction, D'Agostini G., World Scientific, 2003

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Online tutorials

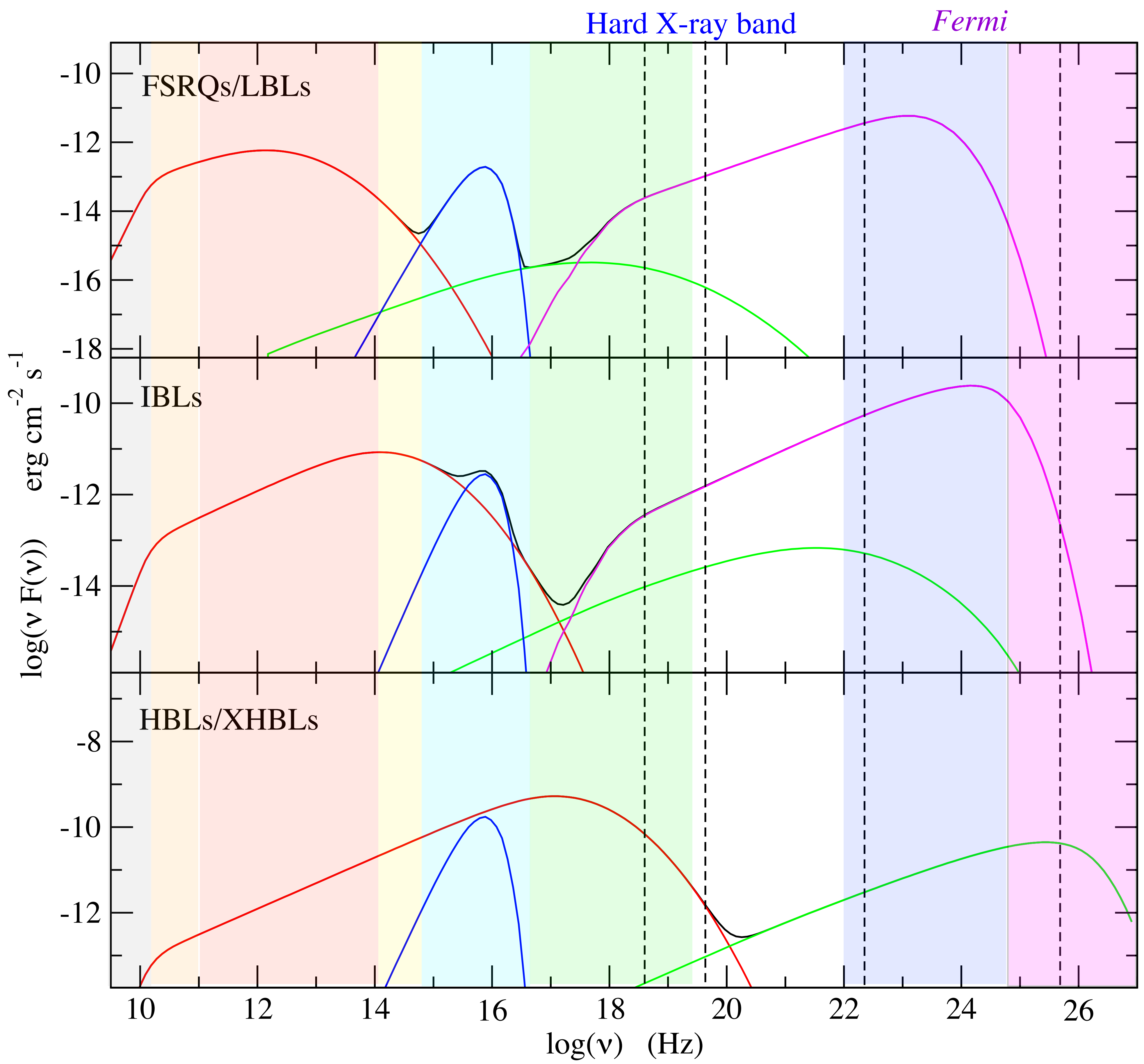
https://github.com/andreatramacere/Geneva_HighEnergy_Course

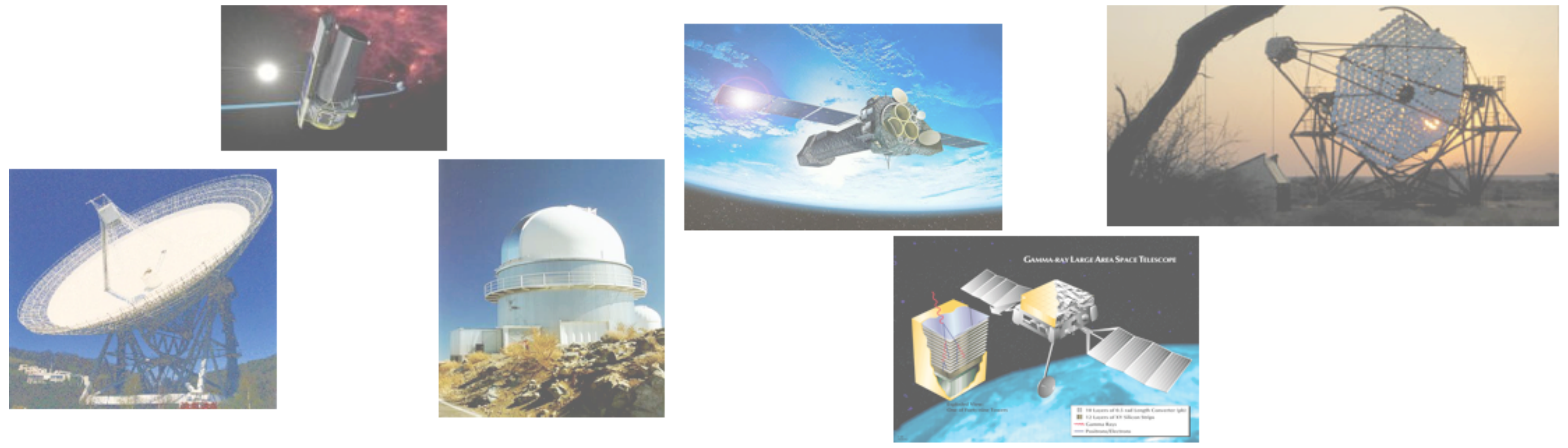
- Tutorial 1: basic operation with jet models
- Tutorial 2: phenomenological trends for synchrotron and SSC emission
- Tutorial 3: phenomenological trends for EC emission
- Tutorial 4: composite models and application to EBL (not covered in these slides)
- Tutorial 5: constraining of the model in the pre-fitting stage, based on accurate and already published phenomenological trends and fitting of multiwavelength SEDs using both frequentist approach (iminuit/scipy) and Bayesian MCMC sampling (emcee)

Today we will focus only on the first two tutorials

Blazars in a nutshell

data



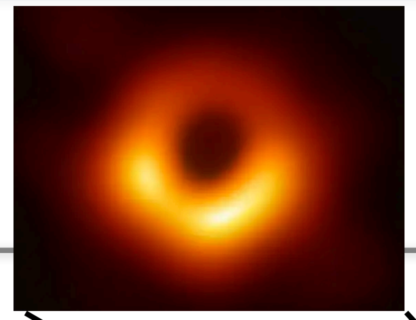


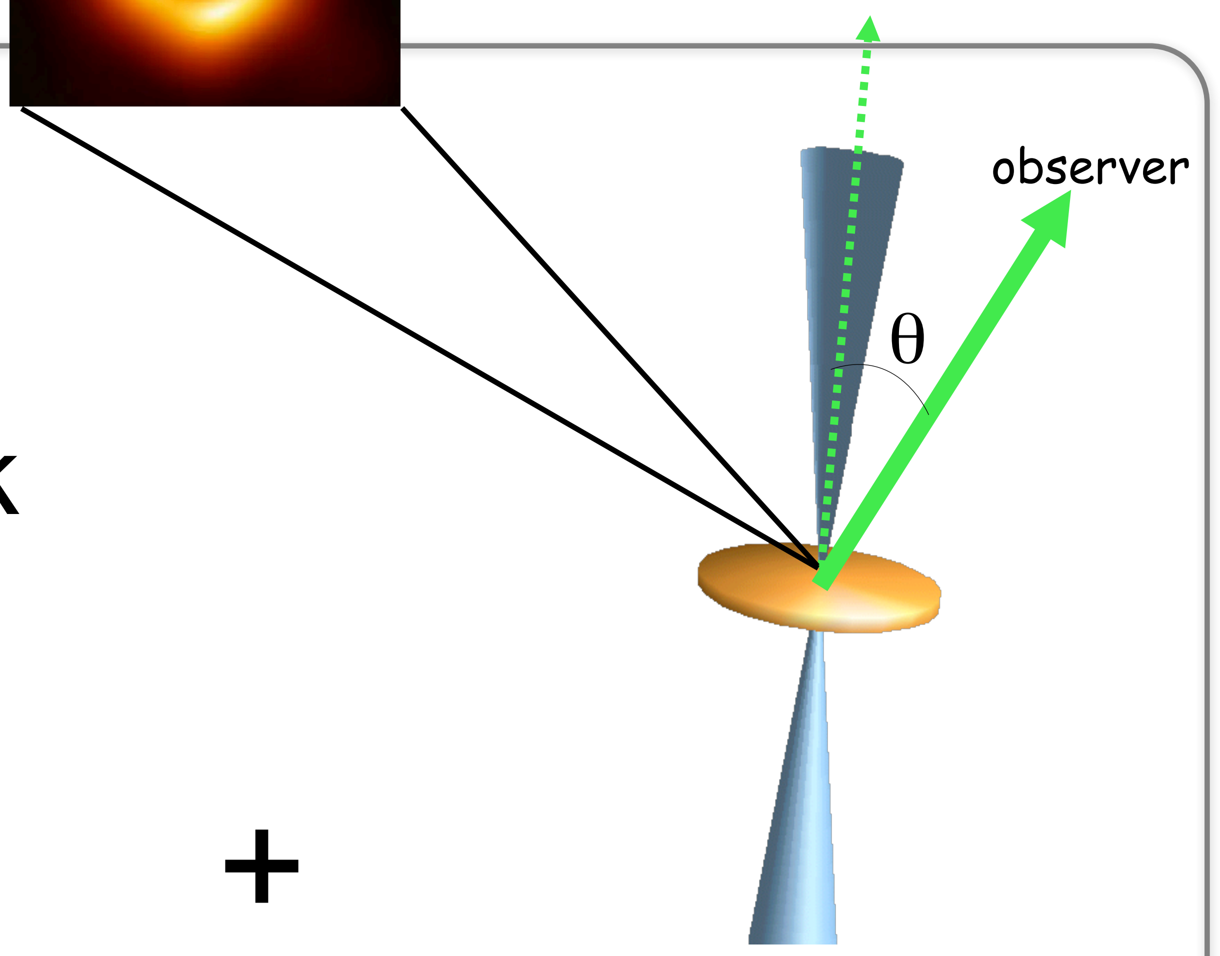
phenomenology

model fit

simulations

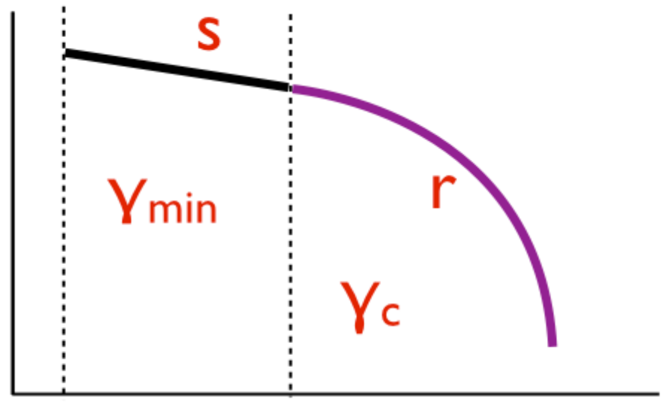
model





jet/disk

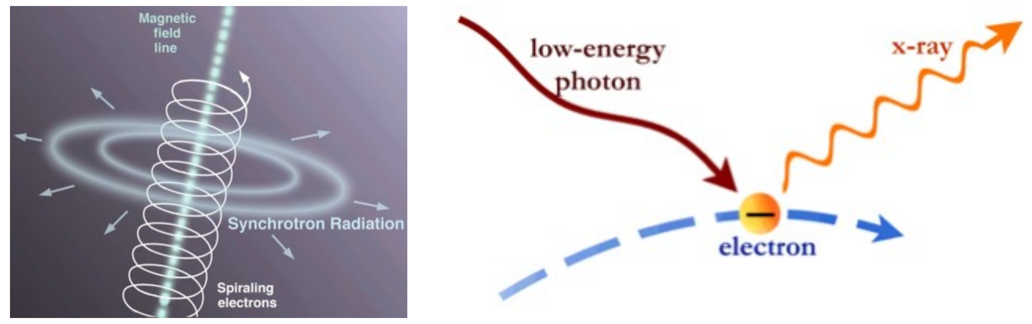
+



acc.

+

em.



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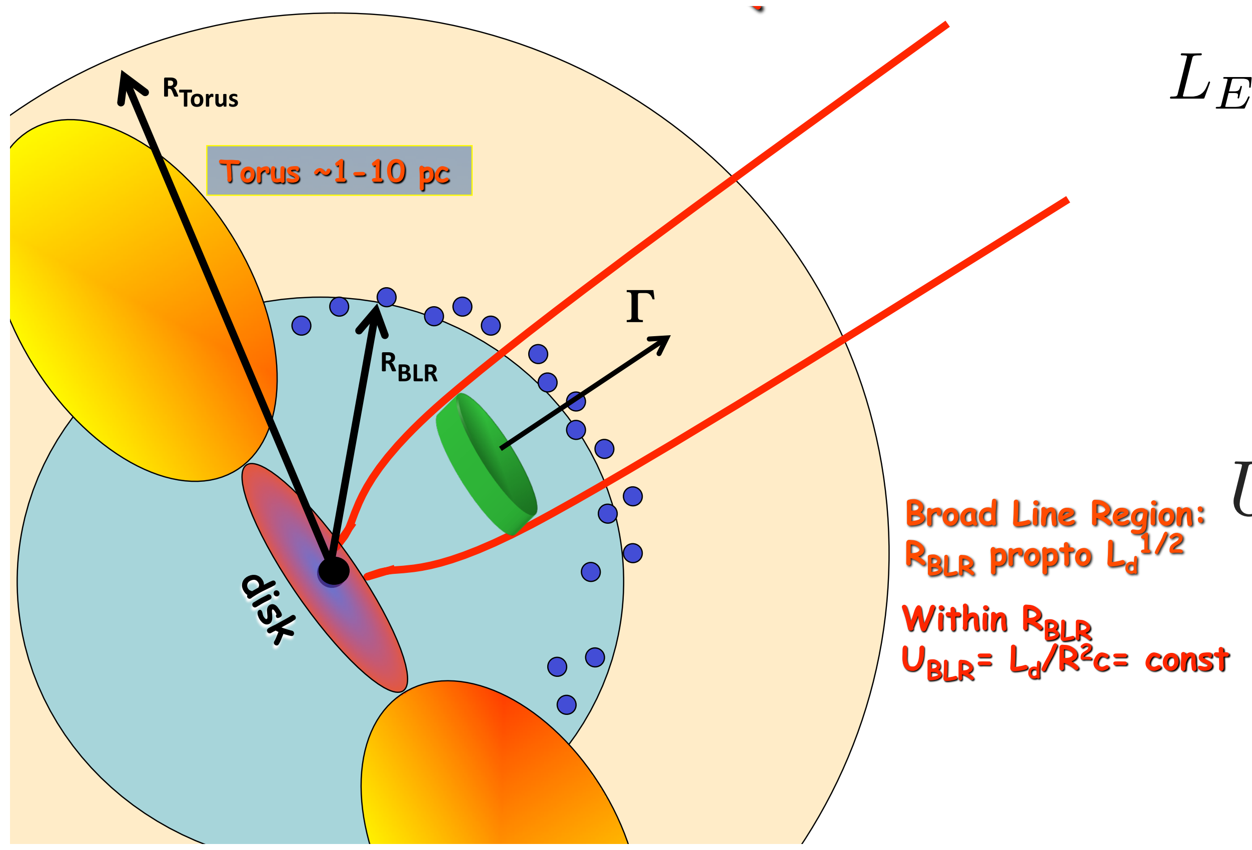
standard picture: acceleration/cooling balance

(Ghisellini,Fossati,Celotti 99-16)

FSRQs
HERGs

rad eff.
$L_d > 10^{-2} L_{EDD}$

$$L_{Edd} \approx 1.4 \times 10^{44} \frac{M_{BH}}{10^6 M_\odot} \mathrm{erg\ s^{-1}}$$

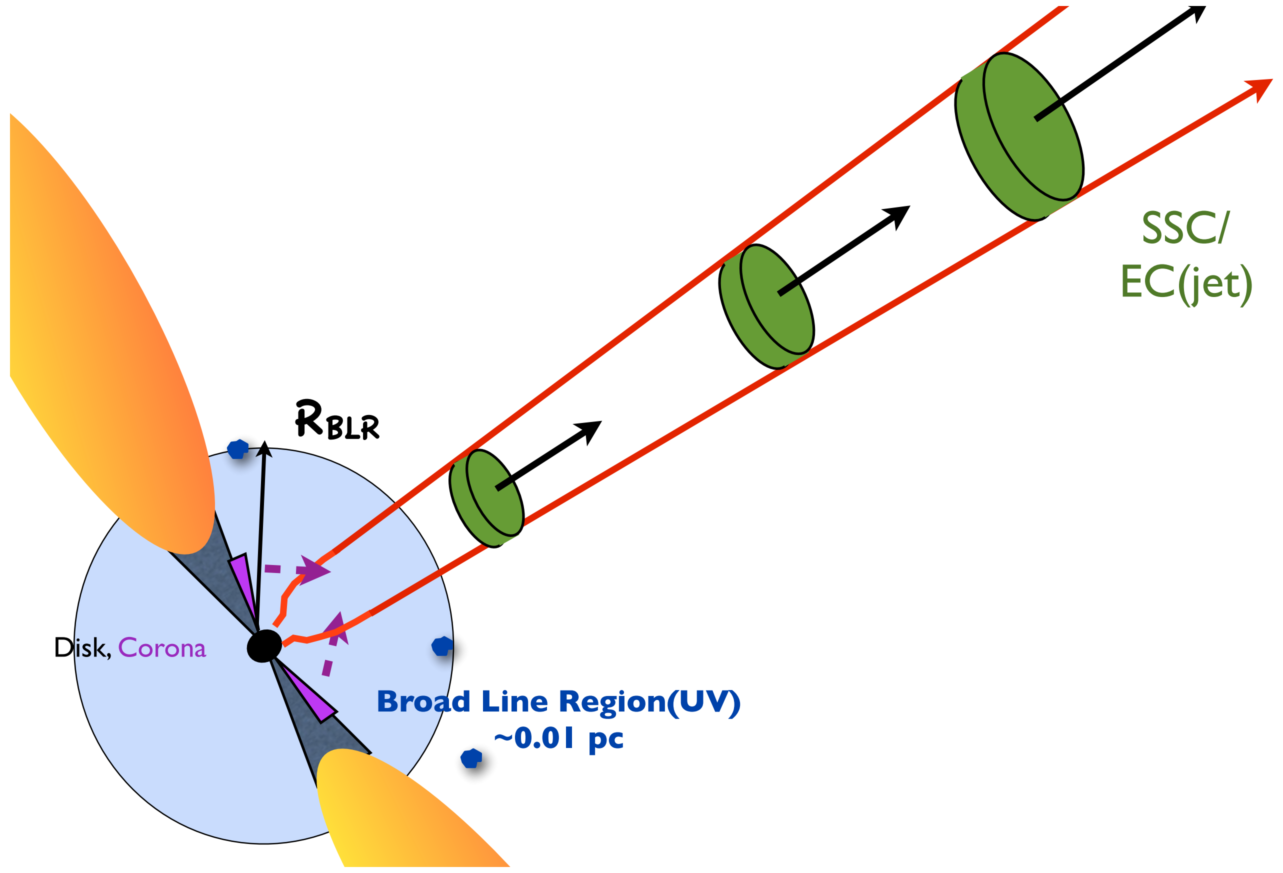


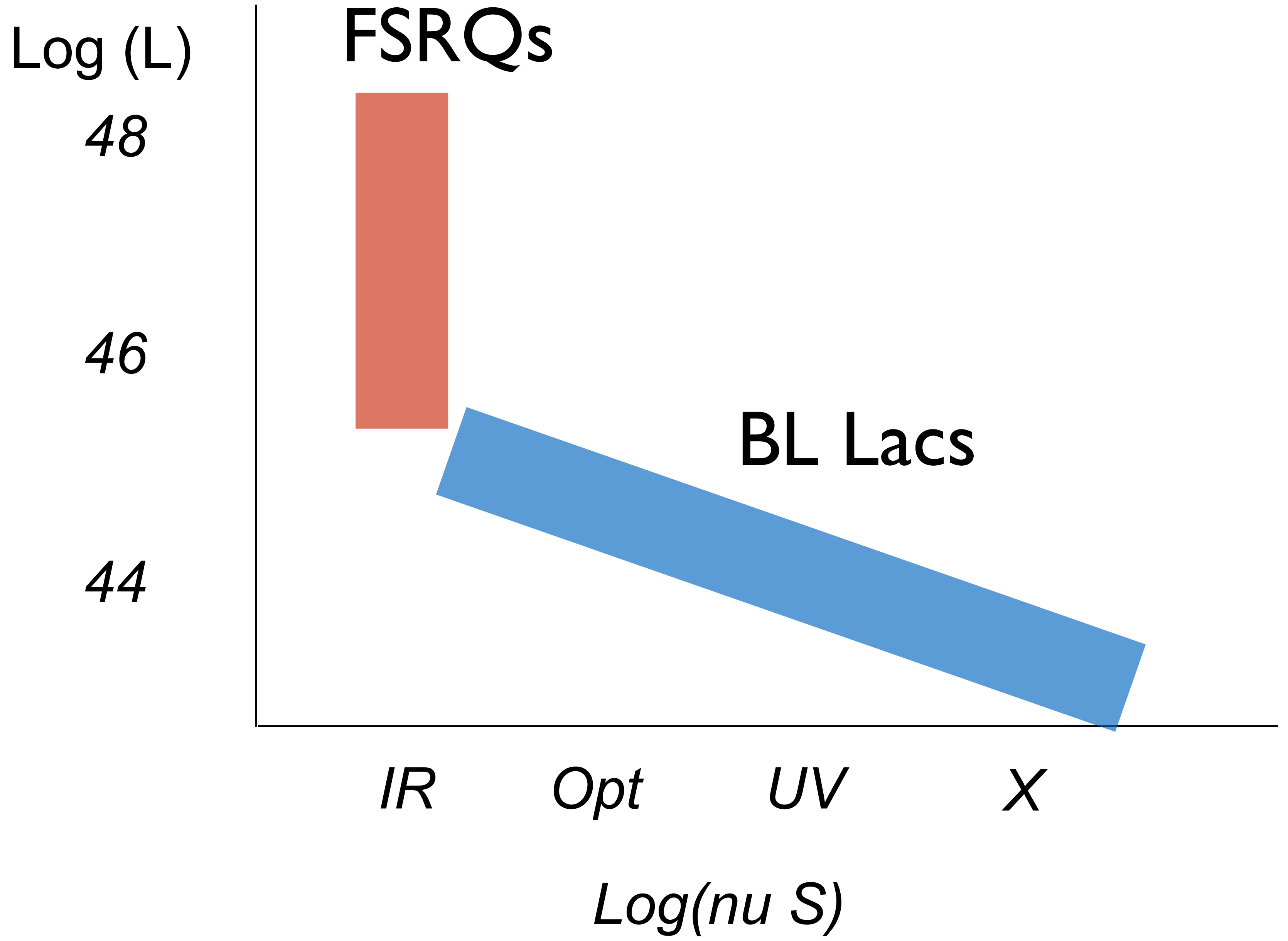
$$\left.\begin{array}{l} U_{ext} \simeq \dfrac{L_d}{R^2{}_{ext} c} \\ R_{ext} \simeq L_d^{1/2} \end{array}\right\} \rightarrow$$

~0.1 erg/cm^3 BLR
~0.01 erg/cm^3 DT

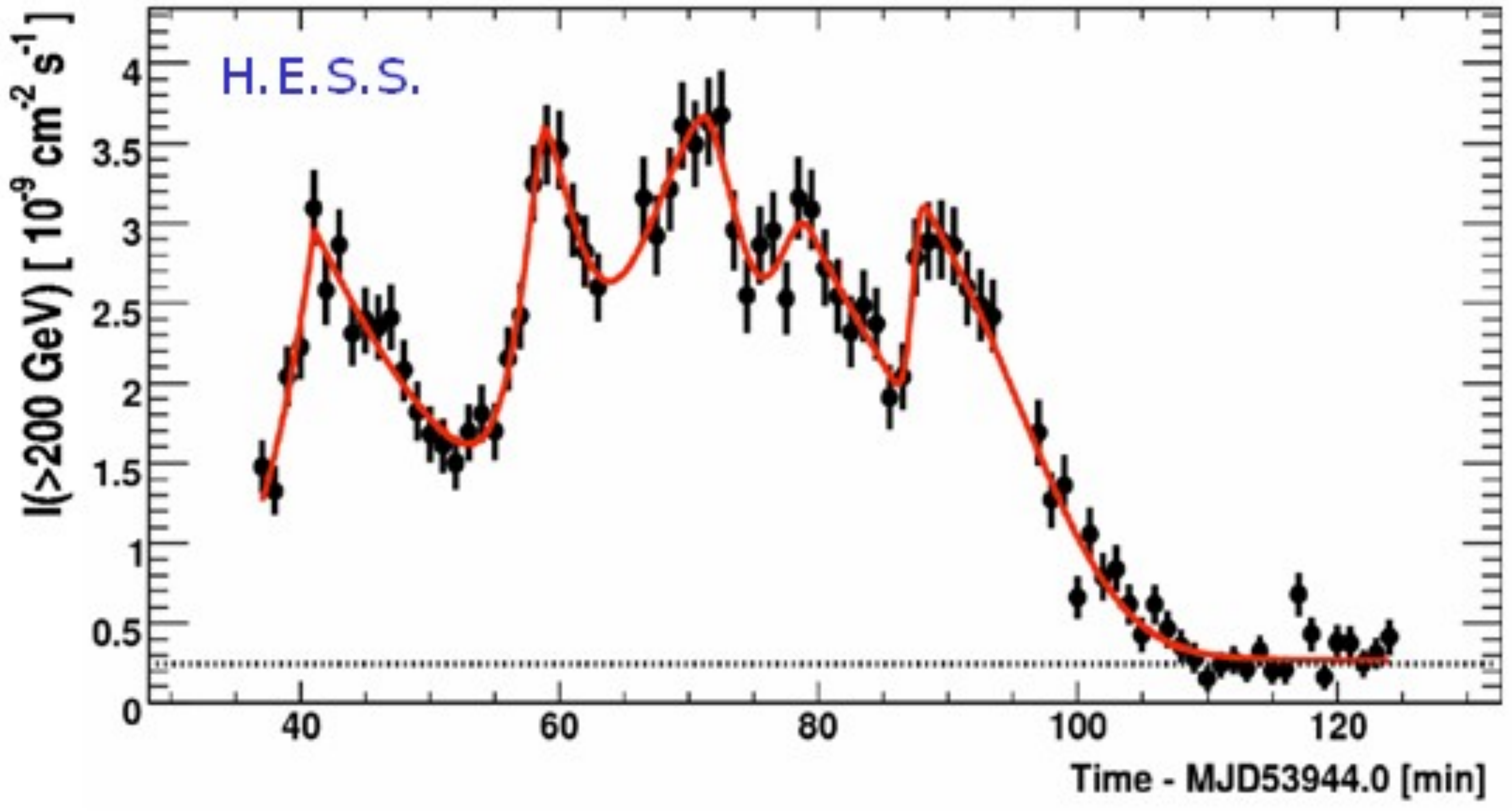
BL Lacs
LERGs

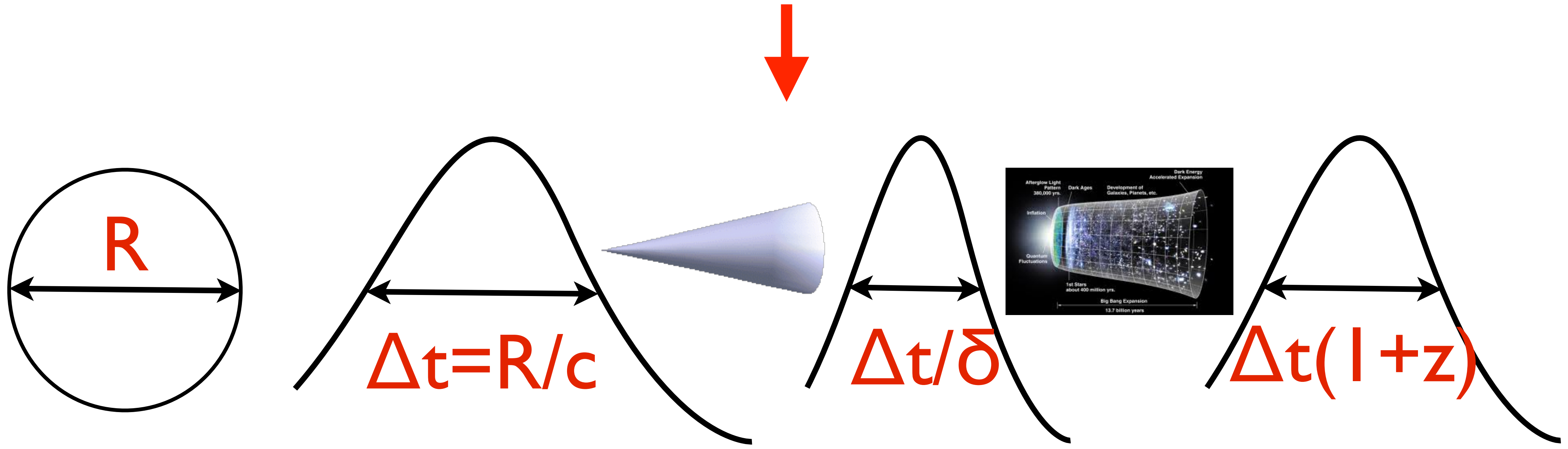
rad ineff.
$L_d < 10^{-2} L_{EDD}$



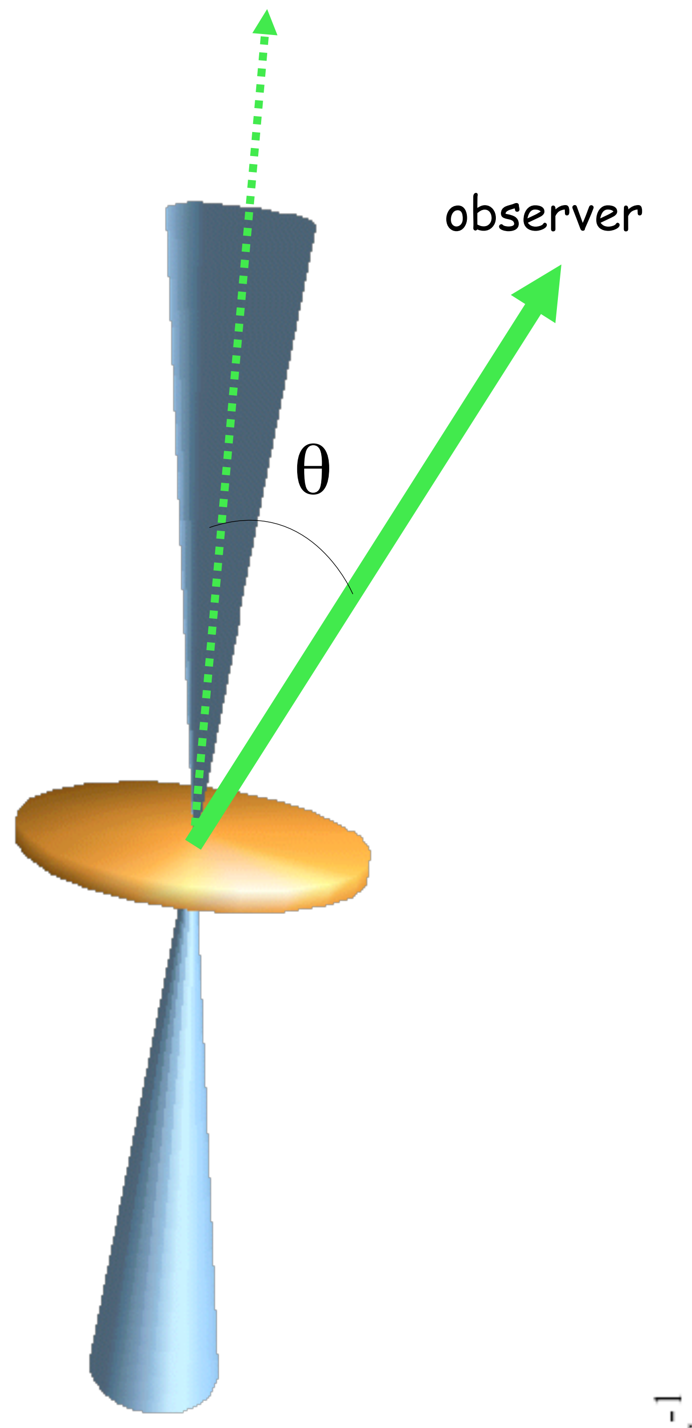


Beamed Emission



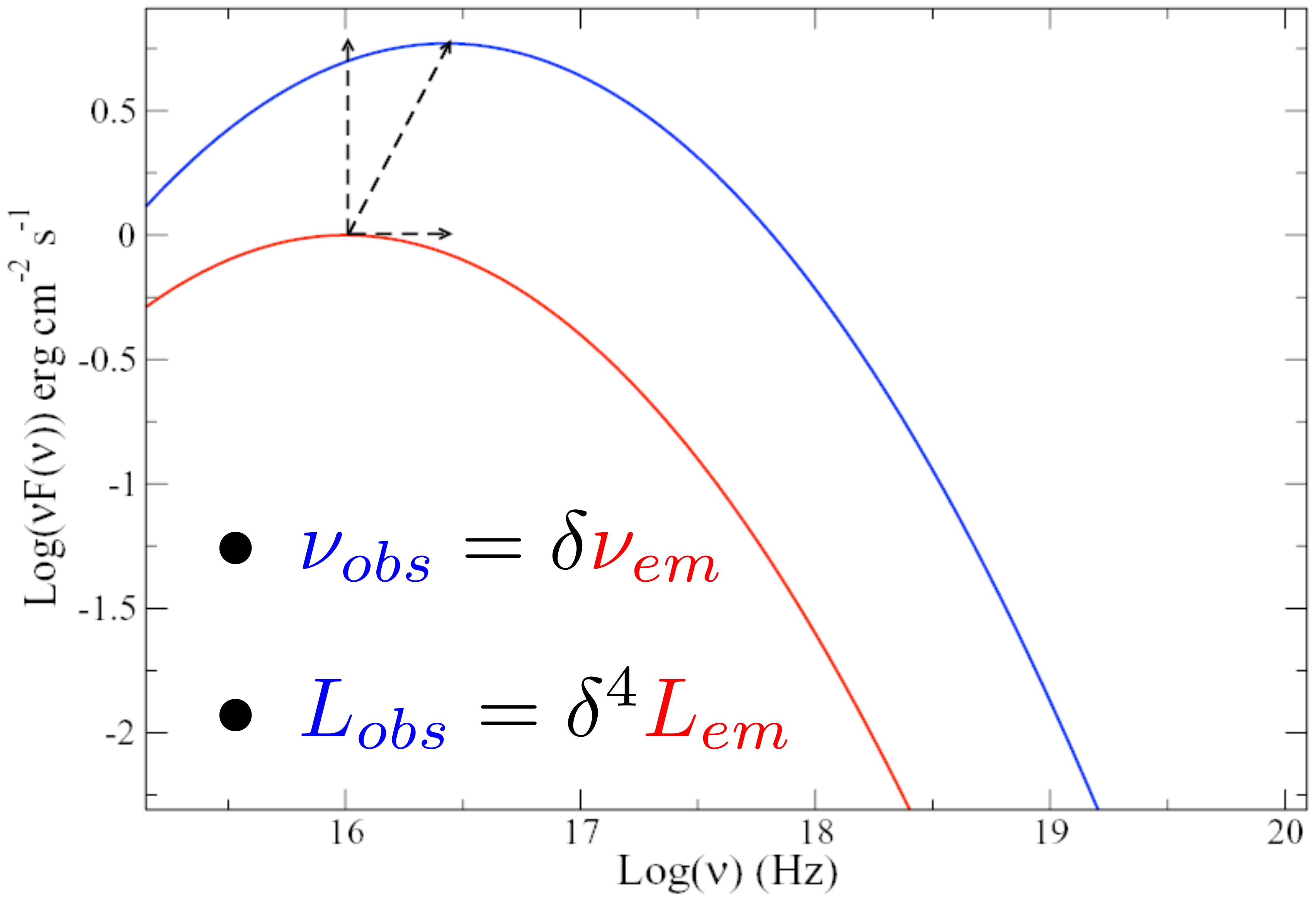


$$R \le c\,\Delta t\,\delta/(1+z)$$



Beaming factor:

- $\delta = \frac{1}{\Gamma(1-\beta cos(\theta))}$
- $\theta = 1/\Gamma$



- $\nu_{obs} = \delta\nu_{em}$
- $L_{obs} = \delta^4 L_{em}$

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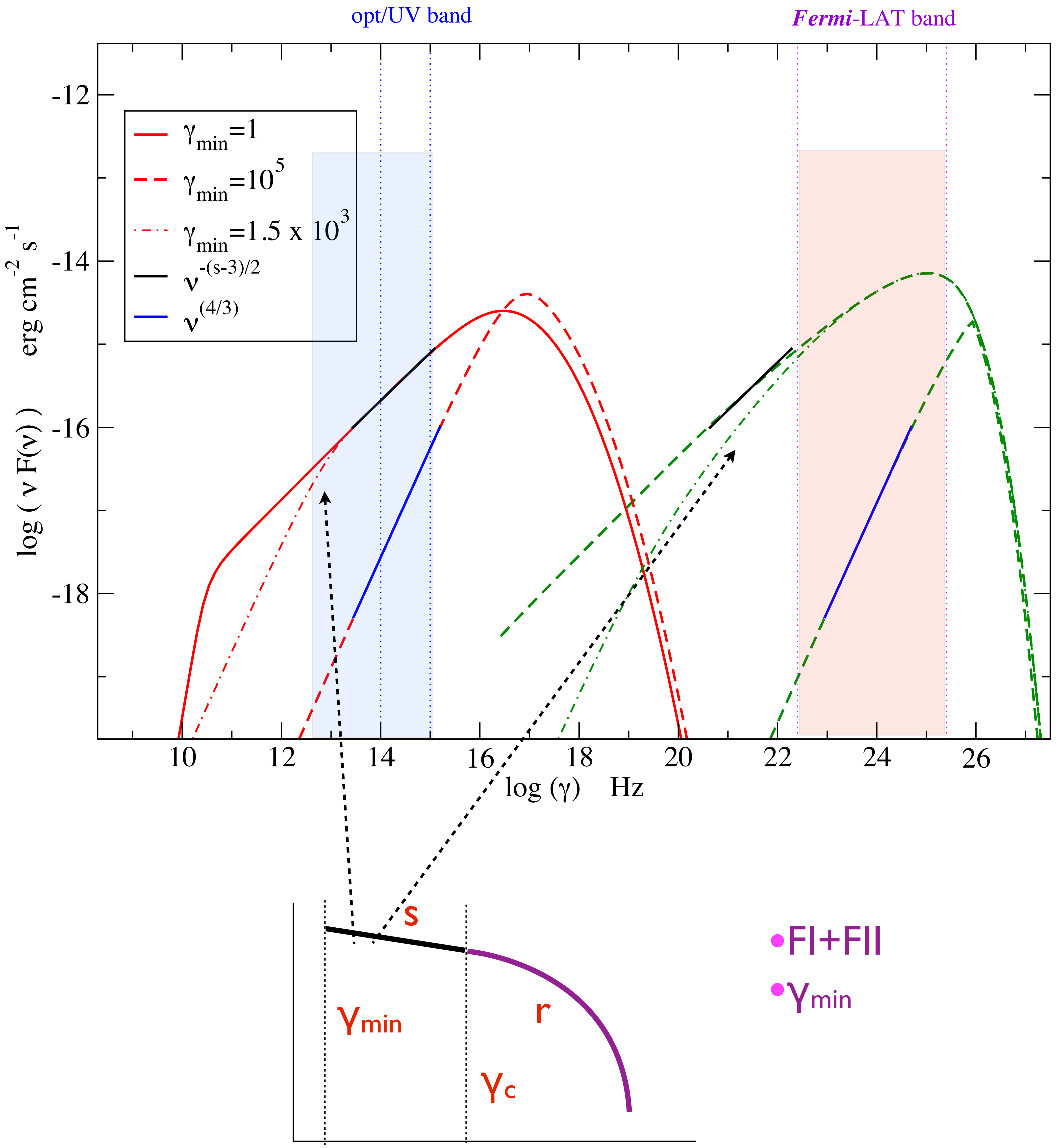
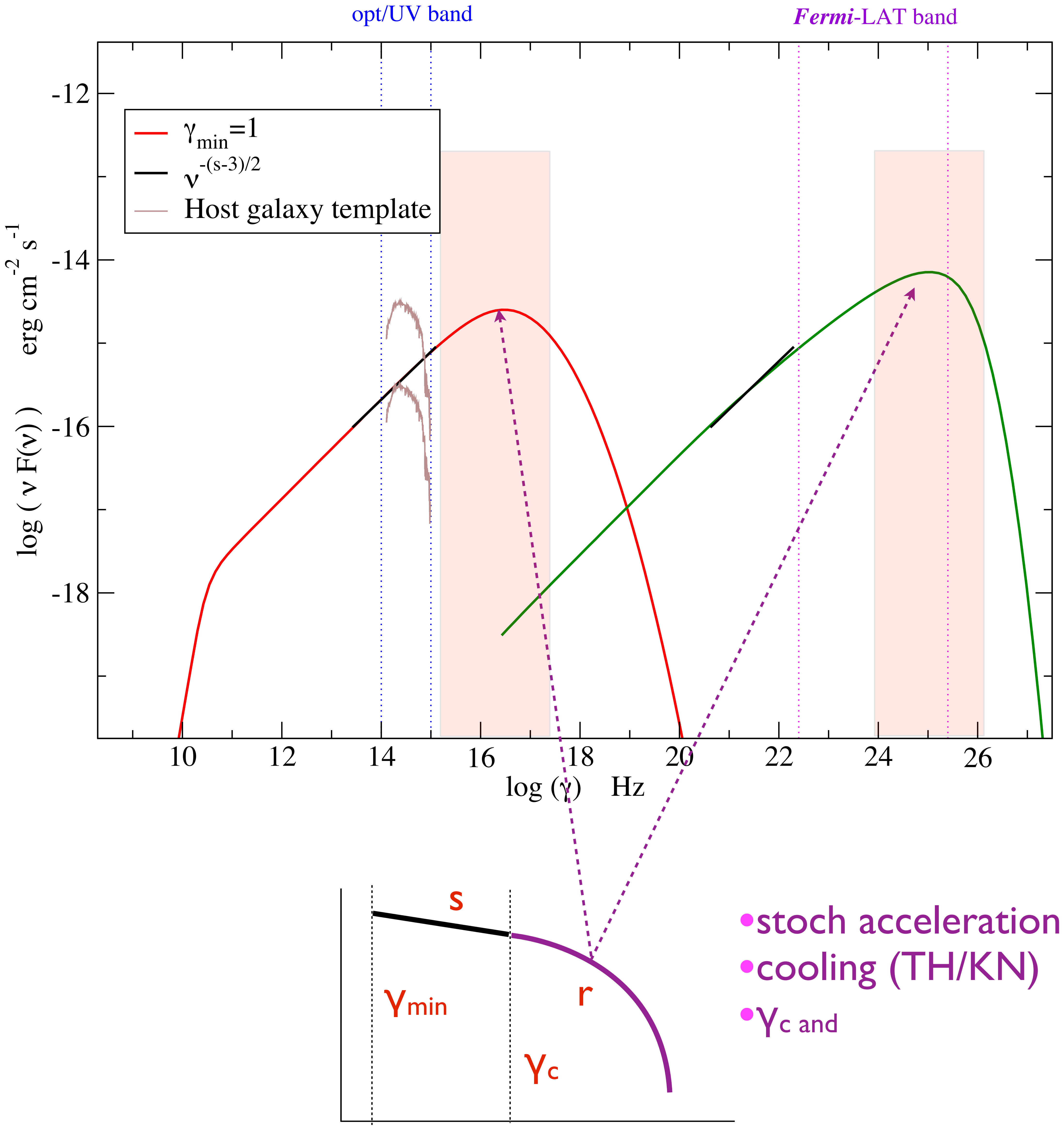
SED shaping and constraining the electron distribution



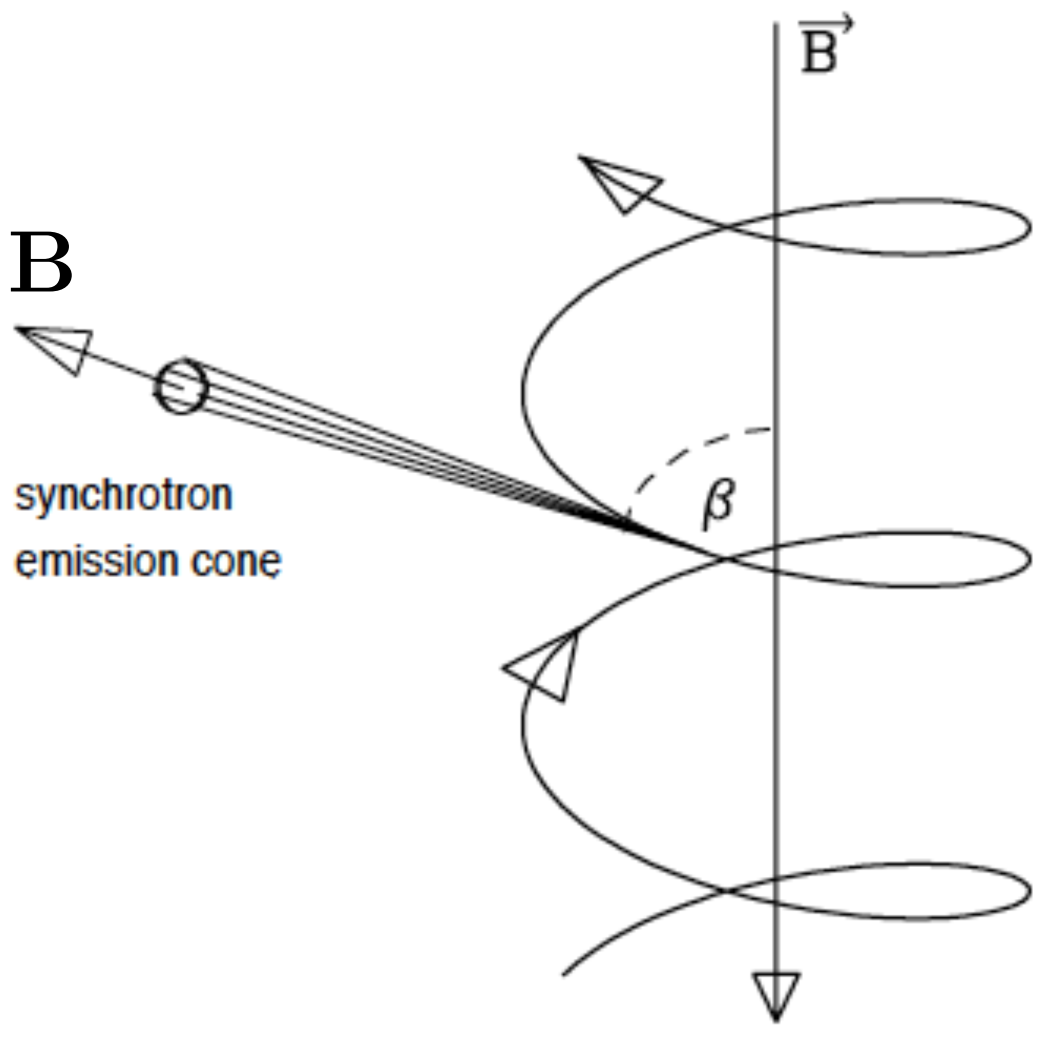
- FI+FII
- γ_{min}



- stoch acceleration
- cooling (TH/KN)
- γ_c and

synchrotron basics single particle

$$m\gamma\frac{d(\mathbf{v})}{dt} = \frac{q}{c}\mathbf{v}\times\mathbf{B}$$



total emitted power

$$a_{\parallel} = 0$$

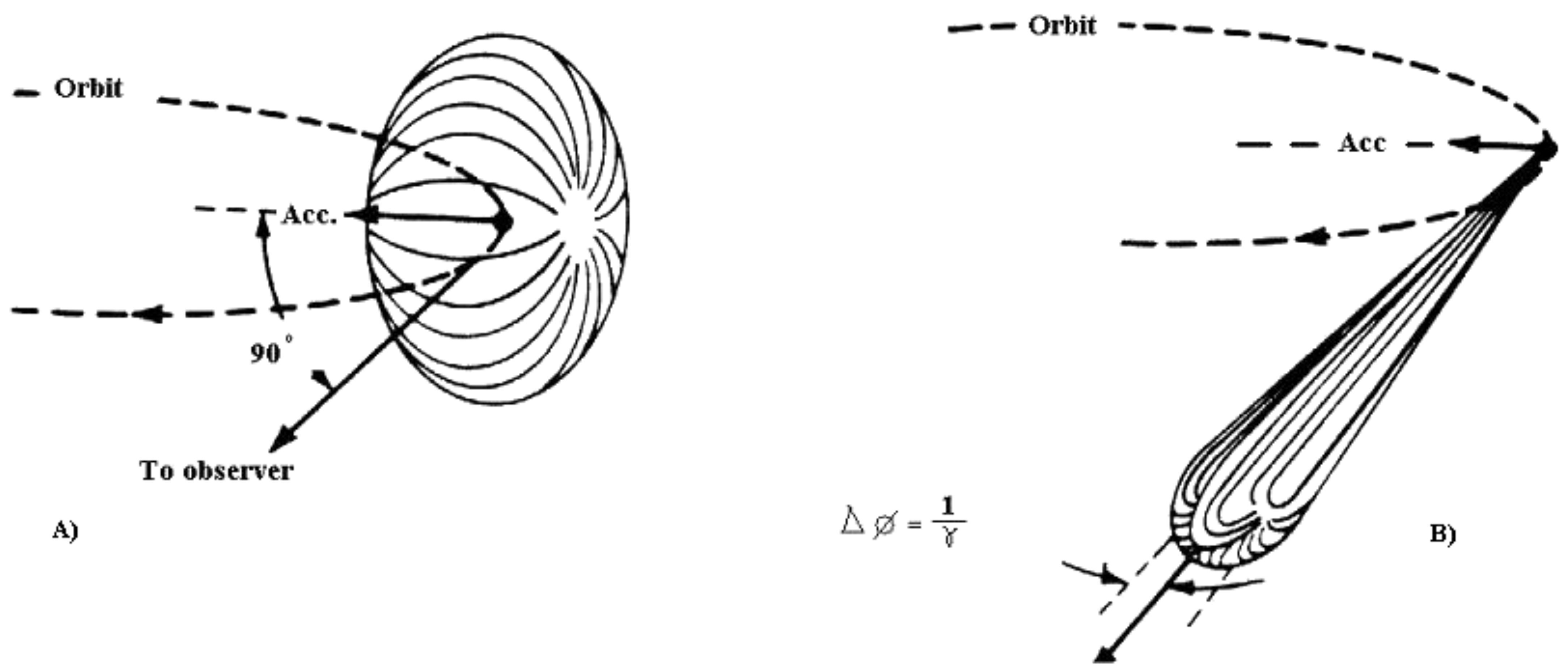
$$a_{\perp} = \frac{evBsin\alpha}{\gamma m_e c}$$

$$r_{\mathrm{L}} = \frac{v_{\perp}^2}{a_{\perp}} = \frac{\gamma mc^2\beta\, sin\alpha}{eB}$$

$$P_{\mathrm{e}} = P'_{\mathrm{e}} = \frac{2e^2}{3c^3}a'^2 = \frac{2e^2}{3c^3}[a'^2_{\parallel} + a'^2_{\perp}]$$

$$\nu_B = \frac{1}{T} = \frac{c\beta sin\alpha}{2\pi r_L}$$

time occurring to complete one orbit (gyration frequency)



$$\nu_{\mathrm{B}} = \frac{eB}{2\pi\gamma mc} = \frac{\nu_{\mathrm{L}}}{\gamma}$$

fundamental frequency

$$P_S = \frac{2e^4}{3m_e c^3}B^2\gamma^2\beta^2 sin^2\alpha$$

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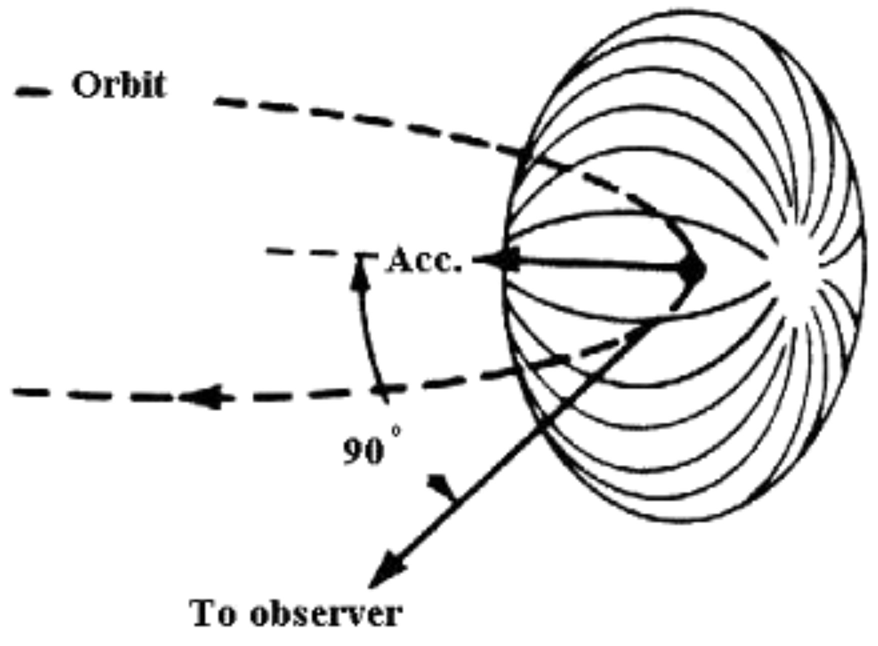
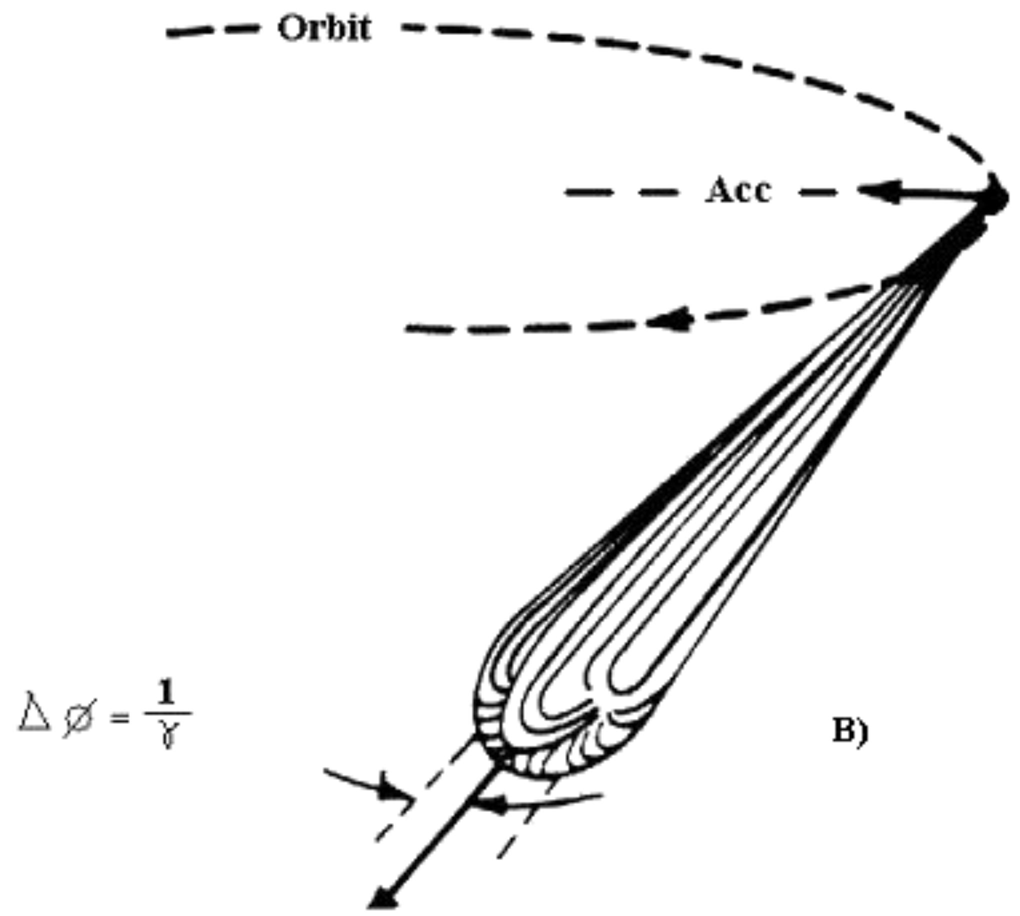
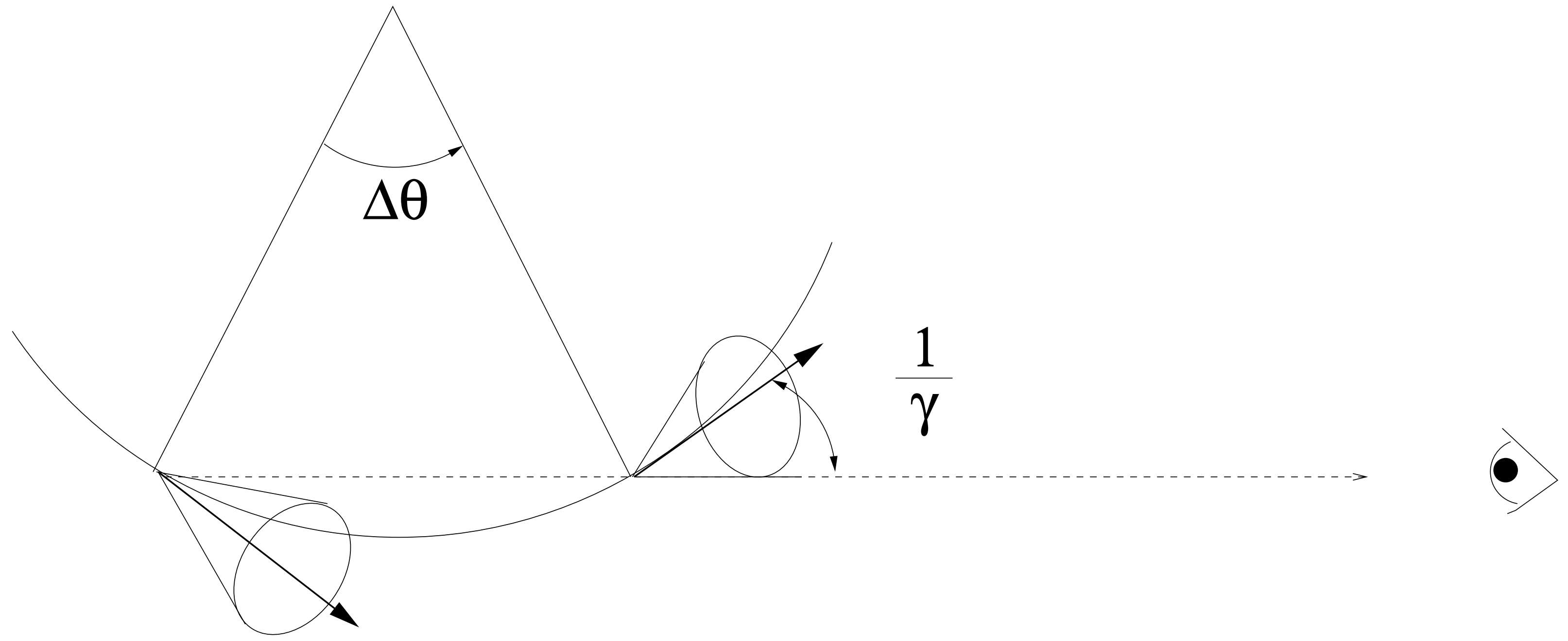
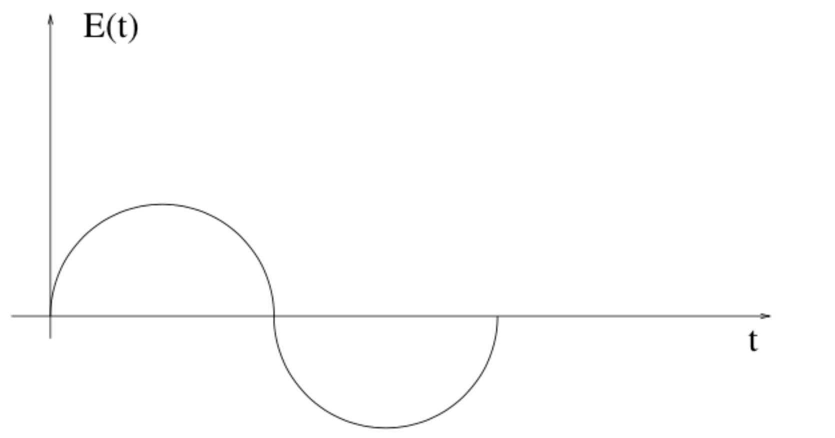
synchrotron basics

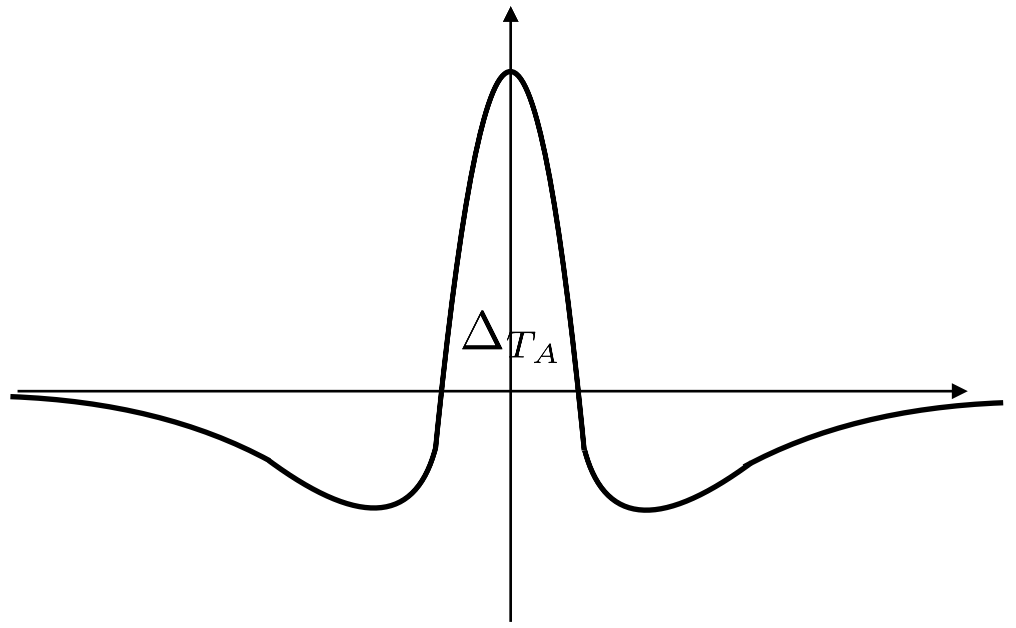


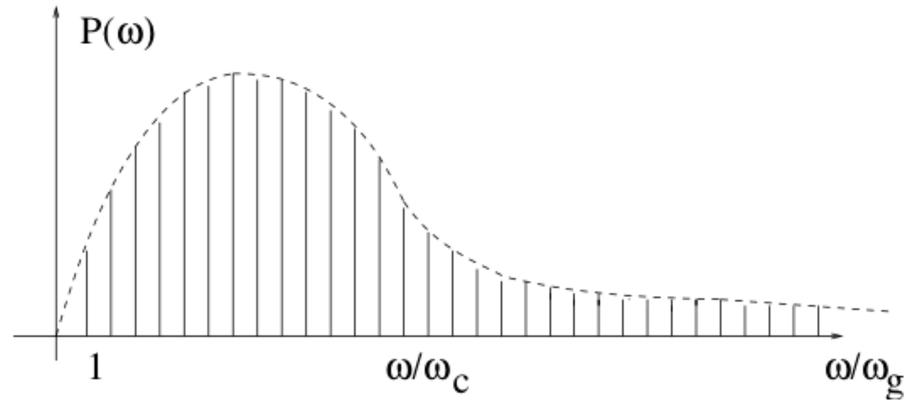
A)



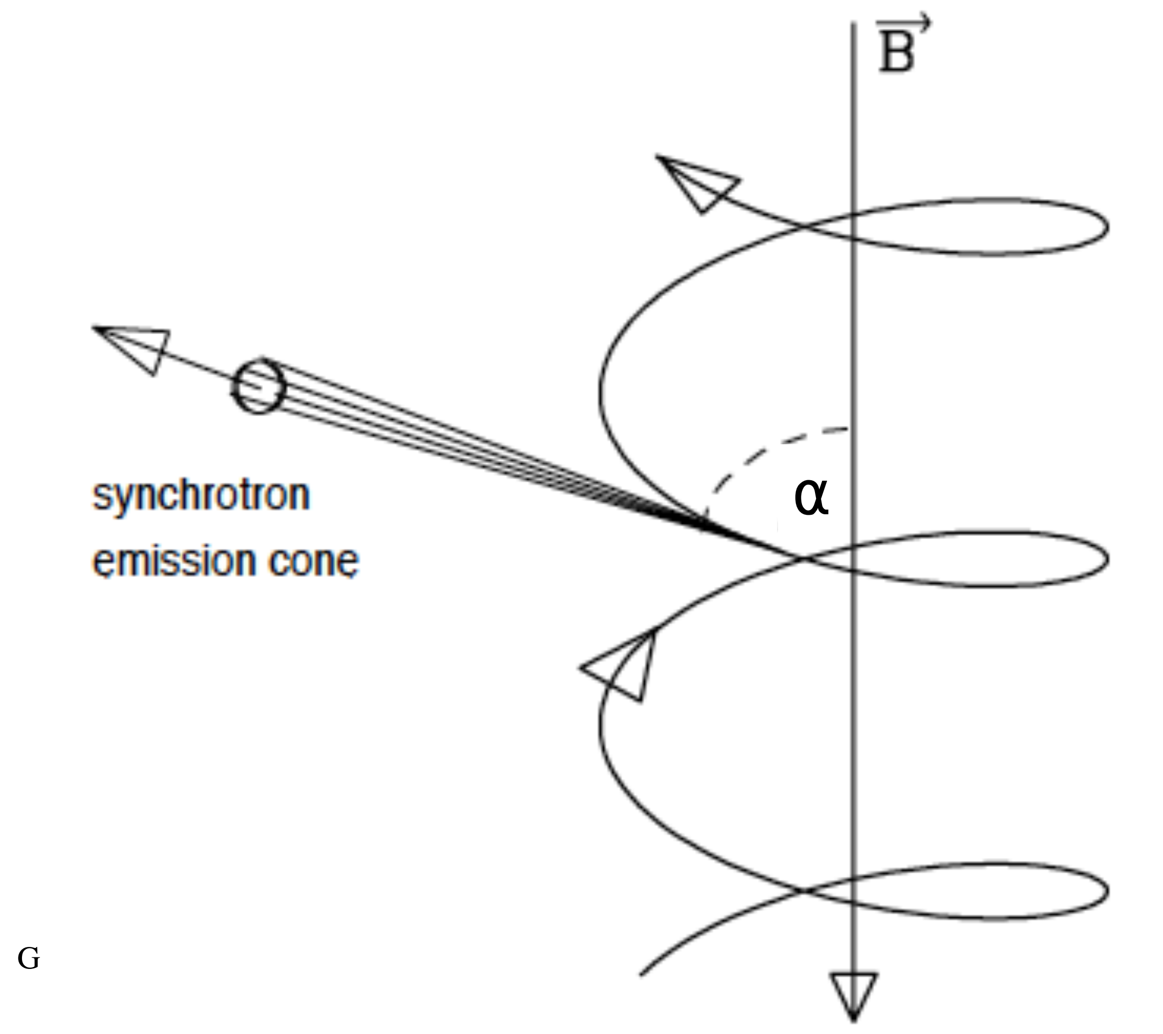


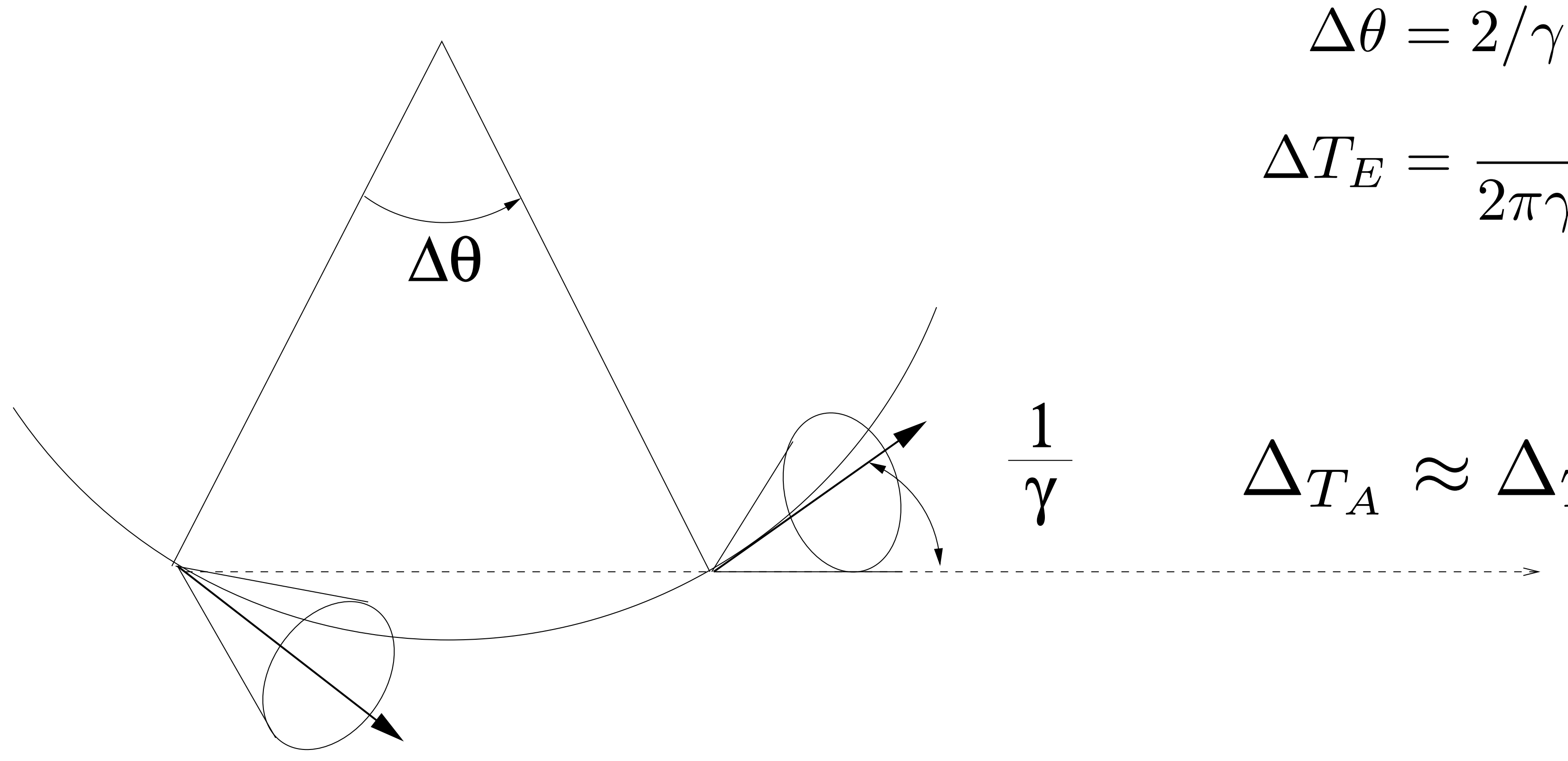






synchrotron basics single particle

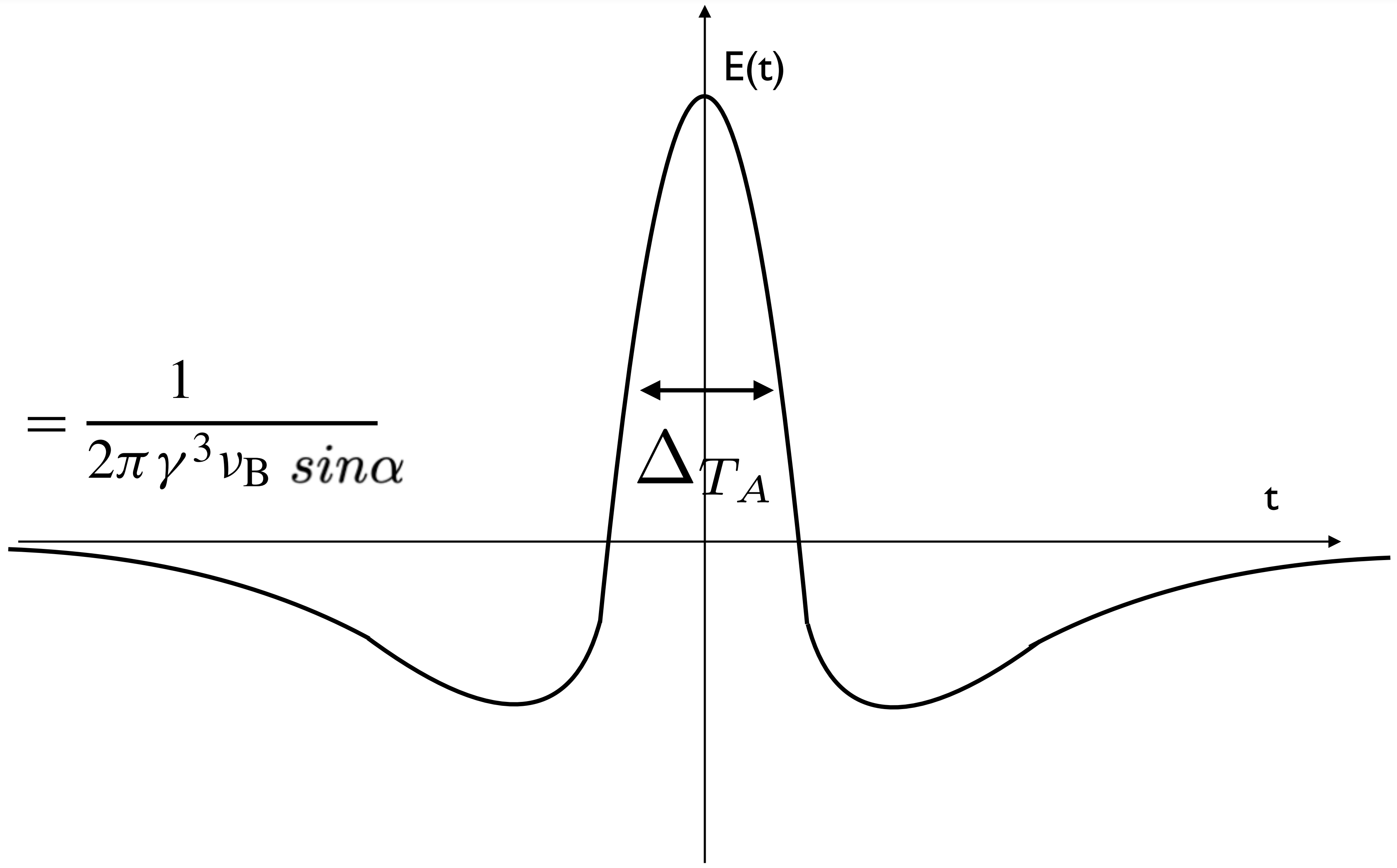




$$\Delta\theta = 2/\gamma$$

$$\Delta T_E = \frac{2}{2\pi\gamma\nu_B sin\alpha}$$

$$\Delta_{T_A} \approx \Delta_{T_E}/(2\gamma^2) = \frac{1}{2\pi\gamma^3\nu_B\ sin\alpha}$$



power spectrum

$$\Delta T_A \Delta\nu \approx \frac{1}{2\pi} \longrightarrow \nu_c = \gamma^3\nu_B\ sin\alpha$$

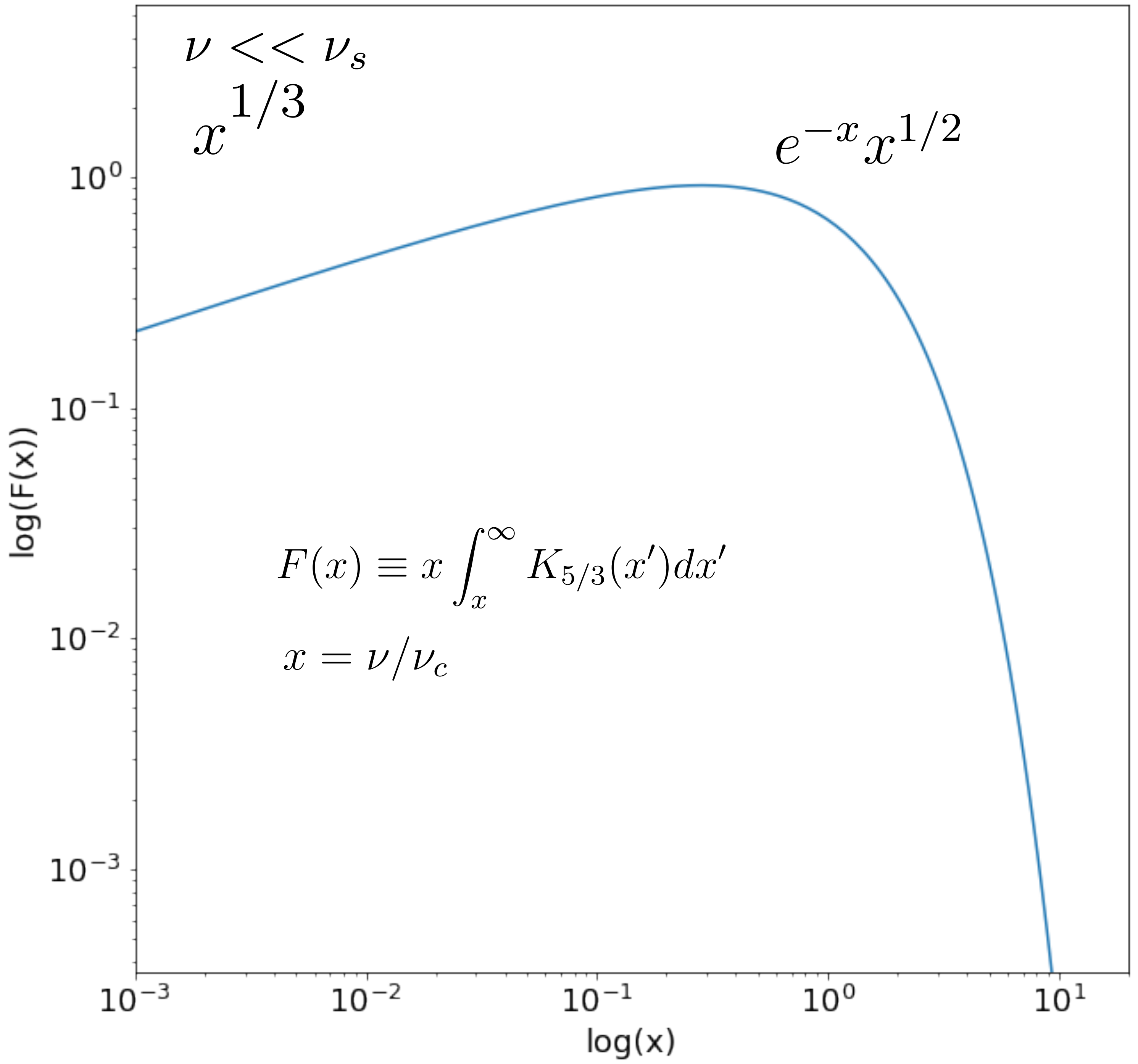
typical frequency

emitted spectrum

$$P_e(\nu,\gamma) = \frac{\sqrt{3}e^3 B\sin\alpha}{2m_e c^2}F(x)$$

$$\nu_c = \frac{3}{2}\nu_B\gamma^3 sin\alpha = \frac{3\gamma^2 eBsin\alpha}{4\pi m_e c}$$

$$\nu_s \propto 10^6 B\gamma^2 sin(\alpha)\ \ \text{Hz}$$



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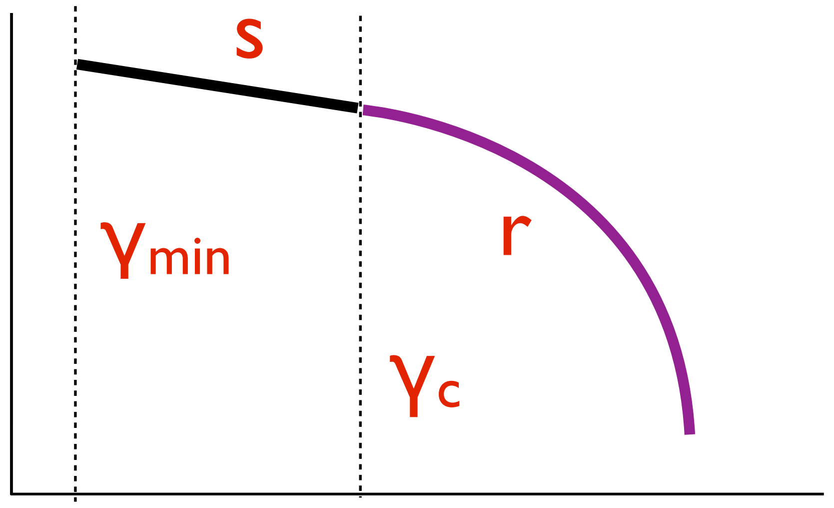
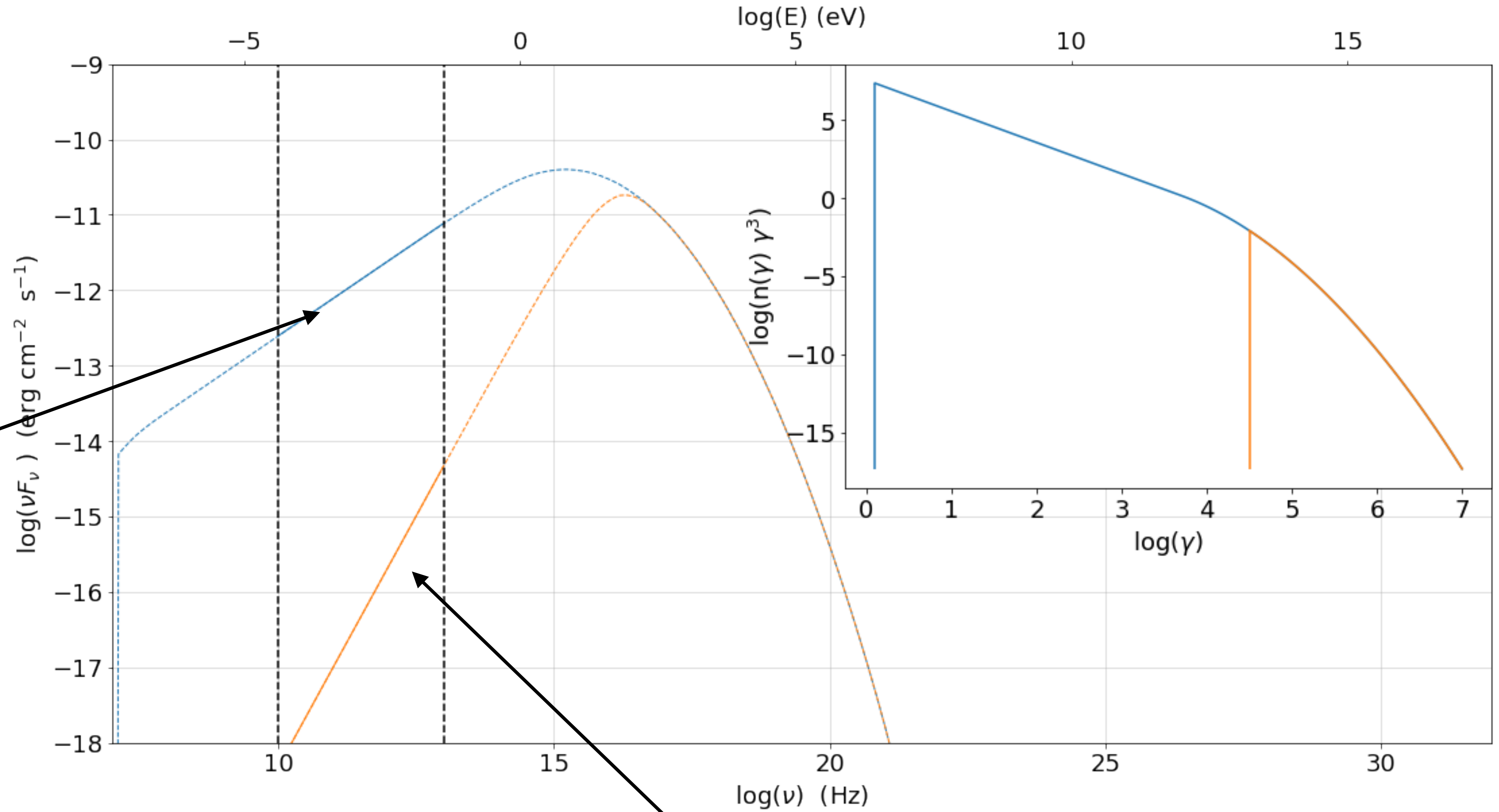
synchrotron basics particle distribution

$$N(\gamma) \propto \gamma^{-s}$$



Tutorial 2

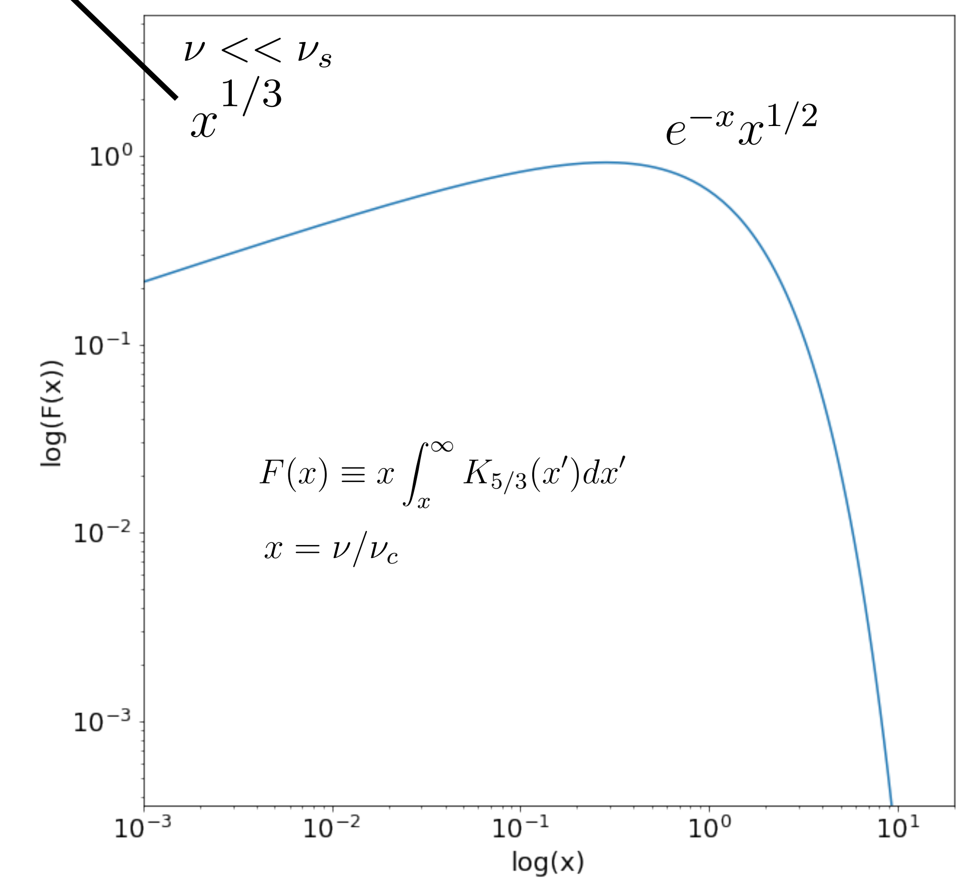


$$j^S_\nu(\nu) = \frac{1}{4\pi}\int_{\gamma_{min}}^{\gamma_{max}} P(\nu,\gamma)N(\gamma)d\gamma \ \propto \nu^{-\frac{s-1}{2}}$$

F_ν	refers to spectral index => SED=	νF_ν
I_ν		νI_ν
L_ν		νL_ν

δ-approx relations $\quad \mathrm{SED} \propto N(\gamma)\gamma^3$

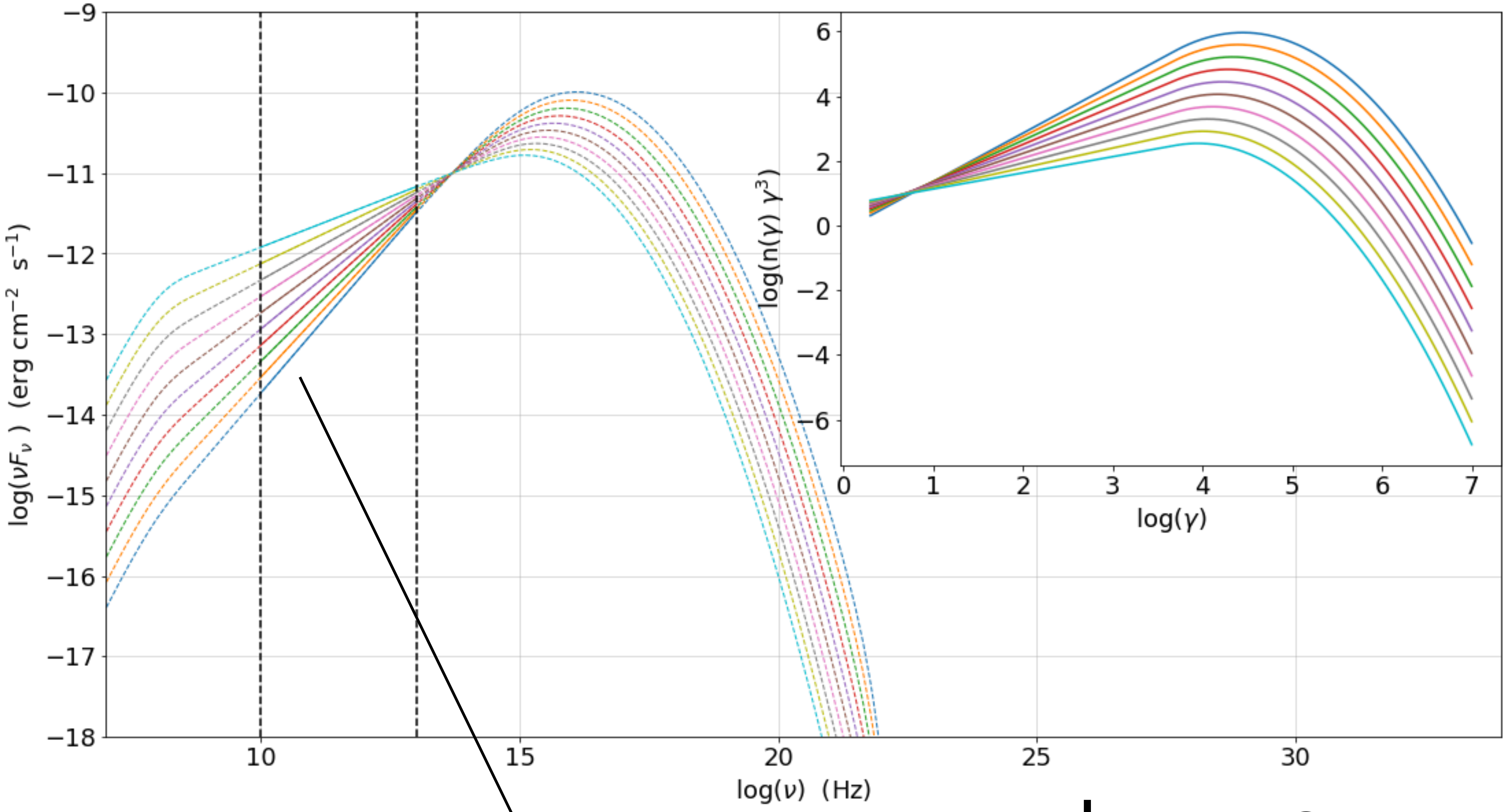
$$S_p^{Sync} \sim \frac{dN(\gamma)}{d\gamma}\gamma_{3p}^3 \ B^2 \ \delta^4 \quad \nu_p^{Sync} \sim 3.2\times 10^6 \ (\gamma_{3p})^2 \ B \ \delta$$

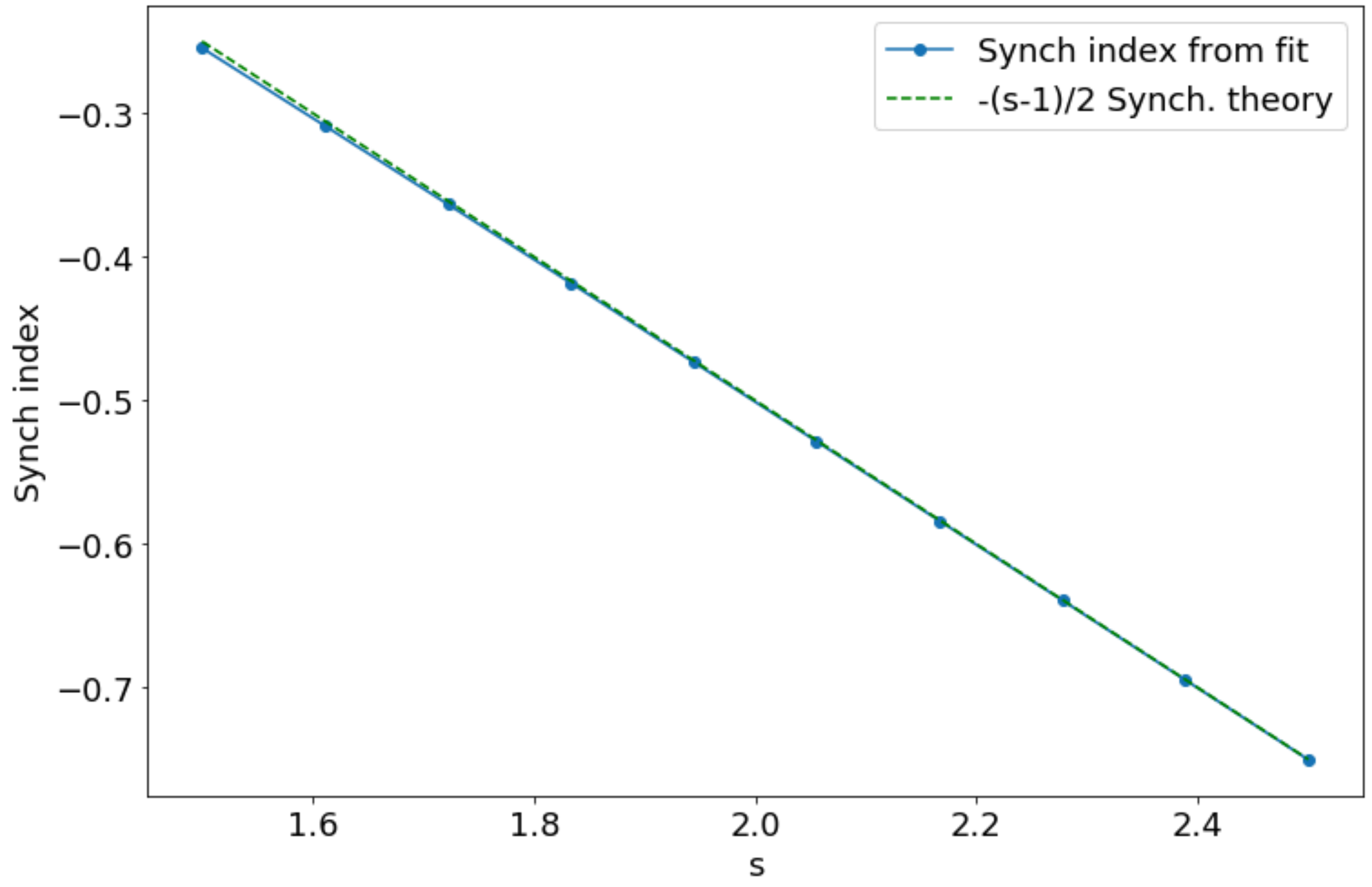


4/3 in SED corresponds to 1/3 in spectral index

Synchrotron emission Estimate of *s* from spectral index

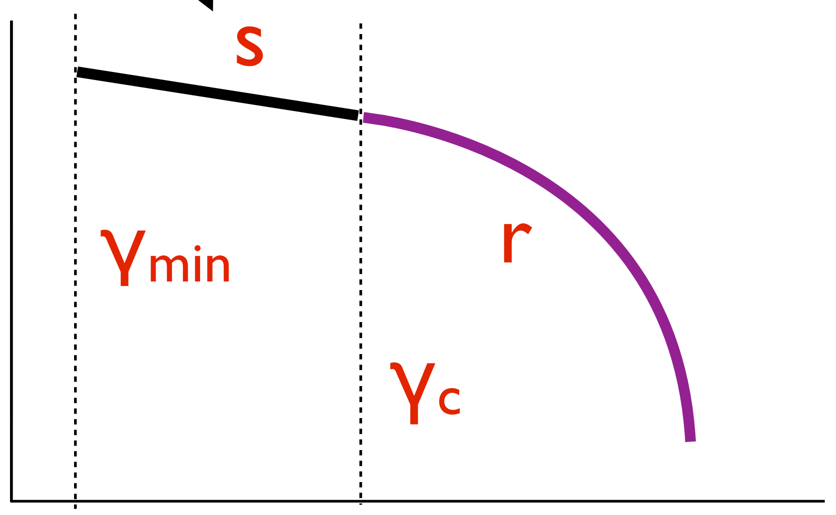
Tutorial 2





$$j^S_\nu(\nu) = \frac{1}{4\pi}\int_{\gamma_{min}}^{\gamma_{max}} P(\nu,\gamma)N(\gamma)d\gamma \propto \nu^{-\frac{s-1}{2}}$$

change s



obs. data → model

- synch. spectral index
- s el. distr.

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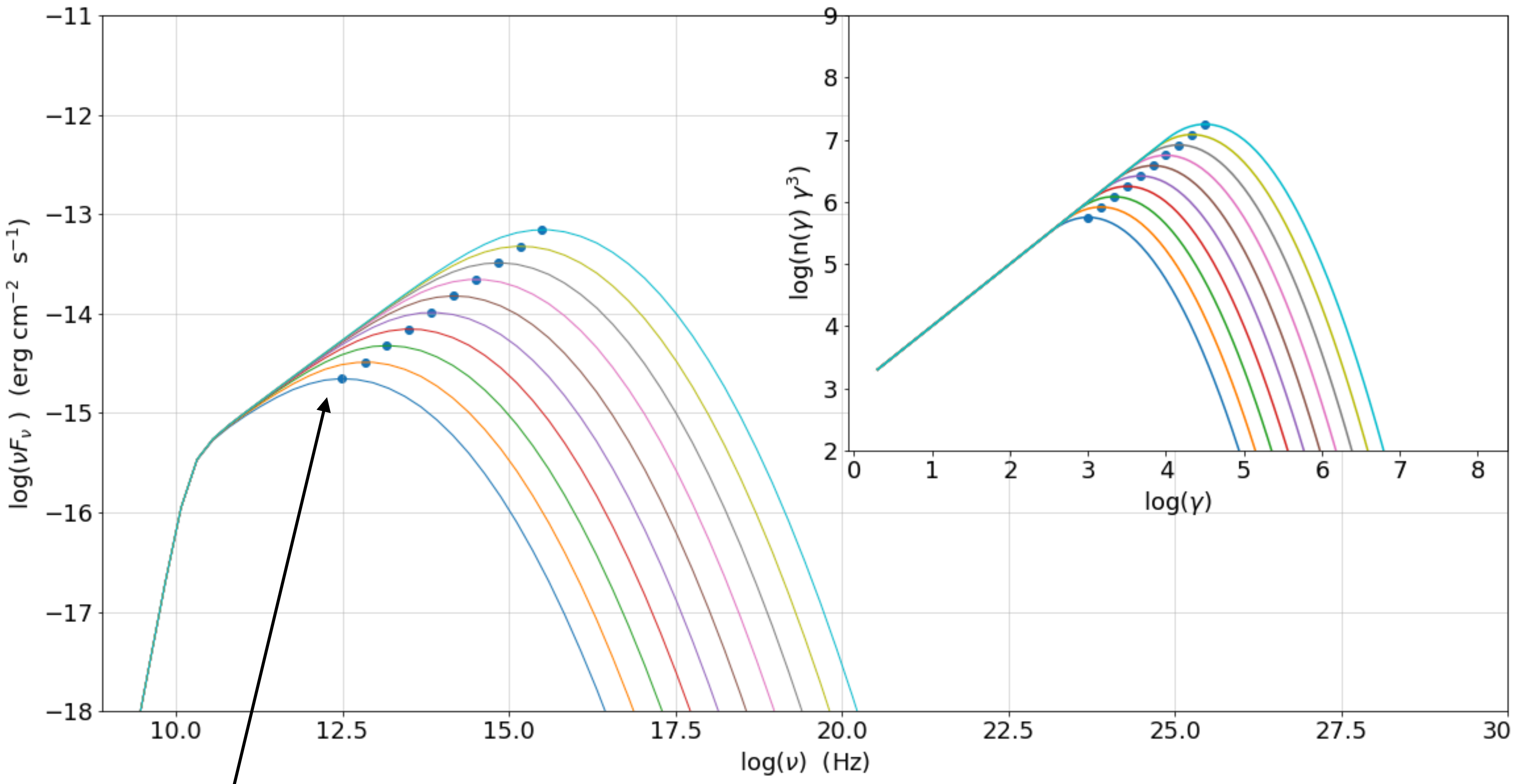
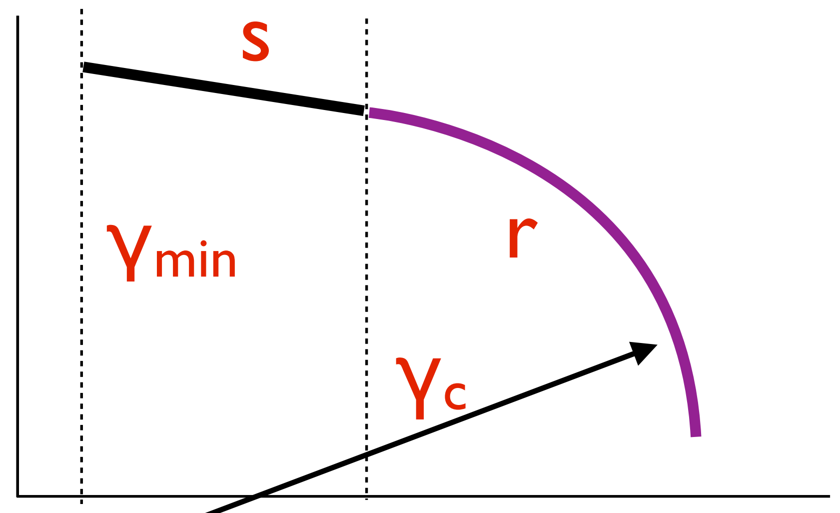
Synchrotron emission Estimate of γ_p^s from ν_p^s

Tutorial 2

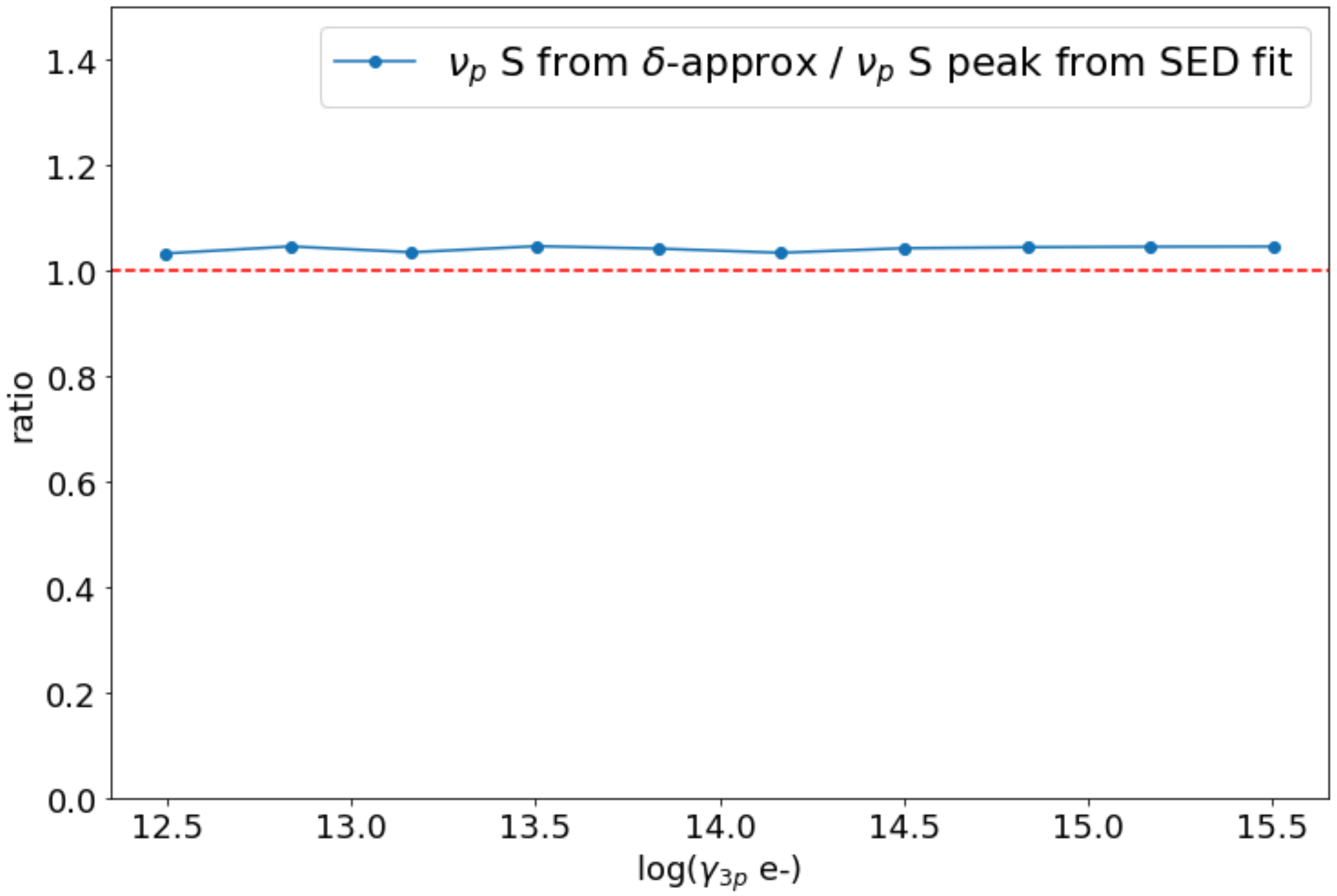




$$\nu_p^{Sync} \sim 3.2 \times 10^6 \ (\gamma_{3p})^2 \ B \ \delta$$

$$S_p^{Sync} \sim \frac{dN(\gamma)}{d\gamma} \gamma_{3p}^3 \ B^2 \ \delta^4$$

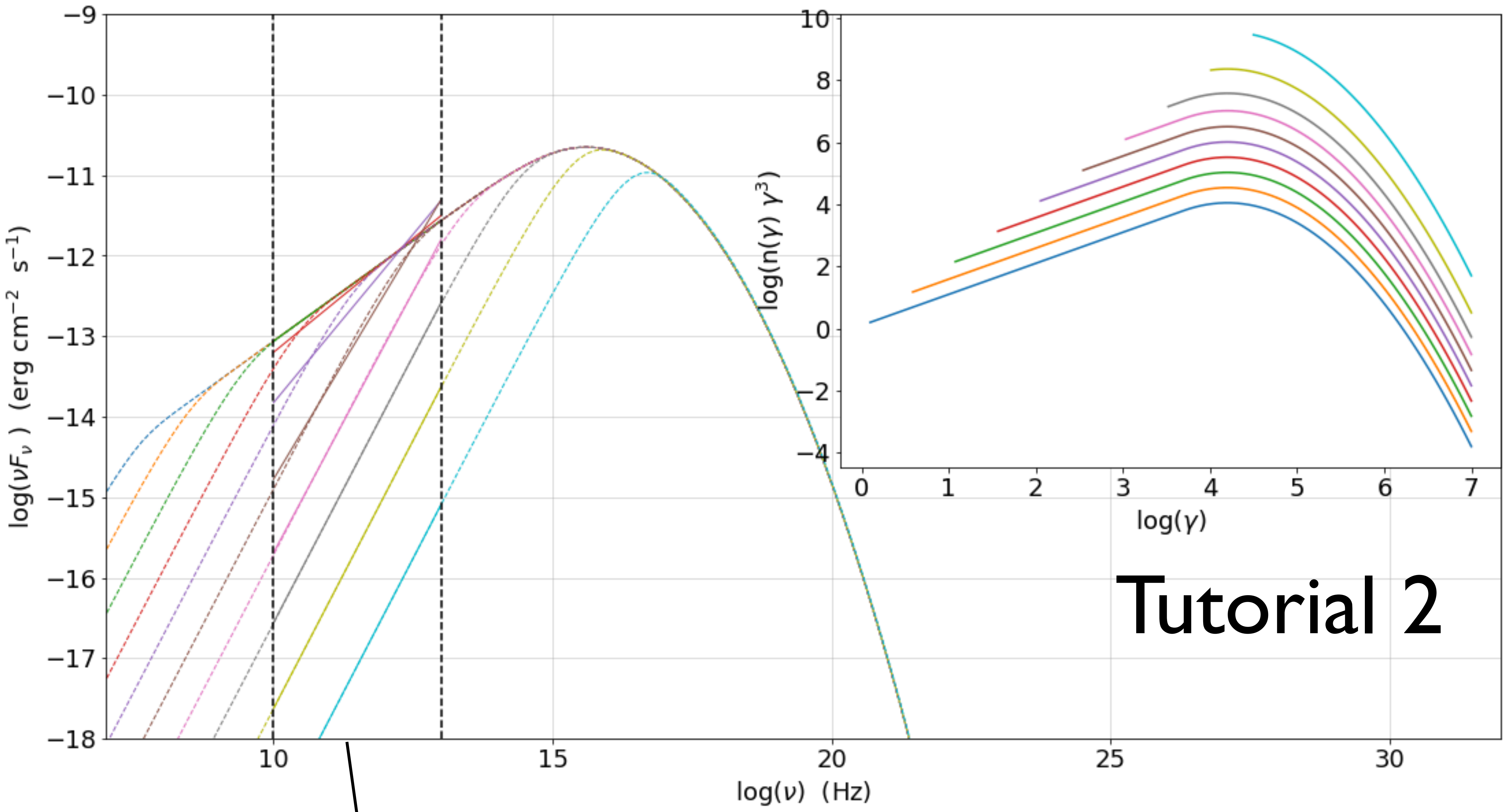
change γ_c

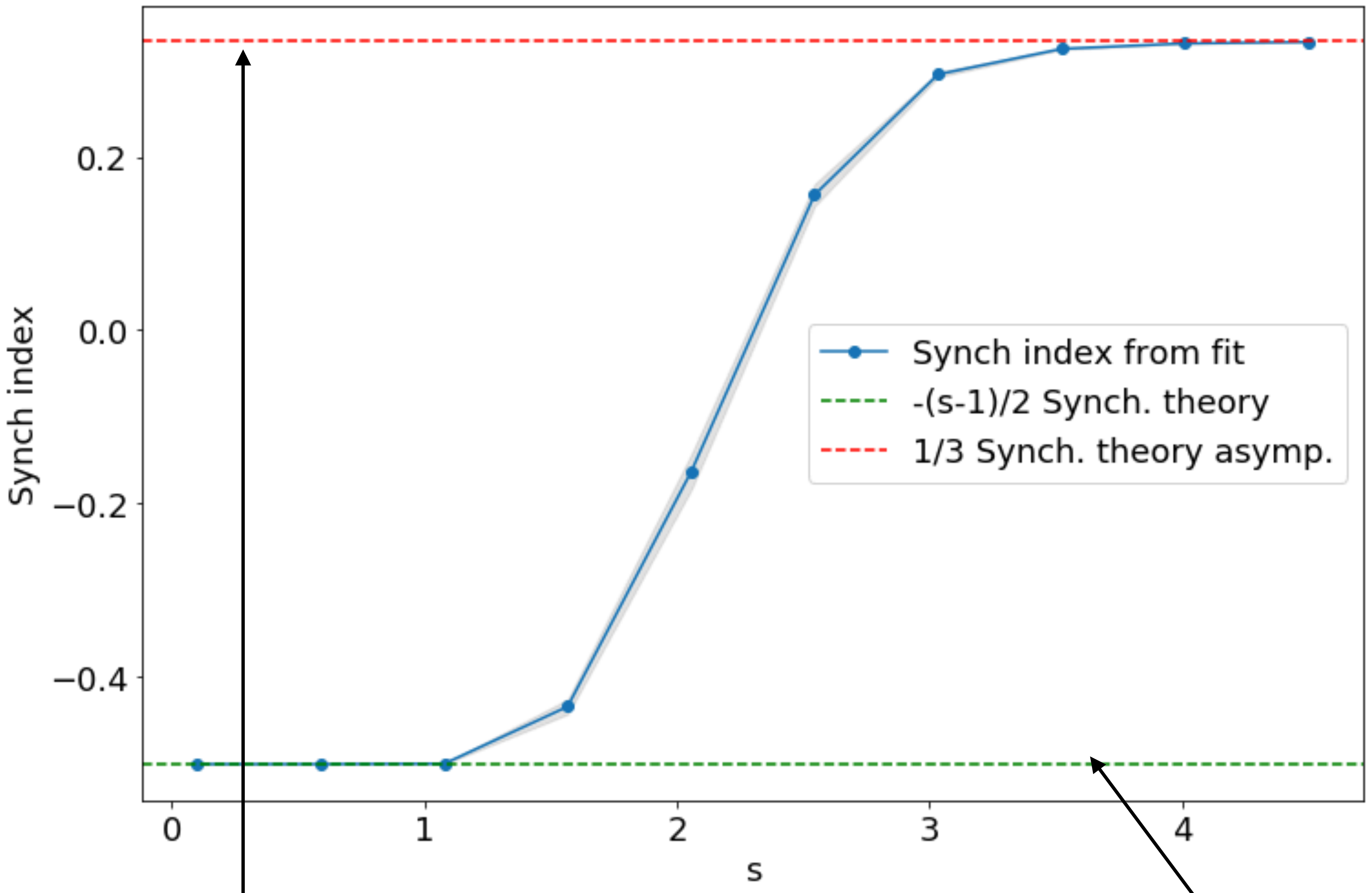


obs. data ⟶ model

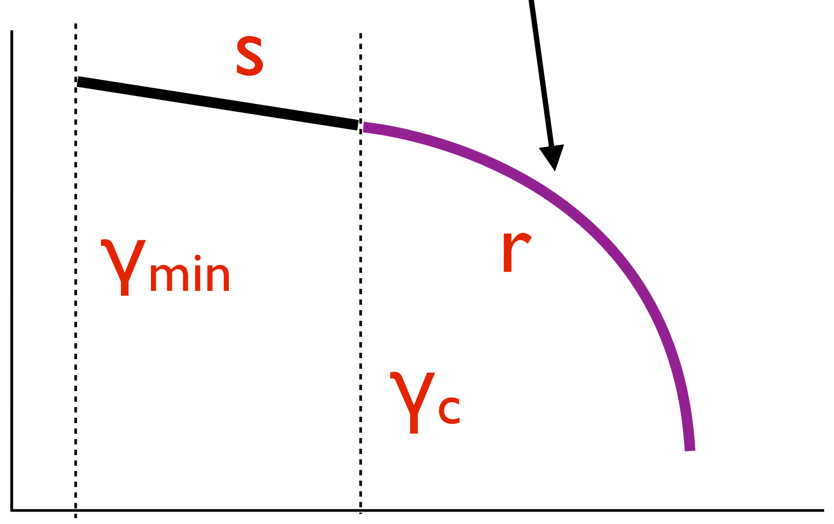
- ν_p^s → γ_p^s

Synchrotron emission Estimate of γ_{min} from spectral index



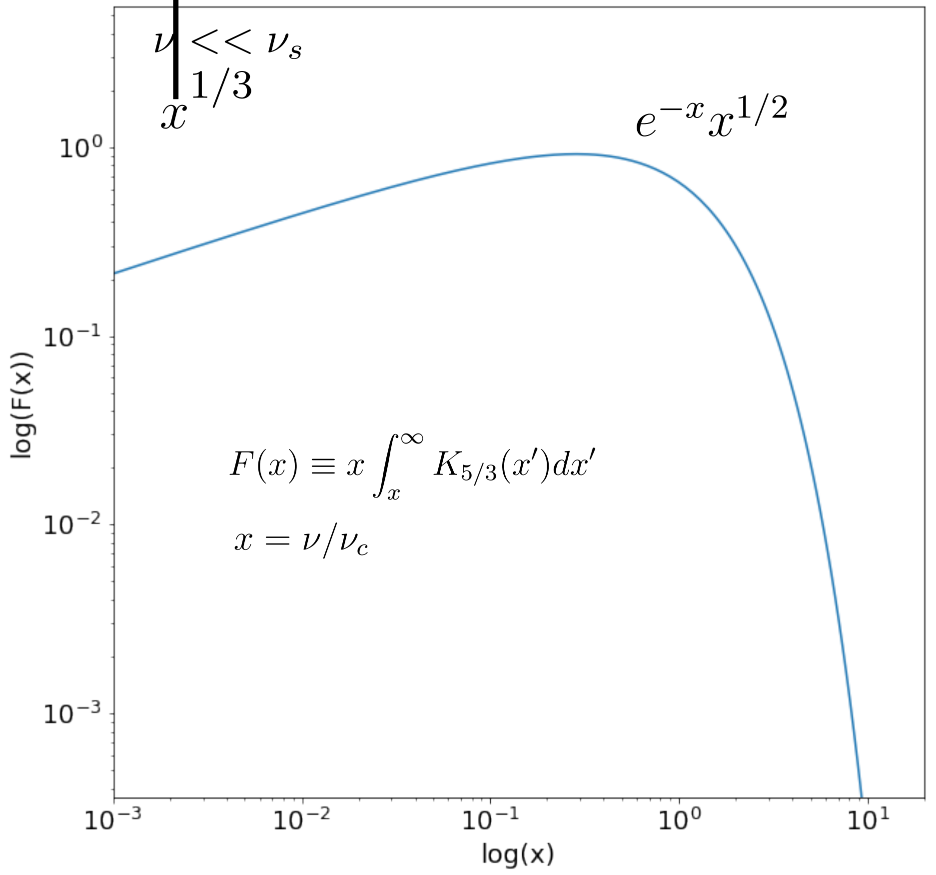


s is fixed



obs. data → model

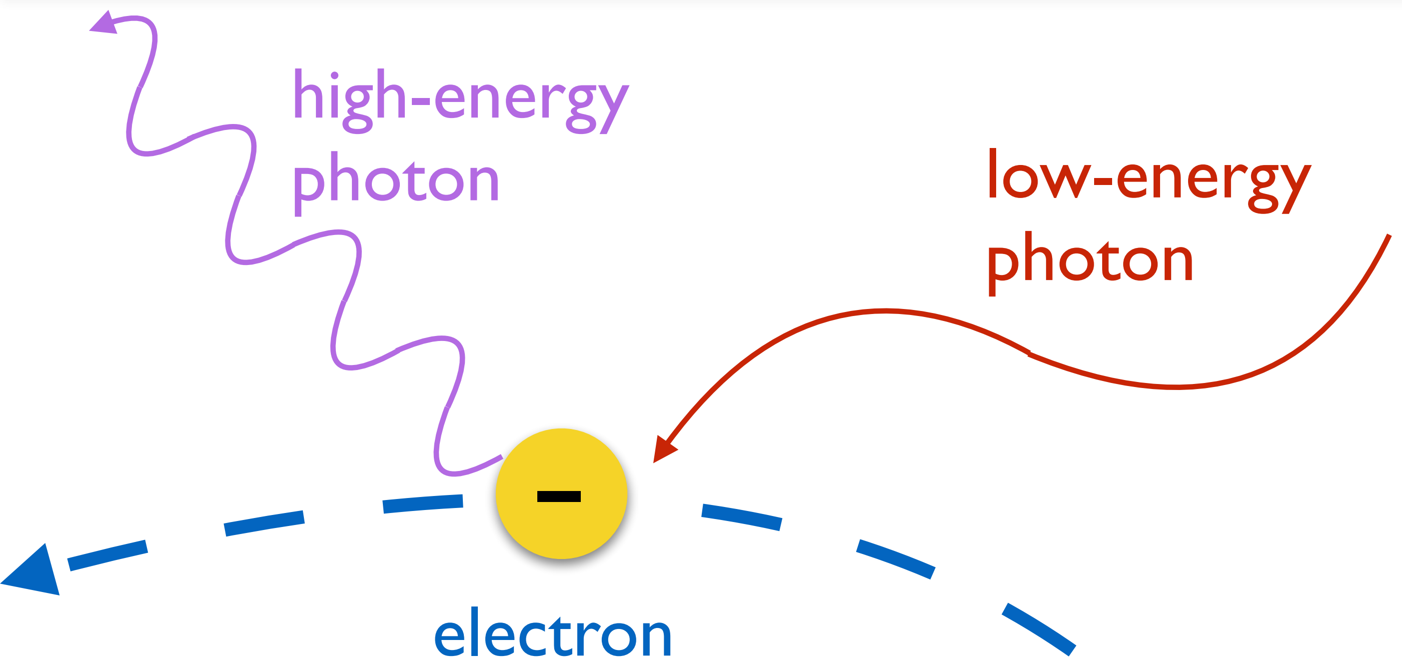
- synch. spectral index
- s el. distr.
- γmin

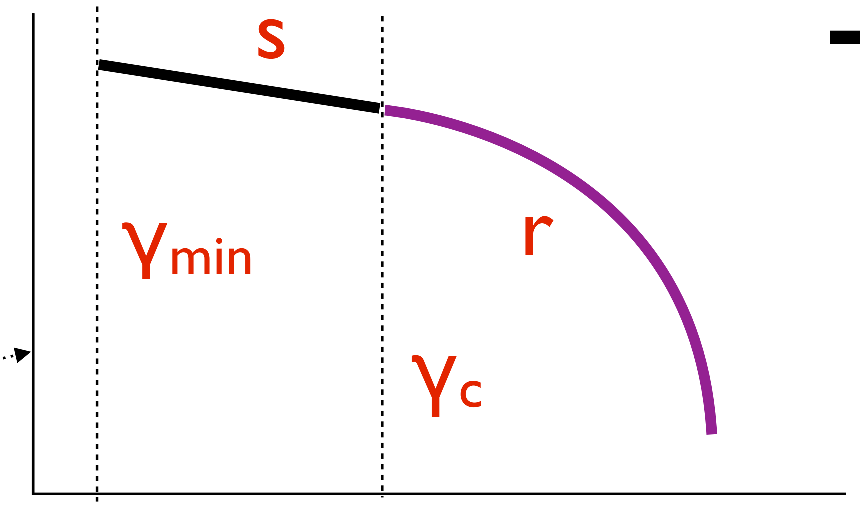


$$j_\nu^S(\nu) = \frac{1}{4\pi}\int_{\gamma_{min}}^{\gamma_{max}} P(\nu,\gamma)N(\gamma)d\gamma \propto \nu^{-\frac{s-1}{2}}$$

IC emission basics

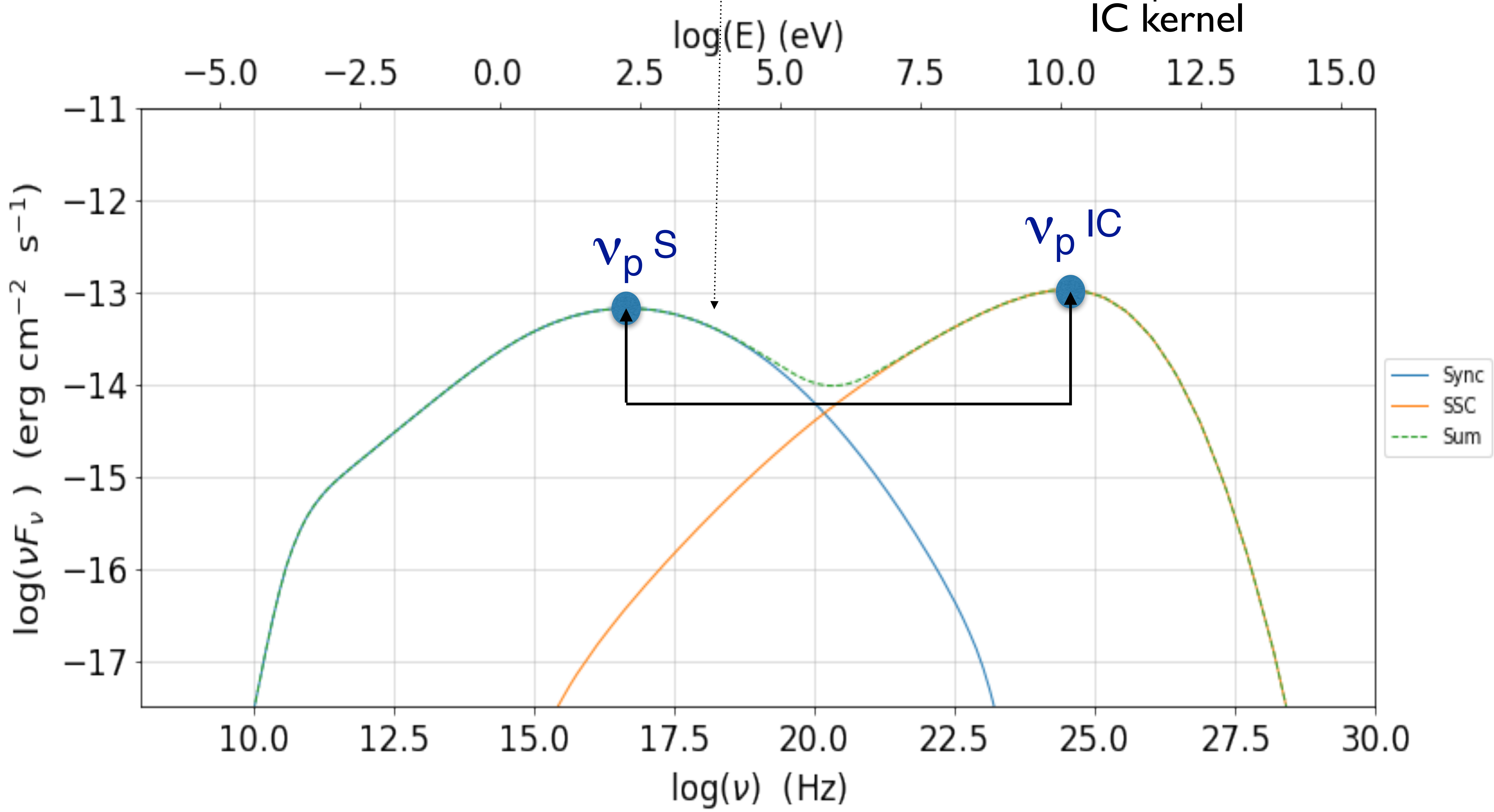
Tutorial 2





$$j_\nu^{SSC} = \frac{3}{4}\frac{h\nu\sigma_T c}{4\pi}\int_0^\infty n(\nu_0)d\nu_0 \int_{\gamma_{min}}^{\gamma_{max}} \frac{N(\gamma)}{\gamma^2} F_c(\gamma,\nu,\nu_0)d\gamma$$

IC kernel



soft photon energy
in the e- rest frame

$$\epsilon' = \frac{h\nu'}{m_e c^2}$$

$\epsilon' << m_e c^2$

TH regime

$(\nu_p^{IC} / \nu_p^{S}) \sim (4/3)\gamma_p^2$

$\epsilon' \geq m_e c^2$

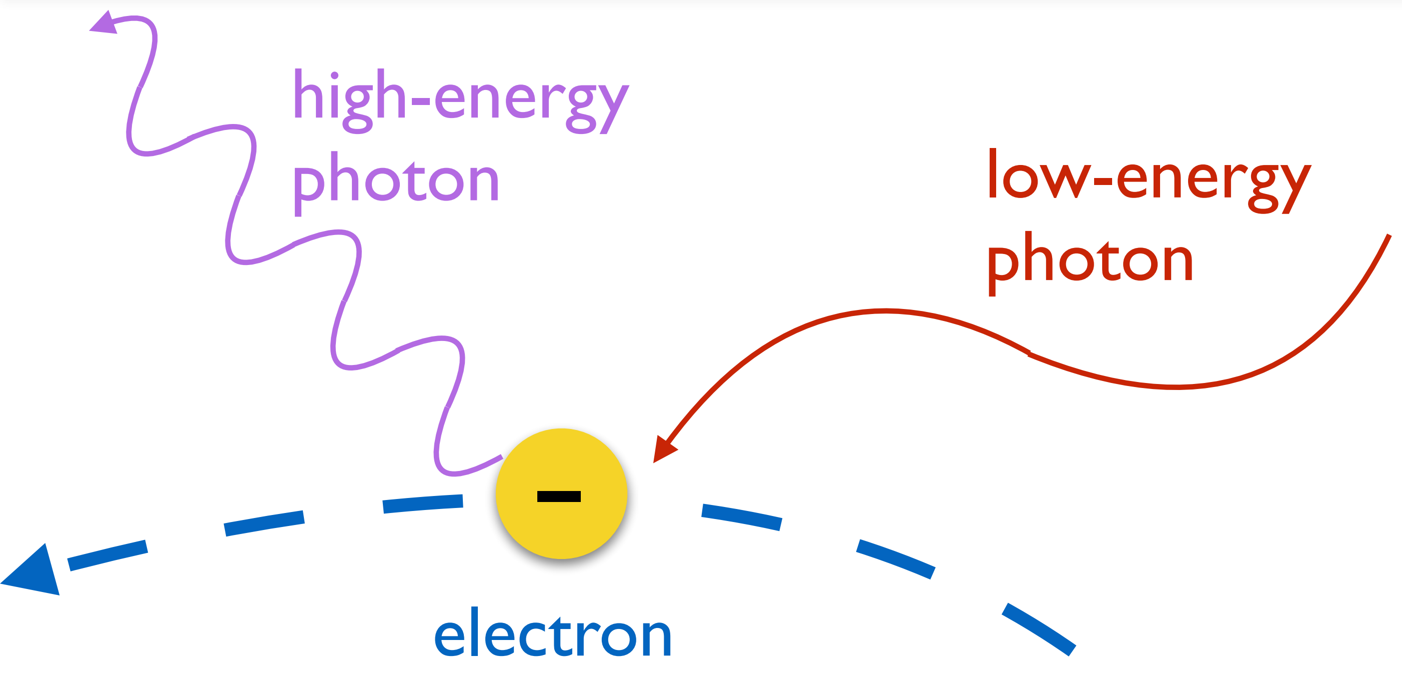
KN regime

$\nu_p^{IC} / \nu_p^{S} \sim \gamma_p$

$h\nu_p^{IC} \sim m_e c^2 \gamma_p$

IC emission basics

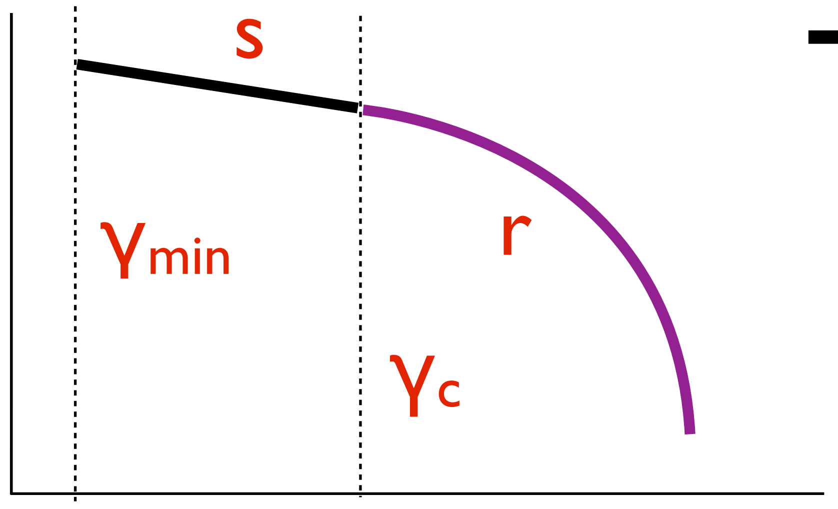
Tutorial 2

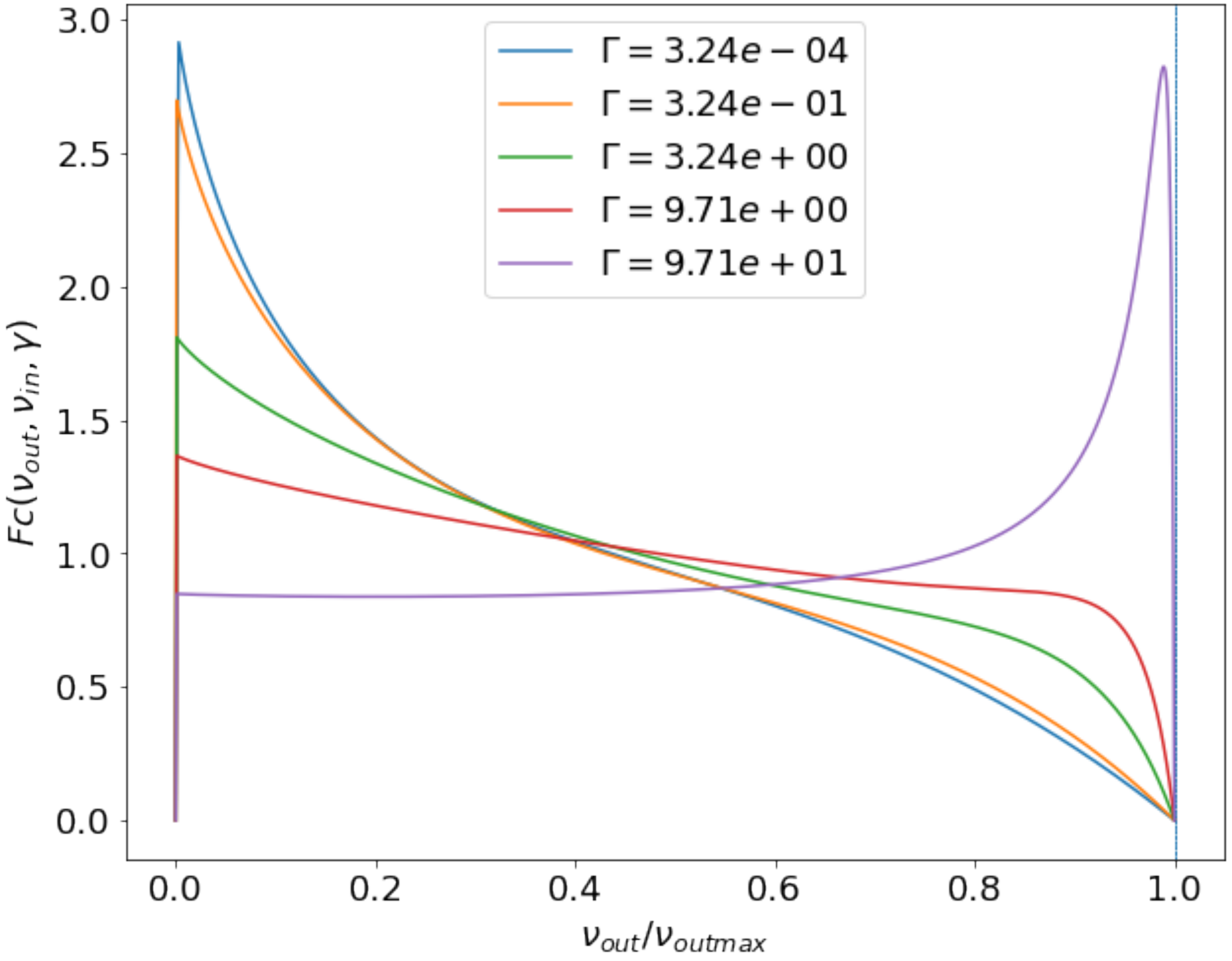


soft photon energy
in the e- rest frame

$$\epsilon' = \frac{h\nu'}{m_e c^2} \qquad \nu_{\mathrm{out\ max}} = \frac{4\nu_{\mathrm{in}}\gamma_e^2}{1+4\gamma_e\epsilon'}$$

$$\Gamma = 4\epsilon\ \gamma_e$$





TH regime

$\epsilon' << m_e c^2$

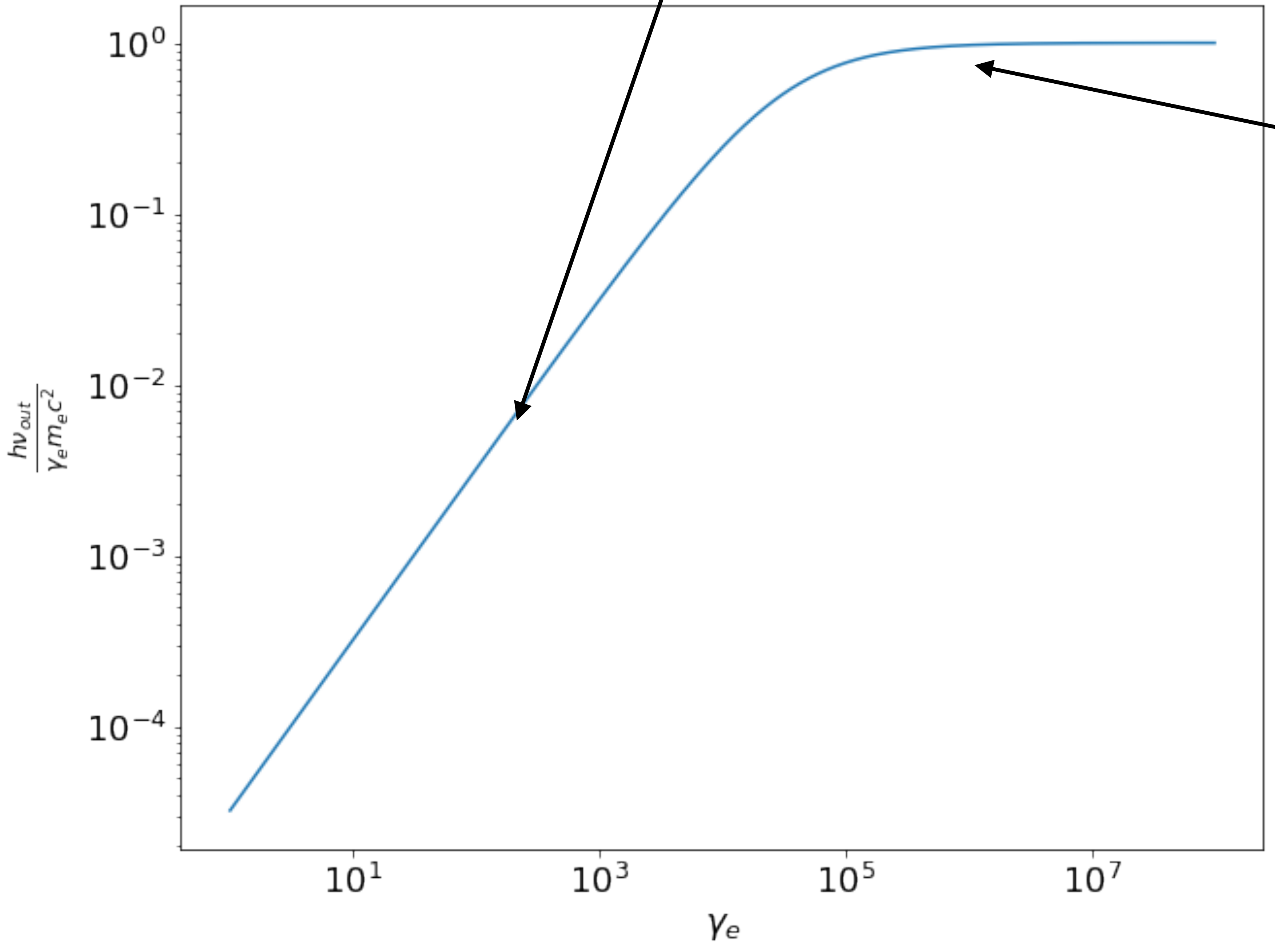
$\nu_p^{\ IC} / \nu_p^{\ S} \sim (4/3)\ \gamma_p^2$

KN regime

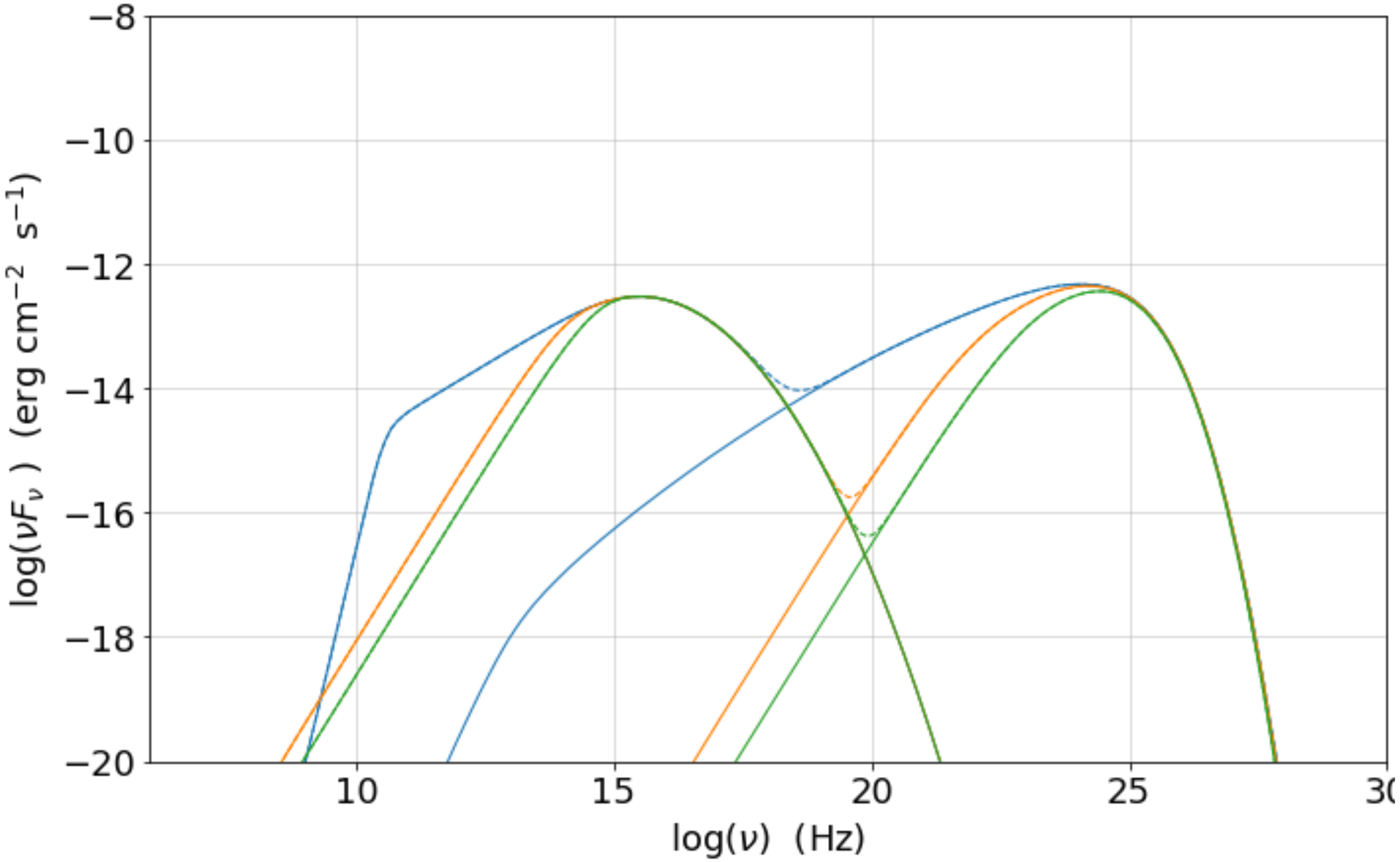
$\epsilon' \geq m_e c^2$

$\nu_p^{\ IC} / \nu_p^{\ S} \sim \gamma_p$

$h\nu_p^{\ IC} \sim m_e c^2 \gamma_p$



Adding the IC emission SSC case



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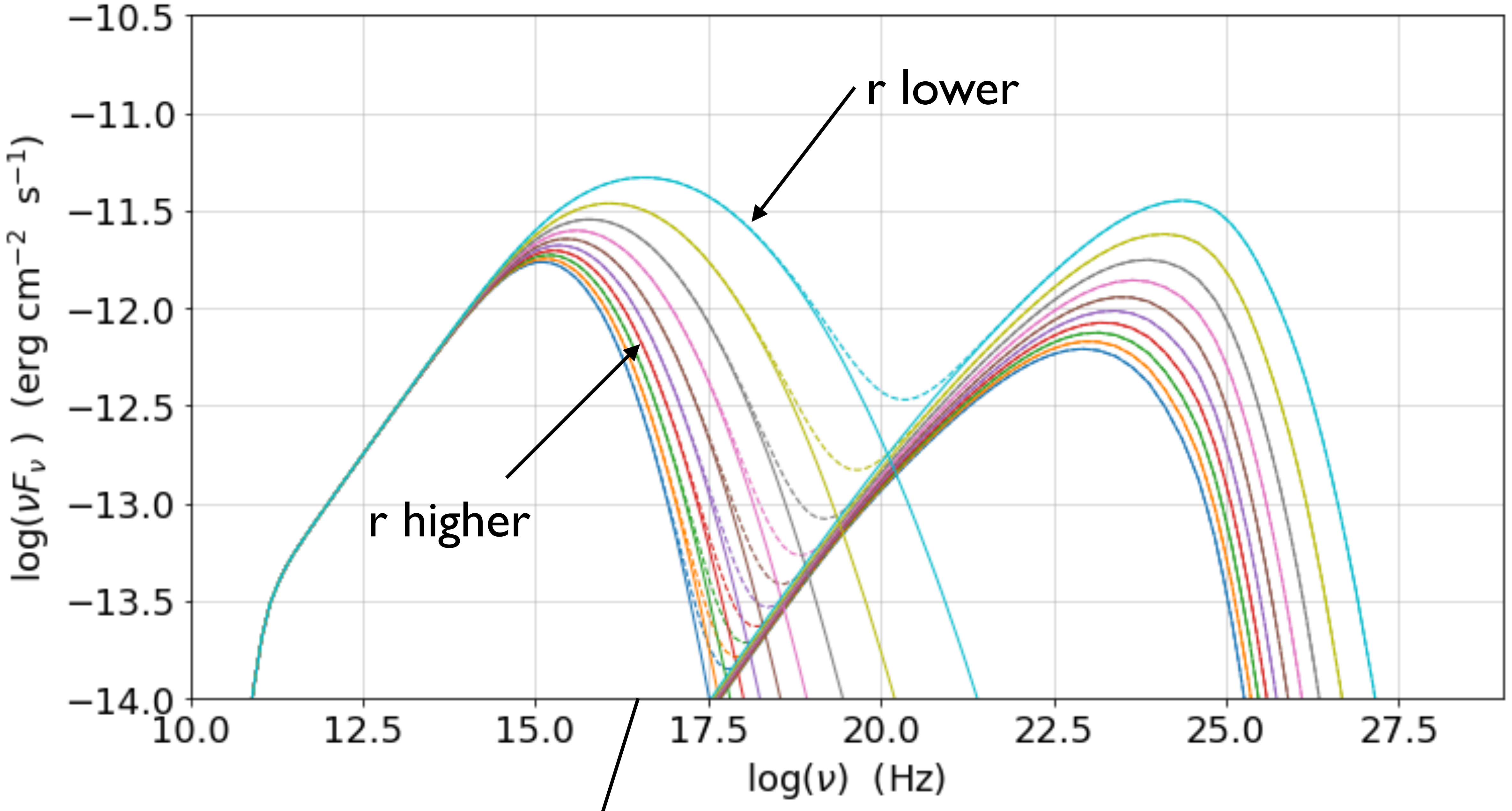
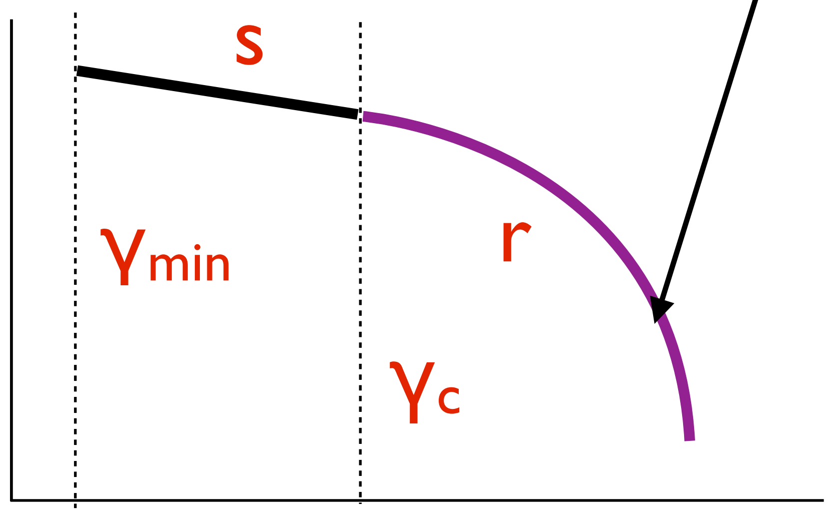
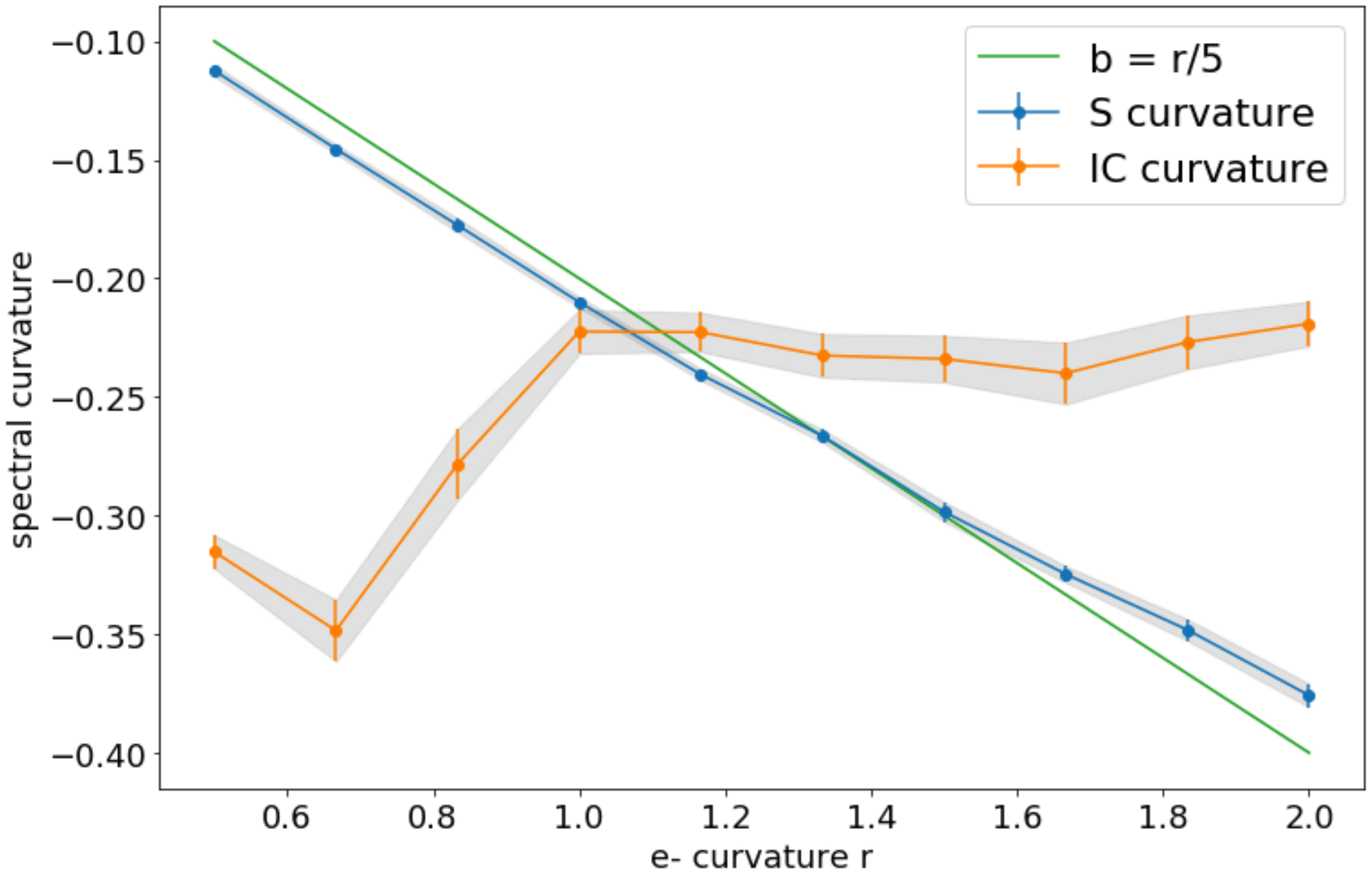
IC emission TH/KN regime and peak curvature



change *r*

Tutorial 2





obs. data → model

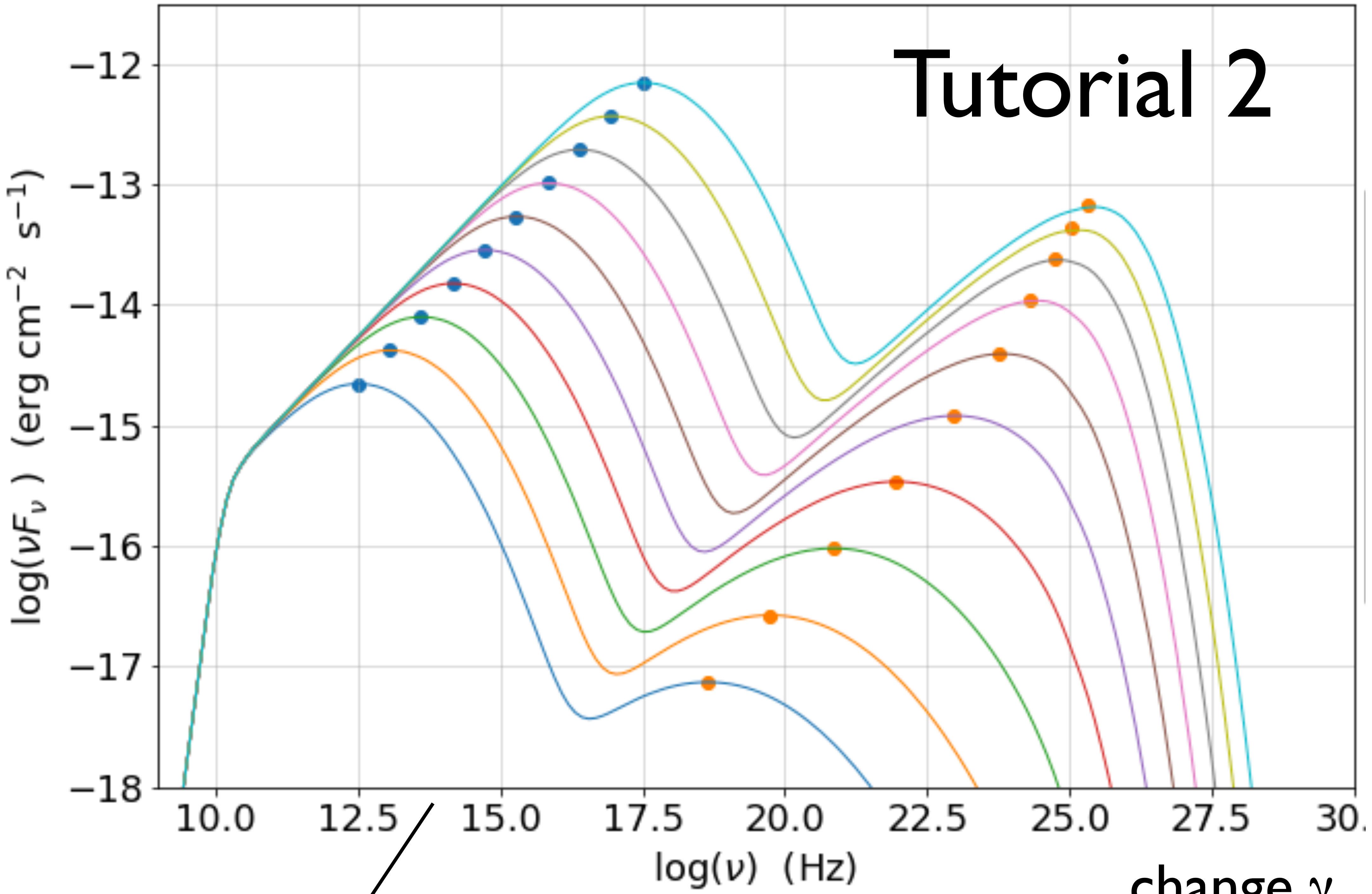
- spectral curvature

- KN regime
- r

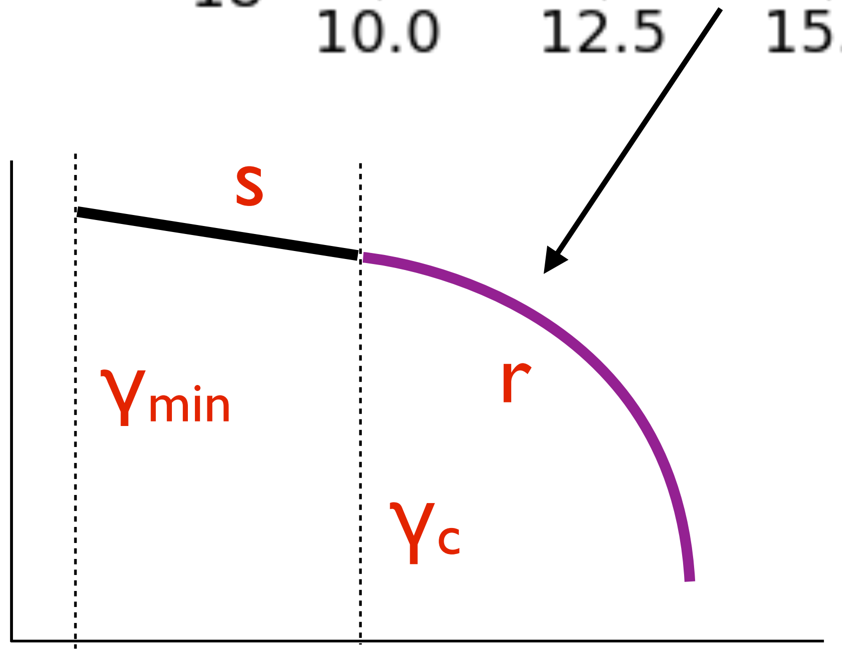
JetSeT

IC emission TH/KN regime and peak freq.

- $\nu_p^{IC} / \nu_p^{S} \sim (4/3)\, \gamma_p^2 \equiv \gamma^2{}_p^{IC}$ is true only in TH regime



change γ_c



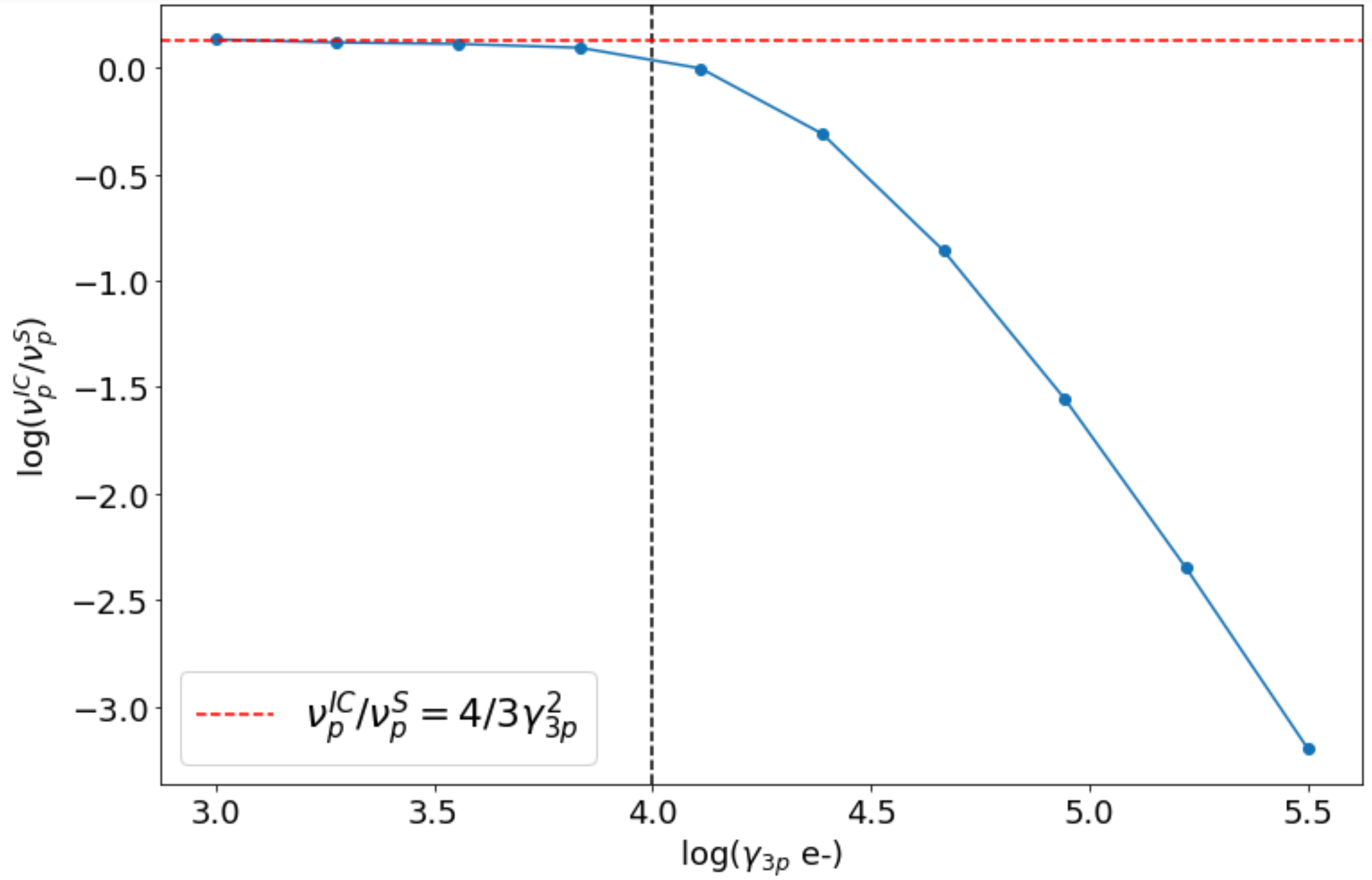
obs. data

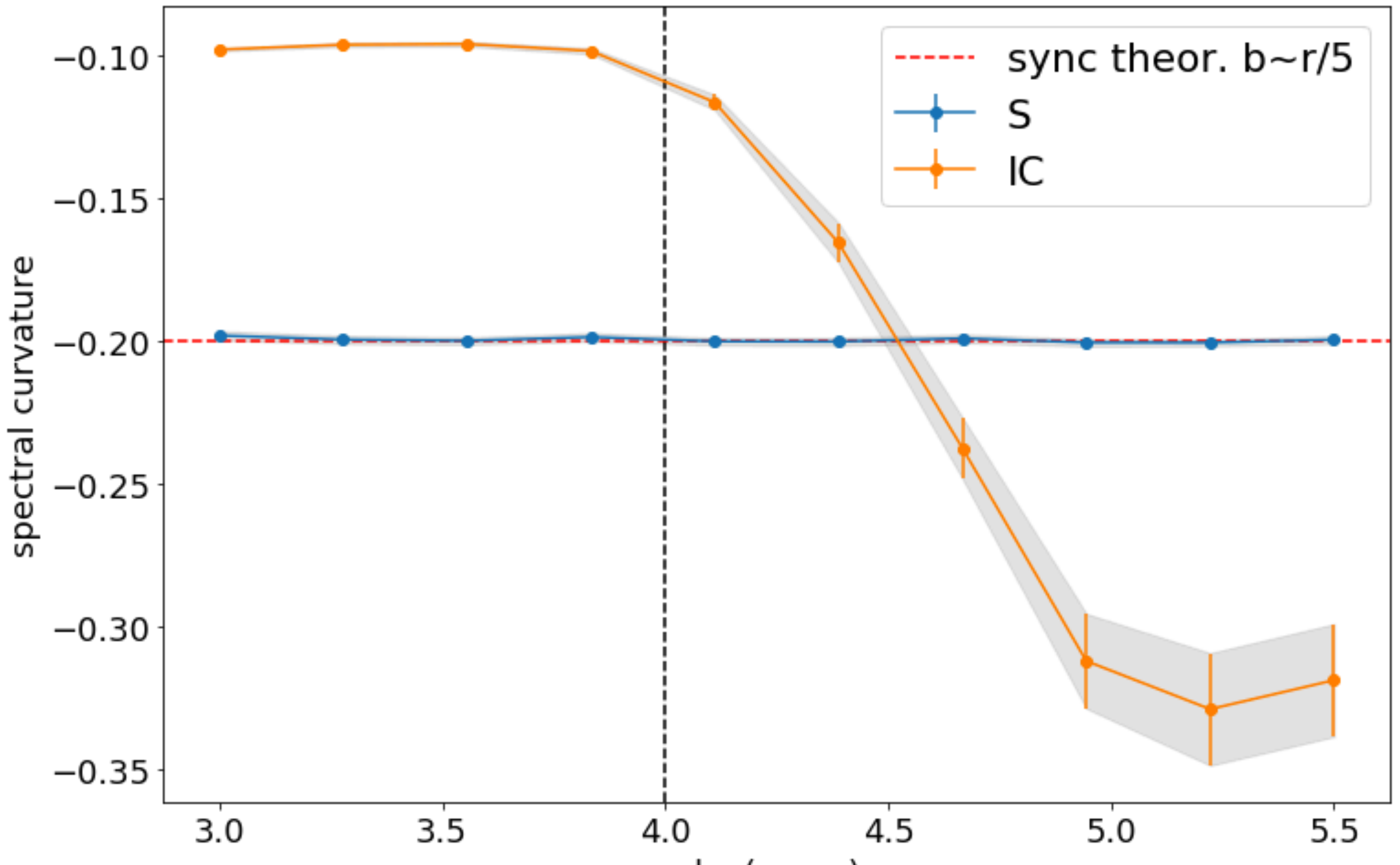
- spectral curvature
- peak freq.

→

model

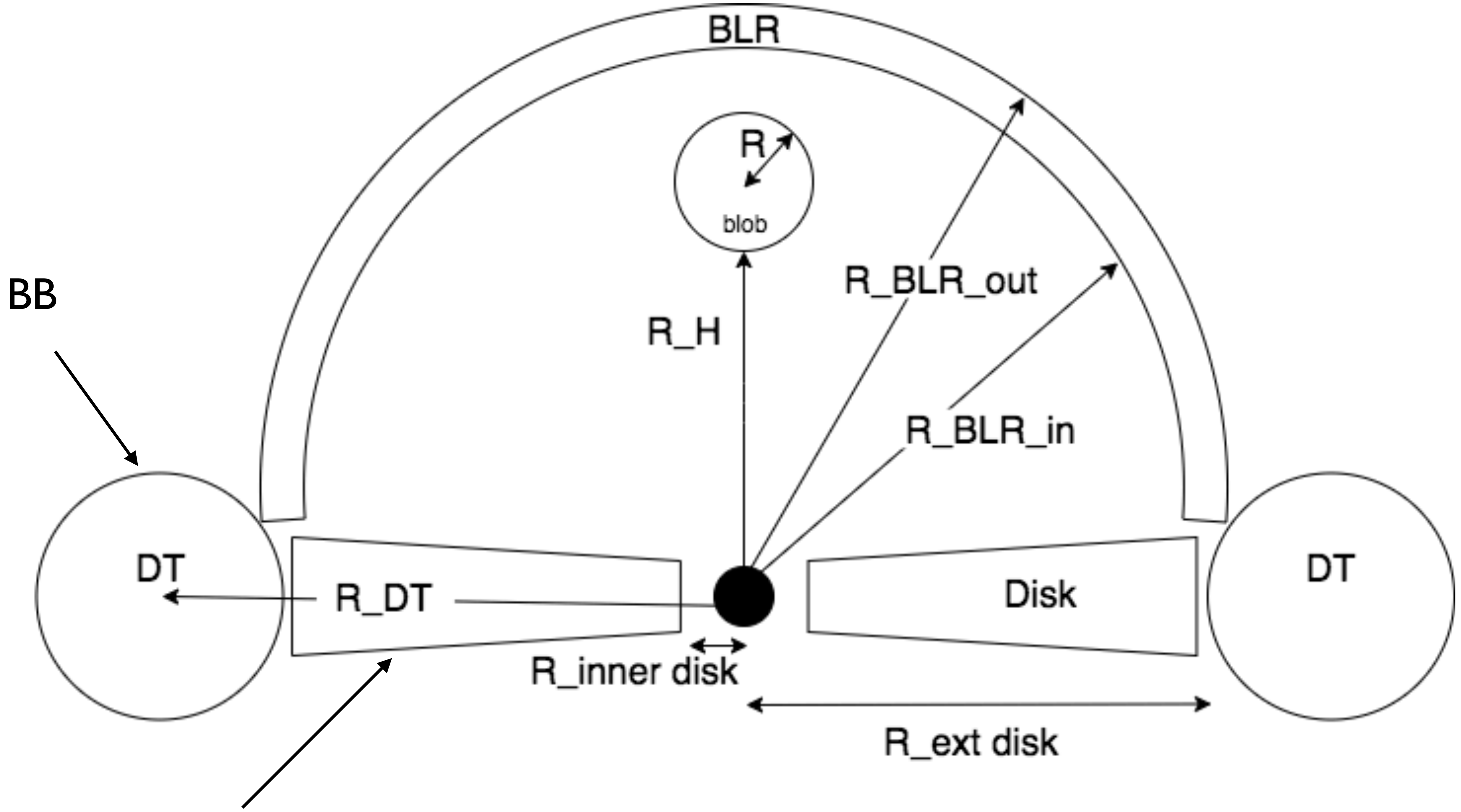
- KN regime
- γ_p, γ_c



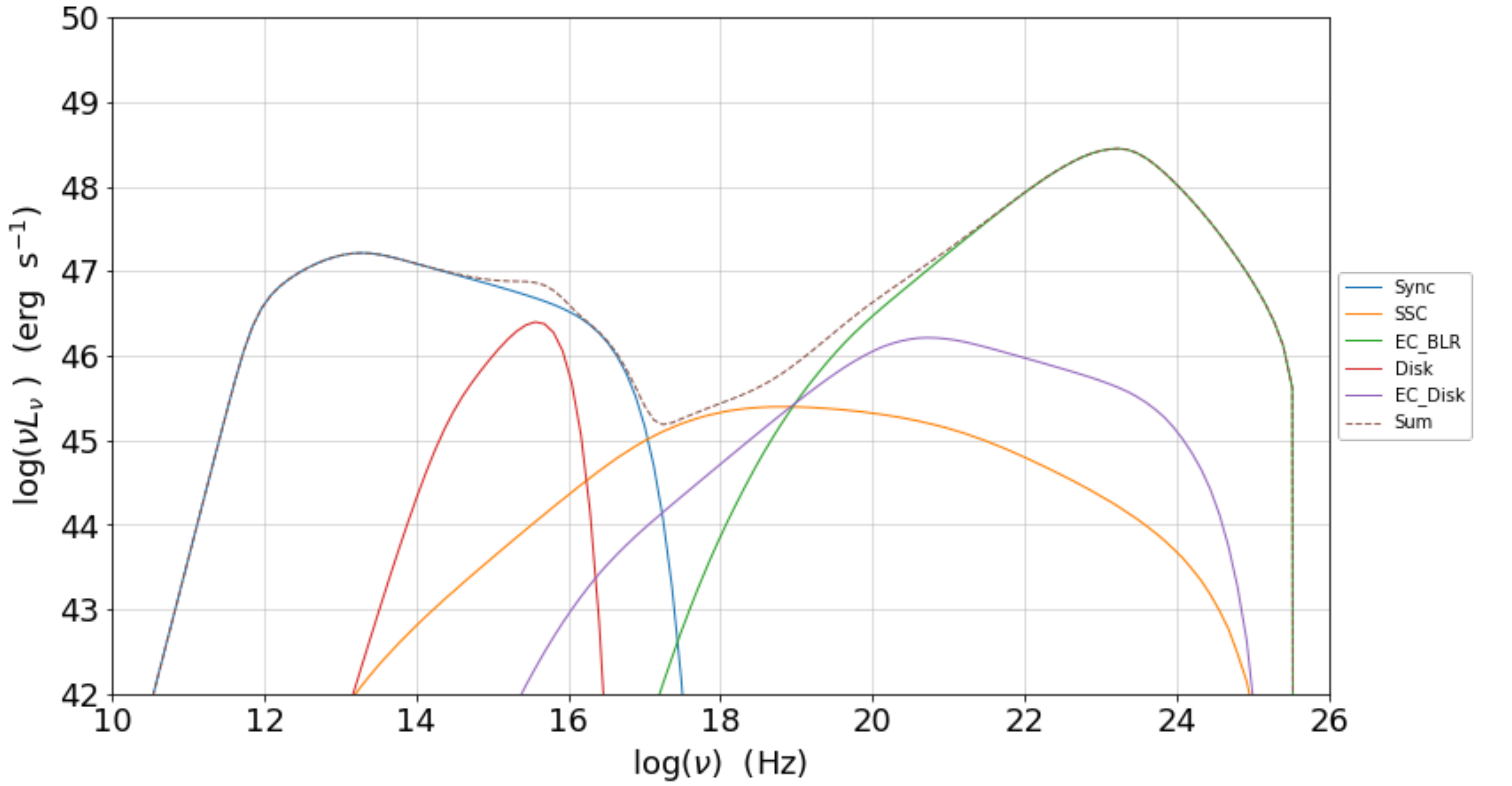


Extra slides

External Compton Scenario



multi T BB $T(R) \approx T_*(R/R*)^{-3/4}$



Transformation of the radiative fields

$$\epsilon^{-3} I_\epsilon \text{ and } \epsilon^{-2} j(\epsilon, \Omega)$$

$$\frac{u(\epsilon, \Omega)}{\epsilon^3} = \frac{u'(\epsilon', \Omega')}{\epsilon'^3} = inv.$$

$$d\Omega' = \frac{d\Omega}{\Gamma^2(1-\beta_\Gamma \cos\theta)^2} = \frac{2\pi d\mu}{\Gamma^2(1-\beta_\Gamma \mu)^2}$$

$$u_\nu(\nu, \Omega) = \frac{I_\nu}{c} \quad (erg\ cm^{-3}\ Hz^{-1}\ sterad^{-1})$$

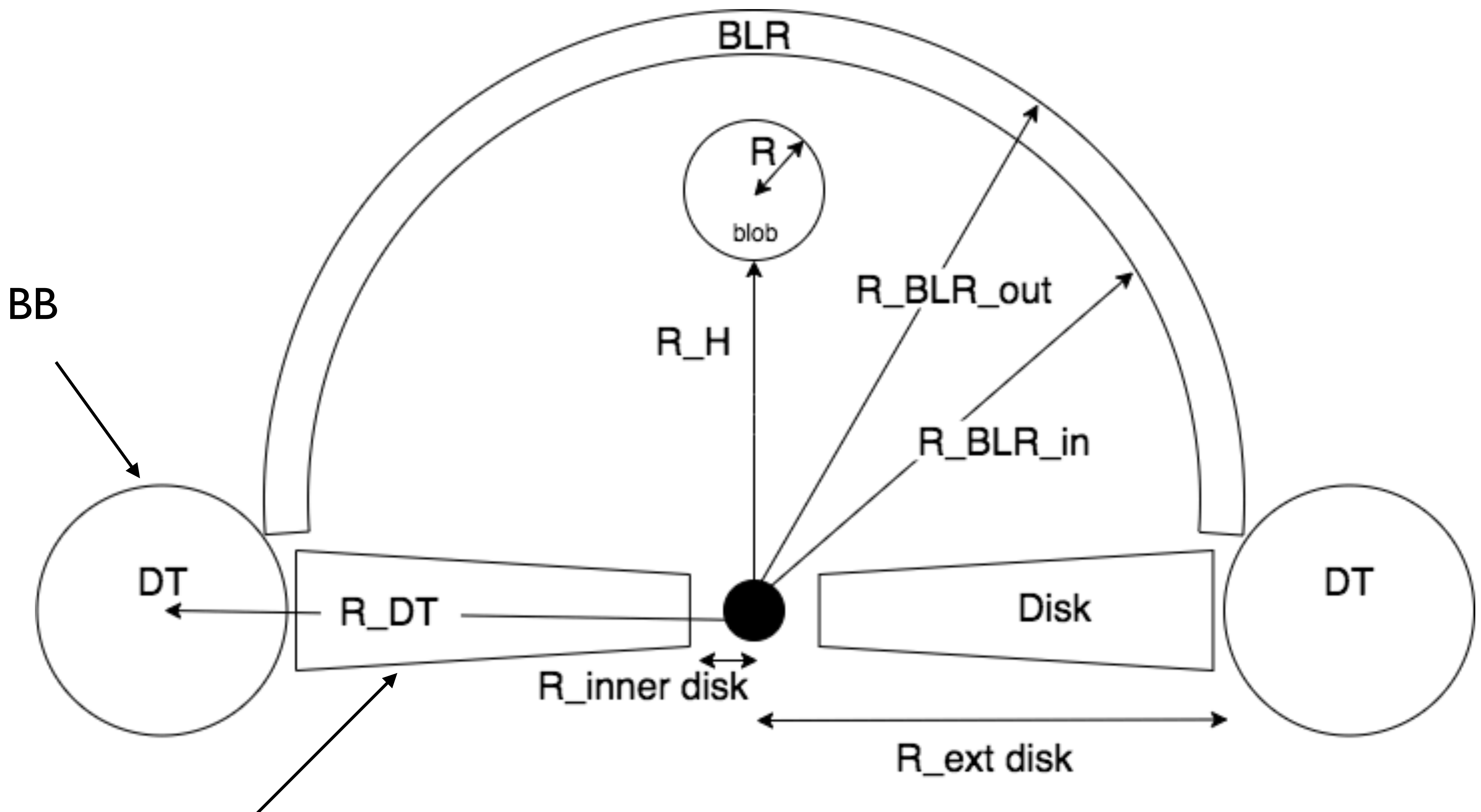
$$I_{\nu'} = \frac{1}{4\pi}\int d\Omega'\, \delta^3 I_{\nu=(\nu'/\Gamma)}$$
$$= \Gamma\tau \frac{L_{\mathrm{nuc}}}{4\pi R^2} f_{\nu=(\nu'/\Gamma)}(T_{\mathrm{ext}})$$

$$u'_{ext} \simeq \Gamma^2 u_{ext}$$

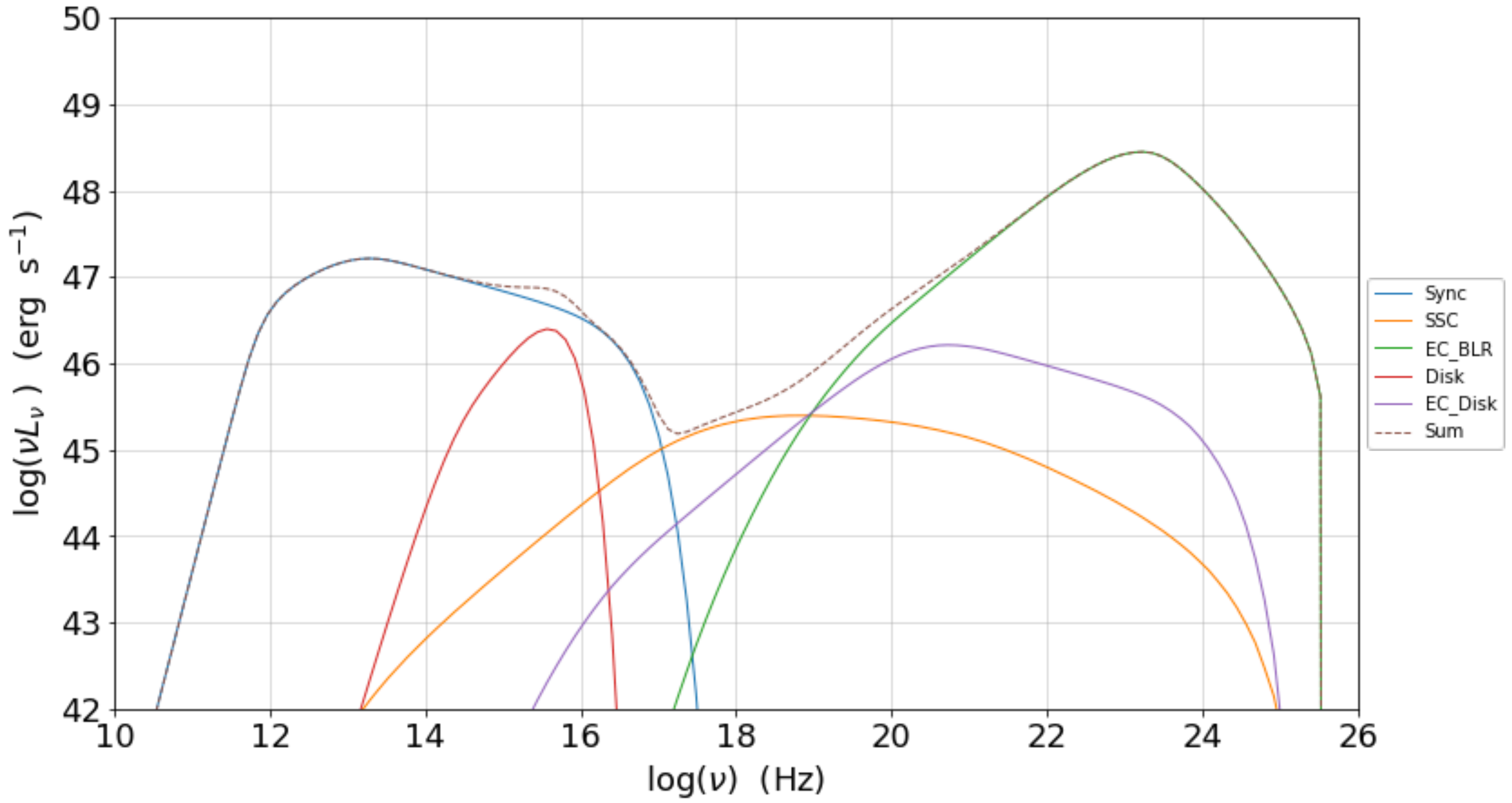
$$L_{ERC} \simeq \Gamma^6 U_{ext}$$

$$\eta = \frac{\dot{\gamma}_{IC}}{\dot{\gamma}_{sync}} = \frac{U_{ph}}{U_B}$$

External Compton Scenario



$T(R) \approx T_*(R/R*)^{-3/4}$



Transformation of the emissters distribution

$$n'(\gamma', \Omega') = n'(\gamma')/4\pi$$ Isotropic emitters distr.

$$n/\gamma^2$$ Invariant.

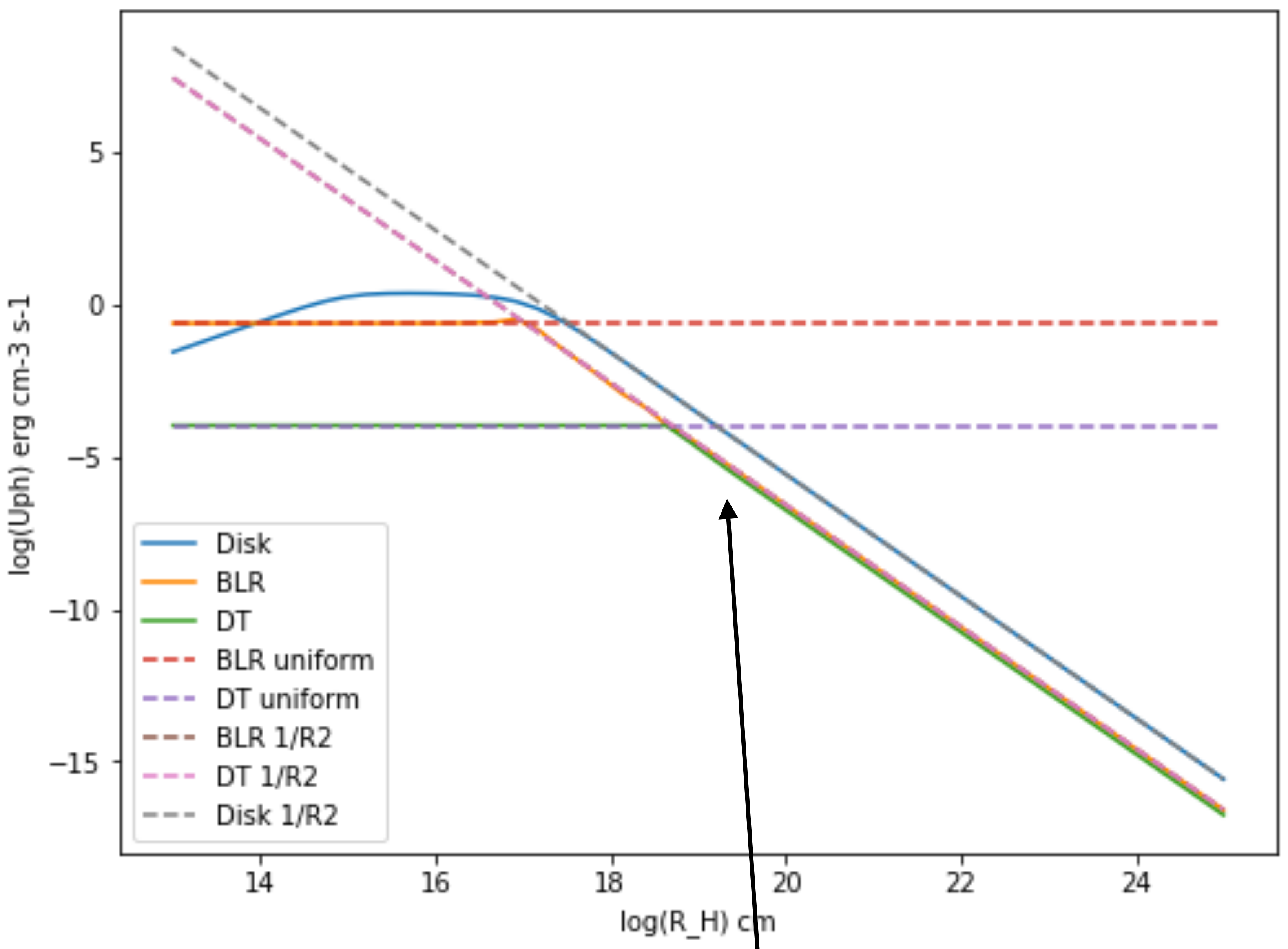
$$n(\gamma, \Omega) = \delta^2 n'(\gamma', \Omega) = \delta^2 n'(\gamma')/4\pi$$

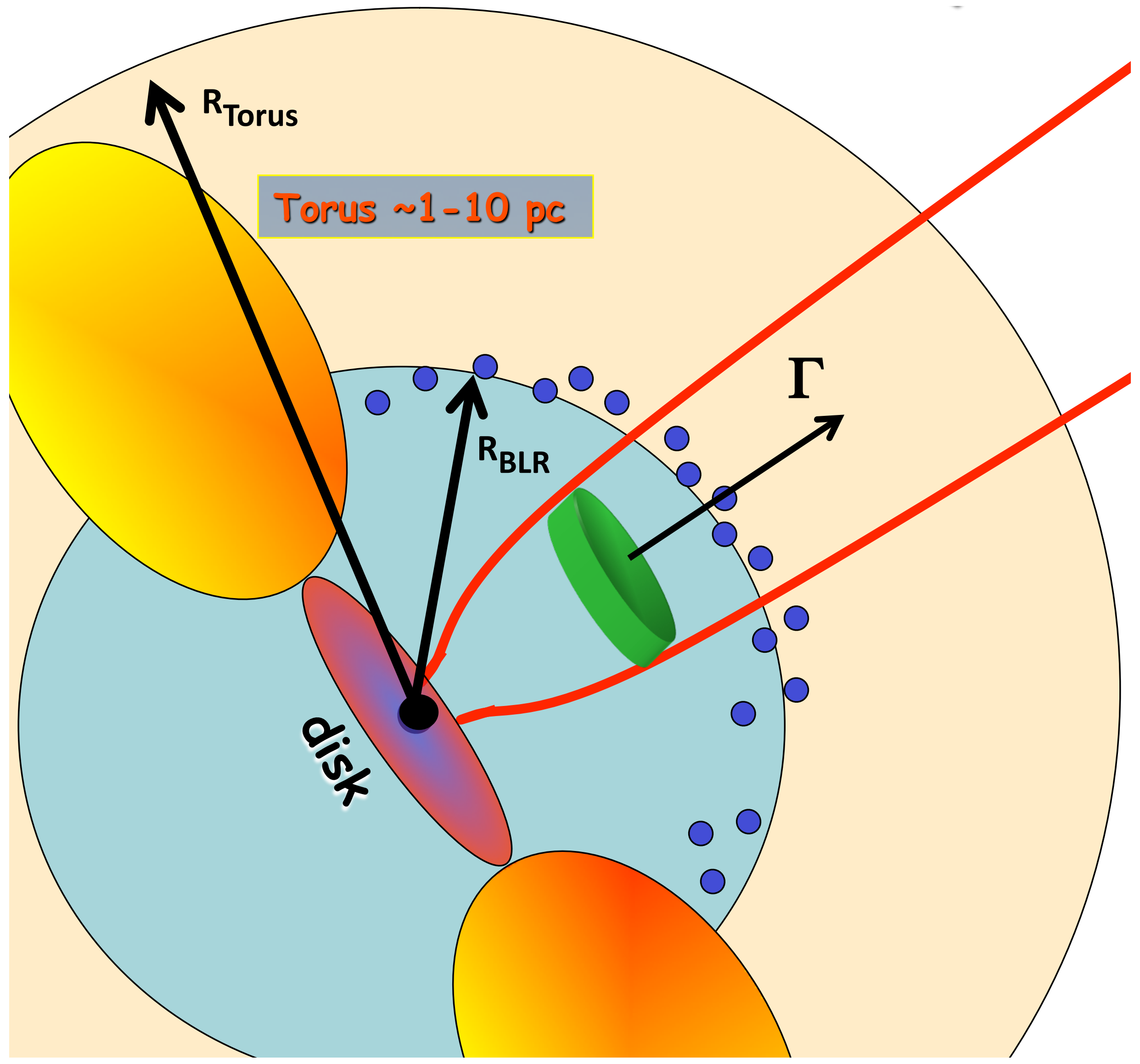
$$\gamma = \delta\gamma'$$

$$V = V'\delta$$

External Compton Scenario

external photon fields in the disk rest frame





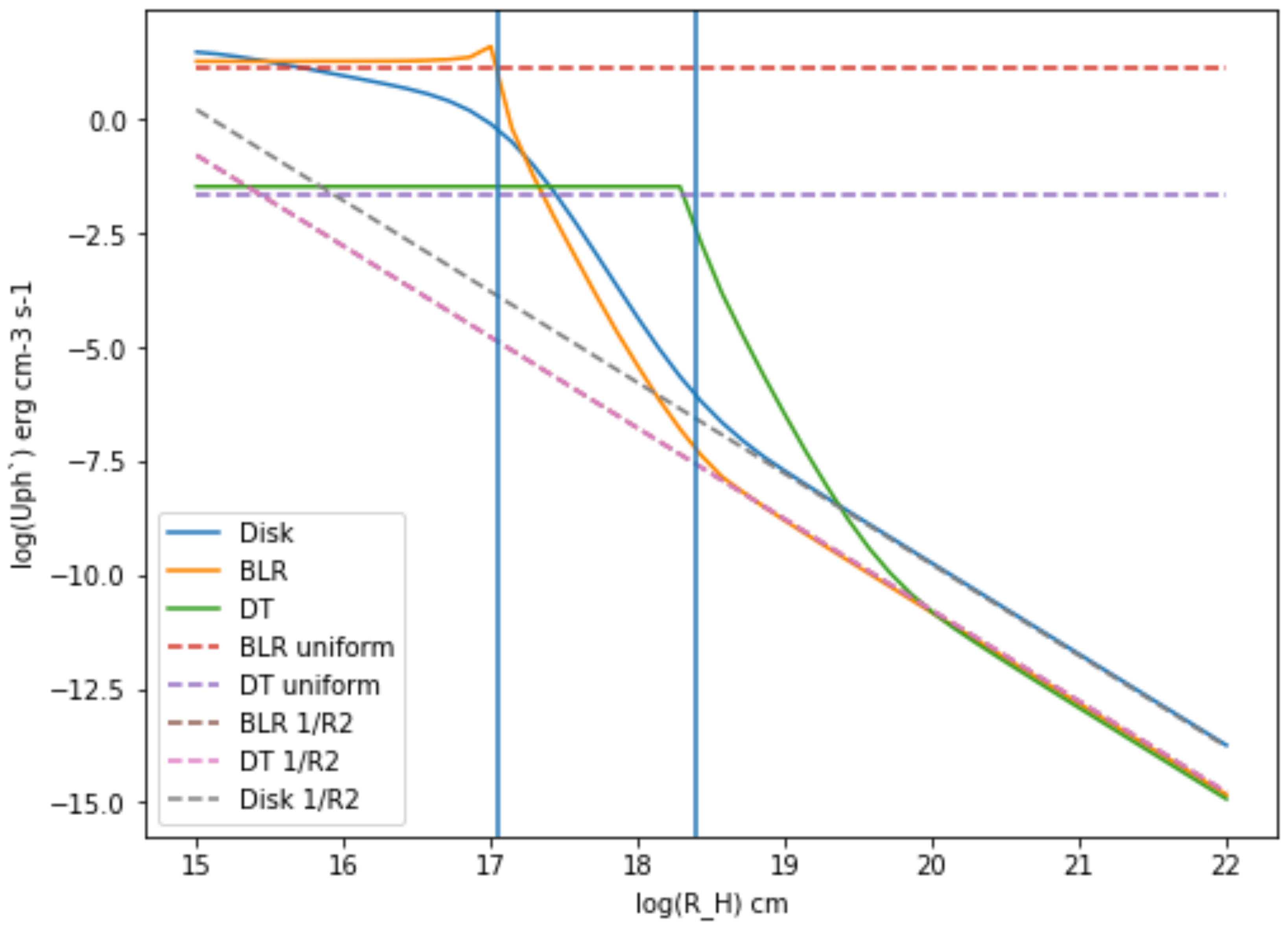
$$U_{ext} \simeq \frac{L_d}{R^2{}_{ext} c}$$

$$R_{ext} \simeq L_d^{1/2}$$

~0.1 erg/cm^3 BLR

~0.01 erg/cm^3 DT

external photon fields in the blob rest frame

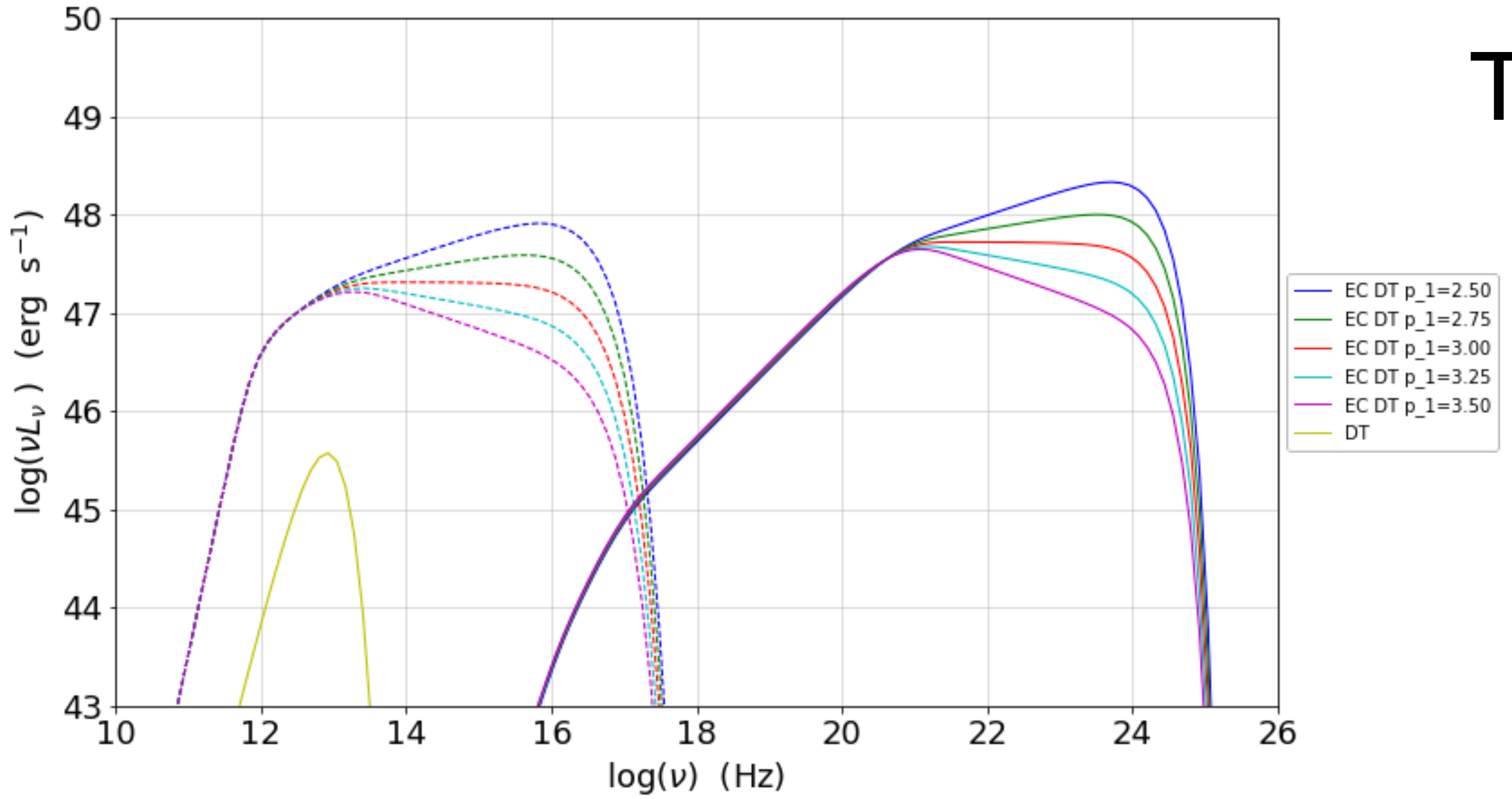


$$u'_{ext} \simeq \Gamma^2 u_{ext}$$

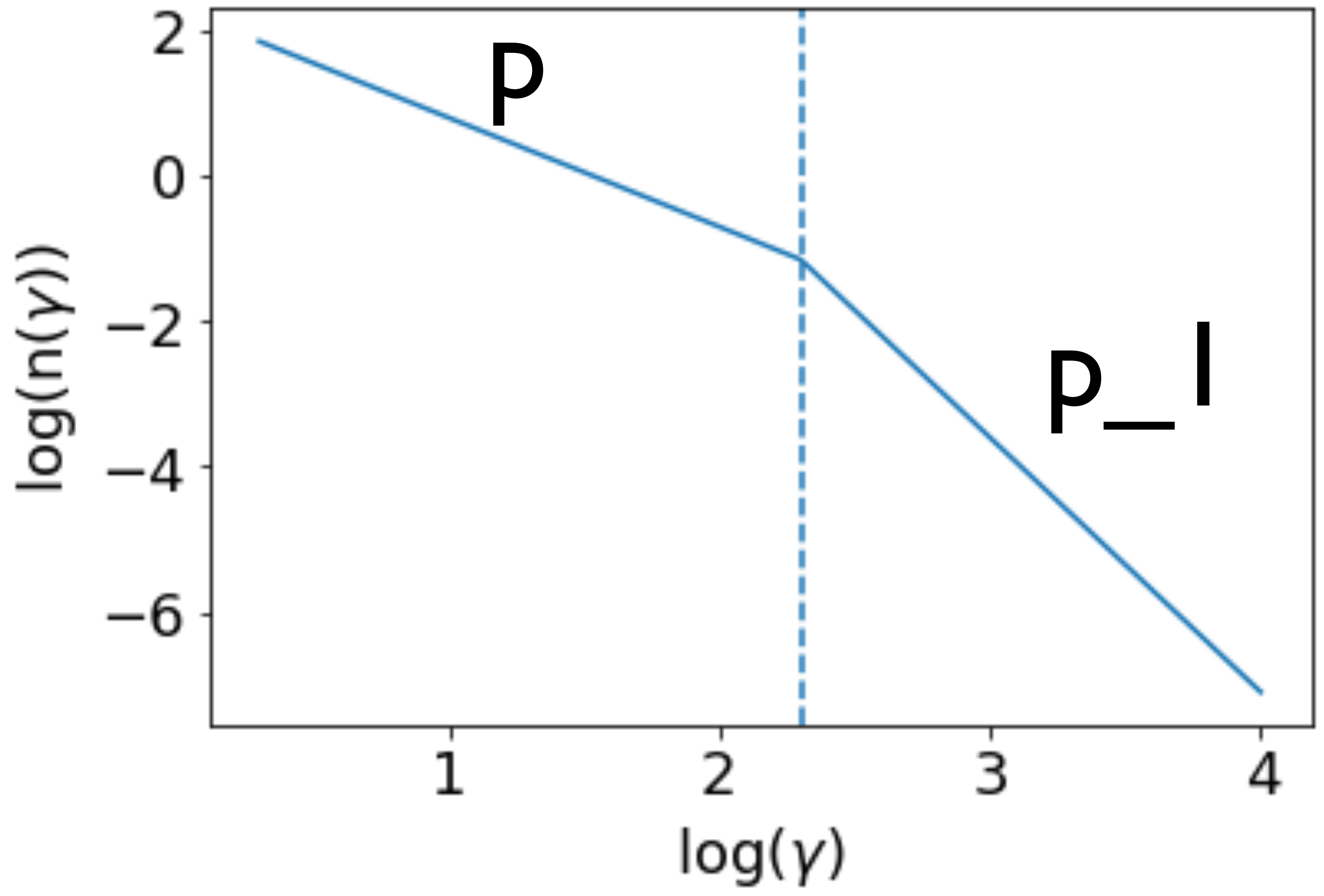
constant within the sphere

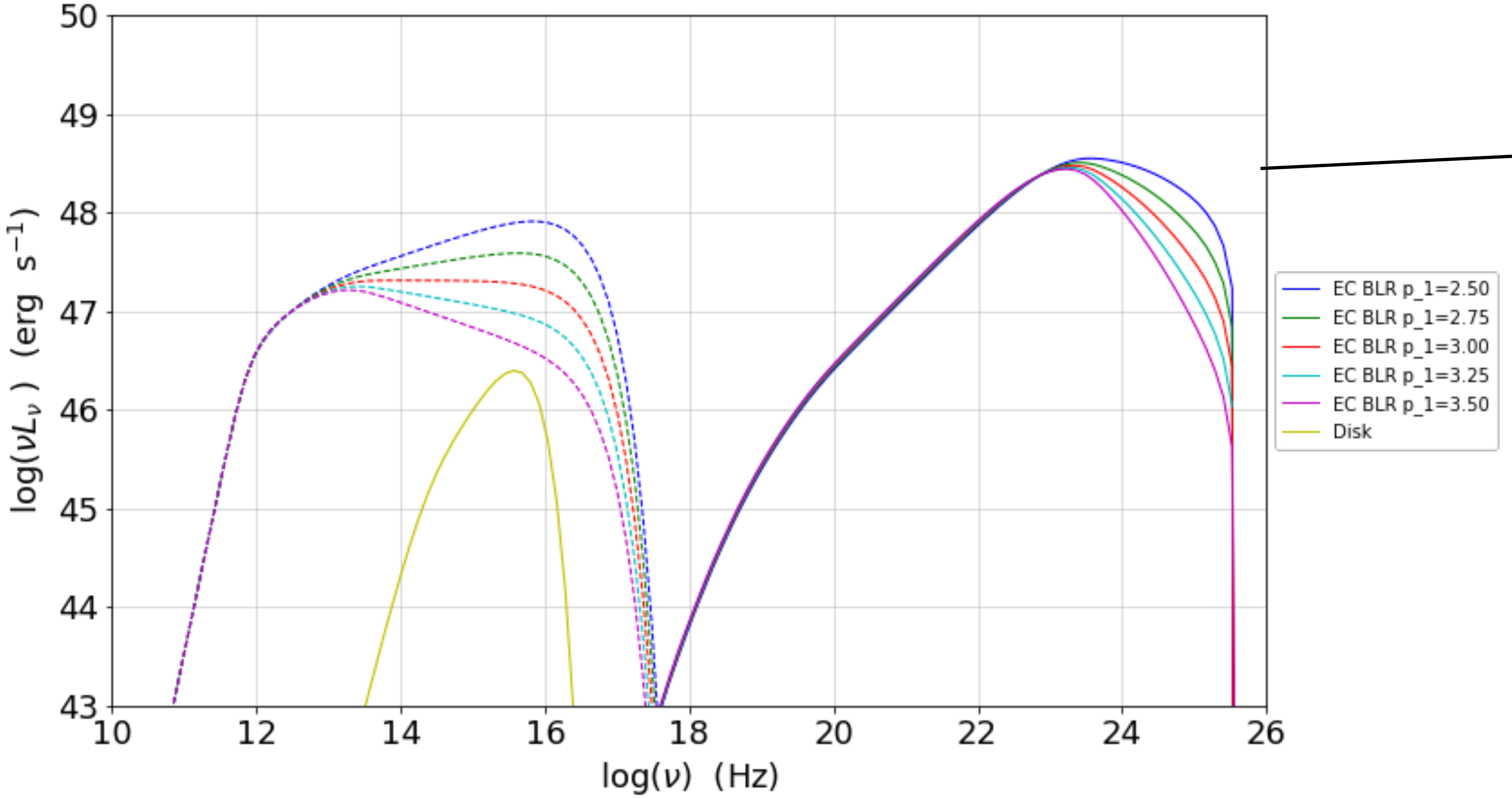
JetSeT

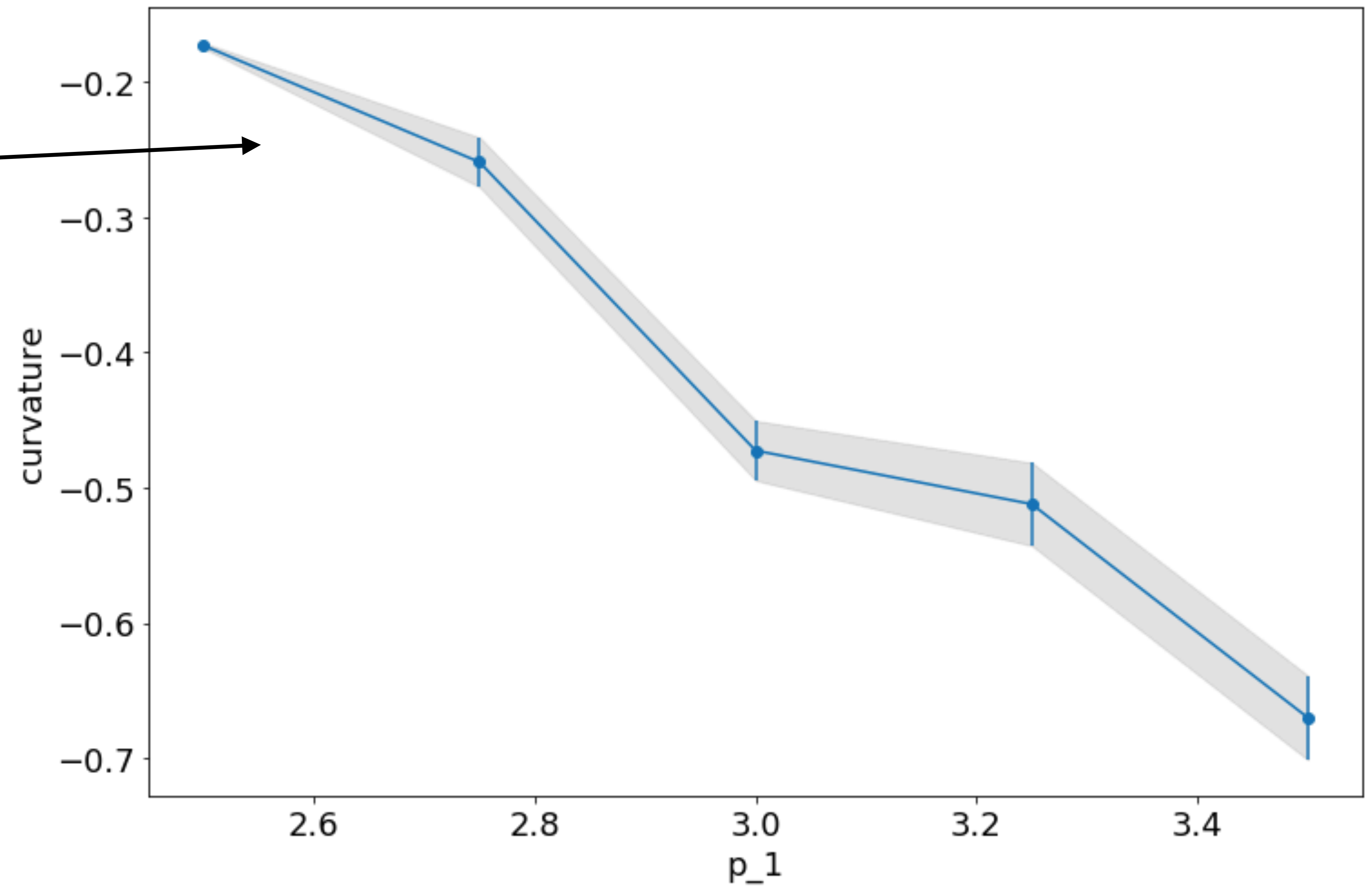
External Compton Scenario and TH/KN



Tutorial 3

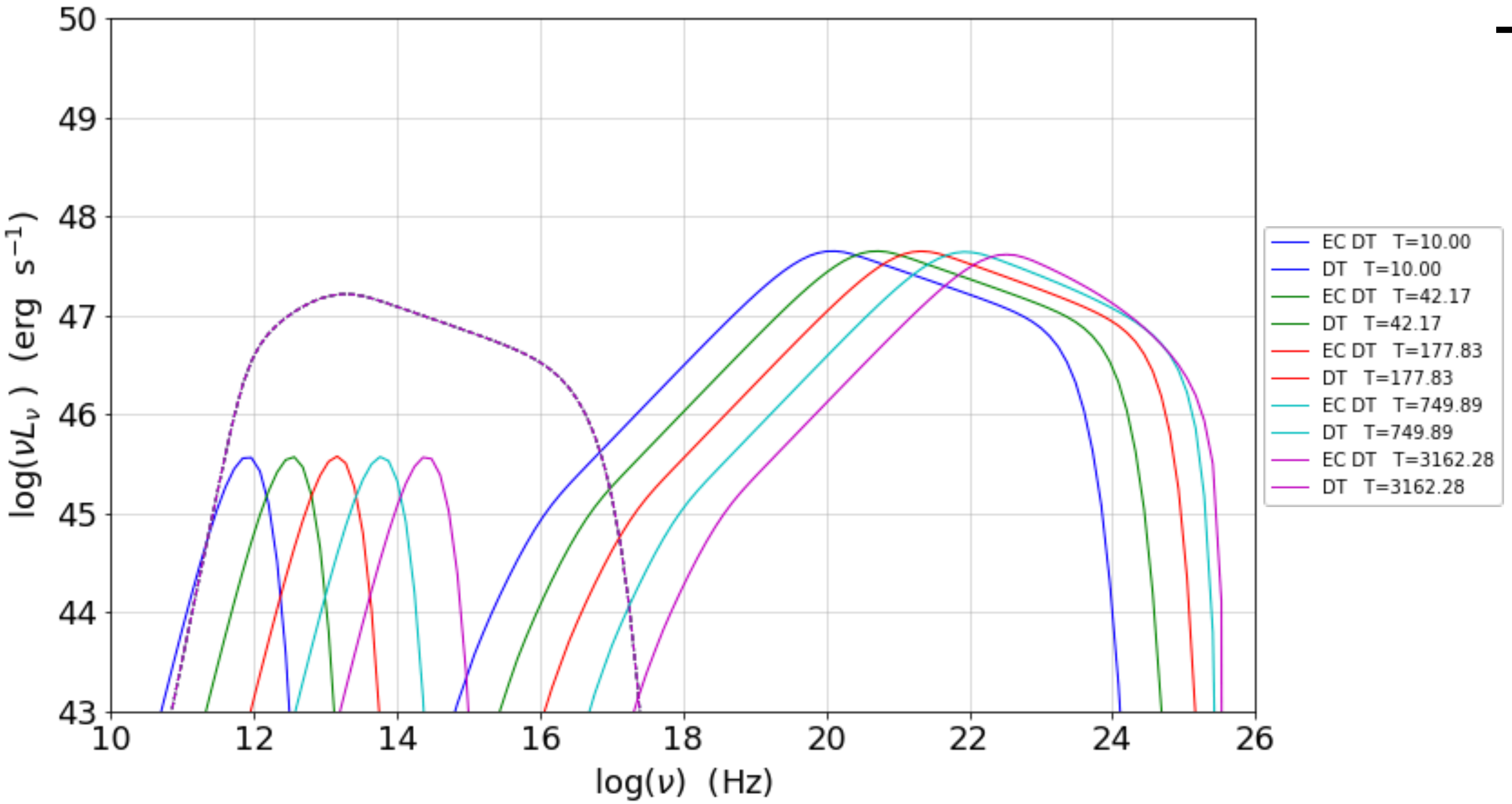






External Compton Scenario and external seed photons energy

$\nu_{p\,EC} \sim (4/3)\,\gamma^2\,\nu''_{p\,ext}\delta\Gamma\,/(1+z)$ $\nu'_{seed\text{-}IC} = \nu''_{p\,ext}\Gamma$



Tutorial 3

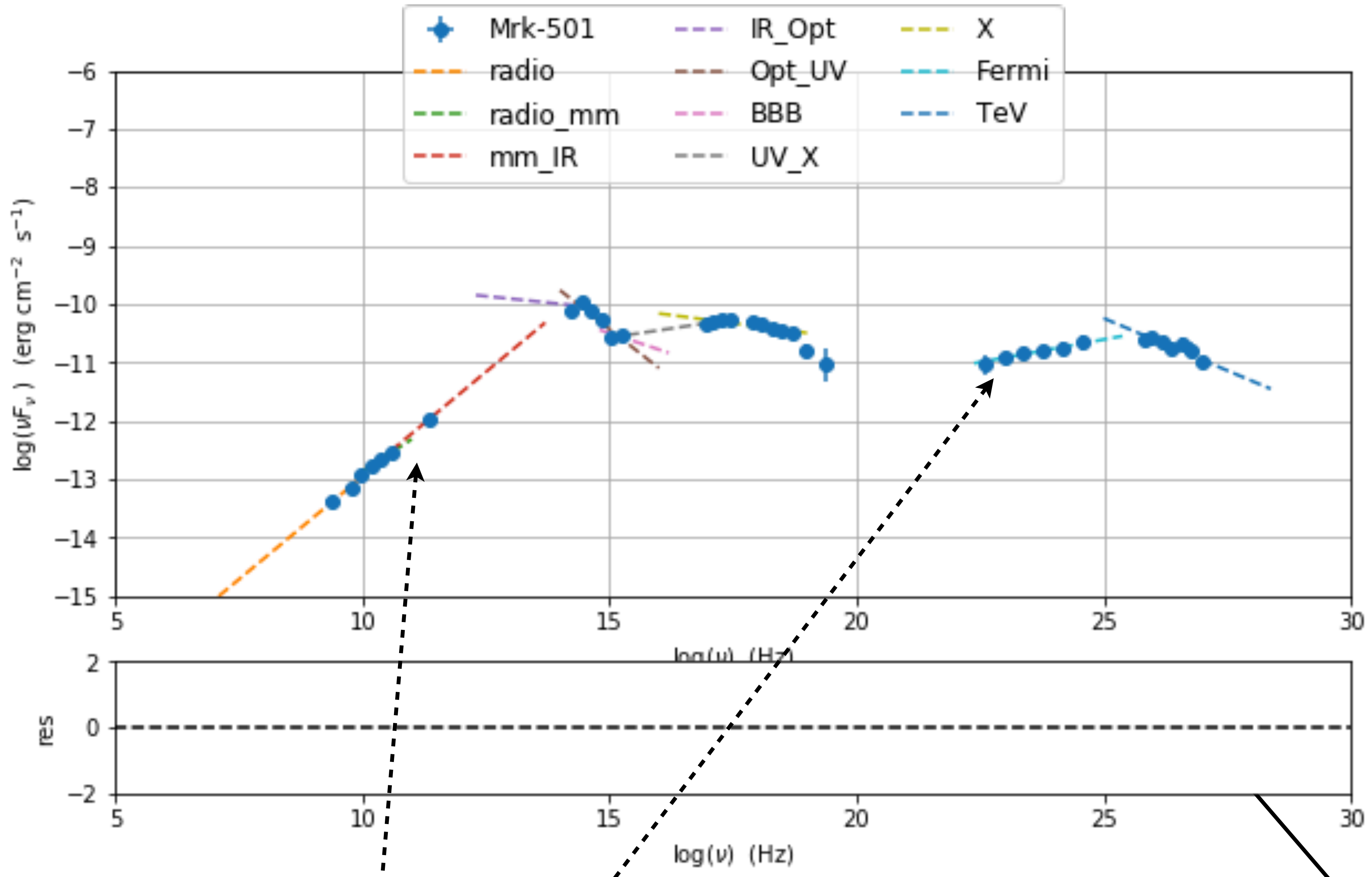
obs. data → model

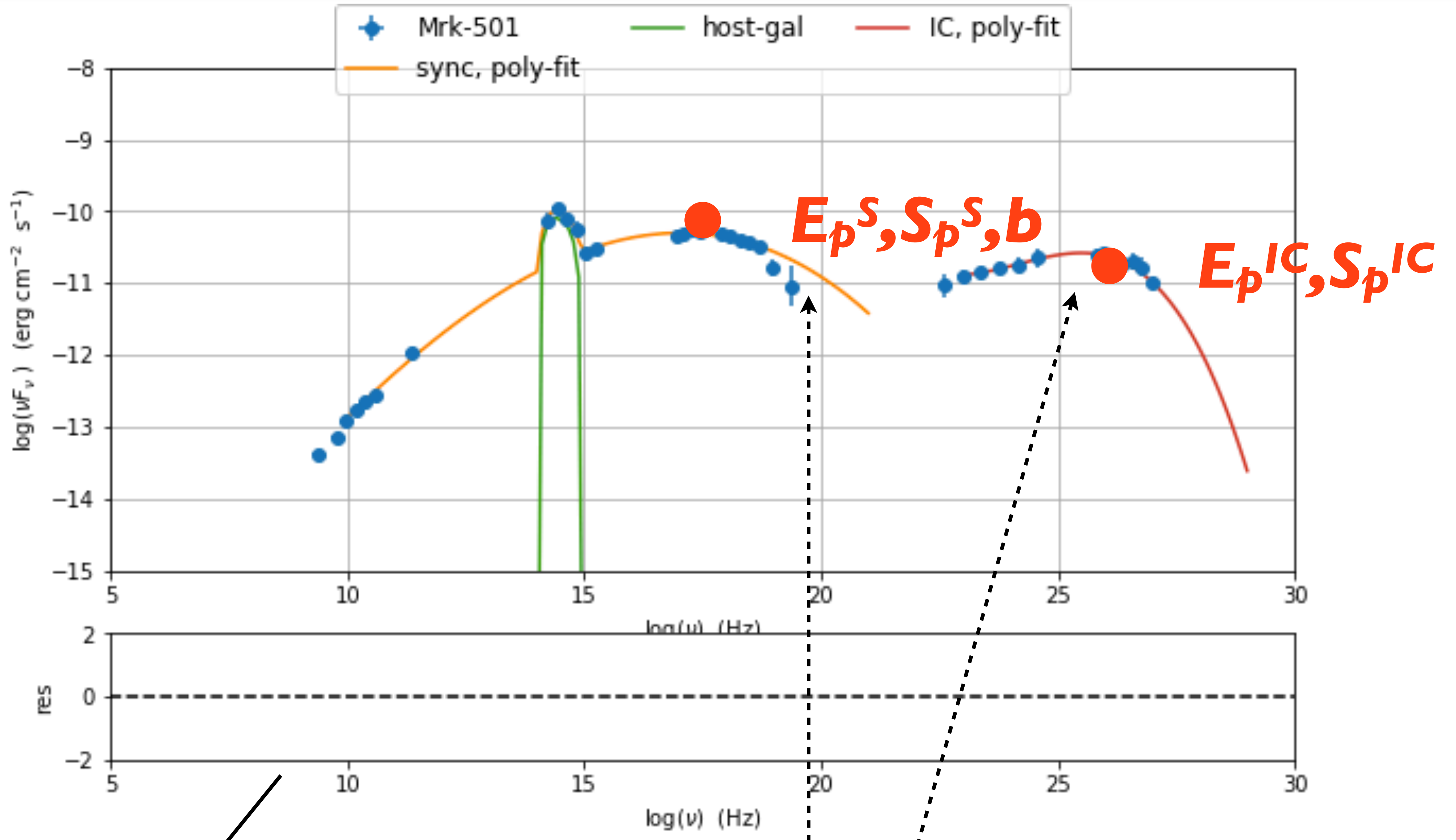
•peak freq. •T seed ext.

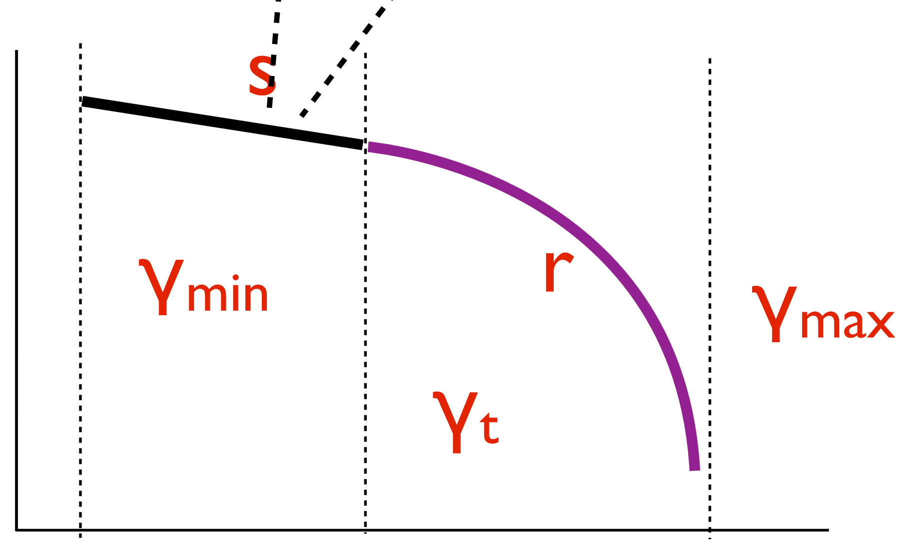
JetSeT

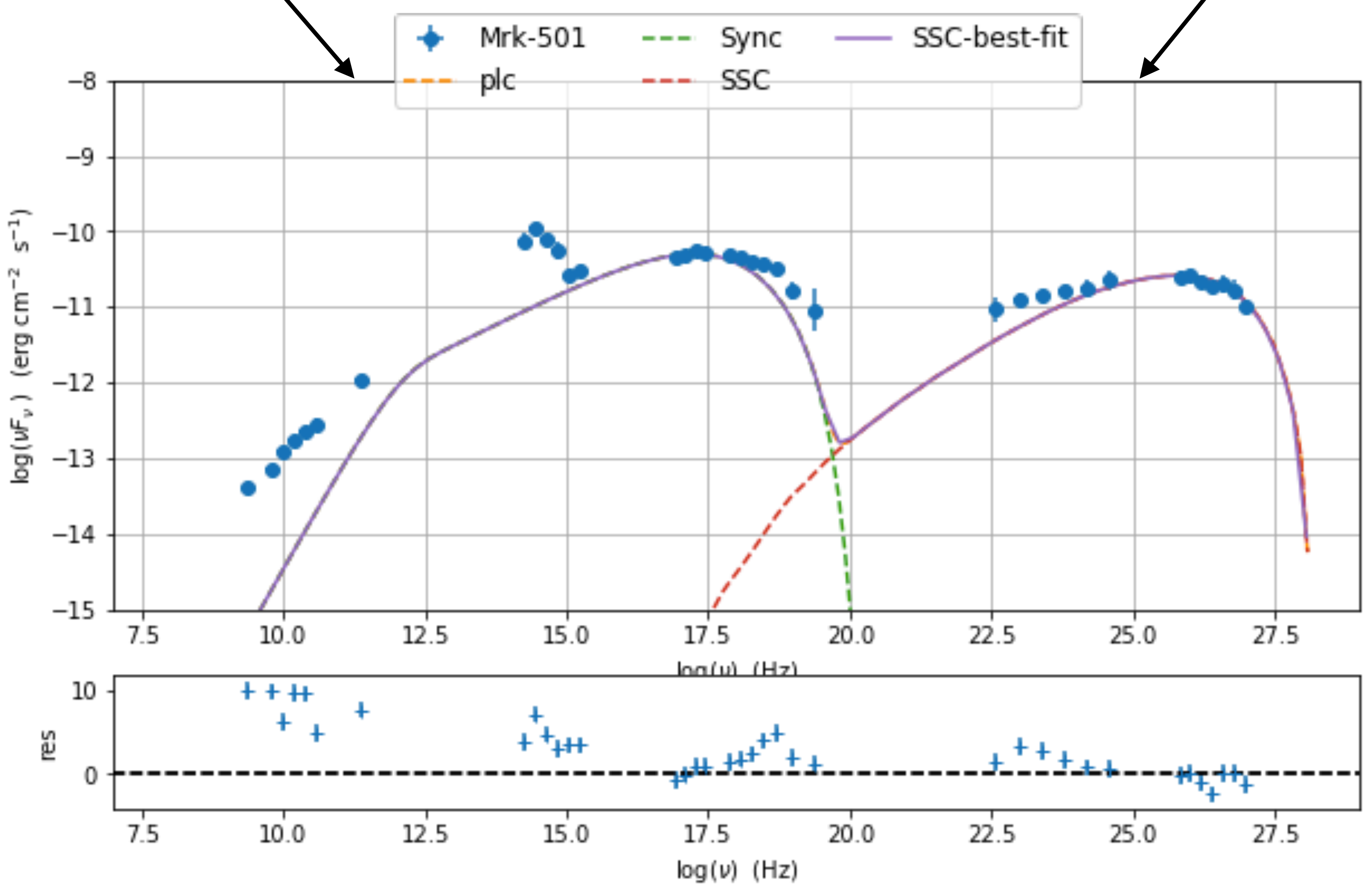
SED shaping and model constraining (Mrk 501)

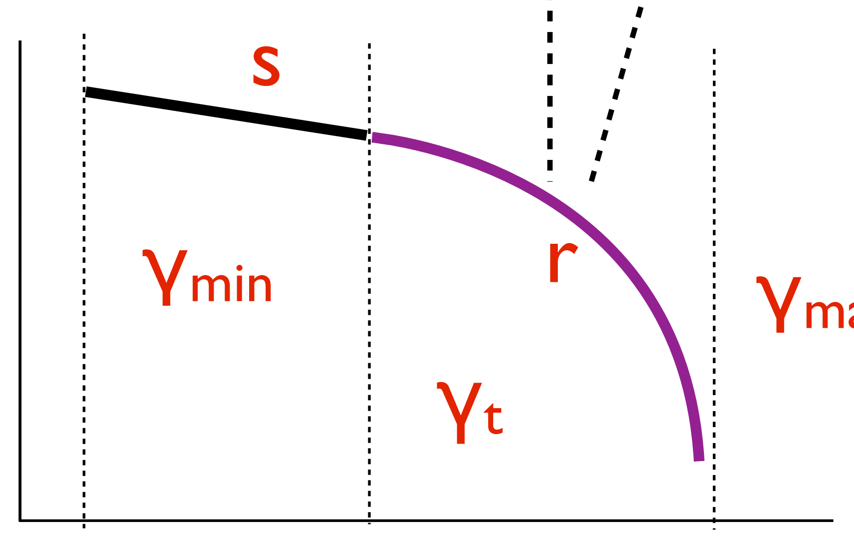
Tutorial 5





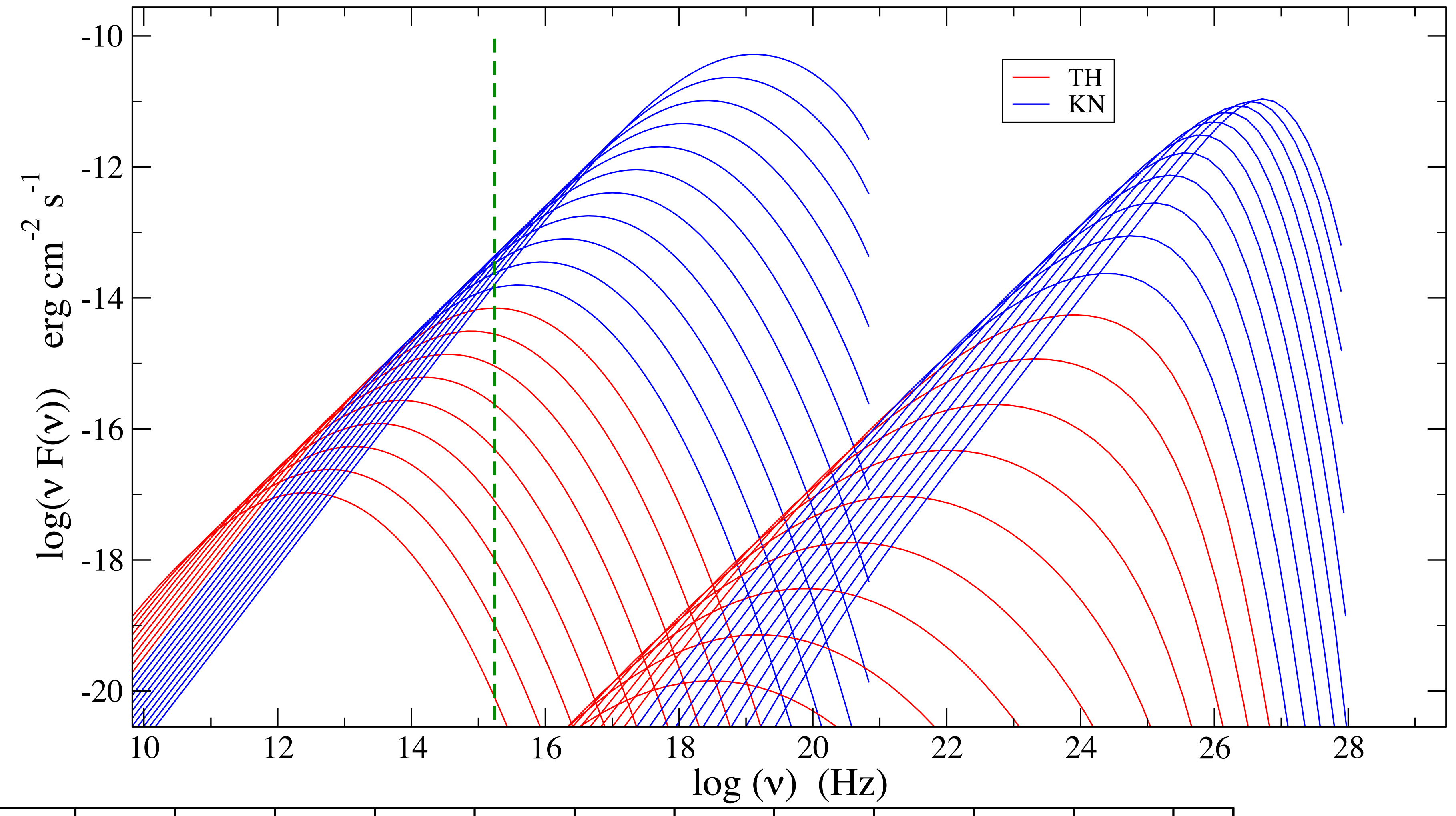


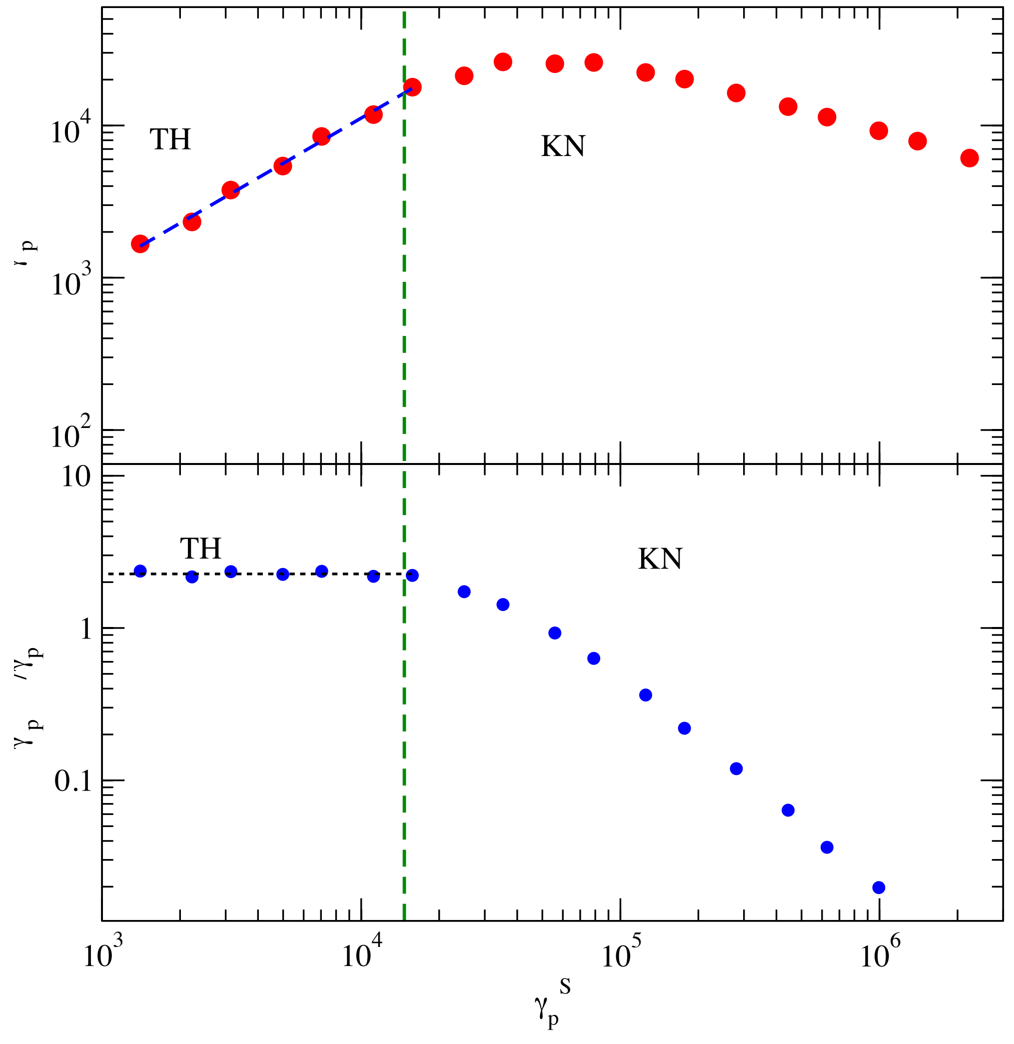


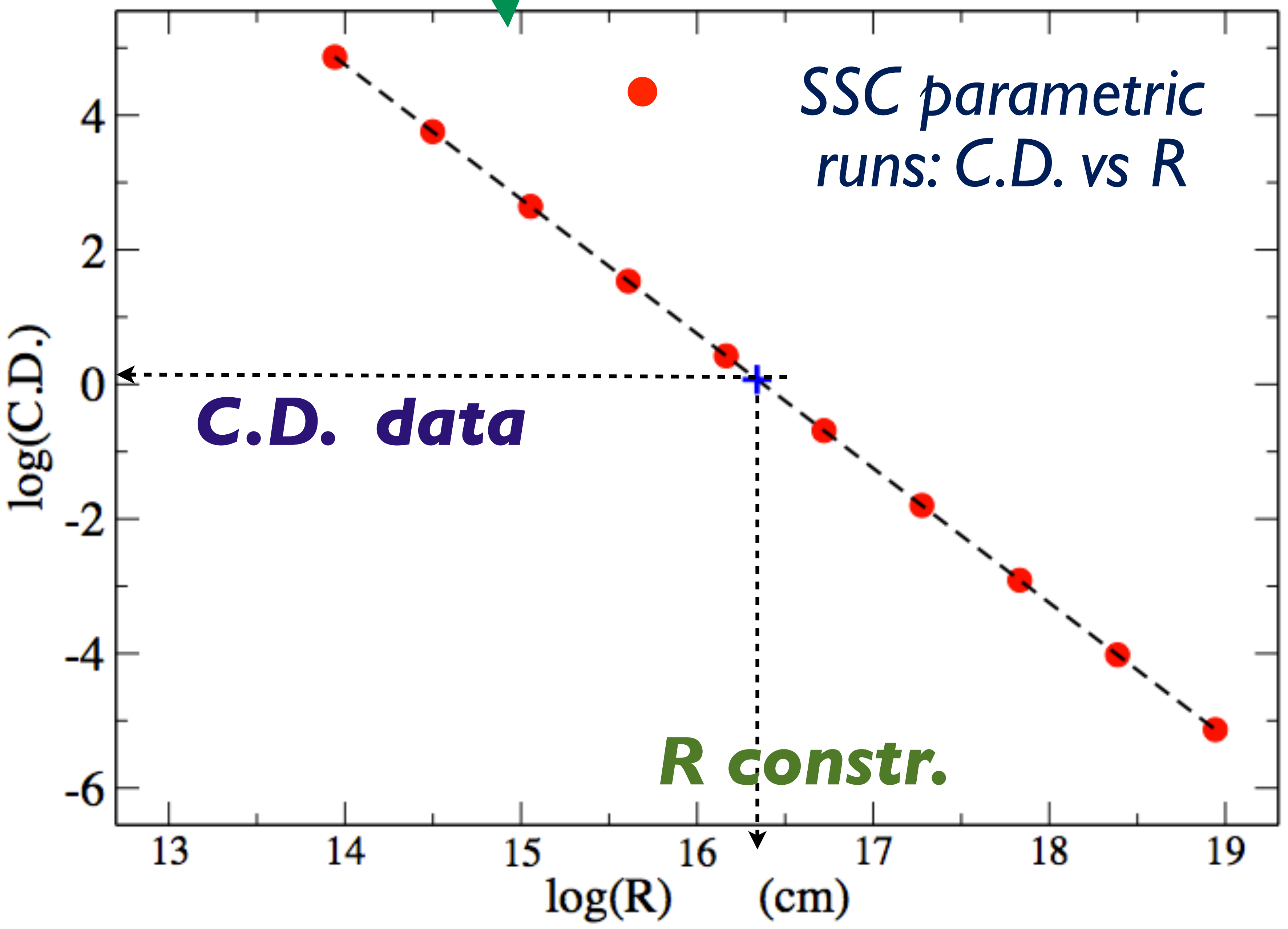


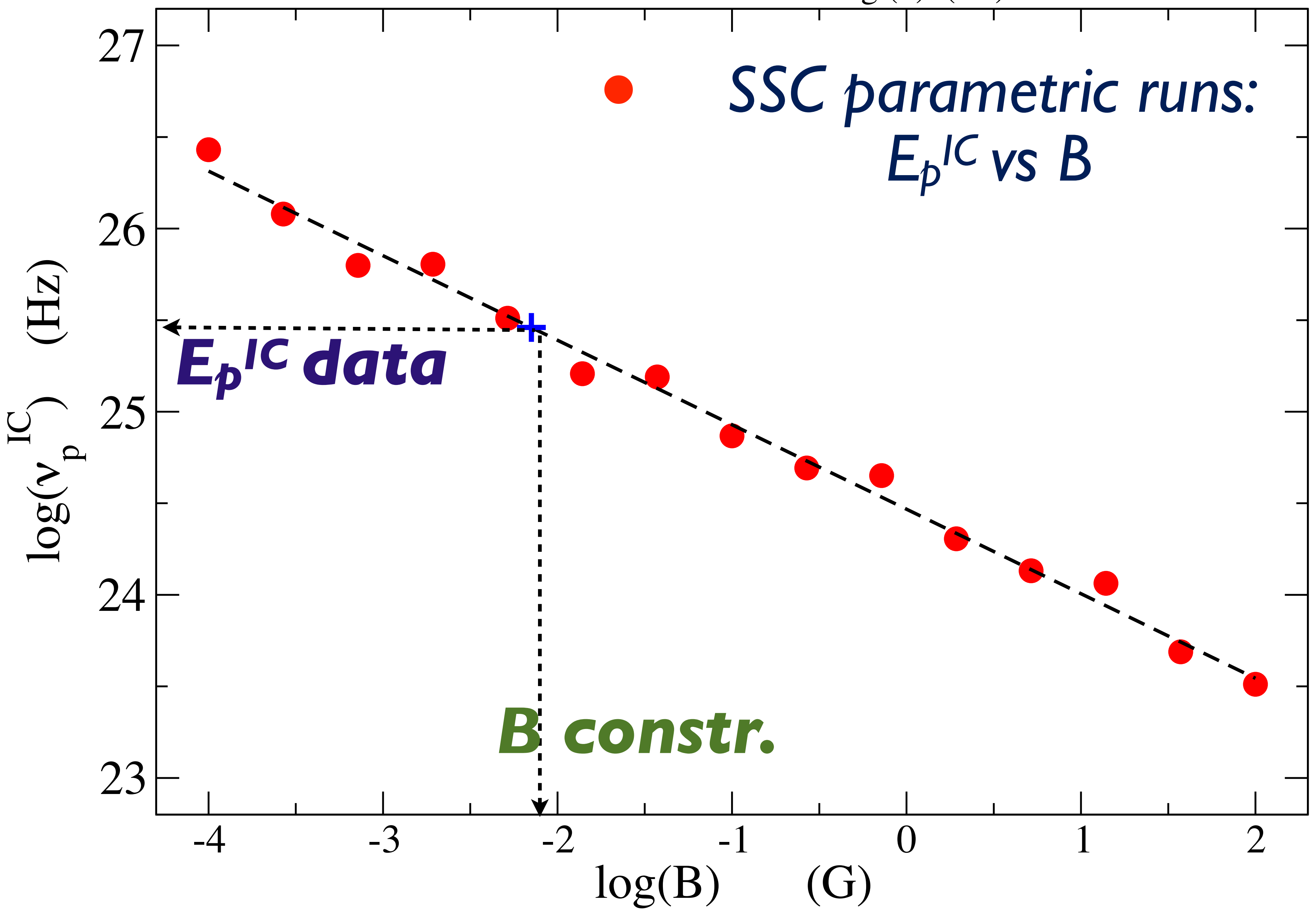
SED shaping and model constraining

- ✓ $\Gamma \Rightarrow s$
- ✓ $b \Rightarrow r$
- ✓ $E_p^S, r, s \Rightarrow \gamma_0$
- ✓ $t_{var}, \delta \Rightarrow R$ u.l.
- ✓ $N \Rightarrow$ best S_p^S match
- • $B \rightarrow$ best E_p^{IC}, S_p^{IC} match
- • $R \rightarrow$ CD





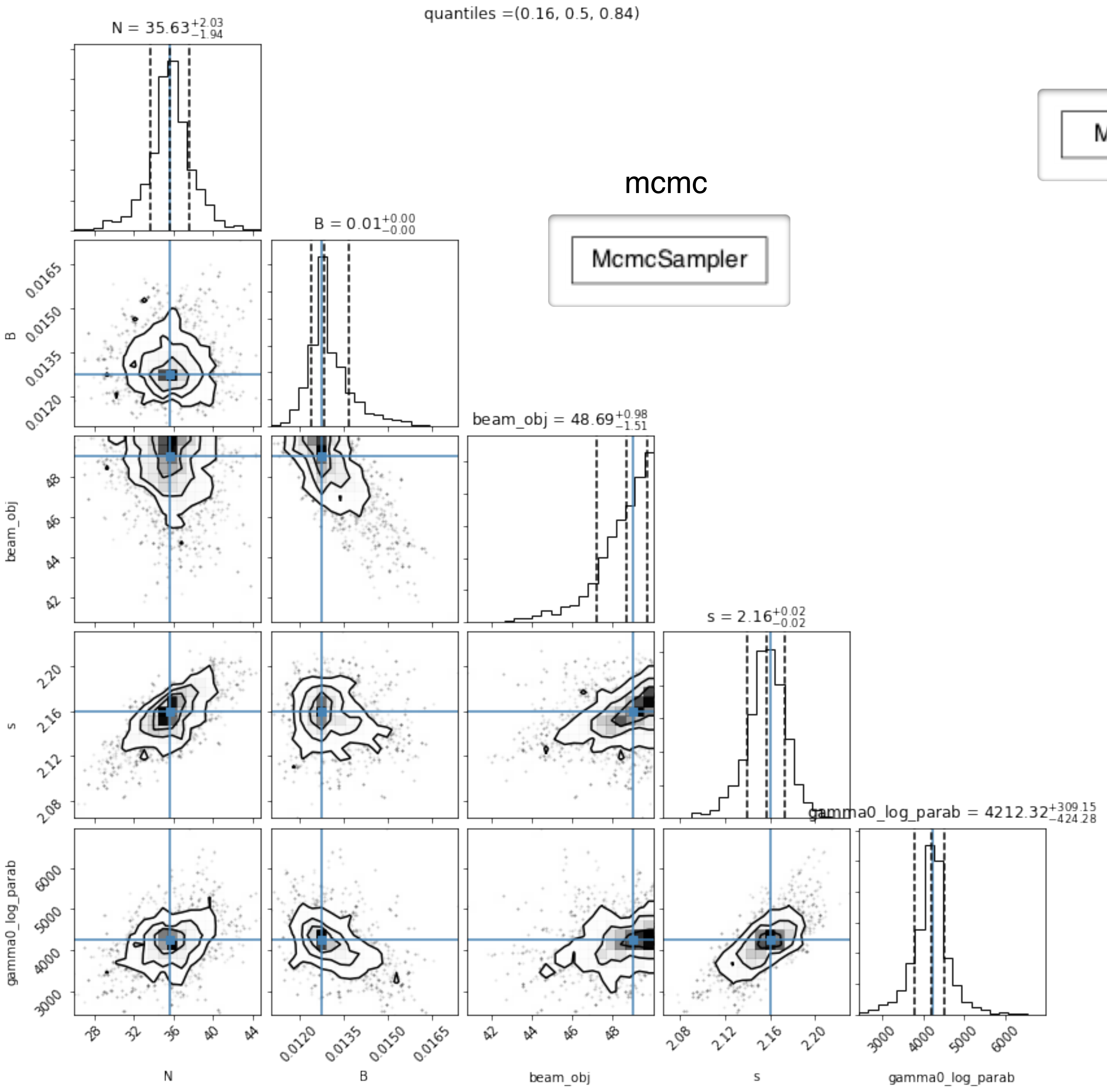




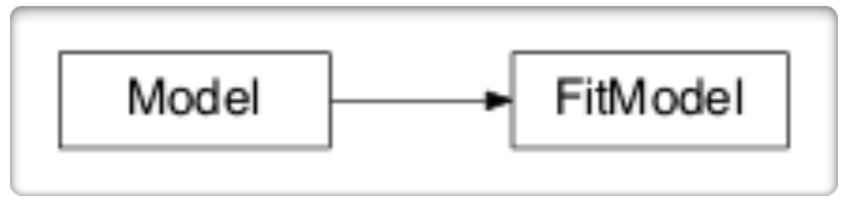
model fitting MCMC

JetSeT

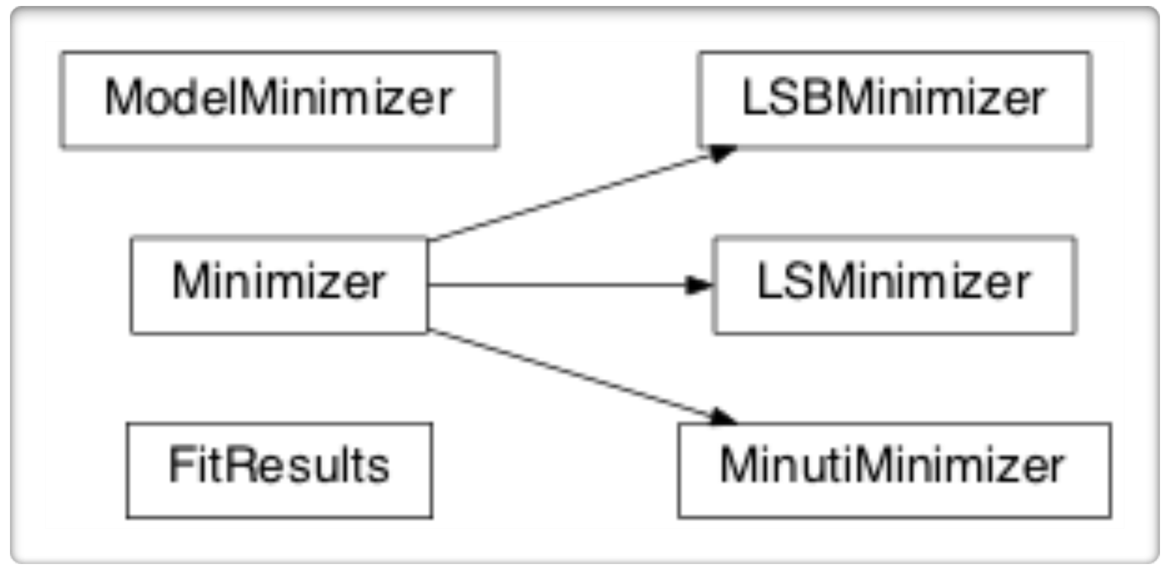
Tutorial 5

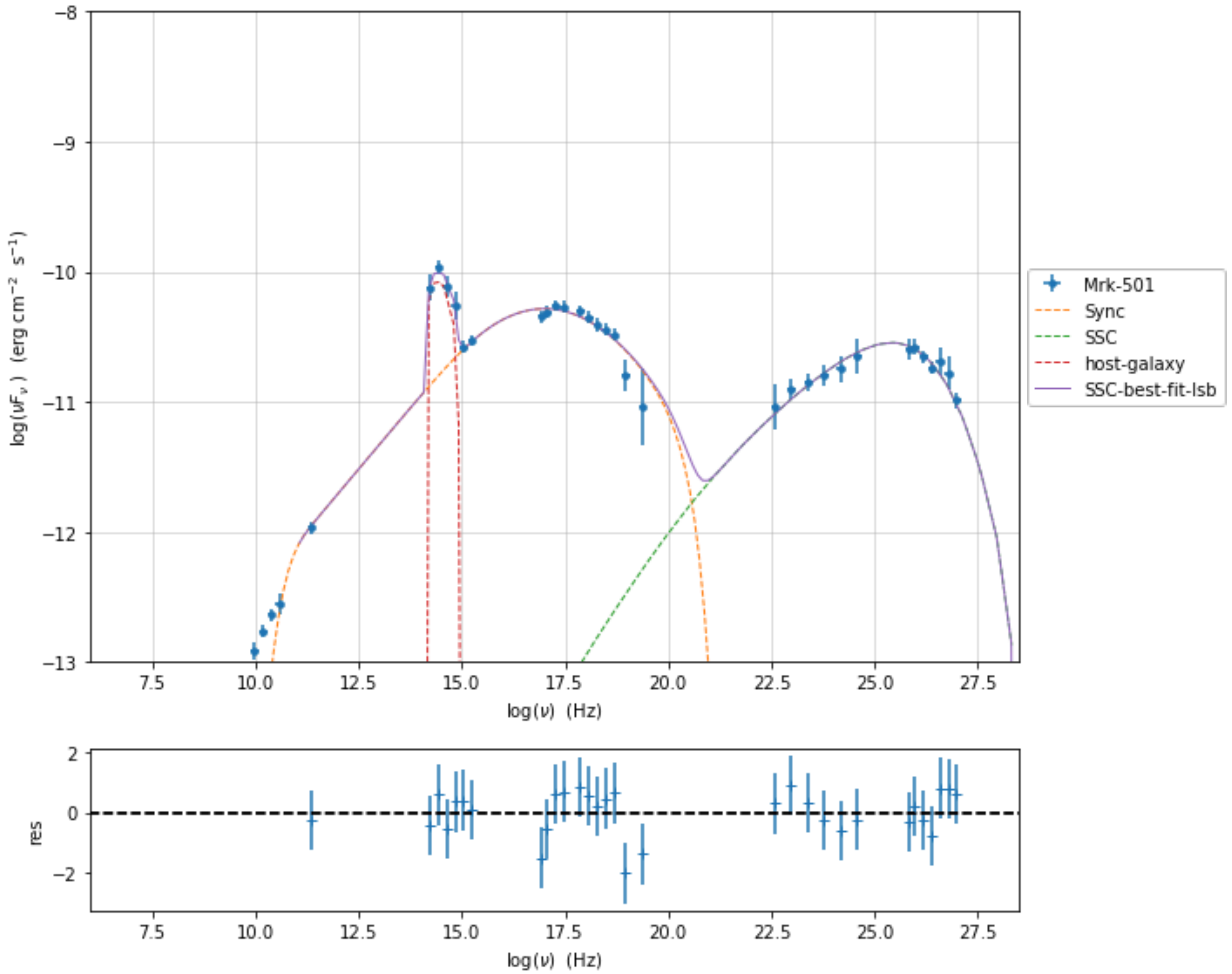


model_manager



minimizer





p(parameters given data) ∝ p(data given parameters)× (parameters)

Posterior = Likelihood × Prior

$$P_{\mathrm{i}} = \pi R^2 \Gamma^2 \beta c U_{\mathrm{i}}'$$

$$U_{\mathrm{e}}' = m_{\mathrm{e}} c^2 \int \gamma N(\gamma) d\gamma$$

$$U_{\mathrm{B}}' = \frac{B'^2}{8\pi}$$

$$U_{\mathrm{rad}}' = \frac{L'}{4\pi R^2 c} = \frac{L}{4\pi R^2 c \delta^4}$$