OBSERVATOIRE DE GENÈVE

UNIVERSITÉ DE GENÈVE

Numerical modelling of radiative and accelerative processes with the JetSeT code

JetSeT

Jets SED modeler and fitting Tool

Andrea Tramacere

https://jetset.readthedocs.io/en/latest/
https://github.com/andreatramacere/jetset
https://www.facebook.com/jetsetastro/

Outline

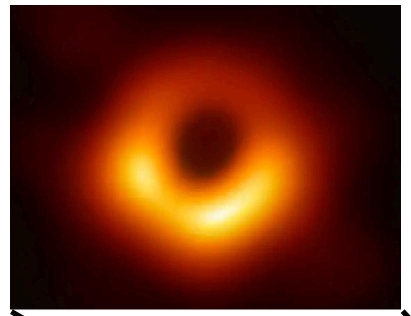
- Theoretical background
- definition of complex radiative **models** SSC/EC IC against CMB/BLR/DT, plus analytical and template models
- handling observed **data** (groping, definition of data sets, etc…)
- **constraining** of the model in the pre-fitting stage, based on accurate and already published **phenomenological trends**
- **fitting of multiwavelength SEDs** using both **frequentist** approach (iminuit/scipy) and Bayesian **MCMC** sampling (emcee)
- Textbooks
  - Radiative Processes in Astrophysics, Ribicky & Lightman, John Wiley & Sons, 1991
  - High Energy Radiation from Black Holes: Gamma Rays, Cosmic Rays, and Neutrinos, Dermer & Menon, Princeton University Press 2009

Online tutorials

https://github.com/andreatramacere/Linnaeus_JetSeT_Lesson

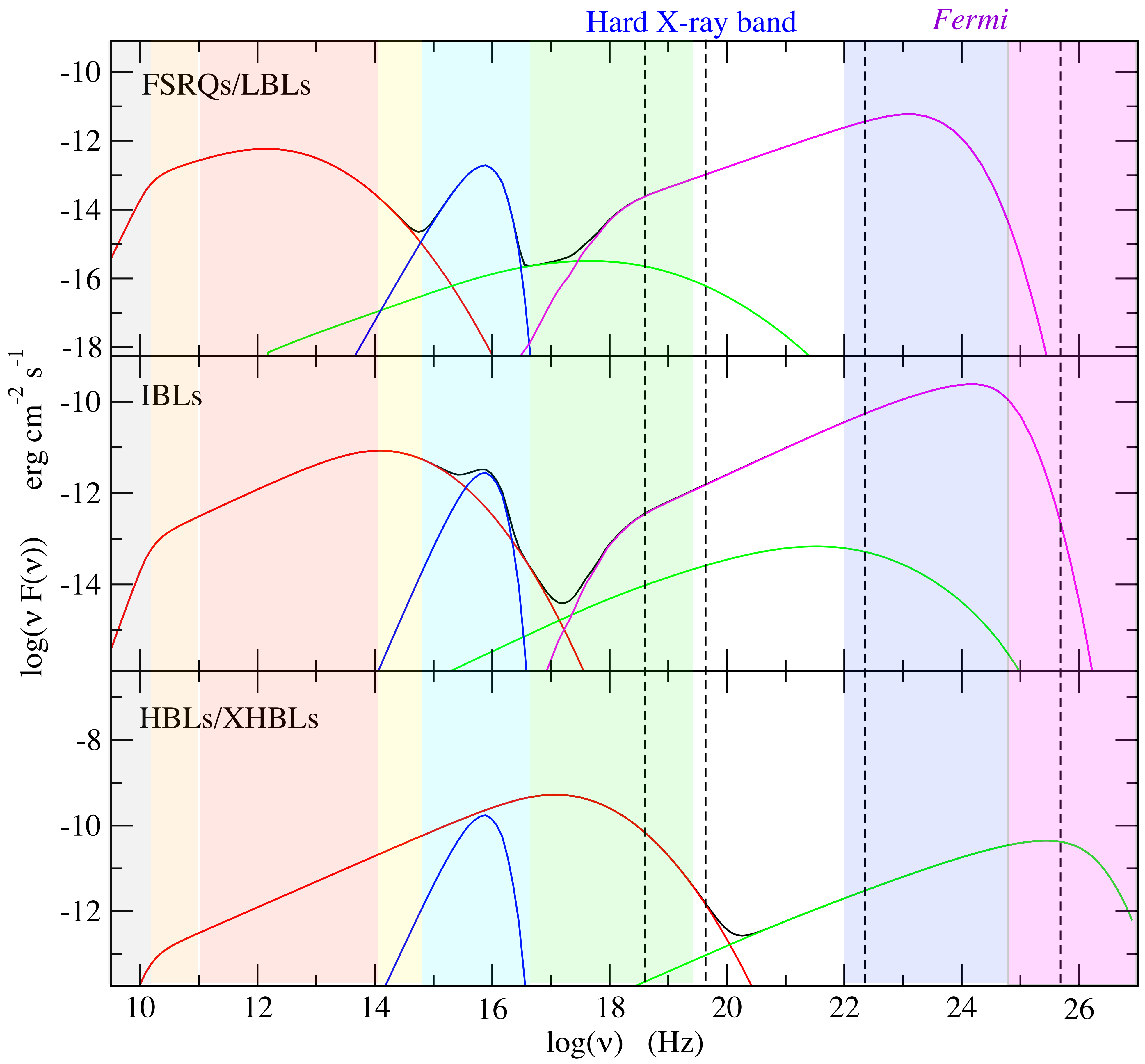
- Tutorial 1: basic operation with jet modesl
- Tutorial 2: phenomenological trends for synchrotron and SSC emission
- Tutorial 3: phenomenological trends for EC emission
- Tutorial 4: composite models and application to EBL (not covered in these slides)
- Tutorial 5: constraining of the model in the pre-fitting stage, based on accurate and already published phenomenological trends and fitting of multiwavelength SEDs using both frequentist approach (iminuit/scipy) and Bayesian MCMC sampling (emcee)

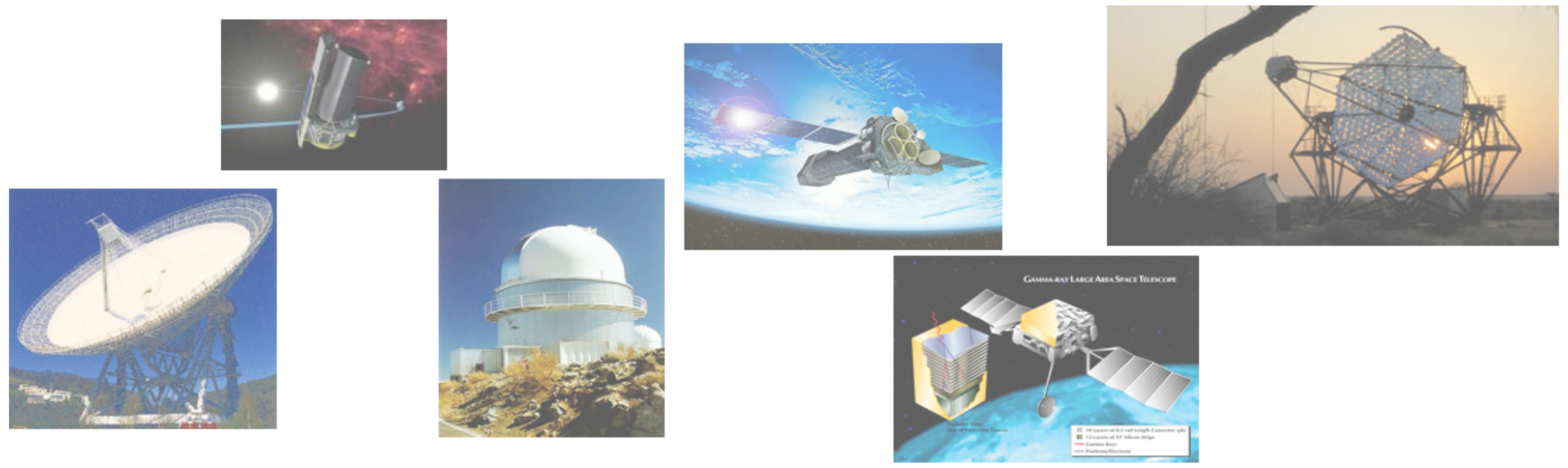
JetSeT :Blazars in a nutshell



model

data



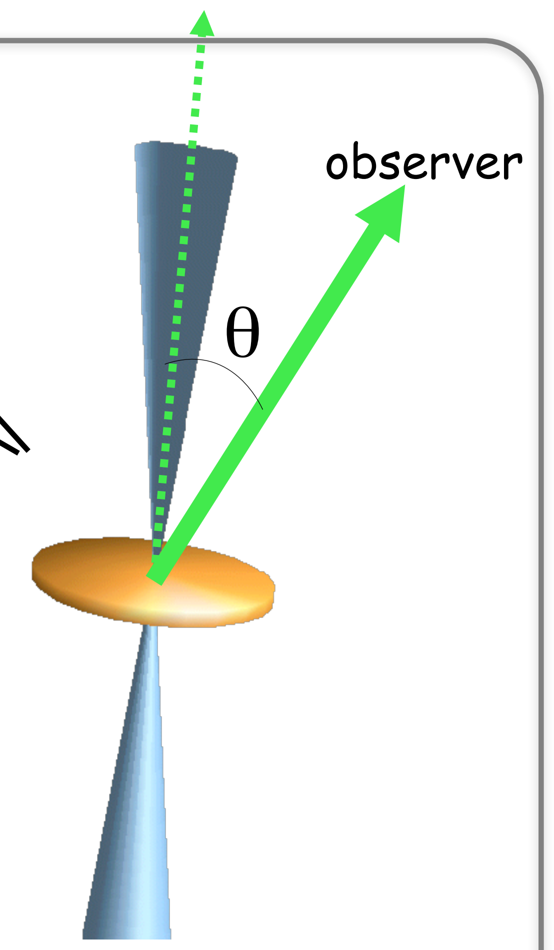


phenomenology

model fit

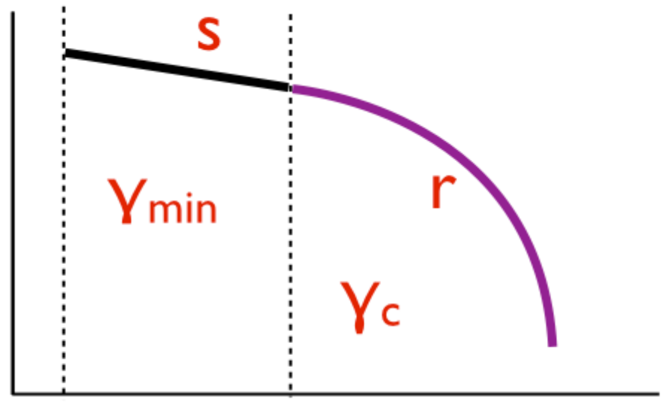
simulations

jet/disk



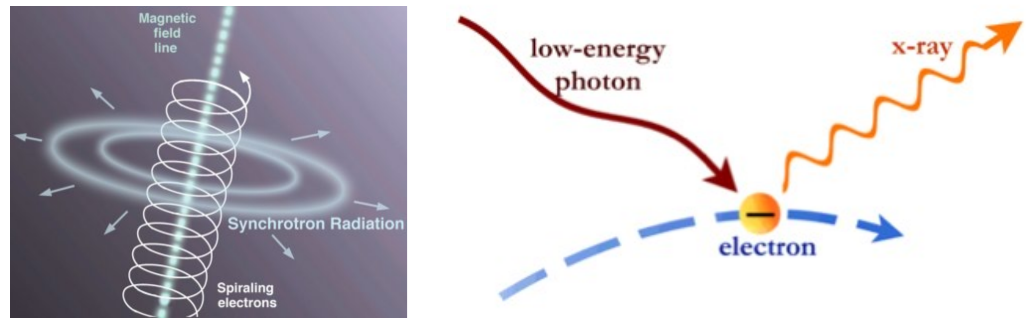
+

acc.



+

em.



standard picture: acceleration/cooling balance

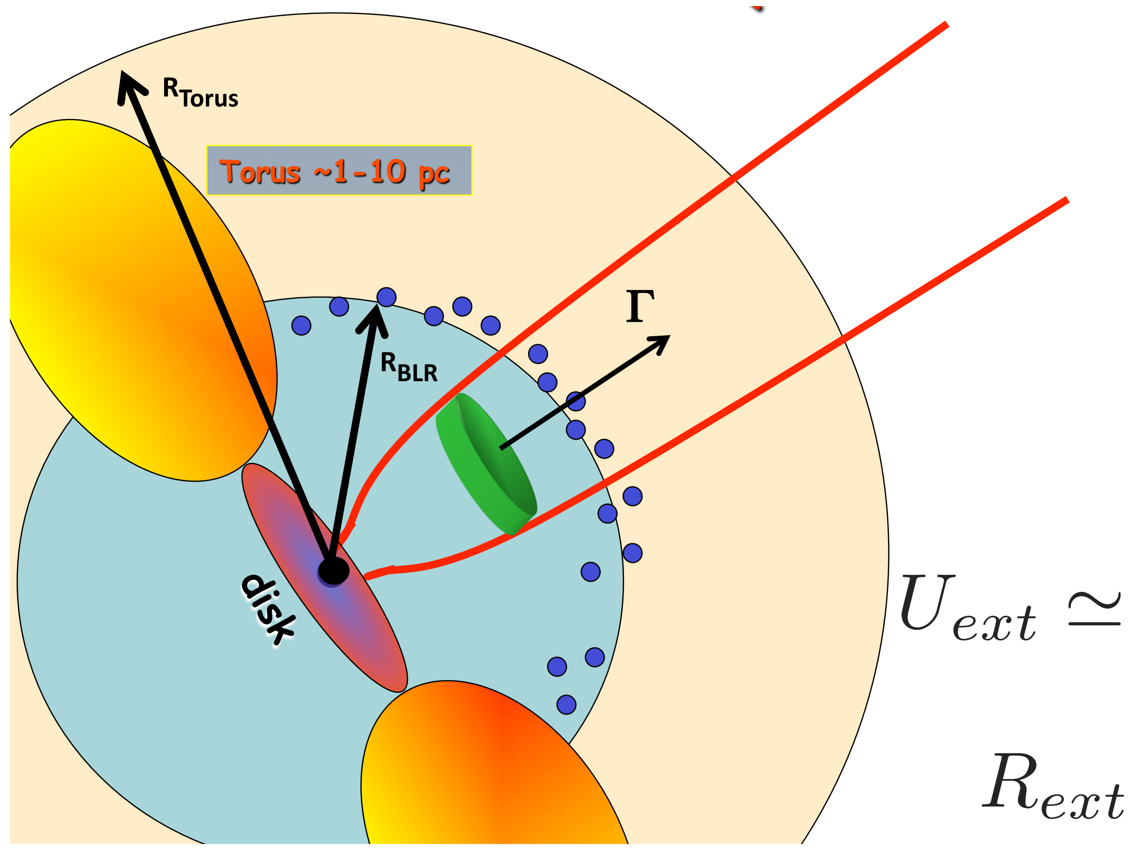
(Ghisellini,Fossati,Celotti 99-16)

FSRQs
HERGs

rad eff.
$L_d > 10^{-2} L_{EDD}$

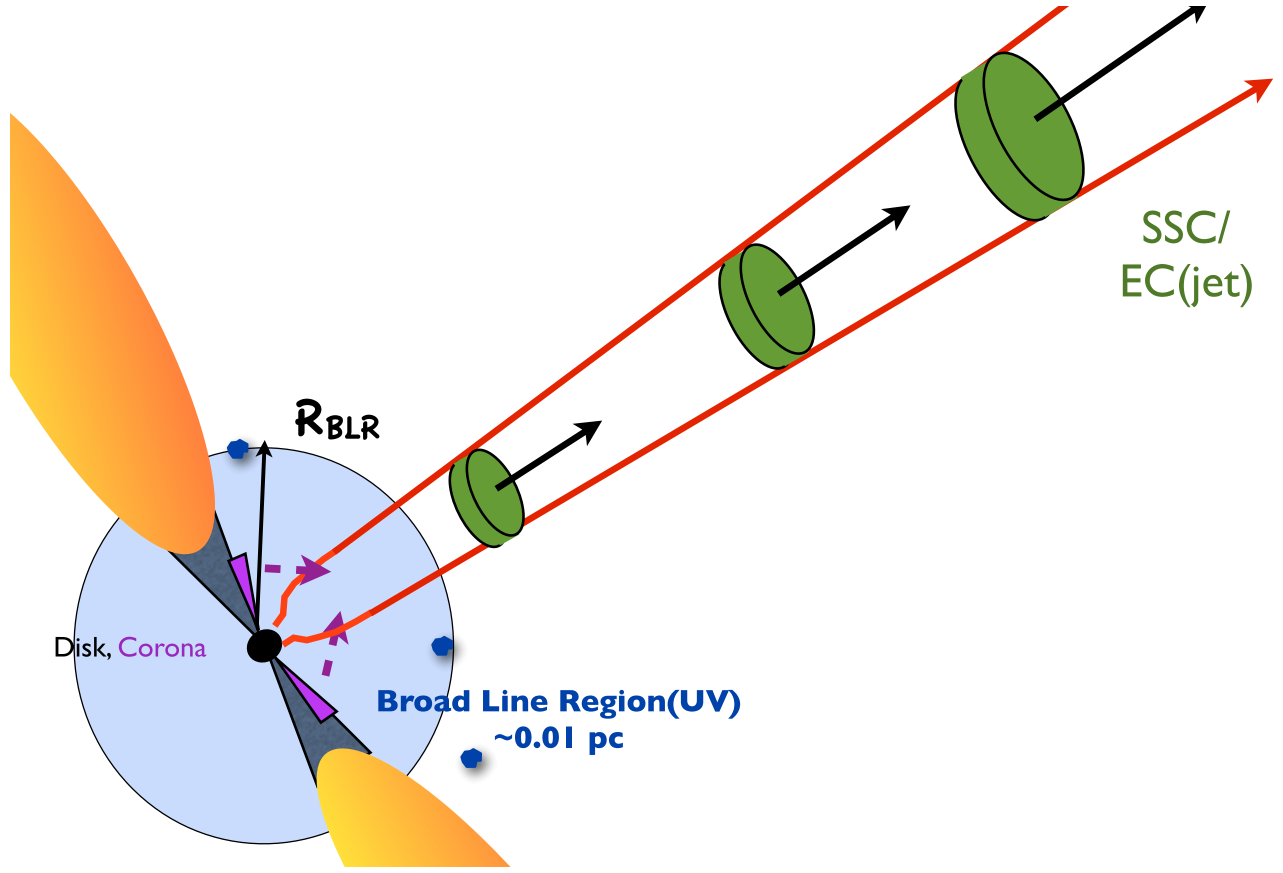
BL Lacs
LERGs

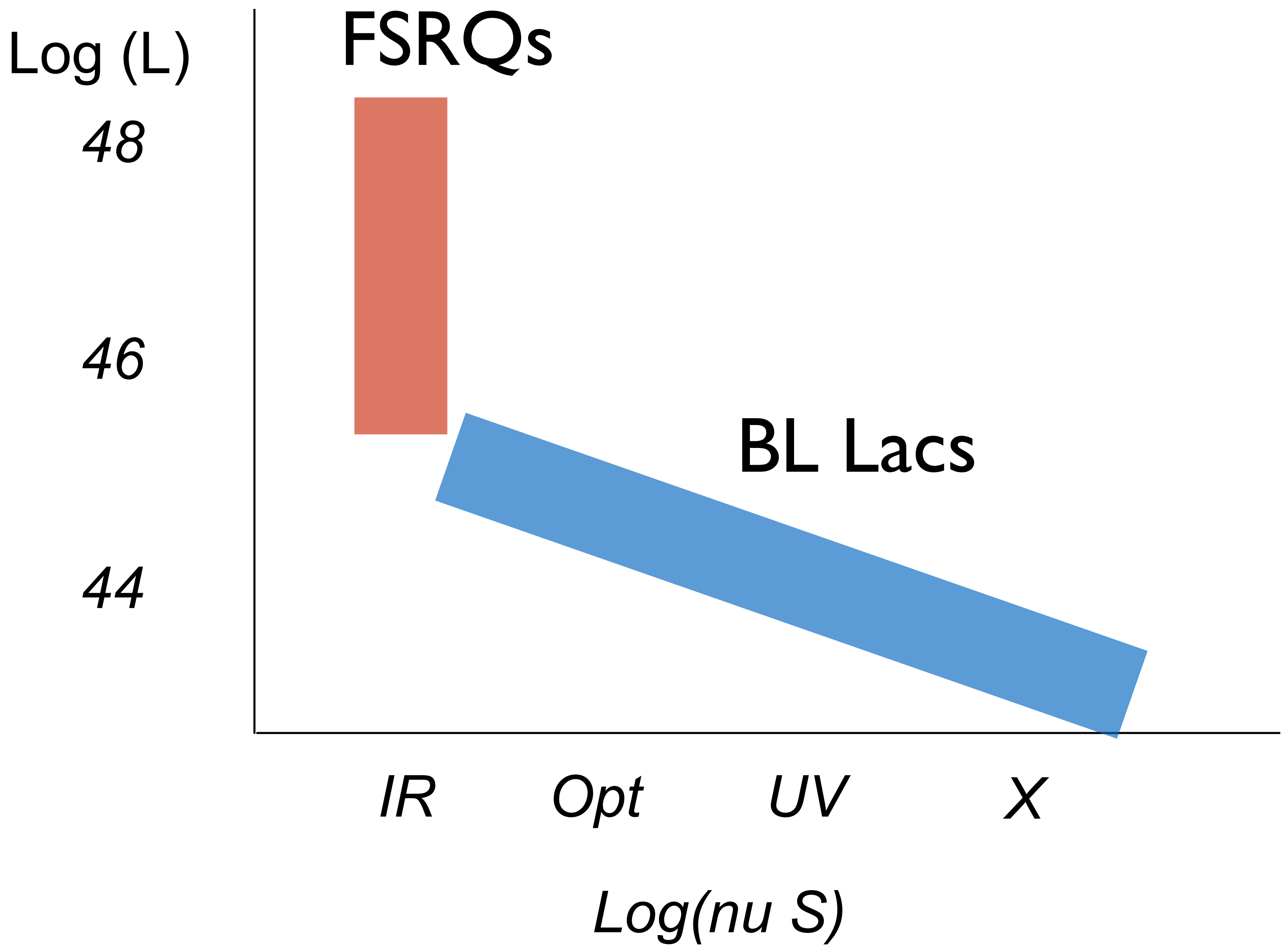
rad ineff.
$L_d < 10^{-2} L_{EDD}$

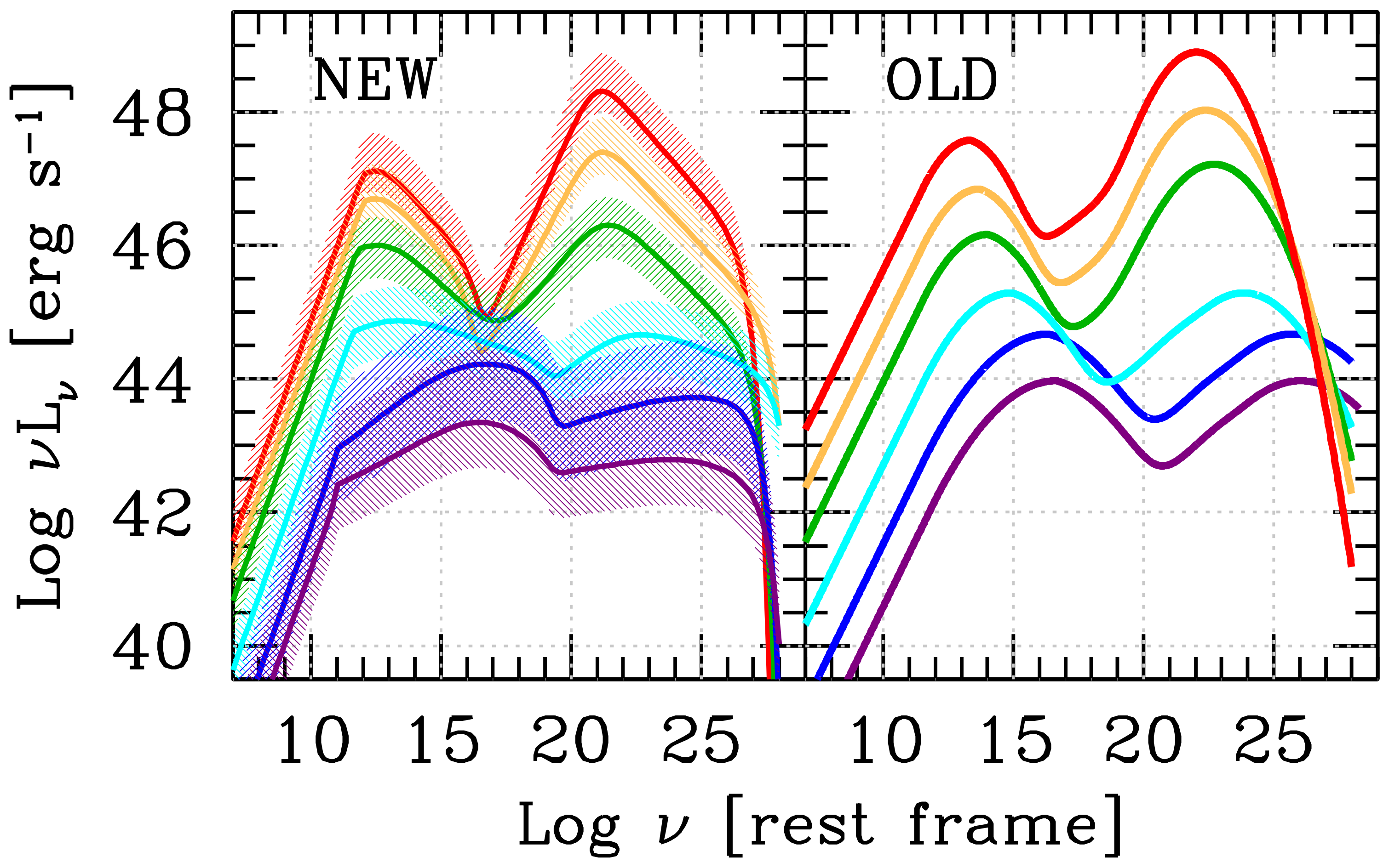


$$\left.\begin{array}{l} U_{ext} \simeq \dfrac{L_d}{R^2_{ext} c} \\ R_{ext} \simeq L_d^{1/2} \end{array}\right\} \rightarrow$$

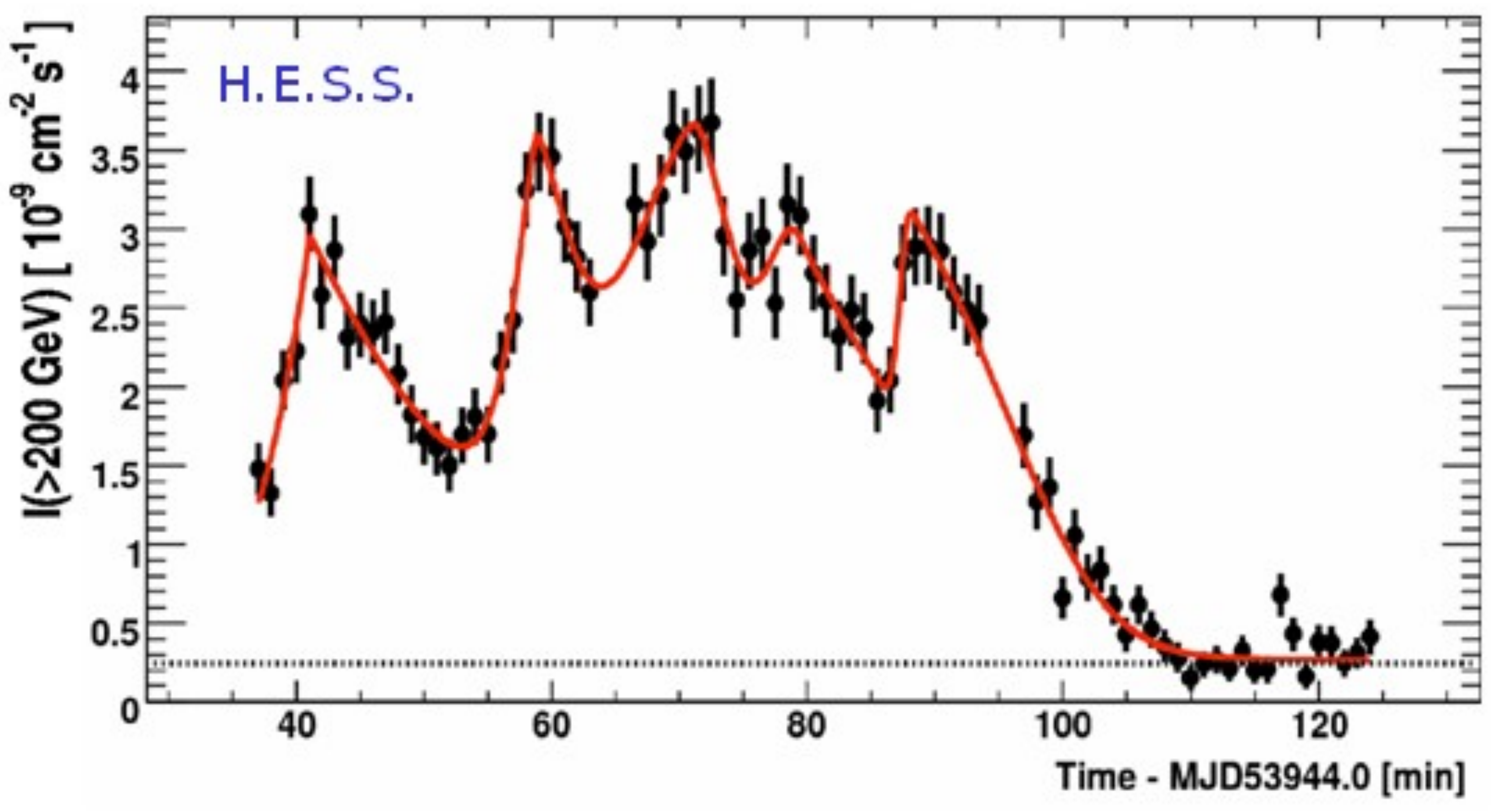
~0.1 erg/cm^3 BLR
~0.01 erg/cm^3 DT

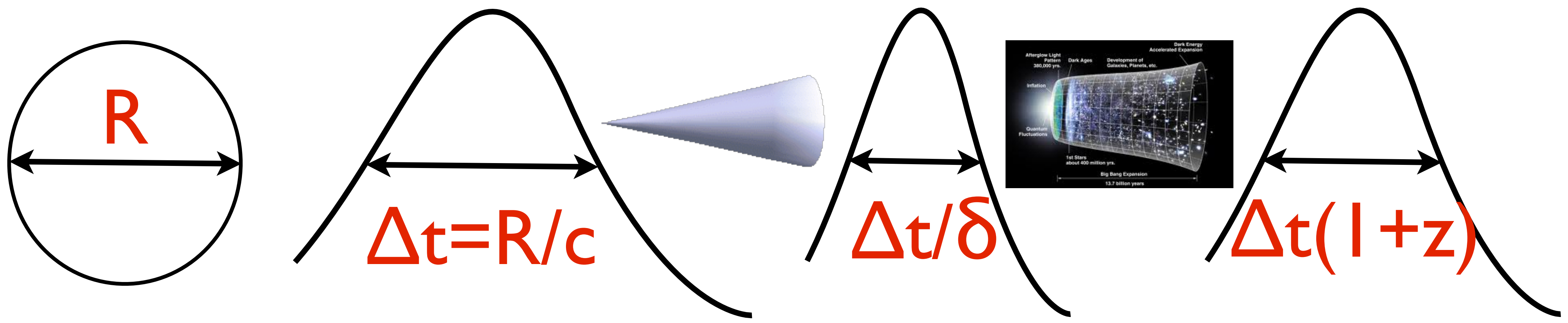




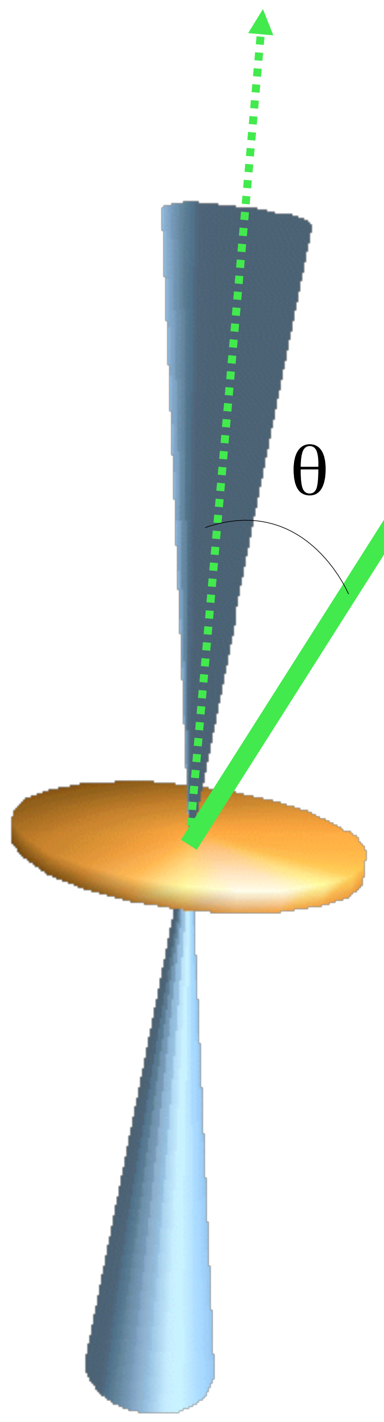


Beamed Emission





R<= c Δtδ/(1+z)

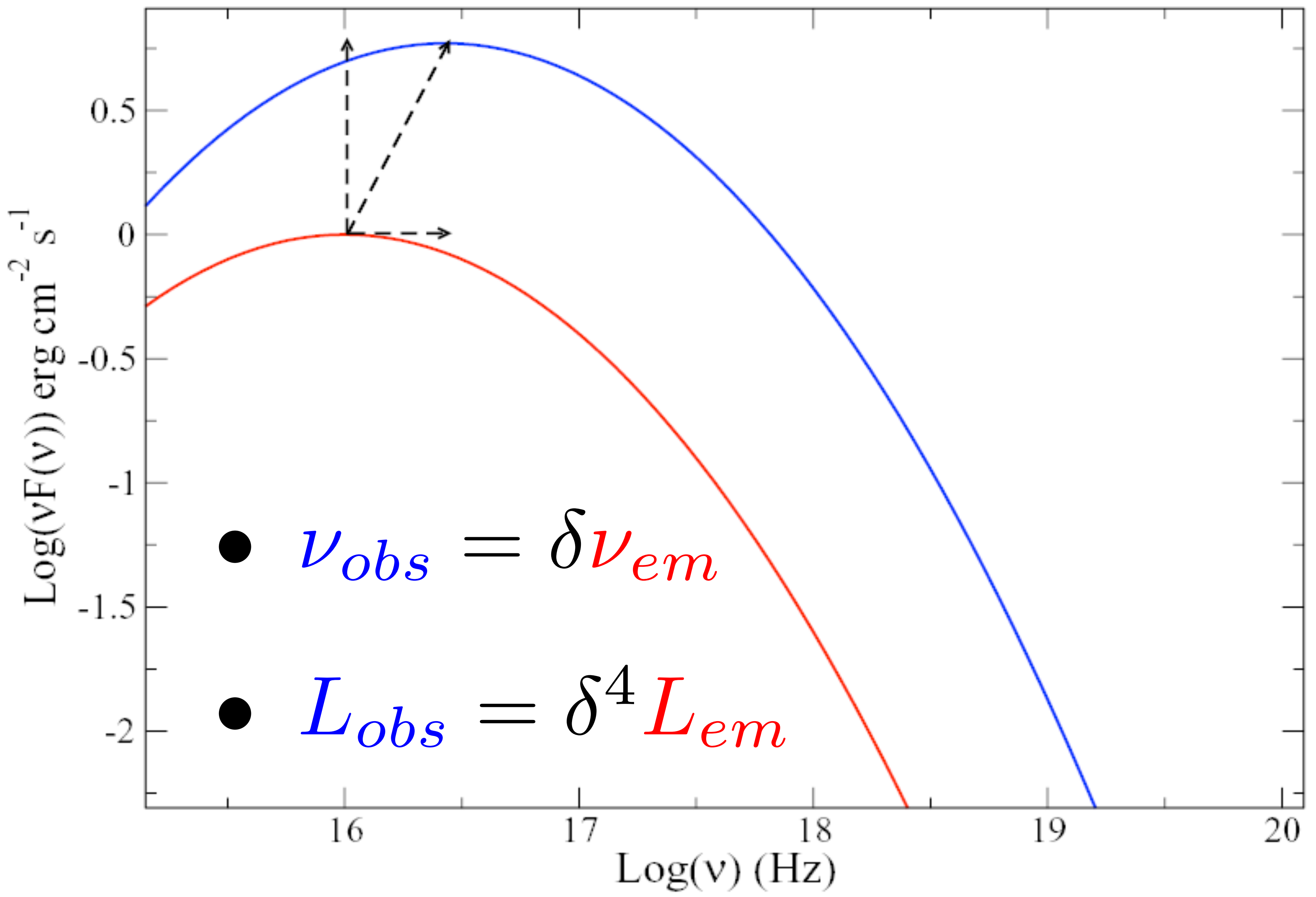


rest frame :
isotropic emission

Observer frame: beamed

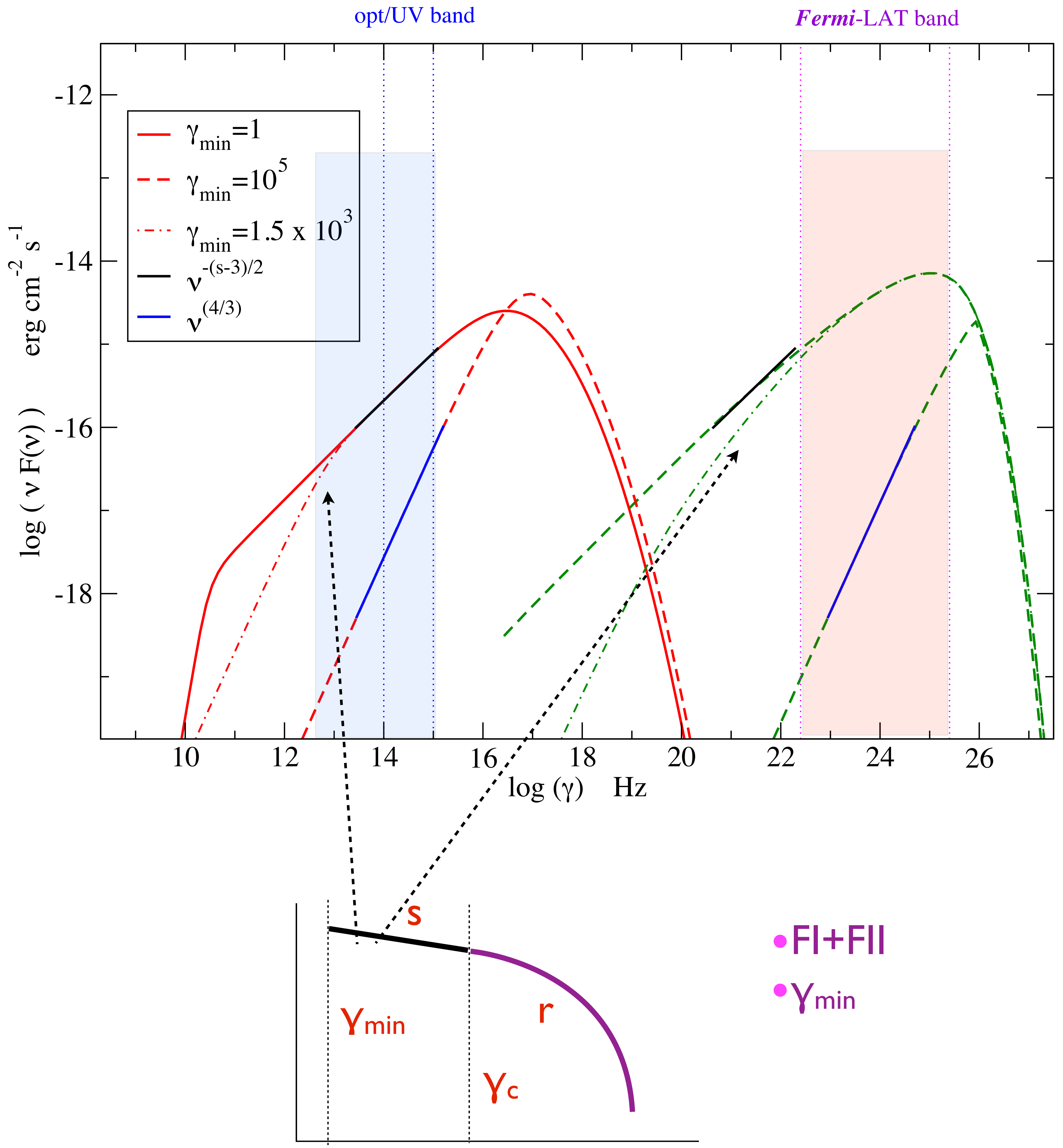
Beaming factor:

- $\delta = \frac{1}{\Gamma(1-\beta cos(\theta))}$
- $\theta = 1/\Gamma$

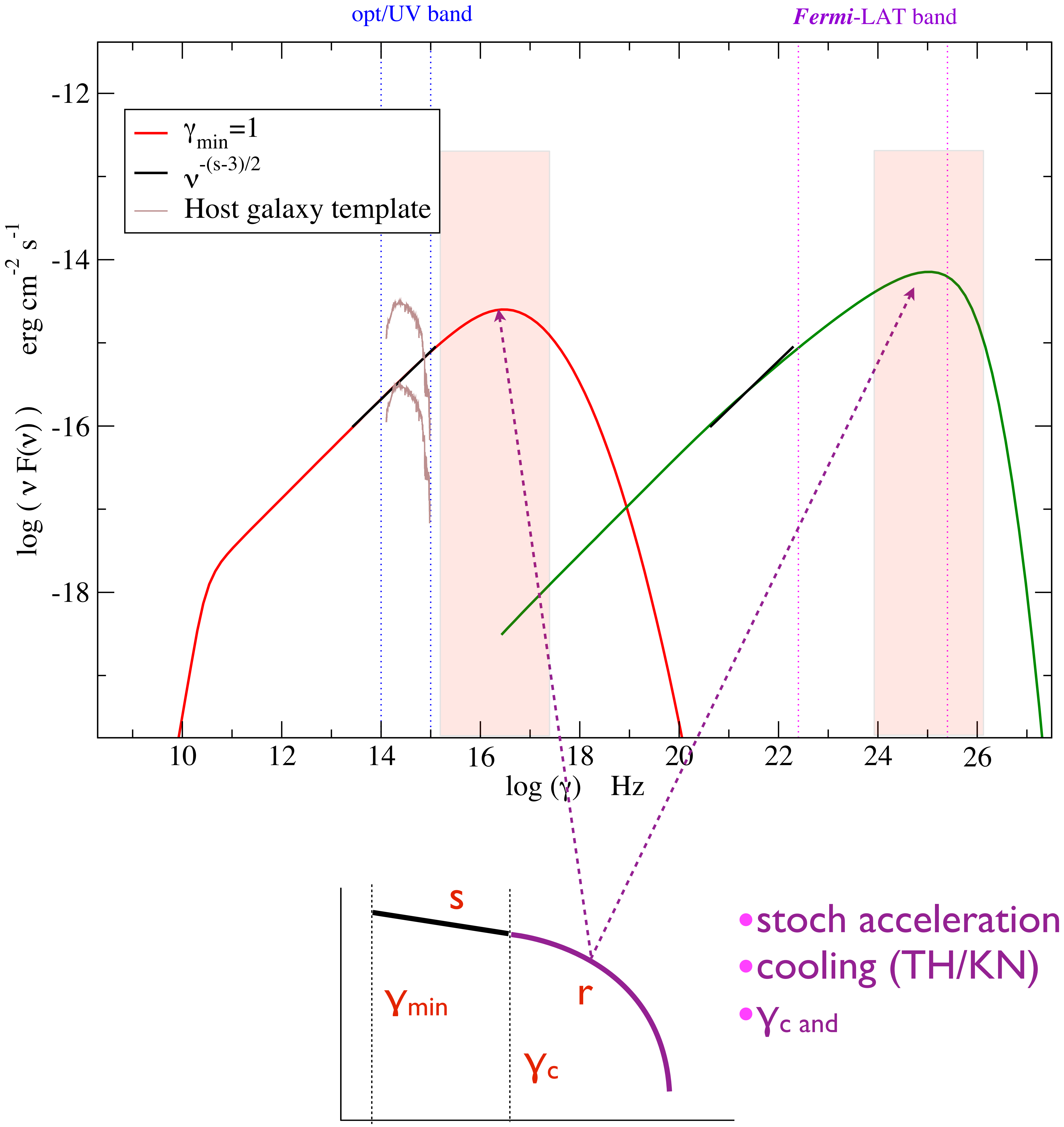


- $\nu_{obs} = \delta\nu_{em}$
- $L_{obs} = \delta^4 L_{em}$

SED shaping and constraining the electron distribution



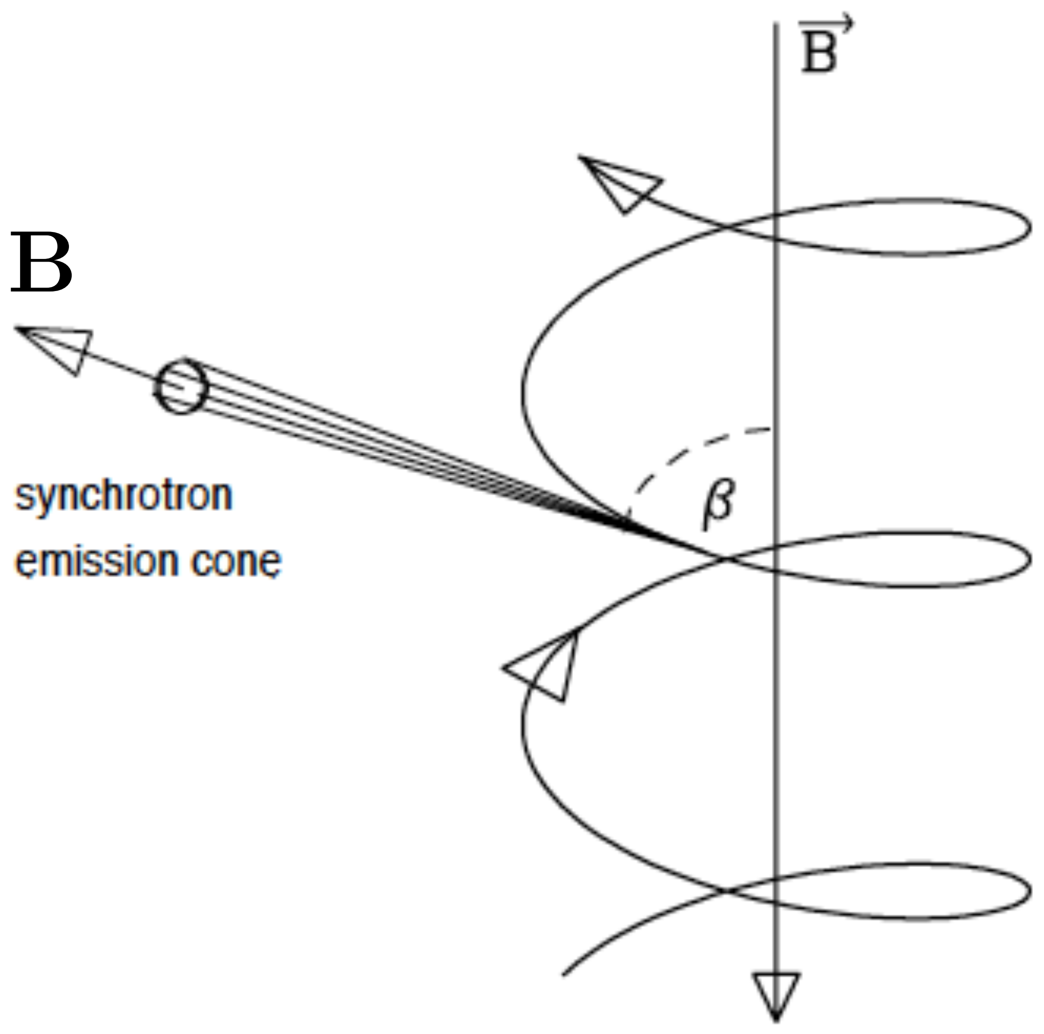
- FI+FII
- γ_{min}



- stoch acceleration
- cooling (TH/KN)
- γ_c and

synchrotron basics single particle

$$m\gamma \frac{d(\mathbf{v})}{dt} = \frac{q}{c}\mathbf{v} \times \mathbf{B}$$



total emitted power

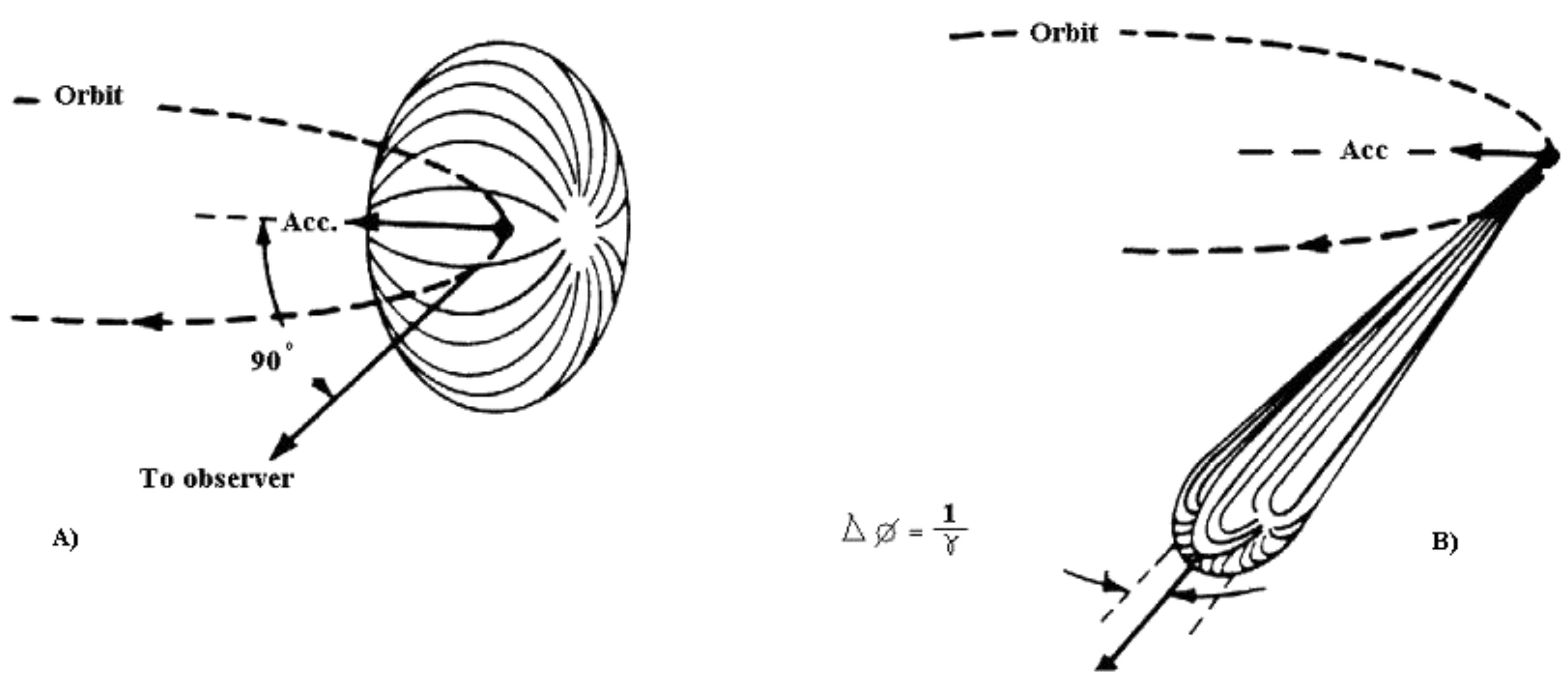
$$a_{\parallel} = 0$$

$$a_{\perp} = \frac{evB sin\alpha}{\gamma m_e c}$$

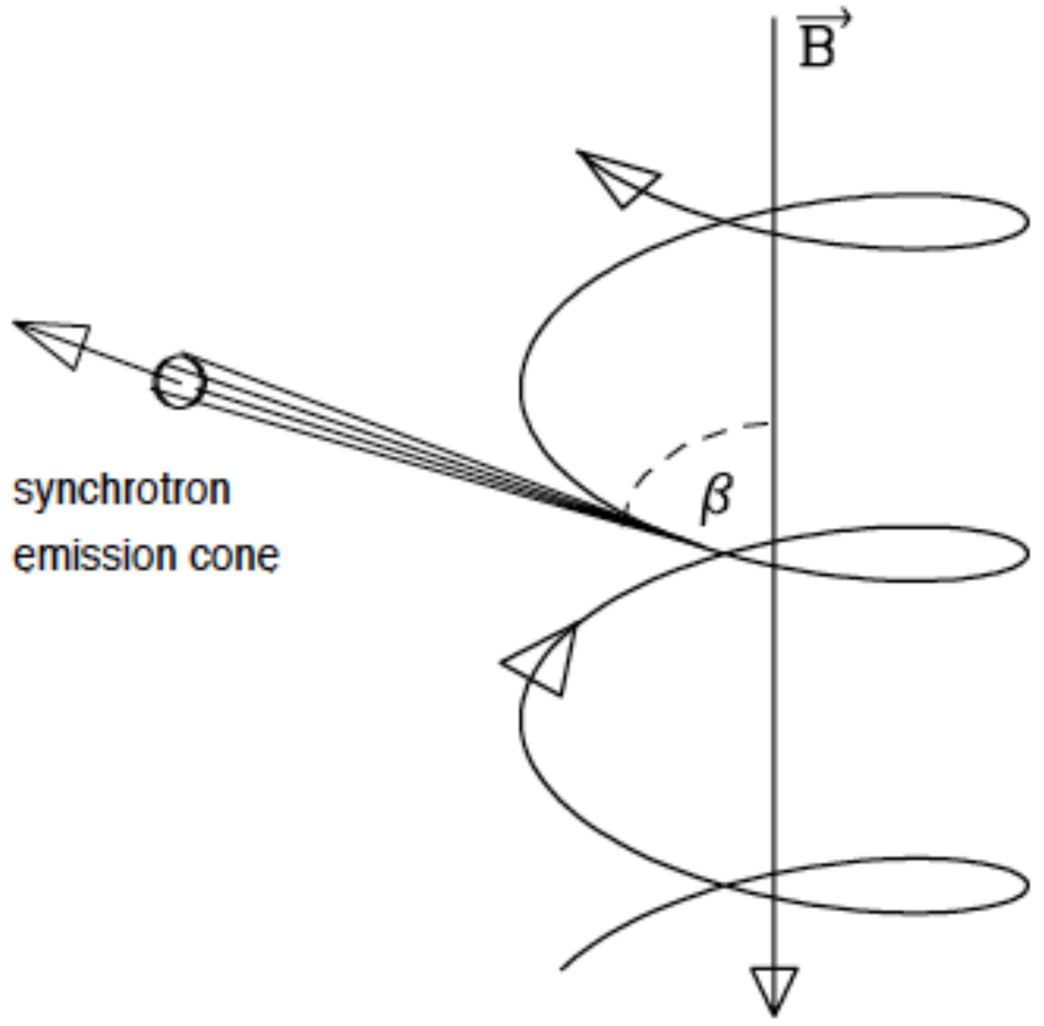
$$\nu_{\mathrm{B}} = \frac{eB}{2\pi\gamma mc} = \frac{\nu_{\mathrm{L}}}{\gamma}$$

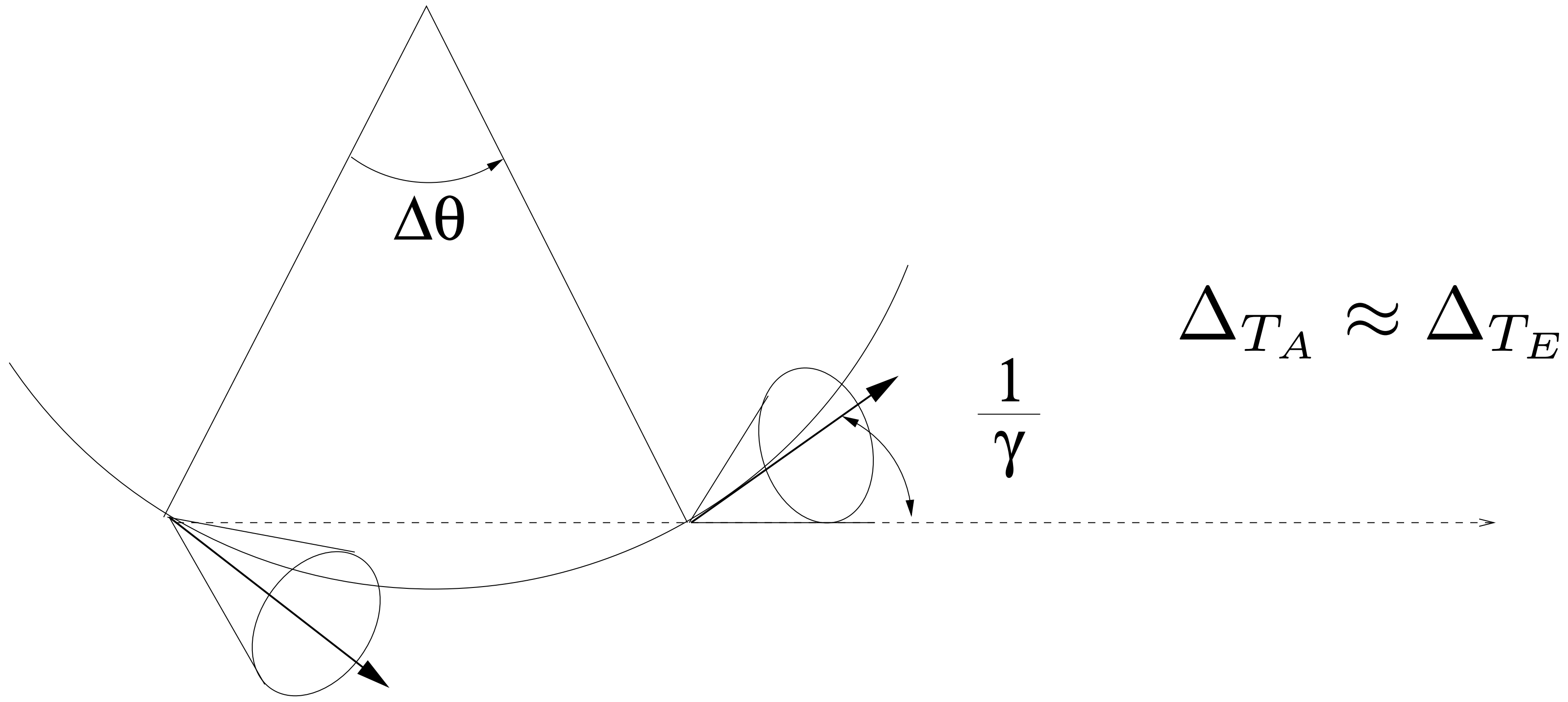
$$P_{\mathrm{e}} = P'_{\mathrm{e}} = \frac{2e^2}{3c^3}a'^2 = \frac{2e^2}{3c^3}[a'^2_{\parallel} + a'^2_{\perp}]$$

$$P_S = \frac{2e^4}{3m_e c^3}B^2\gamma^2\beta^2 sin^2\alpha$$

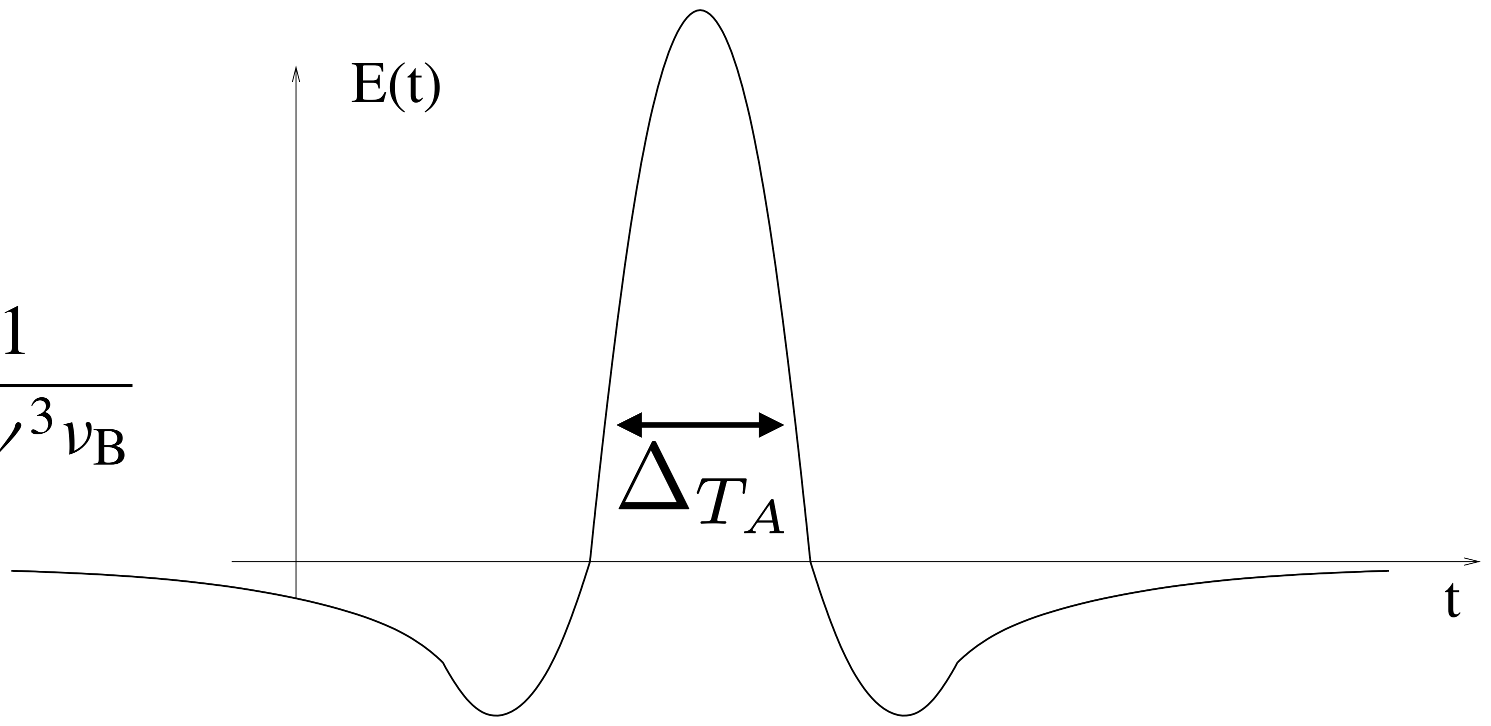


synchrotron basics single particle





$$\Delta_{T_A} \approx \Delta_{T_E}/(2\gamma^2) = \frac{1}{2\pi\gamma^3\nu_B}$$



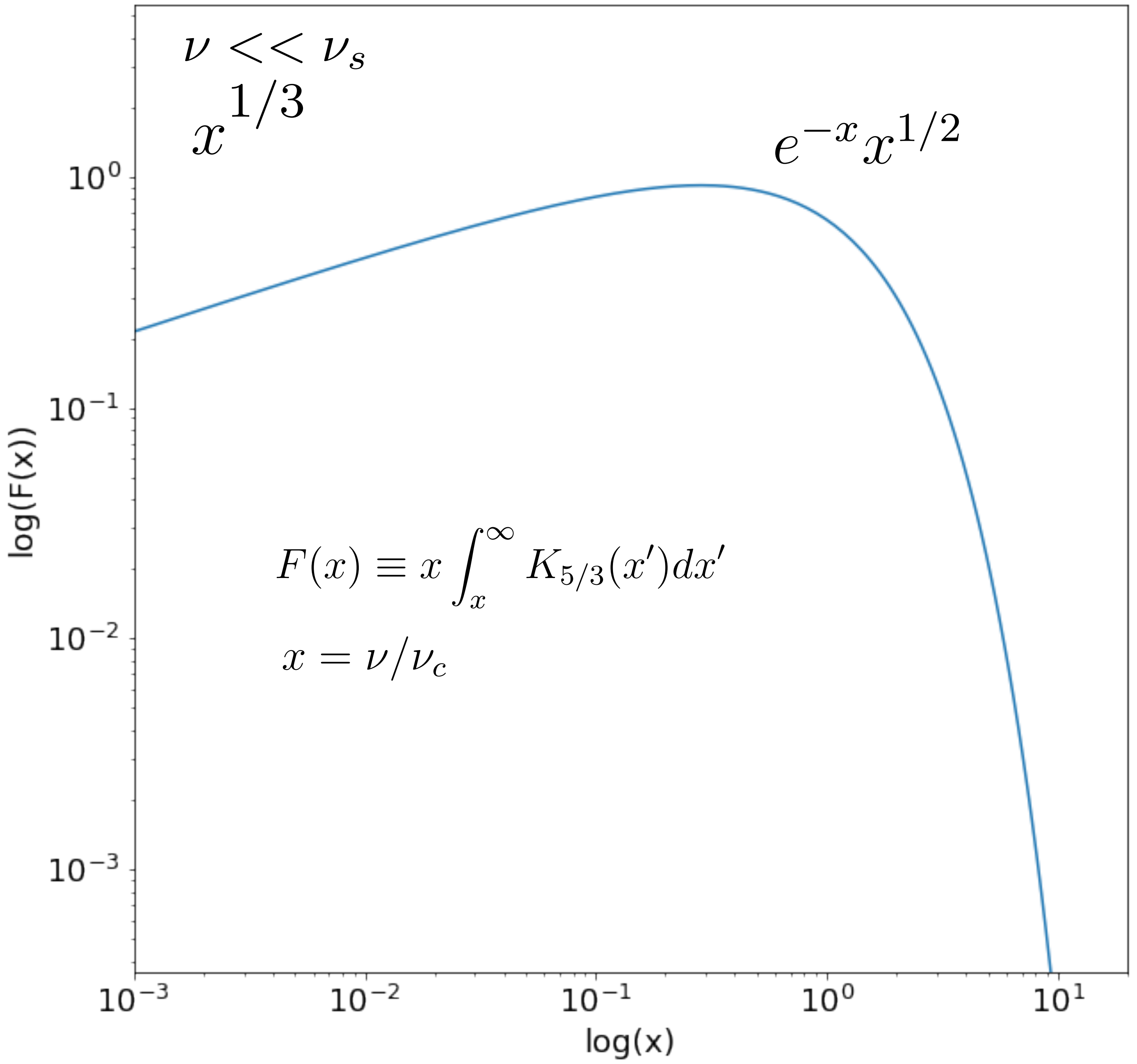
Graphic: Beckmann & Shrader (2012)

power spectrum

$$\Delta_{T_A}\nu_T = \frac{1}{2\pi} \longrightarrow \nu_T = \gamma^3\nu_B$$

emitted spectrum

$$P_e(\nu,\gamma) = \frac{\sqrt{3}e^3 B\sin\alpha}{2m_e c^2}F(x) \longrightarrow$$



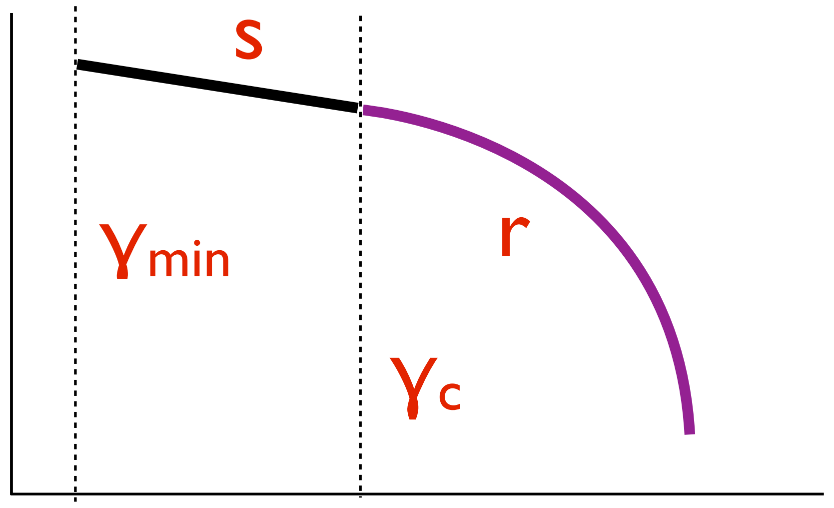
$$\nu_c = \frac{3}{2}\nu_B\gamma^3 sin\alpha = \frac{3\gamma^2 eBsin\alpha}{4\pi m_e c}$$

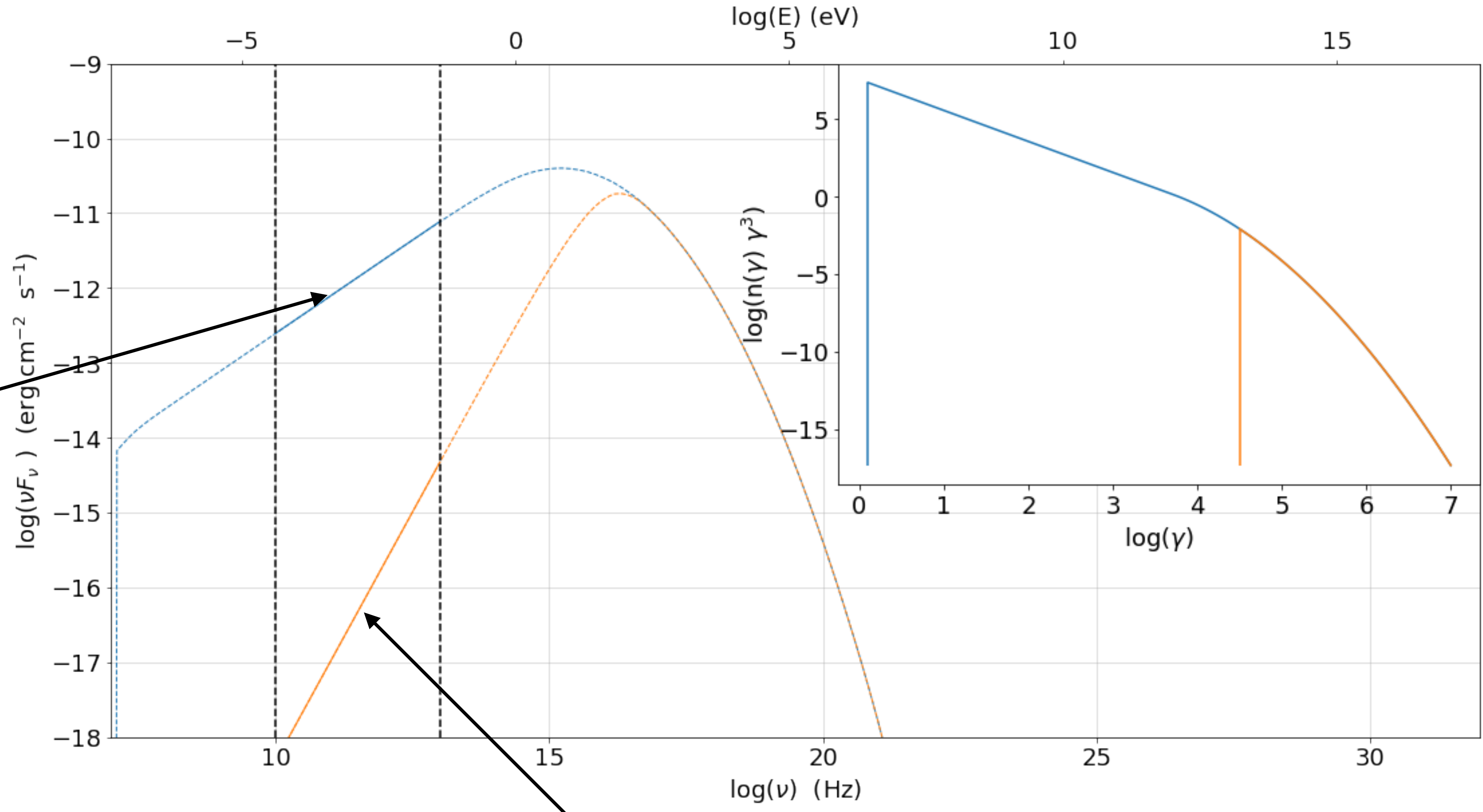
$\nu_s \propto 10^6 B\gamma^2 sin(\alpha)$ Hz

synchrotron basics particle distribution

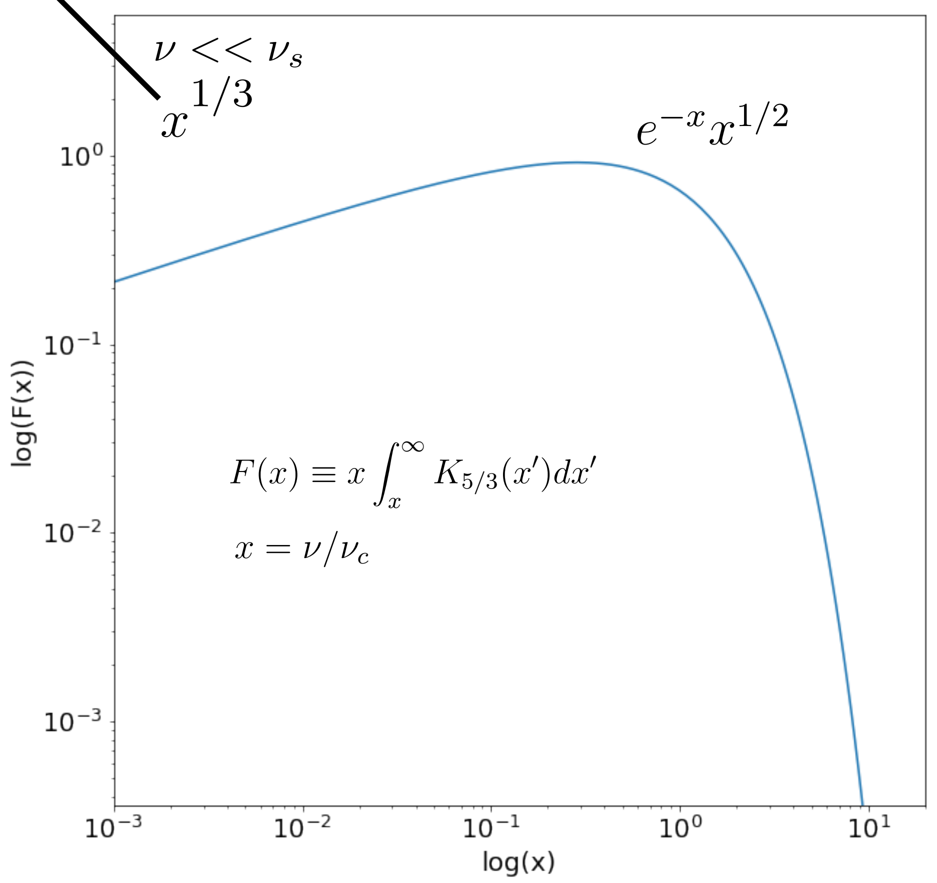
Tutorial 2

$$N(\gamma) \propto \gamma^{-s}$$





$$j_\nu^S(\nu) = \frac{1}{4\pi}\int_{\gamma_{min}}^{\gamma_{max}} P(\nu,\gamma)N(\gamma)d\gamma \;\propto \nu^{-\frac{s-1}{2}}$$

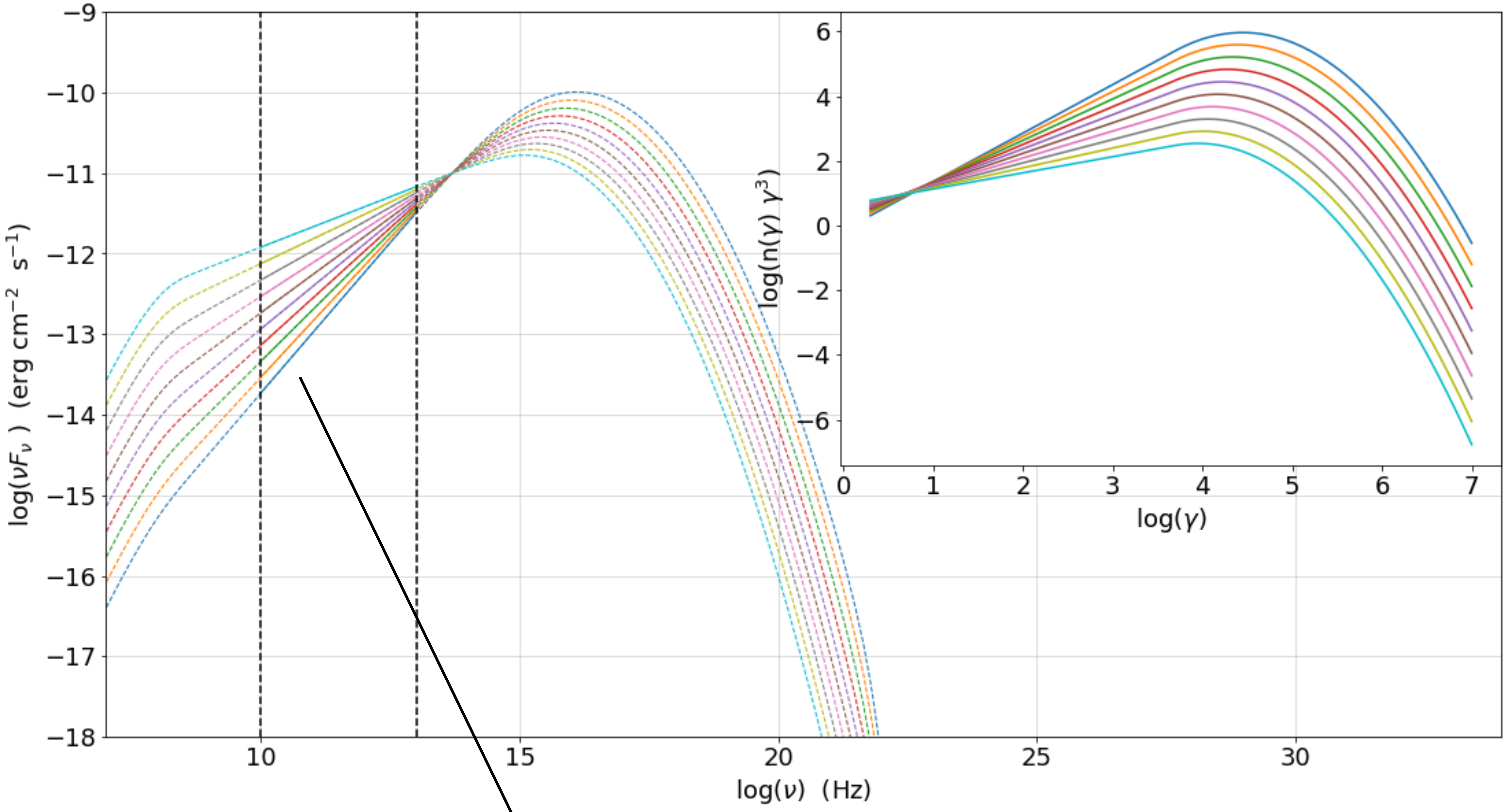


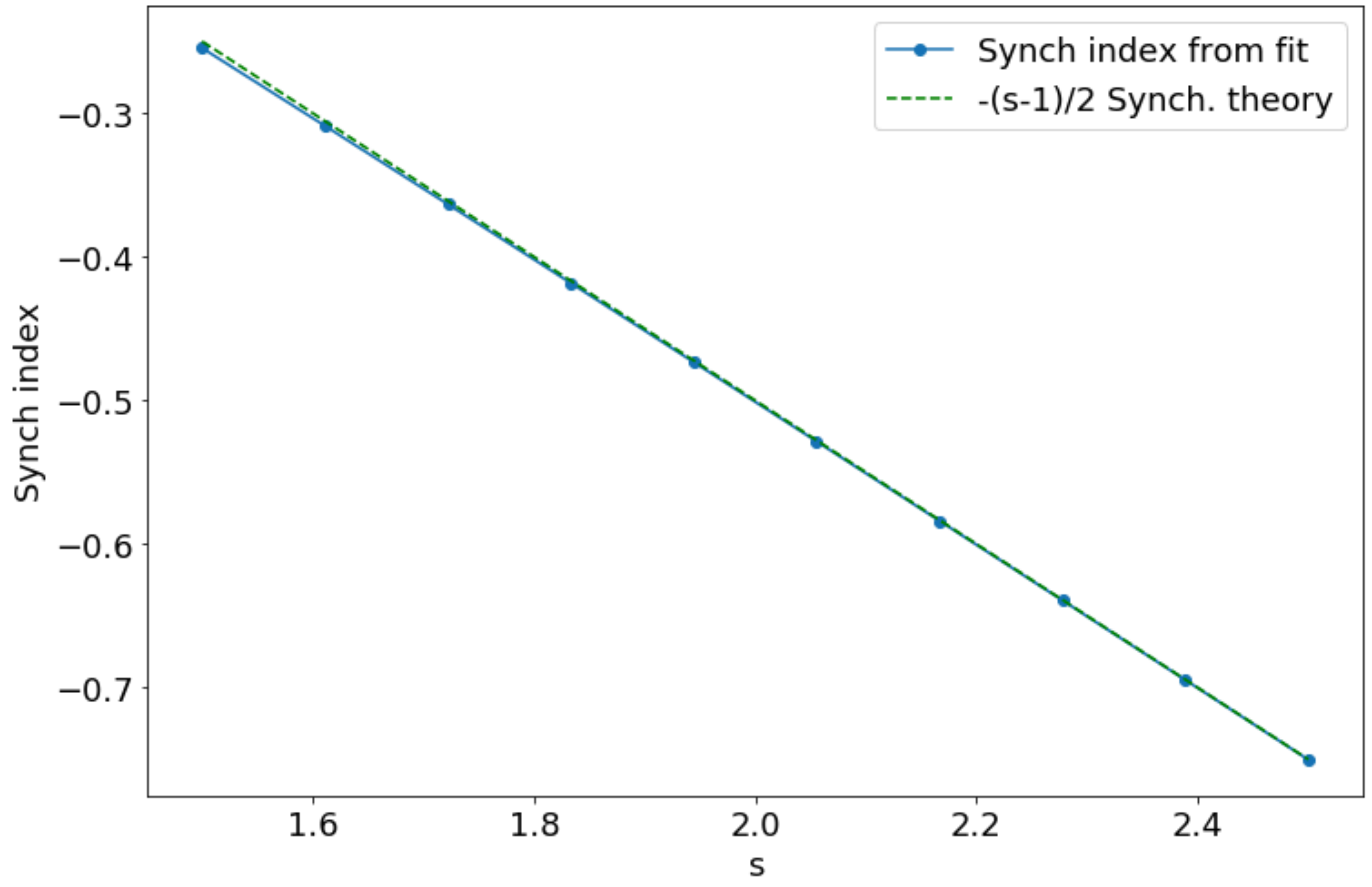
δ-approx relations $\quad \mathrm{SED} \propto N(\gamma)\gamma^3$

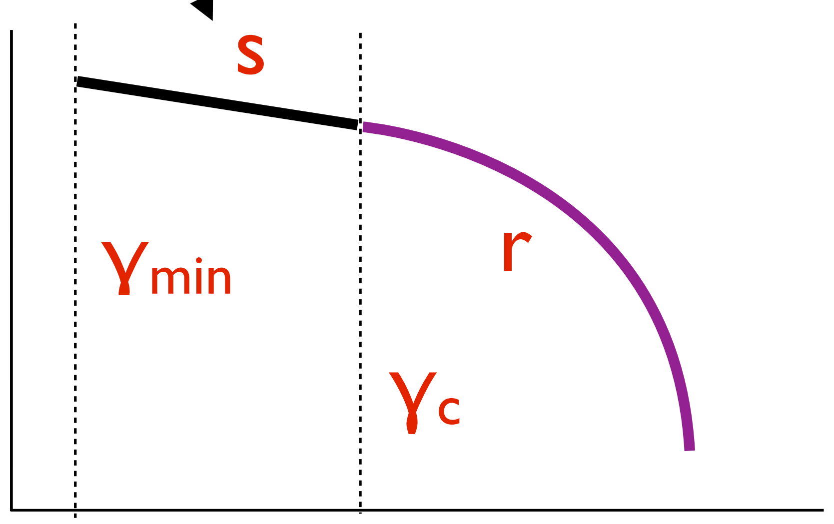
$$S_p^{Sync} \sim \frac{dN(\gamma)}{d\gamma}\gamma_{3p}^3 \; B^2 \; \delta^4 \qquad \nu_p^{Sync} \sim 3.2\times10^6 \; (\gamma_{3p})^2 \; B \; \delta$$

Synchrotron emission Estimate of s from spectral index

Tutorial 2

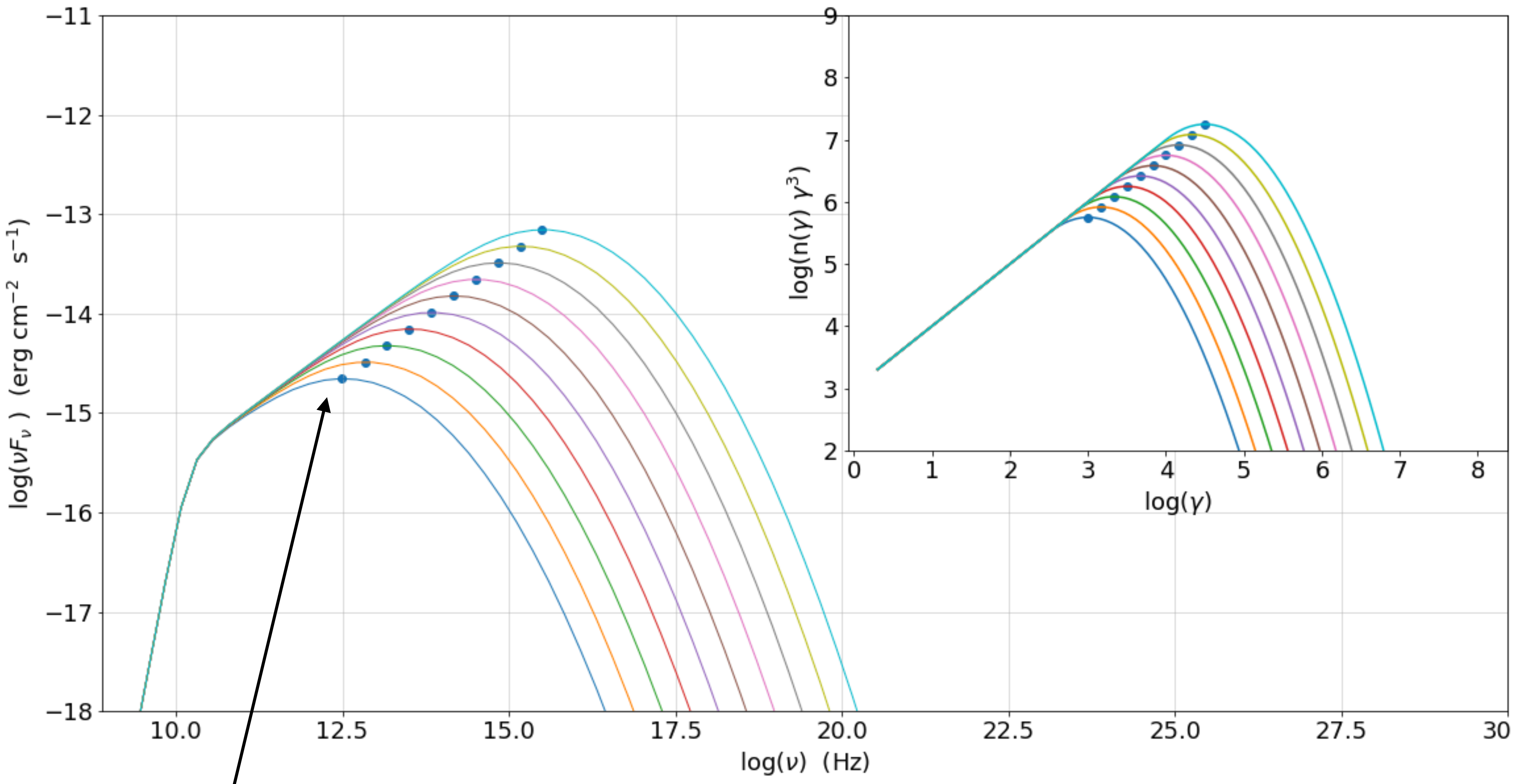






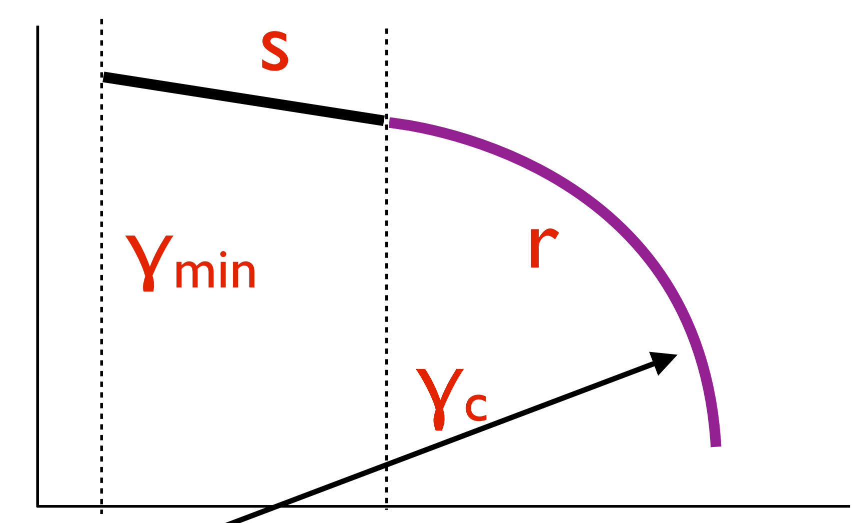
$$j^S_\nu(\nu) = \frac{1}{4\pi}\int_{\gamma_{min}}^{\gamma_{max}} P(\nu,\gamma)N(\gamma)d\gamma \ \propto \nu^{-\frac{s-1}{2}}$$

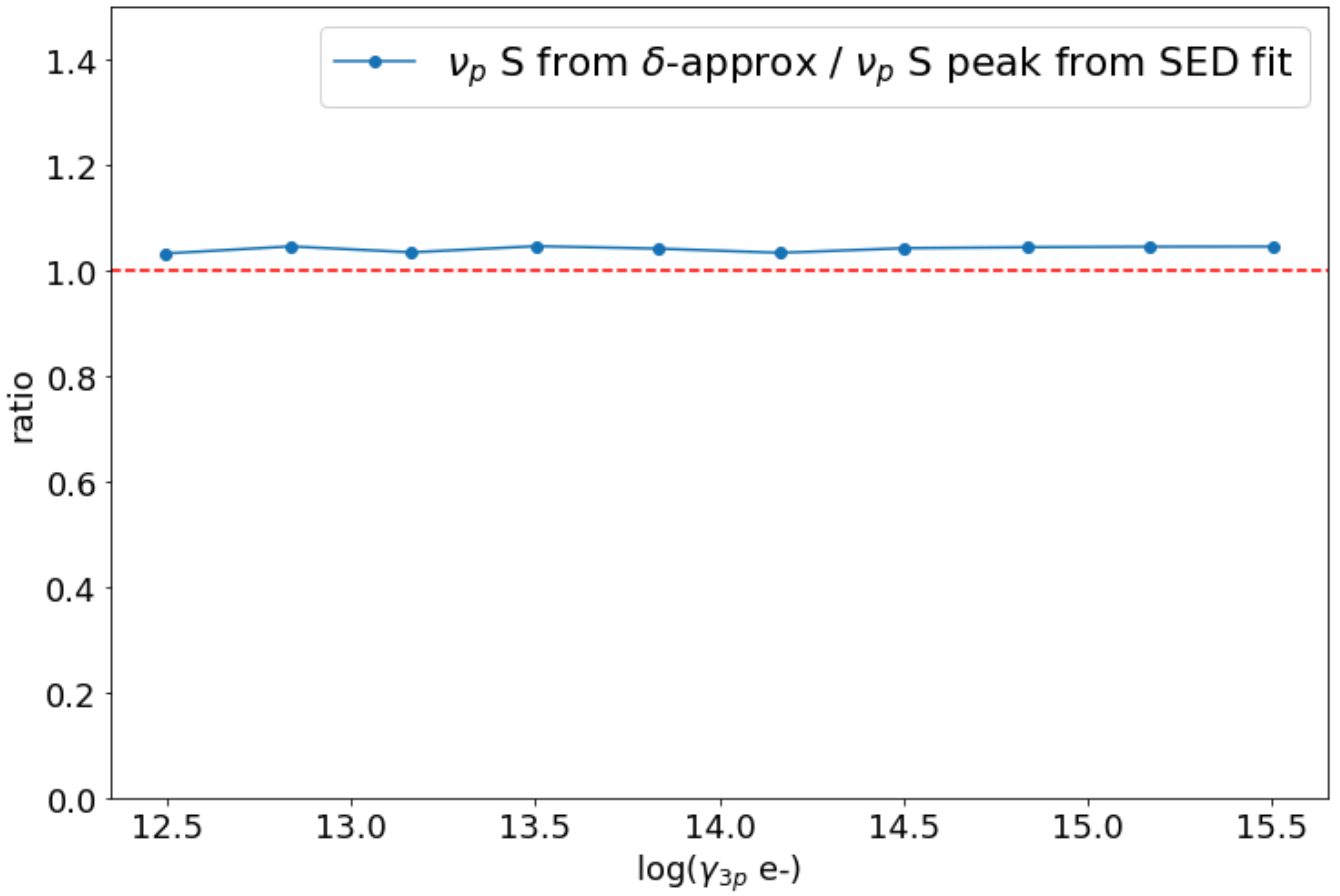
Synchrotron emission Estimate of $\gamma_p{}^s$ from $\nu_p{}^s$ Tutorial 2



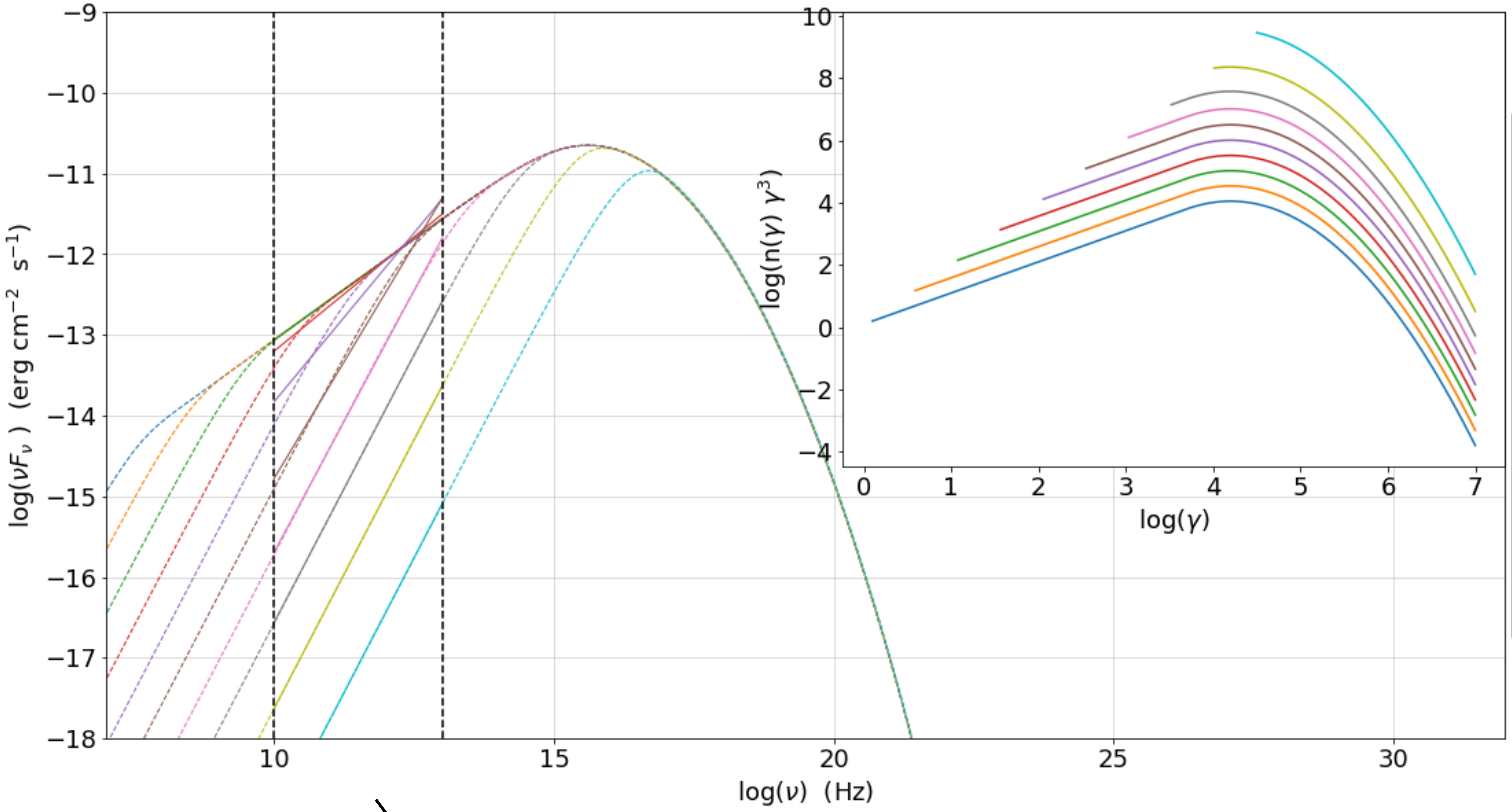
$$\nu_p^{Sync} \sim 3.2 \times 10^6 \ (\gamma_{3p})^2 \ B \ \delta$$

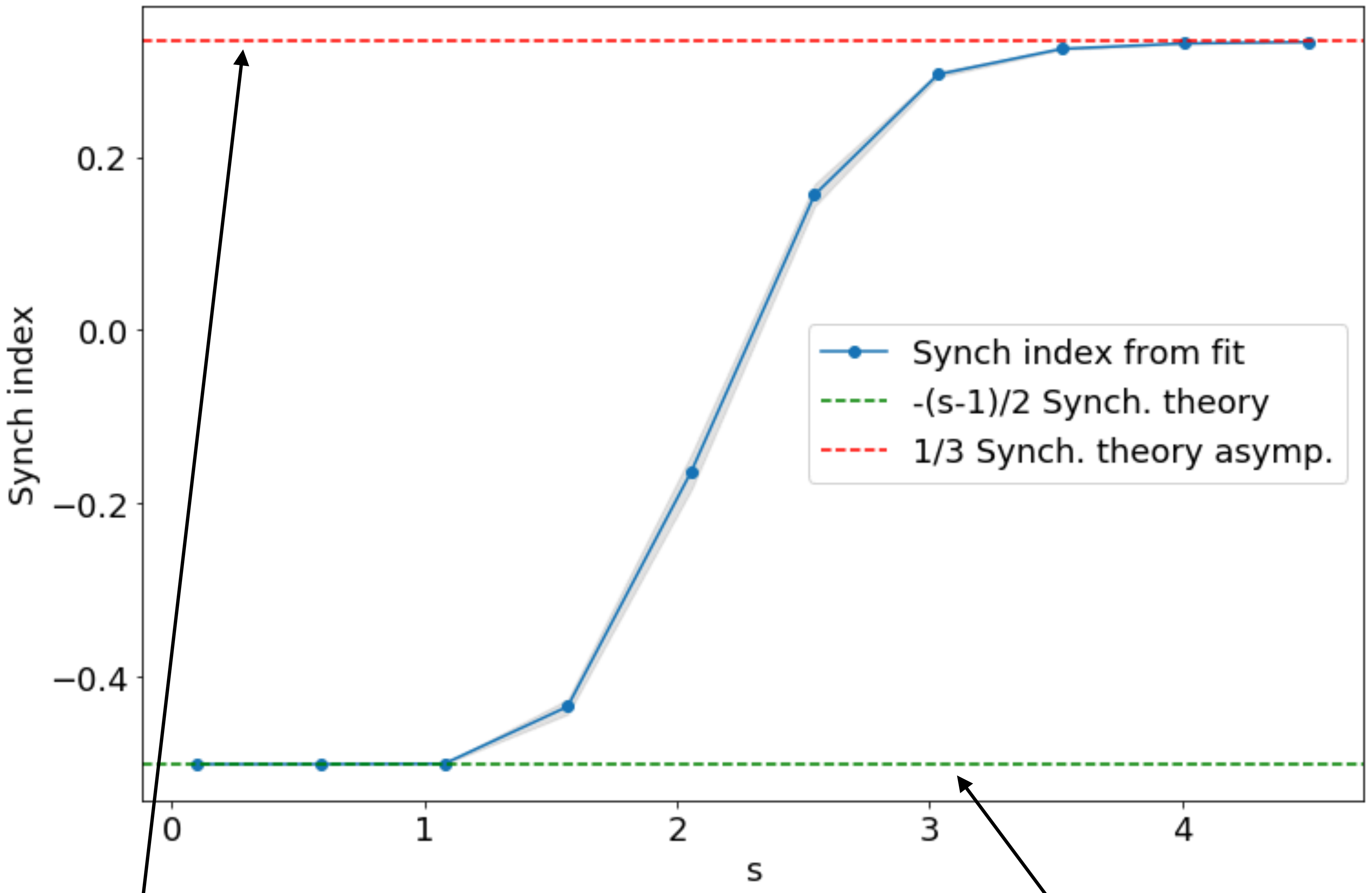
$$S_p^{Sync} \sim \frac{dN(\gamma)}{d\gamma}\gamma_{3p}^3 \ B^2 \ \delta^4$$



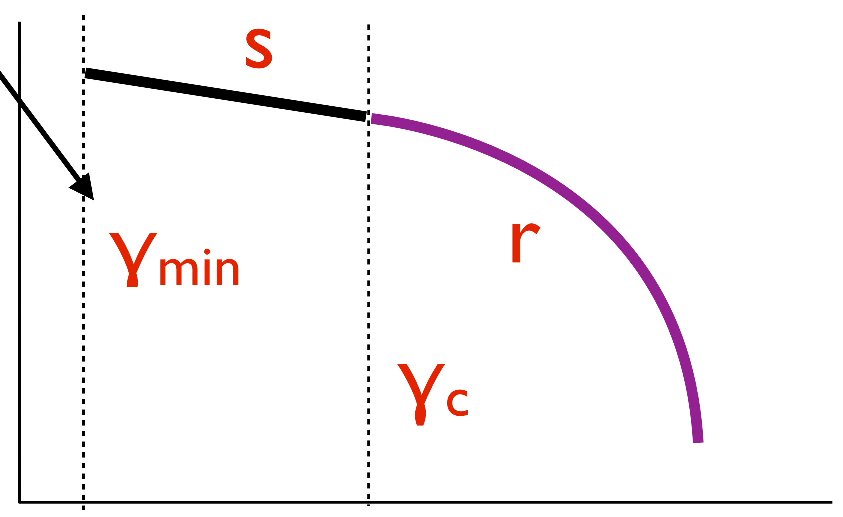


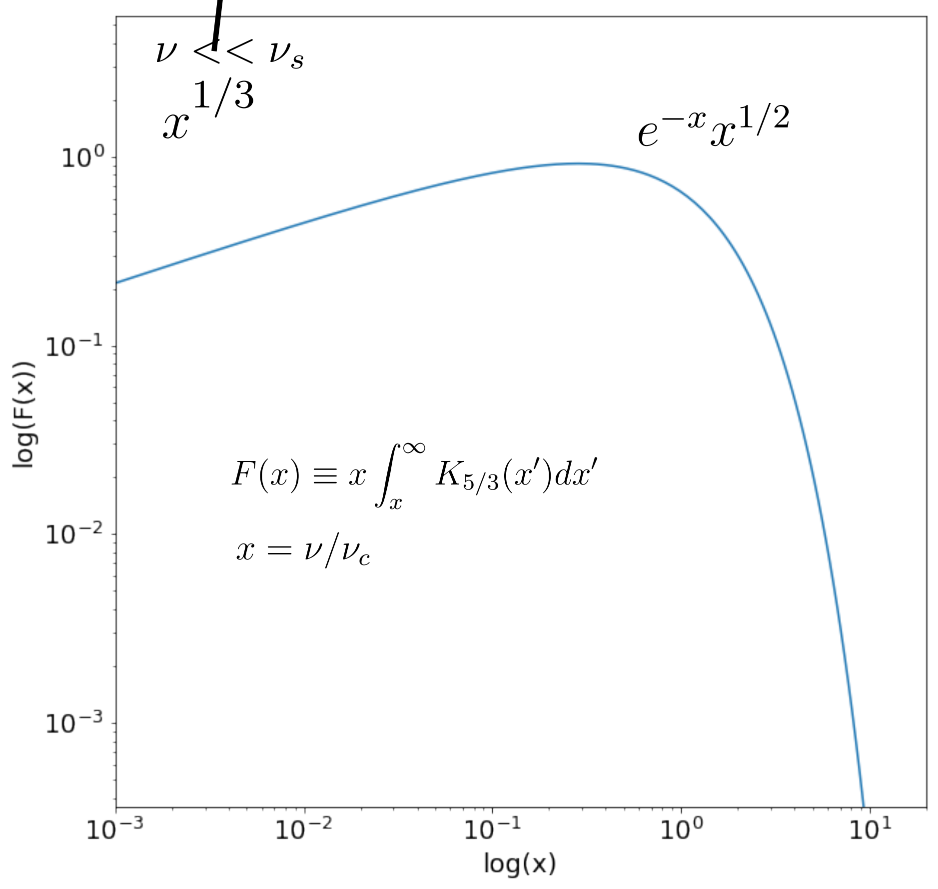
Tutorial 2 Synchrotron emission Estimate of γ_{min} from spectral index





s is fixed

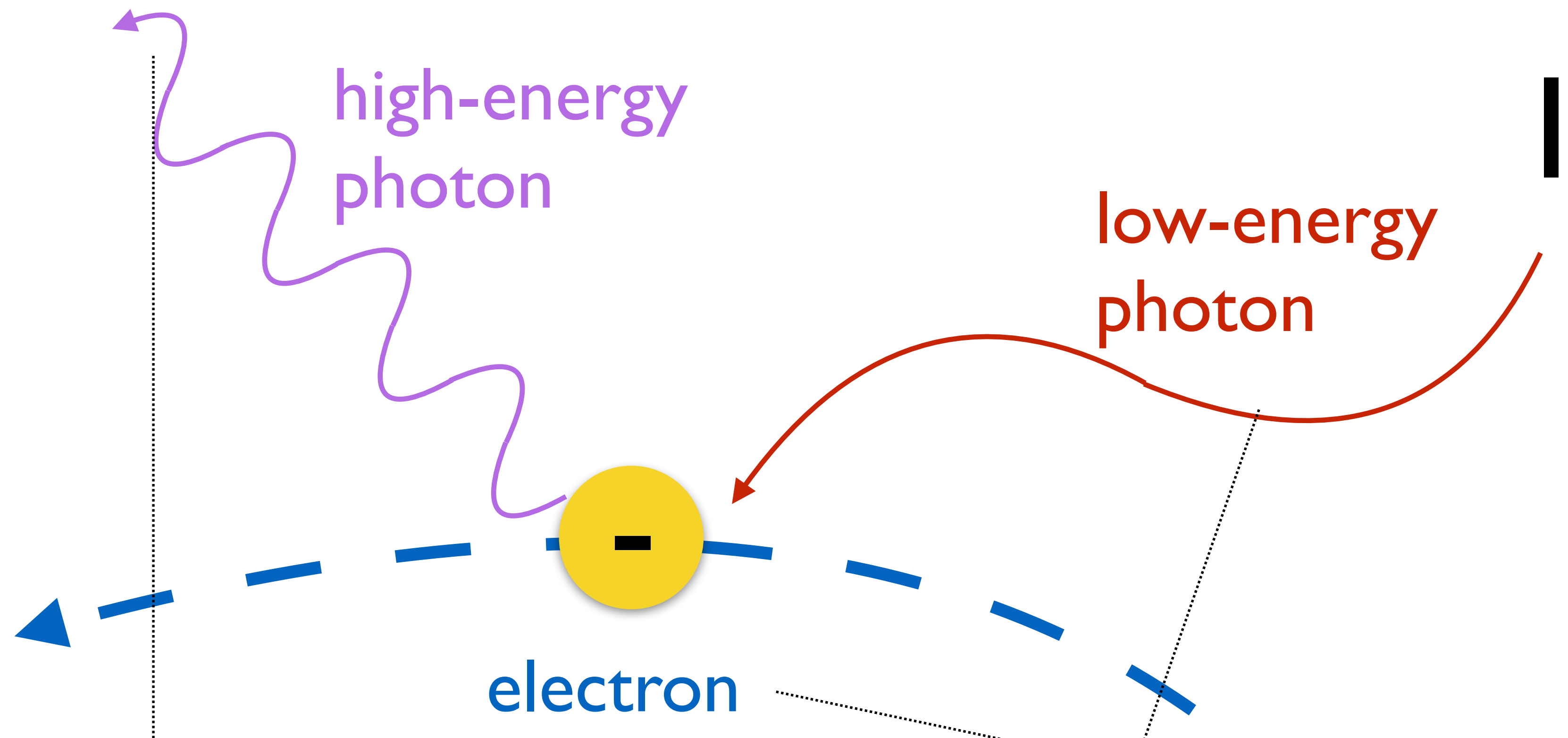


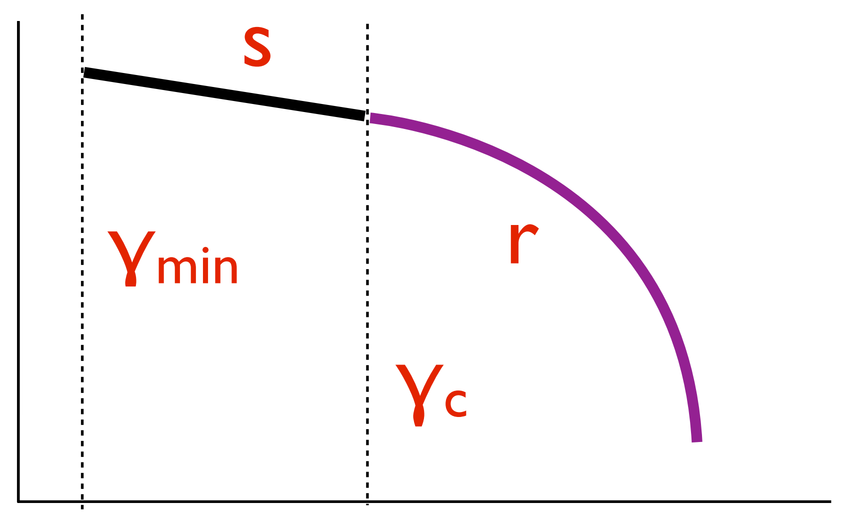


$$j_\nu^S(\nu) = \frac{1}{4\pi}\int_{\gamma_{min}}^{\gamma_{max}} P(\nu,\gamma)N(\gamma)d\gamma \propto \nu^{-\frac{s-1}{2}}$$

IC emission basics

Tutorial 2



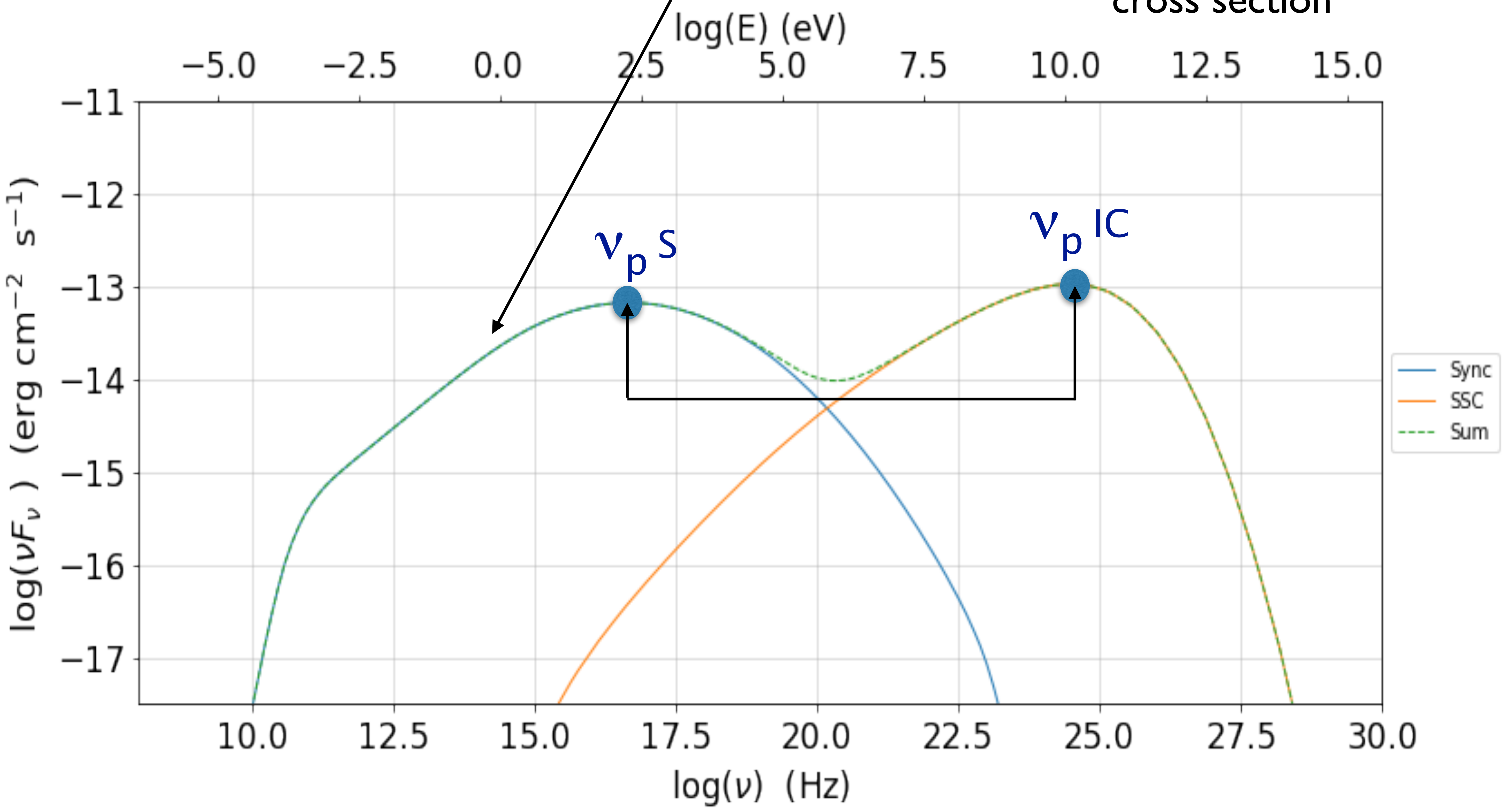


$$j_\nu^{SSC} = \frac{3}{4}\frac{h\nu\sigma_T c}{4\pi}\int_0^\infty n(\nu_0)d\nu_0 \int_{\gamma_{min}}^{\gamma_{max}} \frac{N(\gamma)}{\gamma^2} F_c(\gamma,\nu,\nu_0)d\gamma$$

cross section

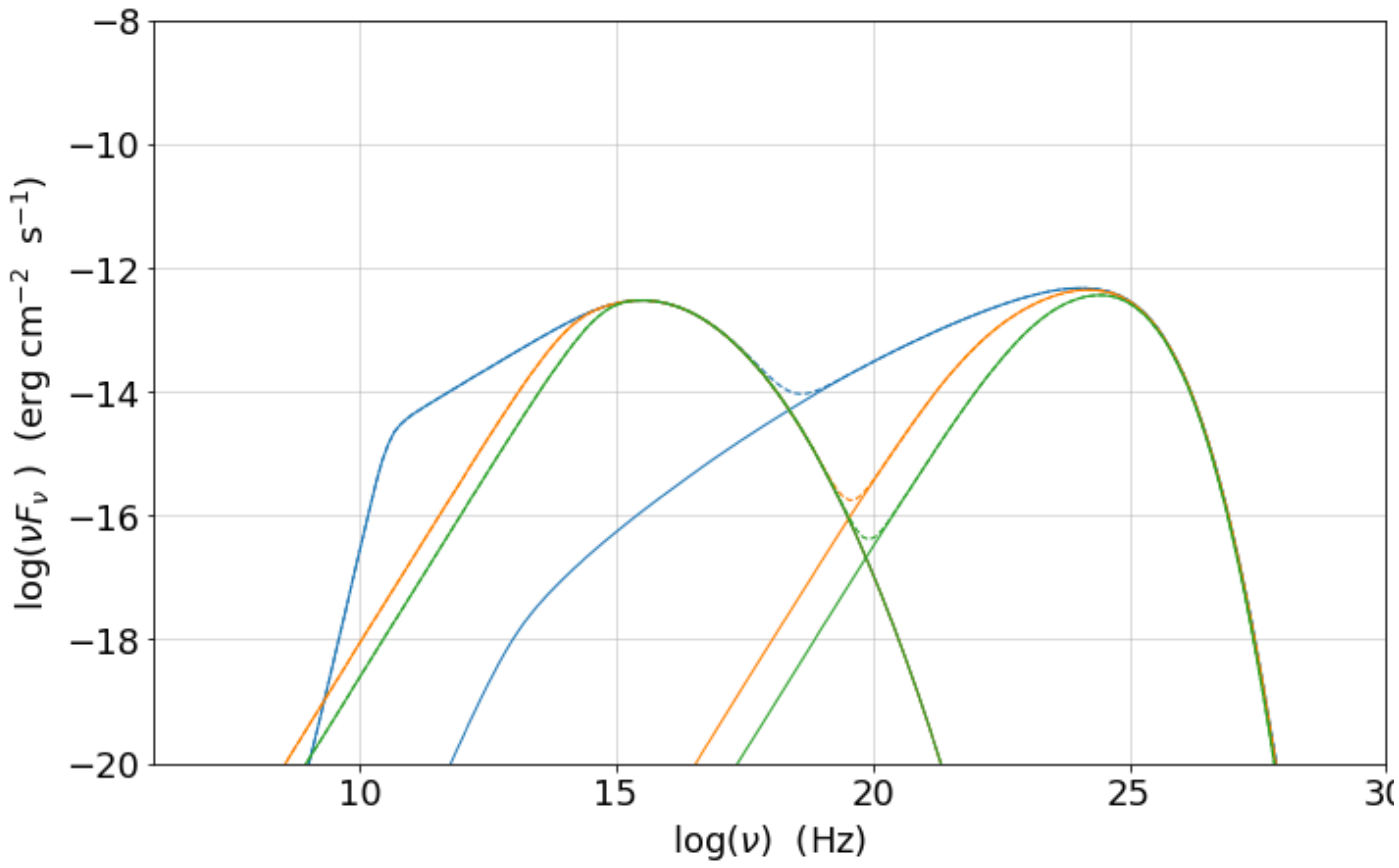
soft photon energy
in the e- rest frame

$$\epsilon' = \frac{h\nu'}{m_e c^2}$$

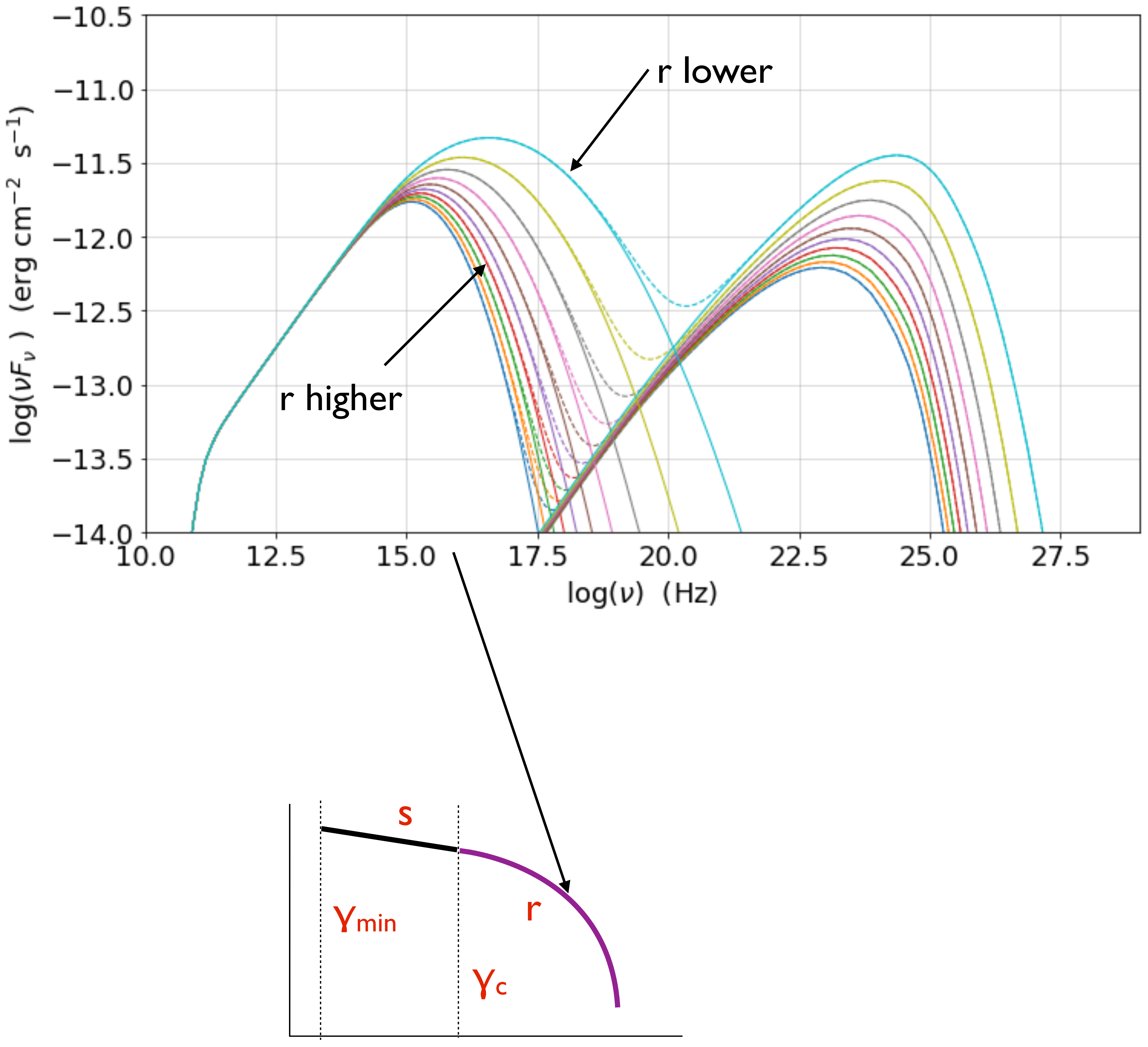


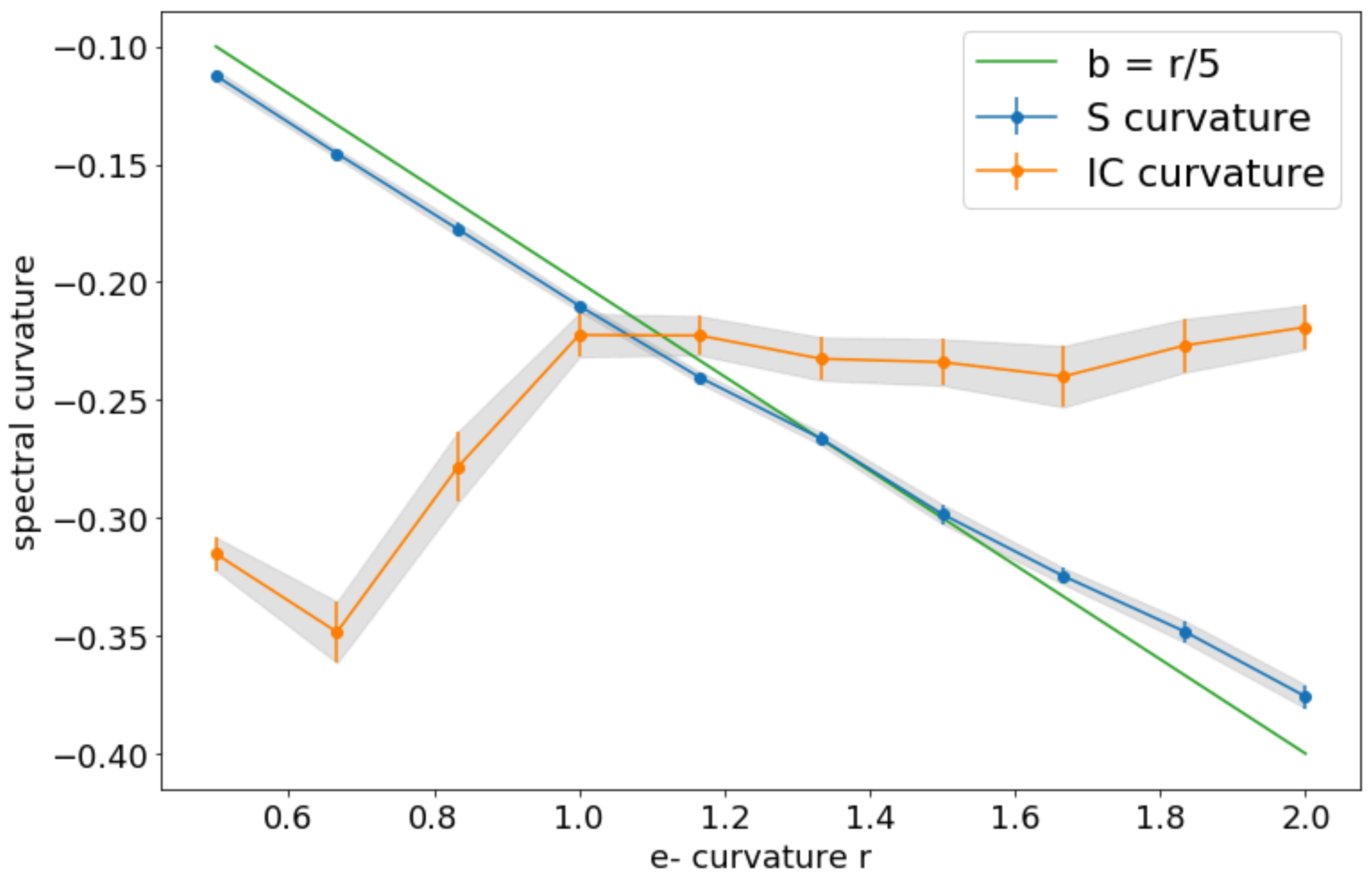
$\epsilon' << m_e c^2$	$\epsilon' \geq m_e c^2$
TH regime	**KN regime**
$\nu_p^{IC} / \nu_p^{S} \sim (4/3)\,\gamma_p^2$	$\nu_p^{IC} / \nu_p^{S} \sim \gamma_p$
	$h\nu_p^{IC} \sim m_e c^2 \gamma_p$

Adding the IC emission SSC case



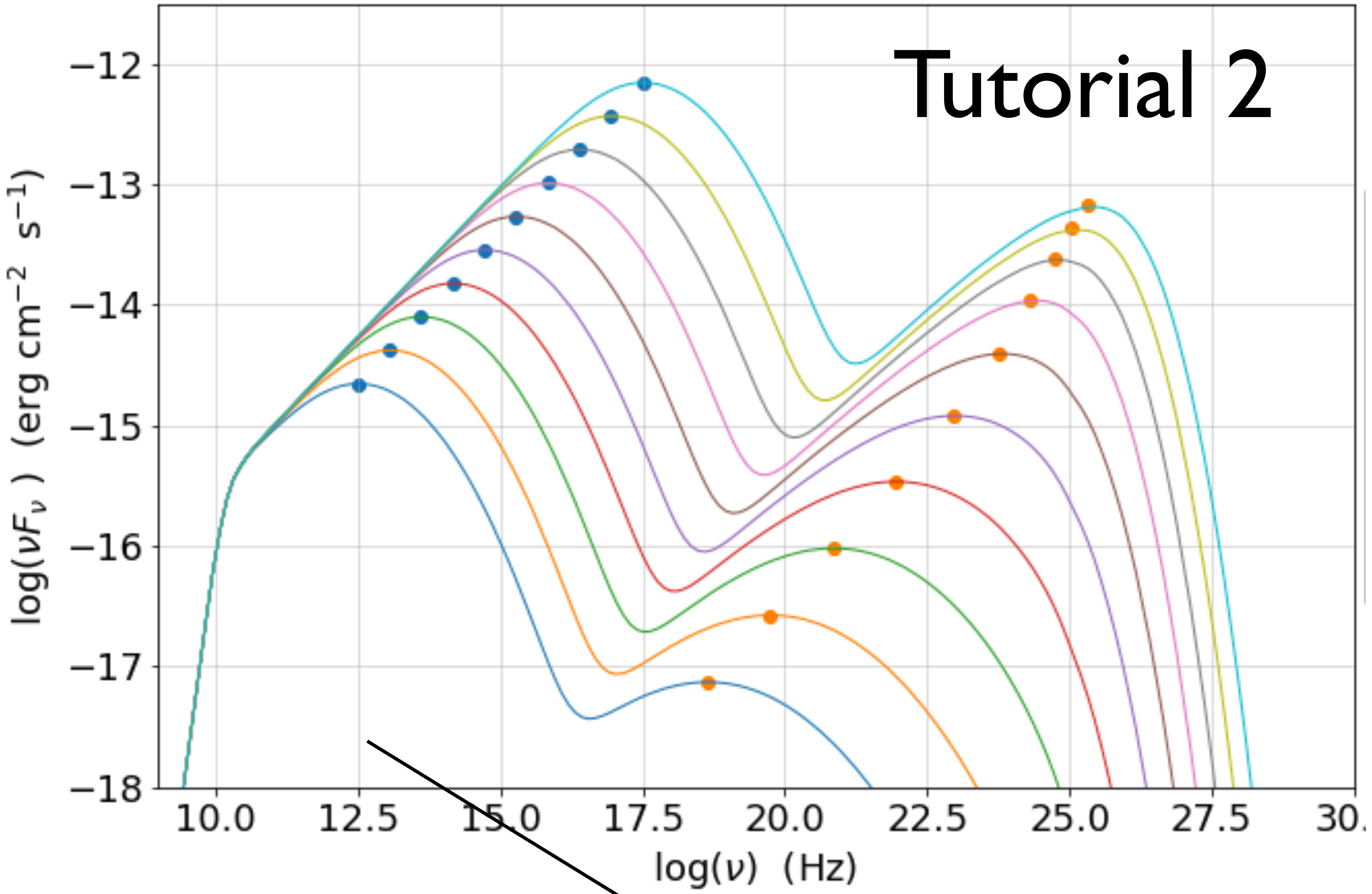
IC emission TH/KN regime and peak curvature Tutorial 2

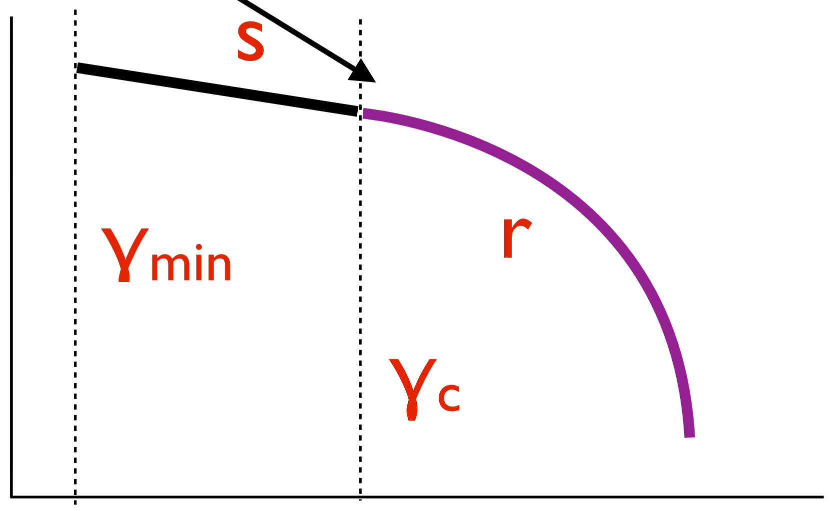


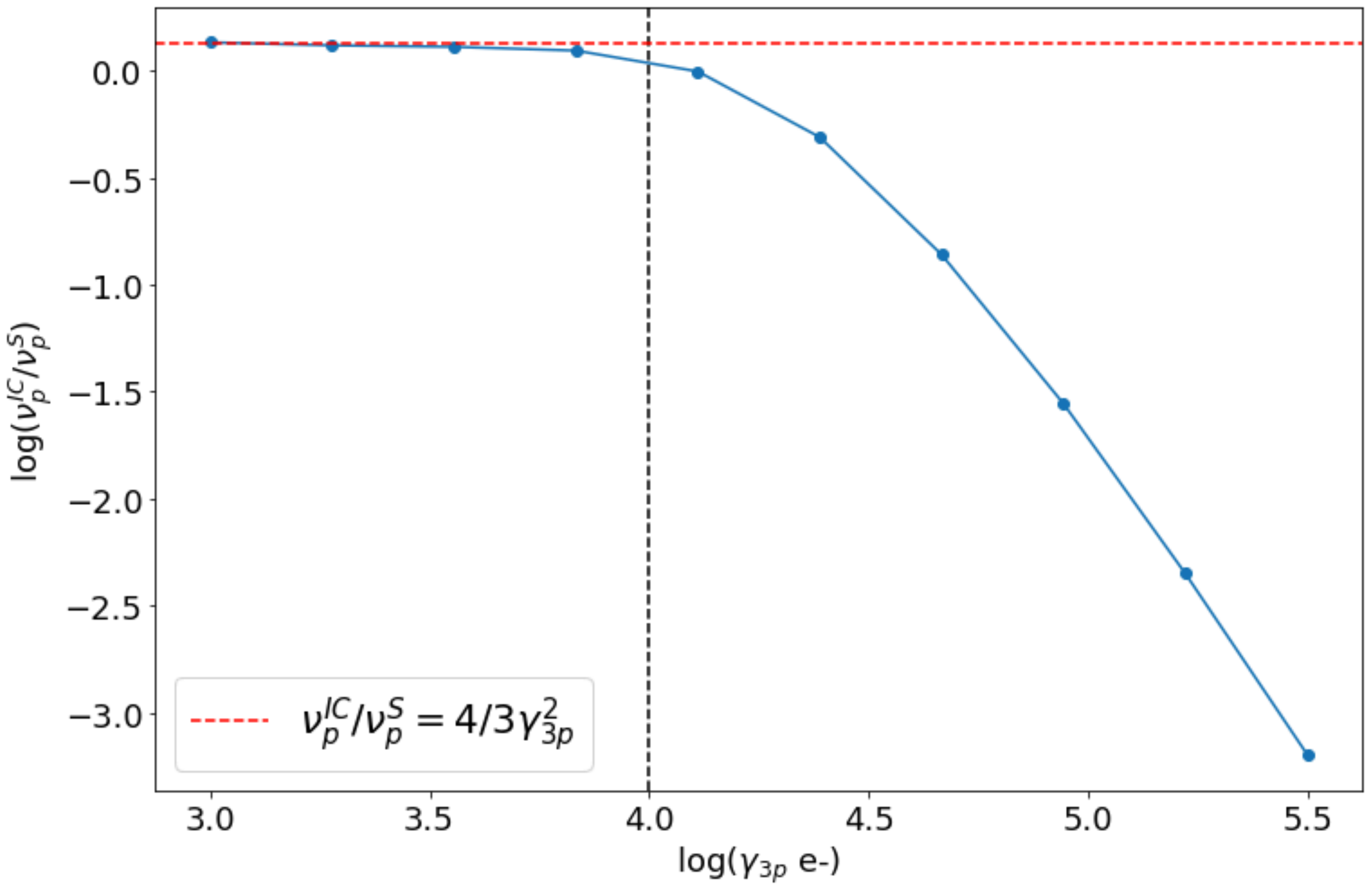


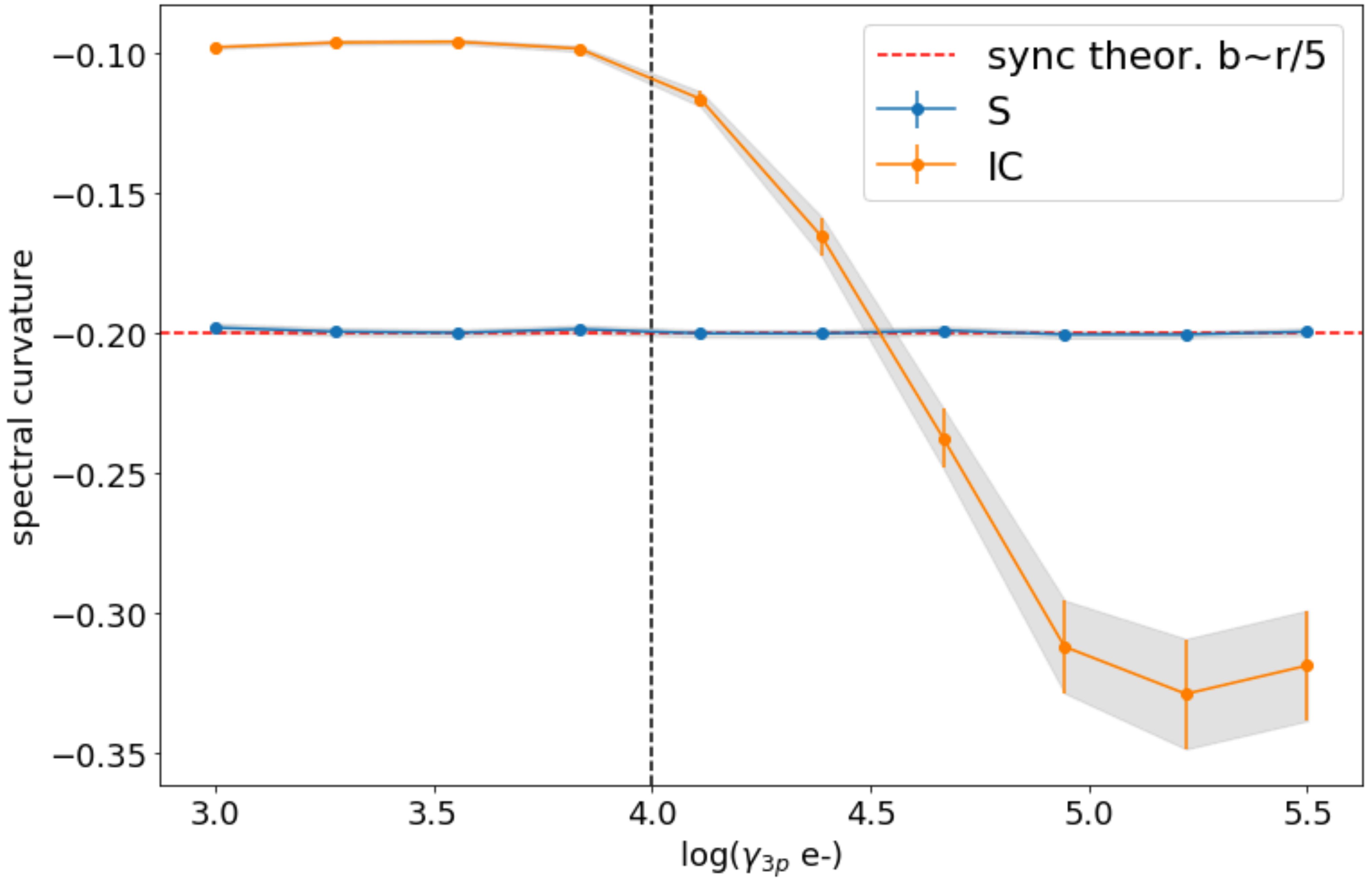
IC emission TH/KN regime and peak freq.

• $\nu_{p}^{IC} / \nu_{p}^{S} \sim (4/3)\,\gamma_p^2 \equiv \gamma^2{}_p^{IC}$ is true only in TH regime

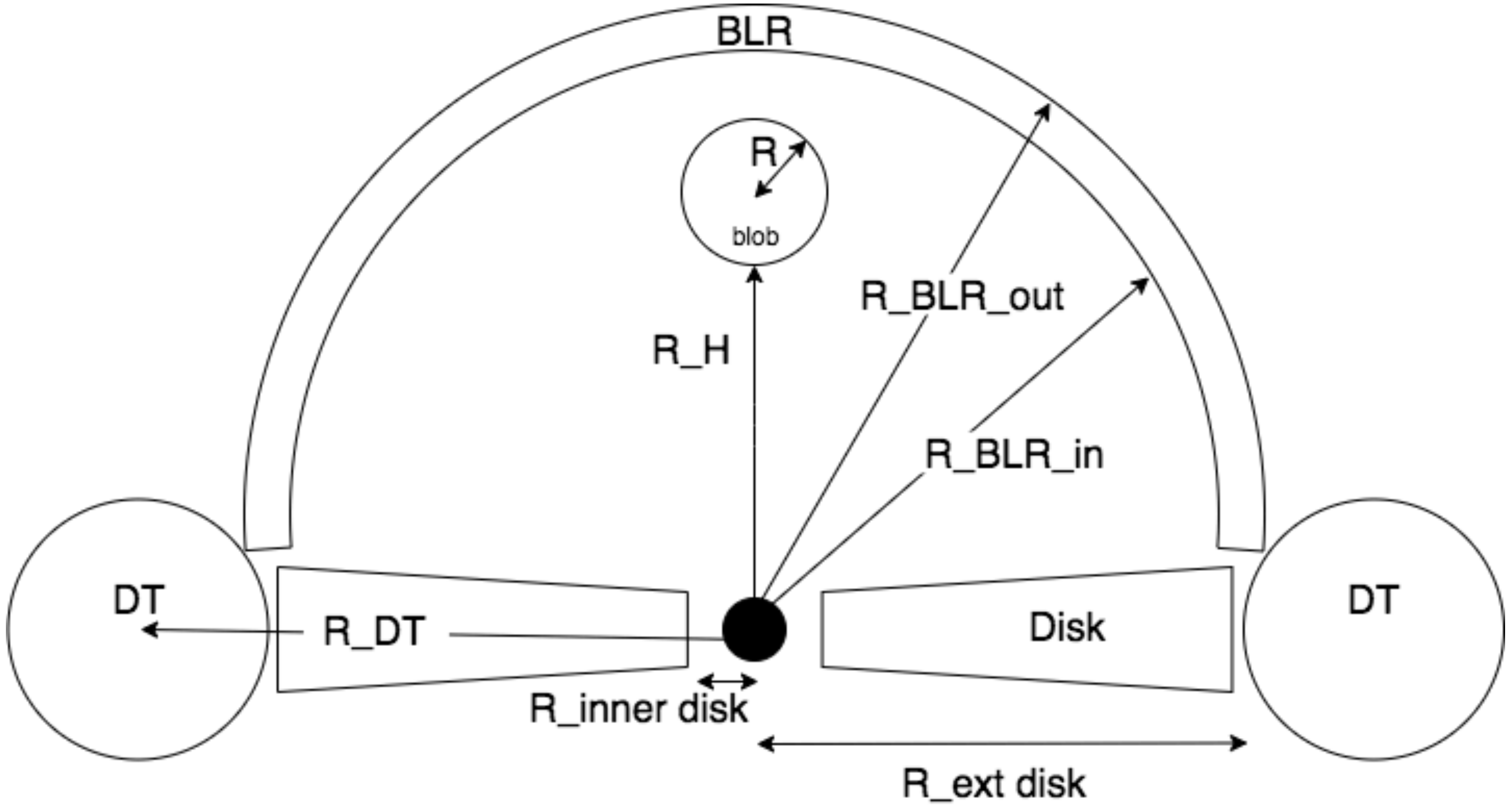


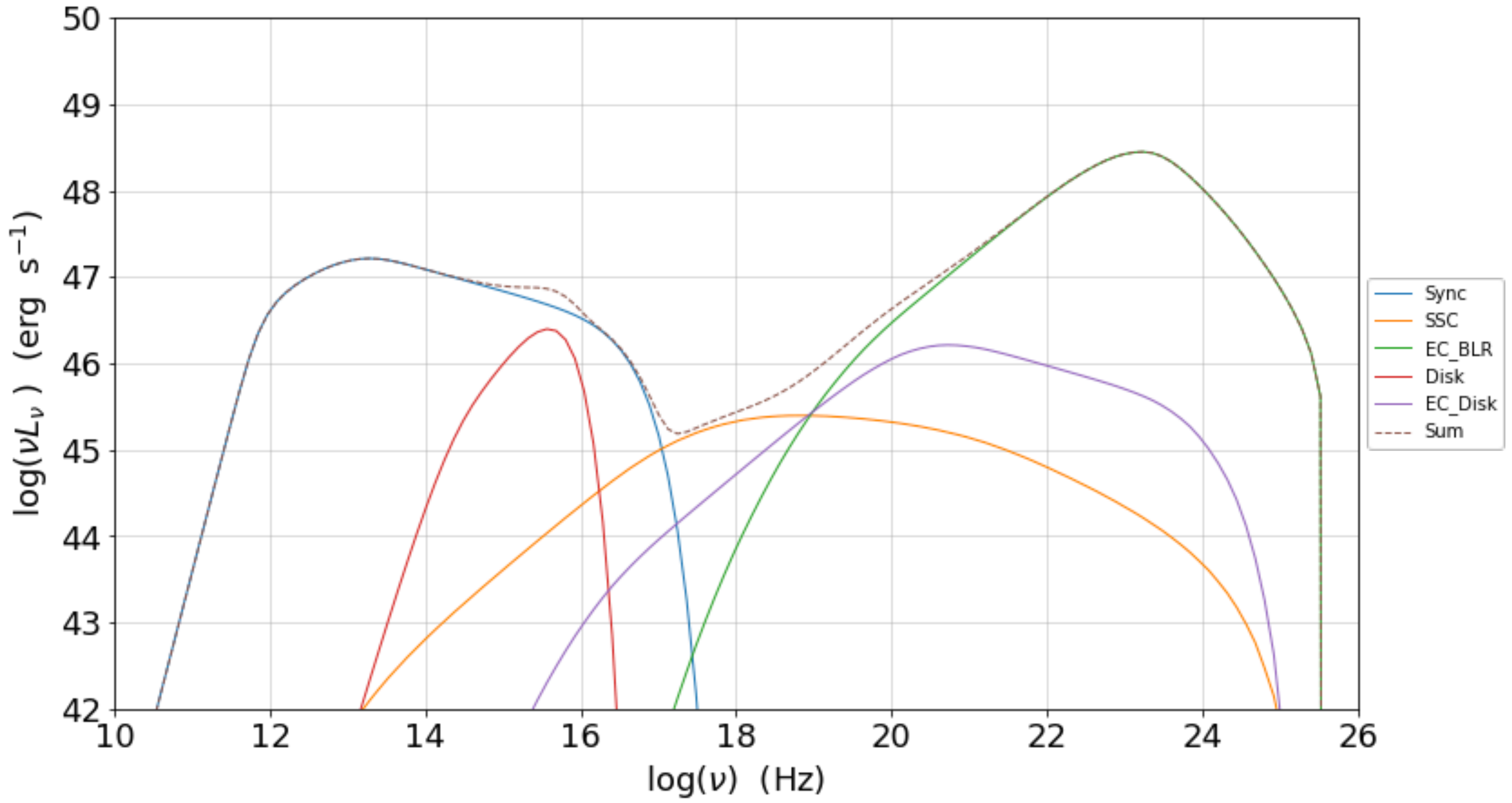






External Compton Scenario





Transformation of the radiative fields

$$\epsilon^{-3} I_\epsilon \text{ and } \epsilon^{-2} j(\epsilon, \Omega)$$

$$I_{\nu'} = \frac{1}{4\pi} \int d\Omega' \, \delta^3 I_{\nu=(\nu'/\Gamma)}$$
$$= \Gamma\tau \frac{L_{\rm nuc}}{4\pi R^2} f_{\nu=(\nu'/\Gamma)}(T_{\rm ext})$$

$$u'_{ext} \simeq \Gamma^2 u_{ext}$$

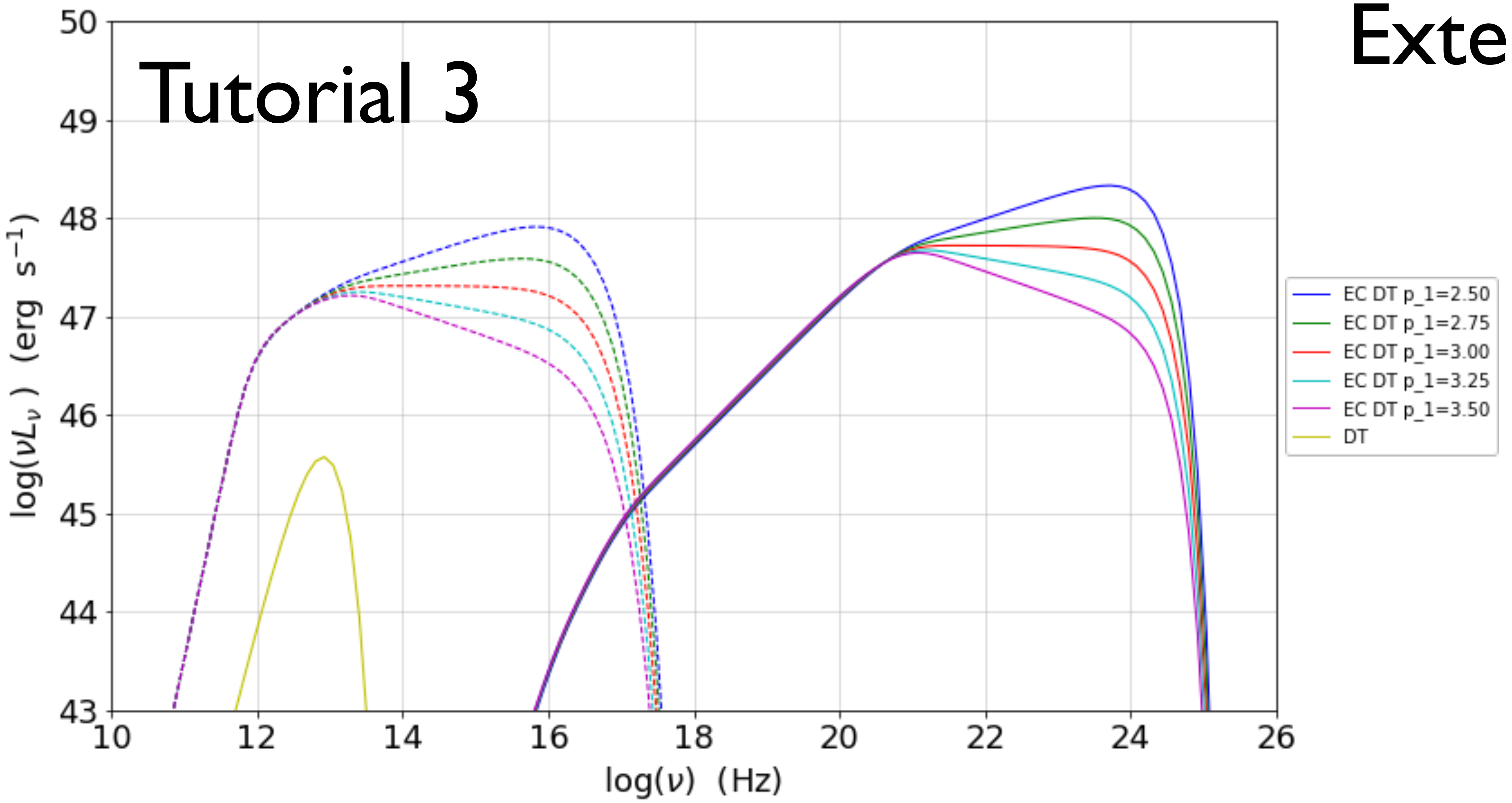
$$L_{ERC} \simeq \Gamma^6 U_{ext}$$

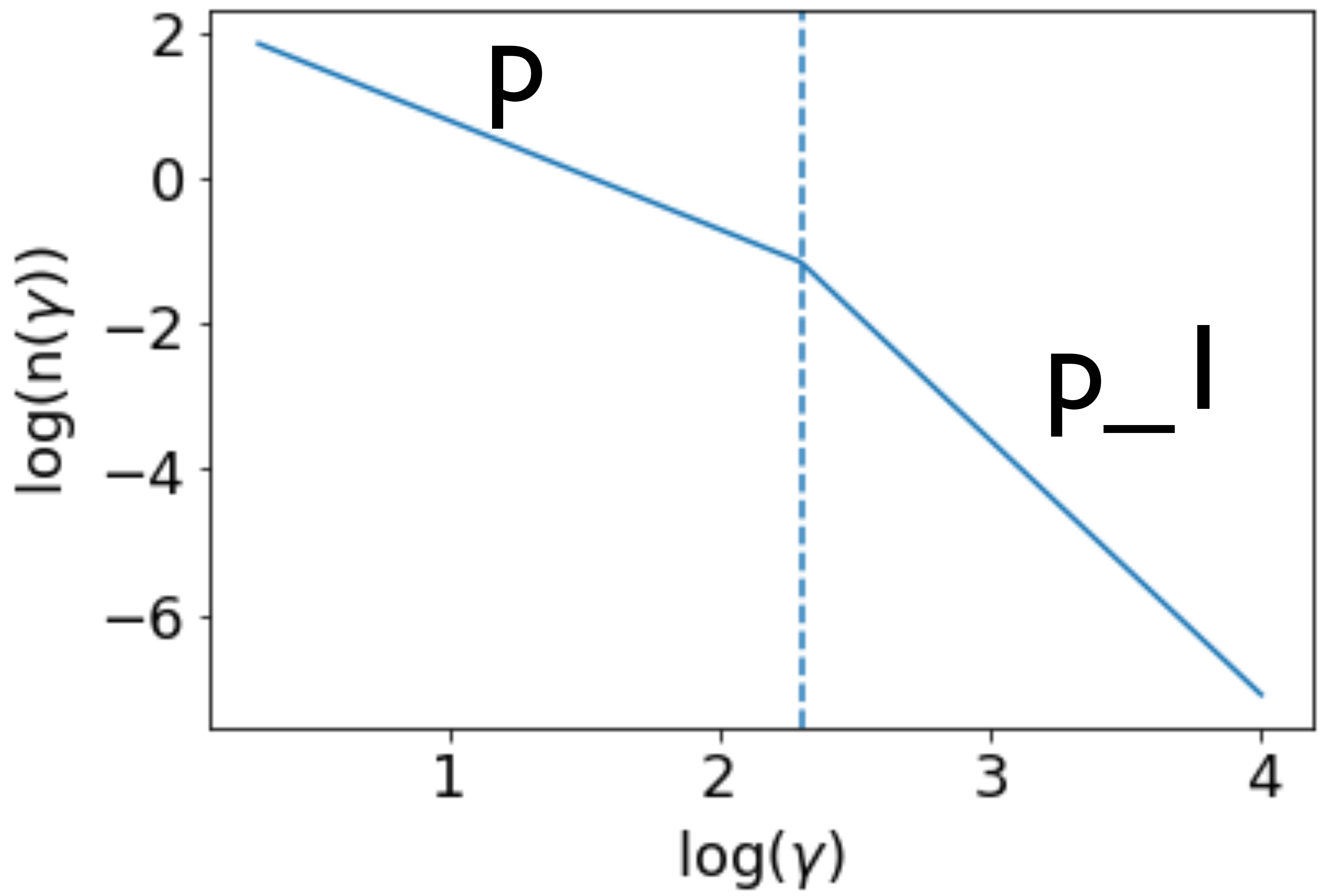
$$\eta = \frac{\dot{\gamma}_{IC}}{\dot{\gamma}_{sync}} = \frac{U_{ph}}{U_B}$$

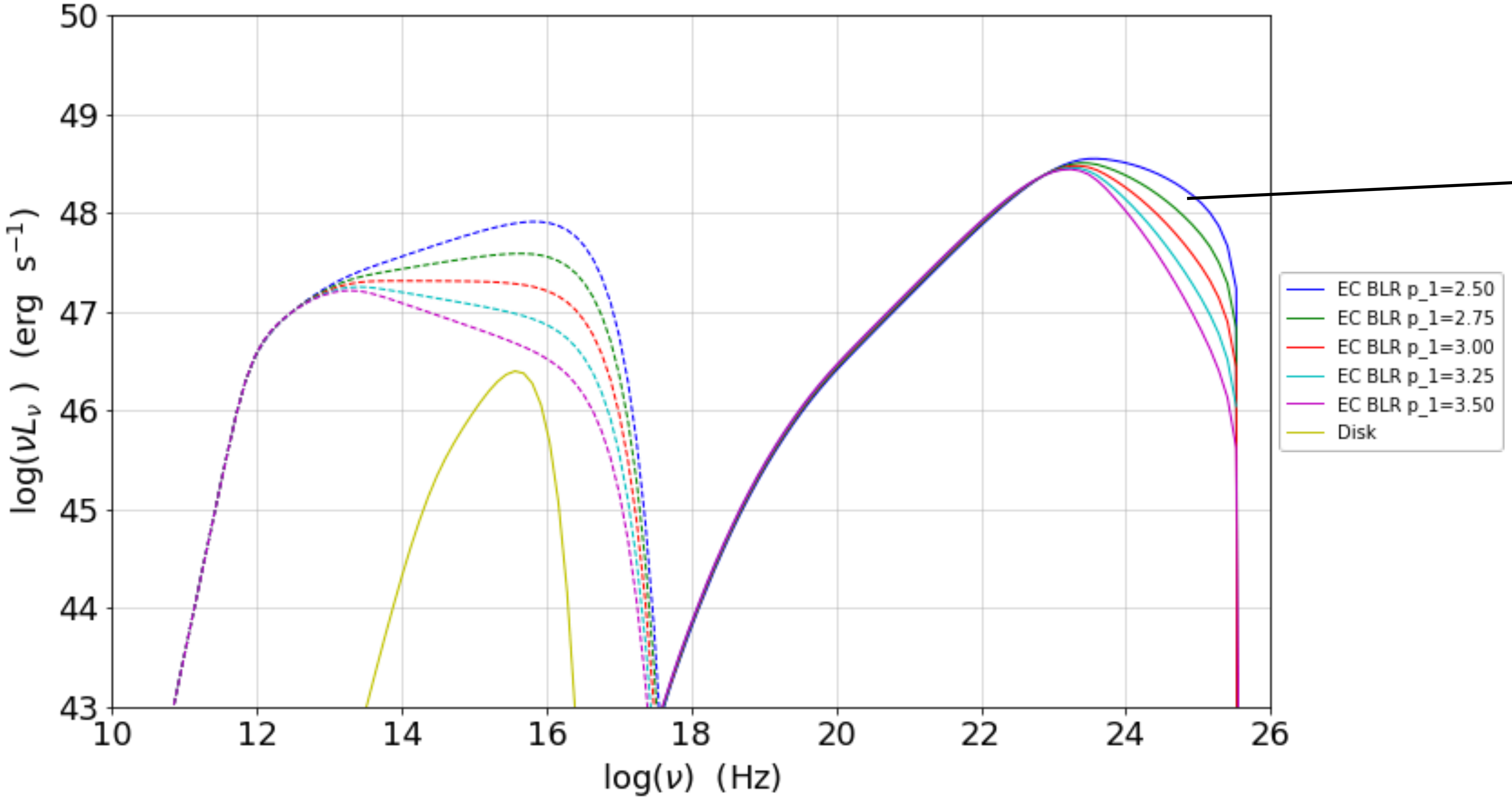
$$\frac{u(\epsilon, \Omega)}{\epsilon^3} = \frac{u'(\epsilon', \Omega')}{\epsilon'^3} = inv.$$

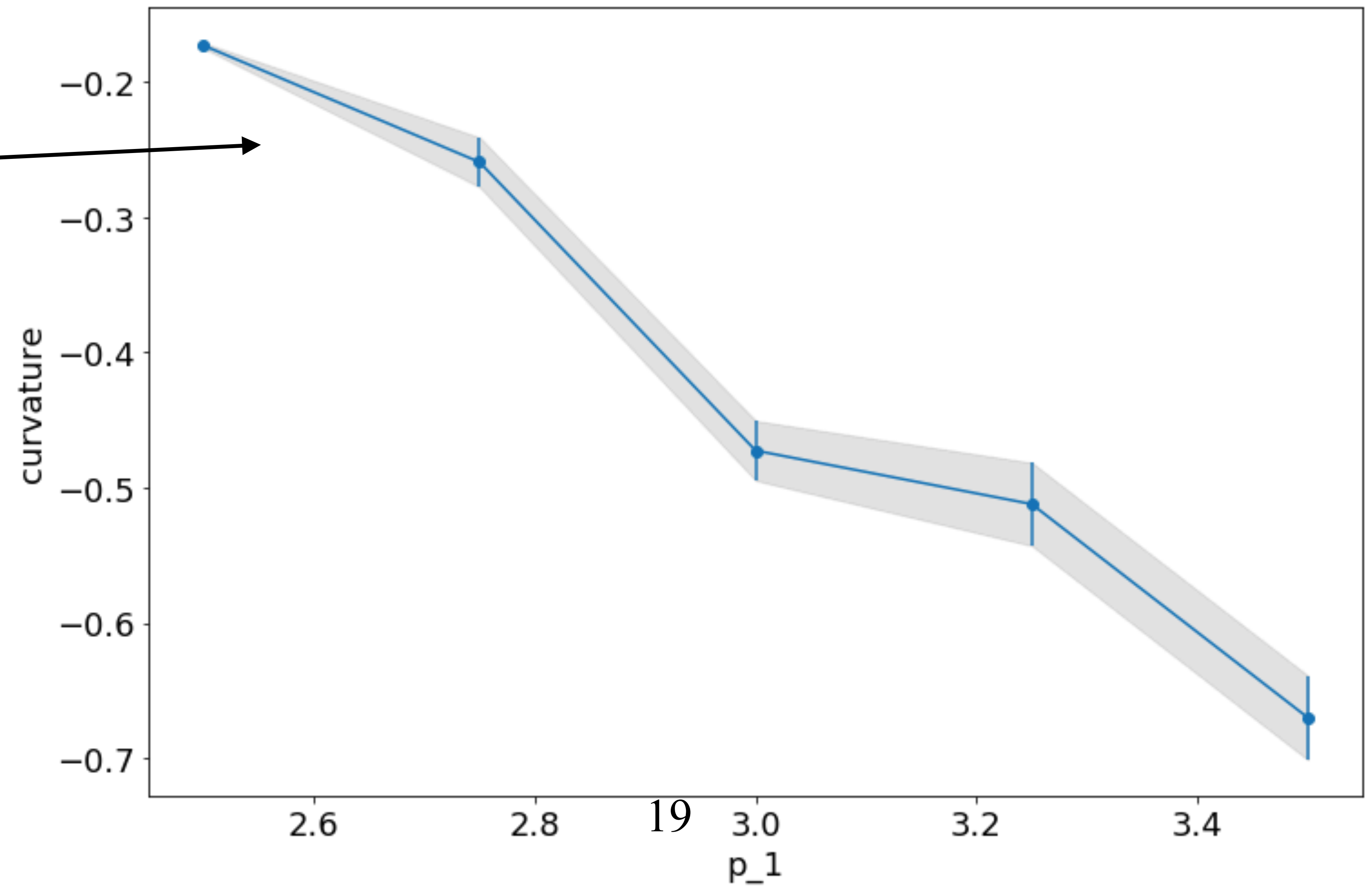
External Compton Scenario and TH/KN

Tutorial 3





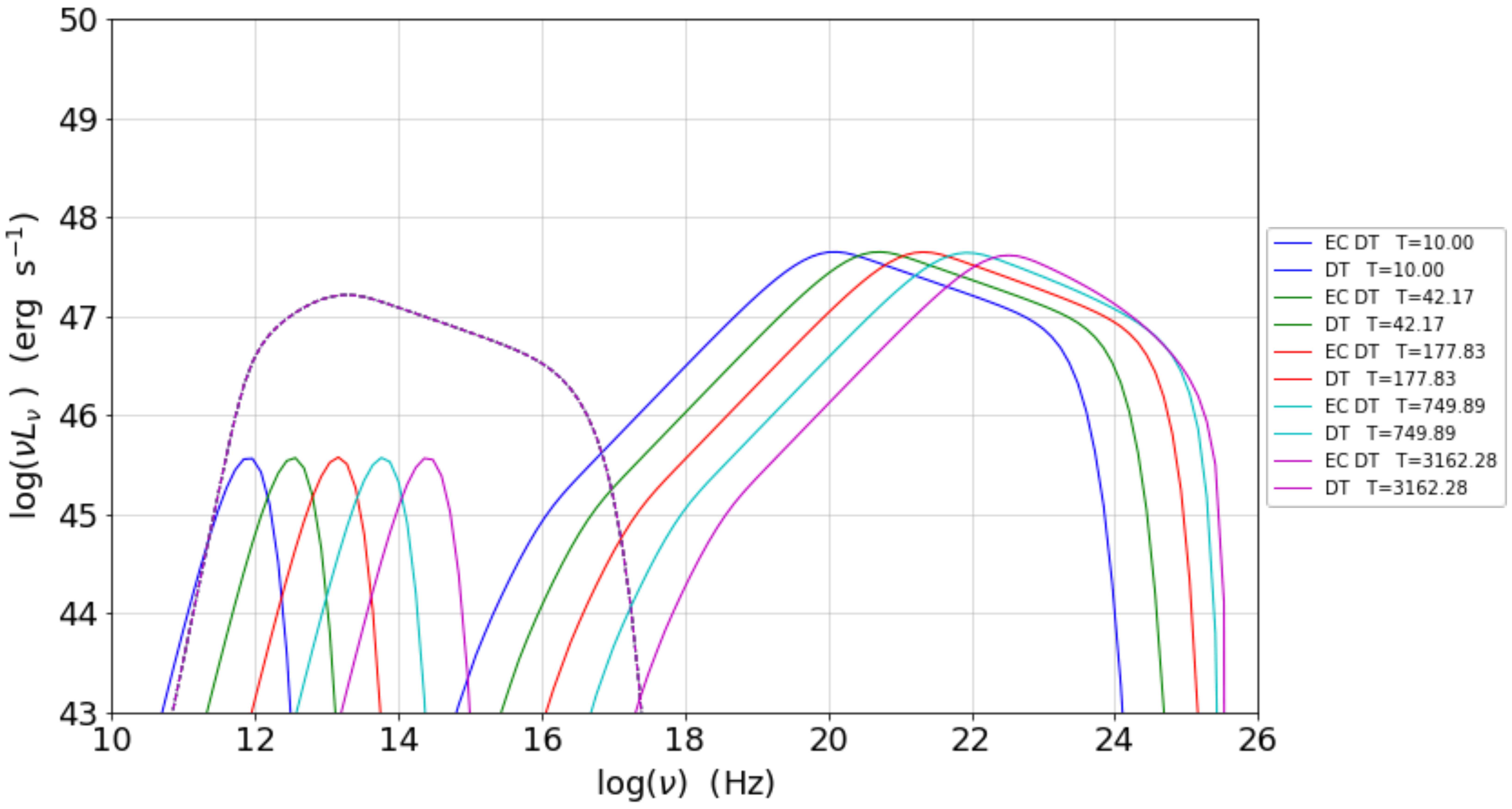




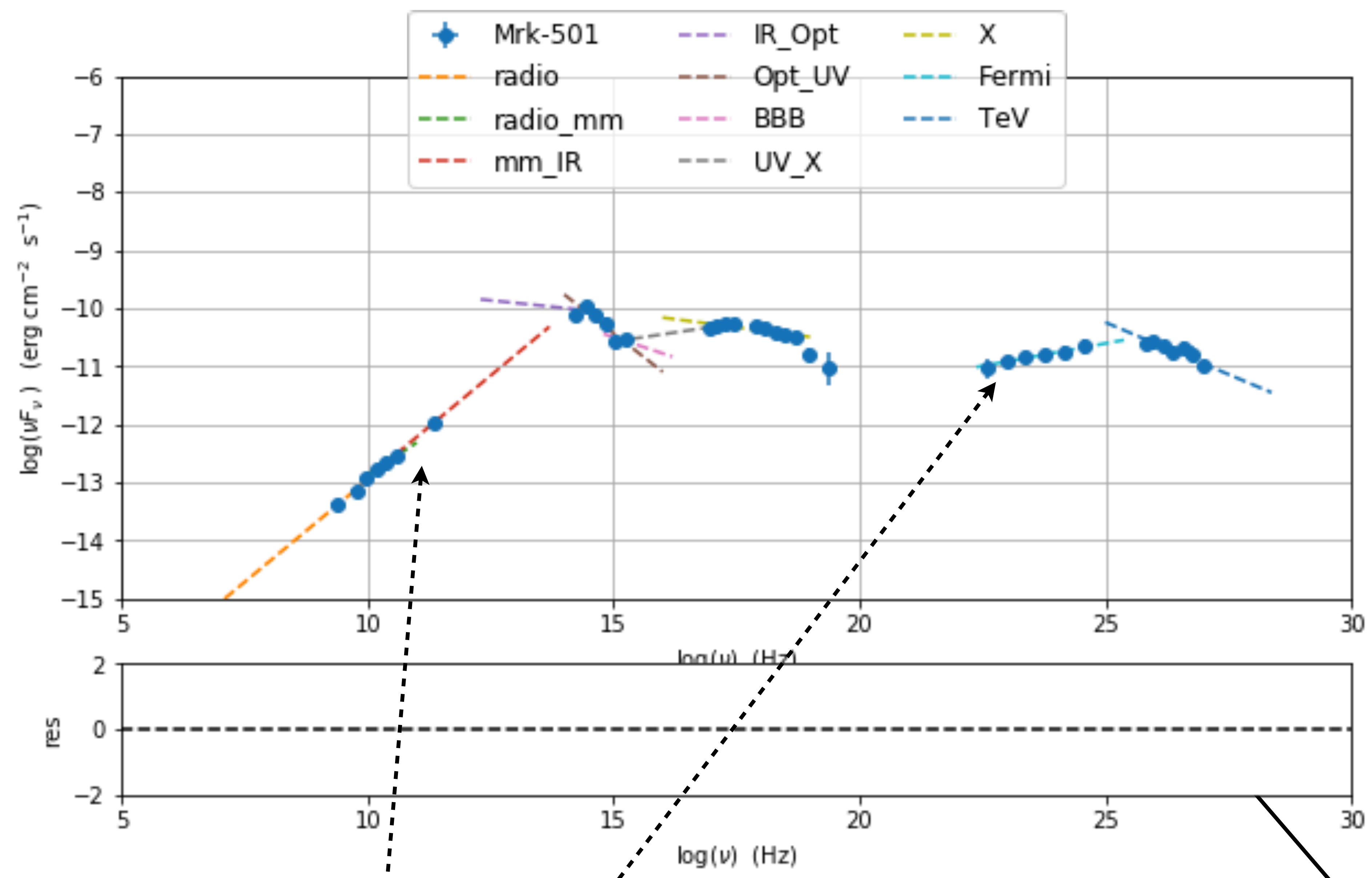
External Compton Scenario and external seed photons energy

$\nu_{p\,EC} \sim (4/3)\,\gamma^2\, \nu''_{p\,ext}\delta\Gamma\,/(1+z)$ $\qquad \nu'_{seed-IC} = \nu''_{p\,ext}\Gamma$

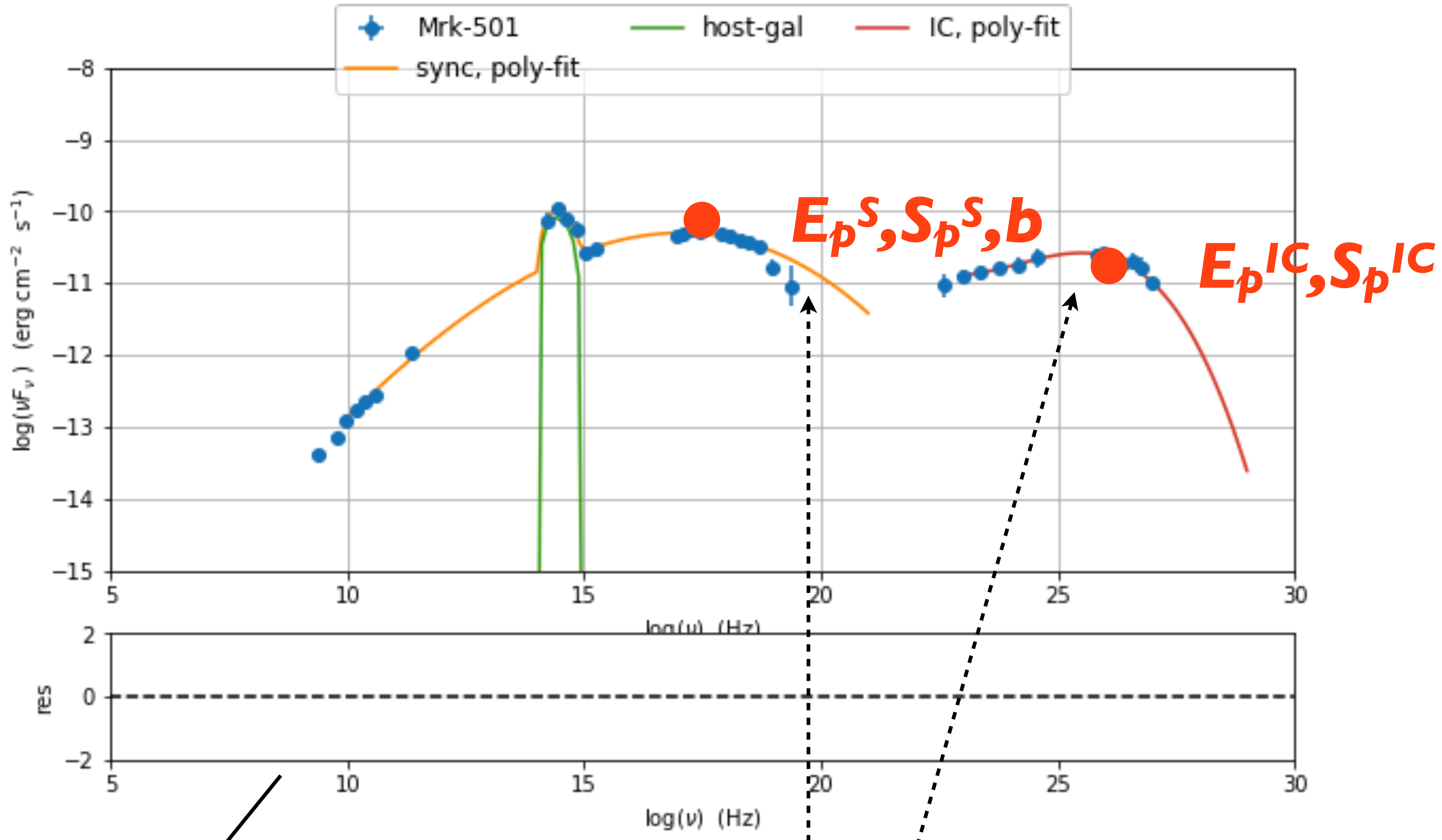
Tutorial 3

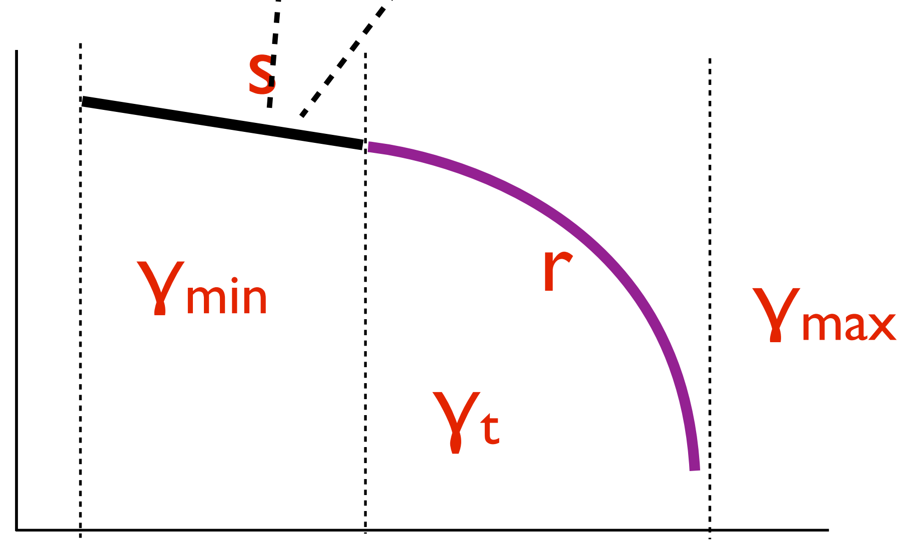


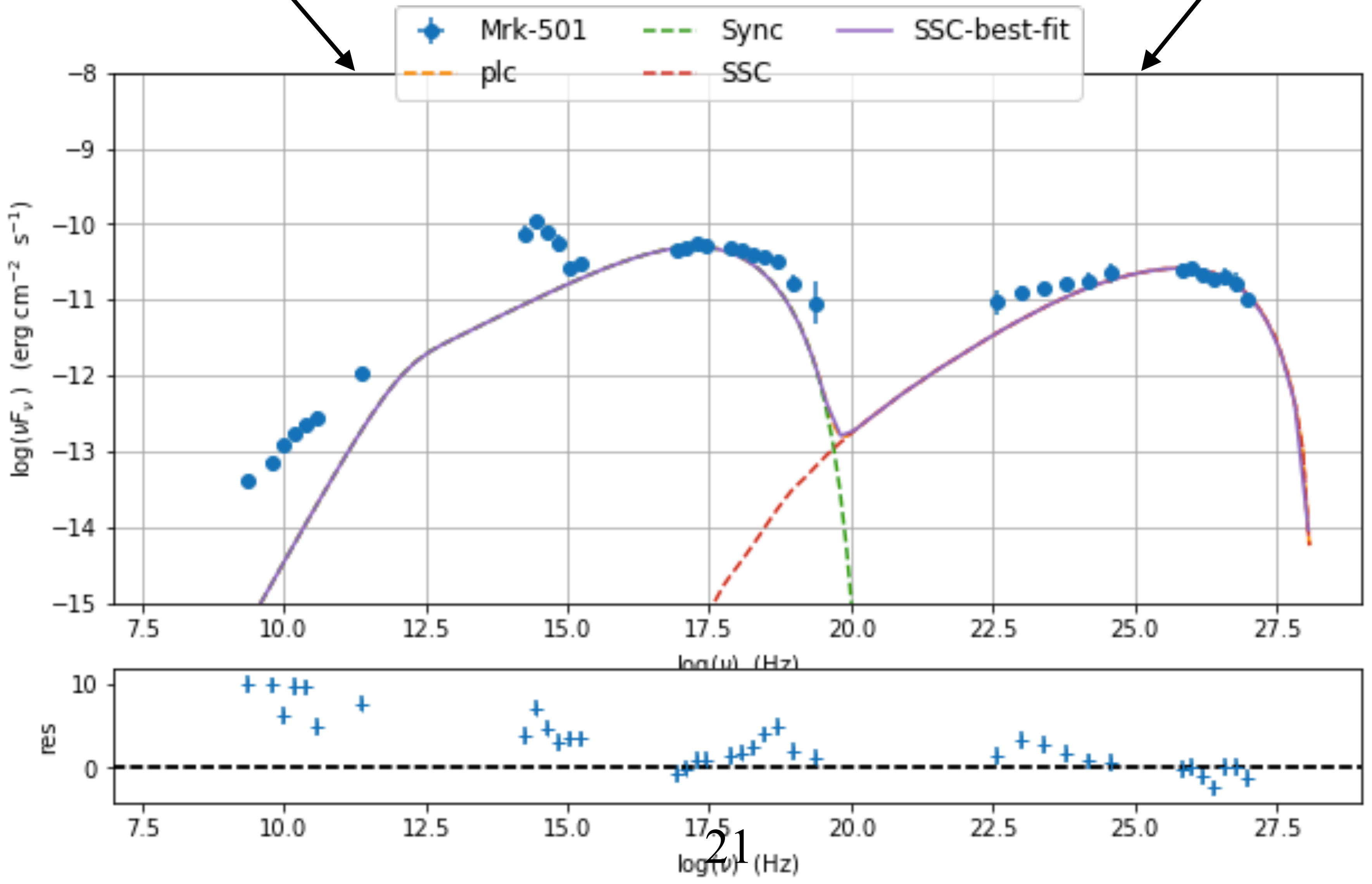
JetSeT SED shaping and model constraining (Mrk 501)

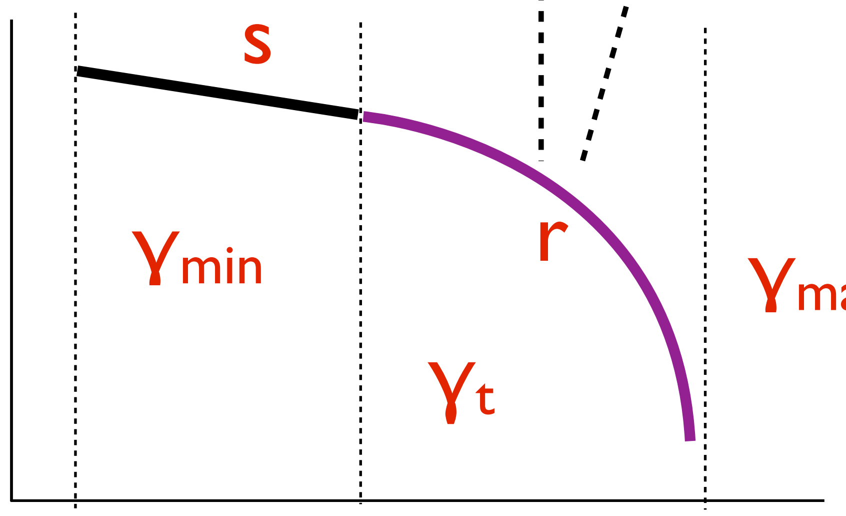


Tutorial 5



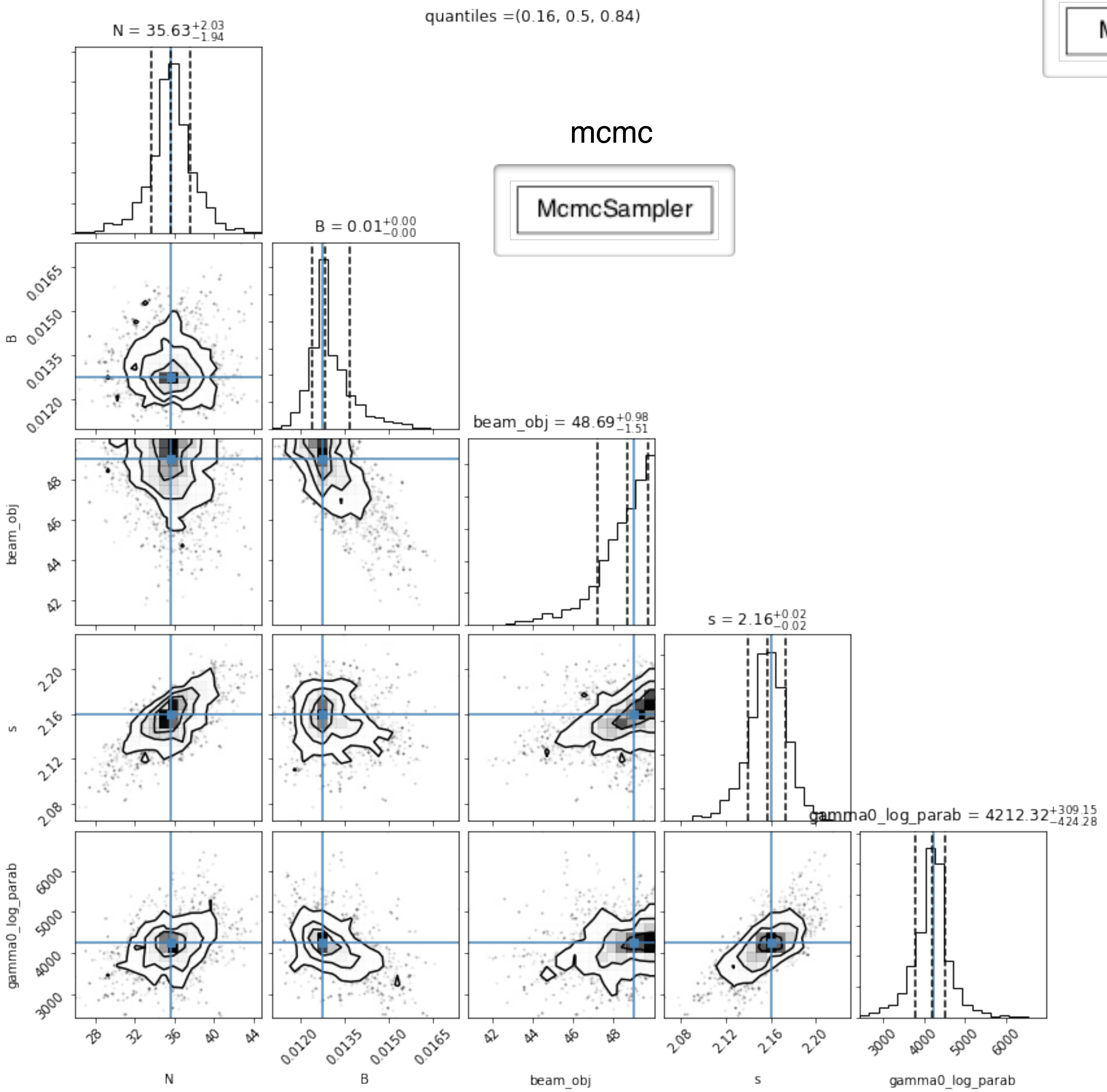


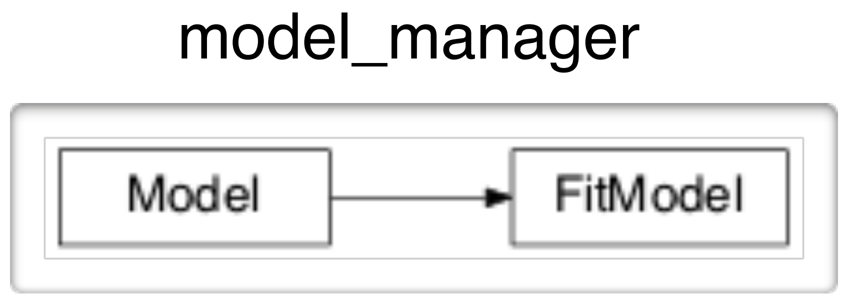


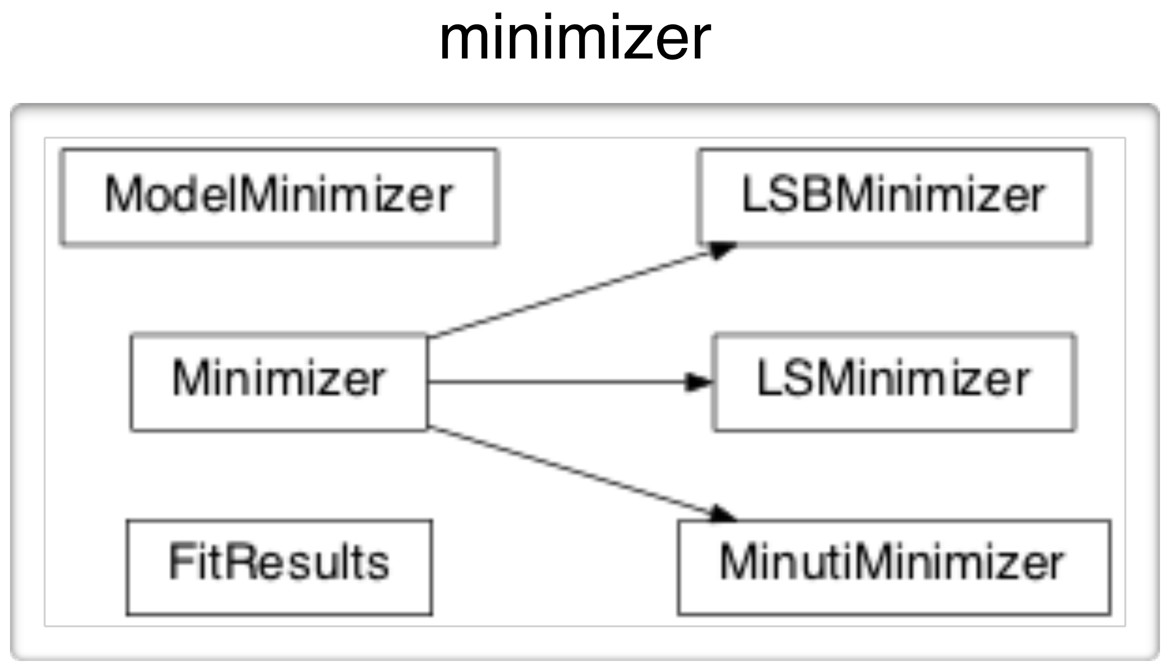


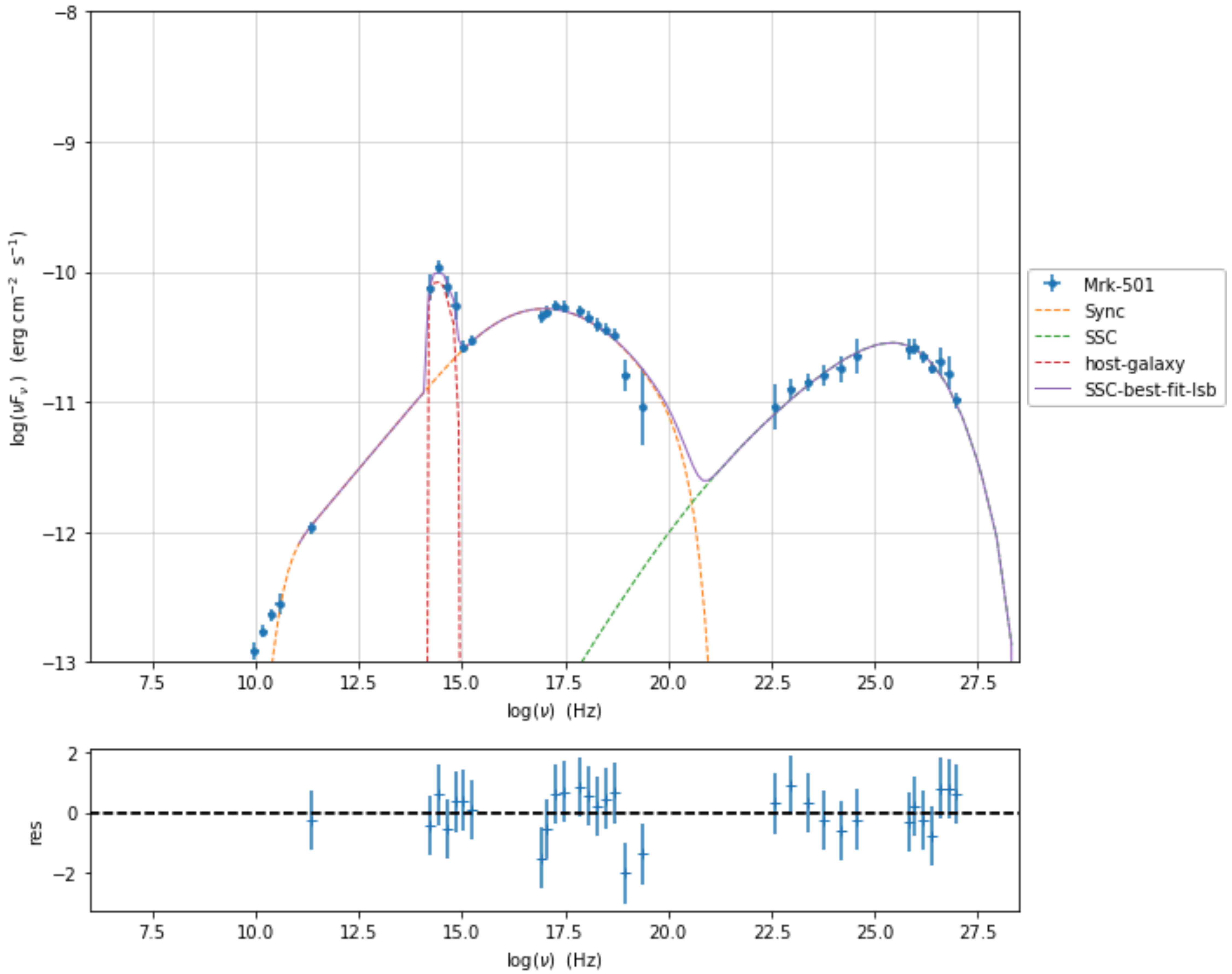
JetSeT model fitting MCMC

Tutorial 5



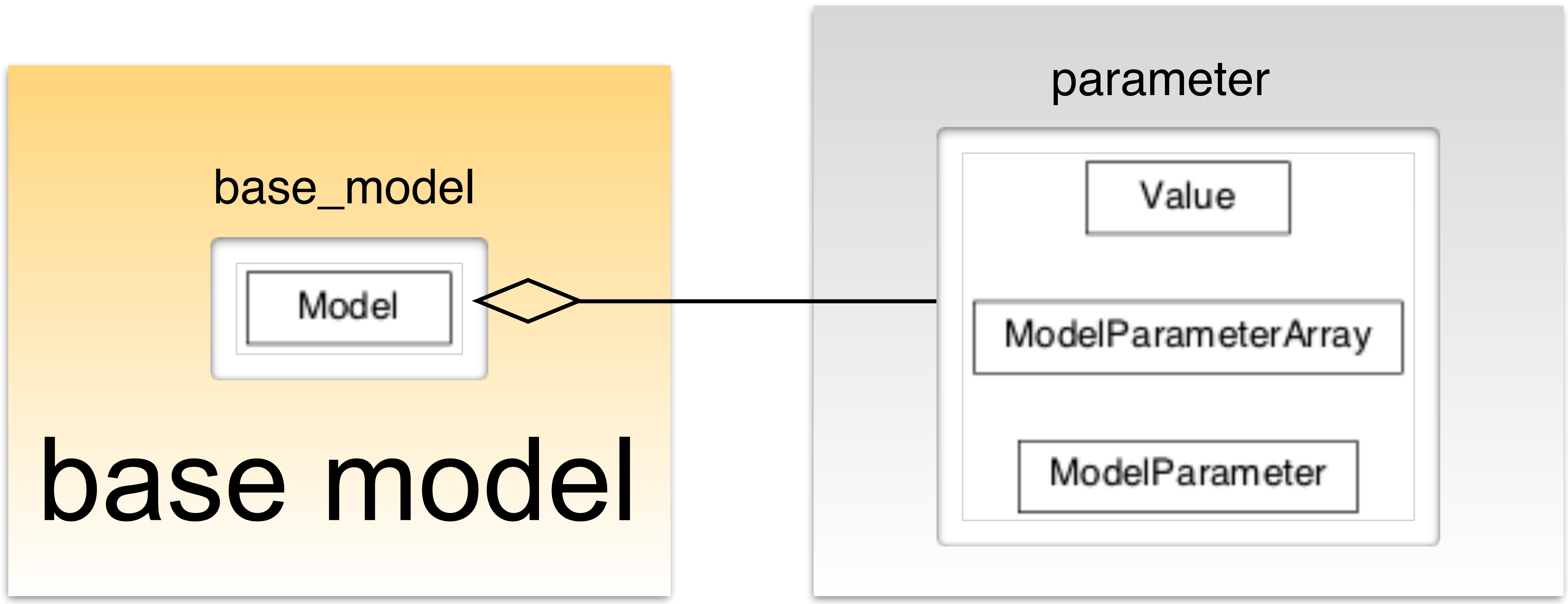


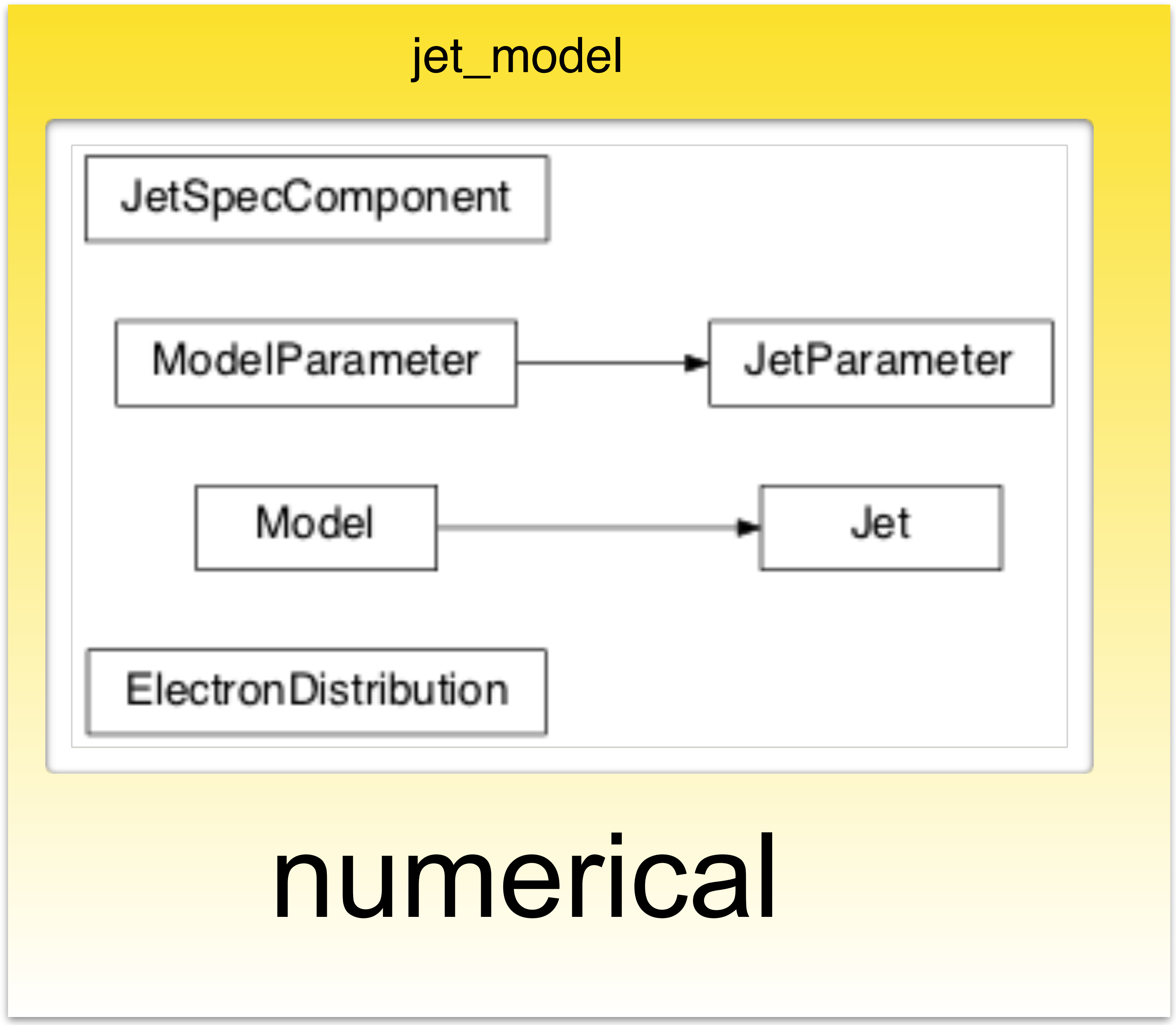


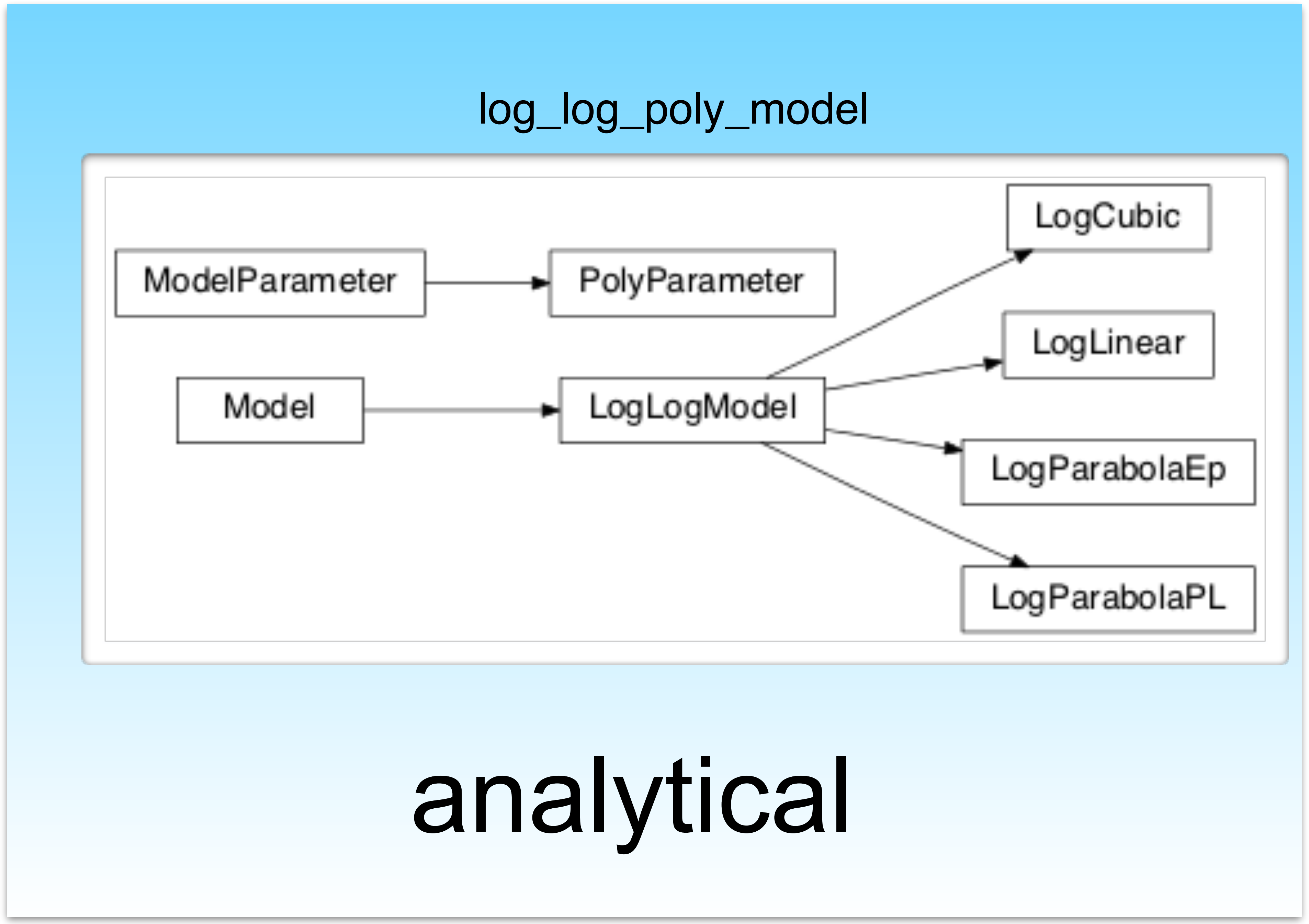


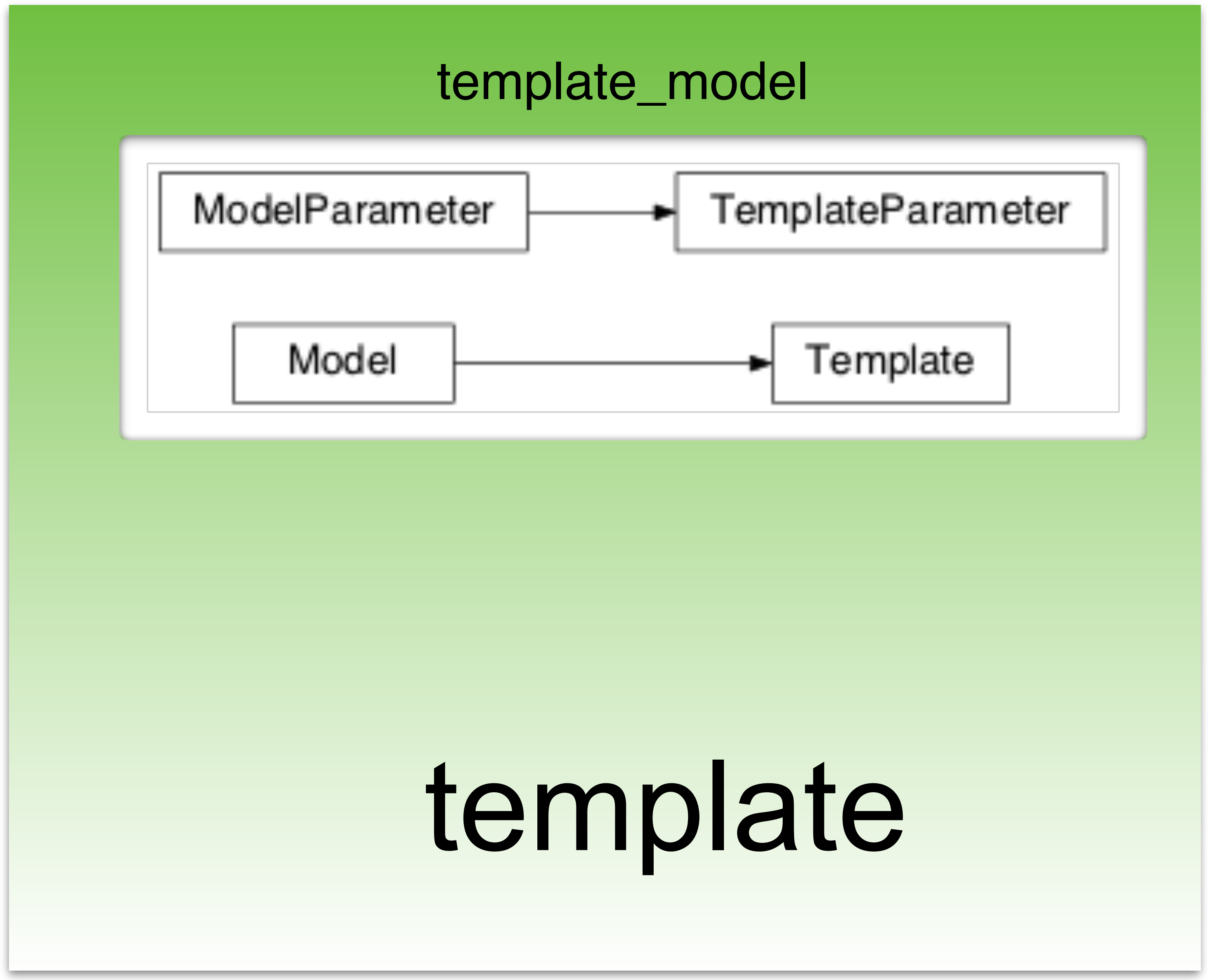
backup slides

JetSeT models

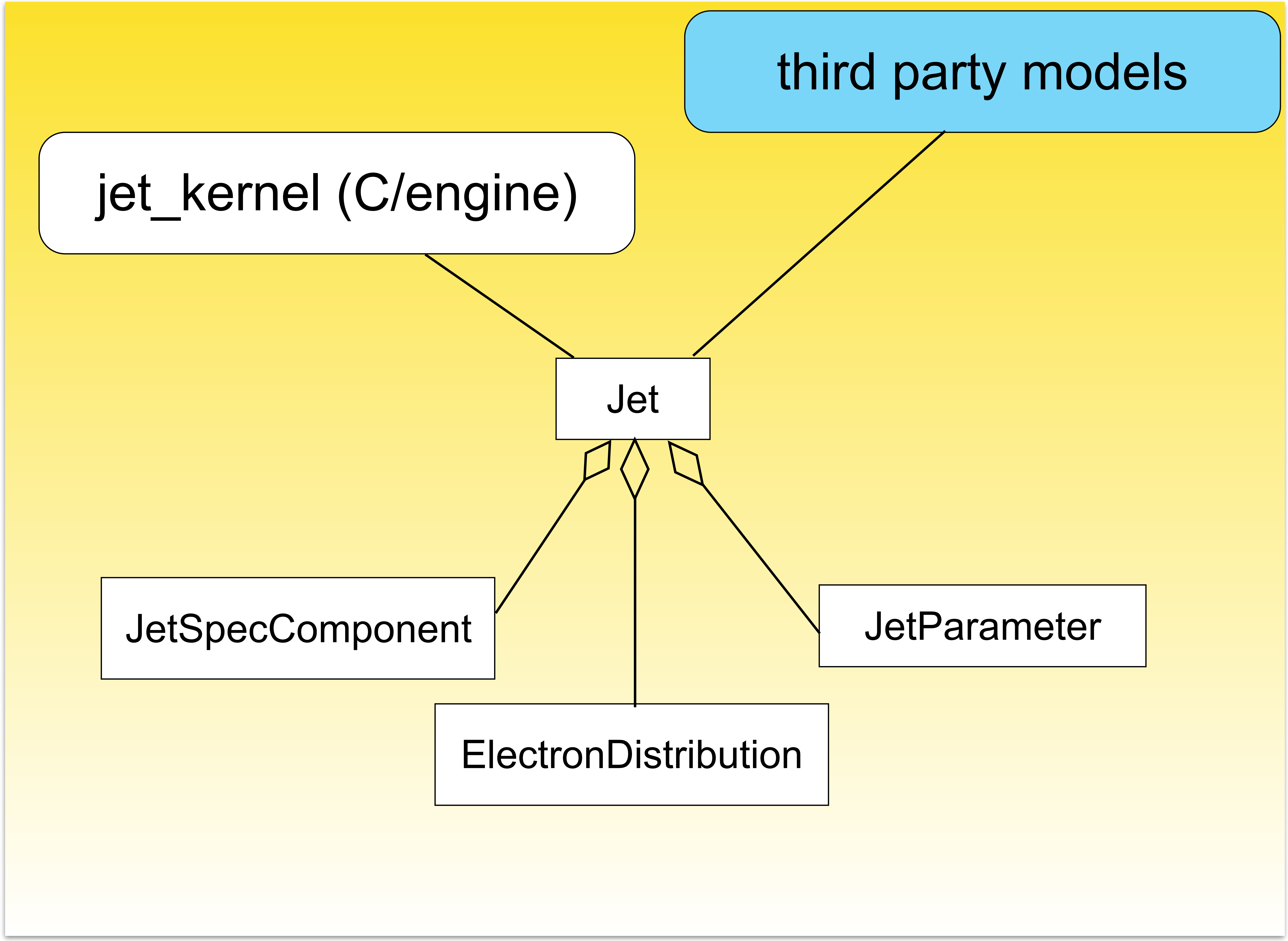




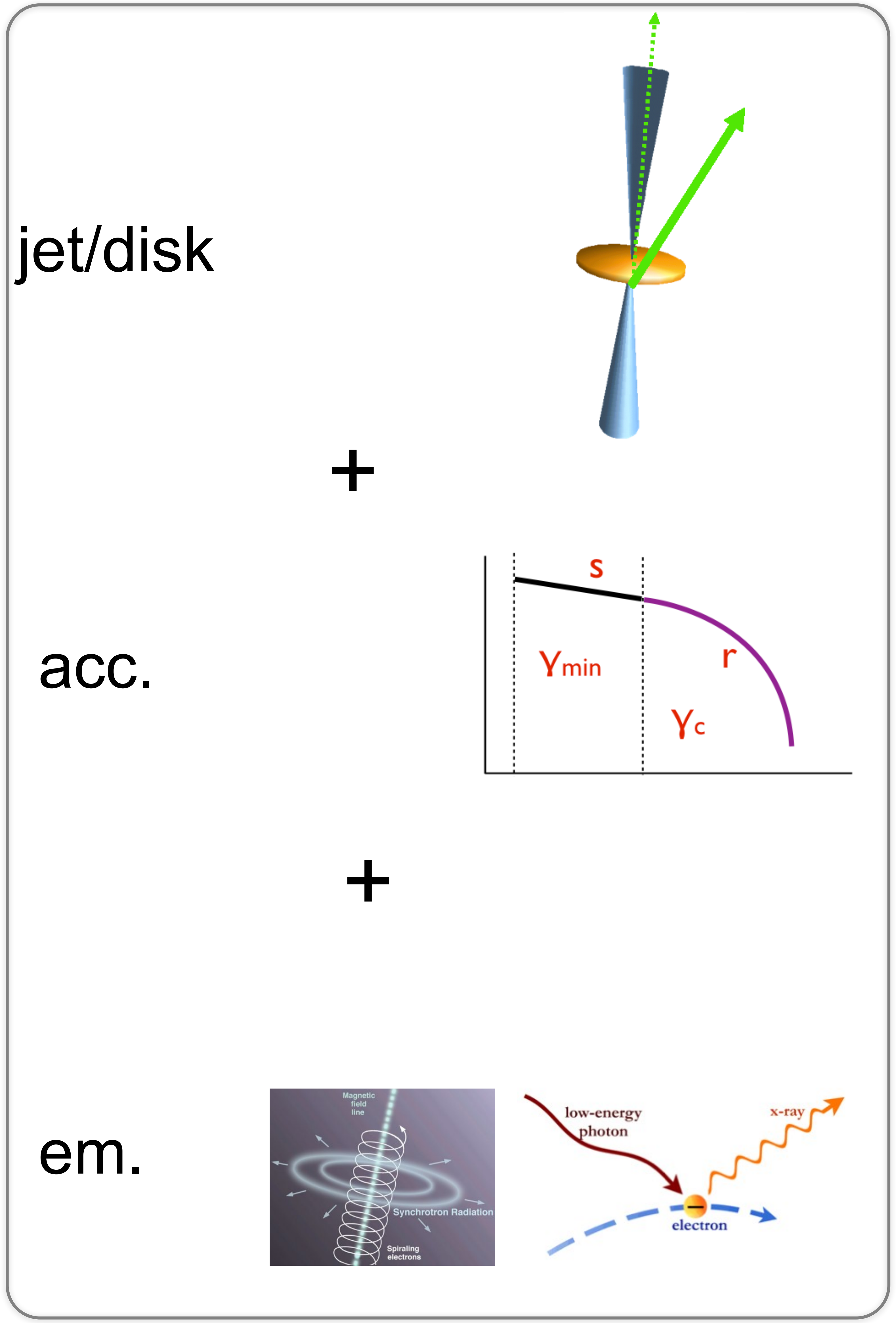




JetSeT jet model definition

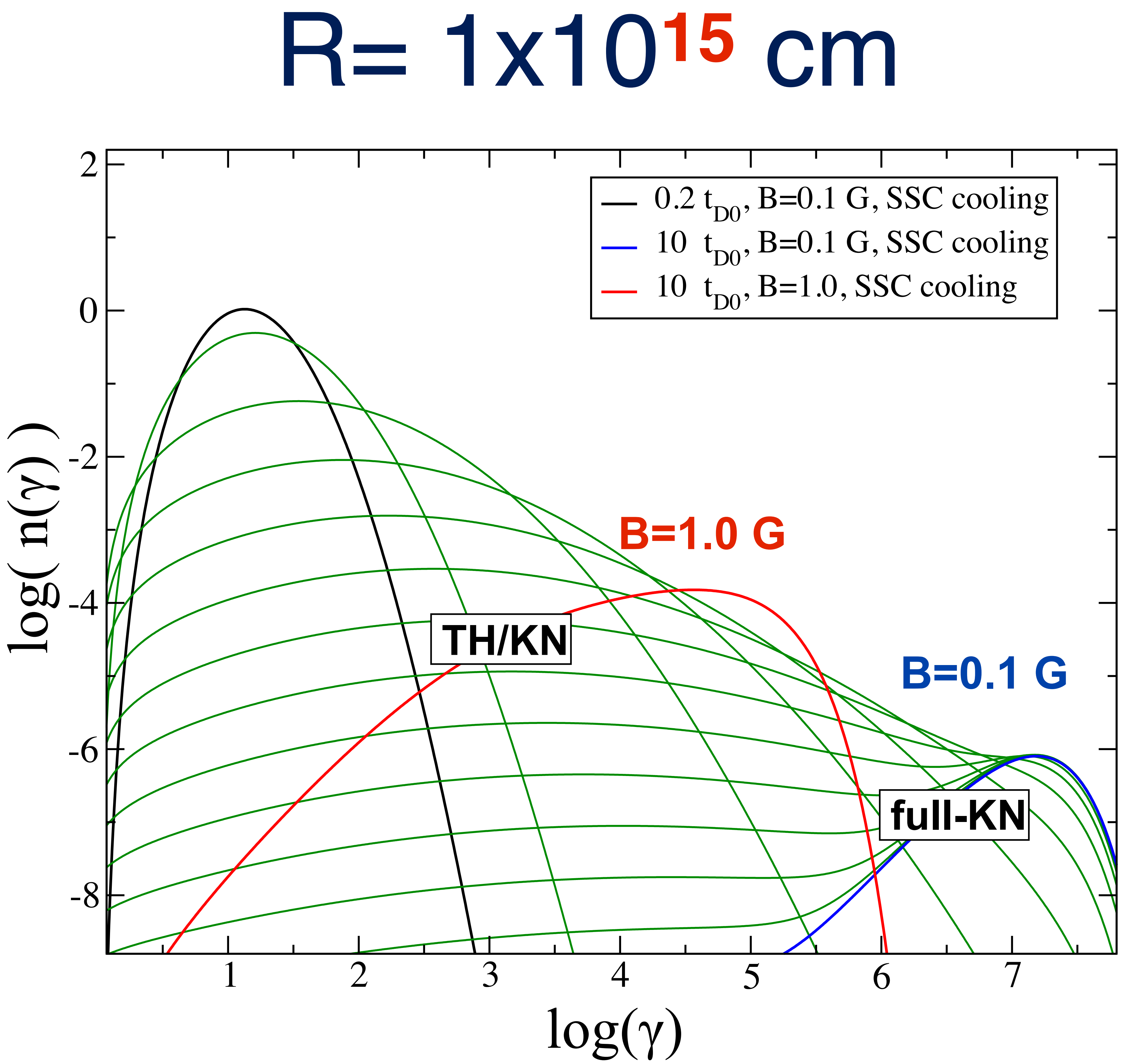


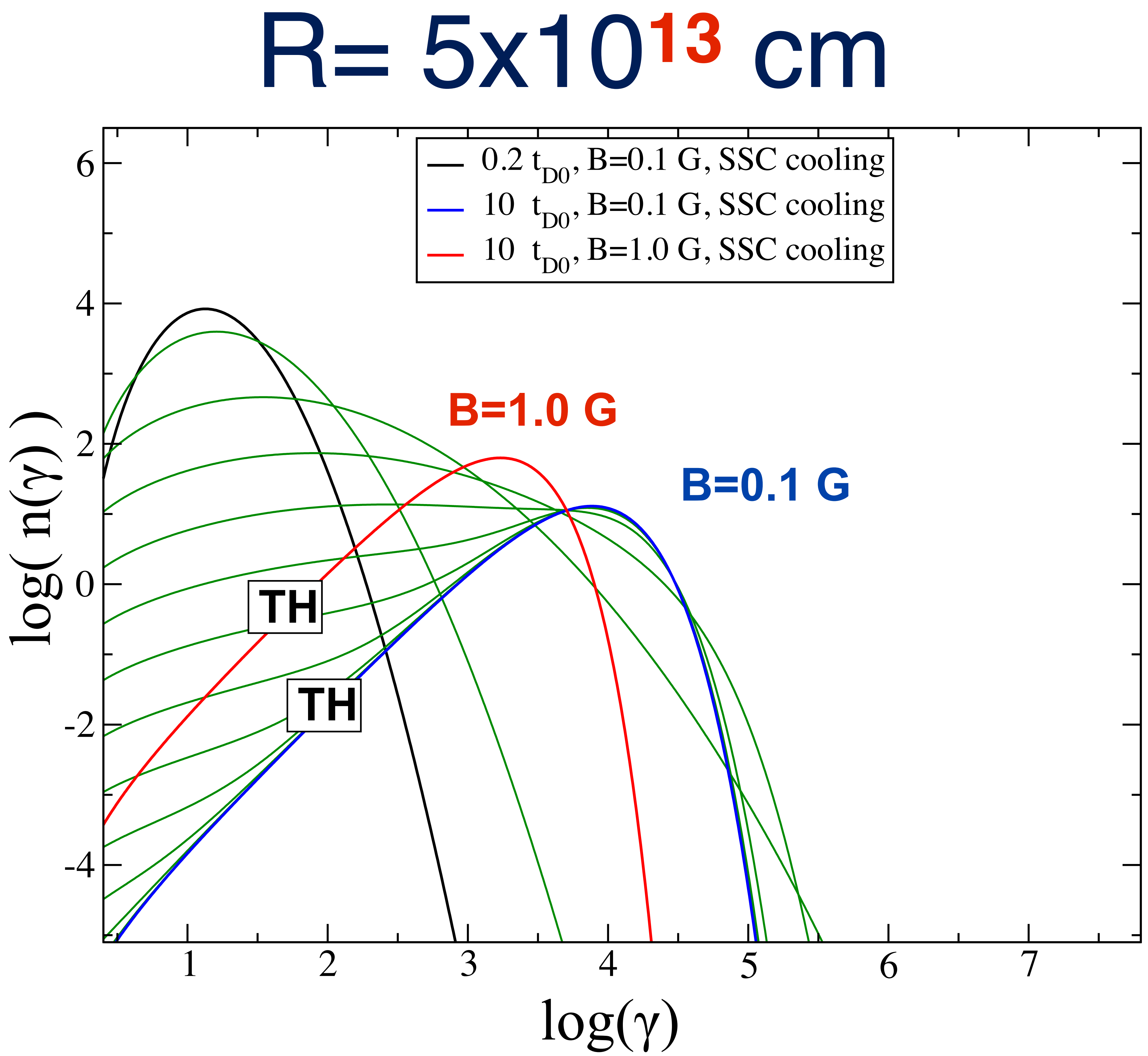
model



JetSeT temporal evolution

IC cooling and equilibrium

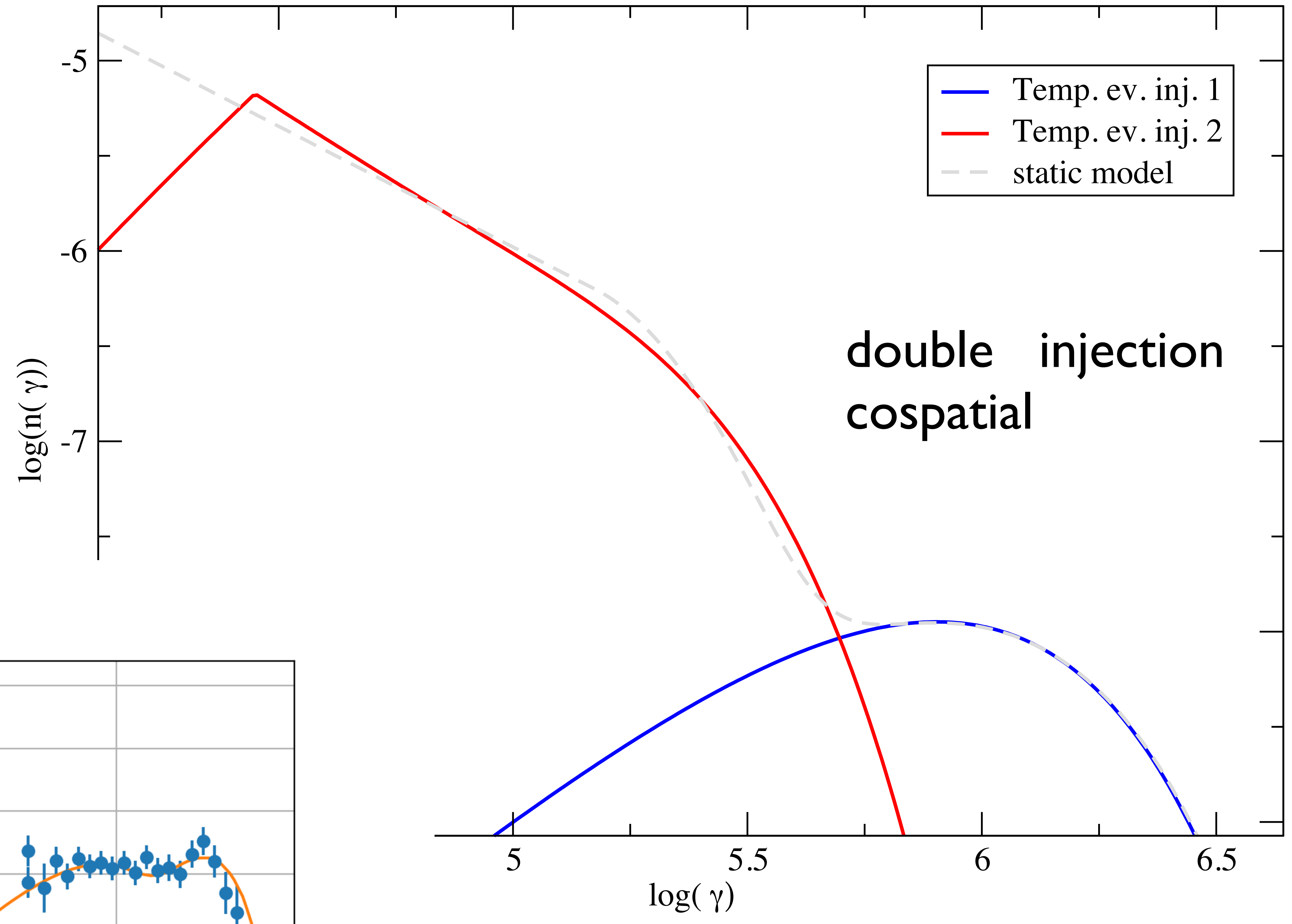


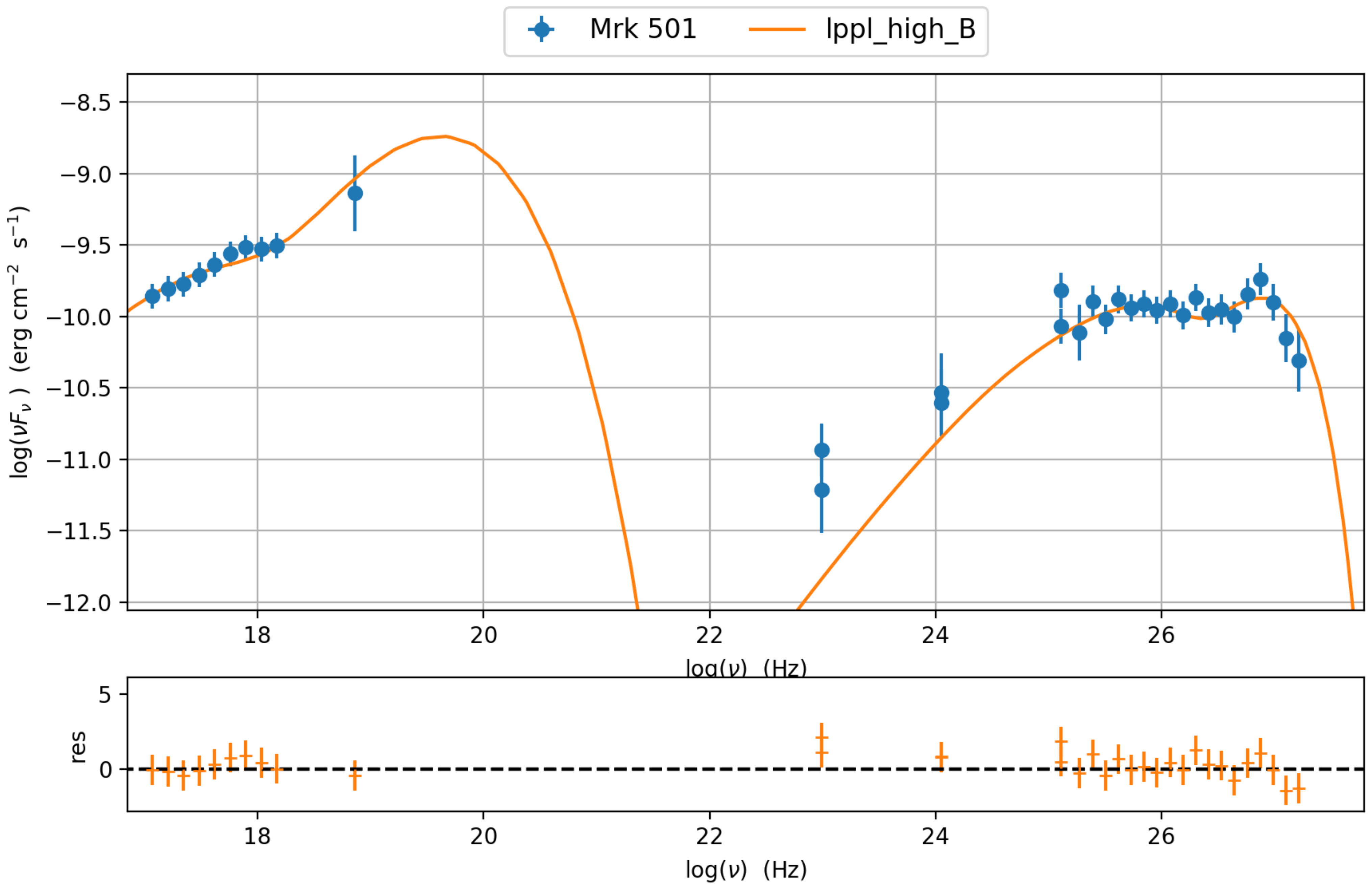


Tramacere +2011

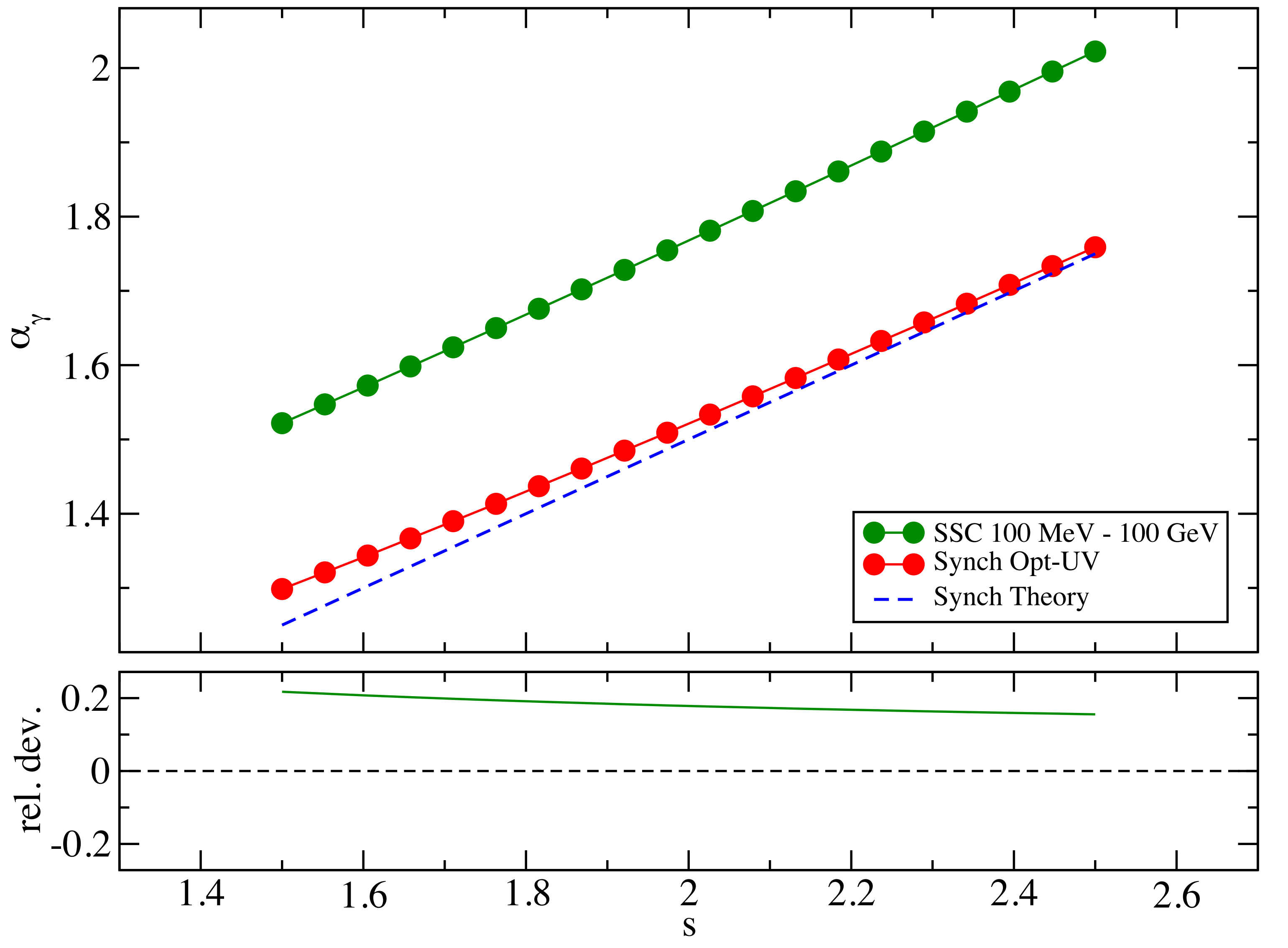
Pile-up and hard spectra

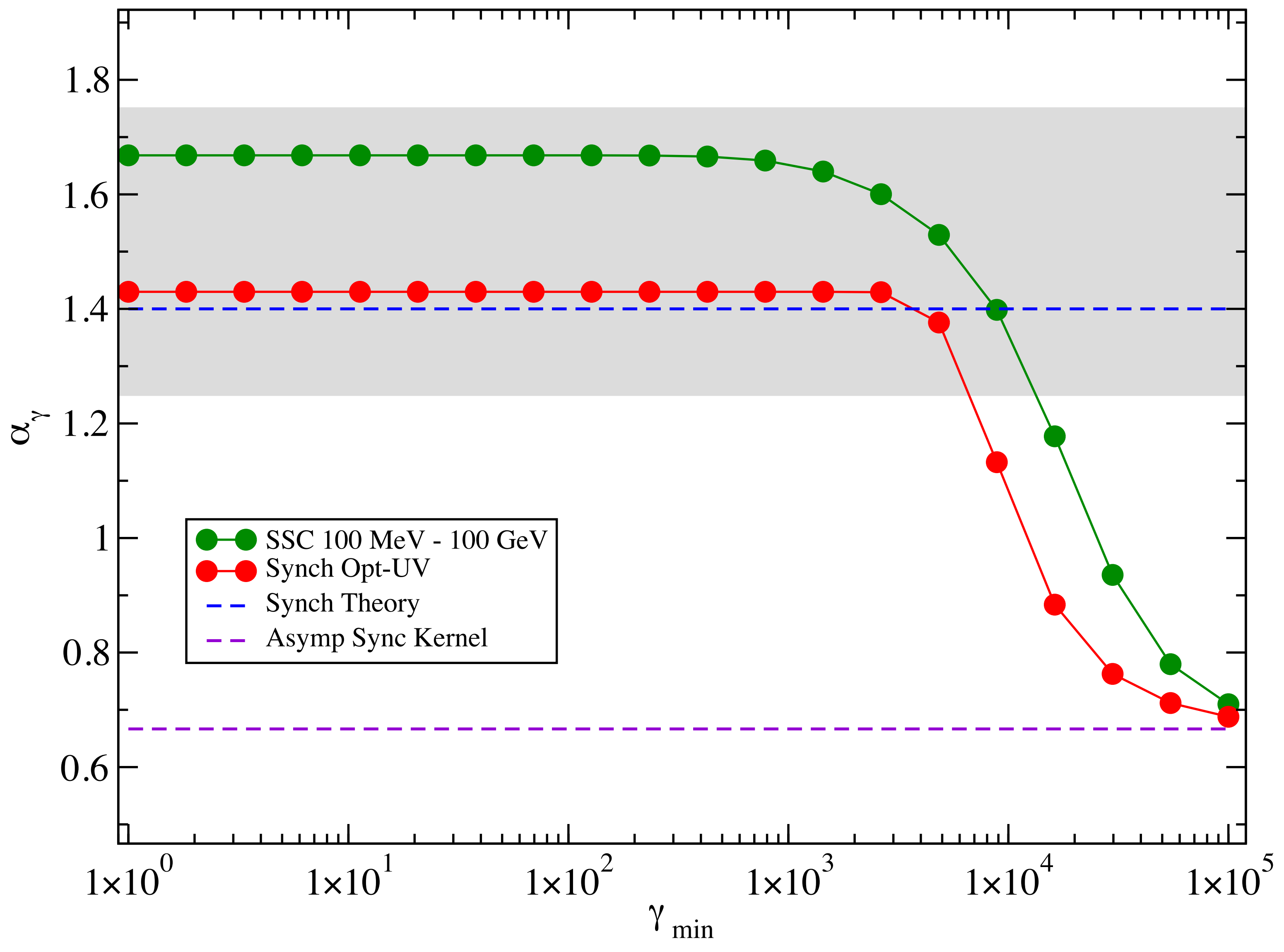
Mrk 501 2014 Flare
MAGIC paper
(accepted)

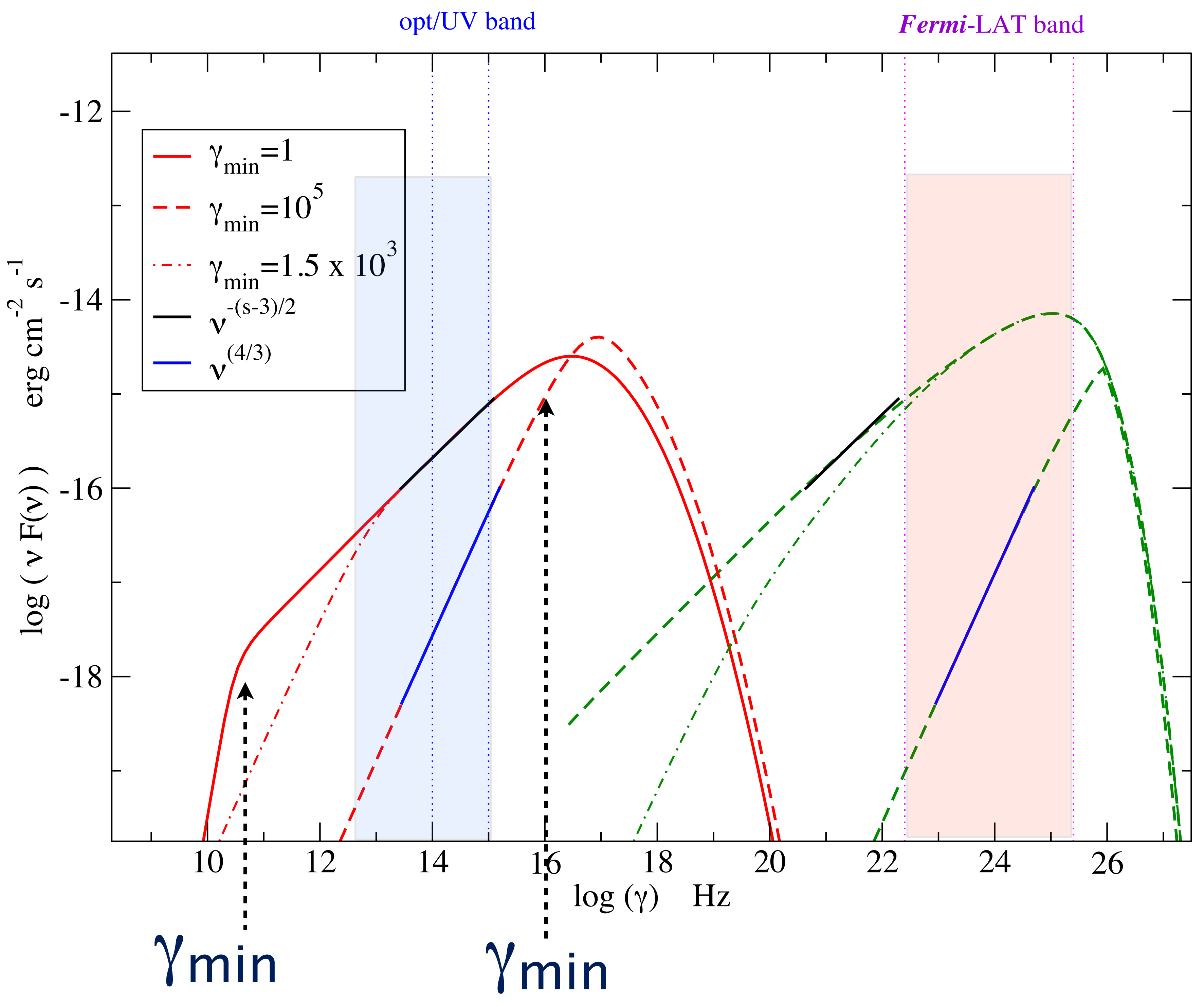




the dependency of s on Γ and γ_{min}

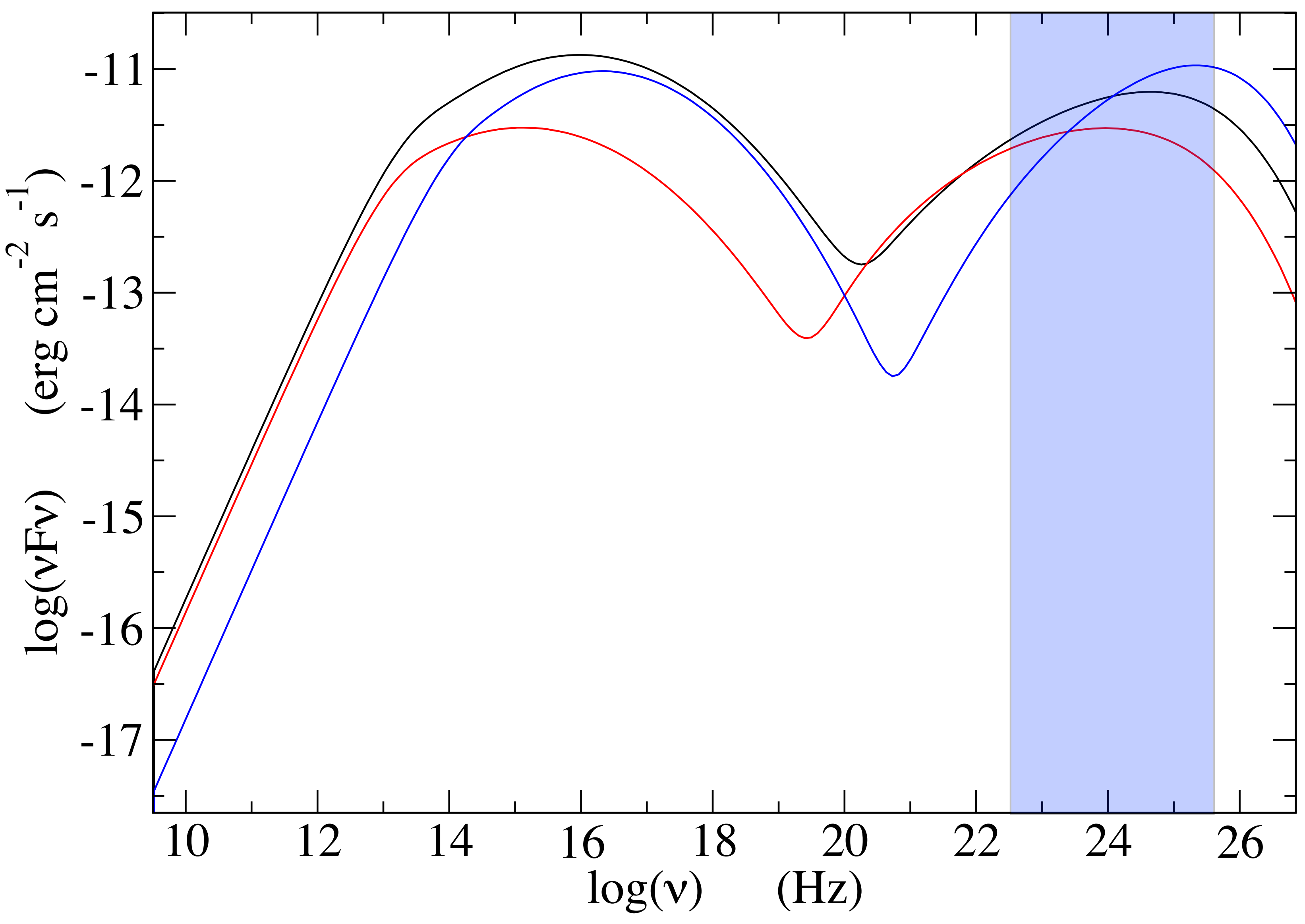




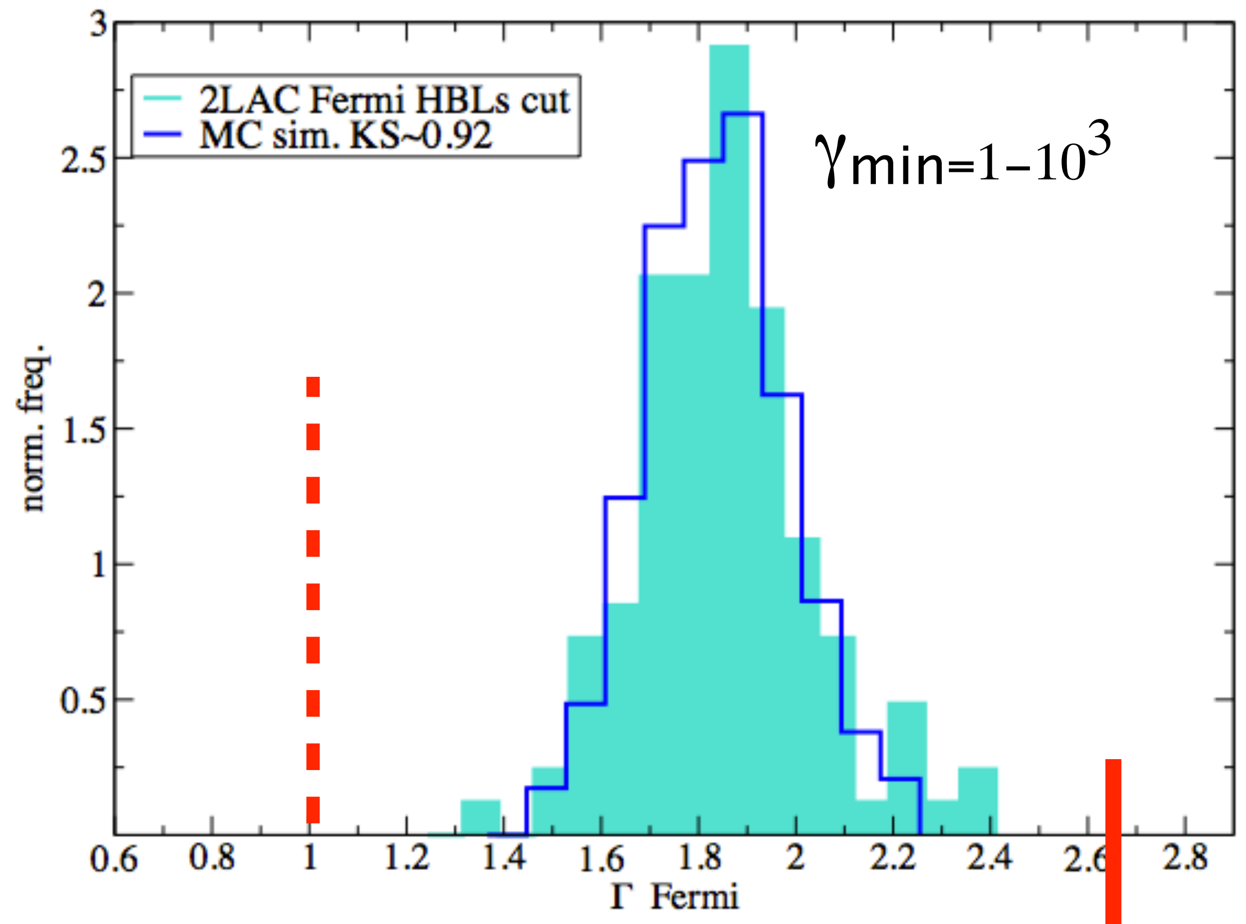


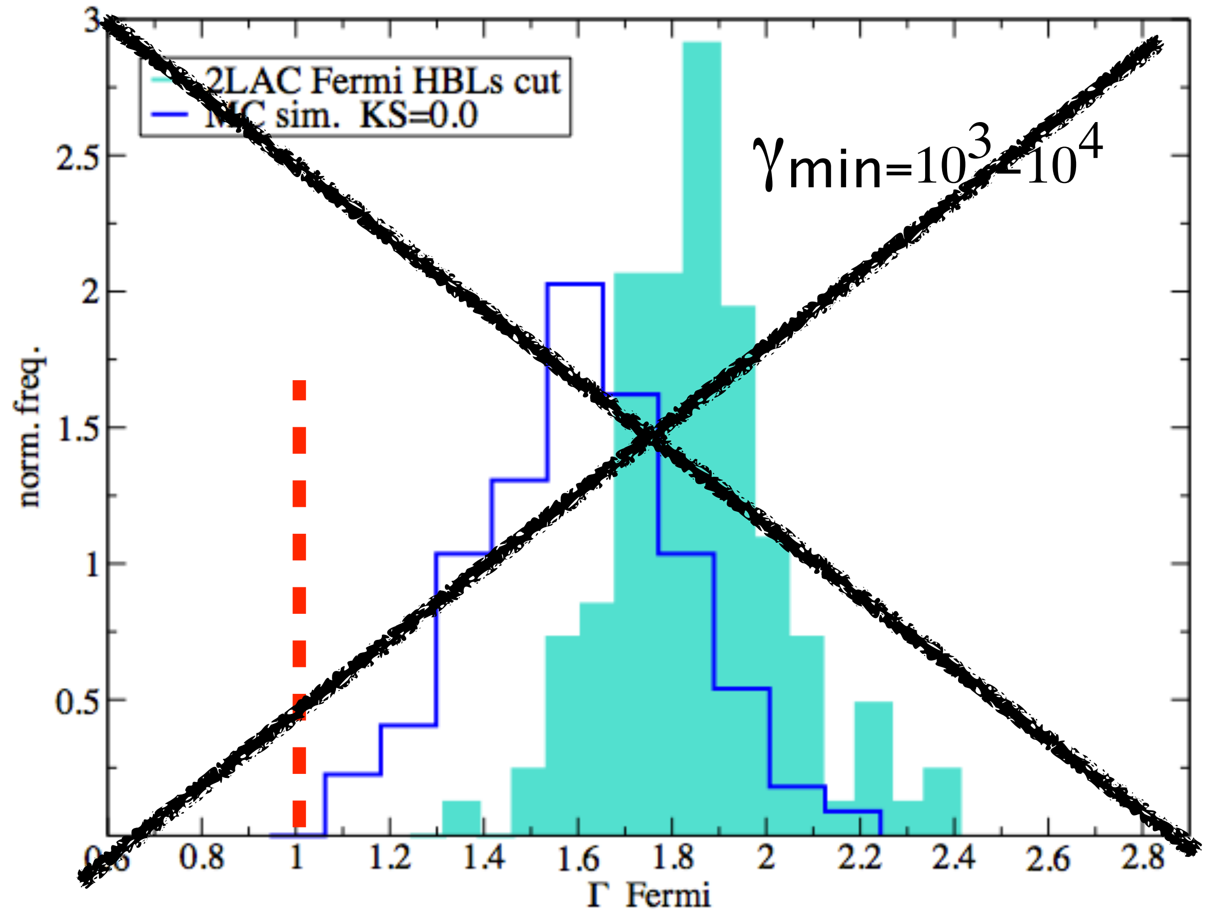
depending on the value of the observed spectral index it is possible to guess the if s or γ_{min} is the dominant driver

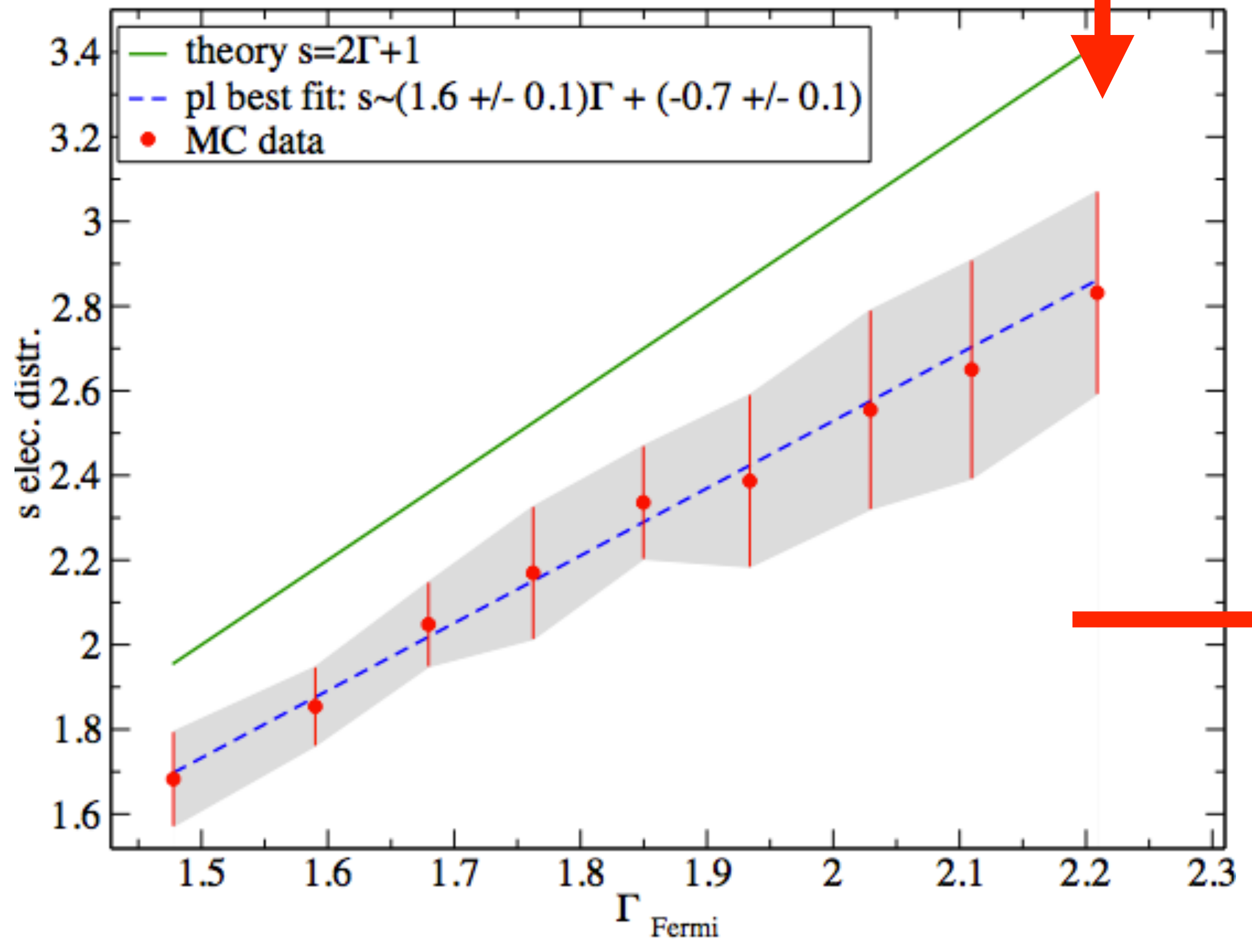
calibrating the Fermi $\Gamma_\gamma - s$: MC approach



The Fermi-LAT window samples different regions of the IC rise, dep. on *B*,δ,*n*(γ)...







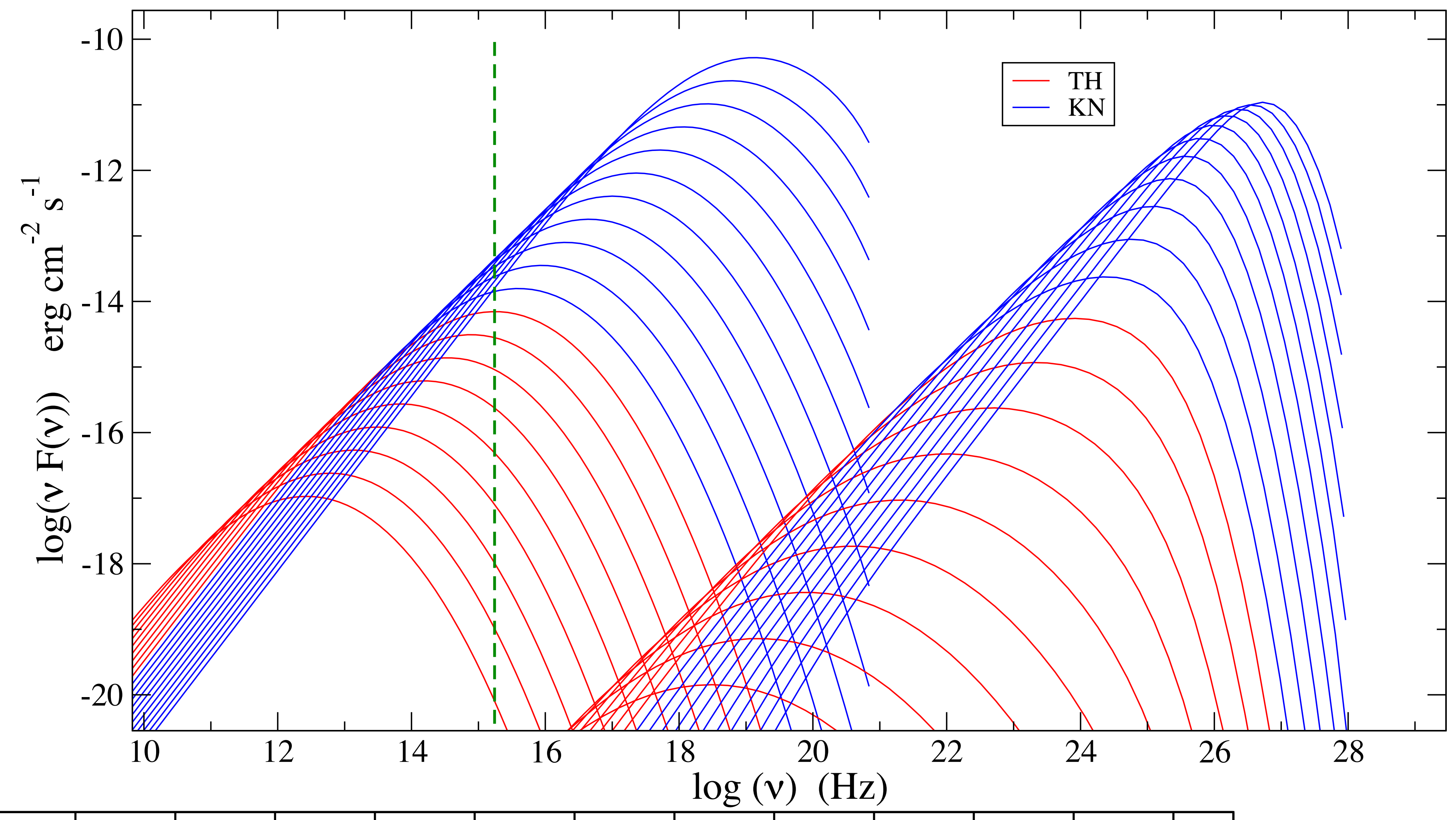
$s = 2\,\Gamma_\gamma - 1$ Theory, biased

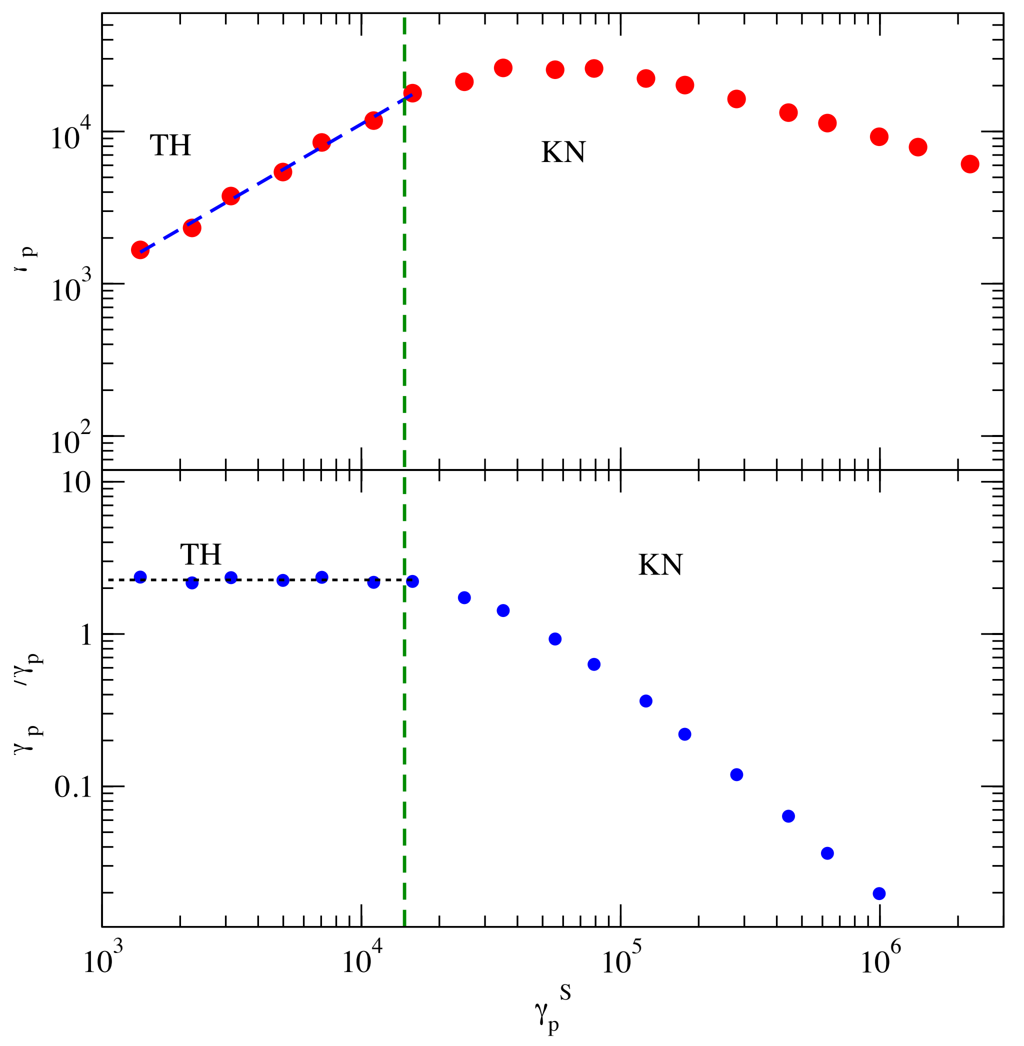
$s \sim 1.6\,\Gamma_\gamma - 0.7$ MC, unbiased

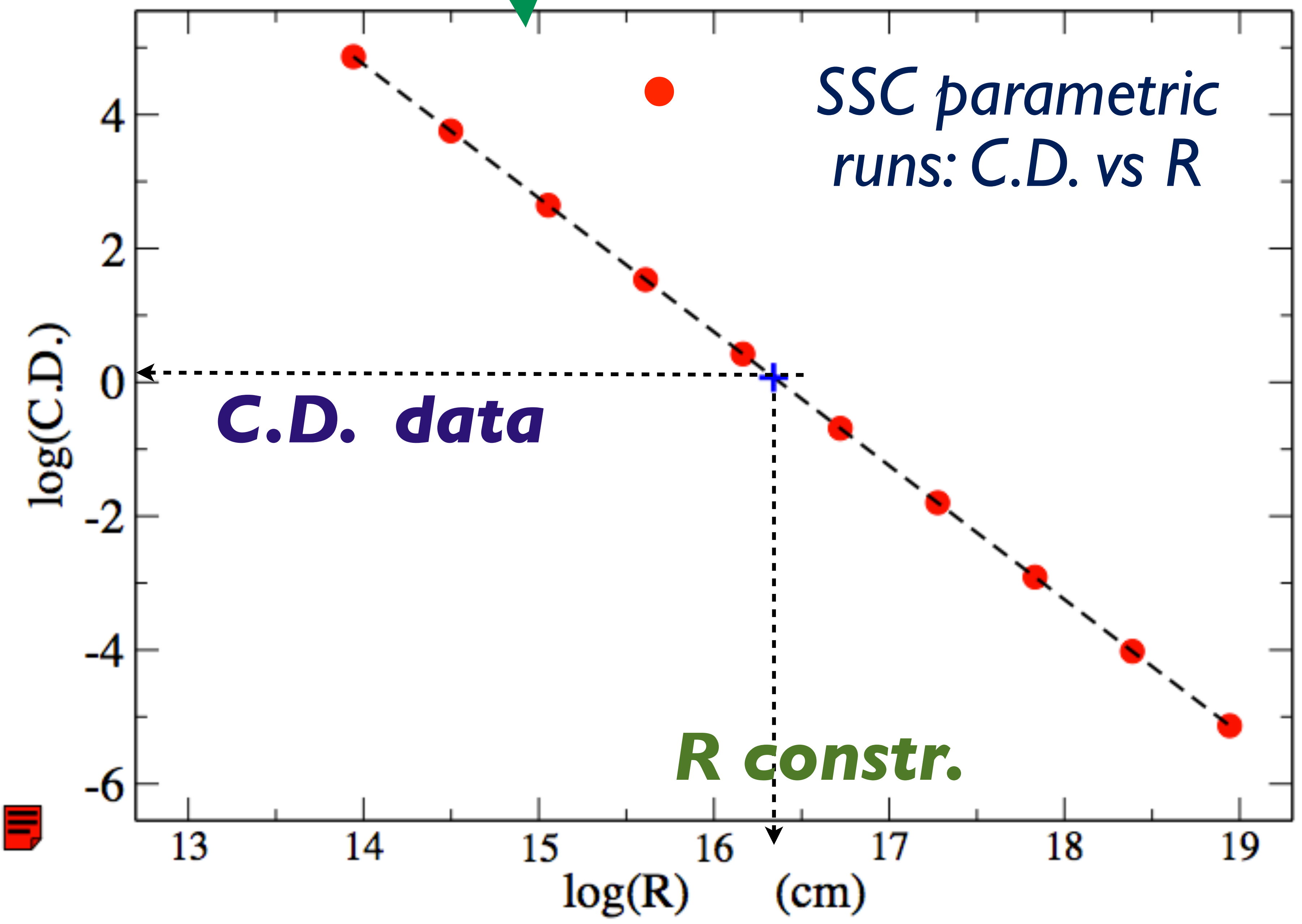
29

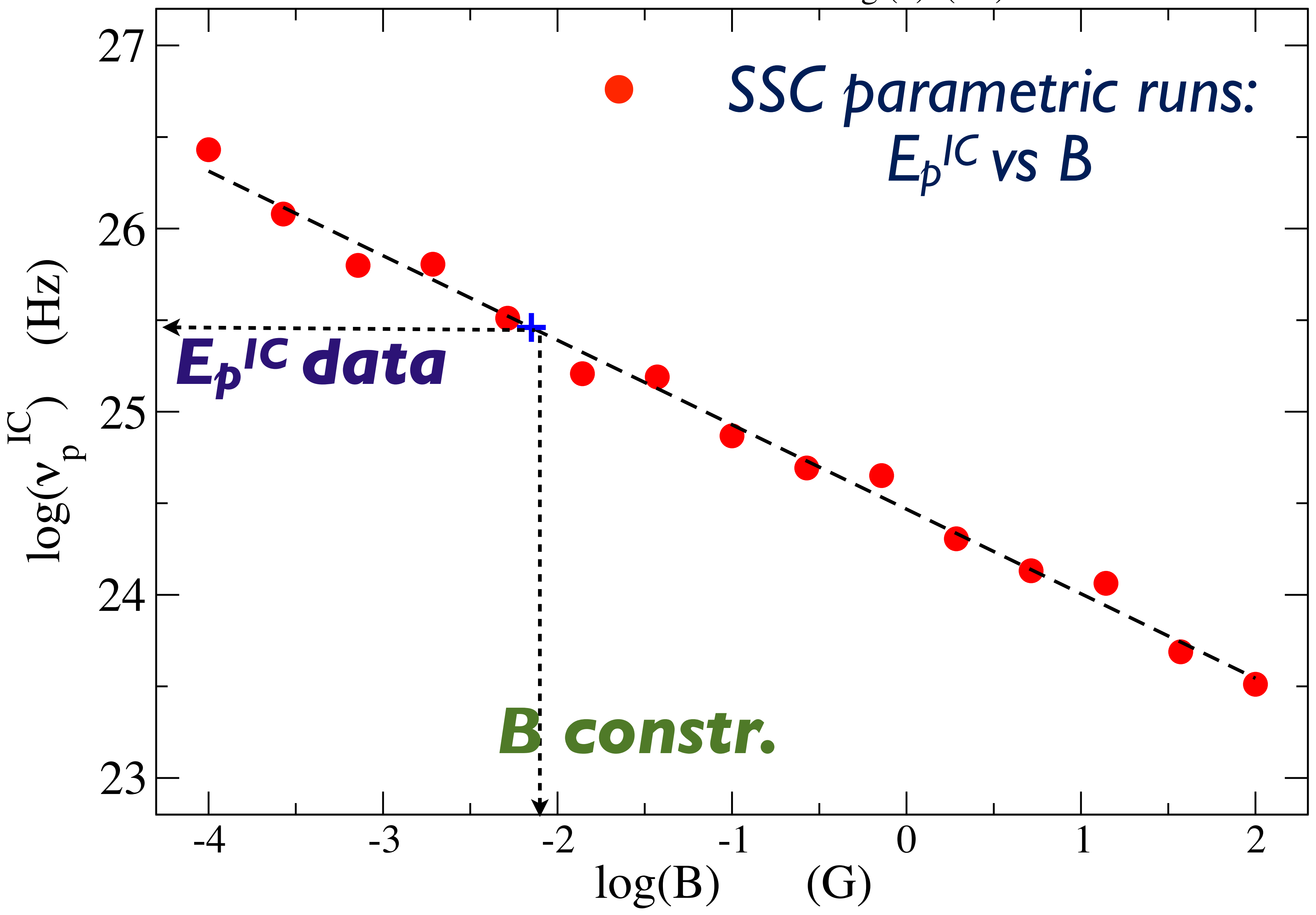
SSC constraints

- ✓ Γ=> s
- ✓ b=>r
- ✓ $E_p{}^S$,r,s=> γ_0
- ✓ t_{var},δ=> R u.l.
- ✓ N=> best $S_p{}^S$ match
- ● B-> best $E_p{}^{IC}$,$S_p{}^{IC}$ match
- ● R -> CD



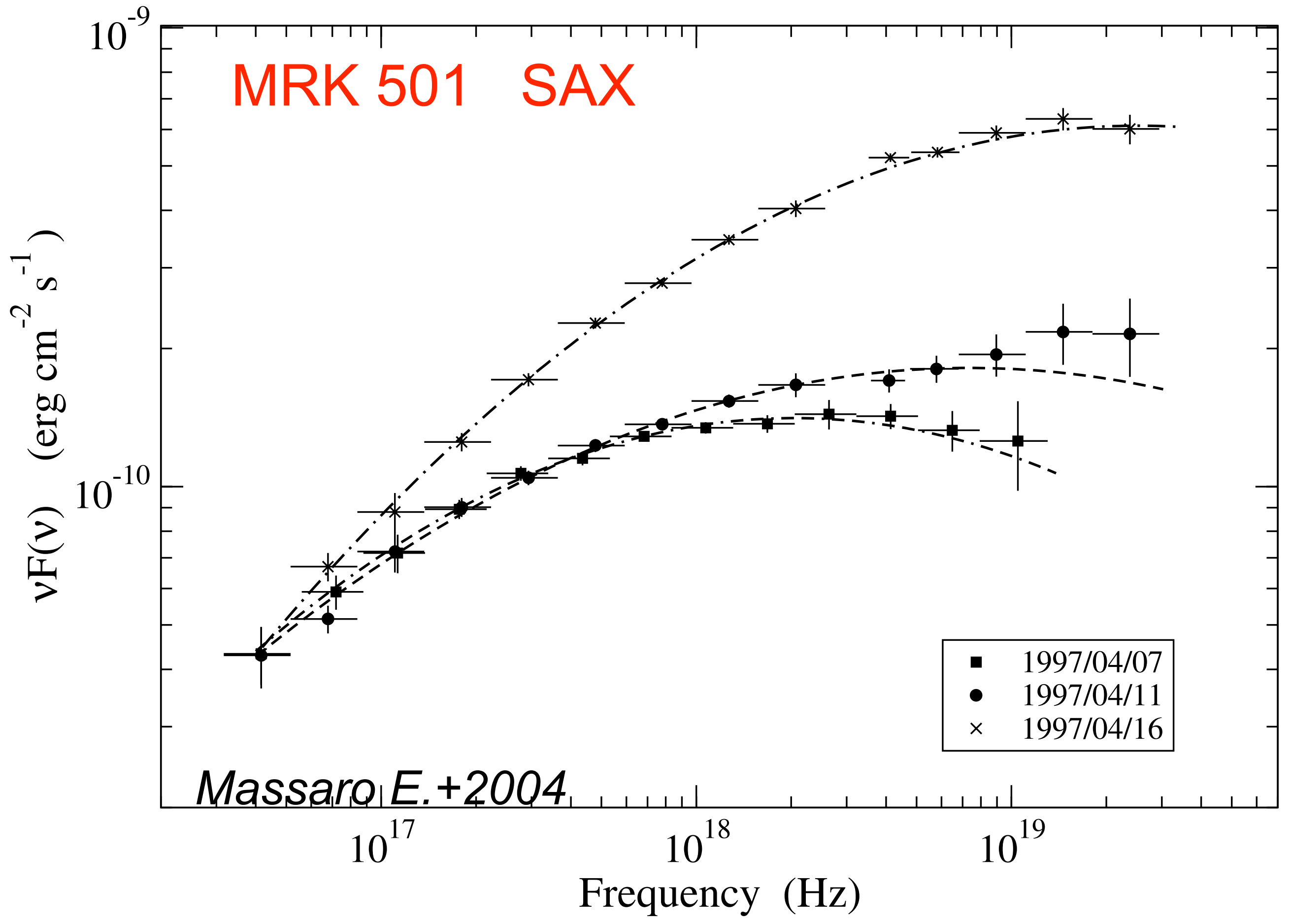


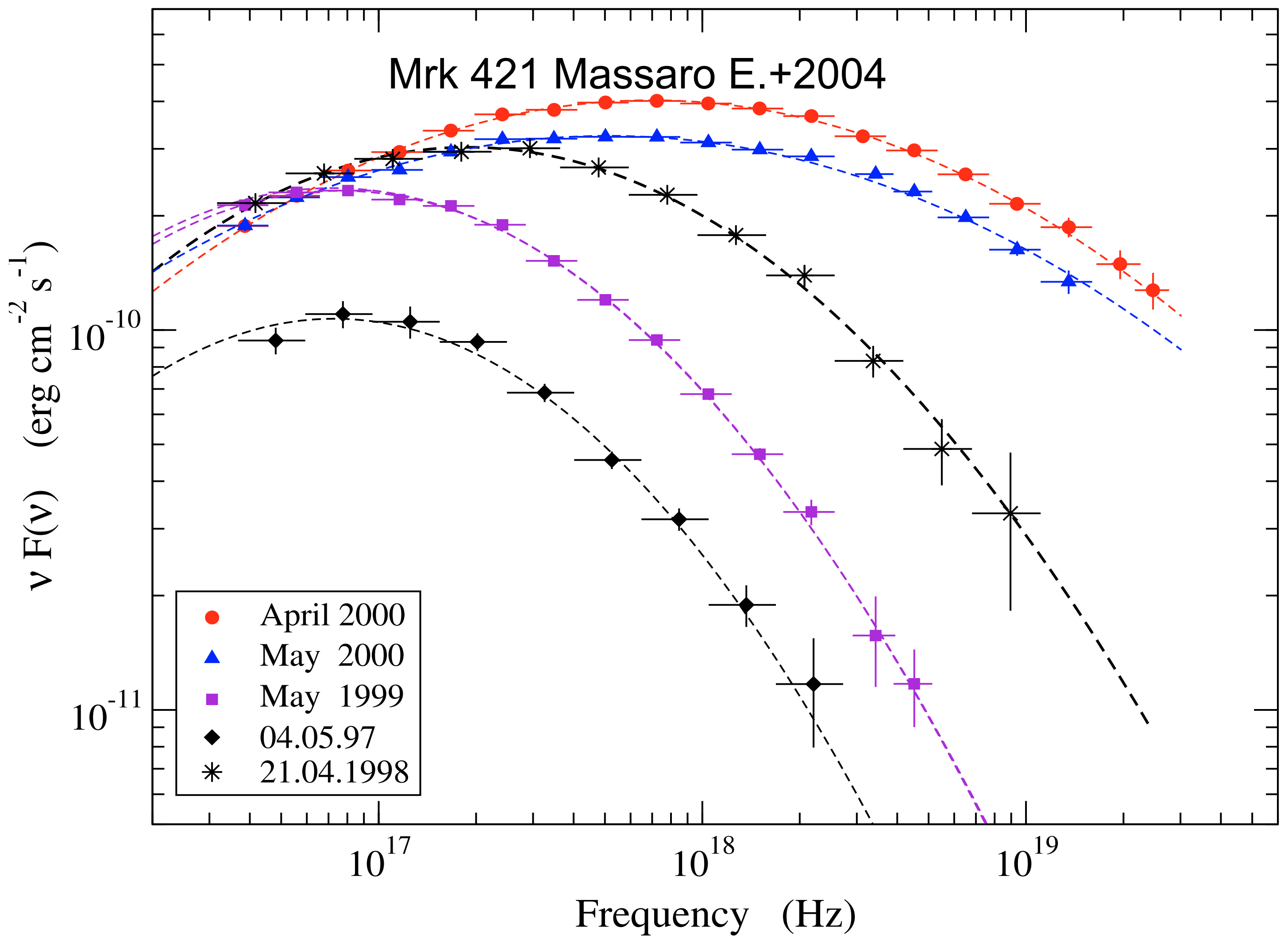


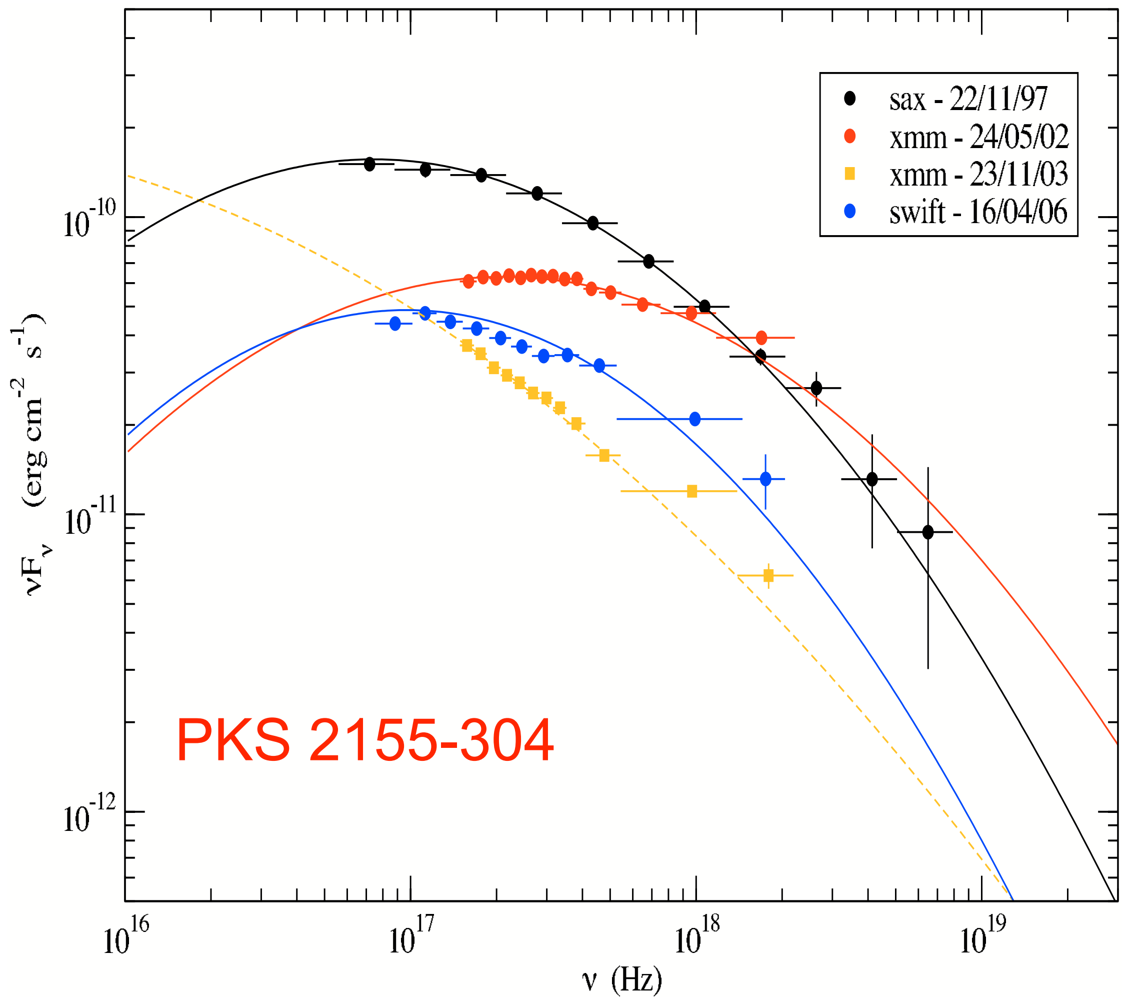


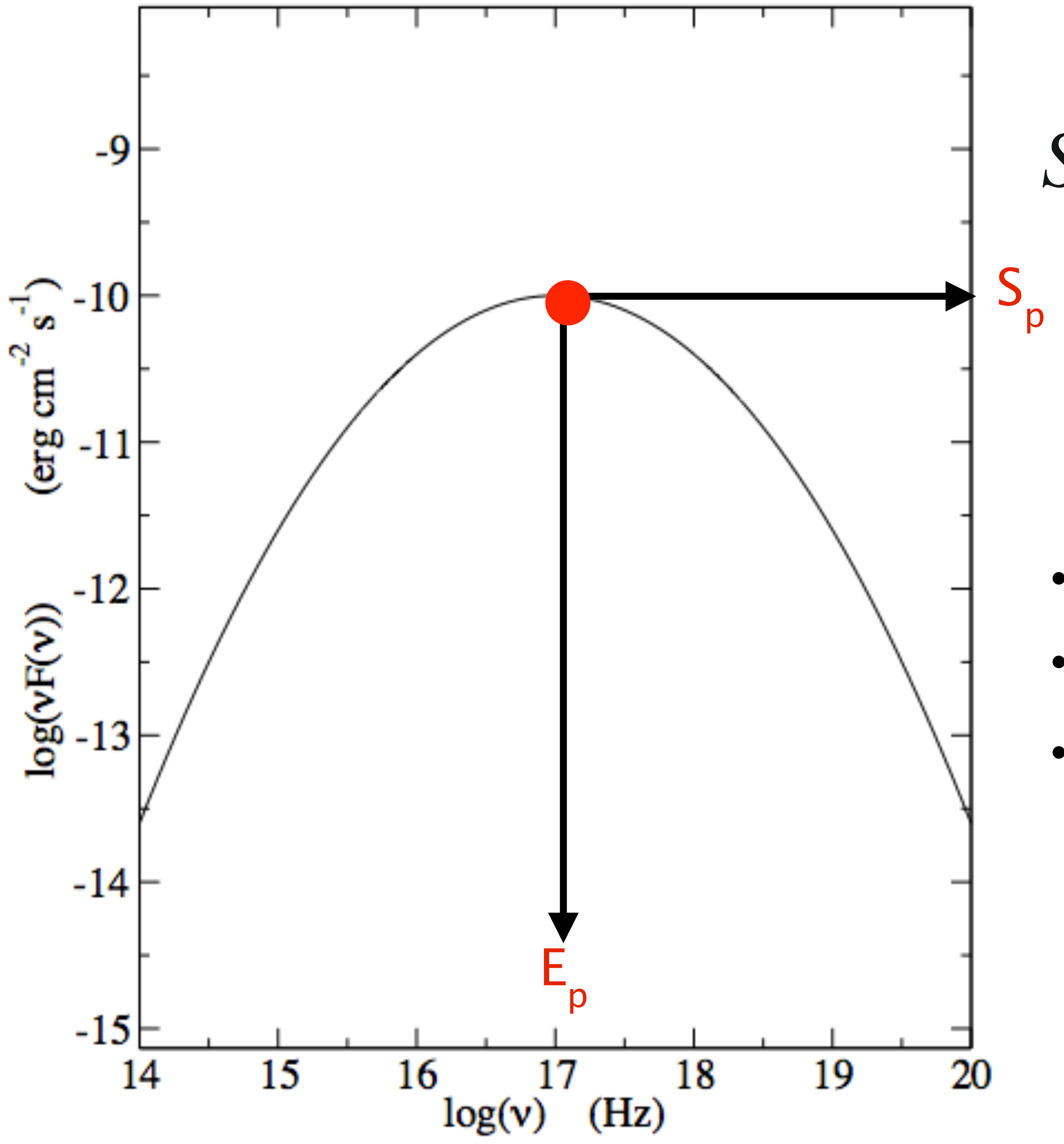
Application to stochastic acceleration: self-consistent modeling

X-ray SPECTRAL DISTRIBUTION OF HBLs (stochastic acceleration)



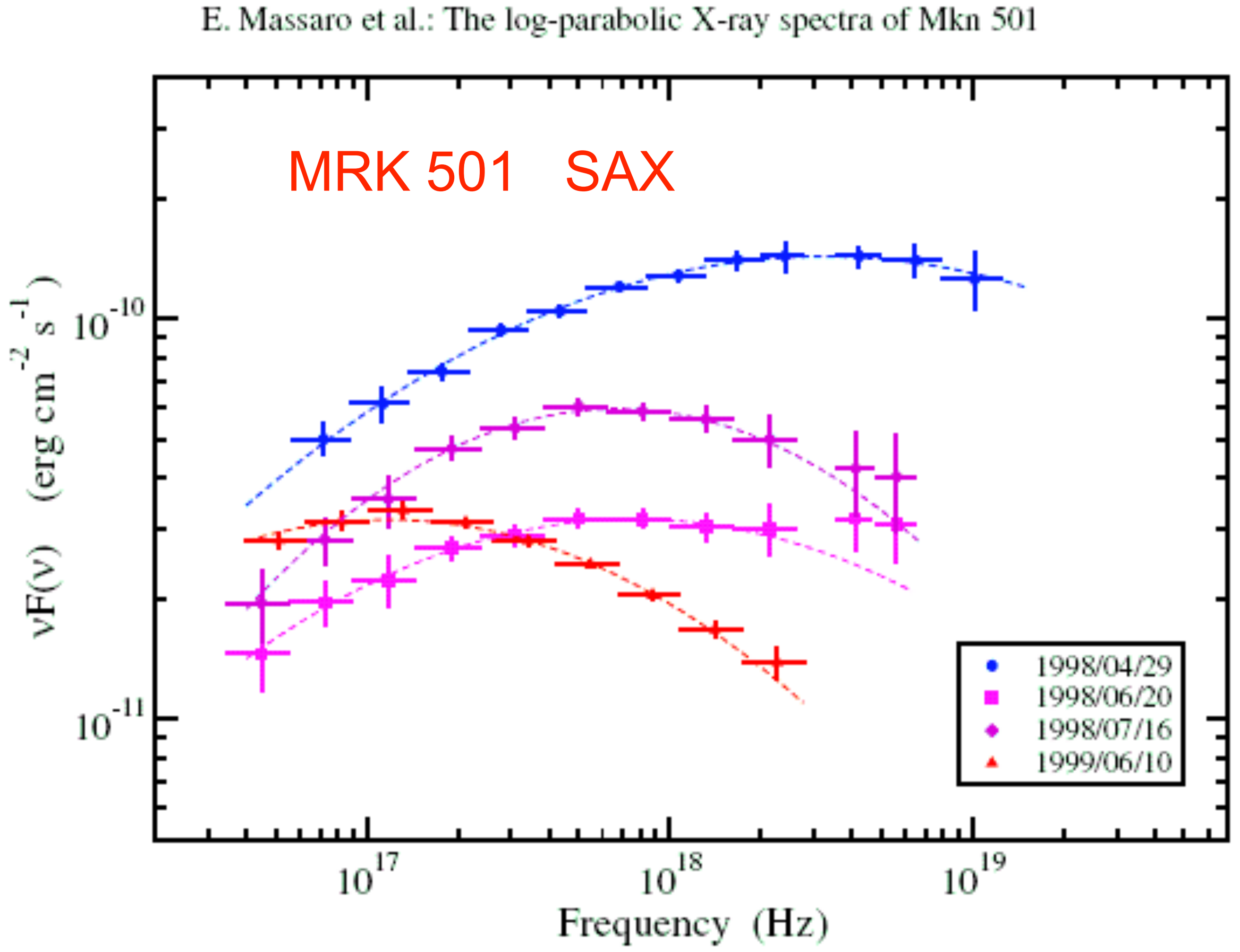


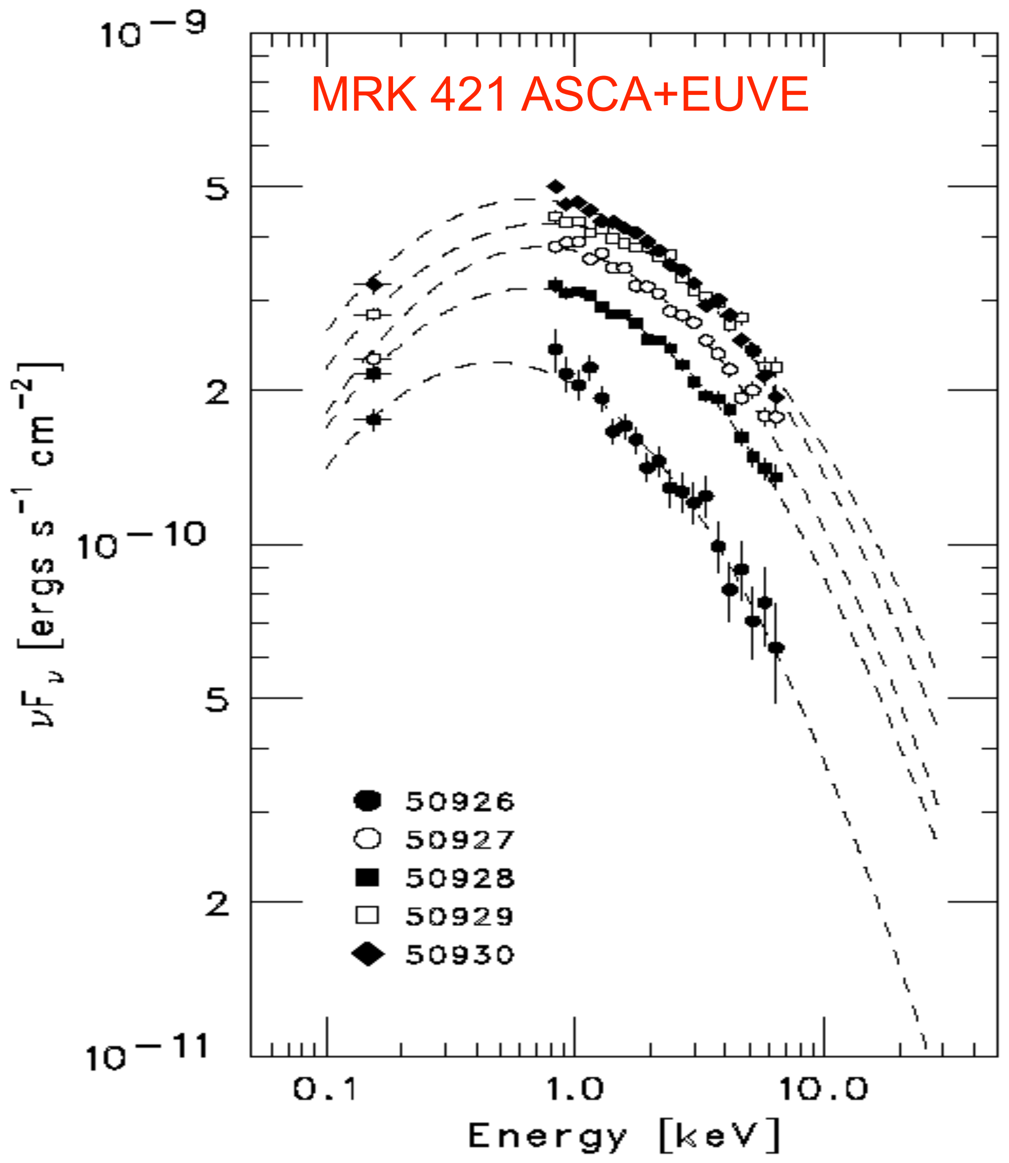




$$S(E) = S_{\rm p}\, 10^{-b\,(\log(E/E_{\rm p}))^2}$$

- b: curvature at peak
- E_p: peak energy
- S_p: SED height @ E_p



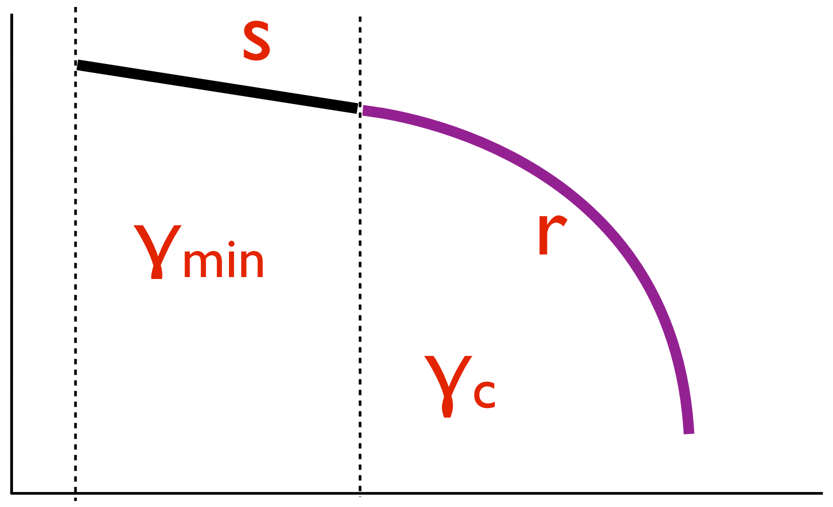


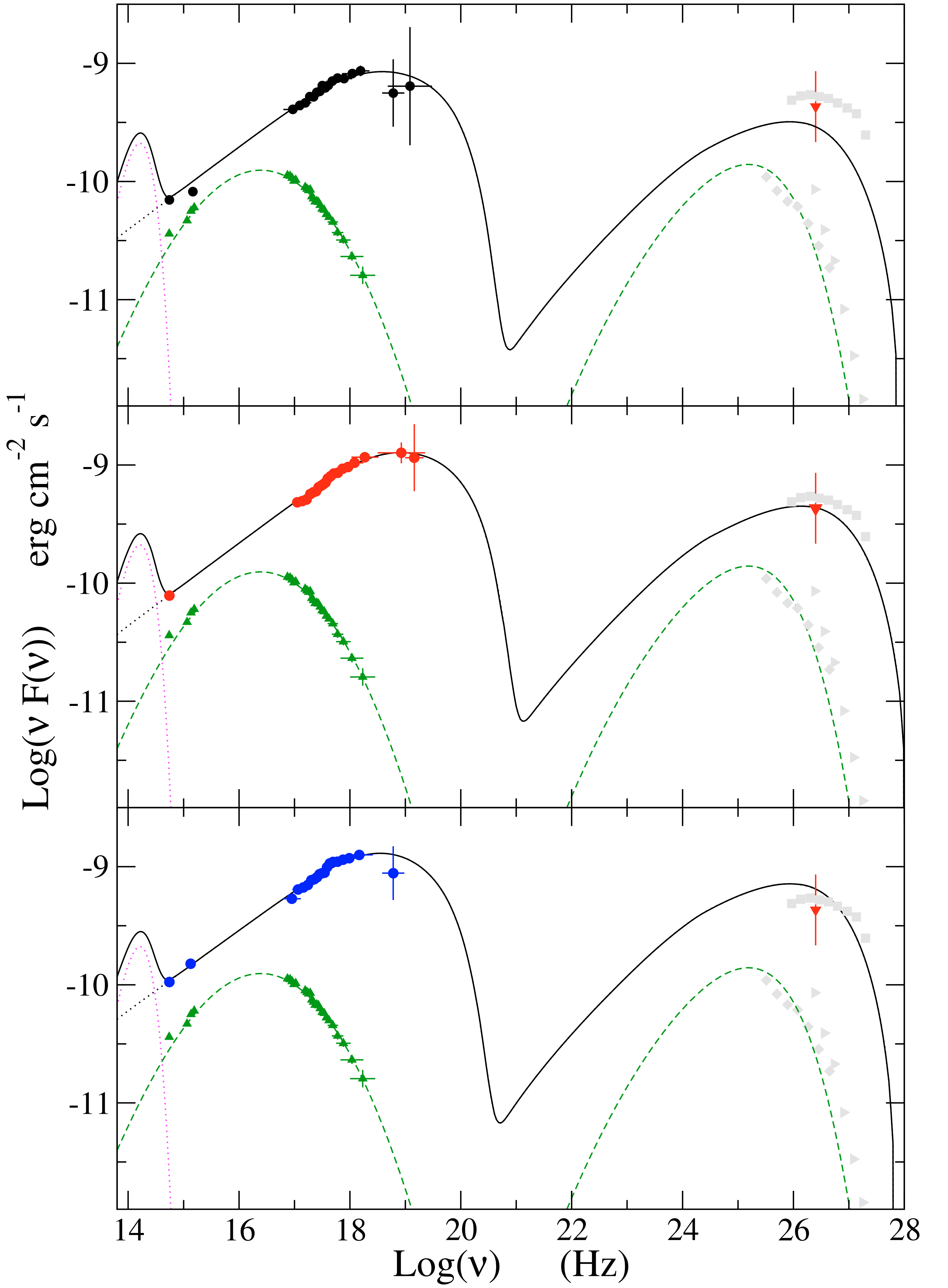
HBL: Fermi I+Fermi II Mrk 421 2006

Tramacere +2009/2011

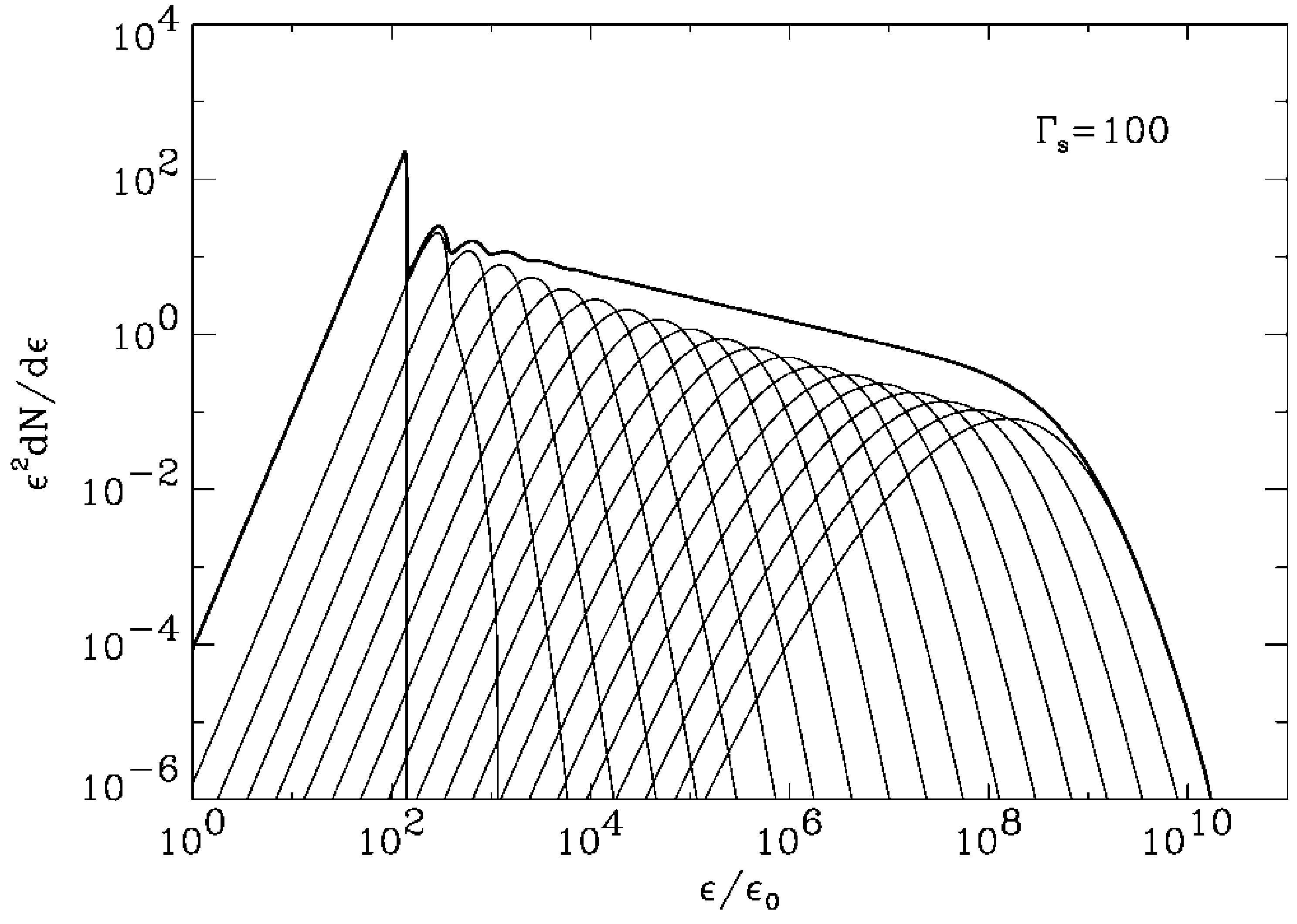
LP+PL spectra

Synch index~[1.6-1.7]=>s~[2.2-2.4]

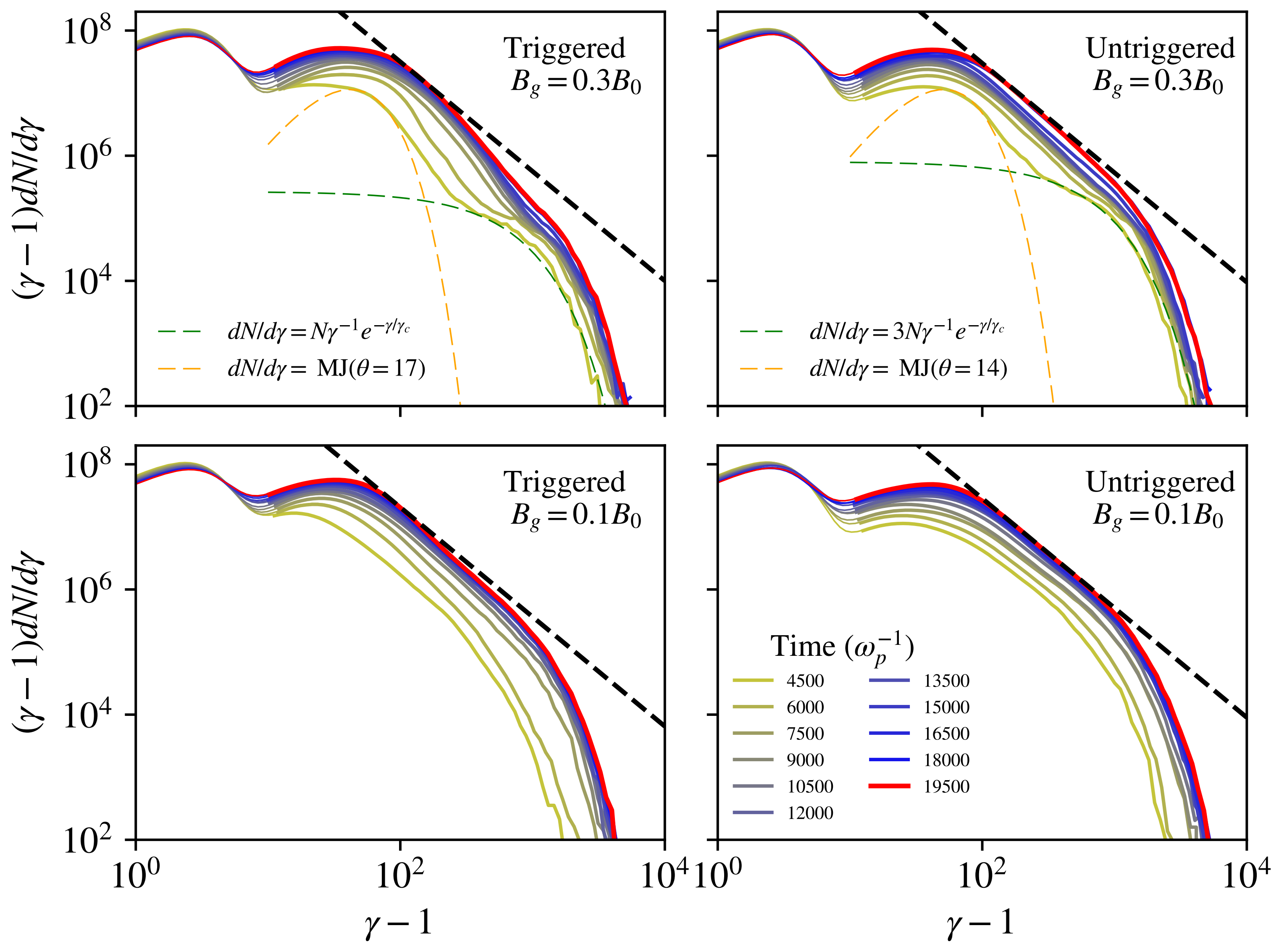




Lemoine,Pelletier 2003 F1 multip.

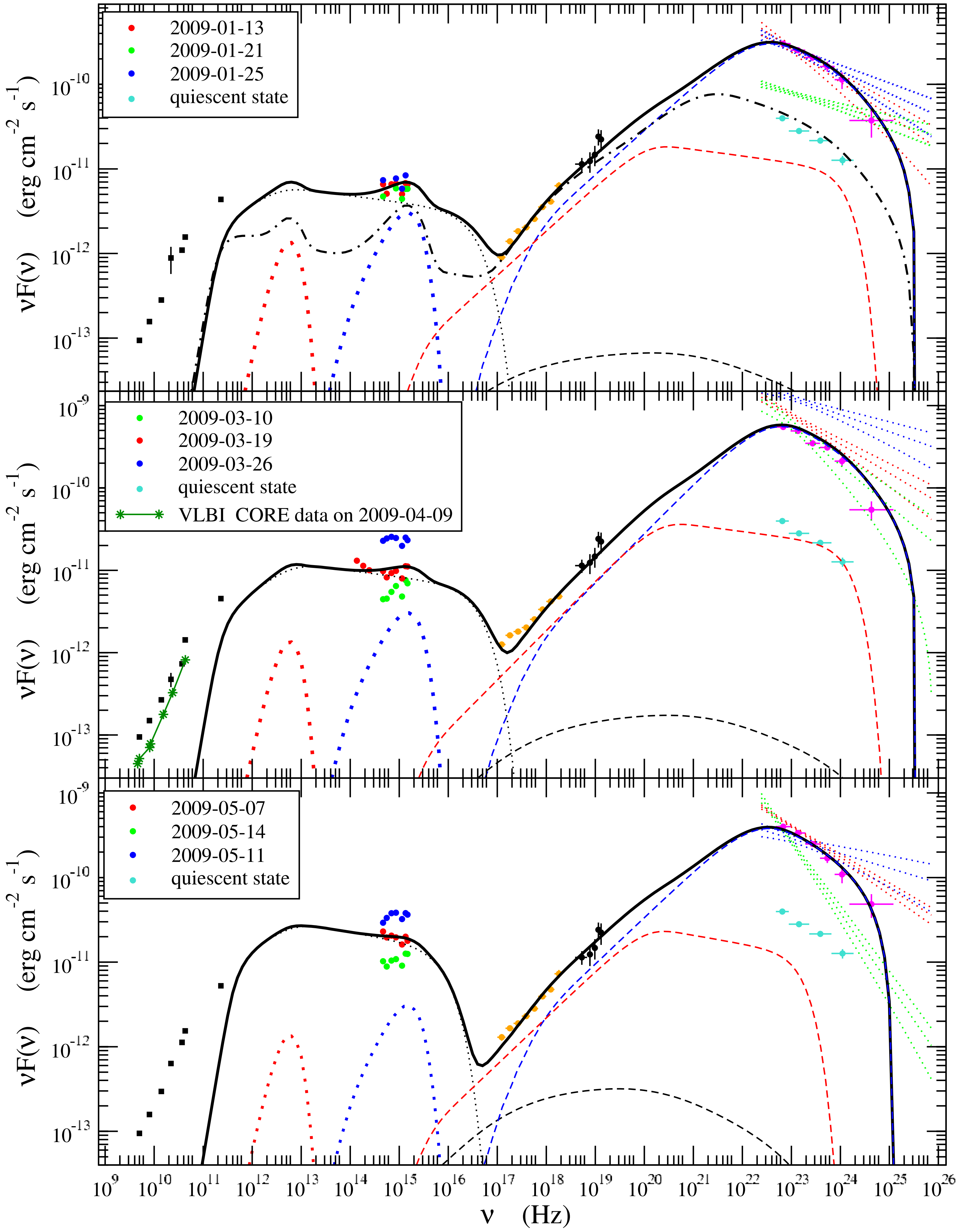


Ball+ 2019 magnetic reconnection (PIC)

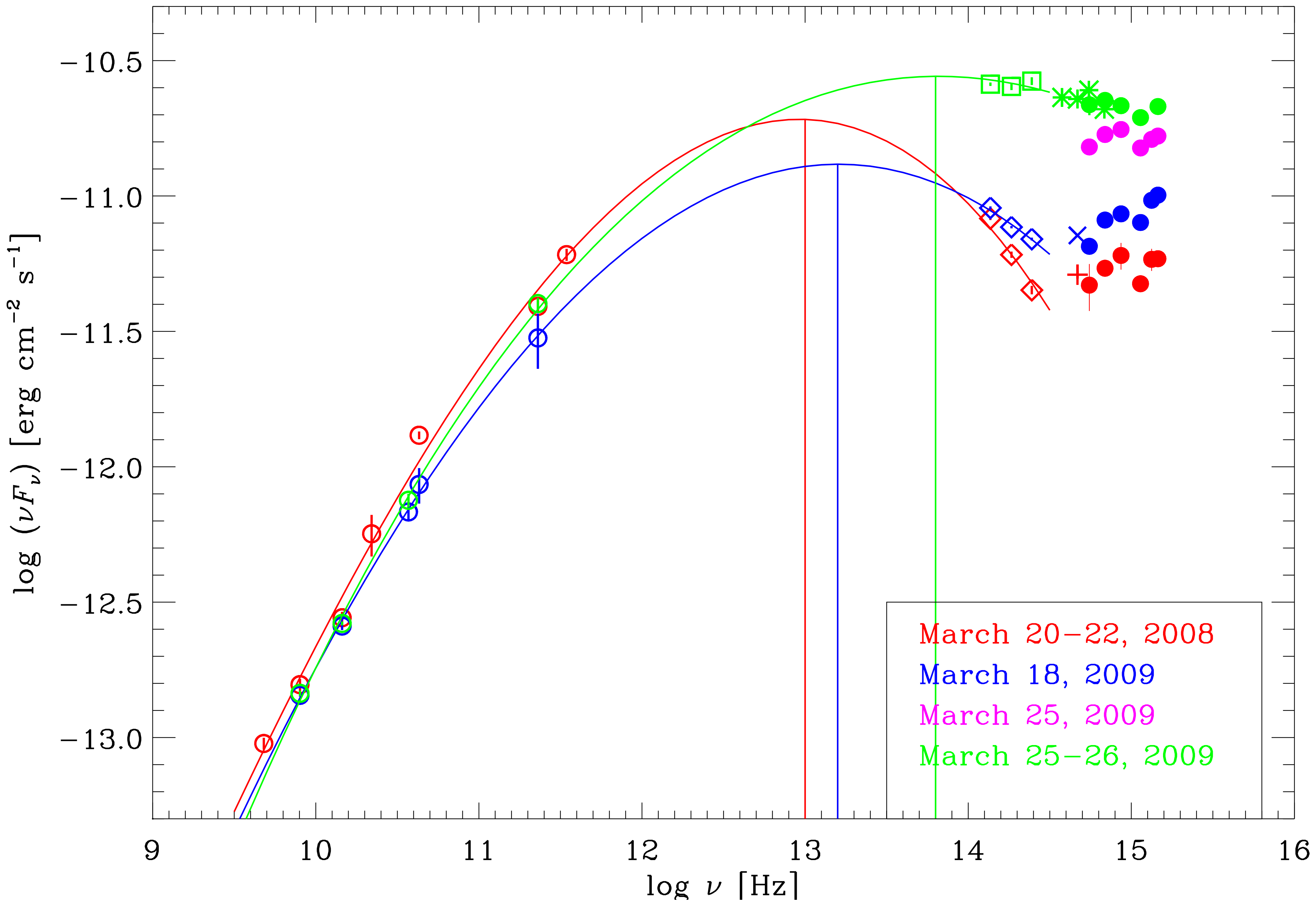


FSRQs

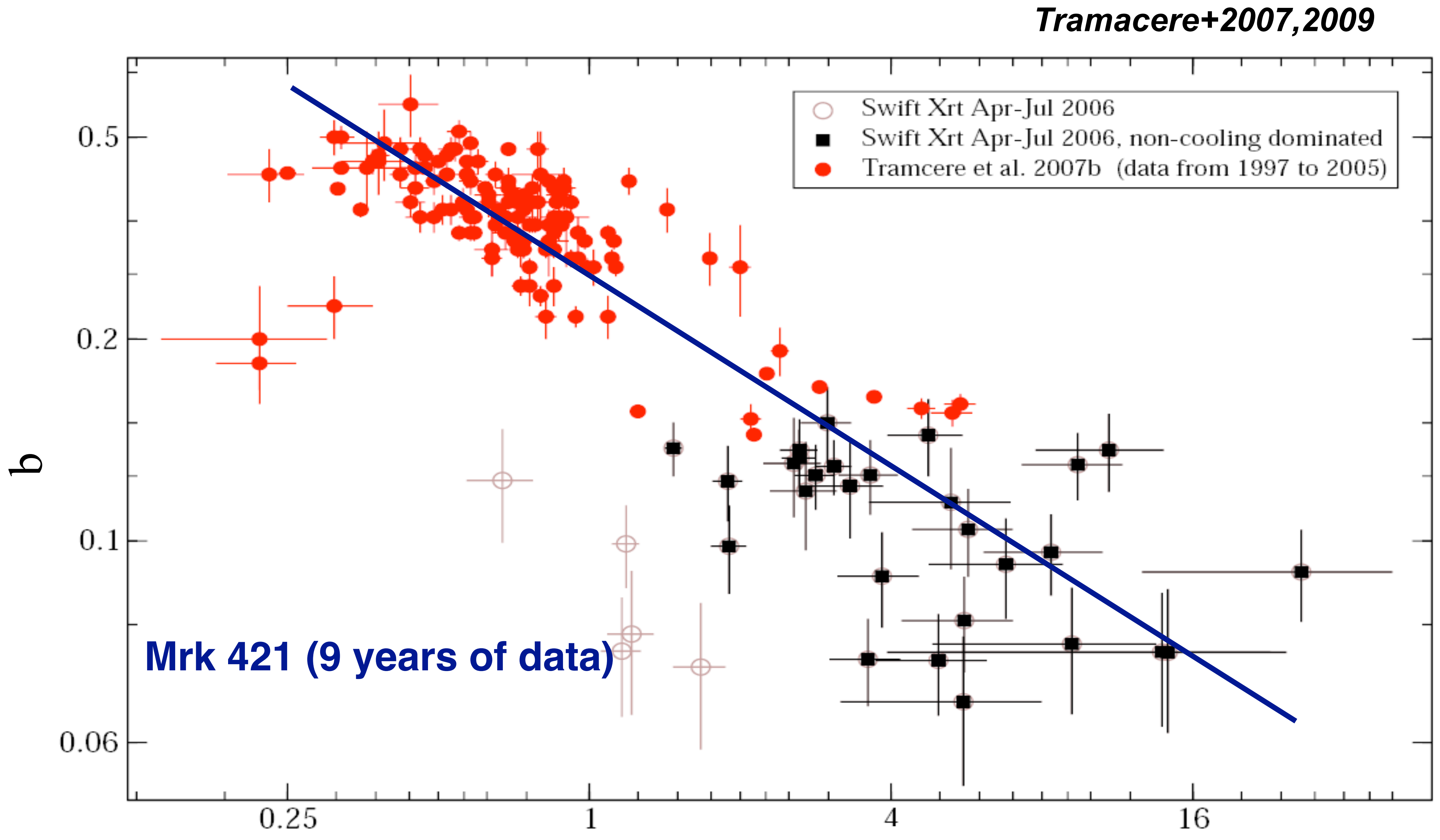
PSK 1510-089 Abdo+ 2010

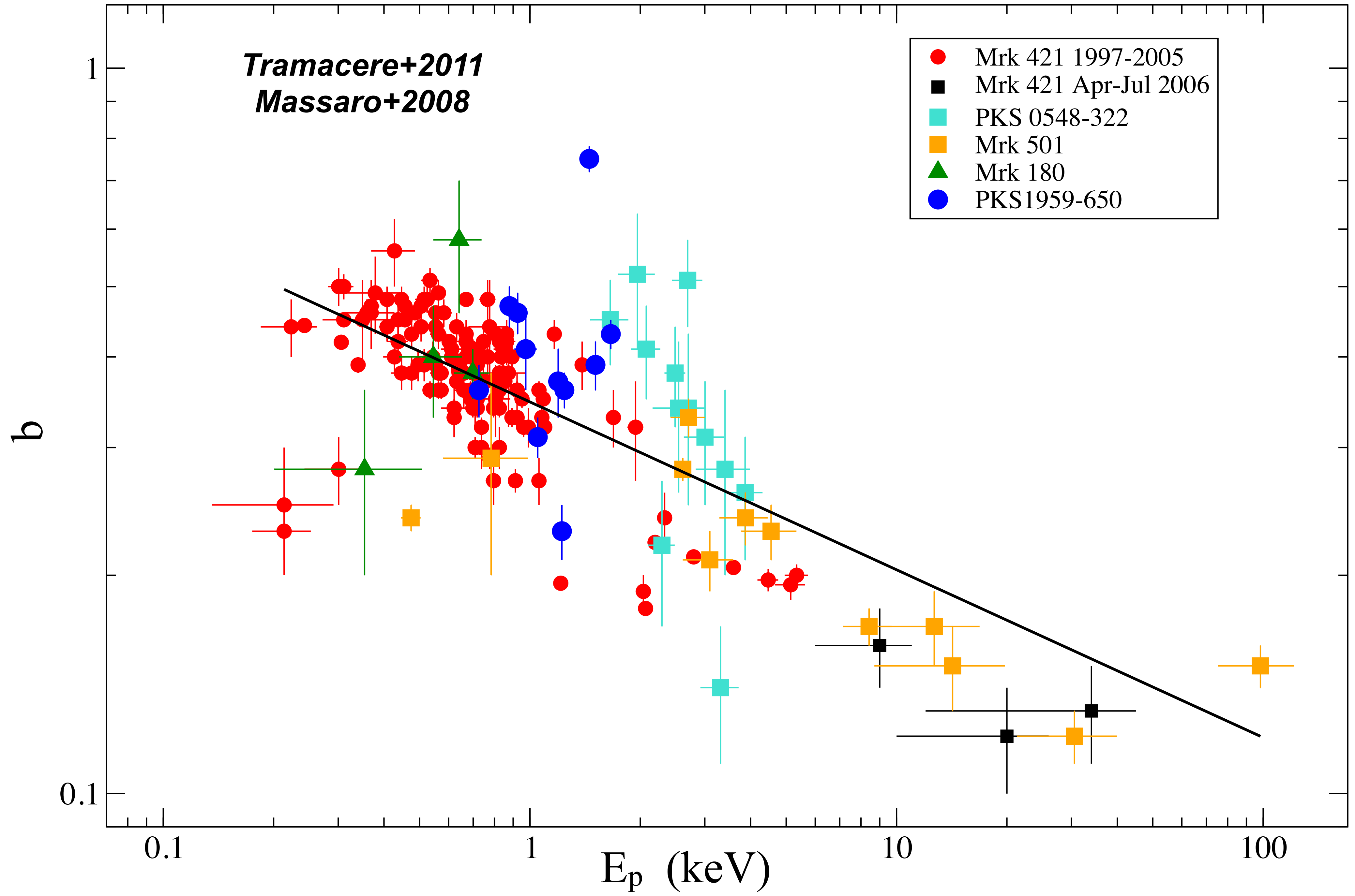


Dammando+ 2012

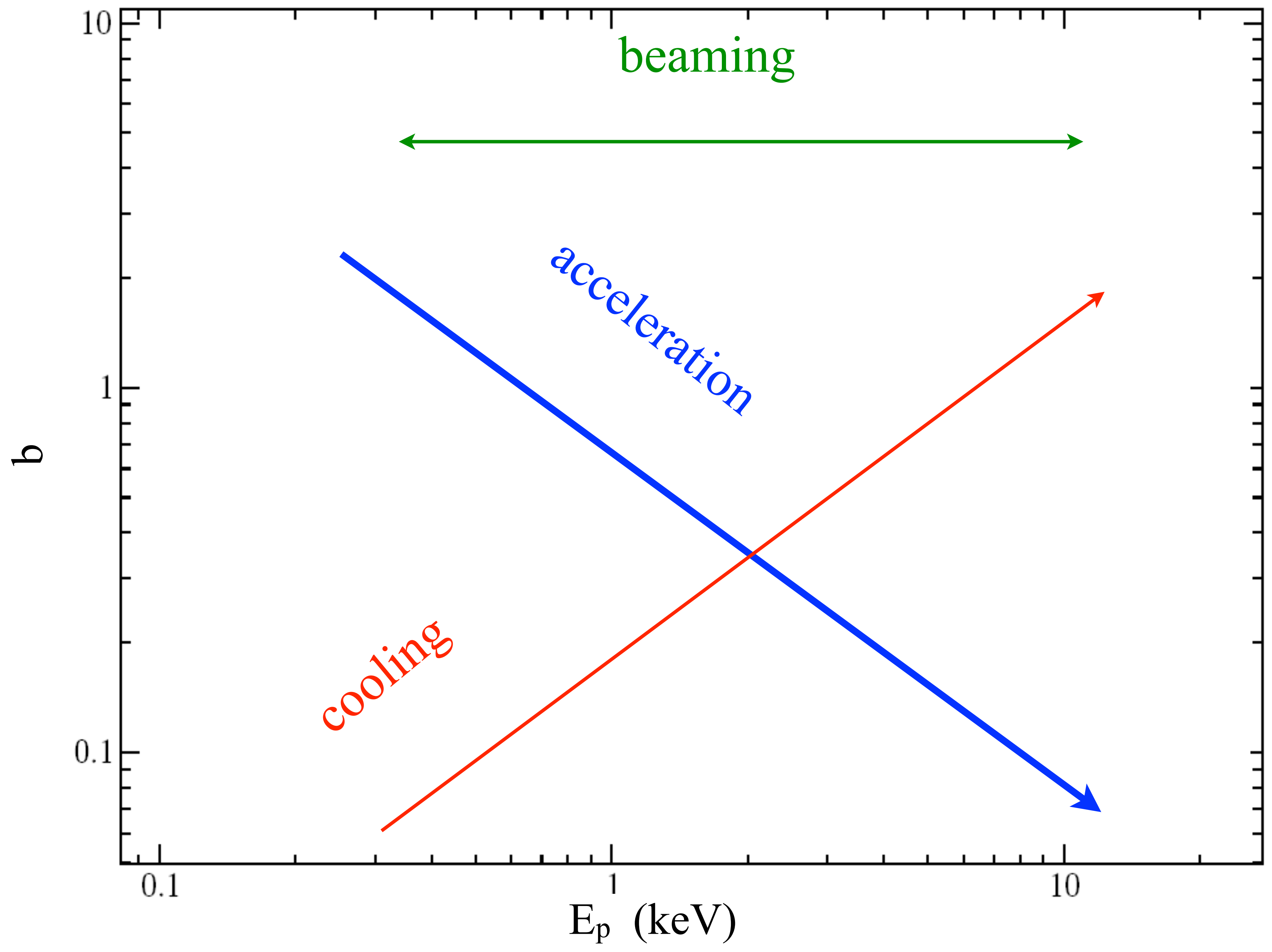


acceleration signature in the E_S-vs-b trend





Ep-vs-b, different scenarios



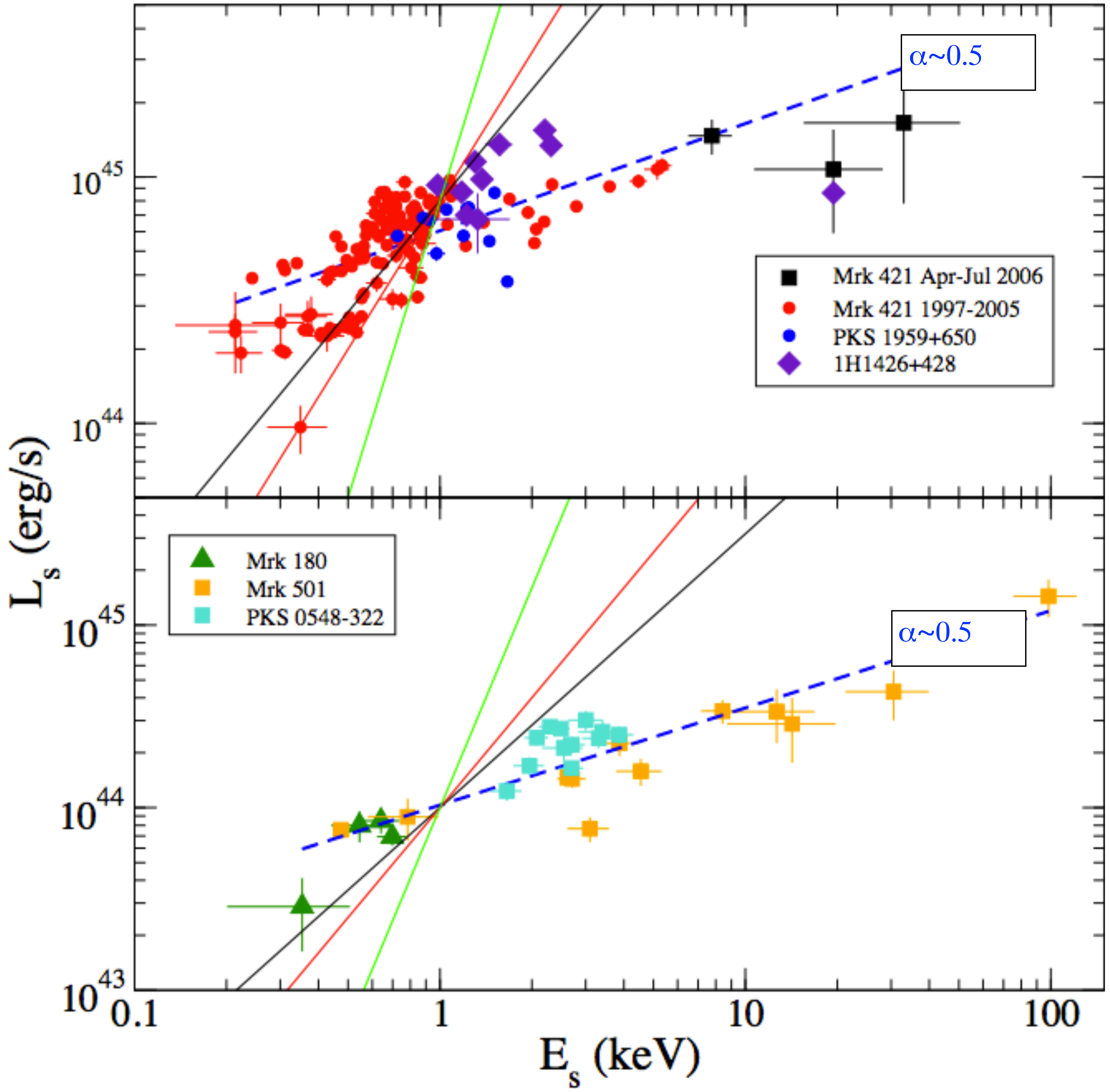
11 years of data:

PKS 0548-322,1H1426+418, Mrk 501 ,1ES1959+650, PKS2155-34

Long term (overall 13 years of data) Ep-vs-b trends hint for an acceleration dominated scenario

acceleration signature in the E_S-vs-L_s trend

long-trend main drivers

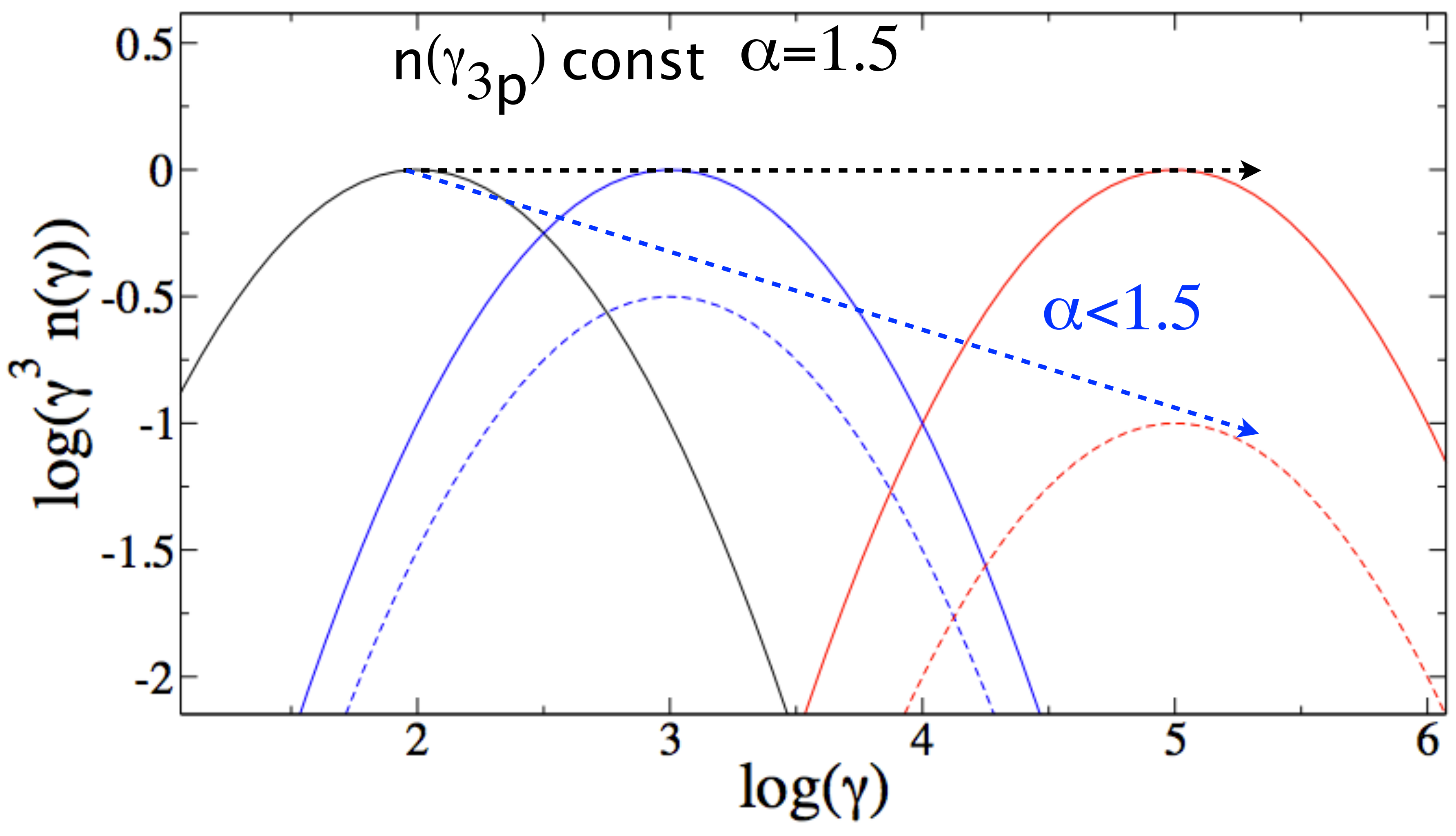


•γ_{3p} ↑ **and n(γ3p)** ↓ **=>** $\alpha<1.5$

acceleration+energy conservation

•B->α=2.0,
•δ->α=4

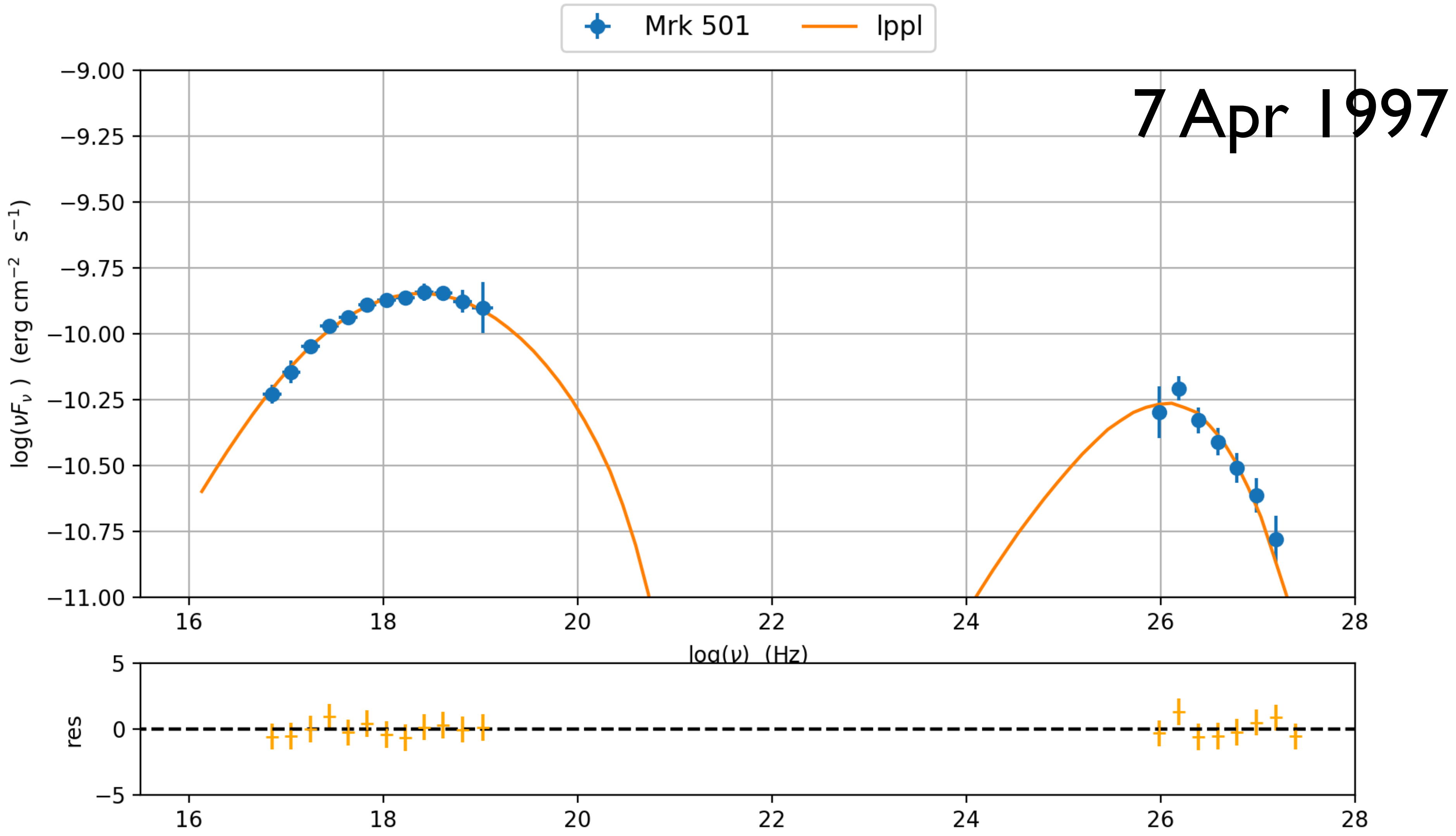
incompatible as long-trend main driver

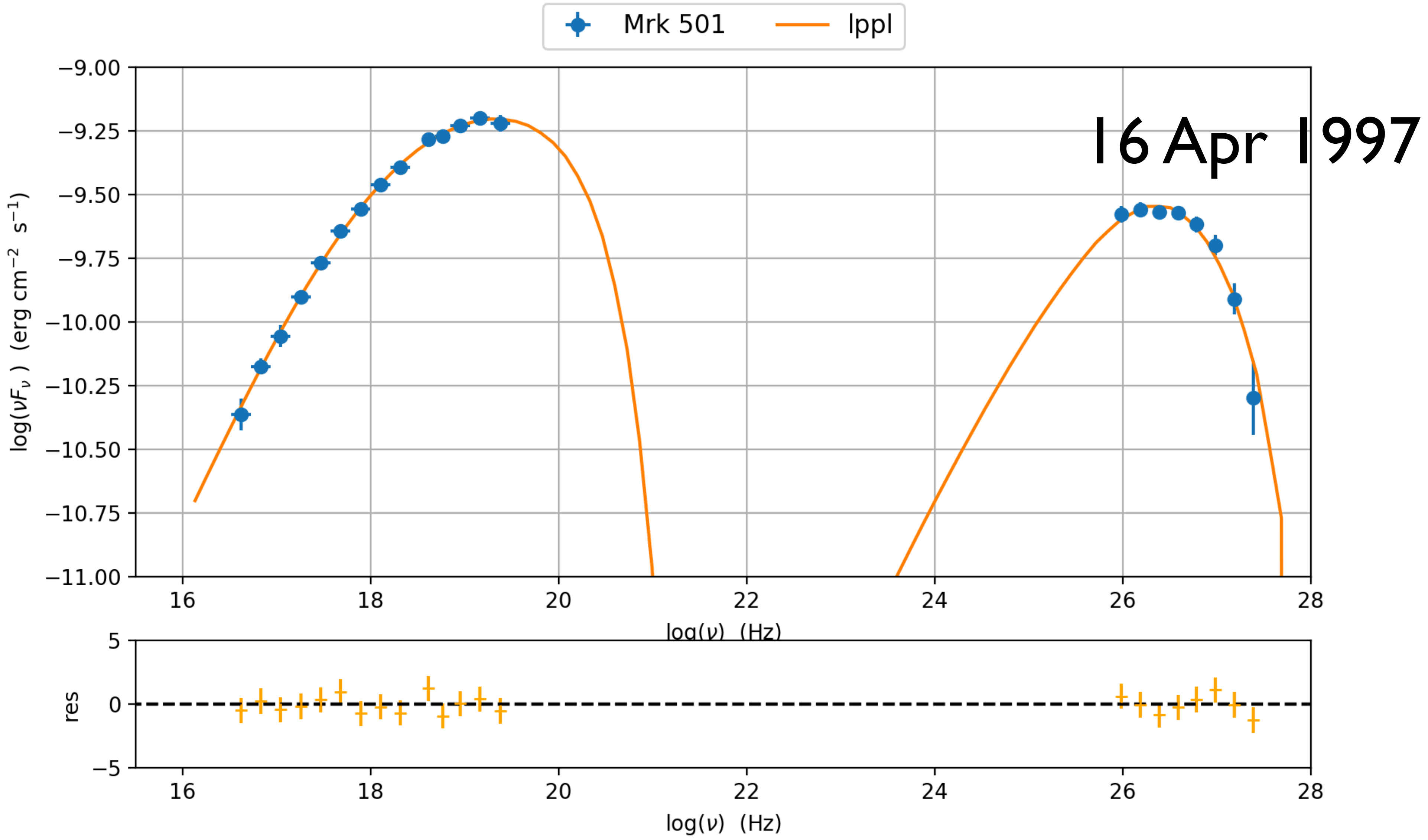


Hard spectra $s \ll 2.00$

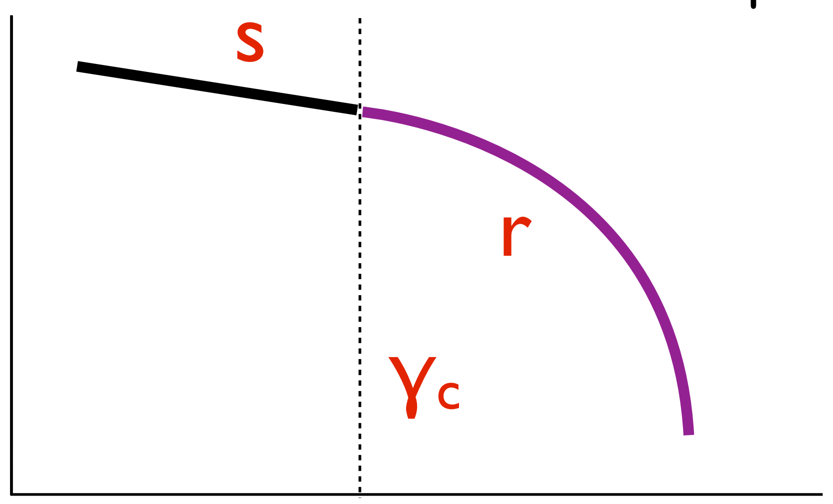
Mrk 501 1997 Flare

Massaro & Tramacere +2006





$$s = 1 + \frac{t_{acc}}{2t_{esc}}$$



best fit pars

best-fit parameters:

Name	best-fit value	best-fit err +
B	+1.072178e-01	+5.436622e-03
N	+4.585348e+00	+4.756569e-01
R	Frozen	Frozen
beam_obj	+2.450884e+01	+7.642113e-01
gamma0_log_parab	+6.609649e+04	+7.427709e+03
gmax	+1.860044e+14	+5.881595e+14
gmin	+1.404527e+03	+2.198648e+02
r	+7.513452e-01	+5.059815e-02
s	+1.638026e+00	+3.170384e-02
z_cosm	Frozen	Frozen

best-fit parameters:

Name	best-fit value	best-fit err +
B	+3.065207e-01	+1.159567e-02
N	+1.079944e+02	+7.375385e+00
R	Frozen	Frozen
beam_obj	+2.722013e+01	+5.889626e-01
gamma0_log_parab	+6.493888e+04	+5.410315e+03
gmax	+1.902146e+06	+2.216666e+02
gmin	+3.003970e+02	+5.686711e+01
r	+6.778727e-01	+3.526656e-02
s	+1.321307e+00	+1.844825e-02
z_cosm	Frozen	Frozen

The origin of the log-parabolic shape: statistical derivation

fluctuation

$$\varepsilon = \bar{\varepsilon} + \chi$$

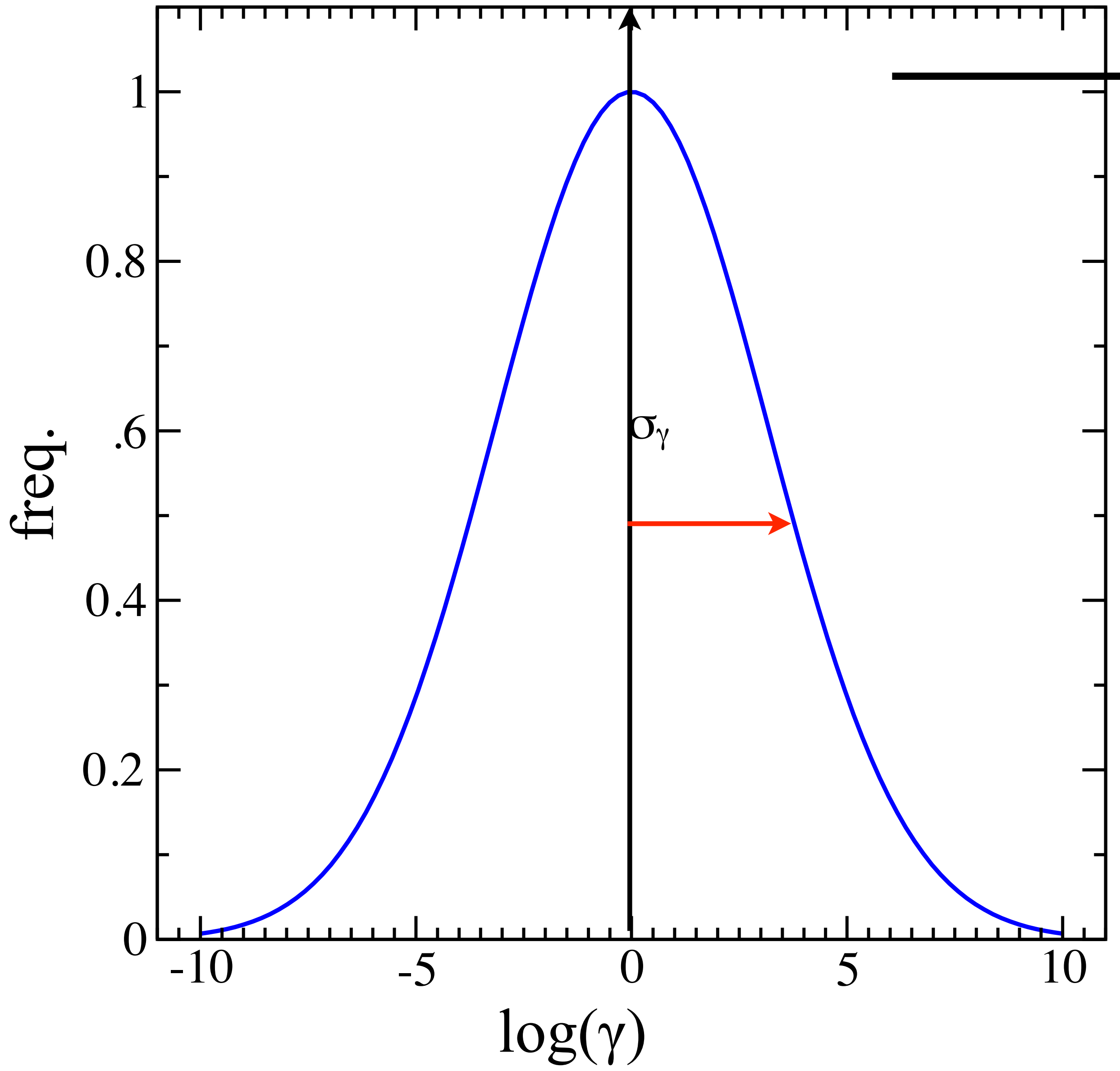
systematic

ε_i is a R.V.

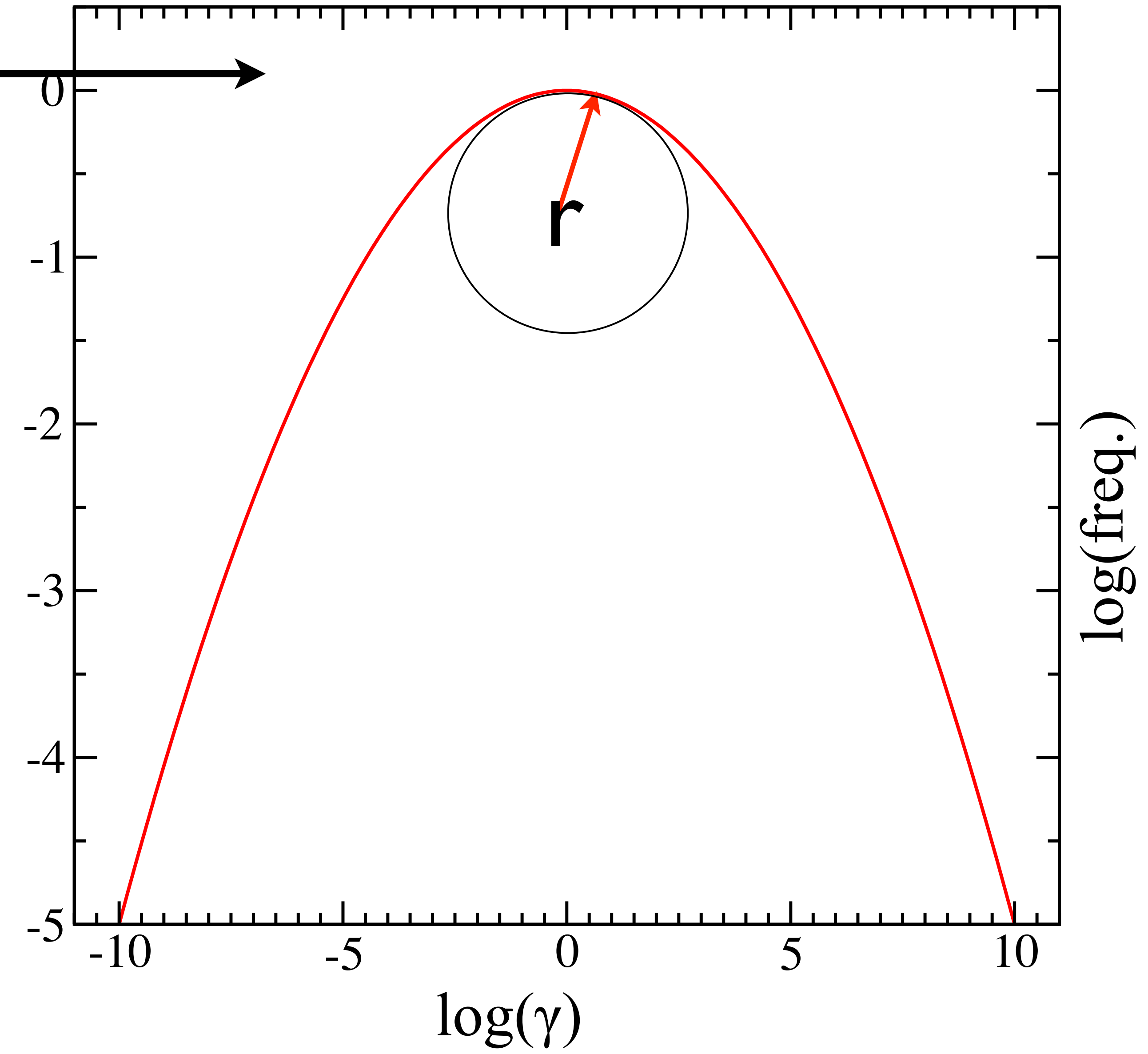
$$\gamma_{n_s} = \gamma_0 \Pi_{i=1}^{n_s} \varepsilon_i$$

C.L. Theorem
multipl. case

log-normal distribution



Log-Parabolic representation



$$\log(n(\gamma)) \propto \frac{(\log\gamma - \mu)^2}{2\sigma_\gamma^2} \propto r\ [\log(\gamma) - \mu]^2$$

$$\sigma_y^2 = \sigma^2(\log(\gamma)) \approx n_s \left(\frac{\sigma_\varepsilon}{\bar{\varepsilon}}\right)$$

curvature = $\dfrac{1}{\sigma_\gamma} = 2r$

$$r = \frac{c_e}{2n_s\left(\frac{\sigma_\varepsilon}{\bar{\varepsilon}}\right)^2}$$

The origin of the log-parabolic shape: diffusion equation approach

$$\frac{\partial n(\gamma,t)}{\partial t} = \frac{\partial}{\partial\gamma}\left\{-[S(\gamma,t)+D_A(\gamma,t)]n(\gamma,t)+D_p(\gamma,t)\frac{\partial n(\gamma,t)}{\partial\gamma}\right\} - \frac{n(\gamma,t)}{T_{esc}(\gamma)} + Q(\gamma,t)$$

CL theorem

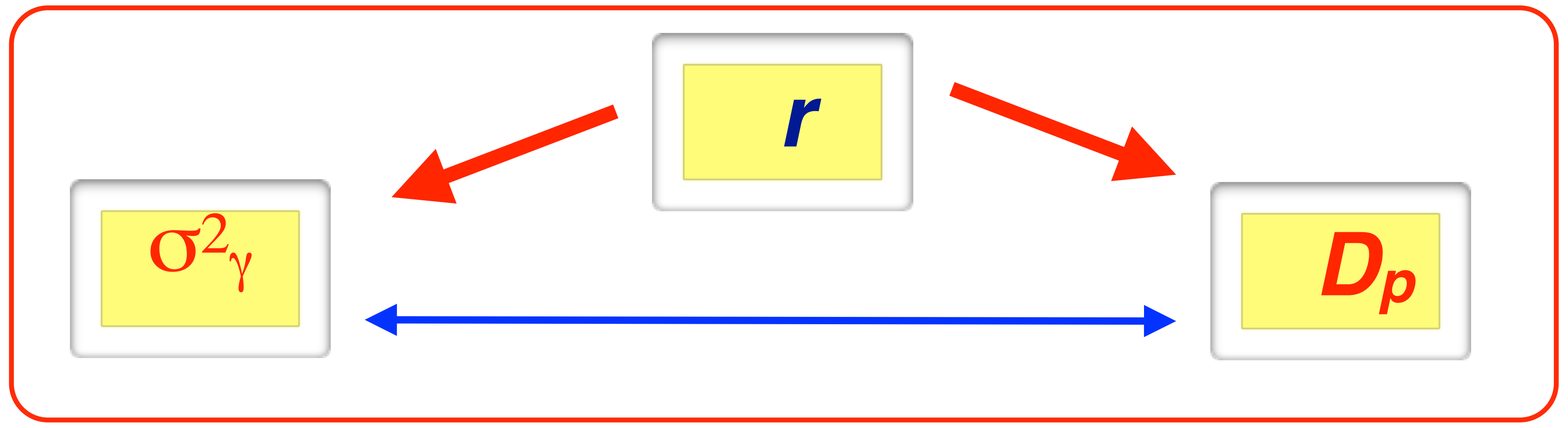
analytical solution for:
$D_p \sim \gamma^q$, $q=2$
"hard-sphere" case
Melrose 1968, Kardashedv 1962

$$\boxed{n(\gamma,t) = \frac{N_0}{\gamma\sqrt{4\pi D_{p0}t}}\exp\left\{-\frac{[\ln(\gamma/\gamma_0)-(A_{p0}-D_{p0})t]^2}{4D_{p0}t}\right\}}$$

$$\boxed{\log(n(\gamma)) \propto \frac{(\log\gamma-\mu)^2}{2\sigma_\gamma^2} \propto r\;[\log(\gamma)-\mu]^2}$$

$$\propto \frac{1}{D_{p0}t} \rightarrow D_{p0} \propto \left(\frac{\sigma_\varepsilon}{\bar{\varepsilon}}\right)^2$$

The curvature *r* is inversely proportional to ***t=>n***$_s$ and ***D***$_p$***=>***σ_ε



JetSeT Temporal evolution

Tramacere +2011

injection term

$$L_{inj} = \frac{4}{3}\pi R^3 \int \gamma_{inj} m_e c^2 Q(\gamma_{inj}, t) d\gamma_{inj} \quad (erg/s)$$

systematic term

$$S(\gamma, t) = -C(\gamma, t) + A(\gamma, t)$$

cooling term

$$C(\gamma) = |\dot{\gamma}_{\rm synch}| + |\dot{\gamma}_{\rm IC}|$$

syst. acc. term

$$A(\gamma) = A_{p0}\gamma, t_A = \frac{1}{A_0}$$

$$\frac{\partial n(\gamma,t)}{\partial t} = \frac{\partial}{\partial \gamma}\left\{ -[S(\gamma,t) + D_A(\gamma,t)]n(\gamma,t) + D_p(\gamma,t)\frac{\partial n(\gamma,t)}{\partial \gamma}\right\} - \frac{n(\gamma,t)}{T_{\rm esc}(\gamma)} + Q(\gamma,t)$$

Turbulent magnetic field $\longrightarrow$ momentum diffusion term

$$W(k) = \frac{\delta B(k_0^2)}{8\pi}\left(\frac{k}{k_0}\right)^{-q}$$

$$D_p \approx \beta_A^2 \left(\frac{\delta B}{B_0}\right)^2 \left(\frac{\rho_g}{\lambda_{\rm max}}\right)^{q-1} \frac{p^2 c^2}{\rho_g c}$$

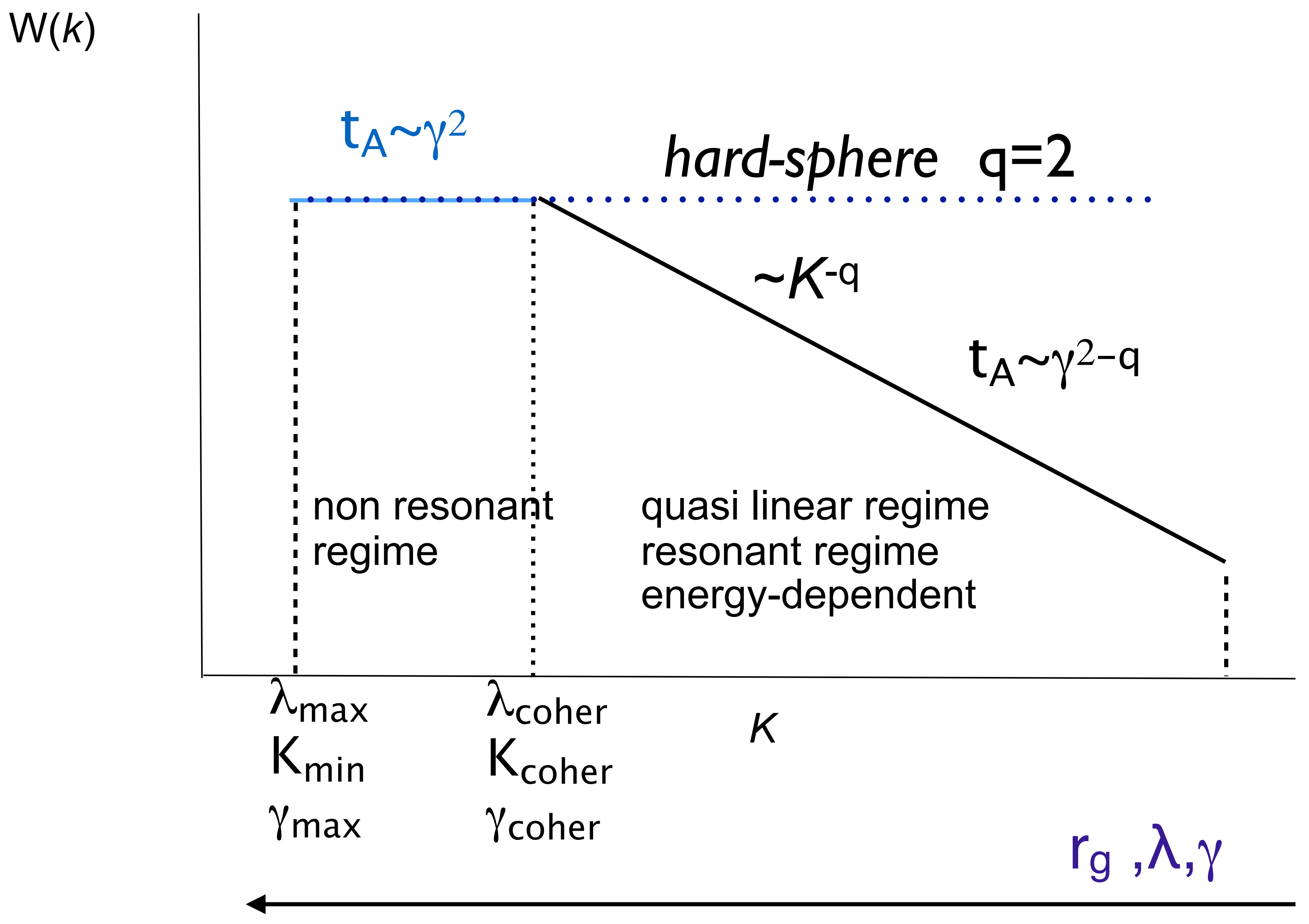
set-up of the accelerator

$$t_D = \frac{1}{D_{p0}}\left(\frac{\gamma}{\gamma_0}\right)^{2-q}$$

$$t_{DA} = \frac{1}{2D_{p0}}\left(\frac{\gamma}{\gamma_0}\right)^{2-q}$$

e-folding time

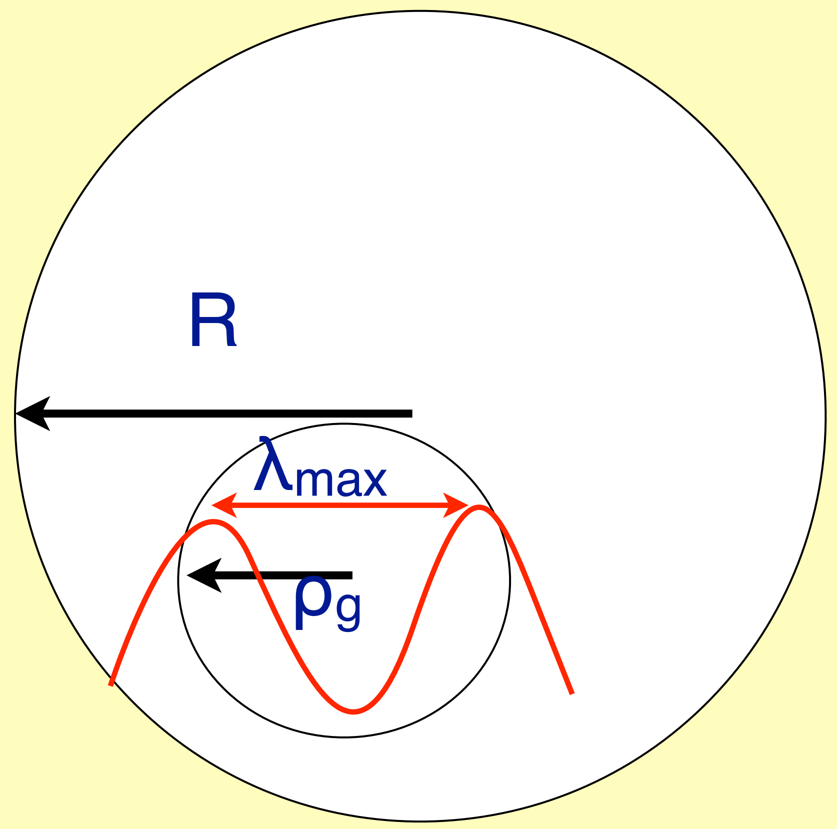
observed values		values compatible with Tammi & Duffy 2009
$E_{p1}/E_{p2} \sim 5$	→	$t_{DA} \sim< 5$ ks
$\Delta t \sim$ few ks		$t_D \sim< 10$ ks



set-up of the accelerator

- $R \sim 10^{13}$-10^{15} cm
- $\delta B/B \ll 1$, $B \sim [0.01$-$1.0]$ G
- $\beta_A \sim 0.1$-0.5
- $\lambda_{max} < R$ => $\sim 10^{[9-15]}$ cm
- $\rho_g < \lambda_{max}$ => $\gamma_{max} \sim 10^{7.5}$

→ **$t_D \sim< 10^4$ ks**

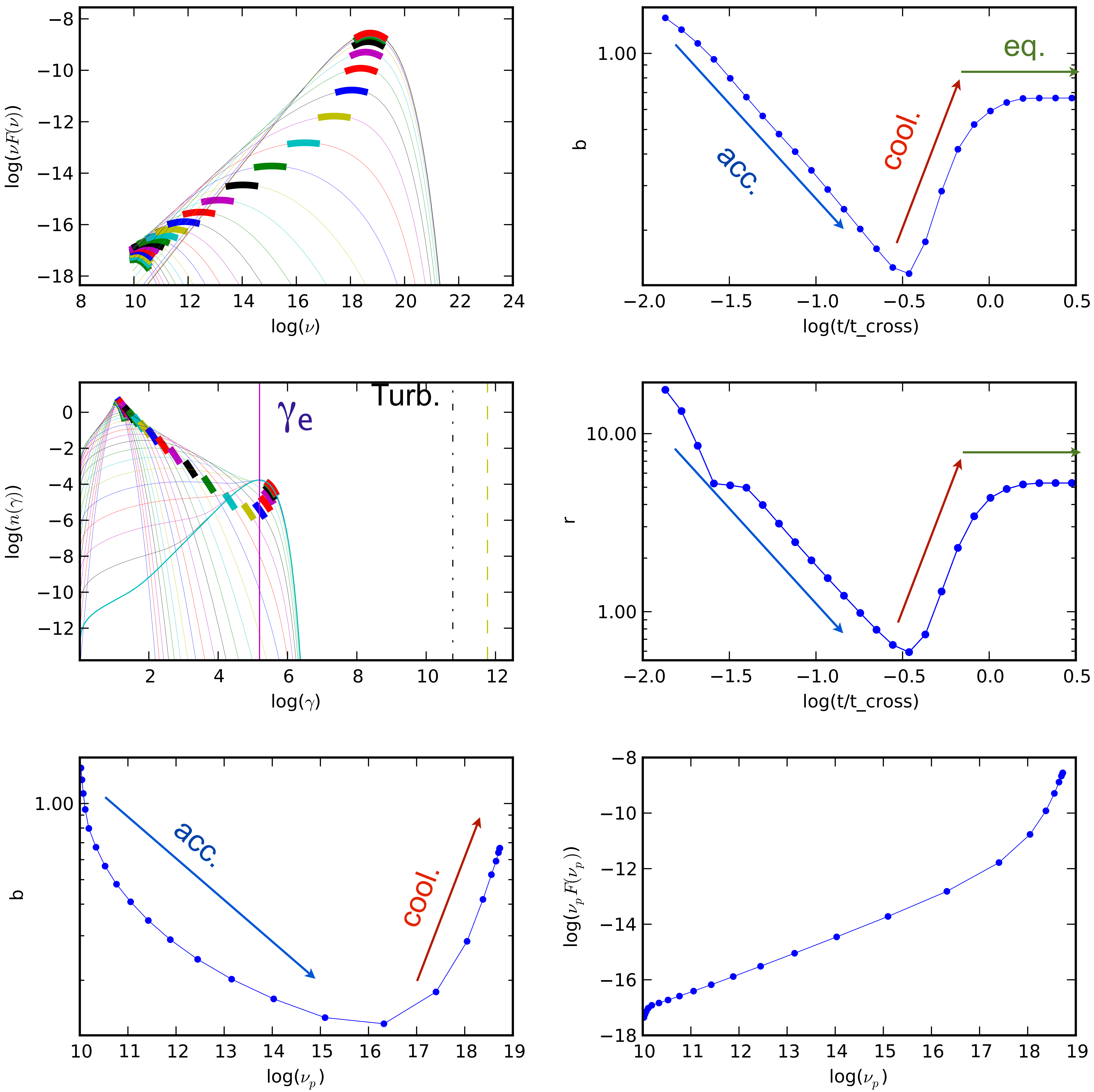


$$D_p \approx \beta_A^2 \left(\frac{\delta B}{B_0}\right)^2 \left(\frac{\rho_g}{\lambda_{\max}}\right)^{q-1} \frac{p^2c^2}{\rho_g c}$$

$$\rho_g = pc/qB$$

acceleration-vs-equil.

Tramacere +2011

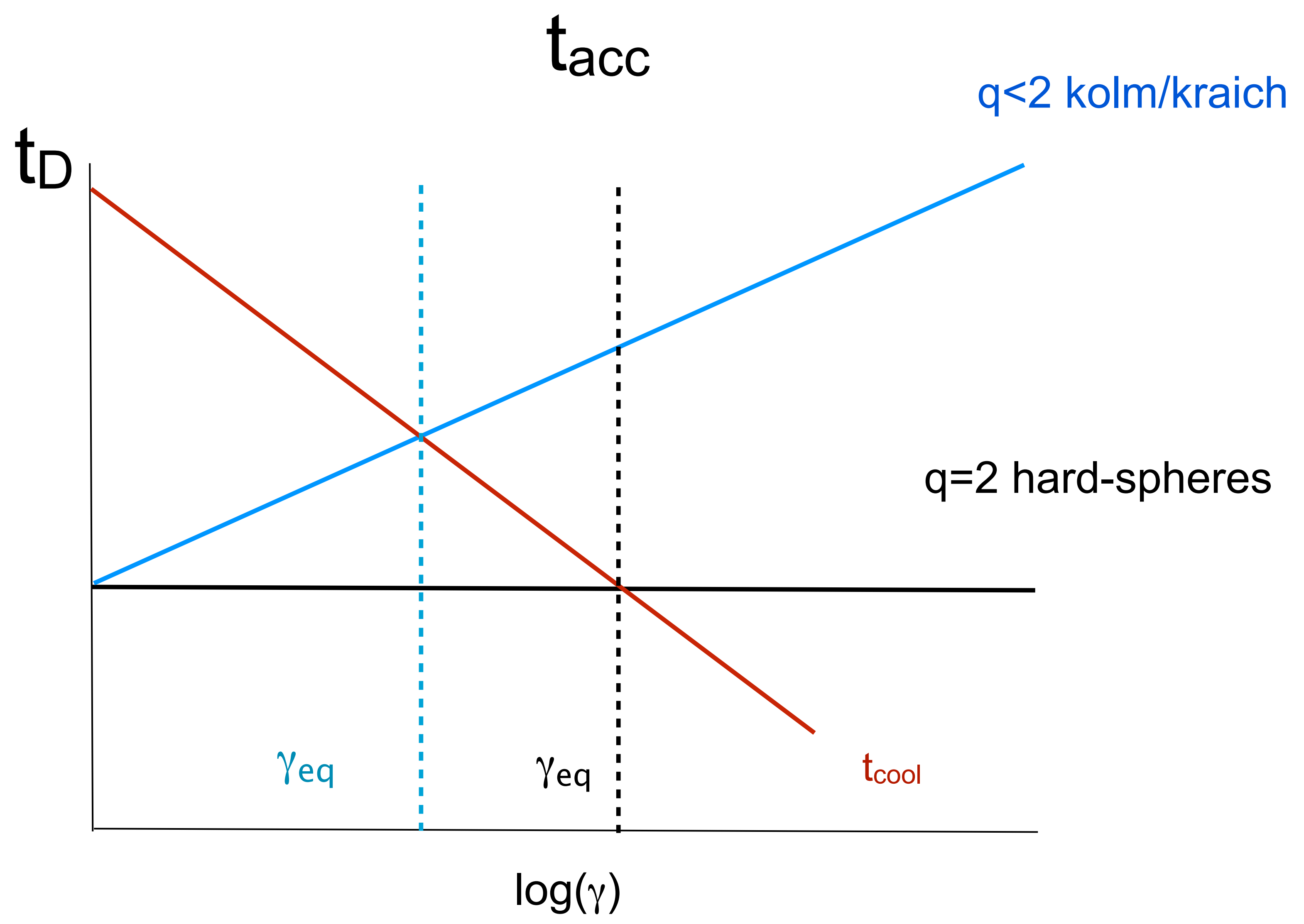


Synchrotron

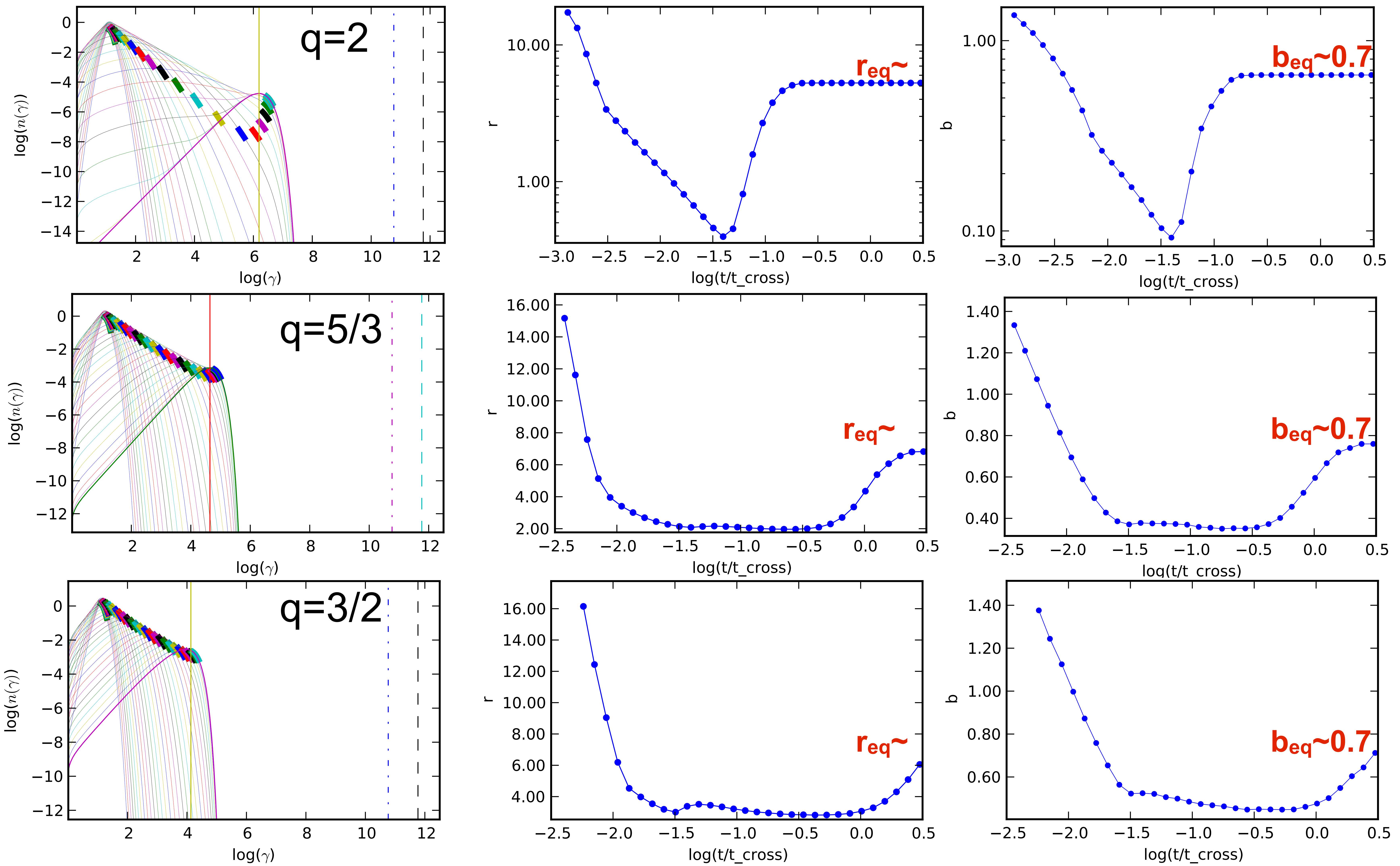
electrons

Synchrotron
spectral trends

effect of the turbulence index q and cooling

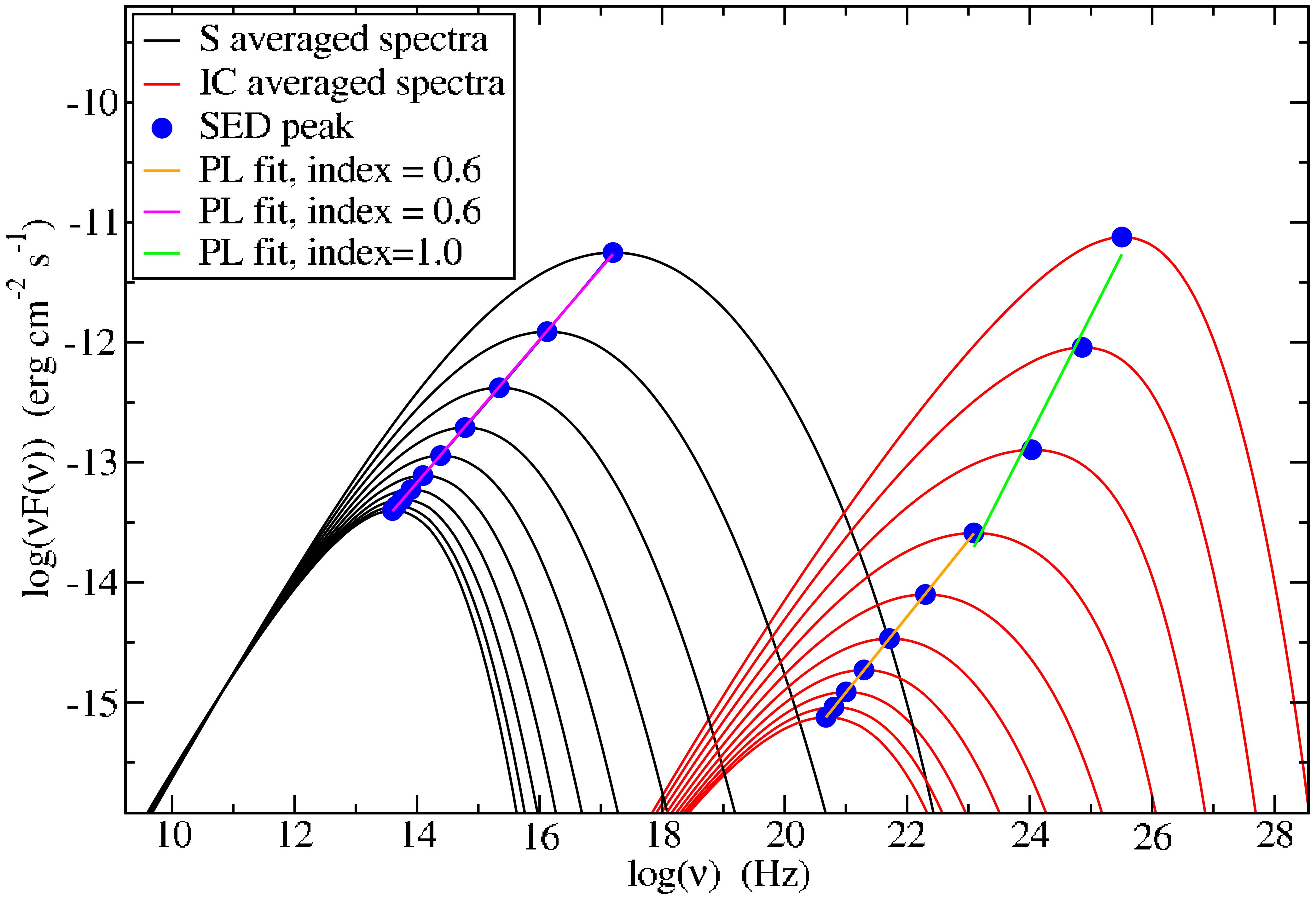


B=1.0 G, $t_{D0}=10^3$, R=5x10^{15} cm

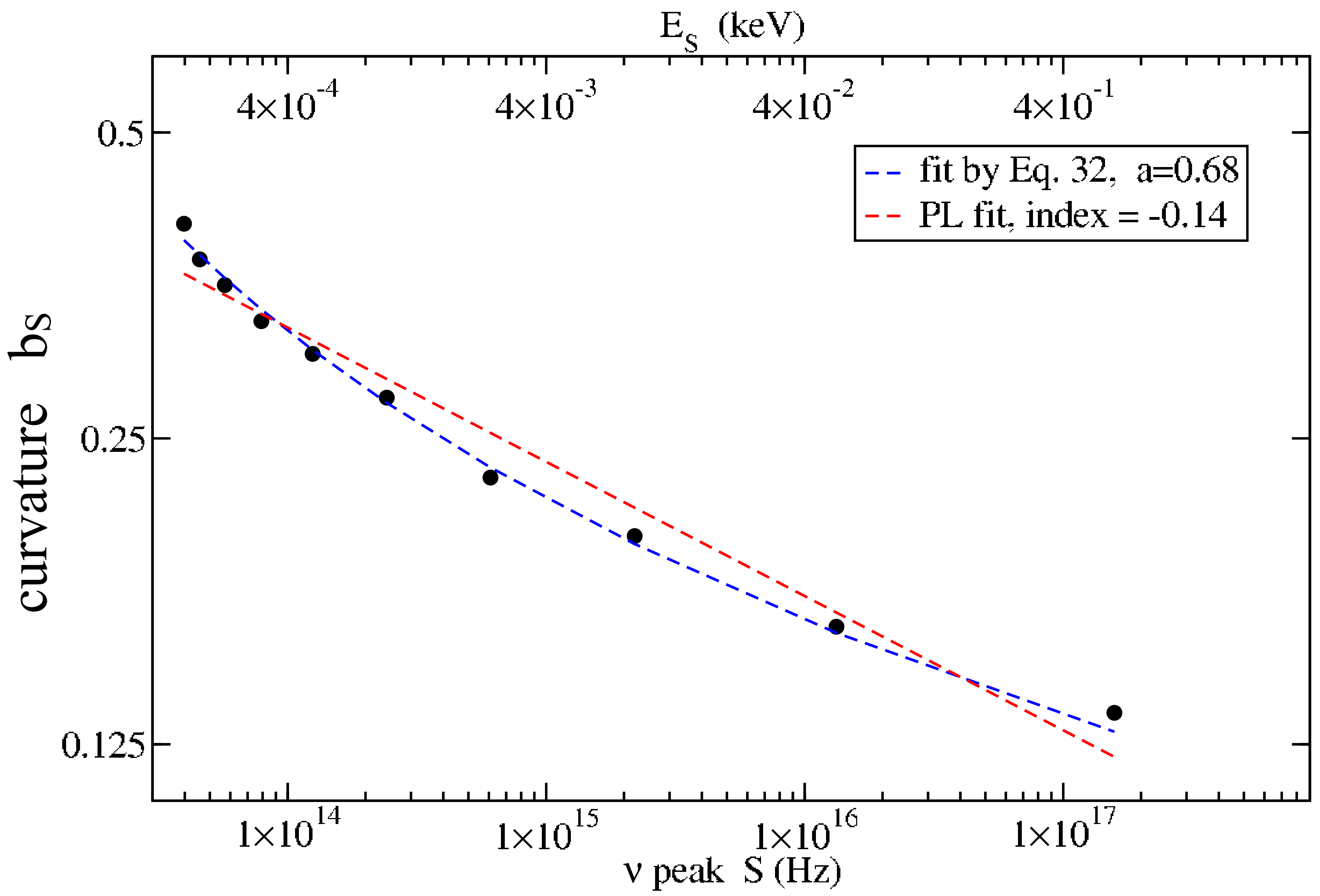


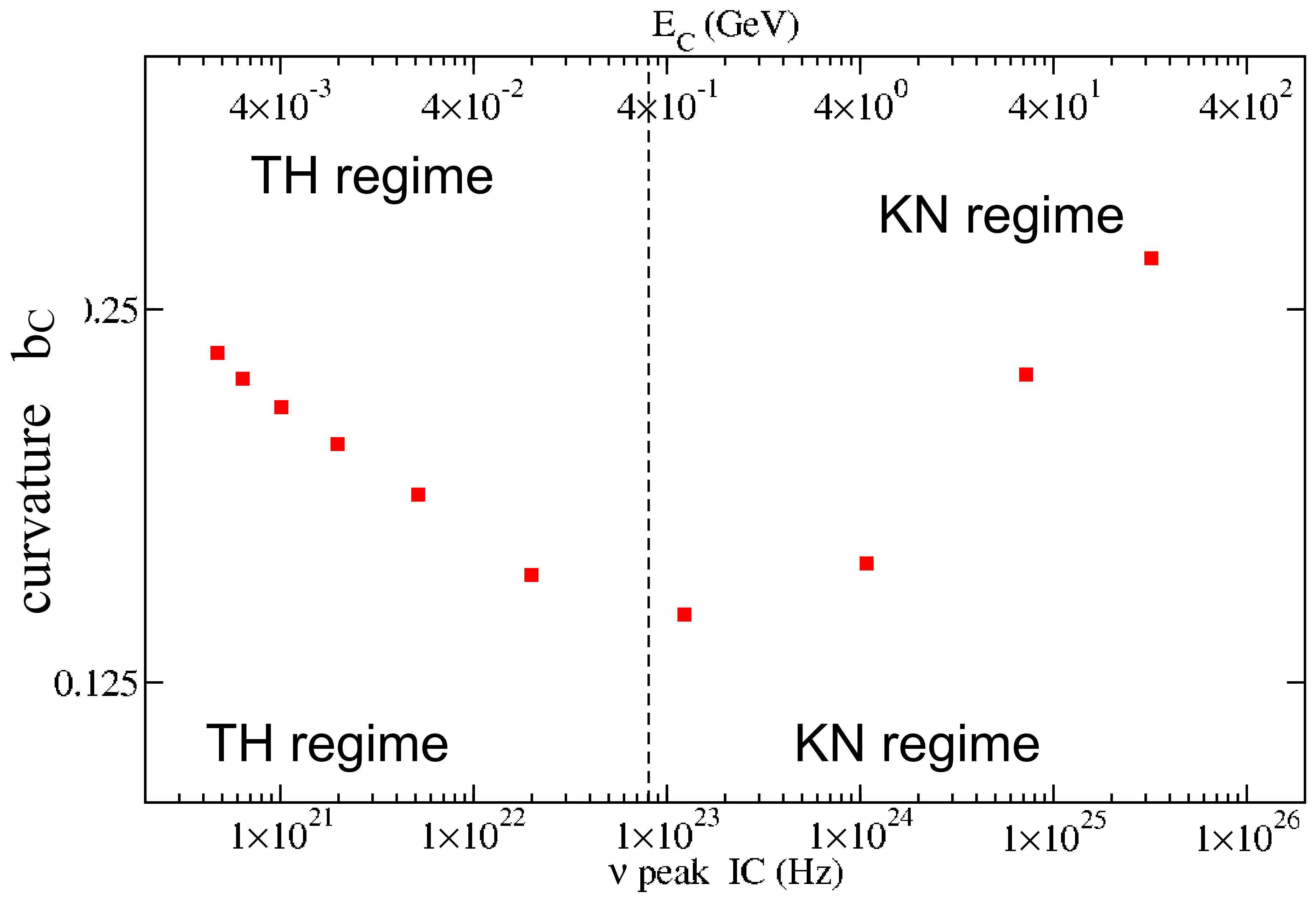
S vs IC

Tramacere+2011



Tramacere +2011

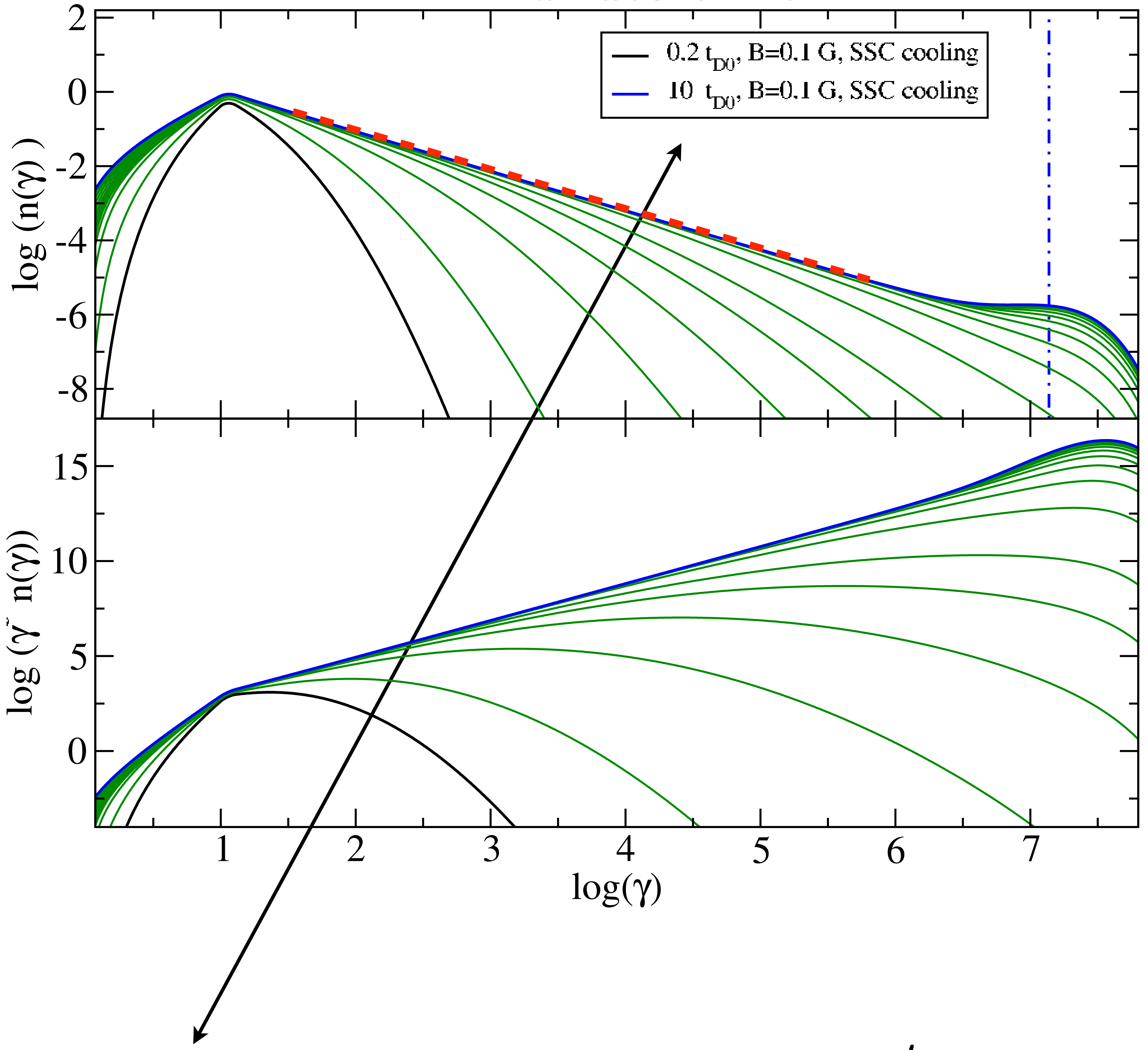




Pile-up and hard spectra

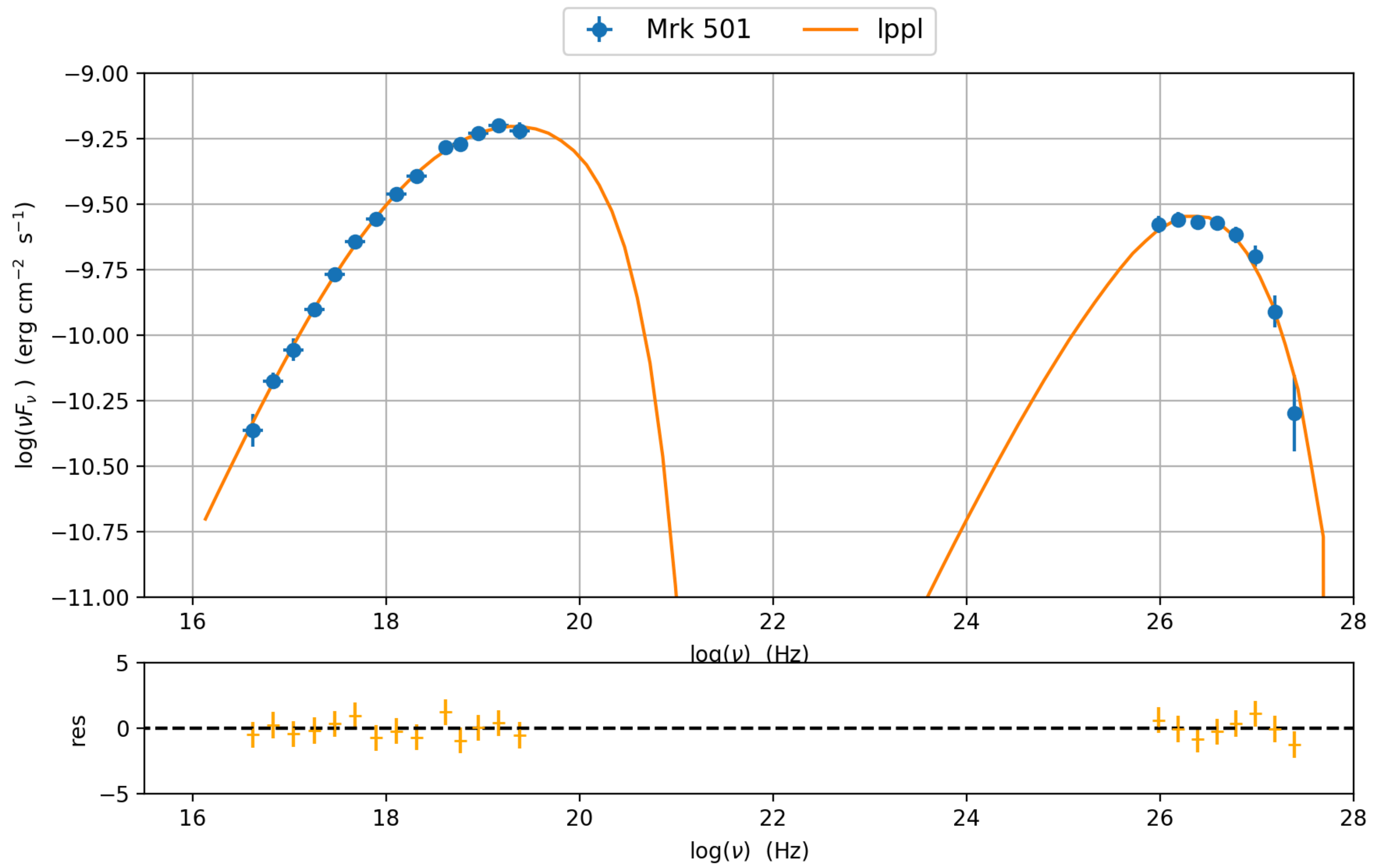
q=2, $R=10^{15}$ cm, B=0.1 G, $t_{inj}=t_D=10^4$ s

Tramacere +2011



s in agreement with $s = 1 + \frac{t_{acc}}{2t_{esc}}$

Mrk 501 1997



Massaro & Tramacere +2006

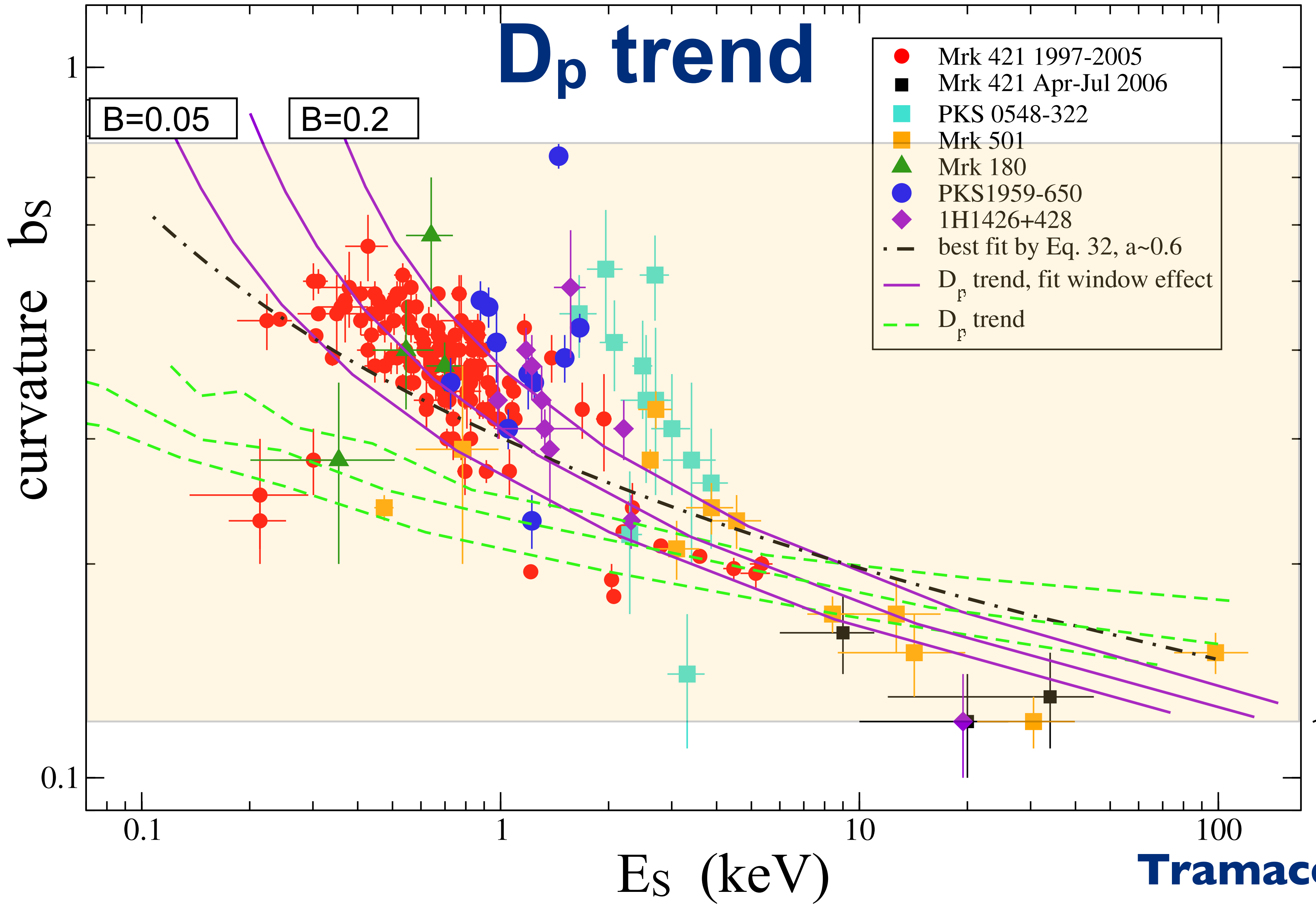
s~1.6

$r\sim0.7\text{-}0.8 \ll r_{eq}\sim6$

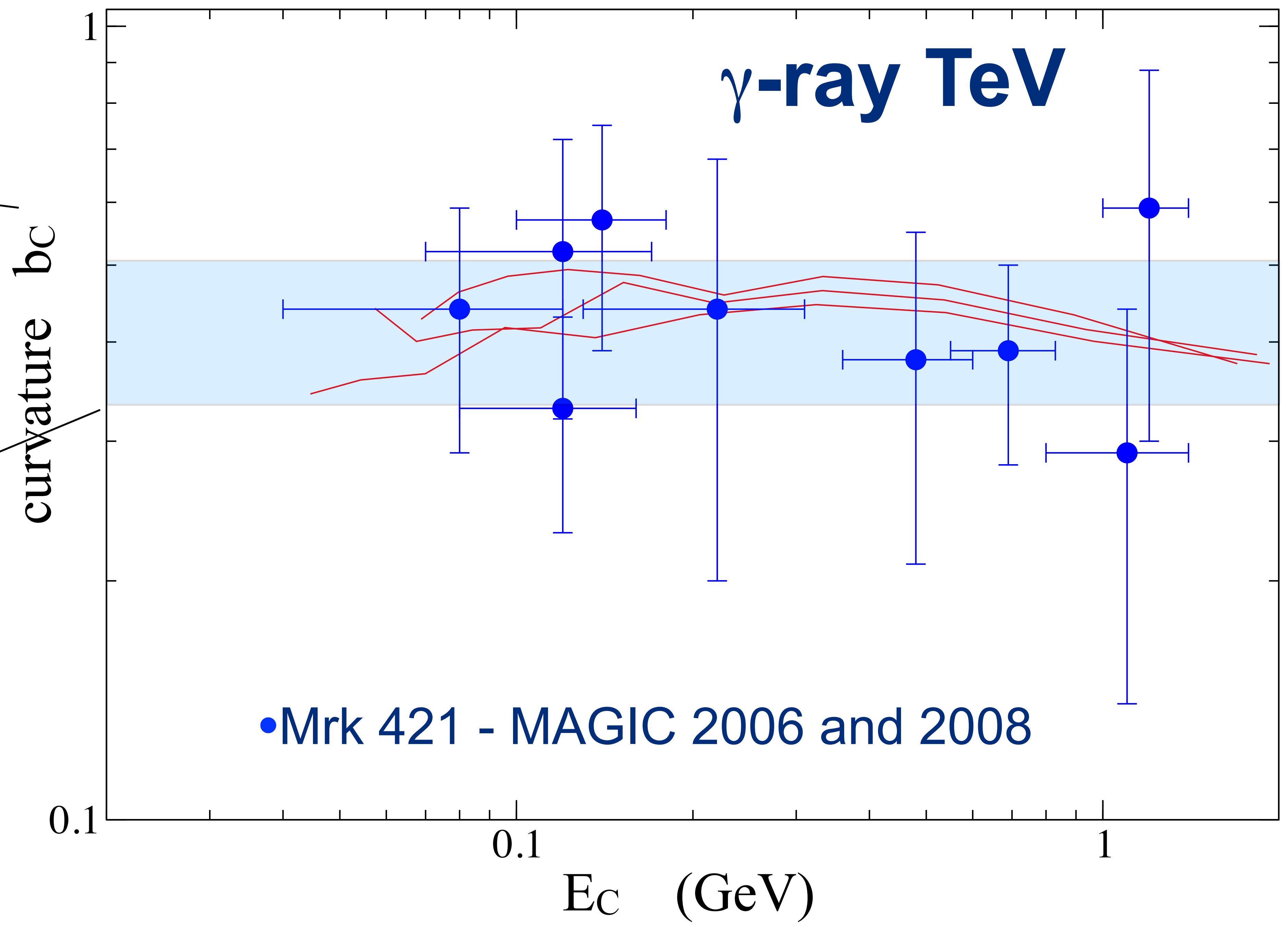
$s \ll s_{FI}\sim2.3$

	Acceleration dominated	Equilibrium
curvature trend	curvature decreasing trend *b-Ep*	curvature stable or increasing ($r \sim 7, b \sim 1.3$)
spectral shape	LPPL or LP	PL+exp-cutoff or Maxwellian

E_s-b_s X-ray trend and γ-ray predictions



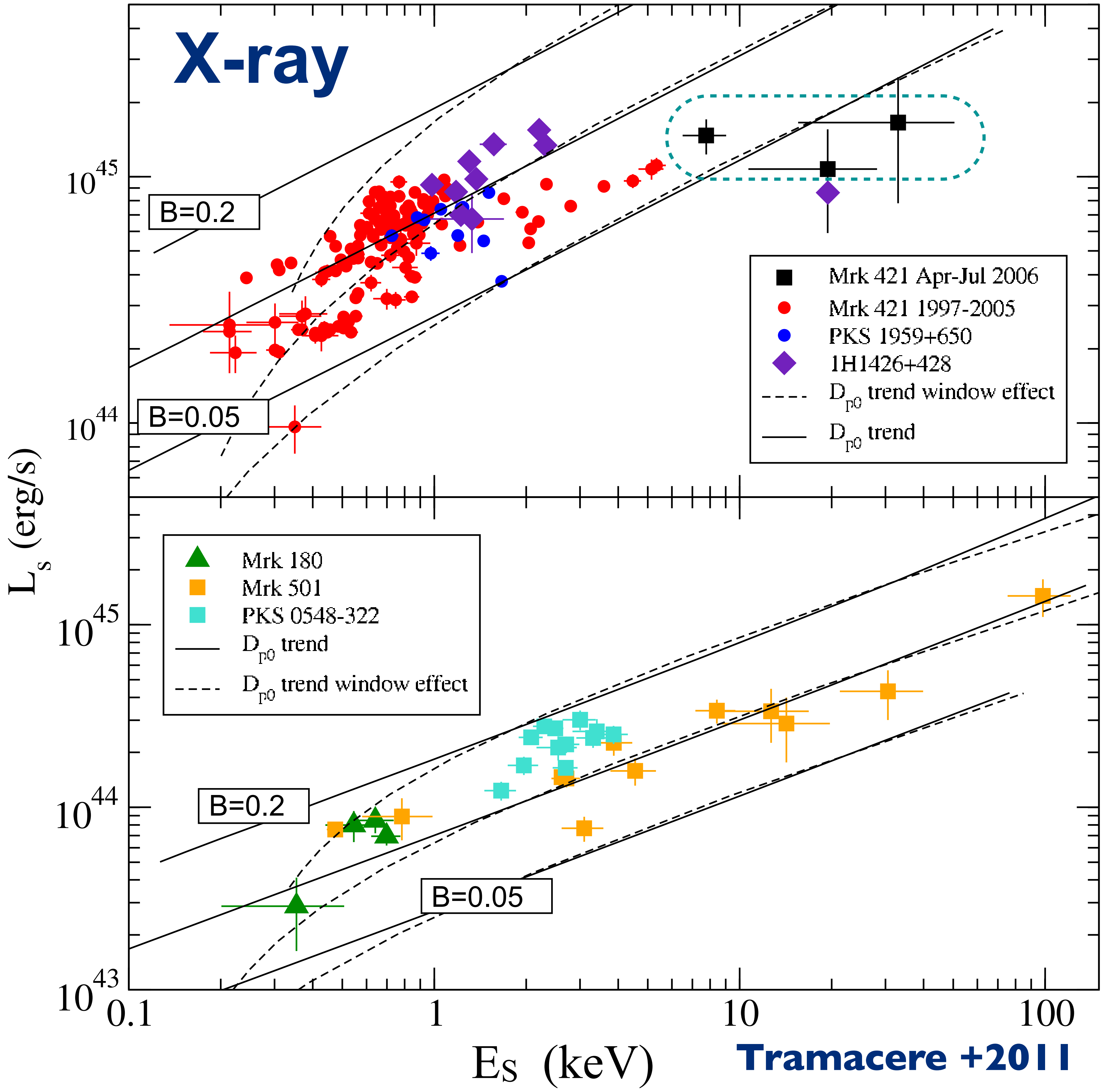
Tramacere +2011

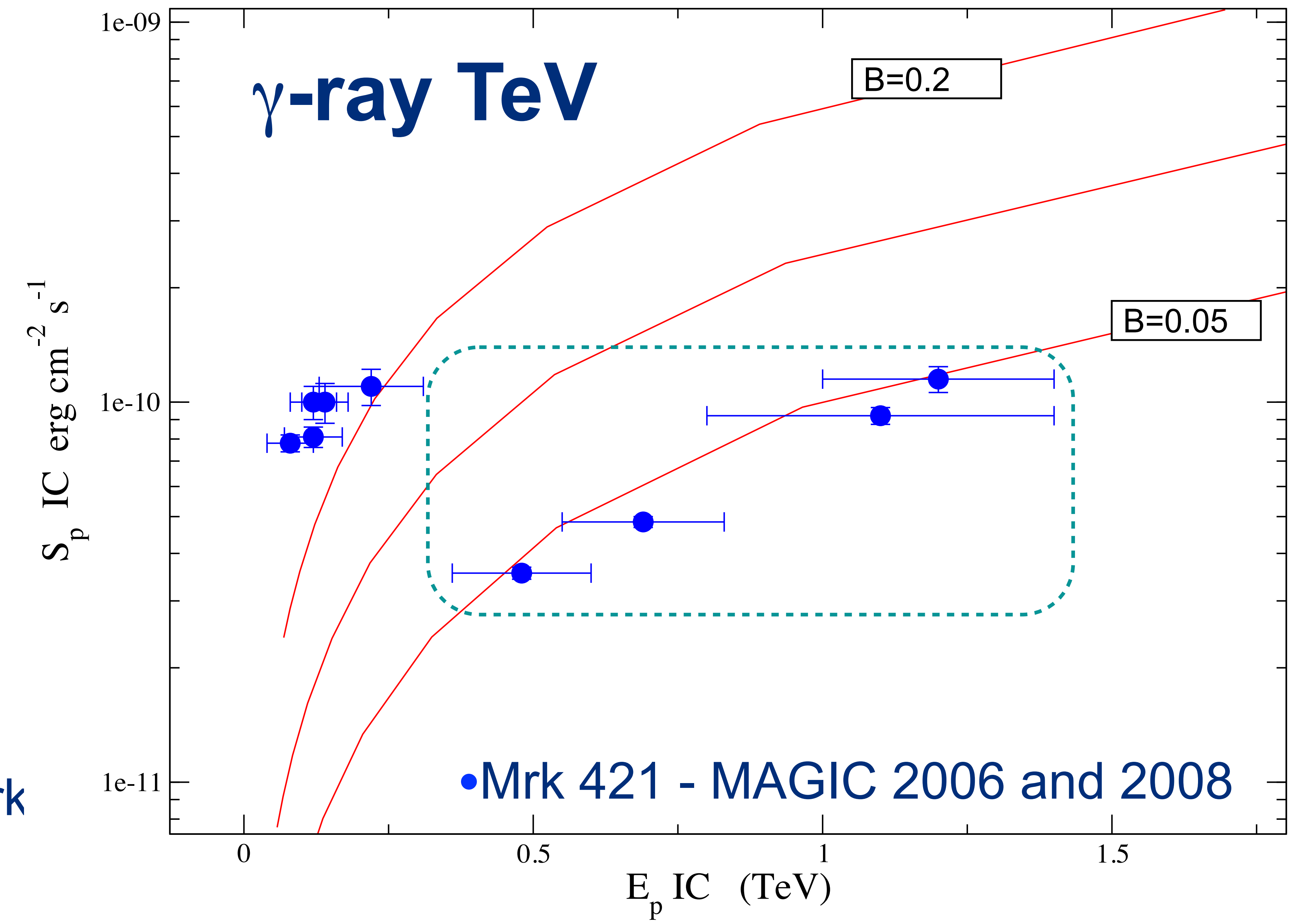


- data span **13 years**, both flaring and quiescent states
- We are able to reproduce these long-term behaviours, by changing the value of only one parameter (D_p)
- for q=2, curvature values imply distribution far from the equilibrium (b~[1.0–0.7])
- More data needed at GeV/TeV, curvature seems to be cooling-dominated
- Similar trend observed in GRBs (Massaro & Grindlay 2001)

$L_{\rm inj}$ (E_s–b_s trend)	(erg s^{-1})	5×10^{39}
$L_{\rm inj}$ (E_s–L_s trend)	(erg s^{-1})	5×10^{38}, 5×10^{39}
q		2
t_A	(s)	1.2×10^3
$t_{D_0} = 1/D_{P0}$	(s)	$[1.5 \times 10^4, 1.5 \times 10^5]$
$T_{\rm inj}$	(s)	10^4
$T_{\rm esc}$	(R/c)	2.0

E_S-L_S X-ray trend and γ-ray predictions

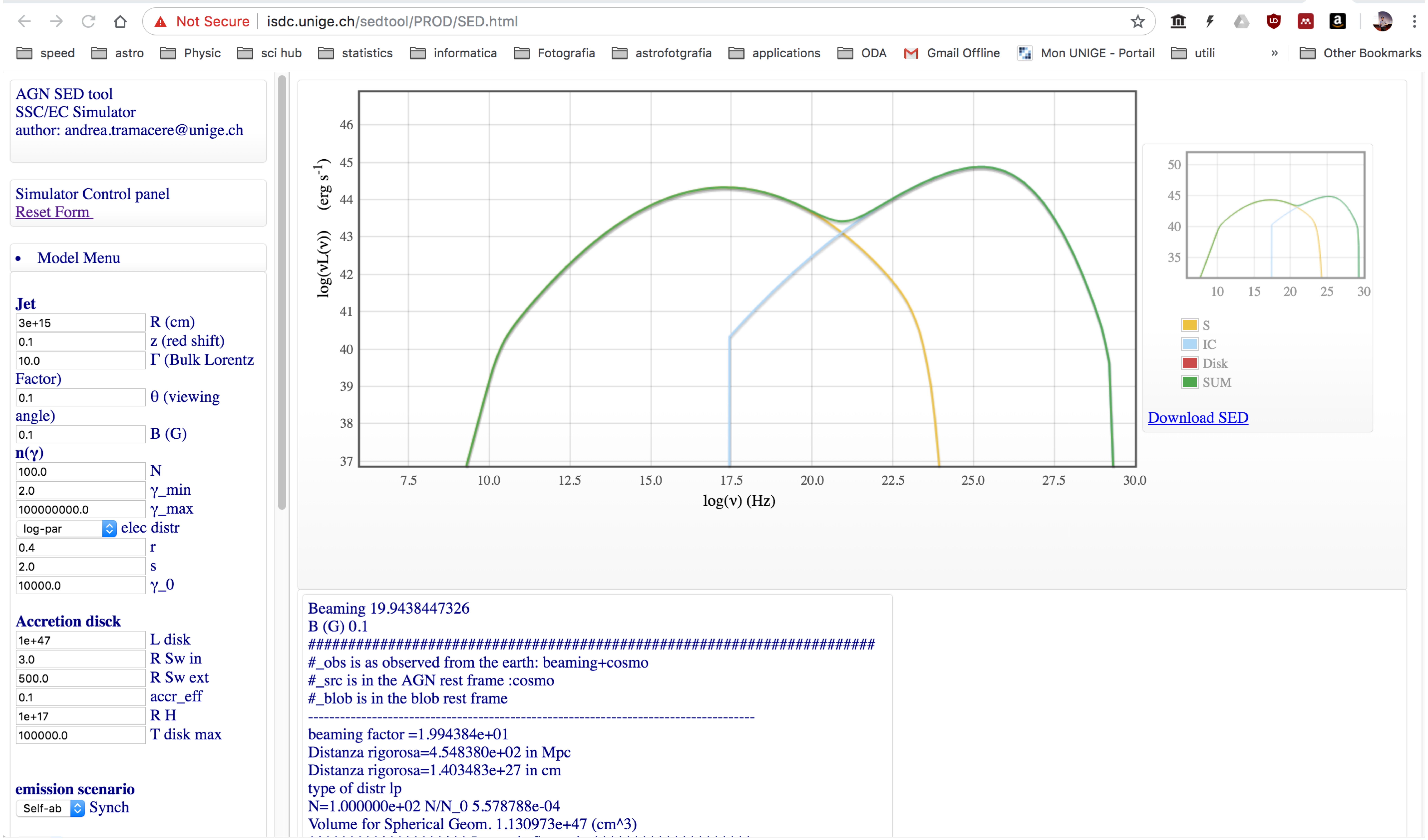




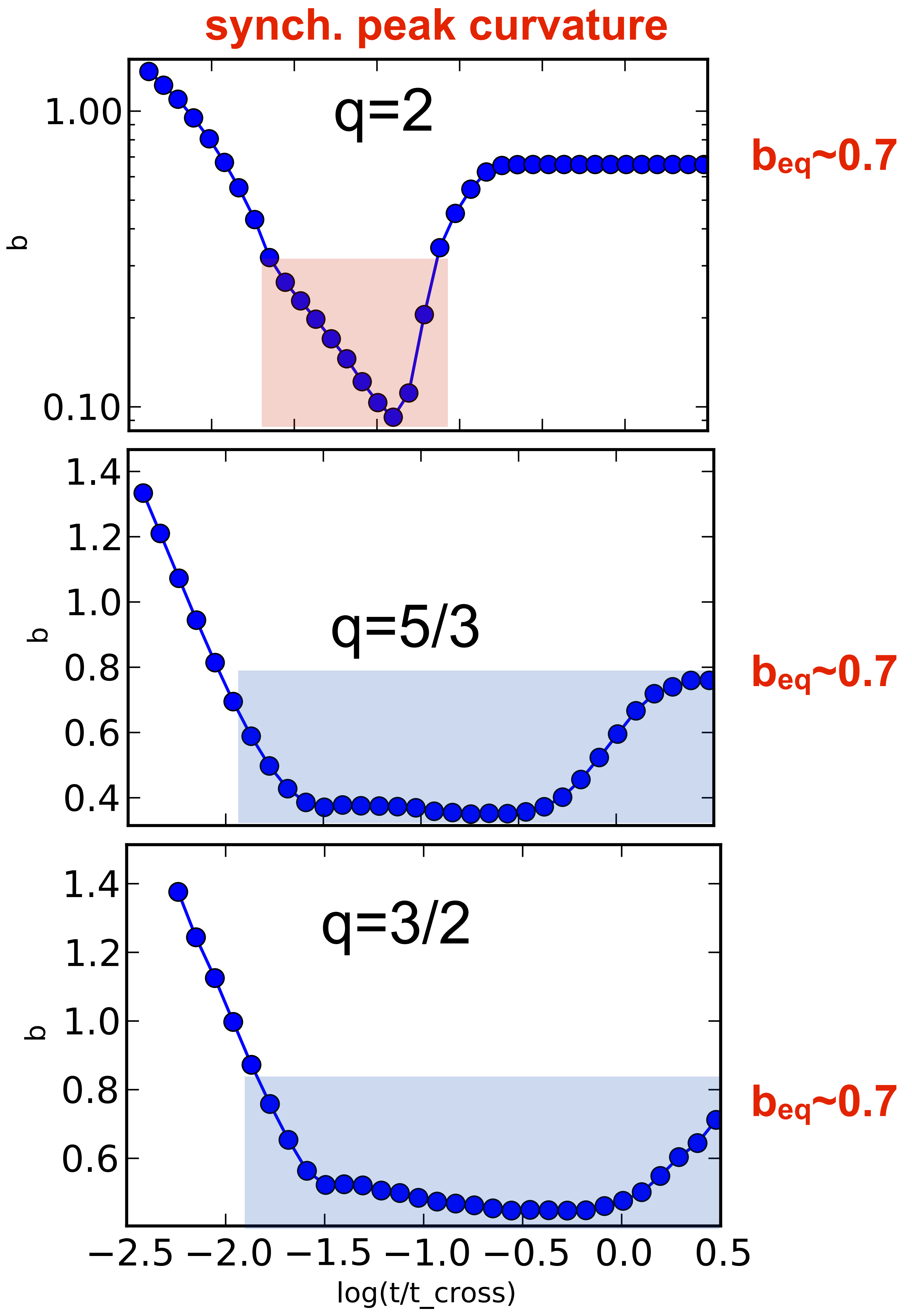
•Mrk

- the E_s–S_s (E_s–L_s) relation follows naturally from that between E_s and b_s
- the low L_{inj} objets (Mrk 501 vs Mrk 421) reach a larger E_S, compatibly with larger γ_{eq}
- Mrk 421 MAGIC data on 2006 match very well the Synchrotron prediction with simultaneous X-ray data
- the average index of the trend $L_s \propto E_S^{\alpha}$ with α~0.6, is compatible with the data, and with a scenario in which a typical constant energy (L_{inj} xt_{inj}) is injected for any flare (jet-feeding problem), whilst the peak dynamic is ruled by the turbulence in the magnetic field.

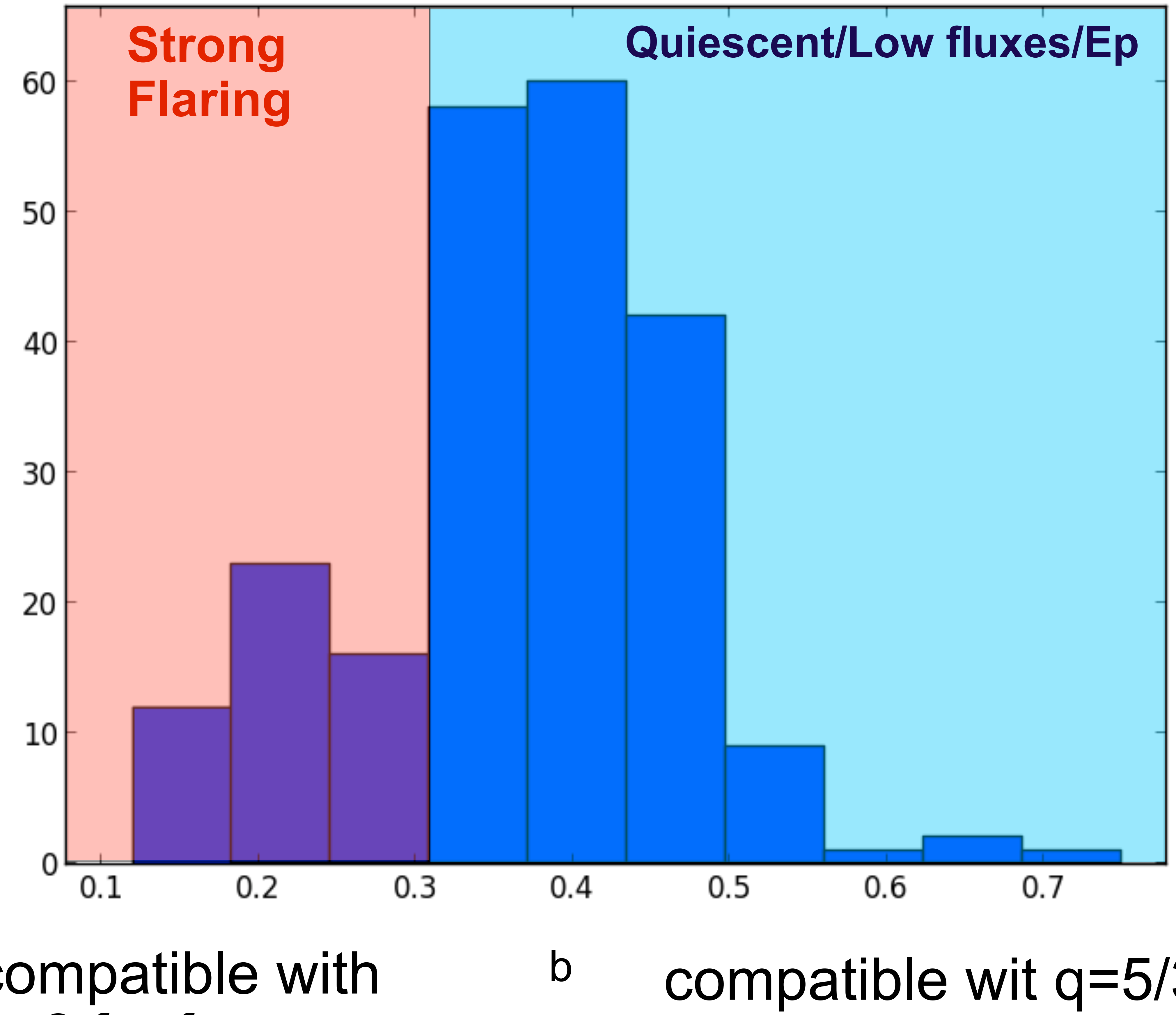
JetSeT web interface: https://www.isdc.unige.ch/sedtool



b distributions and q



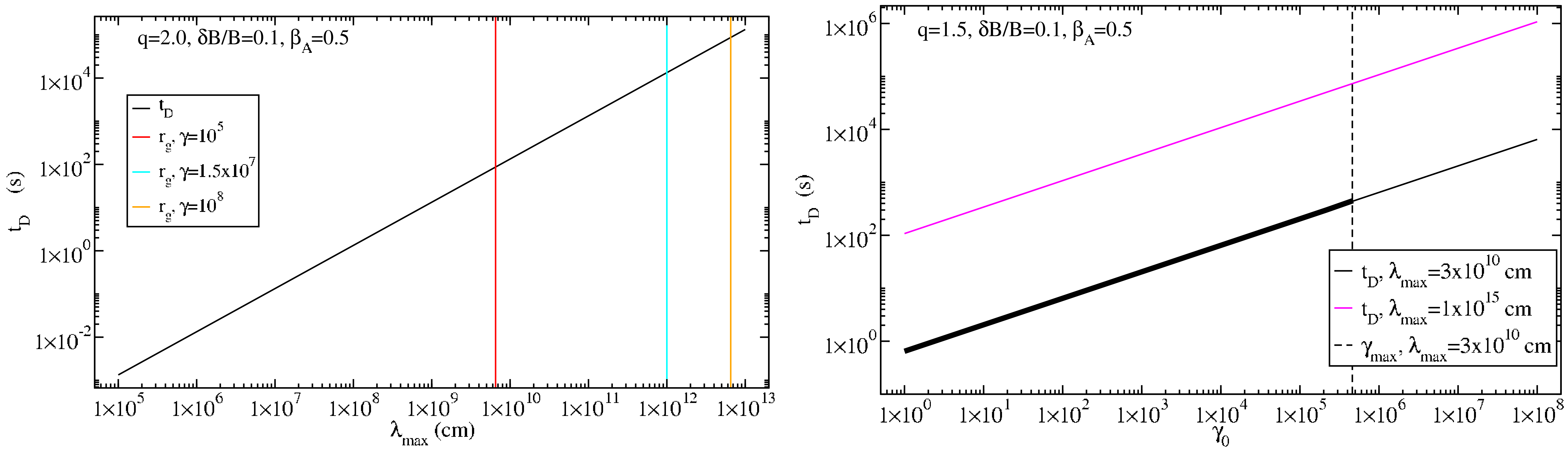
both flaring and quiescent seem to be far from equilibrium b eq.~[0.7-1.0] (if full KN or S)



compatible with q=2 far from equilibrium constraint on B

compatible wit q=5/3 constraint on B, and duration, or TH/KN

q=2 requires more fine tuning, especially on duration



q=2.0, δB/B=0.1, β_A=0.5
t_D
r_g, γ=10^5
r_g, γ=1.5x10^7
r_g, γ=10^8
t_D (s)
λ_max (cm)
q=1.5, δB/B=0.1, β_A=0.5
t_D, λ_max=3x10^10 cm
t_D, λ_max=1x10^15 cm
γ_max, λ_max=3x10^10 cm
t_D (s)
γ_0