Fundamental Theorem of Calculus

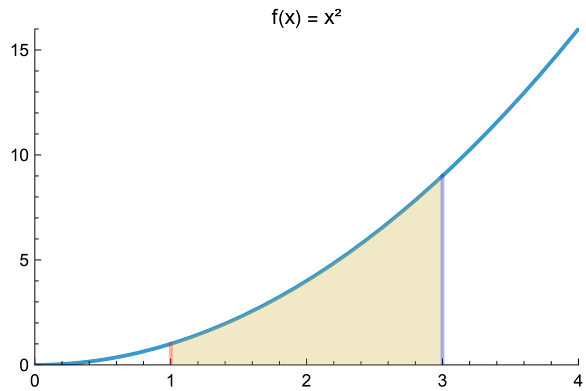
In[ ]:=
```
(*Define a function f(x) and its antiderivative F(x)*)f[x_] := x^2
F[x_] := x^3 / 3

(*Create a visual demonstration of the Fundamental Theorem of Calculus*)
Manipulate[Module[{integralValue, antiderivativeDifference},
  integralValue = NIntegrate[f[t], {t, a, b}];
  antiderivativeDifference = F[b] - F[a];
  f[x_] := x^2;
  F[x_] := x^3 / 3;
  Column[{Style["Fundamental Theorem of Calculus Visualization", Bold, 16],
    Style["∫[a to b] x² dx = (b³/3) - (a³/3)", 14],
    Row[{(*Left plot:The function and the area under it*)
      Plot[f[x], {x, 0, 2 π}, PlotRange → {{0, 4}, {0, 16}},
       PlotLabel → "f(x) = x²", Epilog → {ColorData["Rainbow"][0.7], Opacity[0.3],
         Polygon[{{a, 0}, {a, f[a]}}~Join~Table[{x, f[x]}, {x, a, b, (b - a) / 50}]~
           Join~{{b, f[b]}, {b, 0}}], Red, Thick, Line[{{a, 0}, {a, f[a]}}],
         Blue, Thick, Line[{{b, 0}, {b, f[b]}}]}, ImageSize → 300],
      (*Right plot:The antiderivative and the difference*)
      Plot[F[x], {x, 0, 2 π}, PlotRange → {{0, 4}, {0, 22}}, PlotLabel → "F(x) = x³/3",
       Epilog → {Red, PointSize[0.02], Point[{a, F[a]}], Blue,
         PointSize[0.02], Point[{b, F[b]}], Darker[Green], Dashed, Thick,
         Line[{{a, F[a]}, {b, F[a]}, {b, F[b]}}]}, ImageSize → 300]}],
    (*Display numerical results*)Style[Grid[{{"Bounds", Row[{"a = ", a}],
        Row[{"b = ", b}]}, {"Values", Row[{"F(a) = ", NumberForm[F[a], {5, 4}]}],
        Row[{"F(b) = ", NumberForm[F[b], {5, 4}]}]}, {"Results", Row[
          {"∫[", a, " to ", b, "] f(x) dx = ", NumberForm[integralValue, {5, 4}]}],
        Row[{"F(b) - F(a) = ", NumberForm[antiderivativeDifference, {5, 4}]}]}},
      Alignment → Left, Dividers → All], 14], (*Educational description*)
    Style["The Fundamental Theorem of Calculus states that if F is an
       antiderivative of f, then the definite integral of f from
       a to b equals F(b) - F(a). This visualization shows:", 12],
    Style["• The blue shaded area represents the definite integral
       of f(x) from a to b.", 12],
    Style["• The green dashed line shows the difference F(b) - F(a)
       on the antiderivative graph.", 12],
    Style["• Try moving the sliders to see how the area and the
       difference change together.", 12]}]],
 {{a, 0.5, "Lower bound (a)"}, 0, 4, 0.1, Appearance →
   "Labeled"}, {{b,
```

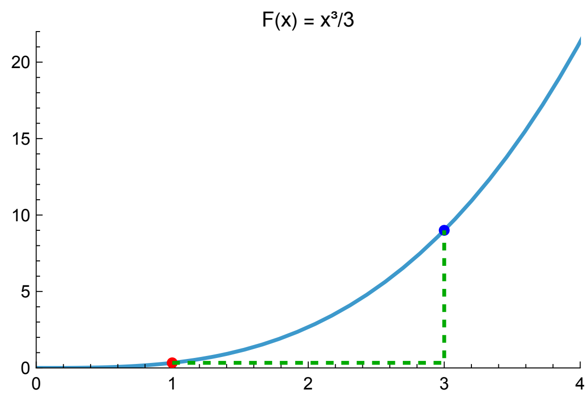
```
    3,
    "Upper bound (b)"},
   0, 4, 0.1, Appearance →
    "Labeled"},
  ControlPlacement → Bottom]

(*To deploy this to Wolfram Cloud*)
(*deployedFTC=CloudDeploy[%,"FundamentalTheoremOfCalculus",Permissions→"Public"]*)
```

Out[ ]=

Fundamental Theorem of Calculus Visualization

∫[a to b] x² dx = (b³/3) - (a³/3)





Bounds	a = 1.	b = 3
Values	F(a) = 0.3333	F(b) = 9
Results	∫[1. to 3] f(x) dx = 8.6667	F(b) - F(a) = 8.6667

The Fundamental Theorem of Calculus states that
if F is an antiderivative of f, then the definite integral
of f from a to b equals F(b) - F(a). This visualization shows:
• The blue shaded area represents the definite integral of f(x) from a to b.
• The green dashed line shows the
difference F(b) - F(a) on the antiderivative graph.
• Try moving the sliders to see how the area and the difference change together.

Lower bound (a) 1.

Upper bound (b) 3

In[ ]:=
```
DeleteObject[CloudObject[
  "https://www.wolframcloud.com/obj/3b2f4345-03e0-42ab-ae14-57f0e4e3ebc7"]]
```

In[ ]:=
```
((2.9)^2)*0.1
```

Out[ ]=
```
0.841
```

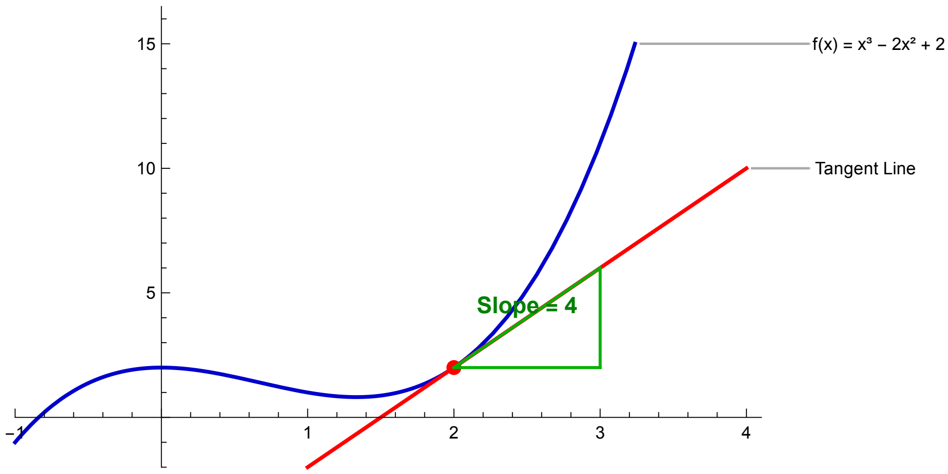
Equation of the Tangent Line

In[ ]:=
```
(*Define the function and its derivative using explicit function definitions*)
f[x_] := x^3 - 2 x^2 + 2
fPrime[x_] := 3 x^2 - 4 x
(*Using symbolic differentiation instead of manual definition*)

(*Define the tangent line equation at point a*)
tangentLine[x_, a_] := fPrime[a] * x + (f[a] - fPrime[a] * a);

(*Create the interactive visualization*)
Manipulate[Module[{derValue, tangentLineEq, secantLineEq, h},
  f[x_] := x^3 - 2 x^2 + 2;
  fPrime[x_] := 3 x^2 - 4 x;
  (*tangentLine[x_,a_]:=f[a]+fPrime[a] (x-a);*)
  tangentLine[x_, a_] := fPrime[a] * x + (f[a] - fPrime[a] * a);
  derValue = fPrime[a];
  tangentLineEq = tangentLine[x, a];
  h = 0.5;
  (*Default value for secant line*)secantLineEq = (f[a + h] - f[a]) / h * (x - a) + f[a];
  Column[{Style["Derivative as the Slope of the Tangent Line", Bold, 16],
    (*Main plot showing function and tangent line*)
    Plot[{f[x], tangentLineEq, If[showSecant, secantLineEq, None]}, {x, -1, 4},
     PlotRange → {{-1, 4}, {-2, 15}}, PlotStyle → {Darker[Blue, 0.2], Red, Dashed},
     Epilog → {(*Mark the point of tangency*)PointSize[0.02], Red,
       Point[{a, f[a]}], (*Highlight the slope triangle if enabled*)
       If[showSlopeTriangle, {Darker[Green, 0.3], Thickness[0.004], Line[
          {{a, f[a]}, {a + 1, f[a]}}], Line[{{a + 1, f[a]}, {a + 1, f[a] + derValue}}],
         Line[{{a, f[a]}, {a + 1, f[a] + derValue}}], (*Label the slope*)
         Text[Style["Slope = " <> ToString[NumberForm[derValue, {4, 2}]], Bold, 12,
           Darker[Green, 0.5]], {a + 0.5, f[a] + derValue / 2 + 0.5}]}, {}], (*Show
        secant line points if enabled*)If[showSecant, {Orange, PointSize[0.015],
         Point[{a + h, f[a + h]}], Dashed, Line[{{a, f[a]}, {a + h, f[a + h]}}]}, {}]},
     PlotLabels → {"f(x) = x³ - 2x² + 2", "Tangent Line",
       If[showSecant, "Secant Line", None]}, ImageSize → 500], (*Information panel*)
    Panel[Grid[{{"Point", Row[{"x = ", NumberForm[a, {4, 2}]}]},
        Row[{"f(", NumberForm[a, {4, 2}], ") = ", NumberForm[f[a], {4, 2}]}]}],
       {"Derivative", Row[{"f'(", NumberForm[a, {4, 2}],
          ") = ", NumberForm[derValue, {4, 2}]}], ""},
```

```
      {"Tangent Line Equation", Row[{"y = ", NumberForm[fPrime[a], {4, 2}],
         "(x - ", NumberForm[a, {4, 2}], ") + ", NumberForm[f[a], {4, 2}]}], ""}},
     Alignment → Left, Dividers → All], ImageSize → 500],
   (*Educational description*)Panel[Column[{Style["Explanation:", Bold, 14],
      "• The derivative of a function at a point equals the slope of the
        tangent line at that point.", "• As you move the slider,
        observe how the tangent line's inclination changes.",
      "• The slope triangle (when enabled) shows rise/run = f'(a).",
      If[showSecant, "• The secant line approaches the tangent line
         as the distance between points decreases.", ""]},
     Alignment → Left], ImageSize → 500]}]], {{a, 1, "Point (x-value)"},
  -0.5, 3, 0.1, Appearance → "Labeled"},
 {{showSlopeTriangle,
   True,
   "Show slope triangle"},
  {True, False}}, {{showSecant,
   False,
   "Show secant line"},
  {True, False}}, ControlPlacement →
  Bottom]

(*To deploy this to Wolfram Cloud,uncomment below*)
(*CloudDeploy[%,"DerivativeAsTangentSlope",Permissions→"Public"]*)
```

Out[∘]=

Derivative as the Slope of the Tangent Line



Point	x = 2	f(2) = 2
Derivative	f' (2) = 4	
Tangent Line Equation	y = 4(x − 2) + 2	

Explanation:
- The derivative of a function at a point equals the slope of the tangent line at that point.
- As you move the slider, observe how the tangent line's inclination changes.
- The slope triangle (when enabled) shows rise /run = f'(a).

Point (x−value) 2

Show slope triangle ☑

Show secant line ☐

In[∘]:=
```
f[1.8]
```
Out[∘]=
```
1.352
```
In[∘]:=
```
g[x_] := 3 x^2 - 4 x;
```
In[∘]:=
```
g[1.8]
```
Out[∘]=
```
2.52
```
In[∘]:=
```
Solve[1.352 == 2.52 * 1.8 + c, c]
```
Out[∘]=
```
{{c → -3.184}}
```
In[∘]:=
```
y == 2.52 x + -3.184
```
Out[∘]=
```
y == -3.184 + 2.52 x
```

```
In[ ]:= Simplify[y == 2.52 (x - 1.8) + 1.35]
Out[ ]=
y == -3.186 + 2.52 x
```