

## Algorithms Lab

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### Exercise – Planet Express

It is year 3000 and you are an employee of the Planet Express delivery company. A customer from planet *Omicron Persei 8* just ordered one yottagram of dark matter using the “same-second” delivery option and you have been given the task of (quickly) planning the delivery.

Planet Express stores dark matter in  $k > 0$  warehouses scattered on planets across the galaxy, and you can initiate the delivery process by choosing any of the warehouses and activating one of its robotic delivery agents.

The galaxy can be modelled as a weighted directed graph  $G = (V, E)$ , where  $V = \{0, \dots, n-1\}$ , and  $|E| = m$ . Each vertex of  $G$  represents a planet while an edge  $(u, v)$  of weight  $c$  means that it is possible to travel from planet  $u$  to planet  $v$  in  $c$  microseconds<sup>1</sup>. The  $k$  warehouses are located on planets  $0, \dots, k-1$  while Omicron Persei 8 always corresponds to vertex  $n-1$ .

In addition,  $T$  planets are part of a convenient teleportation network. Unfortunately, the teleportation technology still requires the source and destination planets to be pairwise reachable by regular means of travel. More precisely, we say that two distinct planets  $u$  and  $v$  that are both part of the teleportation network are linked if it is possible to reach  $v$  from  $u$  and vice-versa without using the teleportation network. An agent on a planet  $u$  can teleport to any another planet  $v$  that is linked with  $u$  in just  $t(u)$  microseconds, where  $t(u)$  is the total number of planets that are linked with  $u$ .

Once activated, the delivery agent will deliver the dark matter to the customer in the shortest possible time, travelling between planets and/or using the teleportation network.

Your task is to determine whether it is possible for Planet Express to complete the delivery within the advertised time of 1 second and, if that is the case, the shortest amount of time required for the delivery. You can assume that the time needed to travel between two locations on the same planet is negligible.

**Input** The first line of the input contains the number  $t \leq 20$  of test cases. Each of the  $t$  test cases is described as follows.

- It starts with a line containing four integers  $n, m, k, T$ , separated by a space. They denote:  
 $n$ : the number of planets in the galaxy, i.e., the number of vertices in  $G$  ( $1 \leq n \leq 10^5$ );  
 $m$ : the number of edges in  $G$  ( $0 \leq m \leq 10^5$ ),  
 $k$ : the number of warehouses, located at the vertices  $0, \dots, k-1$  in  $G$  ( $1 \leq k \leq n$ ),  
 $T$ : the number of planets that are part of the teleportation network ( $0 \leq T \leq n$ ).
- The following line contains  $T$  distinct space-separated integers  $t_0 \dots t_{T-1}$  with  $0 \leq t_i < n$ . They denote the vertices (planets) in  $G$  that are part of the teleportation network.
- The next  $m$  lines define the edges of  $G$ . Each line contains three space-separated integers  $u, v, c$  defining a directed edge  $(u, v)$  in  $G$  of weight  $c$  ( $0 \leq u, v < n, u \neq v, 0 \leq c \leq 10^7$ ).

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<sup>1</sup>A microsecond is equal to  $10^{-6}$  seconds. Spaceships are fast!

**Output** The output consists of one line for each test case.

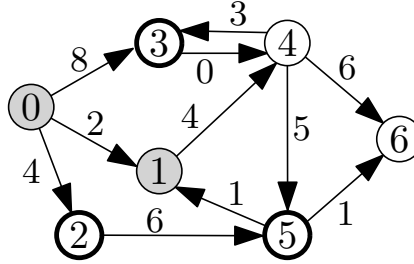
- If the dark matter cannot be delivered within at most 1 second for the  $i$ -th test case, then the  $i$ -th line of the output should contain the string "no".
- Otherwise, the  $i$ -th line of the output should contain a single integer that denotes the **smallest number of microseconds needed to complete the delivery** (i.e., the time a delivery agent needs to reach Omicron Persei 8 from the optimal warehouse).

**Points** There are four groups of test sets worth 20, 30, 30, and 20 points, respectively.

1. For the first group of test sets, you may assume that  $n, m \leq 10^4$ , and  $k, T \leq 20$ .
2. For the second group of test sets, you may assume that  $T \leq 800$ .
3. For the third group of test sets, there are no additional assumptions.
4. For the fourth group of test sets, which is hidden, there are no additional assumptions.

Corresponding sample test sets are contained in `testi.in/out`, for  $i \in \{1, 2, 3\}$ .

**Example** In the following example, corresponding to the first test case of the sample input,  $n = 7$ ,  $m = 11$ ,  $k = 2$ , and  $T = 3$ . Vertices on which a warehouse is located are shown in gray. Vertices that are part of the teleportation network are shown in bold.



An optimal solution delivers the dark matter from the warehouse on the planet 1 via the planets 4, 3, 5, to the planet 6. The teleportation network has been used to travel from the planet 3 to the planet 5. The required delivery time is  $4 + 3 + 1 + 1 = 9$  microseconds.

**Sample Input**

```
2
7 11 2 3
2 3 5
0 1 2
0 2 4
0 3 8
1 4 4
2 5 6
3 4 0
4 3 3
4 5 5
4 6 6
5 1 1
5 6 1
5 6 2 2
2 3
0 1 5
0 3 1000001
1 2 999500
2 0 0
3 0 100
3 4 500
```

**Sample Output**

```
9
no
```