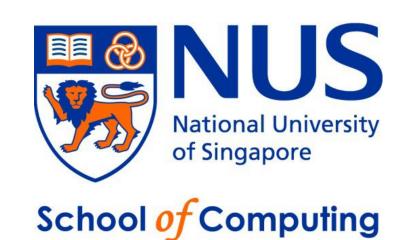
## Synthesis of Programs with Pointers via Read-Only Specifications

Andreea Costea

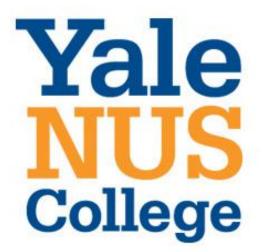
Amy Zhu

Nadia Polikarpova Ilya Sergey





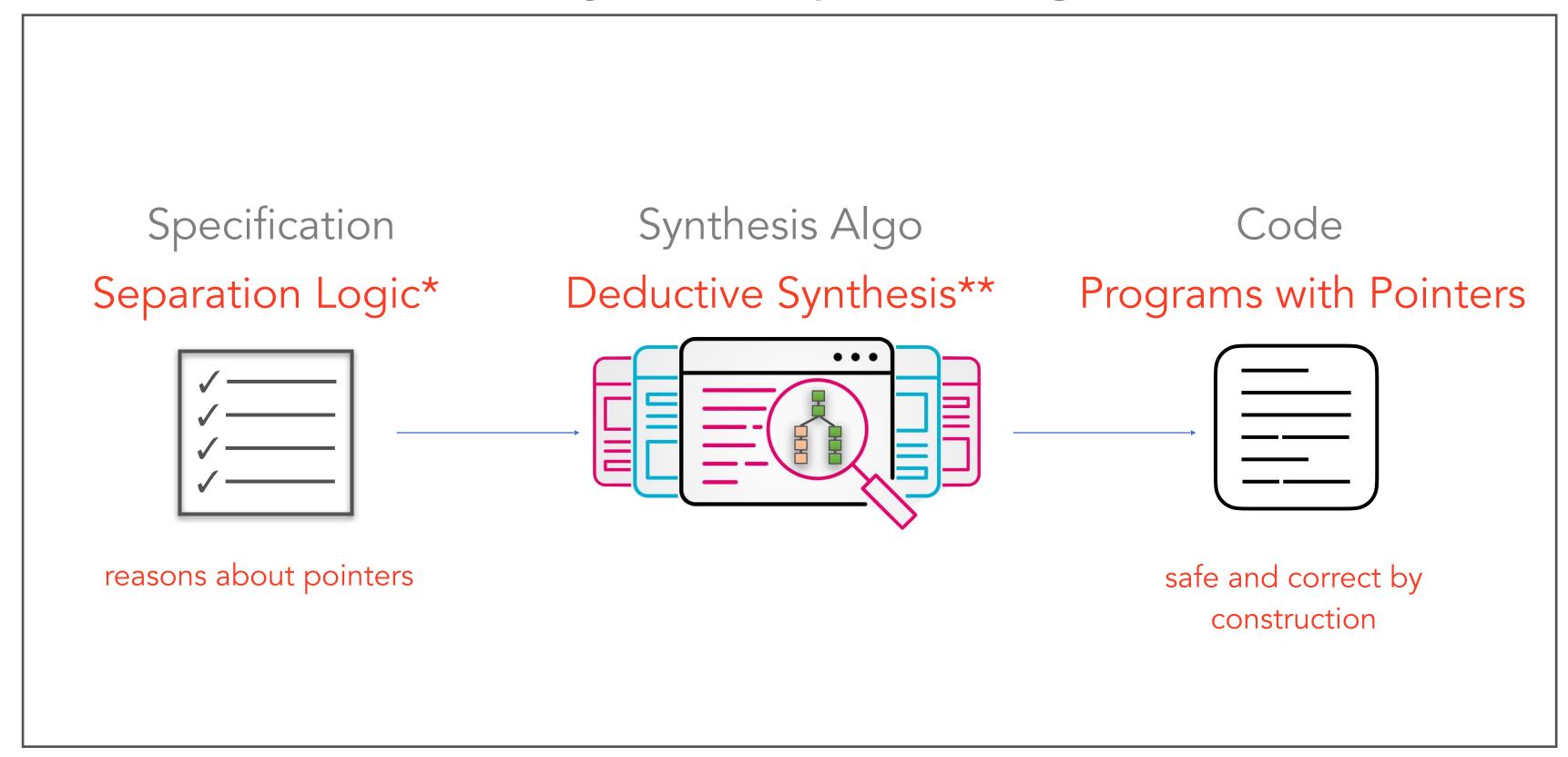




# Synthesis of Programs with Pointers via Read-Only Specifications



#### SSL: Synthetic Separation Logic

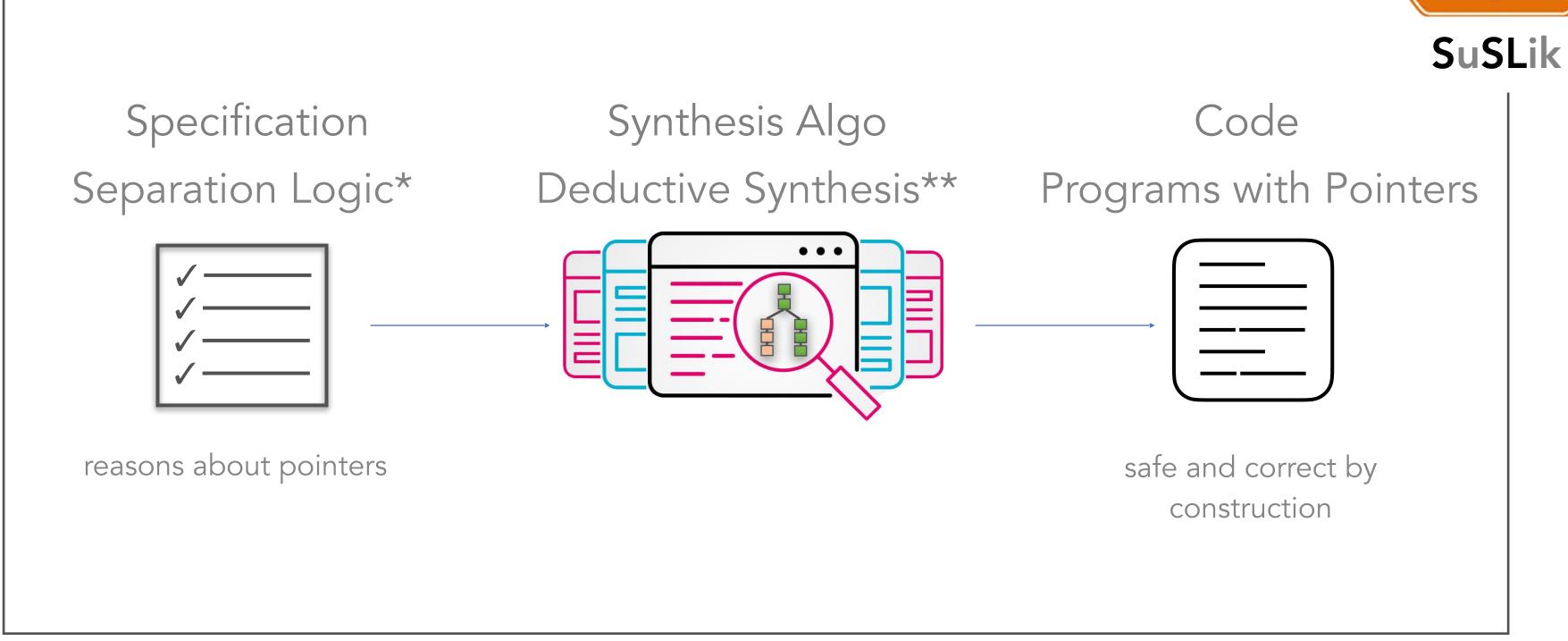


<sup>\*</sup> Local Reasoning about Programs that Alter Data Structures, O'Hearn, Reynolds, Yang:. CSL 2001

<sup>\*\*</sup> Structuring the Synthesis of Heap-Manipulating Programs, Polikarpova & Sergey @POPL'19

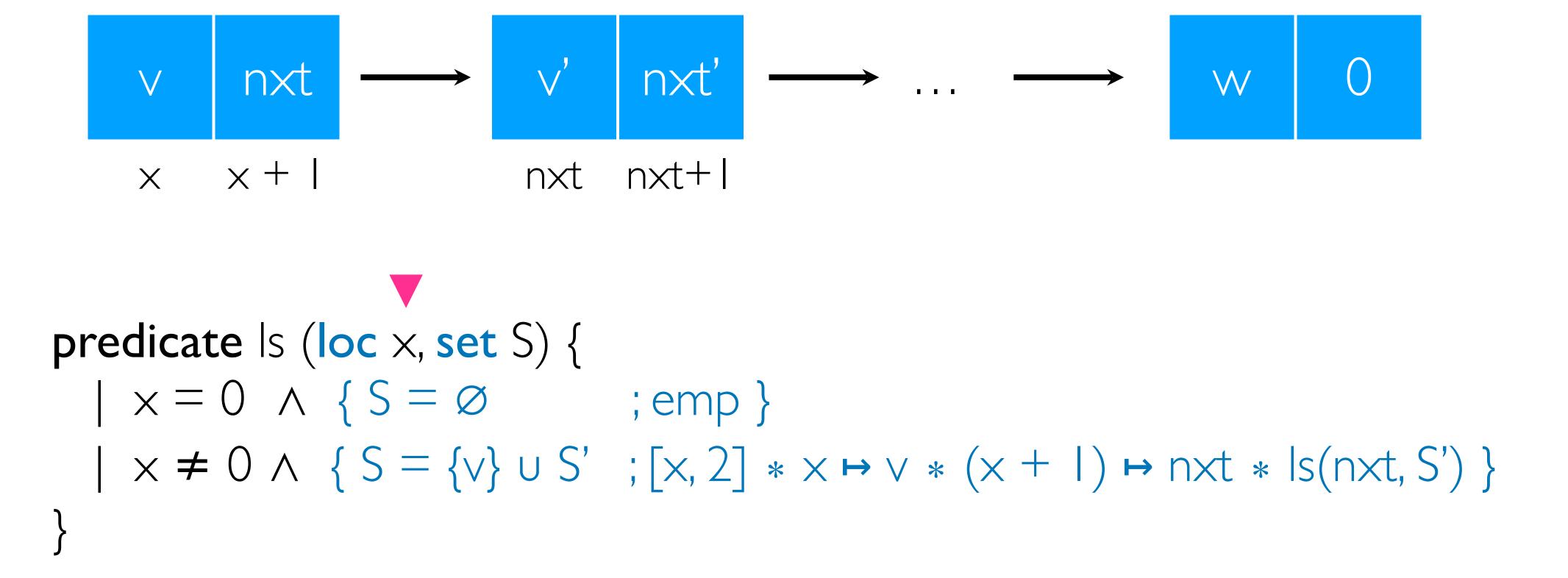
#### **SSL: Synthetic Separation Logic**

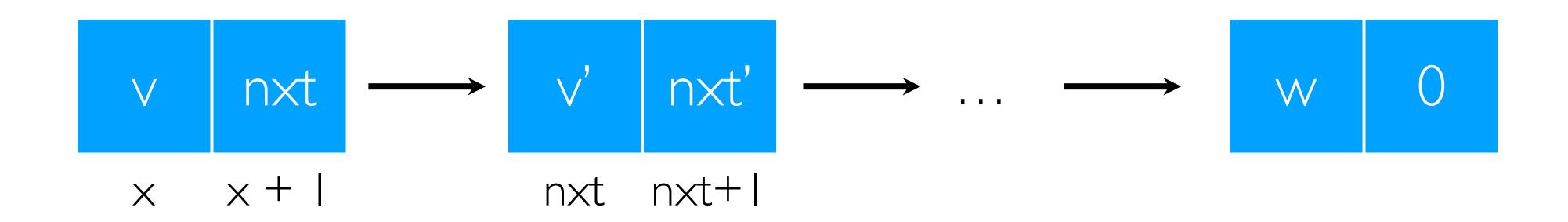




<sup>\*</sup> Local Reasoning about Programs that Alter Data Structures, O'Hearn, Reynolds, Yang:. CSL 2001

<sup>\*\*</sup> Structuring the Synthesis of Heap-Manipulating Programs, Polikarpova & Sergey @POPL'19

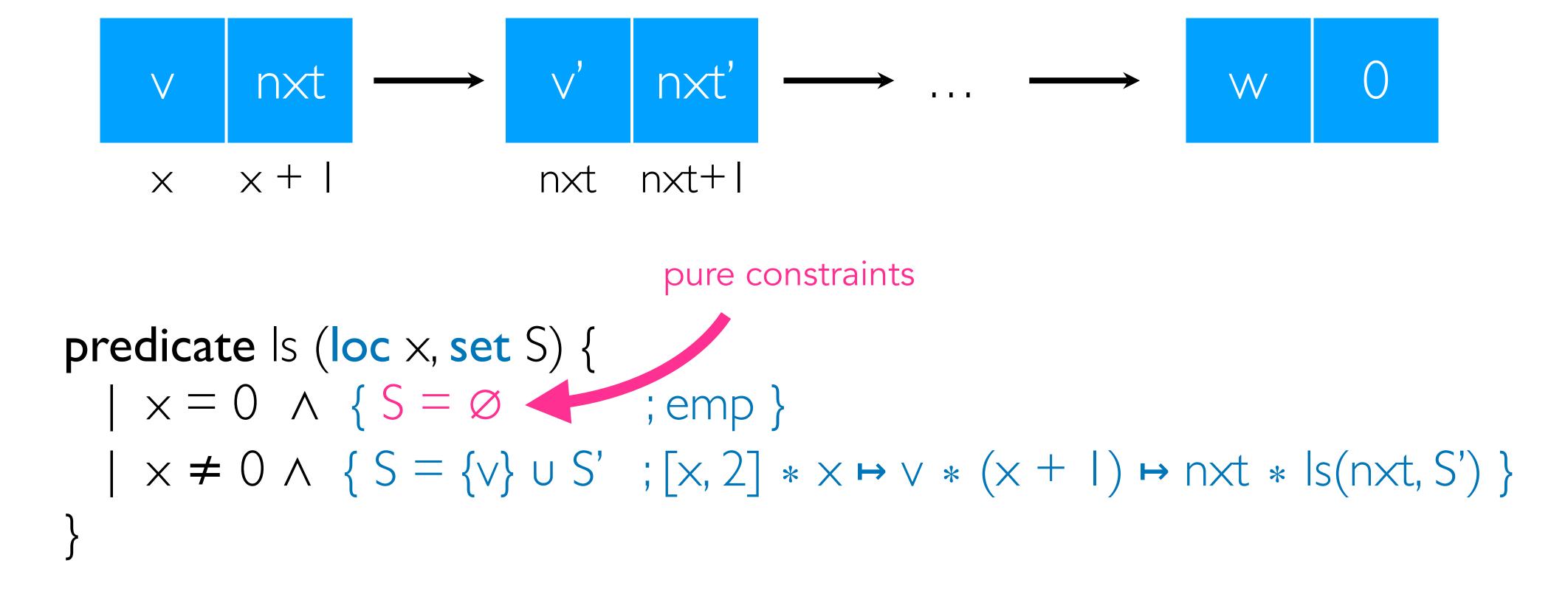


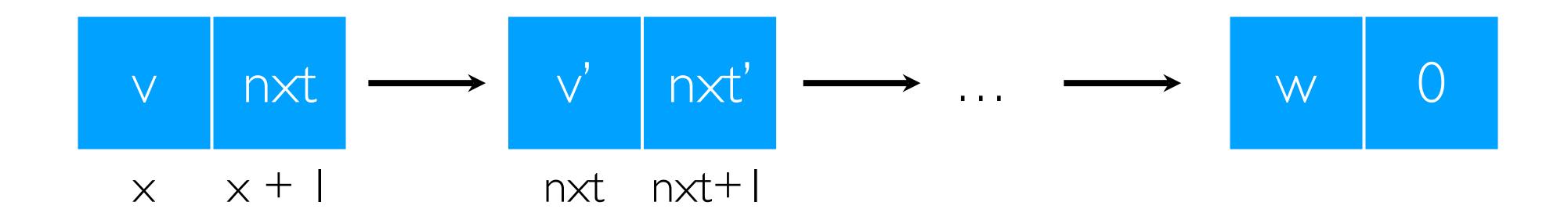


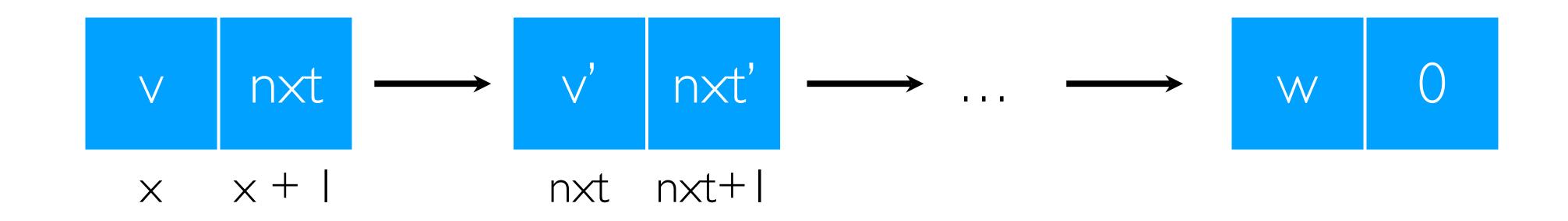
```
predicate |s (loc x, set S) \{

| x = 0 \land \{ S = \emptyset ; emp \}

| x \neq 0 \land \{ S = \{v\} \cup S' ; [x, 2] * x \mapsto v * (x + 1) \mapsto nxt * |s(nxt, S') \}
```



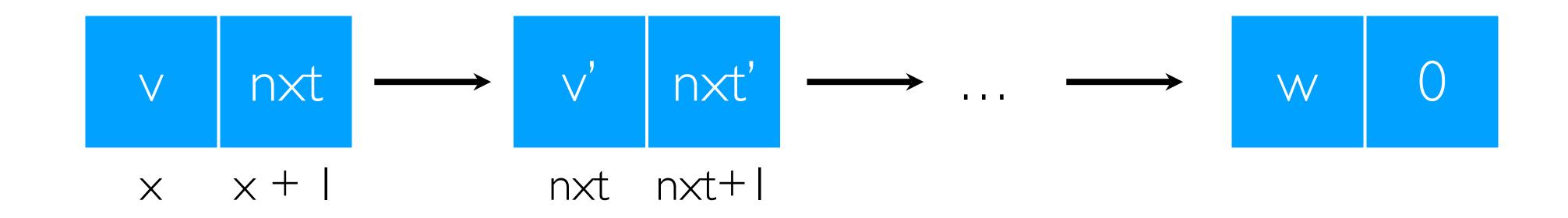




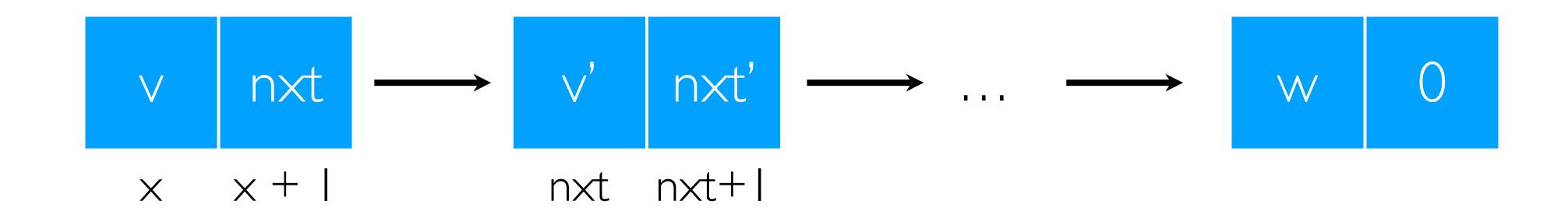
```
predicate |s (loc x, set S) \{

| x = 0 \land \{ S = \emptyset ; emp \}

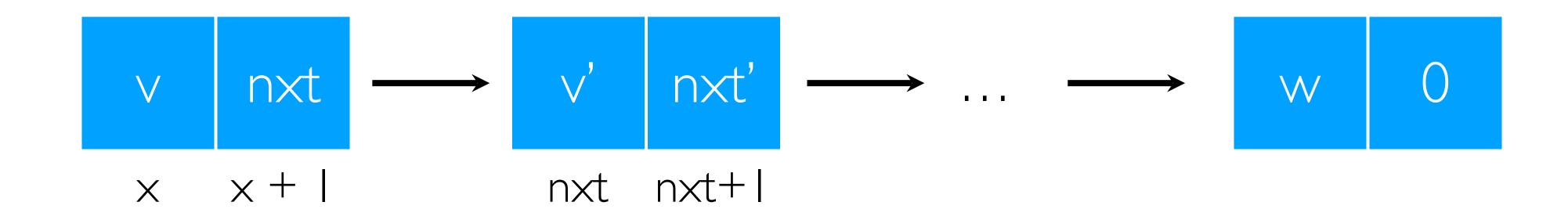
| x \neq 0 \land \{ S = \{v\} \cup S' ; [x, 2] * x \mapsto v * (x + 1) \mapsto nxt * |s(nxt, S') \}
```



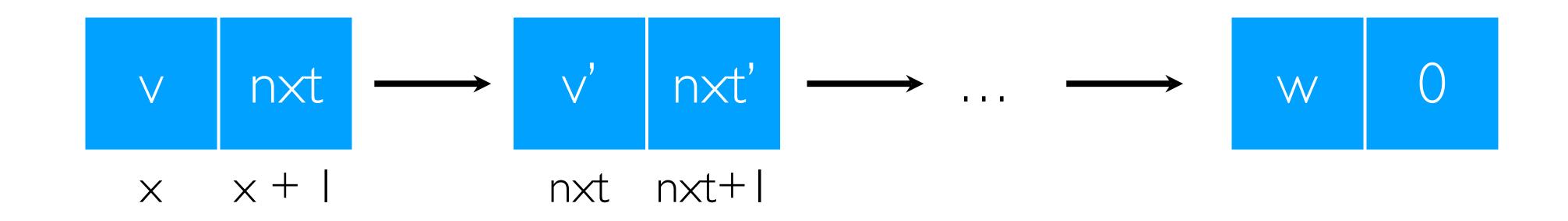
```
predicate ls (loc x, set S) {
 | x = 0 \land \{ S = \emptyset ; emp \} 
 | x \neq 0 \land \{ S = \{v\} \cup S' ; [x, 2] * x \mapsto v * (x + 1) \mapsto nxt * ls(nxt, S') \} 
 | memory block
```



```
predicate ls (loc x, set S) {
 | x = 0 \land \{ S = \emptyset ; emp \} 
 | x \neq 0 \land \{ S = \{v\} \cup S' ; [x, 2] * x \mapsto v * (x + 1) \mapsto nxt * ls(nxt, S') \} 
points-to
```



```
predicate ls (loc x, set S) {
| x = 0 \land \{ S = \emptyset ; emp \} \}
| x \neq 0 \land \{ S = \{v\} \cup S' ; [x, 2] * x \mapsto v * (x + 1) \mapsto nxt * ls(nxt, S') \}
}
```

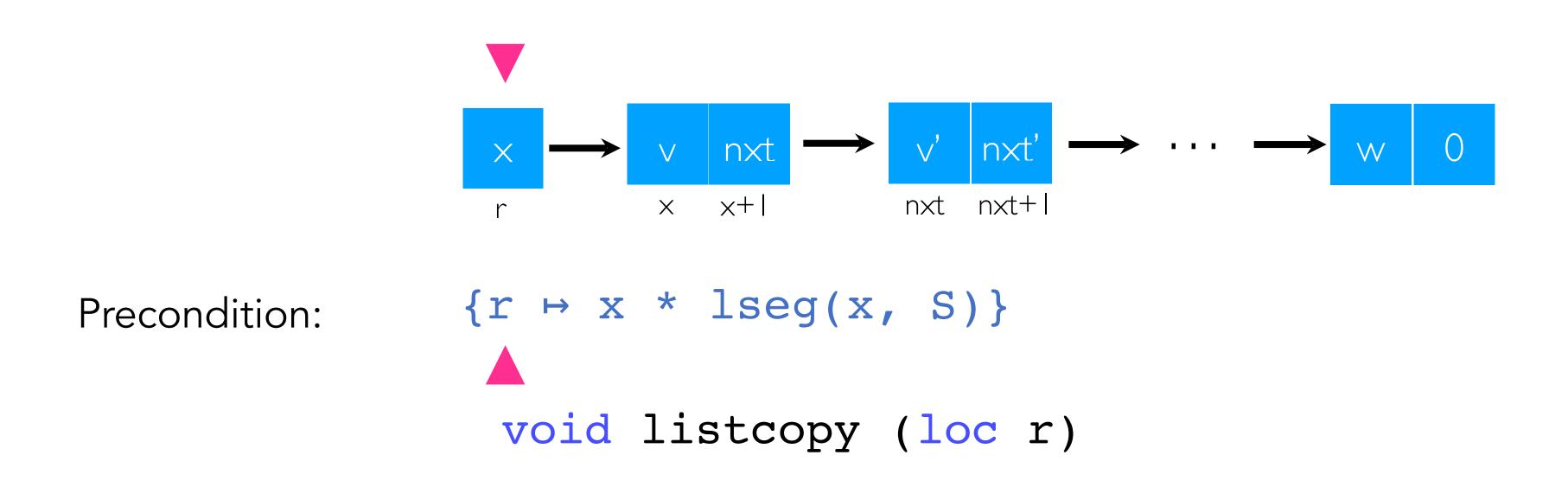


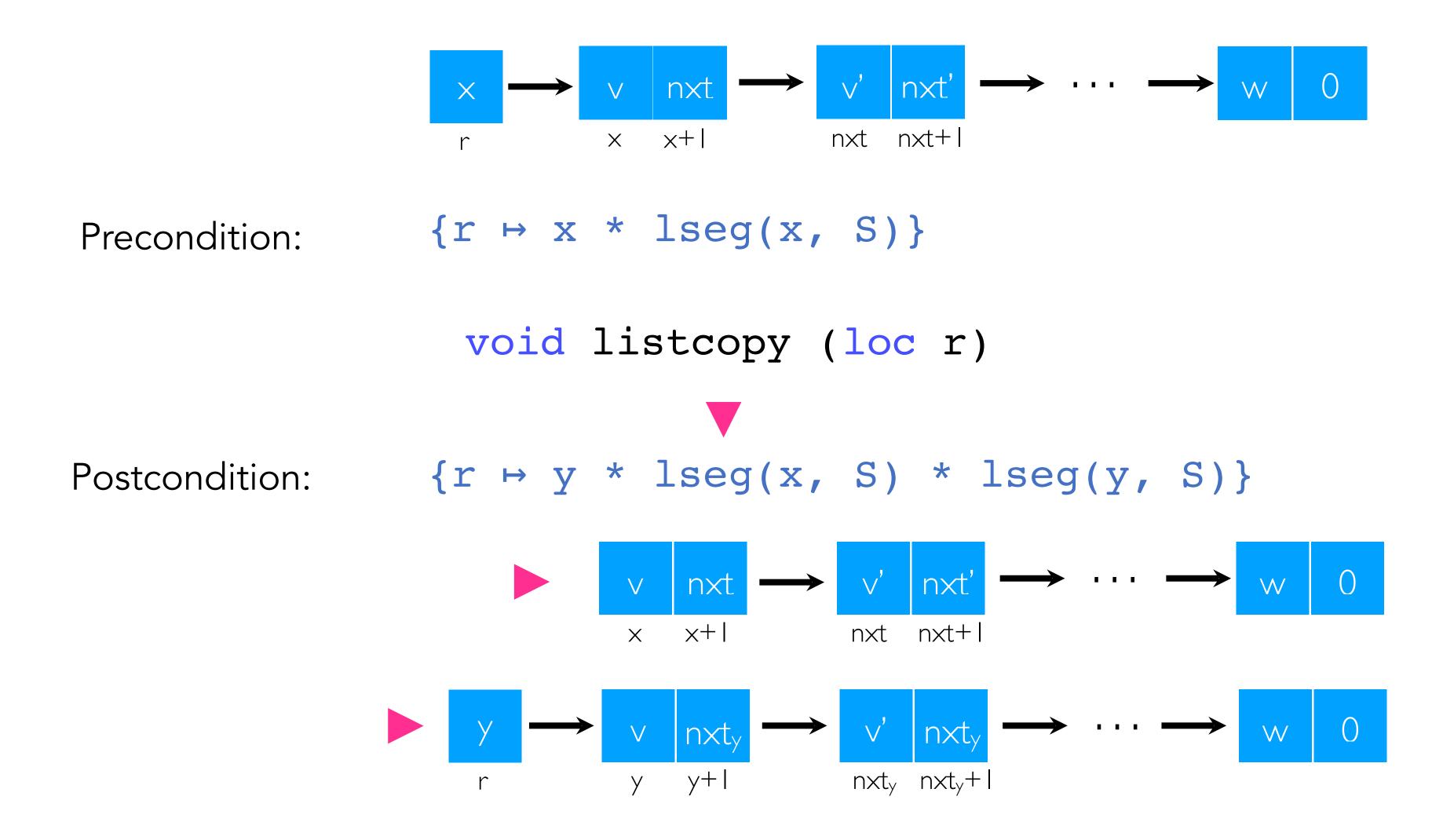
```
predicate |s (loc x, set S) \{

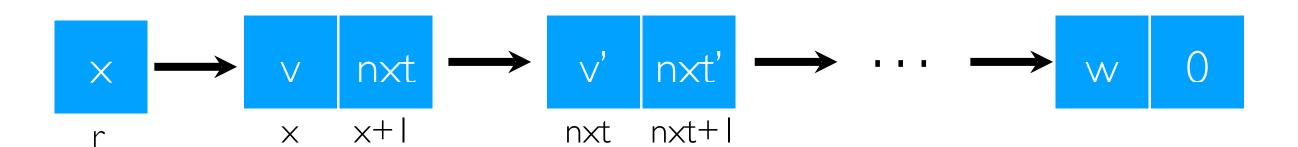
| x = 0 \land \{ S = \emptyset ; emp \}

| x \neq 0 \land \{ S = \{v\} \cup S' ; [x, 2] * x \mapsto v * (x + 1) \mapsto nxt * |s(nxt, S') \}
```

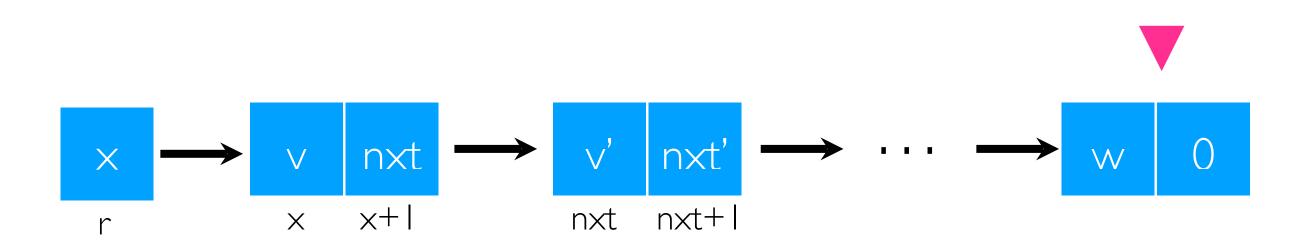
separating conjunction



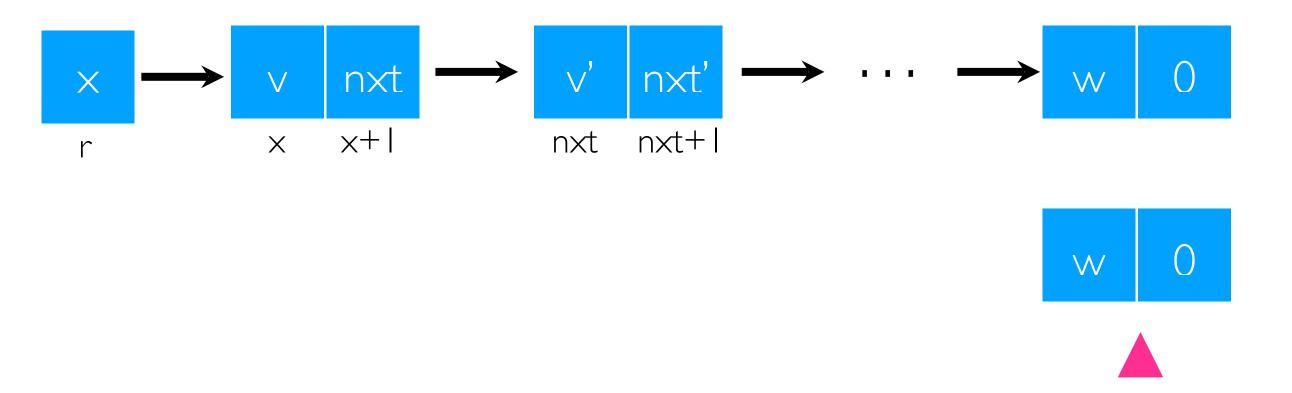




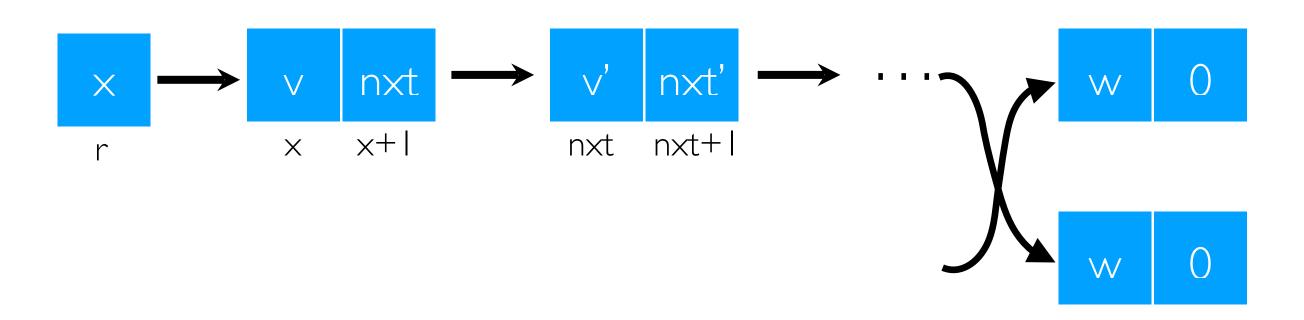
```
\{r \mapsto x * lseg(x, S)\}
       void listcopy (loc r) {
         let x = *r;
     3 	 if (x == 0) {
     4 } else {
           let v = *x;
      let nxt = *(x + 1);
            *r = nxt;
         listcopy(r);
           let y1 = *r;
           let y = malloc(2);
    10
           *(x + 1) = y1;
    11
           *r = y;
           *(y + 1) = nxt;
    13
    14
            *y = v;
    15
\{r \mapsto y * lseg(x, S) * lseg(y, S)\}
```



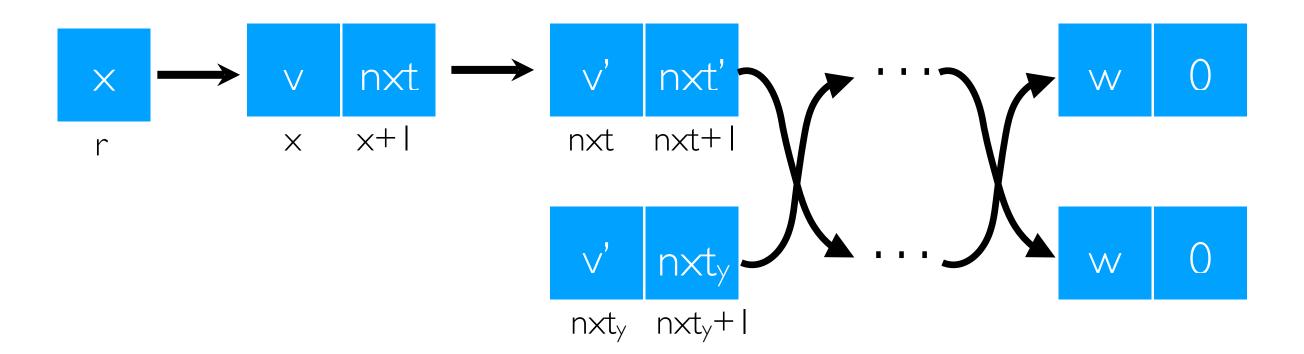
```
\{r \mapsto x * lseg(x, S)\}
     1 void listcopy (loc r) {
     1 et x = *r;
     3 \rightarrow if (x == 0) {
     4 } else {
            let v = *x;
            let nxt = *(x + 1);
            *r = nxt;
            listcopy(r);
            let y1 = *r;
            let y = malloc(2);
    10
            *(x + 1) = y1;
    11
    12
            *r = y;
            *(y + 1) = nxt;
    13
    14
            *y = v;
    15
\{r \mapsto y * lseg(x, S) * lseg(y, S)\}
```



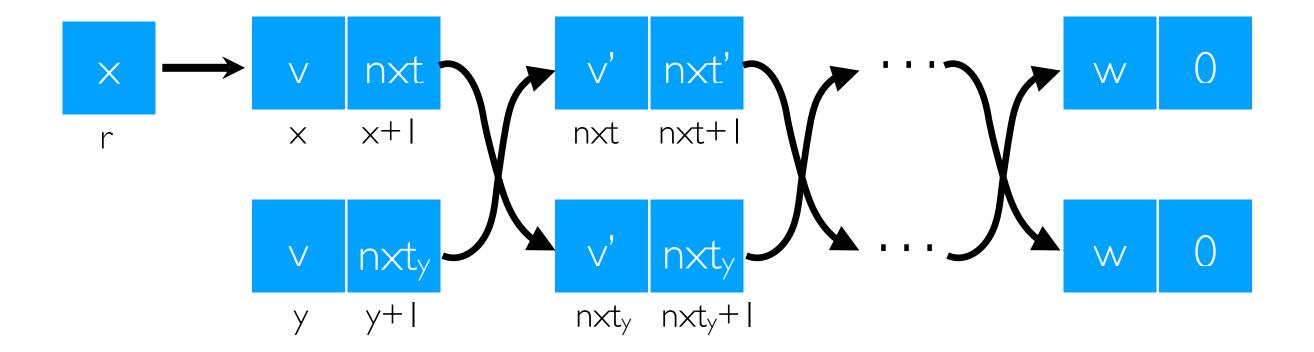
```
\{r \mapsto x * lseg(x, S)\}
       void listcopy (loc r) {
         let x = *r;
     3 	 if (x == 0) {
     4 } else {
           let v = *x;
           let nxt = *(x + 1);
            *r = nxt;
           listcopy(r);
           let y1 = *r;
         let y = malloc(2);
    10
            *(x + 1) = y1;
    11
            *r = y;
           *(y + 1) = nxt;
    13
    14
            *y = v;
    15
\{r \mapsto y * lseg(x, S) * lseg(y, S)\}
```



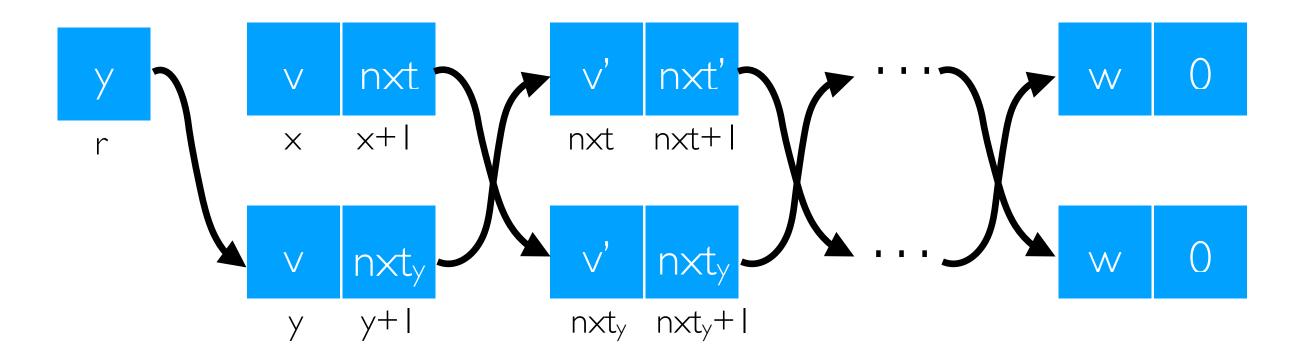
```
\{r \mapsto x * lseg(x, S)\}
       void listcopy (loc r) {
         let x = *r;
     3 	 if (x == 0) {
      } else {
           let v = *x;
           let nxt = *(x + 1);
           *r = nxt;
           listcopy(r);
           let y1 = *r;
           let y = malloc(2);
    10
         *(x + 1) = y1;
    11
           *r = y;
           *(y + 1) = nxt;
    13
    14
            *y = v;
    15
\{r \mapsto y * lseg(x, S) * lseg(y, S)\}
```



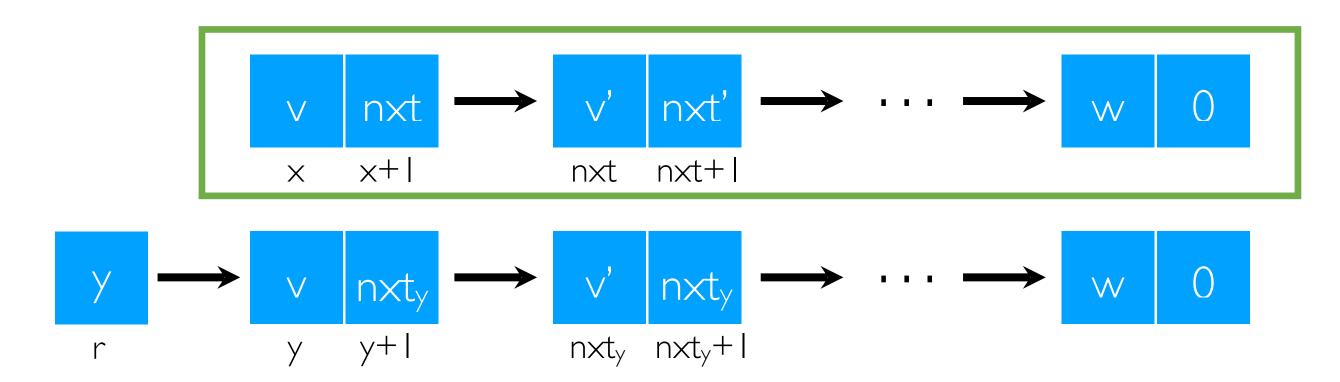
```
\{r \mapsto x * lseg(x, S)\}
        void listcopy (loc r) {
          let x = *r;
     3 	 if (x == 0) {
         } else {
            let v = *x;
            let nxt = *(x + 1);
            *r = nxt;
            listcopy(r);
            let y1 = *r;
            let y = malloc(2);
    10
            *(x + 1) = y1;
    11
            *r = y;
            *(y + 1) = nxt;
    13
    14
            *y = v;
    15
\{r \mapsto y * lseg(x, S) * lseg(y, S)\}
```



```
\{r \mapsto x * lseg(x, S)\}
        void listcopy (loc r) {
          let x = *r;
         if (x == 0) {
         } else {
            let v = *x;
            let nxt = *(x + 1);
            *r = nxt;
            listcopy(r);
            let y1 = *r;
            let y = malloc(2);
    10
            *(x + 1) = y1;
    11
            *r = y;
            *(y + 1) = nxt;
    13
    14
            *y = v;
    15
\{r \mapsto y * lseg(x, S) * lseg(y, S)\}
```

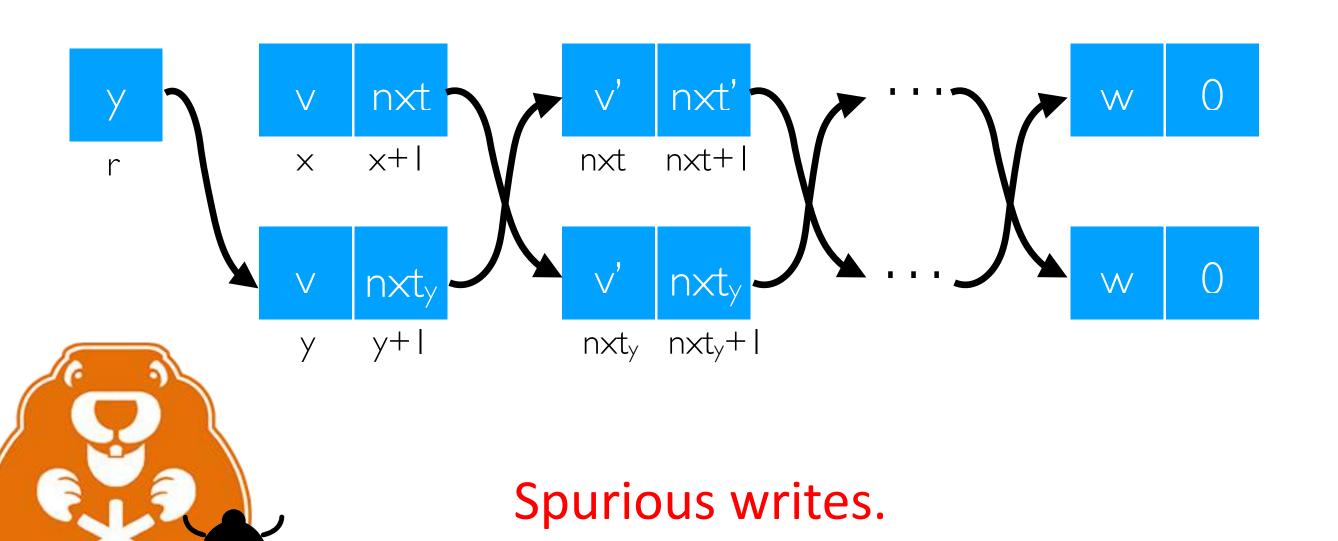


```
\{r \mapsto x * lseg(x, S)\}
        void listcopy (loc r) {
          let x = *r;
         if (x == 0) {
         } else {
            let v = *x;
           let nxt = *(x + 1);
            *r = nxt;
           listcopy(r);
            let y1 = *r;
            let y = malloc(2);
    10
            *(x + 1) = y1;
    11
         *r = y;
            *(y + 1) = nxt;
    14
            *y = v;
    15
\{r \mapsto y * lseg(x, S) * lseg(y, S)\}
```



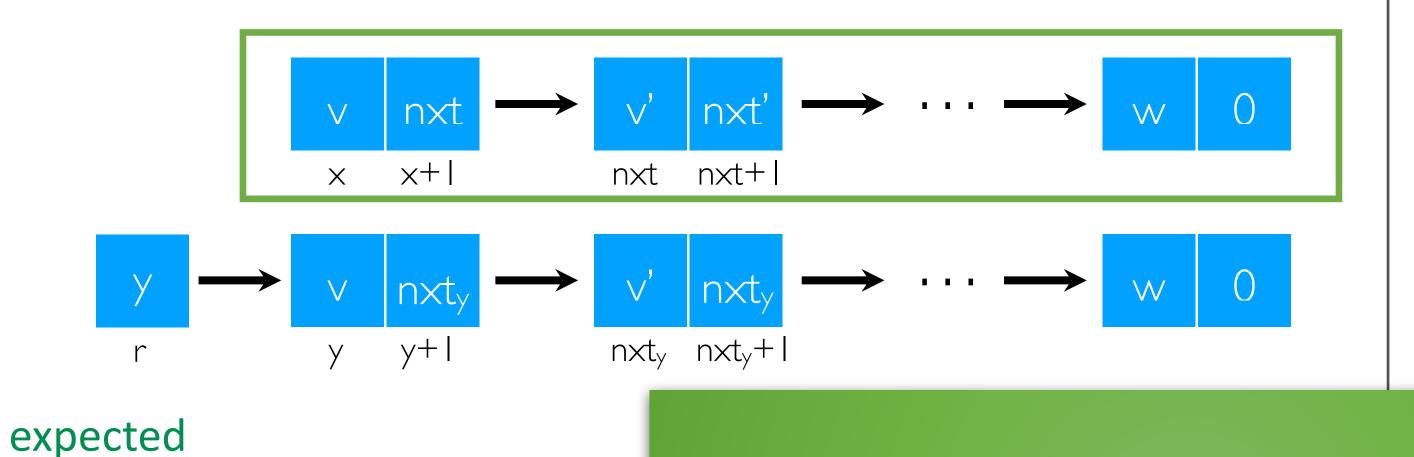
#### expected

#### result

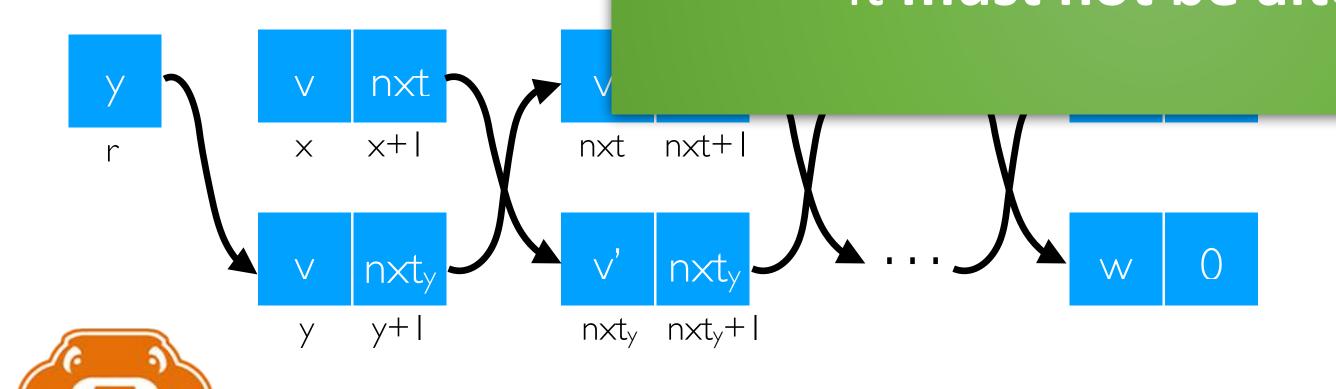


```
\{r \mapsto x * lseg(x, S)\}
       void listcopy (loc r) {
         let x = *r;
          if (x == 0) {
         } else {
            let v = *x;
            let nxt = *(x + 1);
            *r = nxt;
            listcopy(r);
            let y1 = *r;
            let y = malloc(2);
    10
            *(x + 1) = y1;
    11
    12
            *(y + 1) = nxt;
    13
    14
            *y = v;
    15
\{r \mapsto y * lseg(x, S) * lseg(y, S)\}
```

result



```
\{r \mapsto x * lseg(x, S)\}
                               void listcopy (loc r) {
                                 let x = *r;
                                 if (x == 0) {
                                 } else {
                                    let v = *x;
                                    let nxt = *(x + 1);
                                    *r = nxt;
Make the initial list Read-Only:
                                   listcopy(r);
                                    let y1 = *r;
    it must not be altered
                                    let y = malloc(2);
                                    *(x + 1) = y1;
                           12
                                    *(y + 1) = nxt;
                           13
                           14
                                    *y = v;
                           15
                       \{r \mapsto y * lseg(x, S) * lseg(y, S)\}
```



Spurious writes.

# Synthesis of Programs with Pointers via Read-Only Specifications

(our contribution)

Effective: more natural and shorter programs

Efficient: smaller search space—faster synthesis

Robust: better performance in "worst case scenarios"

## A Primer on Synthetic Separation Logic

## Syntactic Separation Logic

#### Syntactic Separation Logic

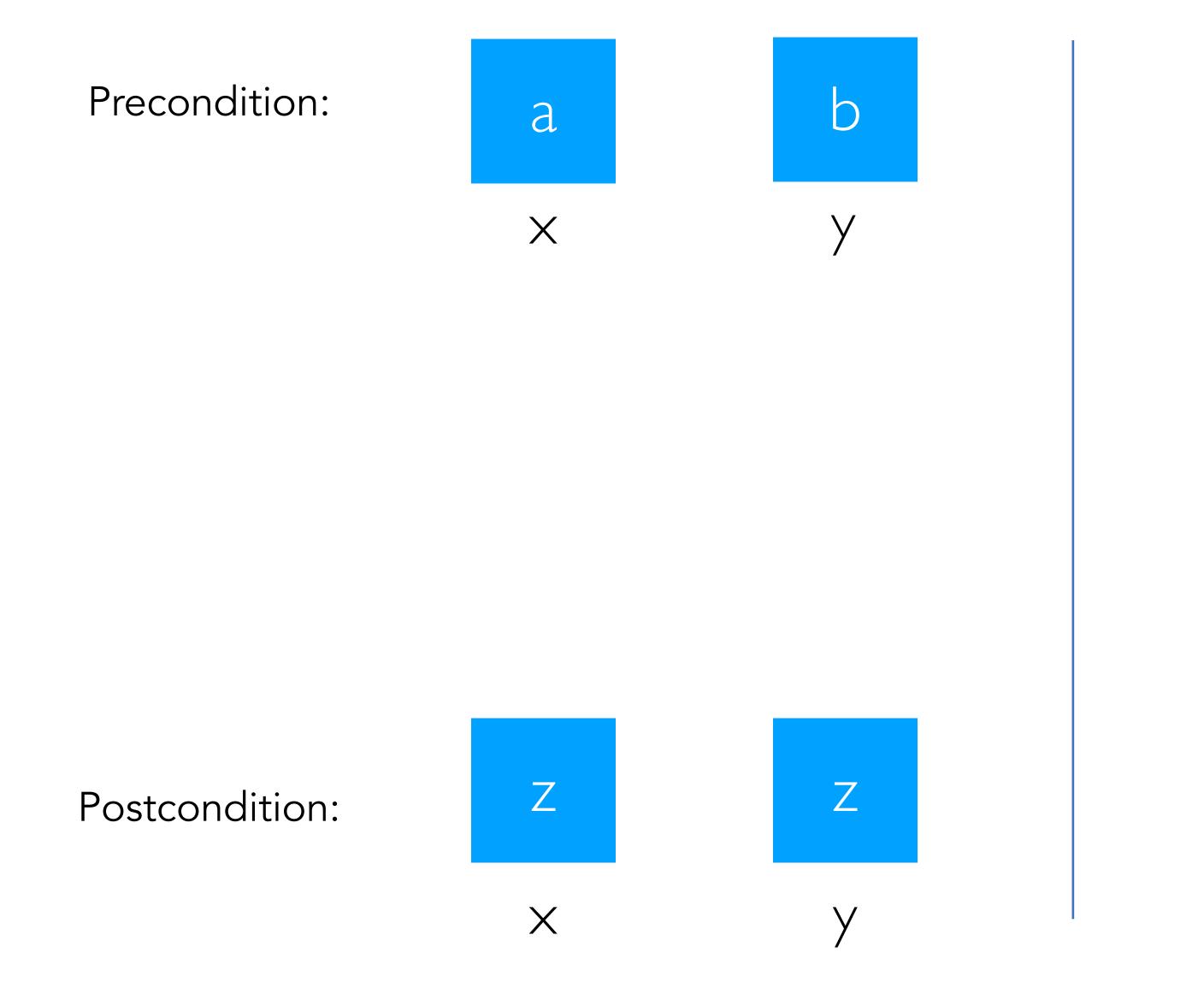
$$\{P\} \ C \ \{Q\}$$

$$P \vdash Q \quad \text{implies} \quad P \rightsquigarrow Q \mid C$$

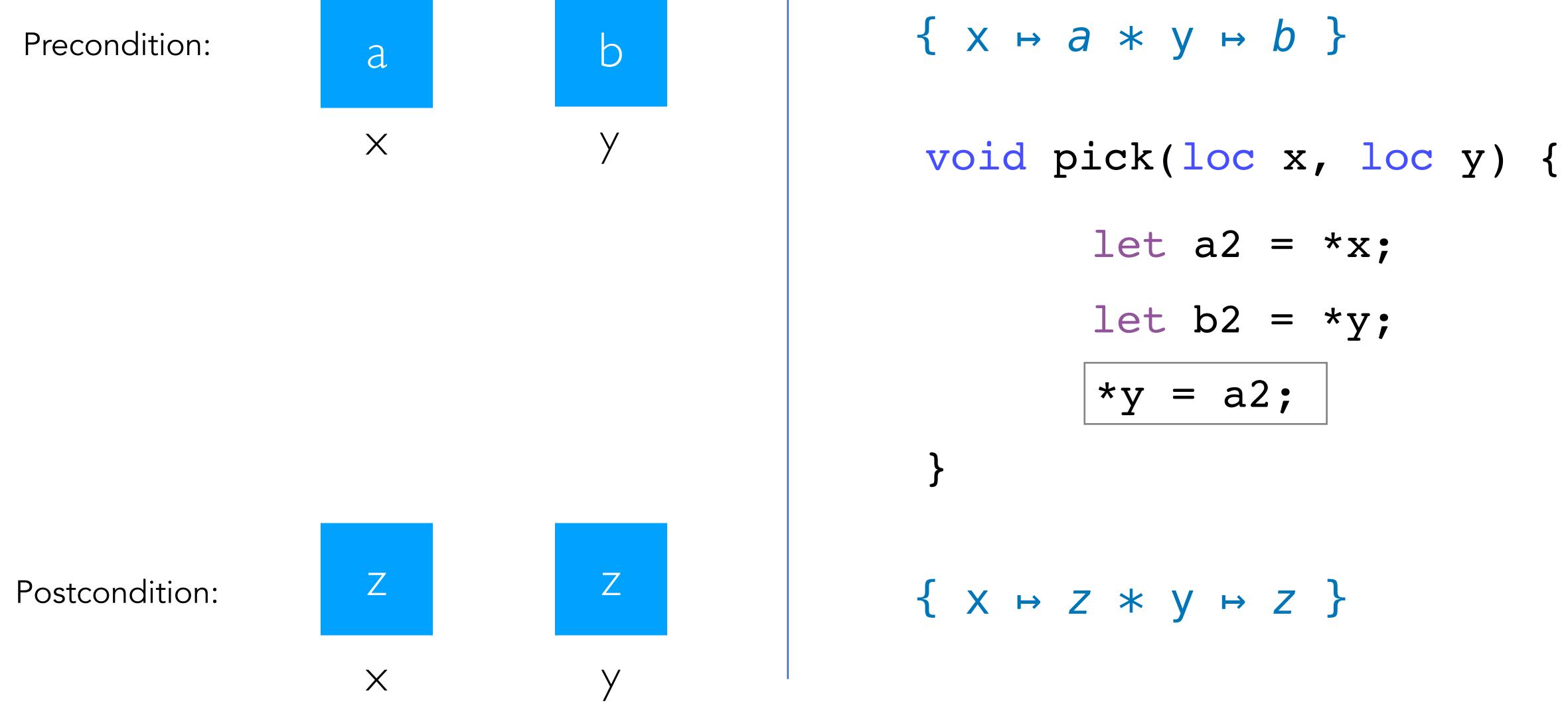
Precondition: Postcondition:

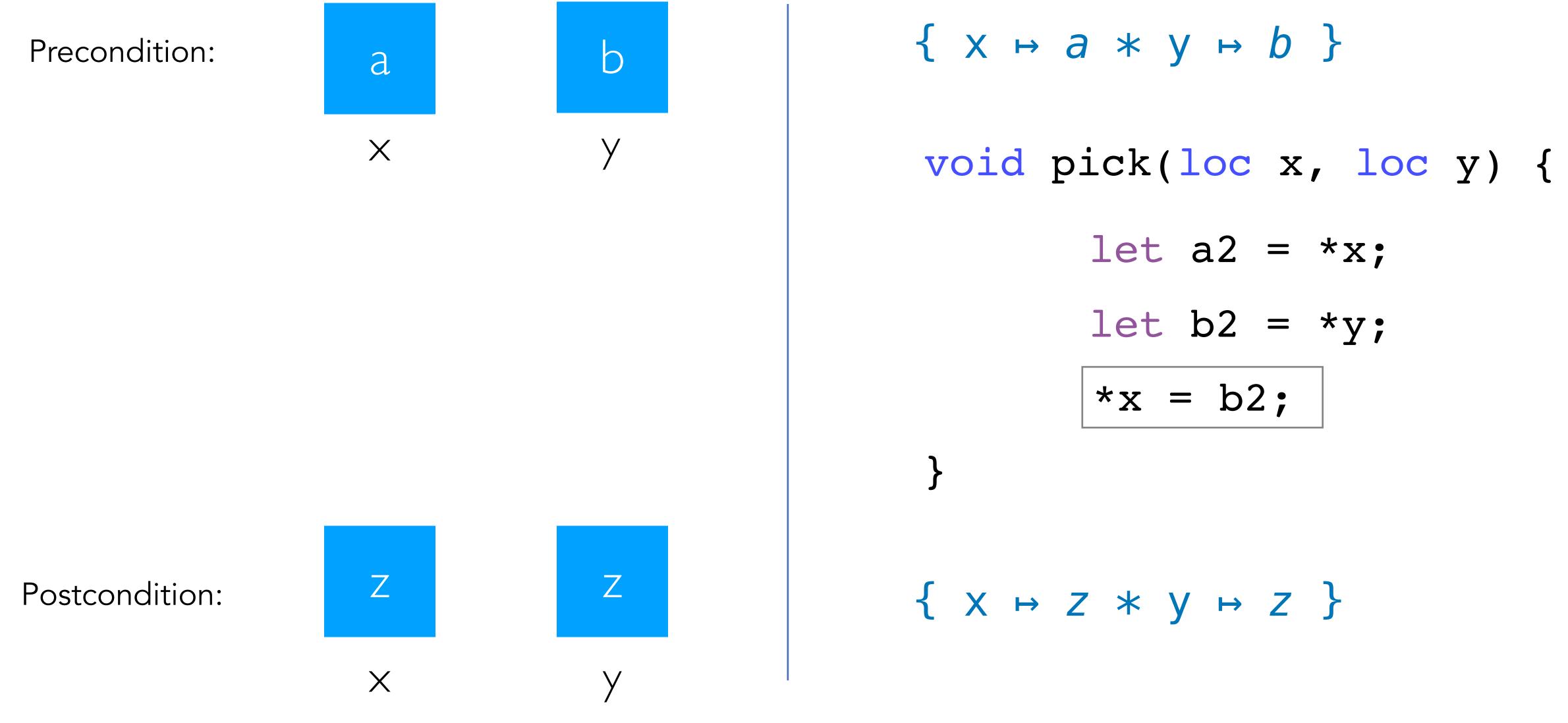
$$\{ x \mapsto a * y \mapsto b \}$$

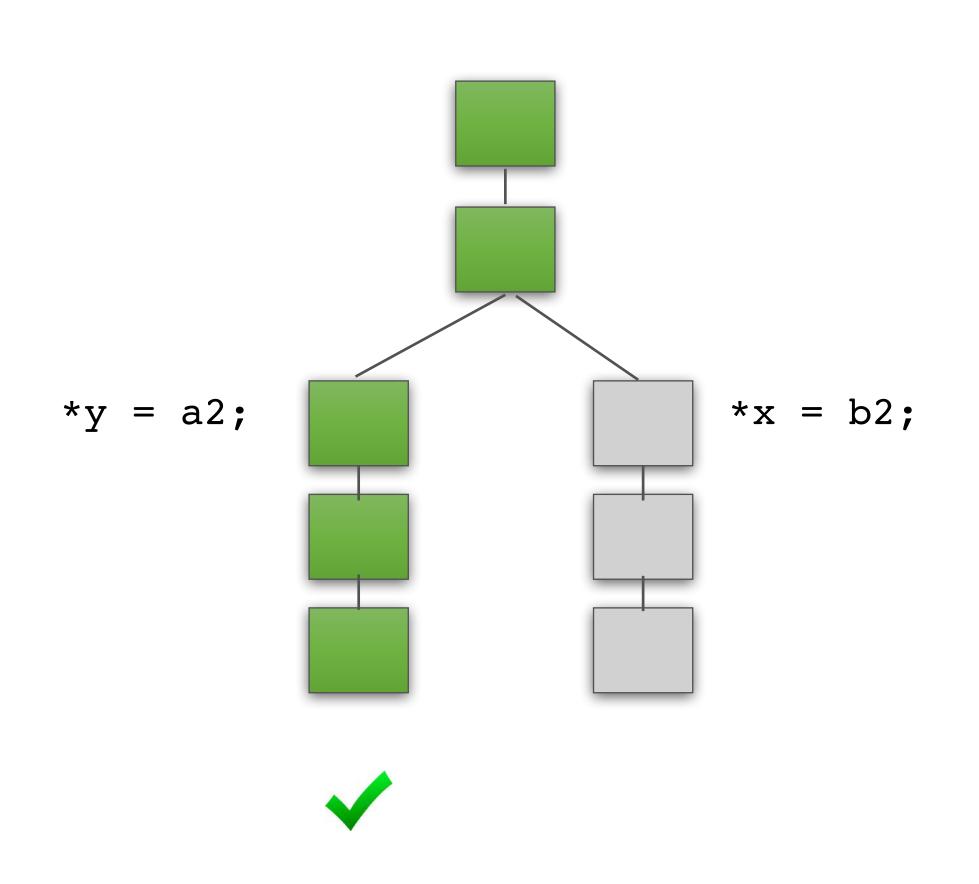
 $\{ x \mapsto z * y \mapsto z \}$ 



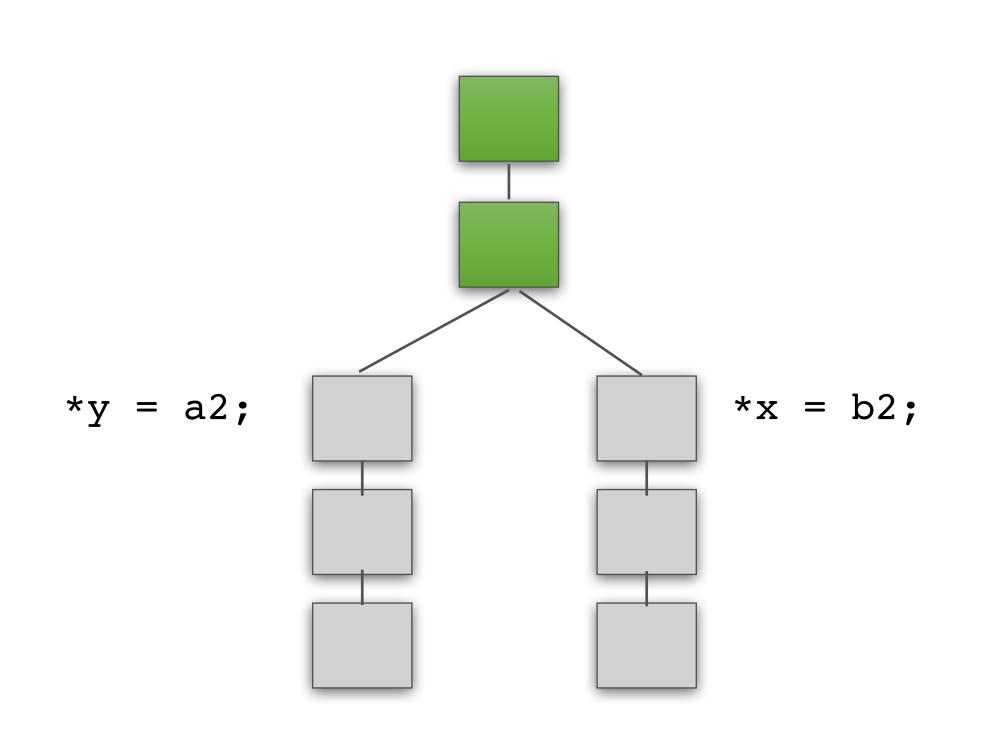
```
\{ x \mapsto a * y \mapsto b \}
void pick(loc x, loc y) {
         let a2 = *x;
         let b2 = *y;
         *y = a2;
\{ X \mapsto Z * Y \mapsto Z \}
```



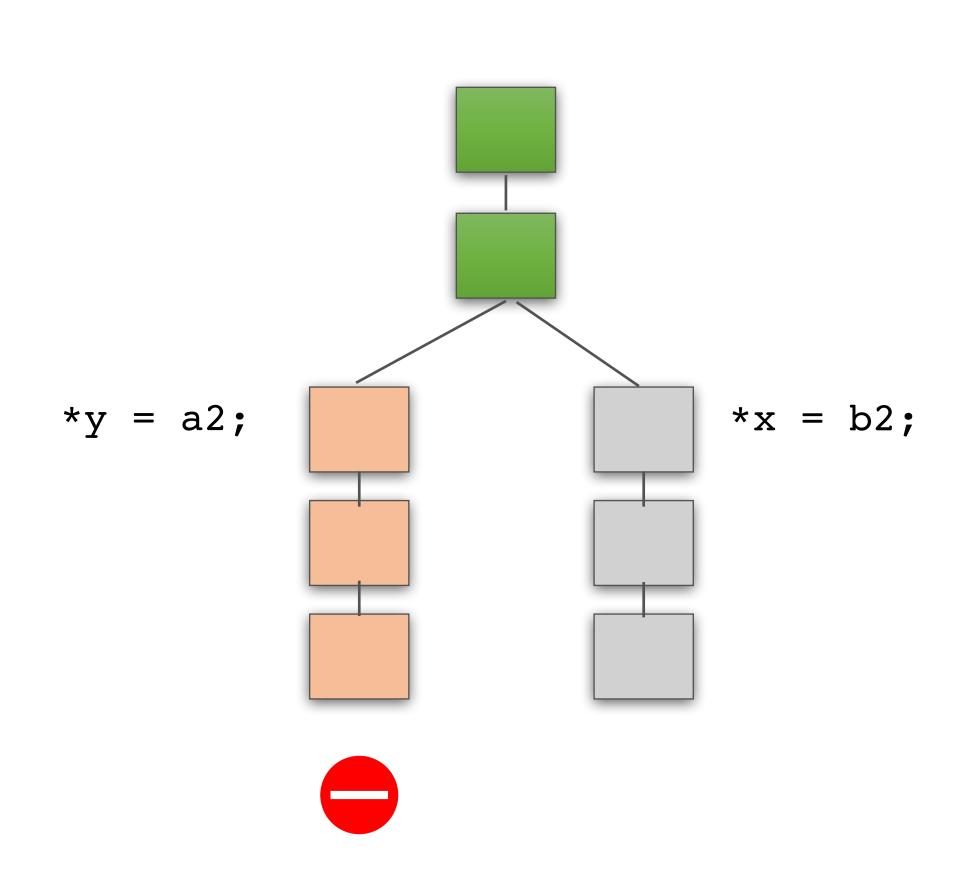




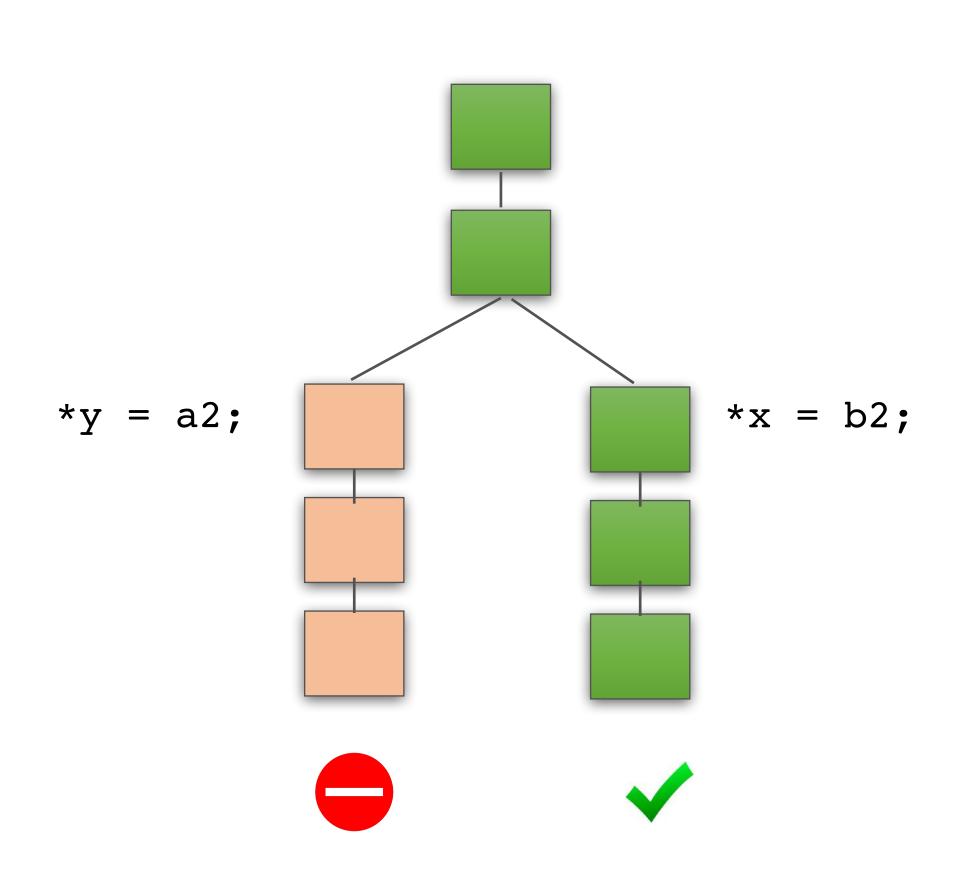
```
\{ x \mapsto a * y \mapsto b \}
void pick(loc x, loc y) {
         let a2 = *x;
         let b2 = *y;
         *y = a2;
\{ X \mapsto Z * Y \mapsto Z \}
```



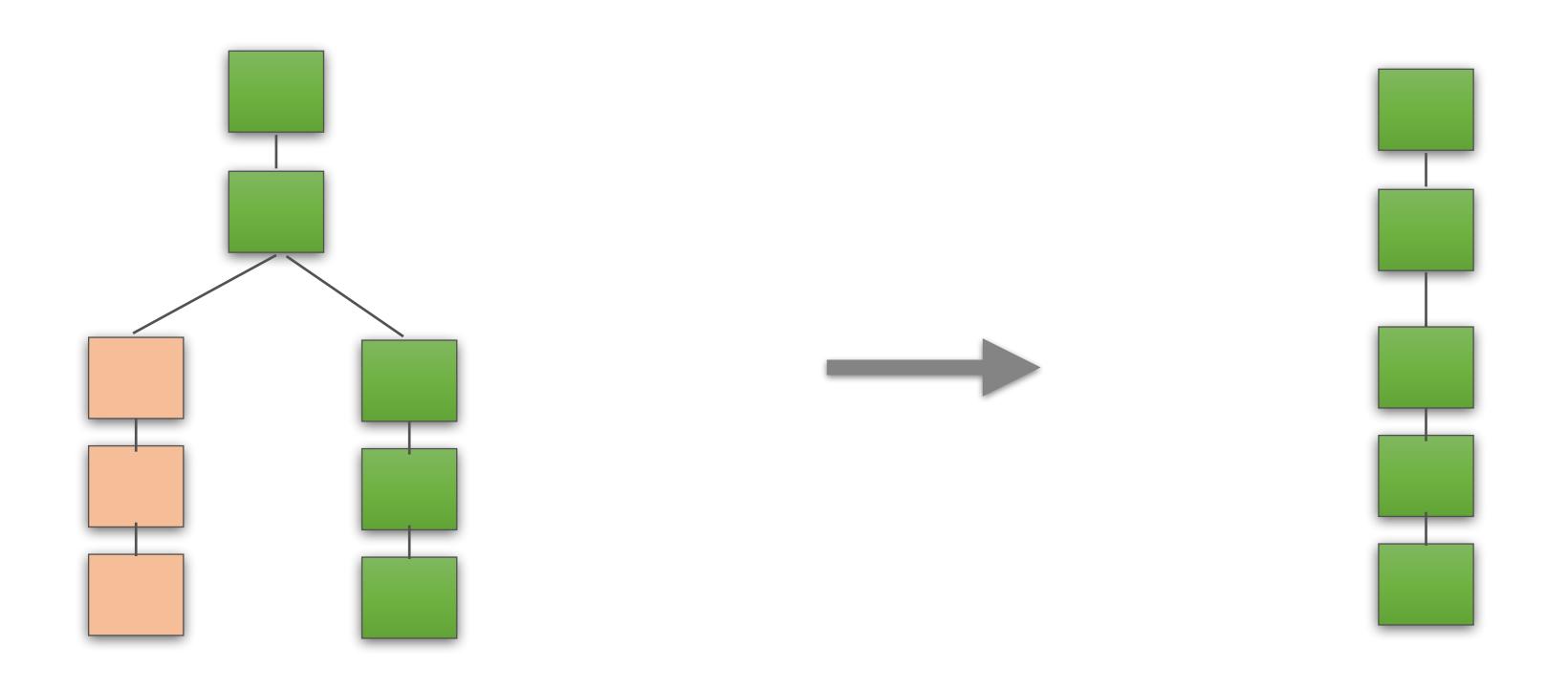
```
\{ x \mapsto a * y \mapsto b_0 \}
void pick(loc x, loc y) {
          let a2 = *x;
          let b2 = *y;
\{ x \mapsto z * y \mapsto z_0 \}
```



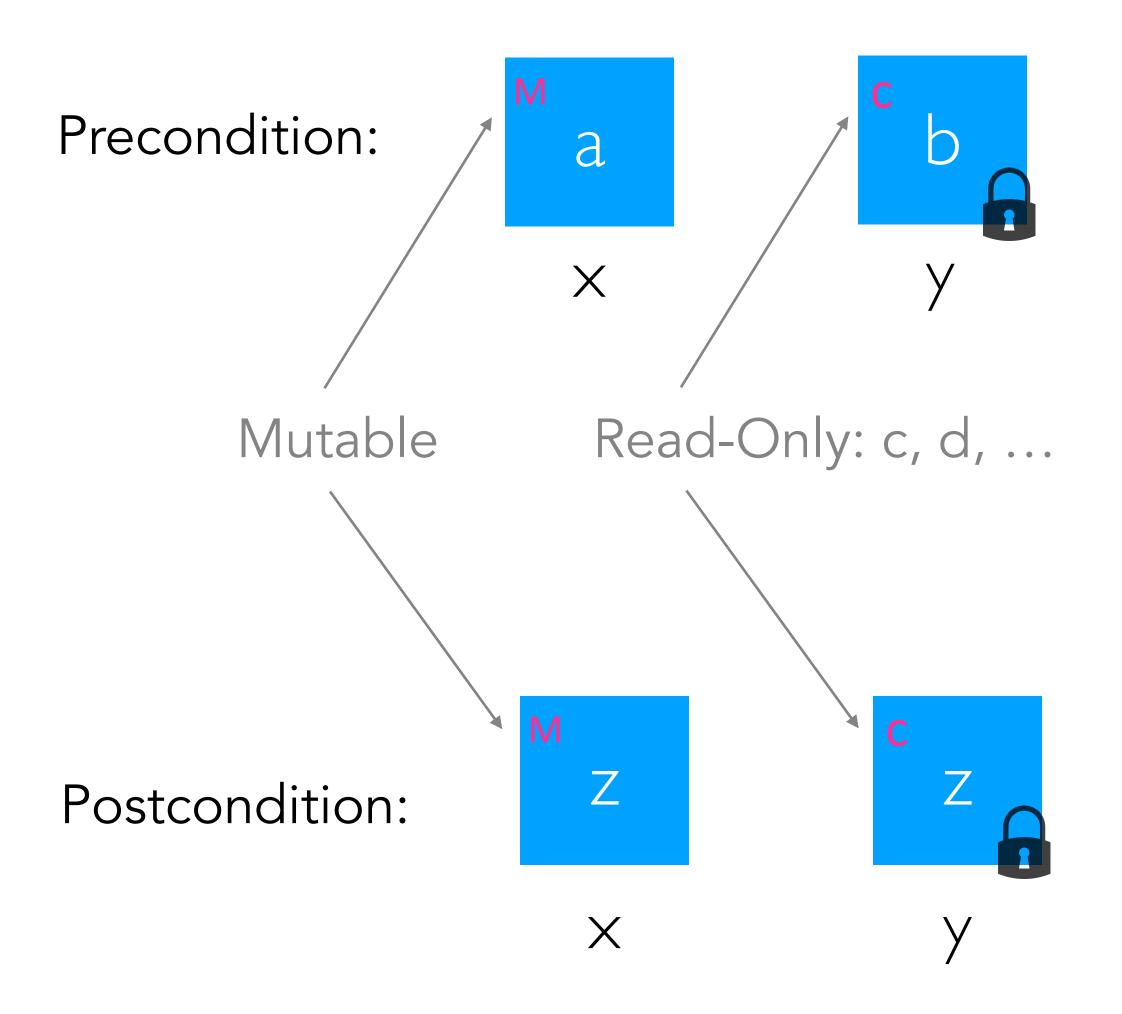
```
\{ x \mapsto a * y \mapsto b_0 \}
void pick(loc x, loc y) {
           let a2 = *x;
           let b2 = *y;
\{ x \mapsto z * | y \mapsto z_{\bigcirc} \}
```



```
\{ x \mapsto a * y \mapsto b_0 \}
void pick(loc x, loc y) {
          let a2 = *x;
          let b2 = *y;
          *x = b2;
\{ x \mapsto z * | y \mapsto z_{\bigcirc} \}
```



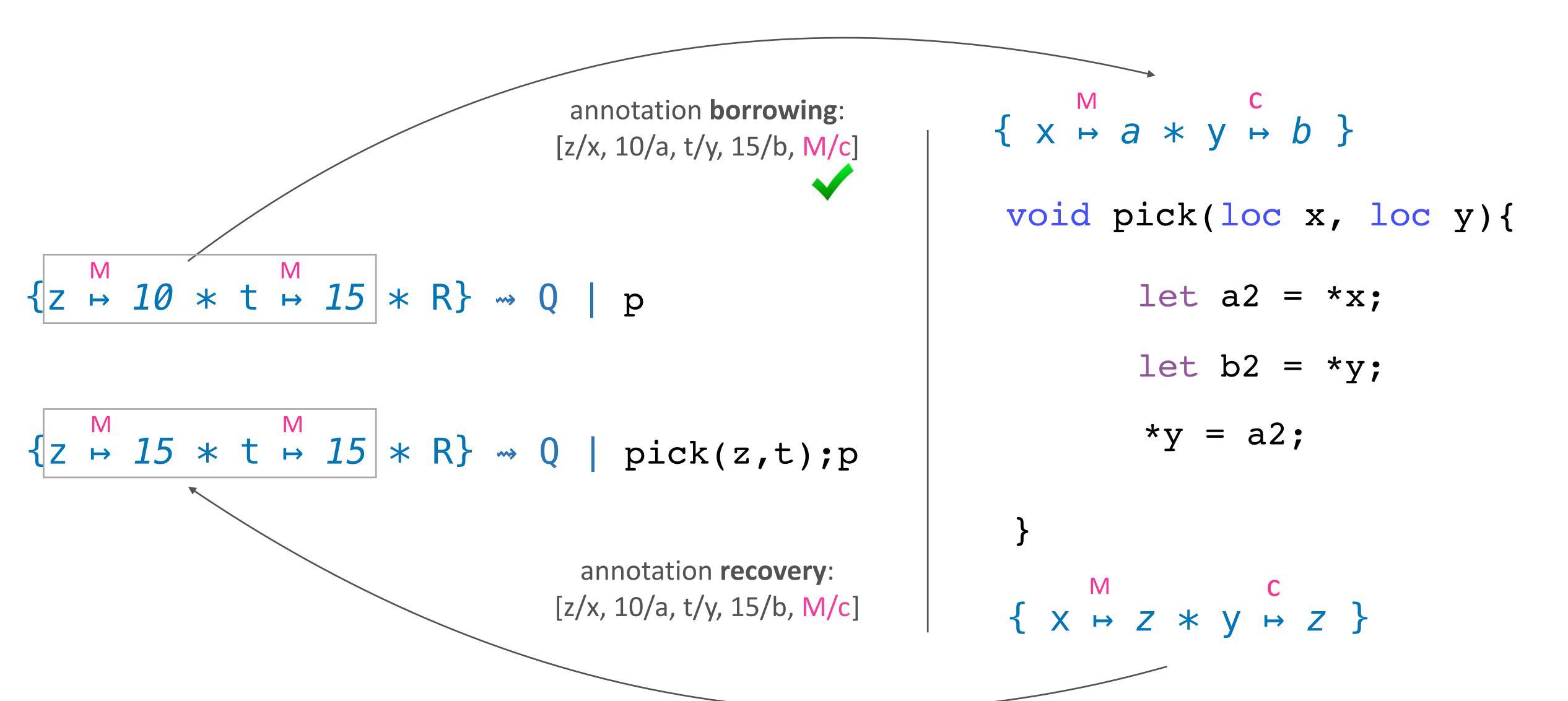
# Read-Only Specifications



```
\{ x \mapsto a * y \mapsto b \}
void pick(loc x, loc y){
        let a2 = *x;
        let b2 = *y;
        *y = a2;
```

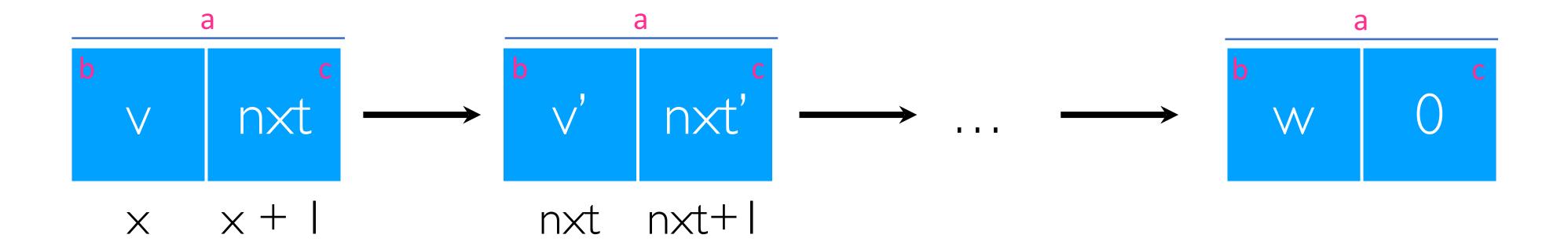
```
\{z \mapsto 10 * t \mapsto 15 * R\} \Rightarrow Q \mid p
```

```
\{ x \mapsto a * y \mapsto b \}
 void pick(loc x, loc y){
        let a2 = *x;
        let b2 = *y;
      *y = a2;
```



```
annotation borrowing:
                             [z/x, 10/a, d/M, t/y, 15/b, M/c]
                                                           void pick(loc x, loc y){
\{z \mapsto 10 * t \mapsto 15 | * R\} \Rightarrow Q \mid p
                                                                   let a2 = *x;
                                                                   let b2 = *y;
                                                                   *y = a2;
```

### Example: copy of a linked list Read-Only Specifications

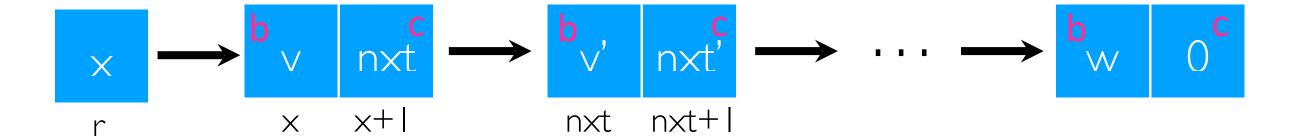


```
{r ↦ x * lseg(x,S,a,b,c) }

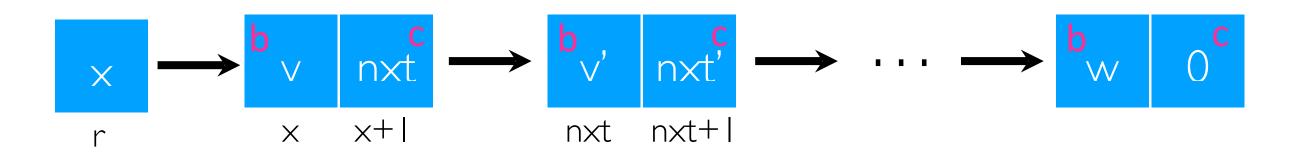
void listcopy (loc r) mutable
{r ↦ y * lseg(x,S,a,b,c) * lseg(y,S,M,M,M)}
```

```
{r ↦ x * lseg(x,S,a,b,c) }
    void listcopy (loc r)

{r ↦ y * lseg(x,S,a,b,c) * lseg(y,S,M,M,M)}
```

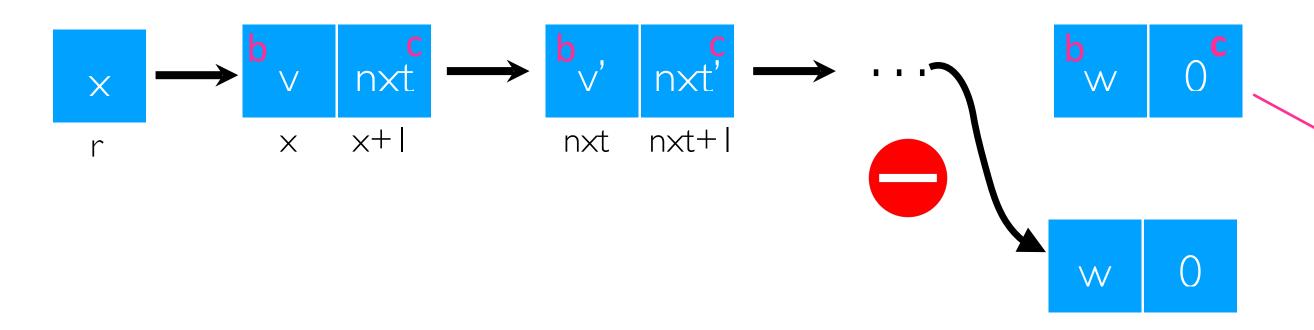


```
\{r \mapsto x * lseg(x,S,a,b,c) \}
         void listcopy (loc r) {
           let x = *r;
           if (x == 0) {
           } else {
              let v = *x;
              let nxt = *(x + 1);
              *r = nxt;
              listcopy(r);
              let y1 = *r;
              let y = malloc(2);
     10
              *(x + 1) = y1;
     12
              *r = y;
              *(y + 1) = nxt;
     13
     14
              *y = v;
     15
\{r \mapsto y * lseg(x,S,a,b,c) * lseg(y,S,M,M,M)\}
```

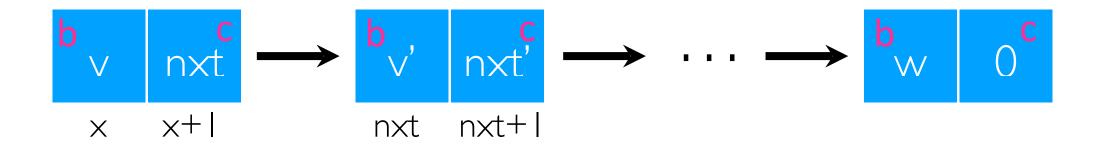


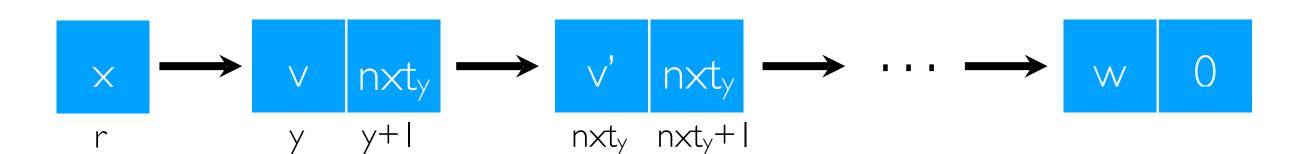


```
\{r \mapsto x * lseg(x,S,a,b,c) \}
         void listcopy (loc r) {
           let x = *r;
           if (x == 0) {
           } else {
              let v = *x;
             let nxt = *(x + 1);
              *r = nxt;
              listcopy(r);
      8
              let y1 = *r;
              let y = malloc(2);
     10
              *(x + 1) = y1;
     12
              *r = y;
              *(y + 1) = nxt;
     13
     14
              *y = v;
     15
\{r \mapsto y * lseg(x,S,a,b,c) * lseg(y,S,M,M,M)\}
```



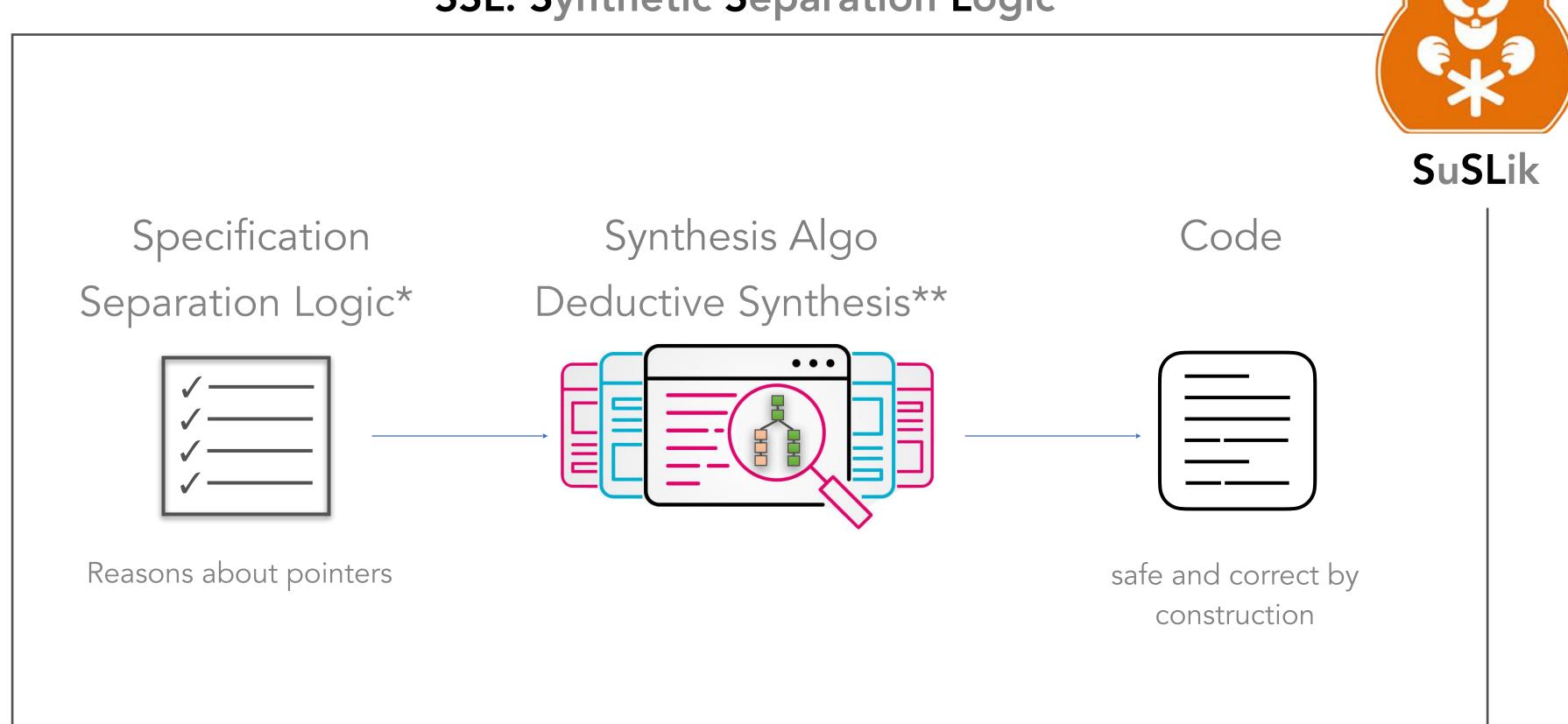
```
\{r \mapsto x * lseg(x,S,a,b,c) \}
         void listcopy (loc r) {
           let x = *r;
           if (x == 0) {
           } else {
              let v = *x;
              let nxt = *(x + 1);
              *r = nxt;
              listcopy(r);
              let y1 = *r;
              let y = malloc(2);
     10
              *(x + 1) = y1;
     11
              *r = y;
     12
              *(y + 1) = nxt;
     13
     14
              *y = v;
     15
\{r \mapsto y * lseg(x,S,a,b,c) * lseg(y,S,M,M,M)\}
                             51
```





```
\{r \mapsto x * lseg(x,S,a,b,c) \}
         void listcopy (loc r) {
           let x = *r;
            if (x == 0) {
            } else {
              let v = *x;
              let nxt = *(x + 1);
      6
              *r = nxt;
              listcopy(r);
      8
              let y1 = *r;
              let y = malloc(2);
     10
     11
     12
              *(y + 1) = y1;
     13
     14
              *y = v;
     15
\{r \mapsto y * lseg(x,S,a,b,c) * lseg(y,S,M,M,M)\}
                             52
```

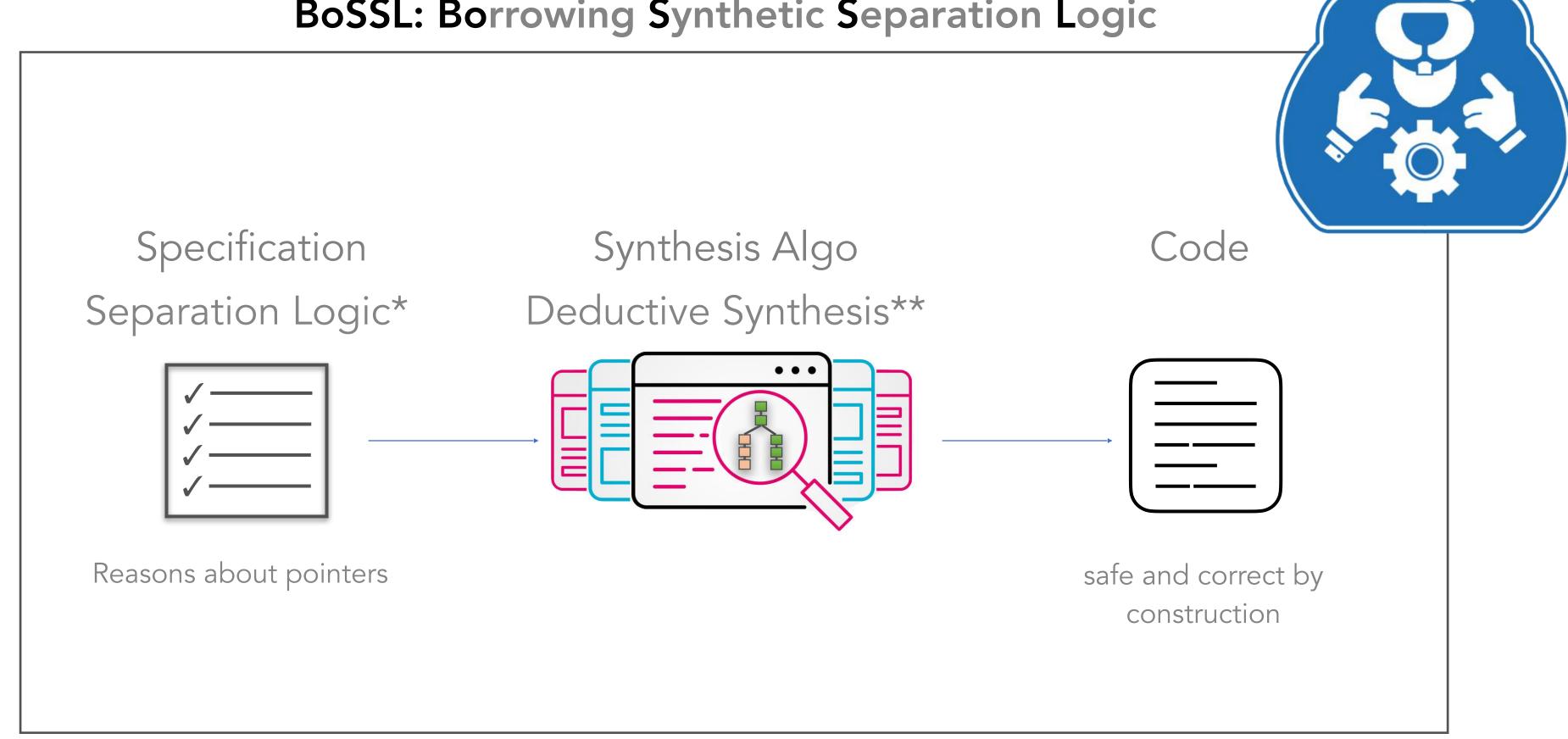
#### **SSL: Synthetic Separation Logic**

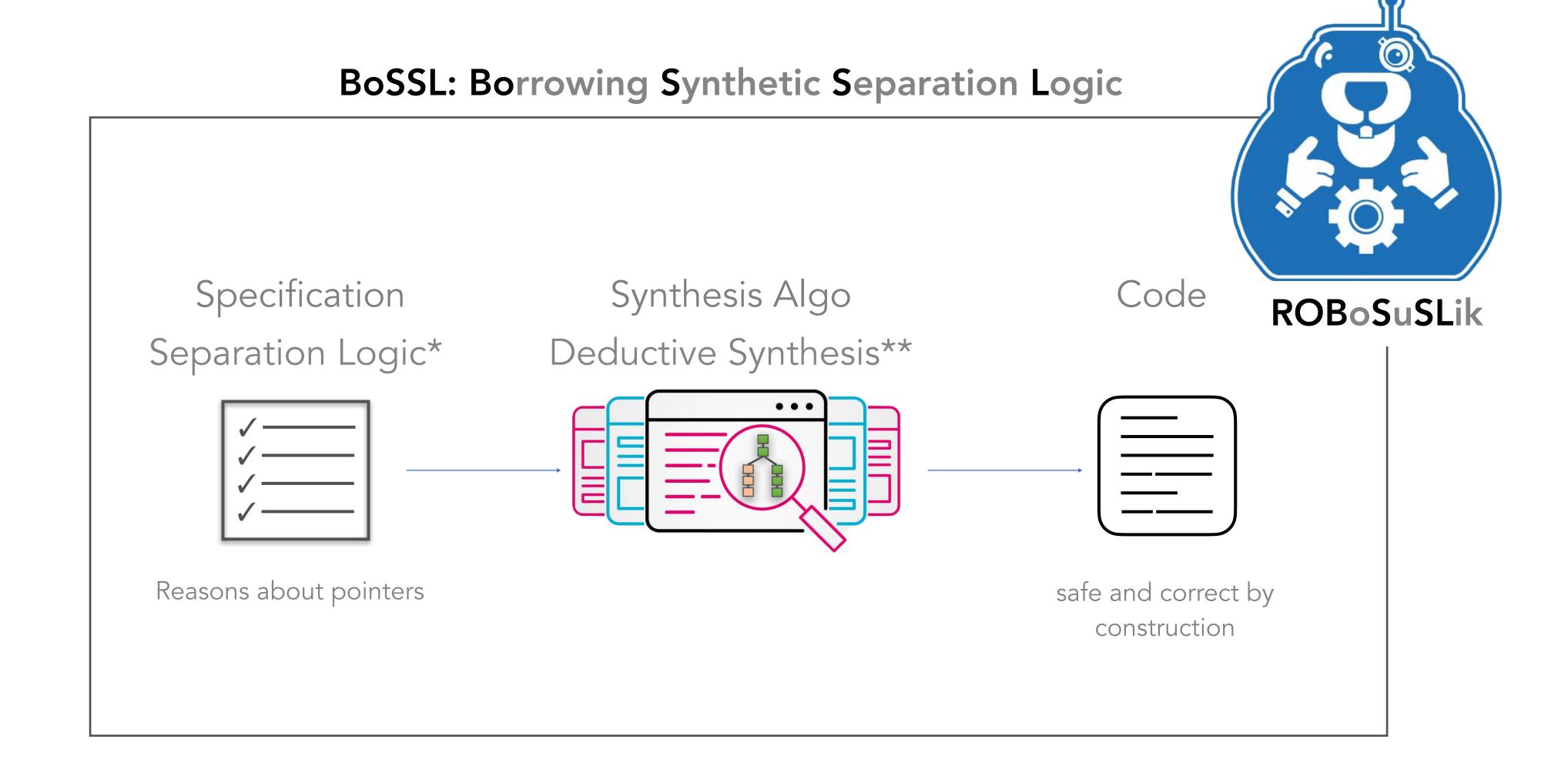


<sup>\*</sup> Local Reasoning about Programs that Alter Data Structures, O'Hearn, Reynolds, Yang:. CSL 2001

<sup>\*\*</sup> Structuring the Synthesis of Heap-Manipulating Programs, Polikarpova & Sergey @POPL'19

#### **BoSSL: Borrowing Synthetic Separation Logic**





https://github.com/TyGuS/robosuslik

# Synthesis of Programs with Pointers via Read-Only Specifications

(our contribution)

Effective: more natural and shorter programs

Efficient: smaller search space—faster synthesis

Robust: better performance in "worst case scenarios"

# Synthesis of Programs with Pointers via Read-Only Specifications

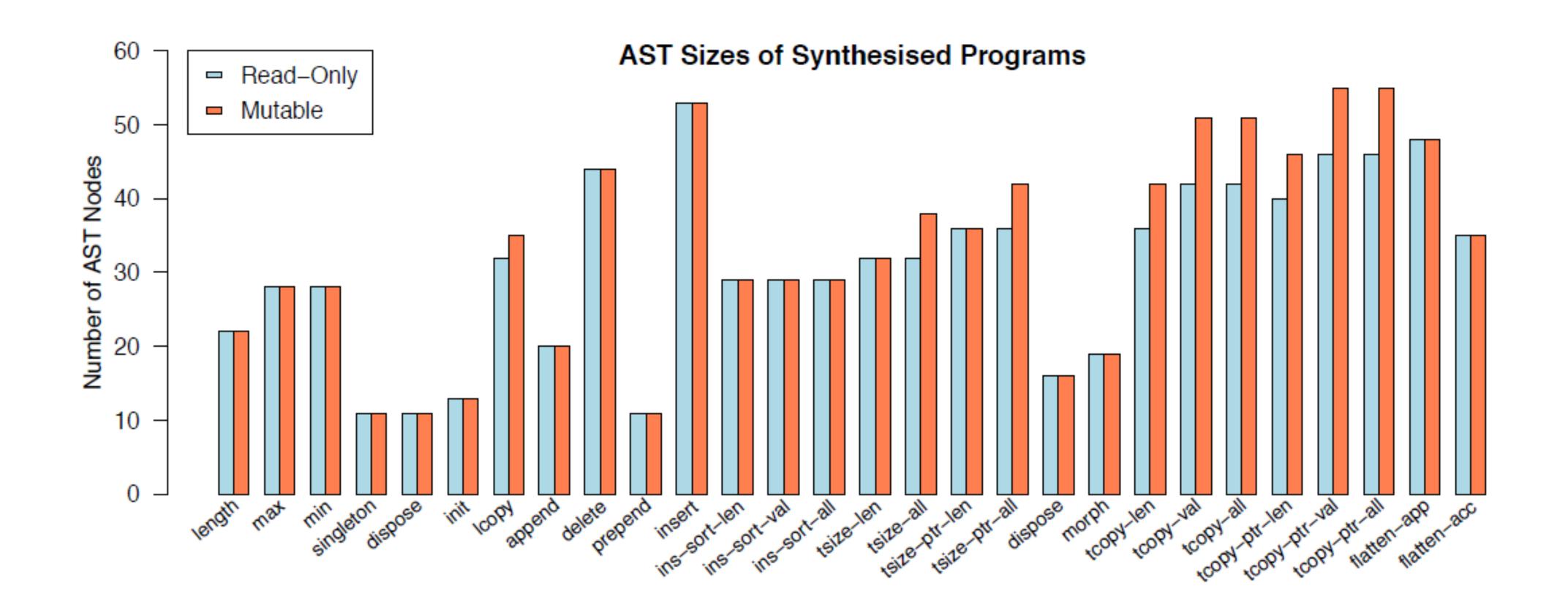
(our contribution)

Effective: more natural and shorter programs

Efficient: smaller search space—faster synthesis

Robust: better performance in "worst case scenarios"

#### Results 1 – AST size



# Synthesis of Programs with Pointers via Read-Only Specifications

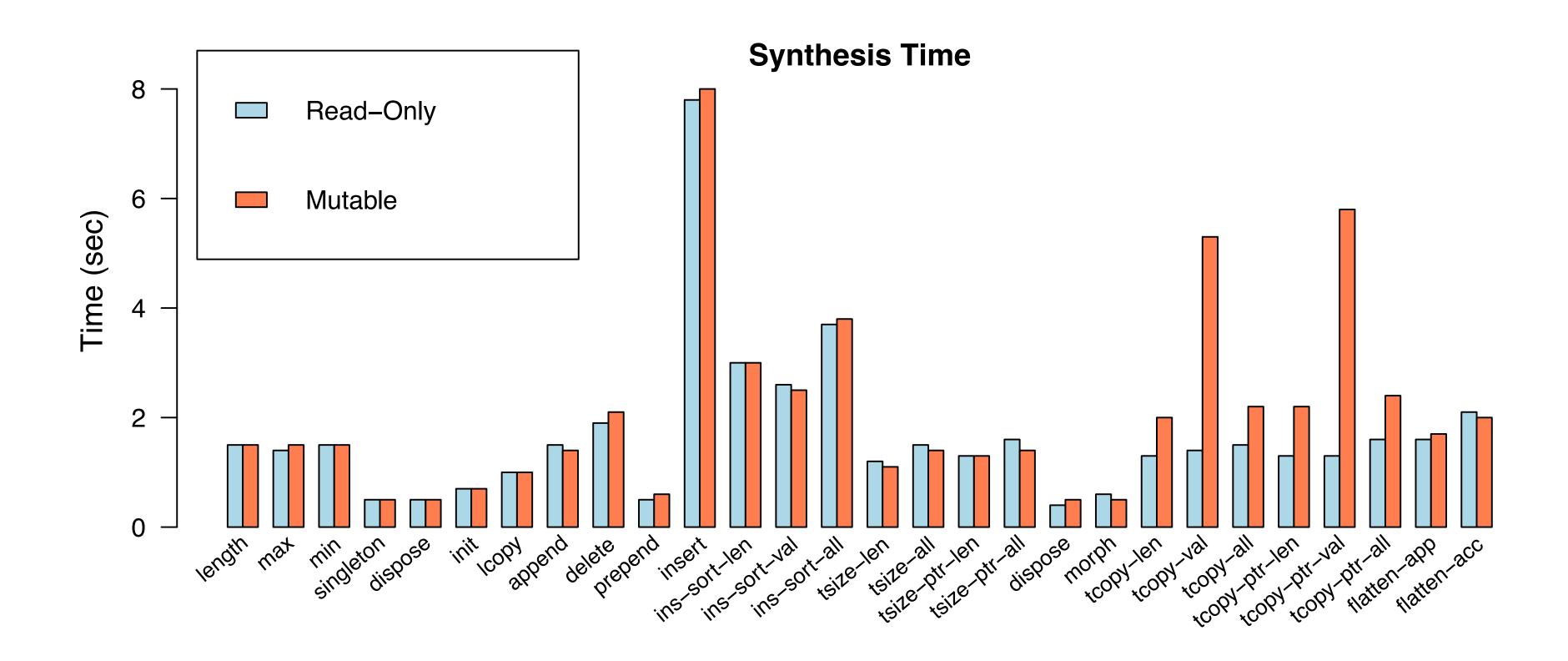
(our contribution)

Effective: more natural and shorter programs

Efficient: smaller search space—faster synthesis

Robust: better performance in "worst case scenarios"

### Results 2 – Synthesis time



# Synthesis of Programs with Pointers via Read-Only Specifications

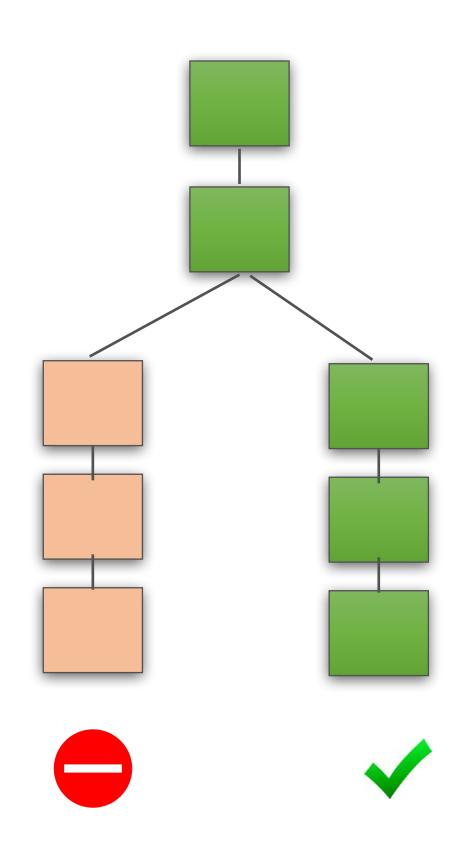
(our contribution)

Effective: more natural and shorter programs

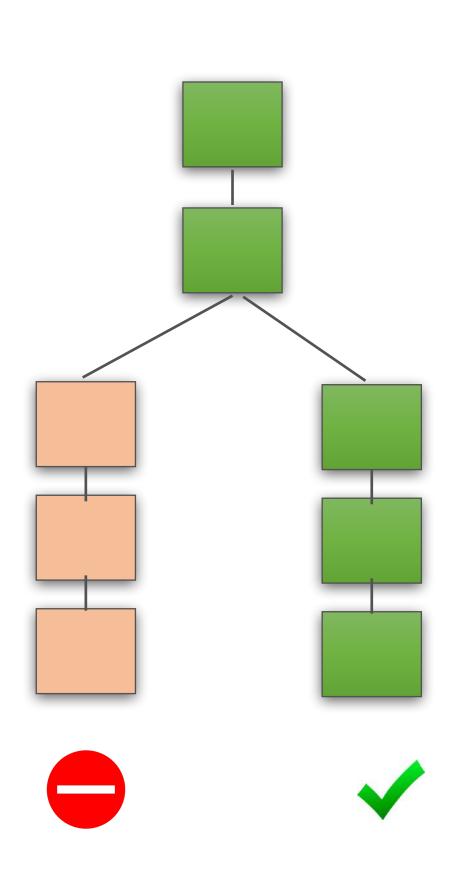
Efficient: smaller search space—faster synthesis

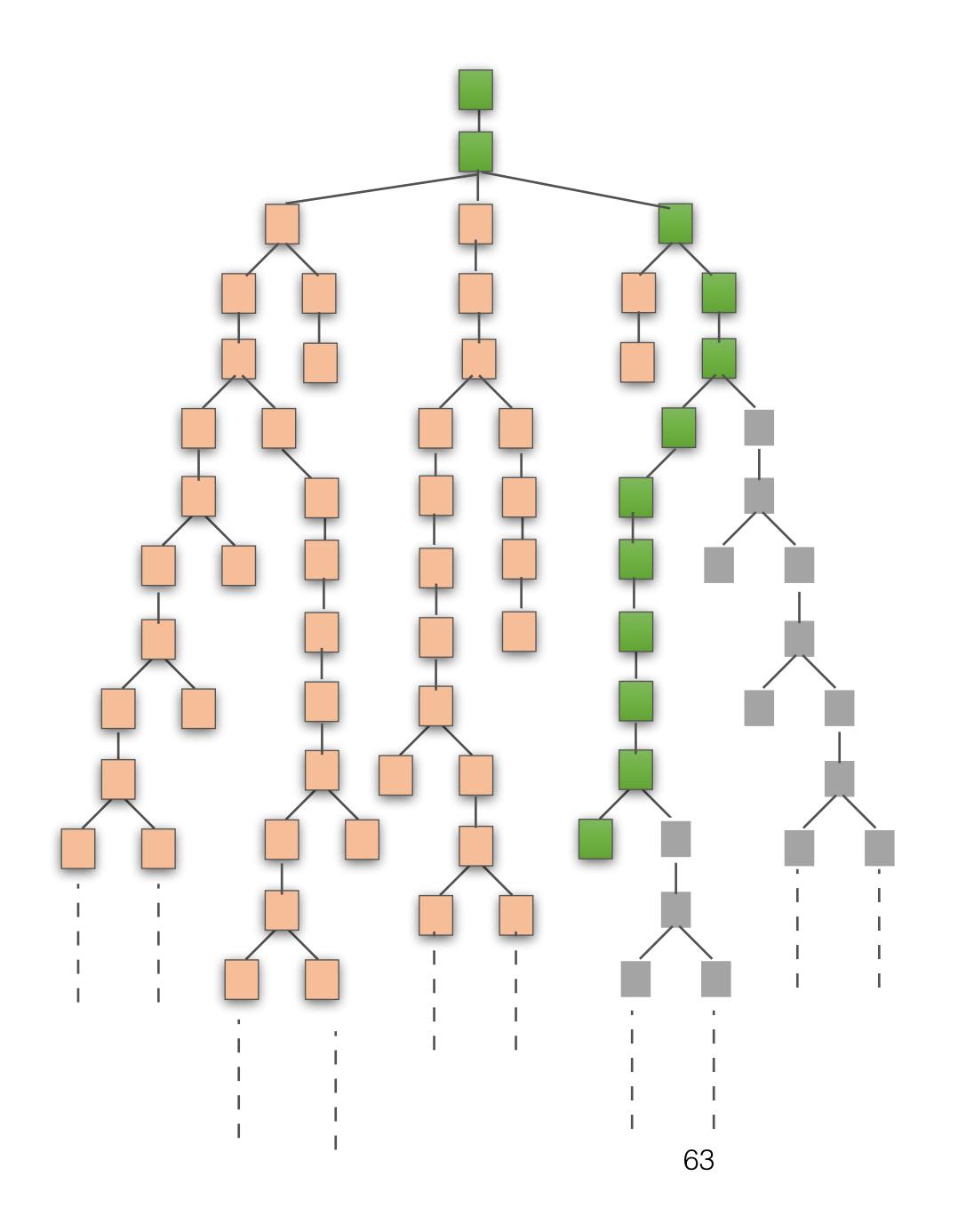
Robust: better performance in "worst case scenarios"

### Robustness



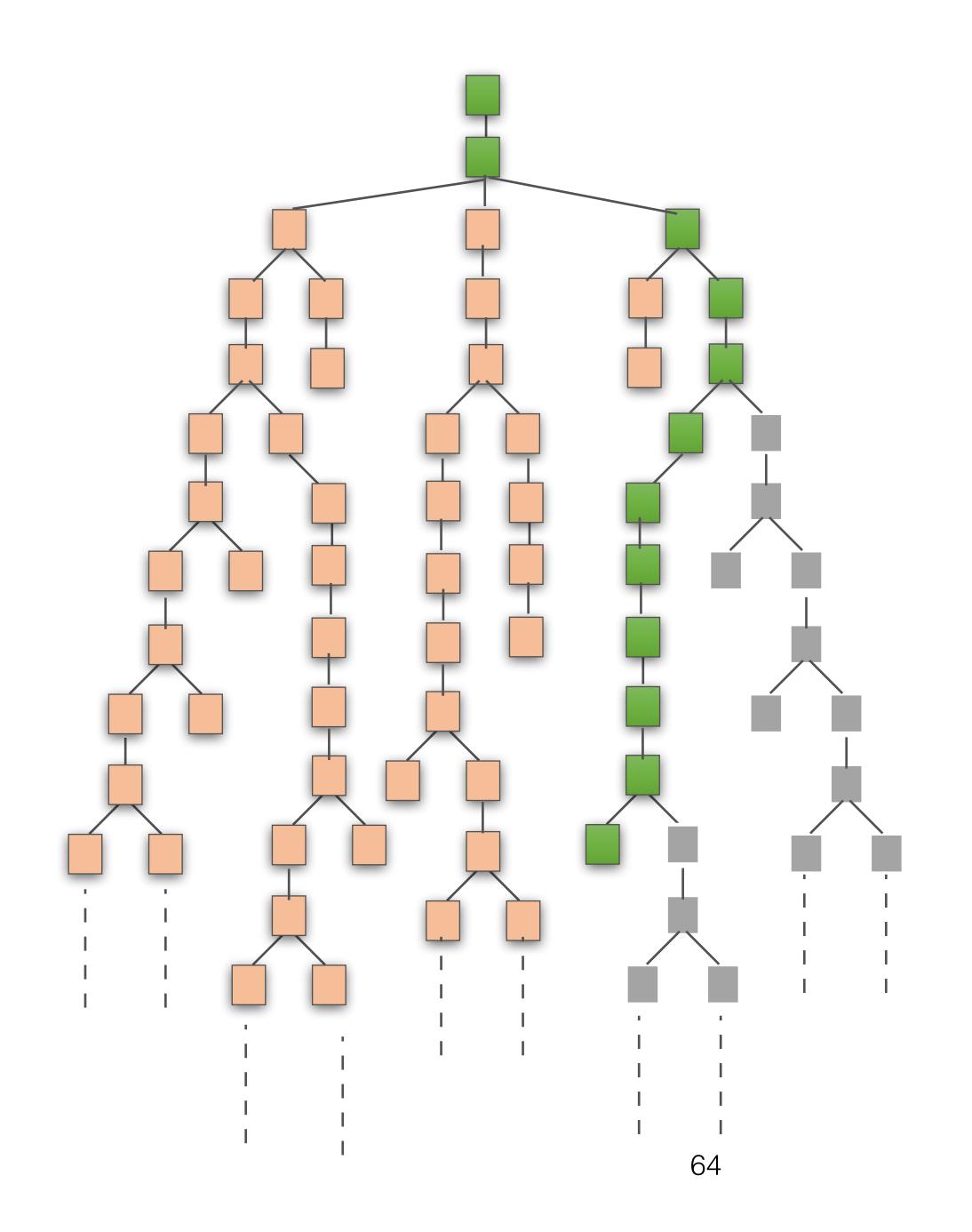
### Robustness





#### Robustness

Is ROBoSuSLik always outperforming SuSLik irrespective of the employed search heuristic?

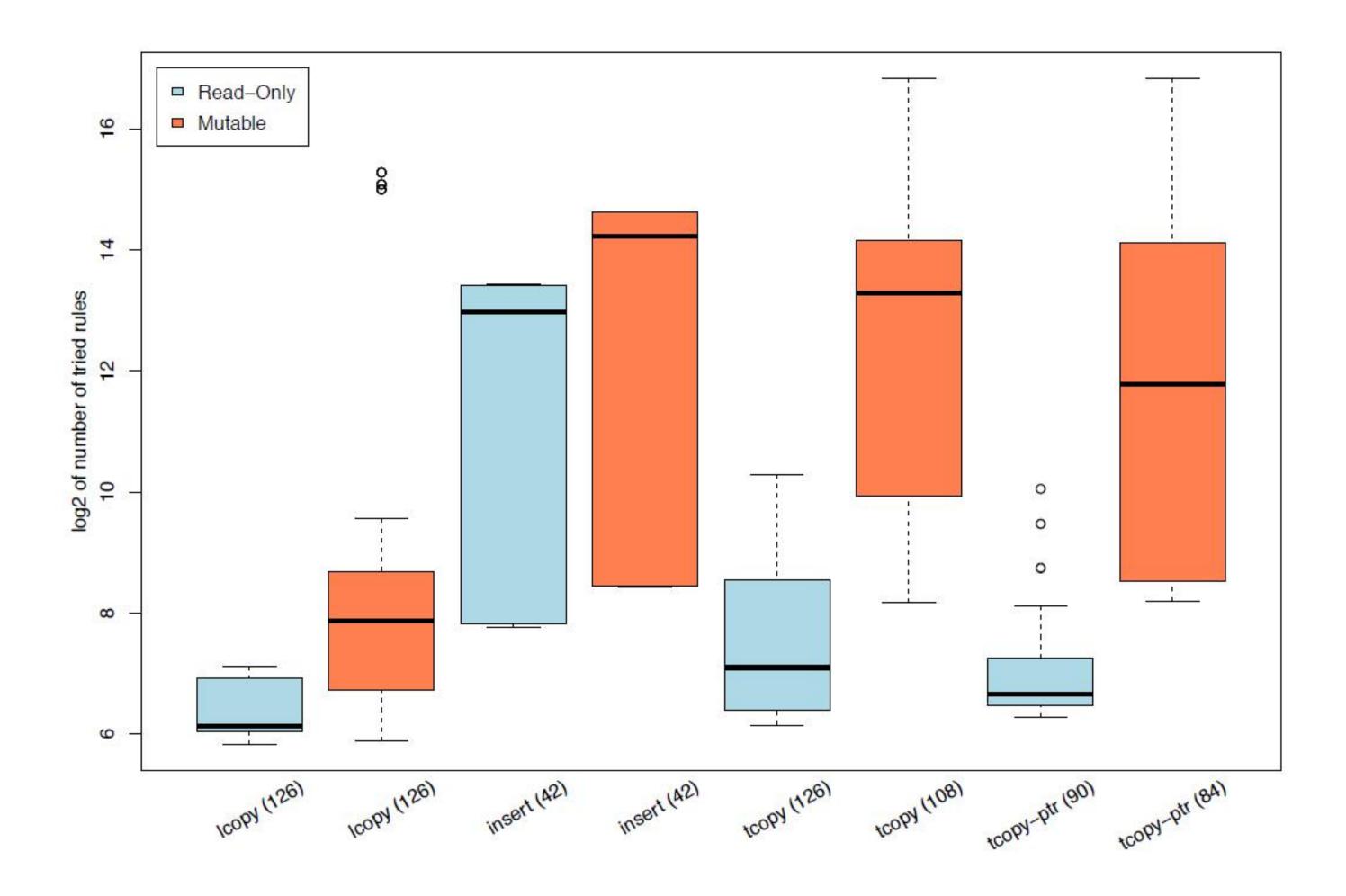


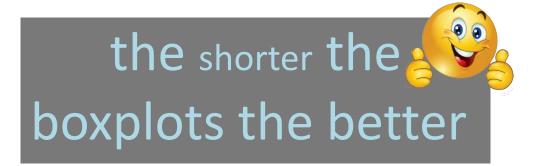
#### Robustness - serach heuristics variations

### We explored:

- 3 variants of specification
- ▶ 6 different unification orders strategies
- ▶ 7 different search strategies

### Results 3 – No of fired rules





# Synthesis of Programs with Pointers via Read-Only Specifications

(our contribution)

Effective: more natural and shorter programs

Efficient: smaller search space—faster synthesis

Robust: better performance in "worst case scenarios"

### Read-Only Specifications: Related Work

Fractional Permissions [Boyland 2003]

Chalice [Leino et al. 2009], Verifast [Jacobs et al. 2011]

Abstract permissions [Heule et al., 2013]

Viper [Muller et al. 2016]

Immutable Specifications [David et al. 2011]

Read-Only Assertions [Chargueraud et al. 2017]

Disjoint Permissions [Bach et al. 2018]



Tailored for verification, not for synthesis!

# Synthesis of Programs with Pointers via Read-Only Specifications

(our contribution)

Effective: more natural and shorter programs

Efficient: smaller search space—faster synthesis

Robust: better performance in "worst case scenarios"



### To Take-Away

Adding borrows to SSL improves the synthesis efficiency:

synthesised programs of better quality improved synthesis performance stronger correctness guarantees robust synthesis



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Bart Jacobs, Jan Smans, Pieter Philippaerts, Frederic Vogels, Willem Penninckx, and Frank Piessens. VeriFast: A Powerful, Sound, Predictable, Fast Verifier for C and Java. In NASA Formal Methods, volume 6617 of LNCS, pages 41-55. Springer, 2011.

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Arthur Chargueraud and Francois Pottier. Temporary Read-Only Permissions for Separation Logic. In ESOP, volume 10201 of LNCS, pages 260 - 286. Springer, 2017.

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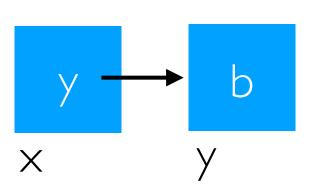
### Proof Search Algorithm

- Goal-driven, with backtracking (in CPS), trying a fixed set of rules;
- Branching: some rules emit many alternatives;
- Along with the program, emits the complete proof tree.
- Optimisations: Invertible Rules (cf. Focusing in Proof Theory),
- phased search, "Early Failure" rules

## Separation Logic

starting in a state that satisfies P, program  $\mathbf{c}$  will execute without memory errors, and upon its termination the state will satisfy  $\mathbf{Q}$ .

empty heap
singleton heap
separating conjunction
memory block
pure constraints



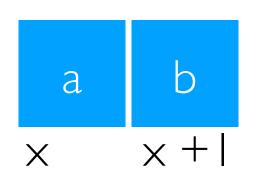
```
{ emp }

{ x \mapsto a }

{ x \mapsto y * y \mapsto b }

{ [x,2] * x \mapsto a * (x+1) \mapsto b }

{ a > 0 ; x \mapsto a }
```



do nothing
read from heap
write to heap
allocate block
free block
procedure call
sequential composition
conditional

# Read-Only Specifications: Related Work

Fractional Permissions [Boyland 2003]

Chalice [Leino et al. 2009], Verifast [Jacobs et al. 2011]

Abstract permissions [Heule et al., 2013]

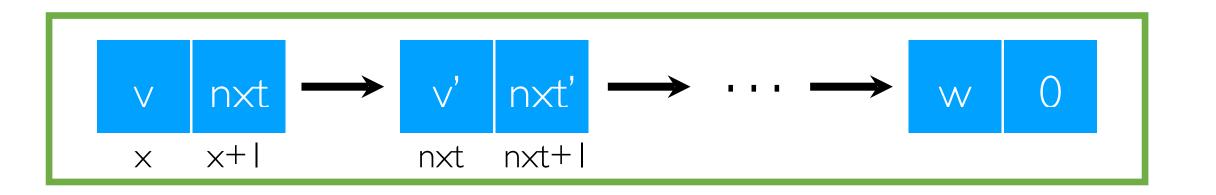
Viper [Muller et al. 2016]

Immutable Specifications [David et al. 2011]

Read-Only Assertions [Chargueraud et al. 2017]



Tailored for verification, not for synthesis!



```
{r \mapsto x * ls(x, S)}
void listcopy (loc r)
{r \mapsto y * ls(x, S) * ls(y, S)}
```



R: Add RO permissions.

```
\{r \mapsto x * ls(x, S)[RO,RO]\}
void listcopy (loc r)
\{r \mapsto y * ls(x, S)[RO,RO] * ls(y, S)[M,M]\}
```



R: Add RO permissions.

```
\{r \mapsto x * ls(x, S)[RO,RO] \}
void listcopy (loc r)
\{r \mapsto y * ls(x, S)[RO,RO] * ls(y, S)[M,M] \}
```

```
\{r \mapsto x * | s(x, S)[RO,RO] \}
void listcopy (loc r)
\{r \mapsto y * ls(x, S)[RO,RO] * ls(y, S)[M,M] \}
// ... <caller>...:
//z \mapsto x' * [s(x', S')[M,M]]
listcopy(z)
//z \mapsto y * ls(x', S')[RO,RO] * ls(y, S')[M,M]
```

```
\{r \mapsto x * ls(x, S)[a,b]\}
void listcopy (loc r)
\{r \mapsto y * ls(x, \{0\})[a,b] * ls(y, S)[M,M] \}
// .... <caller>...:
// r \mapsto x * [s(x, S)[M,M]]
listcopy(z)
// r \mapsto x * ls(x, S)[M,M]
```

# Setup

Varied the properties captured in the inductive definitions.

Applied 42 kinds of *perturbations* to stress the proof search strategy.

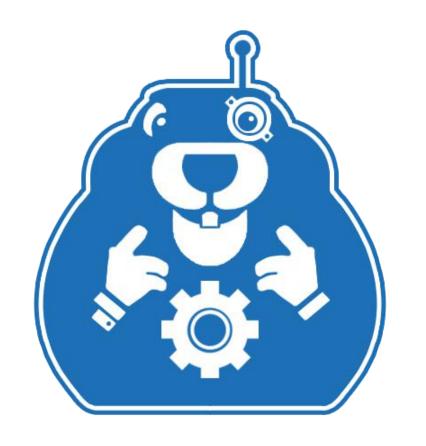
#### SuSLik -> ROBoSuSLik

https://github.com/TyGuS/suslik

https://github.com/TyGuS/suslik/tree/borrows



(Synthesis using Separation Logik)<sup>1</sup>



(Read-Only Borrows for Synthesis using Separation Logik)<sup>2</sup>

- 1. [Polikarpova & Sergey @POPL'19]
- 2. [Costea, Zhu, Polikarpova, Sergey @ESOP'20]

```
(Emp)
{emp} → {emp} | ??
```

```
(Emp)
{emp} → {emp} | skip
```

(Read)

$$\{ x \mapsto A * P \} \rightsquigarrow \{ Q \} |$$

```
\frac{[y/A]\{x\mapsto A*P\} \rightsquigarrow [y/A]\{Q\}|c}{\{x\mapsto A*P\} \rightsquigarrow \{Q\}|let y = *x; c}
(Write)
```

 $\{x \mapsto \_ * P\} \rightsquigarrow \{x \mapsto e * Q\}$ 

```
(Read)
[y/A]\{x\mapsto A*P\} \rightsquigarrow [y/A]\{Q\}\}
  \{x \mapsto A * P\} \rightsquigarrow \{Q\} \mid let y = *x; c
(Write)
          \{x \mapsto e * P\} \rightsquigarrow \{x \mapsto e * Q\} \mid c
    \{x \mapsto *P\} \rightsquigarrow \{x \mapsto e *Q\} \mid *x = e; c
```

$$\frac{\{P\} \rightsquigarrow \{Q\} | c}{\{P*R\} \rightsquigarrow \{Q*R\} | c}$$
 (UnifyHeaps) 
$$[\sigma]R' = R \quad \{P*R\} \rightsquigarrow [\sigma]\{Q*R'\} | c}$$
 
$$\{P*R\} \rightsquigarrow \{Q*R'\} | c$$

void pick(loc x, loc y)

$$\{ \times \mapsto a * y \mapsto b \} \implies \{ \times \mapsto z * y \mapsto z \}$$
??

$$\sigma = [a2/z] \quad \{ \times \mapsto a2 * y \mapsto b2 \} \quad \Rightarrow \quad \{ \times \mapsto a2 * y \mapsto a2 \} \quad | \quad ??$$

$$= \{ \times \mapsto a2 * y \mapsto b2 \} \quad \Rightarrow \quad \{ \times \mapsto z * y \mapsto z \} \quad | \quad ??$$

$$= \{ \times \mapsto a2 * y \mapsto b \} \quad \Rightarrow \quad \{ \times \mapsto z * y \mapsto z \} \quad | \quad \text{let b2} = *y; \quad ??$$

$$= \{ \times \mapsto a * y \mapsto b \} \quad \Rightarrow \quad \{ \times \mapsto z * y \mapsto z \} \quad | \quad \text{let a2} = *x; \quad ??$$
(Read)
$$= \{ \times \mapsto a * y \mapsto b \} \quad \Rightarrow \quad \{ \times \mapsto z * y \mapsto z \} \quad | \quad \text{let a2} = *x; \quad ??$$

```
\{y \mapsto a2\} \Rightarrow \{y \mapsto a2\} ??
                                                                                                   (Write)
                                  \{y \mapsto b2\} \implies \{y \mapsto a2\} \mid *y = a2; ??
                                                                                                            (Frame)
\sigma = [a2/z] \quad \{x \mapsto a2 * y \mapsto b2\} \quad \Rightarrow \quad \{x \mapsto a2 * y \mapsto a2\} \quad | \quad ??
                                                                                                                 (UnifyHeaps)
       \{ \times \mapsto a2 * y \mapsto b2 \} \Rightarrow \{ \times \mapsto z * y \mapsto z \}
                                                                                          ??
                                                                                                                 (Read)
   \{x \mapsto a2 * y \mapsto b\} \Rightarrow \{x \mapsto z * y \mapsto z\} let b2 = *y; ??
                                                                                                                  (Read)
     \{ \times \mapsto a * y \mapsto b \} \implies \{ \times \mapsto z * y \mapsto z \} \qquad \text{let a2} = *x; ??
```

```
(Emp)
                                       {emp} ~ {emp} | skip
                                                                                          (Frame)
                                      \{y \mapsto a2\} \Rightarrow \{y \mapsto a2\} ??
                                                                                              (Write)
                                \{y \mapsto b2\} \implies \{y \mapsto a2\} \mid *y = a2; ??
                                                                                                       (Frame)
\sigma = [a2/z] \quad \{x \mapsto a2 * y \mapsto b2\} \quad \Rightarrow \quad \{x \mapsto a2 * y \mapsto a2\} \quad | \quad ??
                                                                                                           (UnifyHeaps)
       \{x \mapsto a2 * y \mapsto b2\} \Rightarrow \{x \mapsto z * y \mapsto z\}
                                                                                      ??
                                                                                                           (Read)
   \{ \times \mapsto a2 * y \mapsto b \} \Rightarrow \{ \times \mapsto z * y \mapsto z \} let b2 = *y; ??
                                                                                                            (Read)
     \{ \times \mapsto a * y \mapsto b \} \implies \{ \times \mapsto z * y \mapsto z \} \qquad \text{let a2} = *x; ??
```

```
(Emp)
                                        {emp} ~ {emp} | skip
                                                                                           (Frame)
                                       \{y \mapsto a2\} \Rightarrow \{y \mapsto a2\} ??
                                                                                                (Write)
                                 \{ y \mapsto b2 \} \implies \{ y \mapsto a2 \} \mid *y = a2; ??
                                                                                                         (Frame)
\sigma = [a2/z] \quad \{x \mapsto a2 * y \mapsto b2\} \quad \Rightarrow \quad \{x \mapsto a2 * y \mapsto a2\} \quad | \quad ??
                                                                                                             (UnifyHeaps)
       \{x \mapsto a2 * y \mapsto b2\} \Rightarrow \{x \mapsto z * y \mapsto z\}
                                                                                       ??
                                                                                                             (Read)
   \{ \times \mapsto a2 * y \mapsto b \} \implies \{ \times \mapsto z * y \mapsto z \} \qquad | let b2 = *y; ??
                                                                                                              (Read)
                                                                       let a2 = *x; ??
     \{ \times \mapsto a * \vee \mapsto b \} \rightsquigarrow \{ \times \mapsto z * \vee \mapsto z \}
```