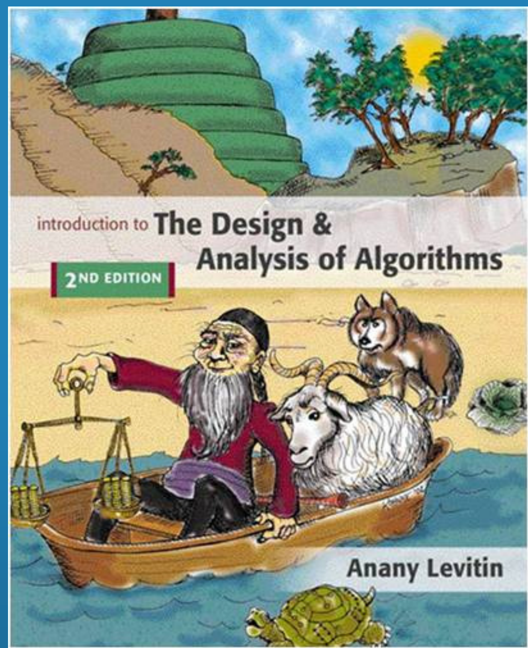
# Chapter 7

## **Space and Time Tradeoffs**




PEARSON
Addison Wesley

# Space-for-time tradeoffs

**Two varieties of space-for-time algorithms:**

- ***input enhancement*** **— preprocess the input (or its part) to store some info to be used later in solving the problem**
  - **counting sorts**
  - **string searching algorithms**

- ***prestructuring*** **— preprocess the input to make accessing its elements easier**
  - **hashing**
  - **indexing schemes (e.g., B-trees)**

# Review: String searching by brute force

*pattern*: a string of *m* characters to search for

*text*: a (long) string of *n* characters to search in

***Brute force algorithm***

**Step 1** Align pattern at beginning of text

**Step 2** Moving from left to right, compare each character of pattern to the corresponding character in text until either all characters are found to match (successful search) or a mismatch is detected

**Step 3** While a mismatch is detected and the text is not yet exhausted, realign pattern one position to the right and repeat Step 2

# Brute-force string matching

```
for (i=0; T[i] != '\0'; i++)
{
for (j=0; T[i+j] != '\0' && P[j] != '\0' && T[i+j]==P[j]; j++) ;
if (P[j] == '\0') found a match
}
```

# String searching by preprocessing

Several string searching algorithms are based on the input enhancement idea of preprocessing the pattern

- Knuth-Morris-Pratt (KMP) algorithm preprocesses pattern left to right to get useful information for later searching

- Boyer -Moore algorithm preprocesses pattern right to left and store information into two tables

- Horspool's algorithm simplifies the Boyer-Moore algorithm by using just one table

# REF: Knuth-Morris-Pratt (KMP)

*http://www.ics.uci.edu/~eppstein/161/960227.html*

| | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| T: | b | a | n | a | n | a | n | o | b | a | n | o |
| i=0: | X | | | | | | | | | | | |
| i=1: | | X | | | | | | | | | | |
| i=2: | | | n | a | n | X | | | | | | |
| i=3: | | | | X | | | | | | | | |
| i=4: | | | | | n | a | n | o | | | | |
| i=5: | | | | | | X | | | | | | |
| i=6: | | | | | | | n | X | | | | |
| i=7: | | | | | | | | X | | | | |
| i=8: | | | | | | | | | X | | | |
| i=9: | | | | | | | | | | n | X | |
| i=10: | | | | | | | | | | | X | |

# KMP

- String matching with skipped outer &inner iterations: KMP, version 1:

```
i=0;
o=0;
while (i<n)
{
for (j=o; T[i+j] != '\0' && P[j] != '\0' && T[i+j]==P[j]; j++) ;
if (P[j] == '\0') found a match;
o = overlap(P[0..j-1],P[0..m]);
i = i + max(1, j-o);
}
```

# KPM

- **KMP, version 2:**

```
j = 0;
for (i = 0; i < n; i++)
for (;;) {      // loop until break
    if (T[i] == P[j]) { // matches?
    j++;        // yes, move on to next state
    if (j == m) {   // maybe that was the last state
        found a match;
        j = overlap[j];
    }
    break;
    } else if (j == 0) break;   // no match in state j=0, give up
    else j = overlap[j];    // try shorter partial match
}
```

# KPM

- KMP overlap computation:

```
overlap[0] = -1;
for (int i = 0; pattern[i] != '\0'; i++) {
overlap[i + 1] = overlap[i] + 1;
while (overlap[i + 1] > 0 &&
      pattern[i] != pattern[overlap[i + 1] - 1])
   overlap[i + 1] = overlap[overlap[i + 1] - 1] + 1;
}
return overlap;
```

Overlap: O(m)

Scan: O(n) time

Total: O(m+n).

# Horspool's Algorithm

A simplified version of Boyer-Moore algorithm:

- preprocesses pattern to generate a shift table that determines how much to shift the pattern when a mismatch occurs

- always makes a shift based on the text's character *c* aligned with the last character in the pattern according to the shift table's entry for *c*

# How far to shift?

**Look at first (rightmost) character in text that was compared:**

- **The character is not in the pattern**

  `.....c......................` (`c` not in pattern)

  `BAOBAB`

- **The character is in the pattern (but not the rightmost)**

  `.....O......................` (`O` occurs once in pattern)

  `BAOBAB`

  `.....A......................` (`A` occurs twice in pattern)

  `BAOBAB`

- **The rightmost characters do match**

  `.....B......................`

  `BAOBAB`

# Shift table

- Shift sizes can be precomputed by the formula

  distance from *c*'s rightmost occurrence in pattern among its first *m*-1 characters to its right end

  $t(c)$ =

  pattern's length *m*, otherwise

  by scanning pattern before search begins and stored in a table called *shift table*

- Shift table is indexed by text and pattern alphabet
  Eg, for **BAOBAB** :

| A | B | C | D | E | F | G | H | I | J | K | L | M | N | O | P | Q | R | S | T | U | V | W | X | Y | Z |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 2 | 6 | 6 | 6 | 6 | 6 | 6 | 6 | 6 | 6 | 6 | 6 | 6 | 3 | 6 | 6 | 6 | 6 | 6 | 6 | 6 | 6 | 6 | 6 | 6 |

# Example of Horspool's alg. application

| A | B | C | D | E | F | G | H | I | J | K | L | M | N | O | P | Q | R | S | T | U | V | W | X | Y | Z | _ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 2 | 6 | 6 | 6 | 6 | 6 | 6 | 6 | 6 | 6 | 6 | 6 | 6 | 3 | 6 | 6 | 6 | 6 | 6 | 6 | 6 | 6 | 6 | 6 | 6 | 6 |

```
BARD LOVED BANANAS
BAOBAB (L=6)
      BAOBAB (B=2)
        BAOBAB (N=6)
              BAOBAB (unsuccessful search)
```

PINGADO em

ALMOCEI PINGA COM LINGUADO PINGADO

- PINGADO  outras
- 6543217  7

- 1234567890123456789012345678901234
- ALMOCEI PINGA COM LINGUADO PINGADO
- PINGADO (I=5)
- -----PINGADO (^G=3)
- ---PINGADO (^C=7)
- -------PINGADO (^G=3)
- ---PINGADO (^D=1)
- -PINGADO (^=ADO, U)
- PINGADO (^U=7)
- -------PINGADO (^D=1)
- -PINGADO (=PINGADO)

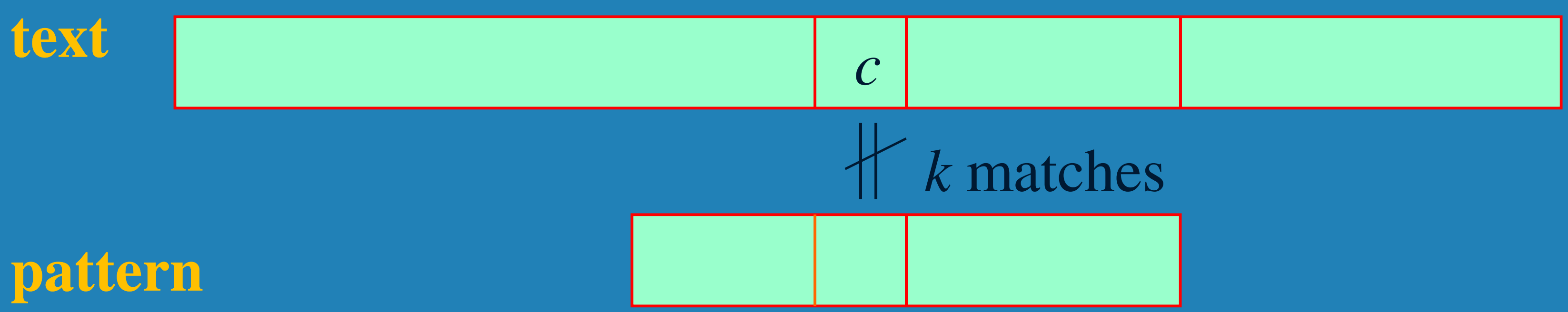
# Boyer-Moore algorithm

**Based on same two ideas:**

- **comparing pattern characters to text from right to left**
- **precomputing shift sizes in two tables**
  - ***bad-symbol table*** **indicates how much to shift based on text's character causing a mismatch**
  - ***good-suffix table*** **indicates how much to shift based on matched part (suffix) of the pattern**

# Bad-symbol shift in Boyer-Moore algorithm

- If the rightmost character of the pattern **doesn't match**, BM algorithm acts as Horspool's
- If the rightmost character of the pattern **does match,** BM compares **preceding characters** right to left until either all pattern's characters match or a mismatch on text's character $c$ is encountered after $k > 0$ matches




bad-symbol shift $d_1 = \max\{t_1(c) - k, 1\}$

# Good-suffix shift in Boyer-Moore algorithm

- Good-suffix shift $d_2$ is applied after $0 < k < m$ last characters were matched

- $d_2(k)$ = the distance between matched suffix of size $k$ and its rightmost occurrence in the pattern that is not preceded by the same character as the suffix

  Example: `CABABA` $d_2(1) = 4$

- If there is no such occurrence, match the longest part of the $k$-character suffix with corresponding prefix;
  if there are no such suffix-prefix matches, $d_2(k) = m$

  Example: `WOWWOW` $d_2(2) = 5$, $d_2(3) = 3$, $d_2(4) = 3$, $d_2(5) = 3$

# Boyer-Moore Algorithm

After matching successfully $0 < k < m$ characters, the algorithm shifts the pattern right by

$$d = \max\{d_1, d_2\}$$

where $d_1 = \max\{t_1(c) - k, 1\}$ is bad-symbol shift

$d_2(k)$ is good-suffix shift

Example: Find pattern `AT_THAT` in

`WHICH_FINALLY_HALTS. _ _AT_THAT`

# Boyer-Moore Algorithm (cont.)

**Step 1** Fill in the bad-symbol shift table

**Step 2** Fill in the good-suffix shift table

**Step 3** Align the pattern against the beginning of the text

**Step 4** Repeat until a matching substring is found or text ends:

Compare the corresponding characters right to left.

If no characters match, retrieve entry $t_1(c)$ from the bad-symbol table for the text's character $c$ causing the mismatch and shift the pattern to the right by $t_1(c)$.
If $0 < k < m$ characters are matched, retrieve entry $t_1(c)$ from the bad-symbol table for the text's character $c$ causing the mismatch and entry $d_2(k)$ from the good-suffix table and shift the pattern to the right by

$$d = \max\{d_1, d_2\}$$

where $d_1 = \max\{t_1(c) - k, 1\}$.

# Example of Boyer-Moore alg. application

| A | B | C | D | E | F | G | H | I | J | K | L | M | N | O | P | Q | R | S | T | U | V | W | X | Y | Z | _ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 2 | 6 | 6 | 6 | 6 | 6 | 6 | 6 | 6 | 6 | 6 | 6 | 6 | 3 | 6 | 6 | 6 | 6 | 6 | 6 | 6 | 6 | 6 | 6 | 6 | 6 |

B E S S _ K N E W _ A B O U T _ B A O B A B S

B A O B A B

$d_1 = t_1(K) = 6$    B A O B A B

$d_1 = t_1(\_)-2 = 4$

$\underline{d_2(2) = 5}$

*max{d1,d2}*

B A O B A B

$\underline{d_1 = t_1(\_)-1 = 5}$

$d_2(1) = 2$

B A O B A B (success)

| $k$ | pattern | $d_2$ |
|---|---|---|
| 1 | BAOB**AB** | 2 |
| 2 | **B**AO**BAB** | 5 |
| 3 | **B**AO**BAB** | 5 |
| 4 | **B**A**OBAB** | 5 |
| 5 | **BAOBAB** | 5 |

# Hashing

- A very efficient method for implementing a *dictionary*, i.e., a set with the operations:
  - find
  - insert
  - delete

- Based on representation-change and space-for-time tradeoff ideas

- Important applications:
  - symbol tables
  - databases (*extendible hashing*)

# Hash tables and hash functions

The idea of *hashing* is to map keys of a given file of size $n$ into a table of size $m$, called the *hash table*, by using a predefined function, called the *hash function*,

$$h: K \rightarrow \text{ location (cell) in the hash table}$$

Example: student records, key = SSN. Hash function:
$h(K) = K \bmod m$ where $m$ is some integer (typically, prime)
If $m = 1000$, where is record with SSN= 314159265 stored?

Generally, a hash function should:

- be easy to compute
- distribute keys about evenly throughout the hash table

# Collisions

If $h(K_1) = h(K_2)$, there is a *collision*

- Good hash functions result in fewer collisions but some collisions should be expected (*birthday paradox*)

- Two principal hashing schemes handle collisions differently:
  - *Open hashing*
    – each cell is a header of linked list of all keys hashed to it
  - *Closed hashing*
    - one key per cell
    - in case of collision, finds another cell by
      - *linear probing:* use next free bucket
      - *double hashing:* use second hash function to compute increment

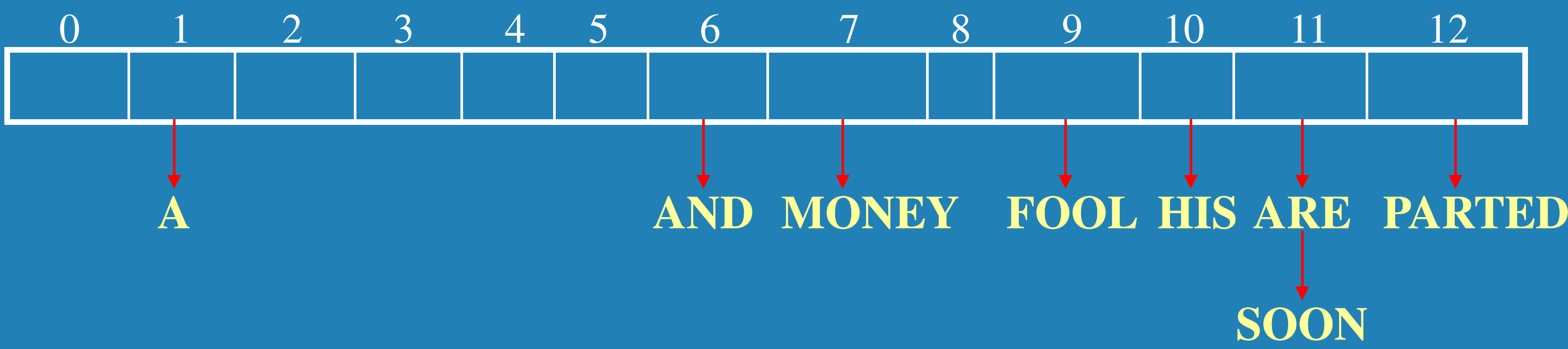
# Open hashing (Separate chaining)

**Keys are stored in linked lists <u>outside</u> a hash table whose elements serve as the lists' headers.**

**Example:** A, FOOL, AND, HIS, MONEY, ARE, SOON, PARTED

***h*(*K*) = sum of *K* 's letters' positions in the alphabet MOD 13**

| Key | A | FOOL | AND | HIS | MONEY | ARE | SOON | PARTED |
|---|---|---|---|---|---|---|---|---|
| *h*(*K*) | 1 | 9 | 6 | 10 | 7 | 11 | 11 | 12 |




**Example:** Search for KID

# Open hashing (cont.)

- If hash function distributes keys uniformly, average length of linked list will be $\alpha = n/m$. This ratio is called *load factor*.
  - file of size $n$ into a table of size $m$

- Average number of probes in successful, $S$, and unsuccessful searches, $U$:

$$S \approx 1+\alpha/2, \qquad U = \alpha$$

- Load $\alpha$ is typically kept small (ideally, about 1)

- Open hashing still works if $n > m$

# Closed hashing (Open addressing)

**Keys are stored <u>inside</u> a hash table.**

| **Key** | **A** | **FOOL** | **AND** | **HIS** | **MONEY** | **ARE** | **SOON** | **PARTED** |
|---|---|---|---|---|---|---|---|---|
| ***h*(*K*)** | **1** | **9** | **6** | **10** | **7** | **11** | **11** | **12** |

| 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | **A** | | | | | | | | | | | |
| | **A** | | | | | | | | **FOOL** | | | |
| | **A** | | | | | **AND** | | | **FOOL** | | | |
| | **A** | | | | | **AND** | | | **FOOL** | **HIS** | | |
| | **A** | | | | | **AND** | **MONEY** | | **FOOL** | **HIS** | | |
| | **A** | | | | | **AND** | **MONEY** | | **FOOL** | **HIS** | **ARE** | |
| | **A** | | | | | **AND** | **MONEY** | | **FOOL** | **HIS** | **ARE** | **SOON** |
| **PARTED** | **A** | | | | | **AND** | **MONEY** | | **FOOL** | **HIS** | **ARE** | **SOON** |

# Closed hashing (cont.)

- Does not work if $n > m$
  - file of size $n$ into a table of size $m$
- Avoids pointers
- Deletions are *not* straightforward
- Number of probes to find/insert/delete a key depends on load factor $\alpha = n/m$ (hash table density) and collision resolution strategy. For linear probing:

$$S = (½)\,(1+ 1/(1- \alpha)) \quad \text{and} \quad U = (½)\,(1+ 1/(1- \alpha)^2)$$

- As the table gets filled ($\alpha$ approaches 1), number of probes in linear probing increases dramatically:

| $\alpha$ | $\frac{1}{2}(1+\frac{1}{1-\alpha})$ | $\frac{1}{2}(1+\frac{1}{(1-\alpha)^2})$ |
|---|---|---|
| 50% | 1.5 | 2.5 |
| 75% | 2.5 | 8.5 |
| 90% | 5.5 | 50.5 |

# indexing schemes (e.g., B-trees)

- Idea
- Example
- Height
- B* : redistribuição
- B+ : chaves nas folhas