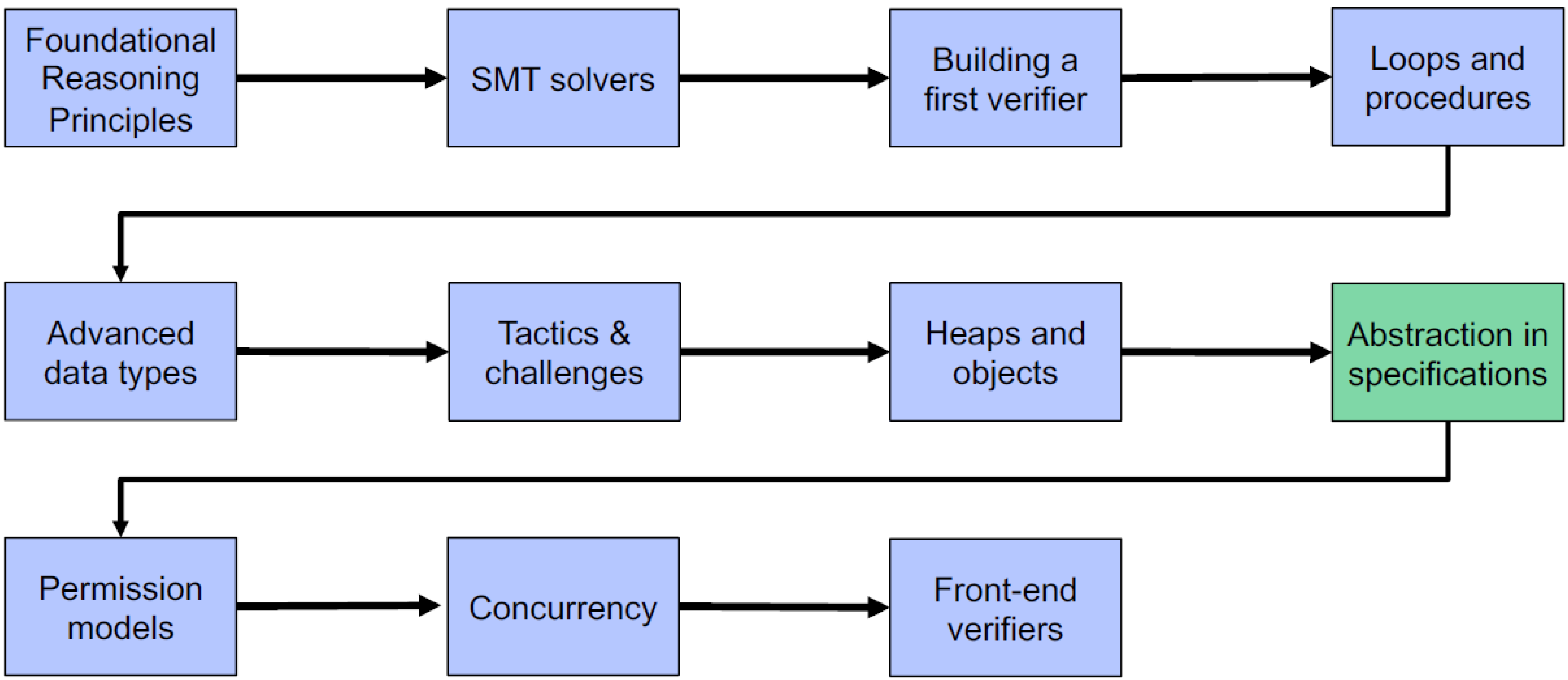
Methodologies for Software Processes

Lecture 10

ABSTRACTION

Tentative course outline



Challenges revisited

Heap data structures pose three major challenges for sequential verification

- Reasoning about aliasing
  - Permissions and separating conjunction
- Framing, especially for dynamic data structures
  - Sound frame rule, but no support yet for unbounded data structures
- Writing specifications that preserve information hiding
  - Not solved

And additional challenges for concurrent programs, e.g., data races

- Permissions are an excellent basis, but see later

Running example: linked lists

➔ 00-lists.vpr

```
field elem: Int
field next: Ref

method head(this: Ref) returns (res: Int)
  requires acc(this.elem)
  ensures  acc(this.elem)
  ensures  res == this.elem
{
  res := this.elem
}
```

```
method append(this: Ref, e: Int)
  requires // permission to all nodes
  ensures  // list was extended
{
  if(this.next == null) {
    var n: Ref
    n := new(*)
    n.next := null
    this.elem := e
    this.next := n
  } else {
    append(this.next, e)
  }
}
```

- Specification reveals implementation details
- Permissions and behavior cannot be expressed so far

Abstraction

1. Predicates: abstraction over permissions

2. Representation invariants

3. Data abstraction: abstraction over values

4. Abstraction functions

User-defined predicates

- User-defined predicates consist of a predicate name, a list of parameters, and a self-framing assertion

Declarations

$\texttt{D ::= ... | } \textbf{predicate}\texttt{ U(}\overline{\texttt{x:T}}\texttt{) \{ P \}}$

```
predicate node(this: Ref) {
  acc(this.elem) && acc(this.next)
}
```

- Predicate instances are predicates

Predicates (or assertions)

$\texttt{P ::= ... | U(}\overline{\texttt{E}}\texttt{)}$

```
method head(this: Ref) returns (res: Int)
  requires node(this)
  ensures  node(this)
{ … }
```

Recursive predicates

→ `01-lists.vpr`

- Predicate definitions may be recursive

Declarations

`D ::= ... | predicate U(`$\overline{x{:}T}$`) { P }`

Predicates (or assertions)

`P ::= ... | U(`$\bar{E}$`)`

- Recursive predicate definitions are interpreted as least fixed points
- All instances of the predicate have finite unfoldings

- Recursive predicates may denote a statically-unbounded number of permissions

```
predicate list(this: Ref) {
  acc(this.elem) && acc(this.next) &&
  (this.next != null ==> list(this.next))
}
```

- If `list(x)` holds, we have `x!=x.next`
- `list` describes a finite linked list

Static verification with recursive predicates

- A program verifier cannot know statically how far to unfold recursive definitions

```
predicate list(this: Ref) {
  acc(this.next) &&
  (this.next != null ==> list(this.next))
}
```

```
inhale list(x)
y.next := null   // do we have permission?
```

- Proving properties of predicates often requires induction proofs
  ➔ difficult to automate

```
predicate step(this: Ref) {
  acc(this.next) &&
  (this.next != null ==> acc(this.next.next)
    (this.next.next != null ==>
      step(this.next.next)
  )
}
```

```
inhale list(x)
exhale step(x)   // should this succeed?
```

```
inhale step(x)
exhale list(x)   // should this succeed?
```

Iso-recursive predicates

- We interpret (non-limited) functions equi-recursively: function = body

- Iso-recursive semantics distinguishes between a predicate instance and its body

```
predicate list(this: Ref) {
  acc(this.elem) && acc(this.next) &&
  (this.next != null ==> list(this.next))
}
```

```
inhale list(x)
x.next := null  // no permission
```

- Intuition: permissions are held by method executions, loop iterations,
  or predicate instances

Folding and unfolding predicates

➔ `02-lists.vpr`

- Exchanging a predicate instance for its body, and vice versa, is done via extra program statements

Statements

```
S ::= ...
    | unfold U(Ē)
    | fold U(Ē)
```

```
predicate list(this: Ref) {
  acc(this.elem) && acc(this.next) &&
  (this.next != null ==> list(this.next))
}
```

- An unfold statement exchanges a predicate instance for its body

```
inhale list(x)
unfold list(x)
x.next := null
```

- A fold statement exchanges a predicate body for a predicate instance

```
inhale list(x)
unfold list(x)
x.next := null
fold list(x)
exhale list(x)
```

Iterative data structure traversals

➔ 04-length.vpr

```
method length(this: Ref) returns (res: Int)
  requires list(this)
  ensures  list(this)
{
  res := 0
  var curr: Ref := this
  unfold list(this)
  while(curr.next != null)
    invariant acc(curr.next)
    invariant (curr.next != null ==> list(curr.next))
  {
    res := res + 1
    curr := curr.next
    unfold list(curr)
  }
}
```

- Postcondition fails
- Loop invariant includes permissions to the list nodes still to be visited …
- … but not to the list nodes already visited
- Entire list is transferred into the loop, but not back

List segments

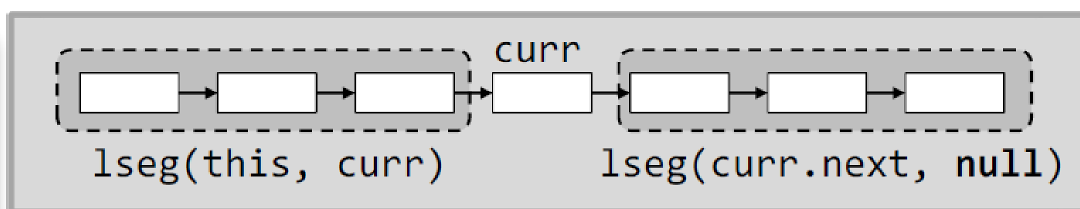
- Describing the “list nodes already visited” requires a way to describe a partial list

- The `lseg` predicate describes (possibly empty) list segments

```
predicate lseg(this: Ref, last: Ref) {
  this != last ==> acc(this.next) &&
                   lseg(this.next, last)
}
```

- Our loop invariant can now include the nodes still to be visited as well as the nodes already visited

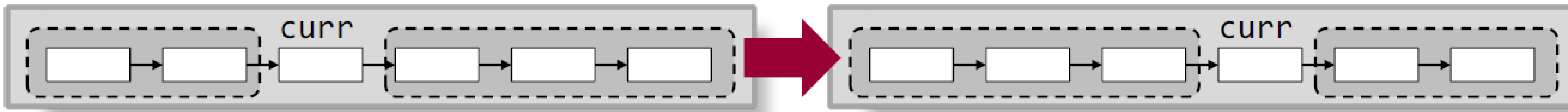
```
while(curr.next != null)
  invariant acc(curr.next)
  invariant lseg(curr.next, null)
  invariant lseg(this, curr)
```



Manipulating recursive definitions

→ `05-addAtEnd.vpr`

- Advancing the `curr` pointer requires extending an `lseg` predicate at the end




```
method addAtEnd(this: Ref, n: Ref, nn: Ref)
  requires lseg(this, n)
  requires acc(n.next) && n.next == nn
  ensures  lseg(this, nn)
{
  if(this == n) {
    fold lseg(nn, nn)  // empty segment
    fold lseg(this, nn)
  } else {
    unfold lseg(this, n)
    addAtEnd(this.next, n, nn)
    fold lseg(this, nn)
  }
}
```

- This requires
  - unfolding the recursive predicate instances all the way to the end
  - folding predicate instances for the extended list segment in the reverse order
- Operation is best implemented as a recursive auxiliary method

Iterative data structure traversals revisited

➔ 06-length.vpr

```
method length(this: Ref) returns (res: Int)
  requires this != null && lseg(this, null)
  ensures  lseg(this, null)
{
  res := 0
  var curr: Ref := this
  unfold lseg(this, null)
  fold lseg(this, curr)  // empty segment to establish loop invariant
  while(curr.next != null)
    invariant acc(curr.next) && lseg(curr.next, null) && lseg(this, curr)
  {
    res := res + 1
    var tmp: Ref; tmp := curr.next  // read here, before permission is folded
    addAtEnd(this, curr, curr.next) // extend predicate
    curr := tmp
    unfold lseg(curr, null)
  }
  addAtEnd(this, curr, null)
}
```

Reminder: ghost code

- Many operations of our example are needed for verification, but not for the execution of the code
  - Fold and unfold operations
  - Entire `addAtEnd` method and calls

```
var tmp: Ref; tmp := curr.next
addAtEnd(this, curr, curr.next)
curr := tmp
unfold lseg(curr, null)
```

- Code that is used for verification only is called ghost code
  - Statements
  - Entire methods
  - Variables, fields, functions, ...

- General rule for ghost code
  The execution of ghost code must not affect the behavior of regular code

- Examples
  - Ghost variables must not occur in conditions of regular conditionals and loops
  - Ghost statements must not assign to regular variables

- Viper does not distinguish regular and ghost code and, thus, does *not* enforce this rule

Abstract predicates

- Predicate bodies reveal implementation details

```
predicate list(this: Ref) {
  acc(this.elem) && acc(this.next) &&
  (this.next != null ==> list(this.next))
}
```

- Predicates can also remain abstract

```
predicate list(this: Ref)
```

- Abstract predicates cannot be folded or unfolded

- Abstract predicates and abstract methods allow one to specify interfaces without revealing implementation details

```
predicate list(this: Ref)

method append(this: Ref, e: Int)
  requires list(this)
  ensures  list(this)
```

- Predicate and method definitions are visible to implementer, but not clients

Encoding of predicates

- Recall that permissions are tracked in a global permission mask

```
type MaskType = Map<T>[(Ref, Field T), Bool]
var Mask: MaskType
```

- We use the same mask to track predicate instances

- For this purpose, we map each predicate instance to a field

```
predicate lseg(this: Ref, last: Ref)
```

```
function
lsegField(this: Ref, last: Ref): Field Int
```

and use that field to index the mask

```
lseg(a, b)
```

```
Mask[null, lsegField(a, b)]
```

- The uninterpreted functions are axiomatized to return unique values

Encoding of unfold and fold

- An unfold statement exchanges a predicate instance for its body

```
unfold U(Ē)
```

```
exhale U(Ē)
inhale body(U(Ē))
```

- A fold statement exchanges a predicate body for a predicate instance

```
fold U(Ē)
```

```
exhale body(U(Ē))
inhale U(Ē)
```

- Actual implementation of fold delays havoc until the predicate instance is exhaled

```
x.next := null
fold list(x)
unfold list(x)
assert x.next == null
```

Abstraction

1. Predicates: abstraction over permissions

2. Representation invariants

3. Data abstraction: abstraction over values

4. Abstraction functions

Representation invariants

→ 08-list.vpr
→ 09-field-len.vpr

- Data structures typically maintain several consistency conditions
  - Value constraints, e.g., references being non-null or integers being positive
  - Structural constraints, e.g., a tree being balanced
- Such representation invariants are
  - Established by constructors
  - Assumed and preserved by all operations
- Representation invariants can be expressed as part of a predicate

```
predicate list(this: Ref) {
  acc(this.elem) && acc(this.next) &&
  (this.next != null ==> list(this.next) &&
                         0 <= this.elem)
}
```

```
method append(this: Ref, e: Int)
  requires list(this)
  ensures  list(this)
{
  unfold list(this)  // assume invariant
  …
  fold list(this)    // check invariant
}
```

Unfolding-expressions

→ `09-field-len.vpr`

- Unfold and fold are statements because they change the state (heap and mask)

- Unfolding-expressions allow one to temporarily unfold a predicate during the evaluation of an expression

Expressions

```
E ::= ... | unfolding U(Ē)
```

- They enable inspecting fields whose permissions are folded inside a predicate

```
predicate list(this: Ref) {
  acc(this.elem) && acc(this.next) && acc(this.len) &&
  (this.next == null ==> this.len == 0) &&
  (this.next != null ==> list(this.next) &&
      unfolding list(this.next) in this.len == this.next.len + 1)
}
```

Abstraction

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Specifying functional behavior

→ `11-list-fun-spec.vpr`

- Using `old`-expressions and `unfolding`-expressions, we can specify some aspects of functional behavior

- But: approach does not work when behavior depends on an unbounded number of fields (e.g., sorting a list)

- And: specifications reveal implementation details

```
predicate list(this: Ref) {
  acc(this.next) && acc(this.len) &&
  (this.next == null ==> this.len == 0) &&
  (this.next != null ==> list(this.next) &&
      unfolding list(this.next) in
      this.len == this.next.len + 1)
}
```

```
method append(this: Ref, e: Int)
  requires list(this)
  ensures  list(this)
  ensures  (unfolding list(this) in this.len) ==
        old(unfolding list(this) in this.len + 1)
```

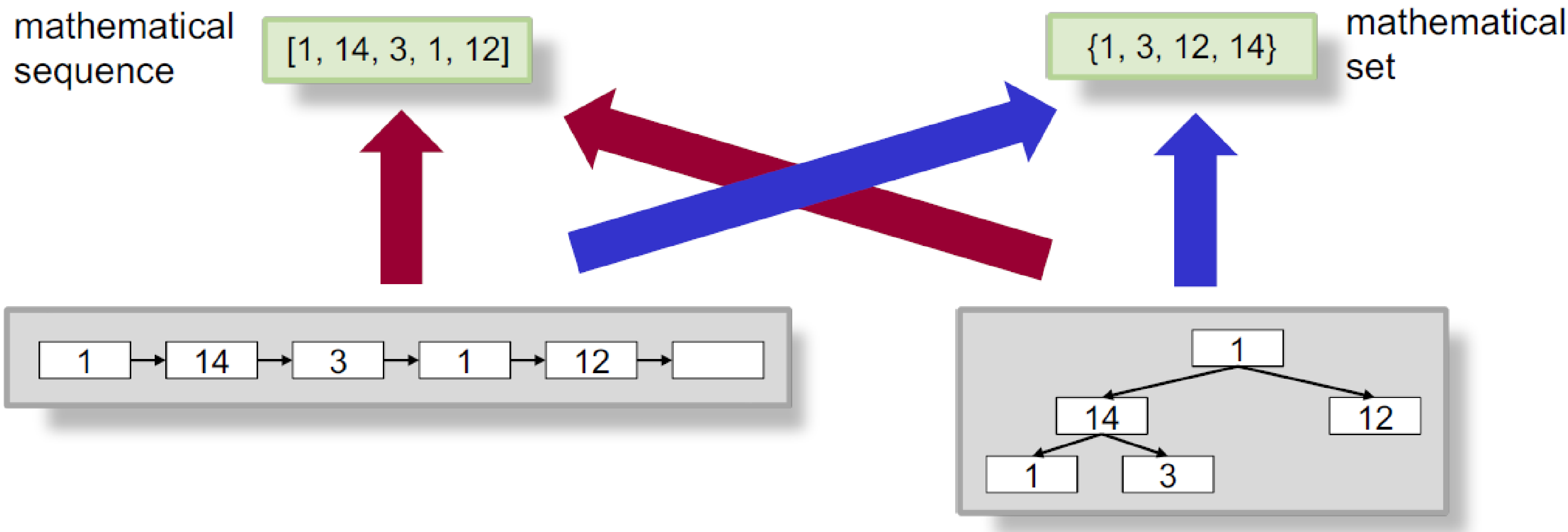
Challenges revisited

Heap data structures pose three major challenges for sequential verification

- Reasoning about aliasing
  - Permissions and separating conjunction
- Framing, especially for dynamic data structures
  - Sound frame rule, predicates
- Writing specifications that preserve information hiding
  - Not solved

Data abstraction

- To write implementation-independent specifications, we map the concrete data structure to mathematical concepts and specify the behavior in terms of those



- Mapping can be a function or a relation

Data abstraction

→ 12-data-abstraction.vpr

- One data abstraction approach is to store the abstract value in a ghost field and express the mapping as representation invariant

```
field content: Seq[Int]  // ghost field

predicate list(this: Ref) {
  acc(this.elem) && acc(this.next) && acc(this.content) &&
  (this.next == null ==> this.content == Seq[Int]()) &&
  (this.next != null ==> list(this.next) &&
                         unfolding list(this.next) in
                         this.content == Seq(this.elem) ++ this.next.content
  )
}
```

Data abstraction (cont'd)

→ 13-data-abstraction.vpr

```
method create() returns (res: Ref)
    ensures  list(res)
    ensures  (unfolding list(res) in res.content) == Seq[Int]()
{
    res := new(*)
    res.next := null
    res.content := Seq[Int]()
    fold list(res)
}
```

- Since the abstract value is stored in a ghost field, mutation of the data structure requires explicit updates of these ghost fields

- Specifications need to unfold predicates to get access to the ghost field

- Information hiding is violated (predicates cannot be abstract)

Data abstraction via predicate arguments

→ 14-pred-arg.vpr

- An alternative is to pass the abstract value as parameter to the predicate
- A representation invariant expresses the mapping between the argument and the data structure

```
predicate list(this: Ref, content: Seq[Int]) {
  acc(this.elem) && acc(this.next) &&
  (this.next == null ==> content == Seq[Int]()) &&
  (this.next != null ==> 0 < |content| &&
                         content[0] == this.elem &&
                         list(this.next, content[1..]))
}
```

Data abstraction via predicate arguments (cont'd)

```
method append(this: Ref, e: Int, c: Seq[Int])
  requires list(this, c)
  ensures  list(this, c ++ Seq(e))
{
  unfold list(this, c)
  if(this.next == null) {
    var n: Ref
    n := new(*)
    n.next := null
    this.elem := e
    this.next := n
    fold list(n, Seq[Int]())
  } else {
    append(this.next, e, c[1..])
  }
  fold list(this, c ++ Seq(e))
}
```

- Solution supports information hiding
- No unfolding-expressions required in specifications
- Preconditions would ideally quantify existentially over abstract value

```
method append(this: Ref, e: Int)
  requires exists c: Seq[Int] ::
             list(this, c)
  ensures  list(this, c ++ Seq(e))
```

- Since existentials are problematic for automation, ghost arguments are required instead (overhead!)

Abstraction

1. Predicates: abstraction over permissions
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Avoiding ghost variables to express abstraction

```
field content: Seq[Int]

method create() returns (res: Ref)
    ensures  list(res)
    ensures  unfolding list(res) in …
{
    res := new(*)
    res.next := null
    res.content := Seq[Int]()
    fold list(res)
}
```

```
method append(this: Ref, e: Int, c: Seq[Int])
  requires list(this, c)
  ensures  list(this, c ++ Seq(e))
```

- Both techniques we have seen so far require ghost variables
- Storing the abstract value in additional variables is unnecessary: all information is already present in the data structure
- Access this information analogously to getter-methods in programs: return the desired information without changing the state

Data abstraction via abstraction functions

→ 16-abs-fun.vpr

- For “getter” methods to be meaningful in assertions, they must
  - be side-effect free,
  - terminate, and
  - be deterministic
- Viper provides heap-dependent functions for this purpose
- Function bodies are expressions
- Functions may be recursive, but termination is not checked by default

```
function content(this: Ref): Seq[Int]
{
  this.next == null
   ? Seq[Int]()
   : Seq(this.elem) ++ content(this.next)
}
```

(incomplete declaration)

Expressions

E ::= f(Ē)

Encoding of heap-dependent functions

- Heap-dependent functions are encoded as uninterpreted functions

- Function body is encoded as a definitional axiom

```
function f(x: T): T' {
  E
}
```

```
function f(x: T, h: HeapType): T'

axiom forall x: T, h: HeapType :: f(x, h) == [[E]]
```

  - [[_]] is the encoding function (omitted for types), parametric in the heap
  - Actual definition is more complex and uses limited functions (see Module 5)
  - A proof obligation checks that the function body is well-defined (omitted here)

- Function calls are applications of these functions in the (current or old) heap

```
f(E)
```

```
f([[E]], Heap)
```

Another frame problem

```
function content(this: Ref): Seq[Int]
{
  this.next == null ?
    Seq[Int]() :
    Seq(this.elem) ++ content(this.next)
}
```

```
// assume we have list(x) && acc(y.f)
tmp := content(x)
y.f := 5
assert tmp == content(x)
```

```
tmp := content(x, Heap)
assert Mask[y,f]
Heap[y,f] := 5
assert tmp == content(x, Heap)
```

- Each heap update modifies the (global) heap
- Any information about heap-dependent functions is lost
- Recovering the information by inspecting the function body would violate information hiding and would not work for abstract functions

Read effects

- Heap-dependent functions must have a precondition that frames the function body, that is, provides all permissions to evaluate the body
- The precondition over-approximates the locations the function value depends on (its read effect)
- If permission to a location is not included in the precondition, modifying it cannot affect the function value, which allows framing

```
function content(this: Ref): Seq[Int]
  requires list(this)
{
  unfolding list(this) in
  (this.next == null ?
    Seq[Int]() :
    Seq(this.elem) ++ content(this.next)
  )
}
```

```
// assume we have list(x) && acc(y.f)
tmp := content(x)
y.f := 5
assert tmp == content(x)
```

Framing axioms

- The read effect is used to generate a framing axiom for the function
- If two heaps agree on a function's read effect then the function yields the same result in both heaps

```
function get(x: Ref): Int
  requires acc(x.elem)
{ … }
```

```
function get(x: Ref, h: HeapType): int

axiom forall x: Ref, h1: HeapType, h2: HeapType ::
  h1[x,elem] == h2[x,elem] ==> get(x, h1) == get(x, h2)
```

- How to express framing axioms if the read effect contains an unbounded set of locations?

```
function content(this: Ref): Seq[Int]
  requires list(this)
{ … }
```

Predicate versions

- Precisely encoding the set of locations contained in a predicate instance is complex and hard to reason about
- We approximate the values stored in these locations by giving each predicate instance a version number

```
function
listField(this: Ref): Field Int
```

```
Heap[null, listField(a)]
```

- When two instances of a predicate have the same version, all contained locations have the same value (but not vice versa)
- Version is changed each time one of these locations might change, during unfold

```
exhale list(x)
havoc Heap[null,listField(x)]
inhale body(list(x))
```

Framing axioms with predicate versions

- Using predicate versions, fields and predicates are handled analogously in the framing axioms

```
function get(x: Ref): Int
  requires list(x)
{ … }
```

```
function get(x: Ref, h: HeapType): int

axiom forall x: Ref, h1: HeapType, h2: HeapType ::
  h1[null,listField(x)] == h2[null,listField(x)] ==>
  get(x, h1) == get(x, h2)
```

Reminder: partial functions

- Preconditions of heap-dependent functions specify the read effect

```
function length(this: Ref): Int
  requires list(this)
{ … }
```

- Like method preconditions, they may also constrain the function arguments (including the heap)

```
function first(this: Ref): Int
  requires list(this) && 0 < length(this)
{
  content(this)[0]
}
```

- Definitional axioms provide a partial definition of the (total) uninterpreted function

```
function f(x: T): T'
  requires P
{ E }
```

```
function f(x: T, h: HeapType): T'

axiom forall x: T, h: HeapType ::
      [[P]] ==> f(x, h) == [[E]]
```

Wrap-up: data abstraction

- Ghost fields + invariants
  - Manual updates of ghost state
  - Predicates cannot be abstract
- Predicate arguments
  - Support information hiding
  - Require ghost parameters
- Heap-dependent functions
  - Support information hiding
  - Typically exist in programs anyway (getters)
  - Separation of concerns: predicates for permissions and invariants, functions for abstraction

```
predicate list(this: Ref) {
  acc(this.elem) && acc(this.next) &&
  (this.next != null ==> list(this.next))
}
```

```
function length(this: Ref): Int
  requires list(this)
{ … }
```

```
function first(this: Ref): Int
  requires list(this) && 0 < length(this)
{ … }
```

```
function content(this: Ref): Seq[Int]
  requires list(this)
{ … }
```

Challenges revisited

Heap data structures pose three major challenges for sequential verification

- Reasoning about aliasing
  - Permissions and separating conjunction
- Framing, especially for dynamic data structures
  - Sound frame rule, predicates
- Writing specifications that preserve information hiding
  - Data abstraction, heap-dependent functions