

# **Methodologies for Software Processes**

## **Lecture 11**

# **PERMISSION MODELS**

## Previously...

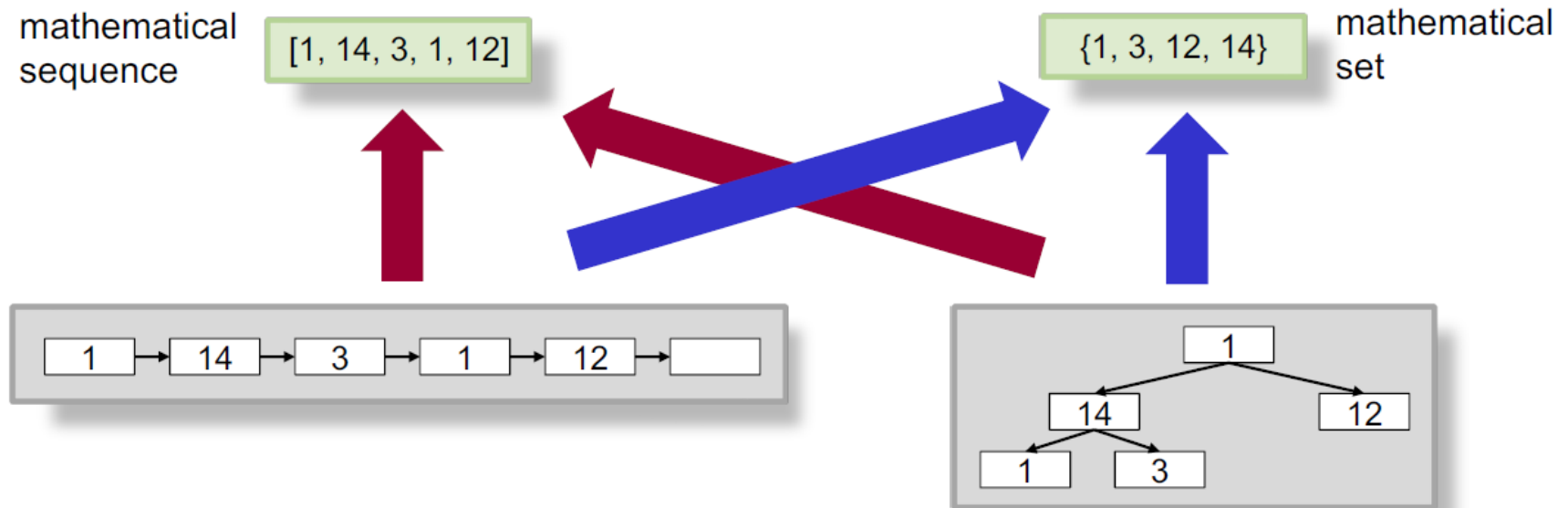
Heap data structures pose three major challenges for sequential verification

- Reasoning about aliasing
  - Permissions and separating conjunction
- Framing, especially for dynamic data structures
  - Sound frame rule, predicates
- Writing specifications that preserve information hiding
  - Data abstraction, heap-dependent functions



# Data abstraction

- To write implementation-independent specifications, we map the concrete data structure to mathematical concepts and specify the behavior in terms of those



- Mapping can be a function or a relation

# Data abstraction via predicate arguments

```
method append(this: Ref, e: Int, c: Seq[Int])
  requires list(this, c)
  ensures  list(this, c ++ Seq(e))
{
  unfold list(this, c)
  if(this.next == null) {
    var n: Ref
    n := new(*)
    n.next := null
    this.elem := e
    this.next := n
    fold list(n, Seq[Int]())
  } else {
    append(this.next, e, c[1..])
  }
  fold list(this, c ++ Seq(e))
}
```

- Solution supports information hiding
- No unfolding-expressions required in specifications
- Preconditions would ideally quantify existentially over abstract value

```
method append(this: Ref, e: Int)
  requires exists c: Seq[Int] ::
    list(this, c)
  ensures  list(this, c ++ Seq(e))
```

- Since existentials are problematic for automation, ghost arguments are required instead (overhead!)

# Data abstraction via abstraction functions

- For “getter” methods to be meaningful in assertions, they must
  - be side-effect free,
  - terminate, and
  - be deterministic
- Viper provides **heap-dependent functions** for this purpose

```
function content(this: Ref): Seq[Int]
{
  this.next == null
    ? Seq[Int]()
    : Seq(this.elem) ++ content(this.next)
}
```

(incomplete declaration)

## Reminder: partial functions

- Preconditions of heap-dependent functions specify the read effect
- Like method preconditions, they may also constrain the function arguments (including the heap)

```
function length(this: Ref): Int  
  requires list(this)  
{ ... }
```

```
function first(this: Ref): Int  
  requires list(this) && 0 < length(this)  
{  
  content(this)[0]  
}
```

## Wrap-up: data abstraction

- Ghost fields + invariants
  - Manual updates of ghost state
  - Predicates cannot be abstract
- Predicate arguments
  - Support information hiding
  - Require ghost parameters
- Heap-dependent functions
  - Support information hiding
  - Typically exist in programs anyway (getters)
  - [Separation of concerns](#):  
predicates for permissions and invariants,  
functions for abstraction

```
predicate list(this: Ref) {  
    acc(this.elem) && acc(this.next) &&  
    (this.next != null ==> list(this.next))  
}
```

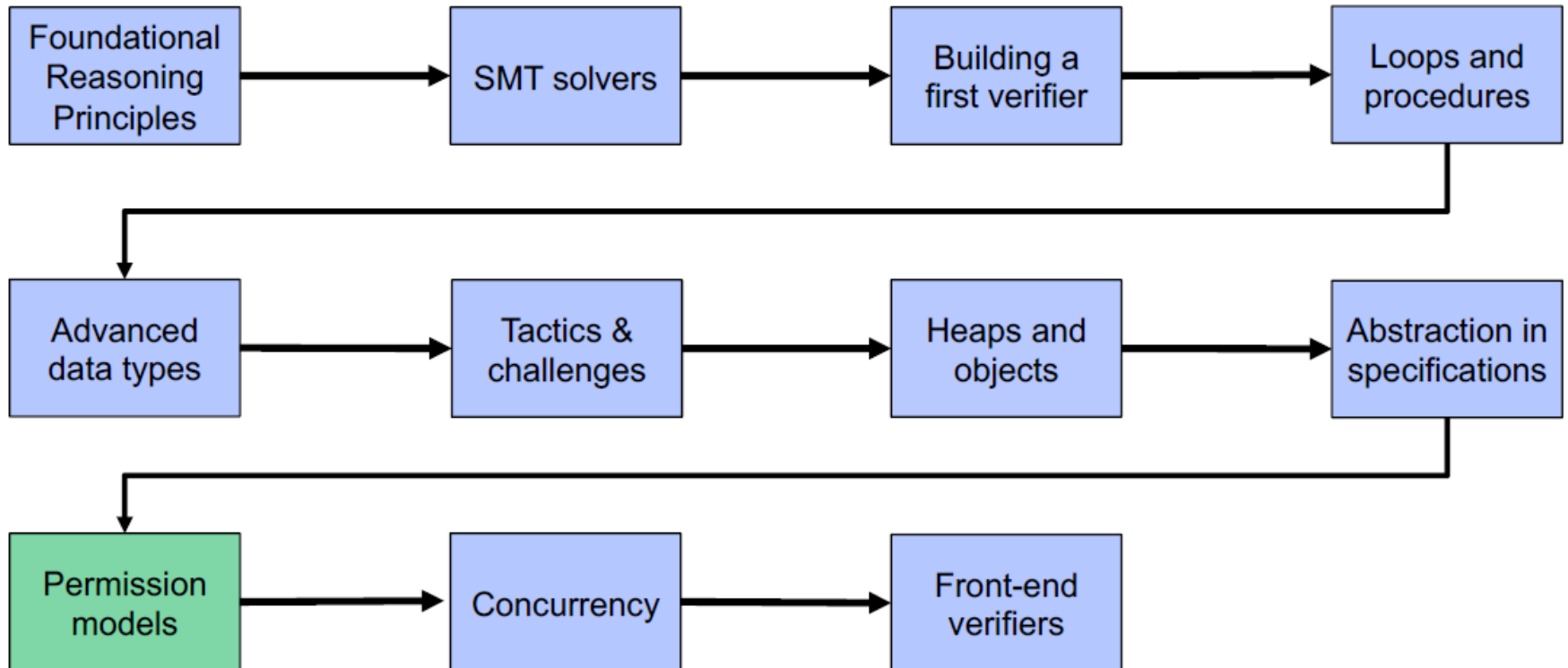
```
function length(this: Ref): Int  
    requires list(this)  
{ ... }
```

```
function first(this: Ref): Int  
    requires list(this) && 0 < length(this)  
{ ... }
```

```
function content(this: Ref): Seq[Int]  
    requires list(this)  
{ ... }
```



# Tentative course outline



# Advanced permission models

1. Fractional permissions
2. Quantified permissions

# Framing revisited

→ 00-clone.vpr

```
method cloneList(this: Ref) returns (res: Ref)
  requires list(this) // read only
  ensures list(this) && list(res)
  ensures content(this) == old(content(this))
{
  res := new(*)
  unfold list(this)
  if(this.next == null) {
    res.next := null
  } else {
    var tmp: Ref
    tmp := cloneList(this.next)
    res.elem := this.elem
    res.next := tmp
  }
  fold list(this)
  fold list(res)
}
```

- Methods that only read a data structure must specify that each abstraction remains **unchanged**
- Adding an abstraction requires changes to existing specifications (**non-modular**)
- Possible solution: specify that predicate version remains unchanged (not possible in Viper)
- We introduce a more expressive solution in the following

# Fractional permissions

- To **distinguish read and write access**, permissions can be split and re-combined
- A **permission amount**  $\pi$  is a rational number in  $[0,1]$
- Viper syntax
  - Permissions are fractions  $n/d$
  - **write** for  $1/1$  and **none** for  $0/1$
  - **acc**(E.f) is a shortcut for **acc**(E.f, **write**)
  - **P**(E) is a shortcut for **acc**(P(E), **write**)
- **Field read** requires a **non-zero** permission
- **Field write** requires **full** (**write**) permission

## Predicates (or assertions)

```
P ::= ...  
    | acc(E.f,  $\pi$ )  
    | acc(P( $\bar{E}$ ),  $\pi$ )
```

```
inhale acc(x.f, 1/2)  
v := x.f
```



```
inhale acc(x.f, 1/2)  
x.f := v
```



# Manipulating fractional permissions

- Separating conjunction sums up permissions of the conjuncts

`acc(x.f, 1/2) && acc(x.f, 1/2)` is equivalent to `acc(x.f, 1/1)`

- inhale *adds* permissions
- exhale *subtracts* permissions and *havocs* only when *all* permission to a location or predicate instance is removed
- Values are *framed* as long as *some* permission is held

```
method cloneList(this: Ref) returns (res: Ref)
  requires acc(list(this), 1/2) // read only
  ensures acc(list(this), 1/2) && list(res)
  { ... }
```

```
method frameList(this: Ref) returns (l: Ref)
  requires list(this)
  {
    var tmp1: Seq[Int]
    tmp1 := content(this)
    l := cloneList(this) // no havoc of version
    assert tmp1 == content(this)
  }
```

# Predicates and fractional permissions

→ 02-clone.vpr  
→ 02-fractions.vpr

- Predicates may contain fractional permissions, e.g. to permit sharing
- Field locations with more than full permission are infeasible (magic)
- Predicate instances with more than full permission are feasible (no magic)
- **Unfold and fold multiply** the fraction of the predicate with the fractions in the predicate body

```
predicate readCell(this: Ref) {  
  acc(this.cell) && acc(this.cell.val, 1/2)  
}
```

```
predicate P(this: Ref) {  
  acc(this.val, 1/2)  
}
```

```
inhale acc(x.val)  
fold P(x)  
fold P(x)  
exhale P(x) && P(x) // not false
```



```
inhale acc(readCell(x), 1/4)  
unfold acc(readCell(x), 1/4)  
exhale acc(x.cell.val, 1/8)
```



## Heap-dependent functions

- Heap-dependent functions may only read the heap
- Hence, an arbitrarily small fraction would be sufficient
- Problem: we don't know how often permissions are split
- Possible solution: use **wildcard** to avoid concrete fraction

```
function length(this: Ref): Int
  requires list(this)
{
  unfolding list(this) in
    (this.next == null ? 0 : length(this.next) + 1)
}
```

```
inhale acc(list(this), 1/2)
x := length(this)
```



```
function length(this: Ref): Int
  requires acc(list(this), wildcard)
{
  unfolding acc(list(this), wildcard) in
    (this.next == null ? 0 : length(this.next) + 1)
}
```

# Adjusted encoding: permissions and field access

- Permissions are tracked in a global permission mask

```
type MaskType = Map<T>[(Ref, Field T), Real]  
var Mask: MaskType
```

- Convention:  $\text{Mask}[\text{null}, f] == 0.0$  for all fields  $f$

- Field access

```
v := x.f
```

```
assert Mask[x,f] > 0.0  
v := Heap[x,f]
```

```
x.f := E
```

```
assert Mask[x,f] == 1.0  
Heap[x,f] := E
```

- Field access requires permission!



## Adjusted encoding: inhale

- **inhale** A means:
  - obtain all permissions required by assertion A
  - assume all logical constraints
- Encoding is defined recursively over the structure of A

**inhale** acc(E.f,  $\pi$ )

Mask[[E]],f := Mask[[E]],f +  $\pi$   
**assume** Mask[[E]],f <= 1.0

Reaching more than full permission  
for a **field** location goes to magic

**inhale** acc(P(E),  $\pi$ )

Mask[null,PField([E])] := Mask[null,PField([E])] +  $\pi$

**inhale** A && B

[[**inhale** A]]; [[**inhale** B]]

Separating conjunction:  
add sum of permissions

- The encoding also asserts that E and  $\pi$  are well-defined (omitted here)

## Adjusted encoding: exhale

- **exhale** A means:
  - assert all logical constraints
  - check and remove all permissions required by assertion A
  - havoc any locations to which all permission is lost
- Encoding is defined recursively over the structure of A

**exhale** acc(E.f,  $\pi$ )

```
assert Mask[[[E]],f] >=  $\pi$   
Mask[[[E]],f] := Mask[[[E]],f] -  $\pi$ 
```

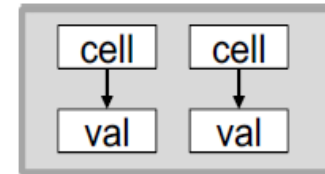
**exhale** A

```
var oldMask: MaskType  
var newHeap: HeapType  
oldMask := Mask  
[[exhale A]]  
assume forall y,g :: Mask[y,g] > 0.0 ==> newHeap[y,g] == Heap[y,g]  
Heap := newHeap // effectively havocs all locations to which all  
                  permission was lost
```

# Sharing in data structures

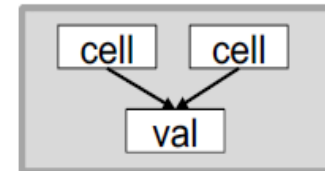
- Full permissions can describe tree-shaped data structures only

```
predicate exclusiveCell(this: Ref) {  
  acc(this.cell) && acc(this.cell.val)  
}
```



- Fractional permissions allow sharing

```
predicate sharedCell(this: Ref) {  
  acc(this.cell) && acc(this.cell.val, 1/2)  
}
```



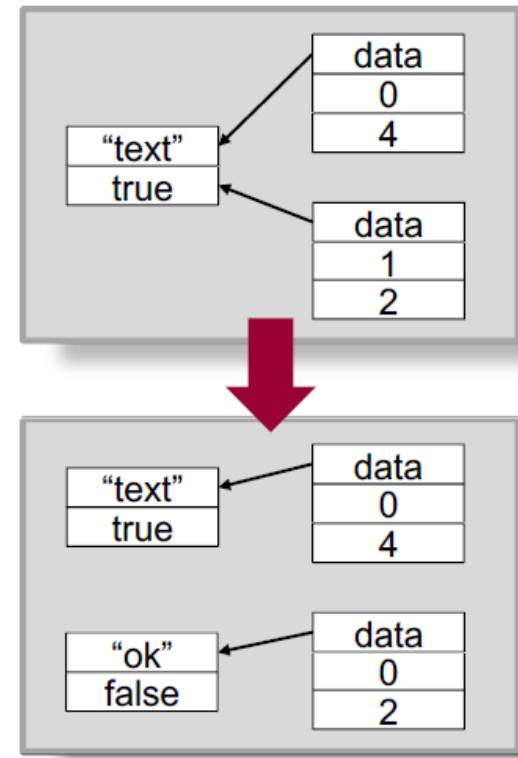
- Sharing is used in many data structures
  - Examples: doubly-linked lists, global data, caches, graphs, ...

# Case study: binary reference counting

→ 04-rcbox.vpr

- Binary reference counting optimizes code that uses immutable data
- Keep track whether the data is shared
- Updates on shared data perform a copy (“copy on write”)
- Updates on unshared data perform a destructive update
- Once shared, the data does not go back to unshared (unlike with full reference counting)

- Example: text segments



# The Perm type

- The demo uses a ghost field of type `Perm`
- Values of type `Perm` include:
  - constants `none`, `write`, `wildcard`, and fractions
  - expressions, e.g., `write - x.frac` or `2*write`
- `Perm` is typically used for ghost variables
  - Parameterize methods that require read permission
  - Perform permission accounting when permissions are distributed and later re-collected
- Type `Perm` is encoded as a real

```
field frac: Perm
```

```
method cloneList(this: Ref, p: Perm)
    returns (res: Ref)
  requires acc(list(this), p)
  ensures  acc(list(this), p) &&
           list(res)
```

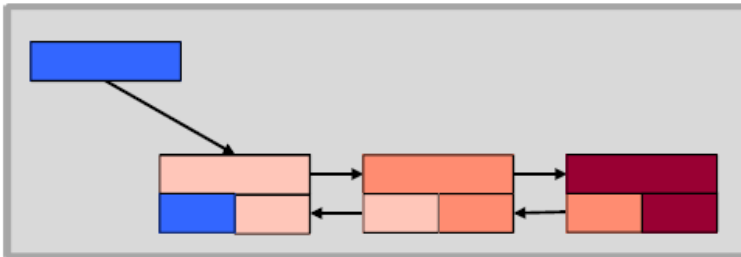
# Sharing in mutable data structures

- Previously: immutable shared objects
- To specify mutable data structures with sharing, we arrange fractional permissions such that they can be combined to obtain a full permission

- Example: doubly-linked list

```
predicate nodes(this: Ref) {  
  acc(this.next) && acc(this.prev, 1/2) &&  
  (this.next != null ==>  
    acc(this.next.prev, 1/2) &&  
    this.next.prev == this &&  
    nodes(this.next)  
  )  
}
```

```
predicate dlist(this: Ref) {  
  acc(this.head) &&  
  (this.head != null ==>  
    acc(this.head.prev, 1/2) &&  
    this.head.prev == null &&  
    nodes(this.head)  
  )  
}
```



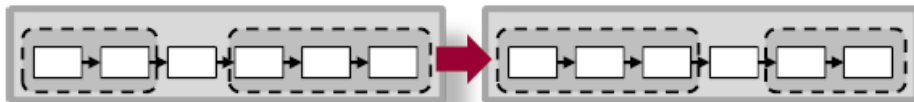
# Advanced permission models

1. Fractional permissions

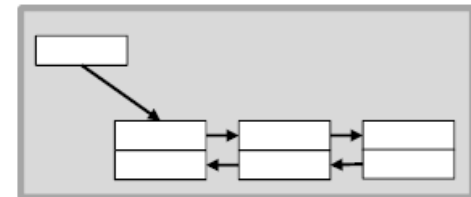
2. Quantified permissions

# Limitations of recursive predicates

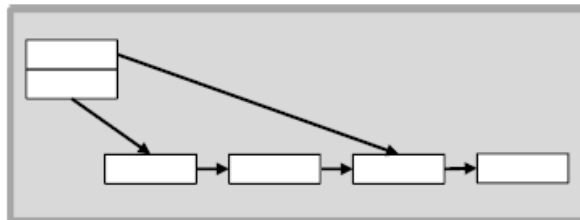
- Recursive predicates allow one to specify unbounded data structures
  - Traversals happen **in the order** in which the predicate needs to be unfolded
- Predicates are not ideal for many other use cases



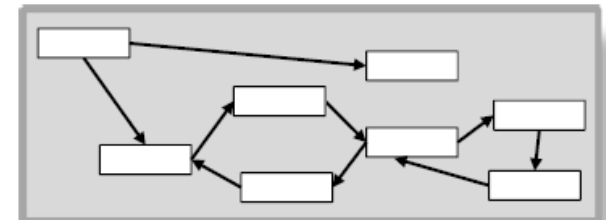
Iterative traversals



Other traversal orders



Random-access data structures



Arbitrary cyclic data structures



# Quantified permissions

- To denote permission to an unbounded set of locations without prescribing a traversal order, we allow permissions and predicates in universal quantifiers

## Permissions

$P ::= \dots \mid \text{forall } \overline{x:T} :: P$

- Universal quantifiers can be thought of as a possibly-infinite iterated conjunction

$\text{forall } x:T :: P \iff P[x/v1] \wedge P[x/v2] \wedge \dots$

- Viper's forall represents a possibly-infinite **iterated separating conjunction**

$\text{forall } x:T :: P \iff P[x/v1] * P[x/v2] * \dots$

# Explicit footprints

→ 06-qp-list-iter.vpr

- As alternative to predicates, we can specify permission to an unbounded set of locations by
  - maintaining an explicit set of references as ghost state (the explicit footprint)
  - quantifying over the set elements in specifications



```
field head: Ref
field nodes: Set[Ref] // explicit footprint
```

```
predicate list(this: Ref) {
  acc(this.head) && acc(this.nodes) &&
  (forall n: Ref :: n in this.nodes ==> acc(n.elem) && acc(n.next) &&
    (n.next != null ==> n.next in this.nodes)) &&
  (this.head != null ==> this.head in this.nodes)
}
```

## Well-formed quantified permissions

- Viper requires for each assertion `acc(E.f)` under a `forall x:T` that E is *injective* for all instantiations  $x_1, x_2$ :

$$x_1 \neq x_2 \implies E[x/x_1] \neq E[x/x_2]$$

- Analogous rule applies to predicates (for parameter tuples)
- Examples

```
forall x: Ref :: x in s ==> acc(x.f) // s has type Set[Ref]
```



```
forall x: Ref :: acc(y.f) && (y.f != x ==> P(x))
```

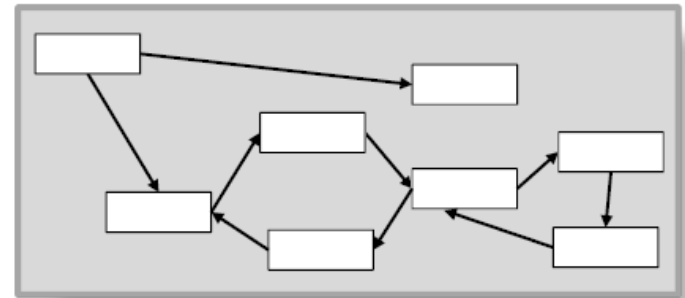


```
forall i: Int :: 0 <= i && i < |t| ==> acc(t[i].f) // t has type Seq[Ref]
```

well-formed if t  
contains no  
duplicates

## Complex sharing: graph marking

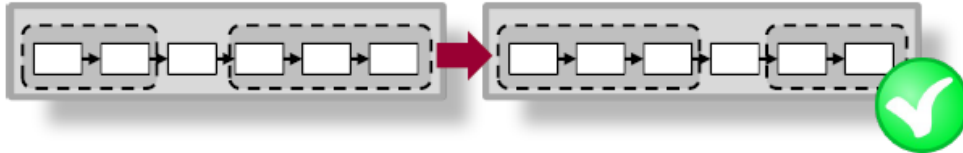
- We represent a graph as a set of nodes
- Each node stores a (possibly empty) set of successors
- Each node contains a flag that is set during marking



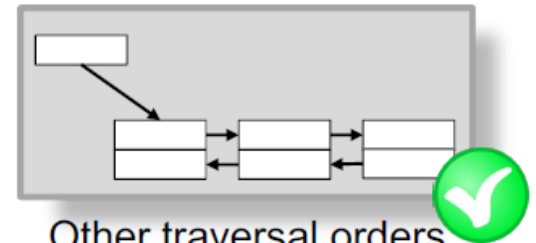
```
field next: Set[Ref]
field flag: Bool

define graph(nodes) (
  forall n: Ref :: n in nodes ==> acc(n.next) && acc(n.flag) && (n.next subset nodes)
)
```

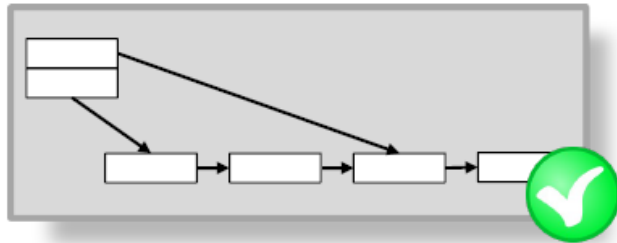
# Quantified permissions address the limitations of predicates



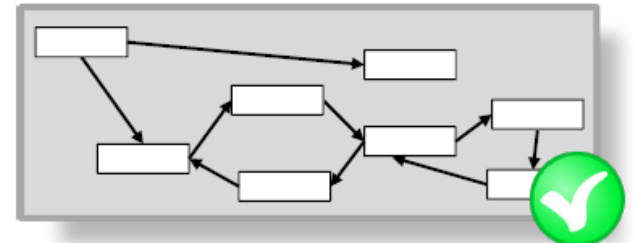
Iterative traversals



Other traversal orders



Random-access data structures



Arbitrary cyclic data structures

# Arrays

- Viper does not have built-in arrays
- In contrast to sequences, arrays are mutable heap data structures
- We model arrays by a set of disjoint references that can be accessed via an index
- `loc(a, i).val` models `a[i]`
- More-dimensional arrays can be encoded analogously

```
field val: Int // for integer arrays

domain Array {
  function loc(a: Array, i: Int): Ref
  function len(a: Array): Int
  function first(r: Ref): Array
  function second(r: Ref): Int

  axiom injectivity {
    forall a: Array, i: Int :: {loc(a, i)}
      first(loc(a, i)) == a &&
      second(loc(a, i)) == i
  }

  axiom length_nonneg {
    forall a: Array :: len(a) >= 0
  }
}
```

# Accessing array locations

- Arrays are random-access data structures
- We can express permissions using quantified permissions

```
forall i: Int :: 0 <= i && i < len(a) ==> acc(loc(a, i).val)
```

- Similarly for sub-ranges of the array

- We define macros for convenient access

```
define lookup(a, i)  
  loc(a, i).val
```

```
define update(a, i, e) {  
  loc(a, i).val := e  
}
```

- Bounds are checked implicitly via permissions

## Wrap-up: advanced permission models

- Fractional permissions
  - Distinguish between read and write permission
  - Are useful to express sharing, to strengthen framing, and for concurrency (see later)
- Quantified permissions
  - Complement predicates for the specification of unbounded data structures
  - Are especially useful for random-access structures, complex sharing, and flexible traversals
  - Inherit challenges of quantification (controlling instantiations, performance)
- Other permission models exist
  - Magic wands (permission-aware implication): useful to specify partial data structures
  - Counting permissions are related to fractional permissions, but use units