Programming Paradigms

Lecture 3

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Statements, Kernel Language, Abstract Machine

Reminder of last lecture

- Kernel language
  - linguistic abstraction
  - data types
  - variables and partial values
  - unification

Overview

- Some Oz concepts
  - Pattern matching
  - Tail recursion
  - Lazy evaluation
- Kernel language
  - statements and expressions
- Kernel language semantics
  - Use *operational semantics*
    - Aid programmer in reasoning and understanding
  - The model is a sort of an *abstract machine*, but leaves out details about registers and explicit memory address
    - Aid implementer to do an efficient execution on a real machine

Pattern-Matching on Numbers

```
fun {Fact N}
    case N
    of 0 then 1
    [] N then N*{Fact (N-1)} end
end
```

Pattern Matching on Structures

```
fun {Depth T}
case T
   of  leaf(value:_)   then 1
    [] node(left:L right:R value:_)
            then 1+{Max {Depth L} {Depth R}}
    end
end
```

Compared to Conditional

```
fun {SumList Xs}
   case Xs
   of nil  then 0
   [] X|Xr then X + {SumList Xr} end
end
```

Using only Conditional

```
fun {SumList Xs}
    if {Label Xs}=='nil' then 0
    elseif {Label Xs}=='|' andthen {Width Xs}==2
                then Xs.1 +{SumList Xs.2}
    end
end
```

Linear Recursion

```
fun {Fact N}
   case N
   of 0  then 1
   [] N then N * {Fact (N-1)} end
end
```

```
{Fact 3}
⇒ 3*{Fact 2}
⇒ 3*(2*{Fact 1})
⇒ 3*(2*(1*{Fact 0}))
⇒ 3*(2*(1*1))
⇒ 3*(2*1)
⇒ 3*2
⇒ 6
```

going down recursion

return from recursion

Accumulating Parameter

```
fun {Fact N } {FactT N 1} end
```

Accumulating Parameter

```
fun {FactT N Acc}
    case N
    of 0  then Acc
    [] N then {FactT (N-1) N*Acc} end
end
```

Accumulating Parameter

```
{Fact 3}
⇒ {FactT 3 1}
⇒ {FactT 2 3*1}
⇒ {FactT 2 3})
⇒ {FactT 1 2*3}
⇒ {FactT 1 6}
⇒ {FactT 0 1*6}
⇒ {FactT 0 6}
⇒ 6
```

going down
recursion and accumulating
result in parameter

Accumulating Parameter = Tail Recursion = Loop!

Tail Recursion = Loop

```
fun {FactT N Acc}
    case N
    of 0  then Acc
    [] N then N=N-1
              Acc=N*Acc
              {FactT N Acc}
    end
end
```

jump

Last call = Tail call

Lazy Evaluation

Infinite list of numbers!

```
fun lazy {Ints N} N|{Ints N+1} end
```

```
{Ints 2}
⇒ 2|{Ints 3}
⇒ 2|(3|{Ints 4})
⇒ 2|(3|(4|{Ints 5}))
⇒ 2|(3|(4|(5|{Ints 6})))
⇒ 2|(3|(4|(5|(6|{Ints 7}))))
        :
```

What if we were to compute : `{SumList {Ints 2}}` ?

Taking first N elements of List

```
fun {Take L N}
   if N<=0 then nil
   else case L of
               nil then nil
          [] X|Xs then X|{Take Xs (N-1)} end end
end
```

```
      {Take [a b c d] 2}
      ⇒ a|{Take [b c d] 1}
      ⇒ a|b|{Take [c d] 0}
      ⇒ a|b|nil

      {Take {Ints 2} 2}
      ⇒  ?
```

Eager Evaluation

Loop as Infinite list eagerly evaluated!

```
{Take {Ints 2} 2}
⇒ {Take 2|{Ints 3} 2}
⇒ {Take 2|(3|{Ints 4}) 2}
⇒ {Take 2|(3|(4|{Ints 5}))} 2}
⇒ {Take 2|(3|(4|(5|{Ints 6}))) 2}
⇒ {Take 2|(3|(4|(5|(6|{Ints 7})))) 2}
        :
```

Lazy Evaluation

Evaluate the lazy argument only as needed

```
{Take {Ints 2} 2}
⇒ {Take 2|{Ints 3} 2}
⇒ 2|{Take {Ints 3} 1}
⇒ 2|{Take 3|{Ints 4} 1}
⇒ 2|(3|{Take {Ints 4} 0})
⇒ 2|(3|nil)
```

terminates despite infinite list

Kernel Concepts

- Single-assignment store
- Environment
- Semantic statement
- Execution state and Computation
- Statements Execution for:
  - skip and sequential composition
  - variable declaration
  - store manipulation
  - conditional

Procedure Declarations

- Kernel language

  $\langle x \rangle$ = `proc {$` $\langle y_1 \rangle \ldots \langle y_n \rangle$`}` $\langle s \rangle$ `end`

  is a legal statement
  - binds $\langle x \rangle$ to procedure value
  - declares (introduces a procedure)
- Familiar syntactic variant

  `proc {`$\langle x \rangle$ $\langle y_1 \rangle \ldots \langle y_n \rangle$`}` $\langle s \rangle$ `end`

  introduces (declares) the procedure $\langle x \rangle$
- A procedure declaration is a value, whereas a procedure application is a statement!

What Is a Procedure?

- It is a **value** of the **procedure type**.
  - Java: methods with `void` as return type

```
declare
X = proc {$ Y}                $ is the nesting operator
        {Browse 2*Y}
    end
{X 3}                         6
{Browse X}                    <P/1 X>
```

- But how to return a result (as parameter) anyway?
  - Idea: use an unbound variable
  - Why: we can supply its value after we have computed it!

Operations on Procedures

- Three basic operations:
  - Defining them (with `proc` statement)
  - Calling them (with `{ }` notation)
  - Testing if a value is a procedure
    - `{IsProcedure P}` returns `true` if `P` is a procedure, and `false` otherwise

```
declare
X = proc {$ Y}
        {Browse 2*Y}
    end
{Browse {IsProcedure X}}
```

Towards Computation Model

- Step One: Make the language small
  - Transform the language of function on partial values to a small kernel language
- Kernel language
  - procedures
  - records
  - local declarations

no functions

no tuple syntax

no list syntax

no nested calls

no nested constructions

From Function to Procedure

```
fun {Sum Xs}
    case Xs
    of nil then 0
    [] X|Xr then X+{Sum Xr}
    end
end
```

- Introduce an output parameter for procedure

```
proc {SumP Xs N}
    case Xs
    of nil then N=0
    [] X|Xr then N=X+{Sum Xr}
    end
end
```

Why we need `local` statements?

```
proc {SumP Xs N}
   case Xs
   of nil then N=0
   [] X|Xr then
      local M in {SumP Xr M} N=X+M end
   end
end
```

- Local declaration of variables supported.
- Needed to allow kernel language to be based entirely on procedures

How `N` was actually transmitted?

- Having the call `{SumP [1 2 3] C}`, the identifier `Xs` is bound to `[1 2 3]` and `C` is unbound.
- At the callee of `SumP`, whenever `N` is being bound, so will be `C`.
- This way of passing parameters is called **call by reference**.
- Procedures output are passed as references to unbound variables, which are bound inside the procedure.

Local Declarations

`local` `X` **`in`** … **`end`**

Introduces the variable identifier `X`

- visible between `in` and `end`
- called scope of the variable/declaration

Creates a new store variable

Links environment identifier to store variable

Abbreviations for Declarations

- Kernel language
  - just one variable introduced at a time
  - no assignment when first declared
- Oz language syntax supports:
  - several variables at a time
  - variables can be also assigned (initialized) when introduced

Transforming Declarations Multiple Variables

```
local X Y in
   ⟨statement⟩
end
```

⇨

```
local X in
   local Y in
      ⟨statement⟩
   end
end
```

Transforming away Declarations' Initialization

```
local
    X=⟨expression⟩
in
    ⟨statement⟩
end
```

⇨

```
local X in
    X=⟨expression⟩
    ⟨statement⟩
end
```

Transforming Expressions

- Replace function calls by procedure calls
- Use local declaration for intermediate values
- Order of replacements:
  - left to right
  - innermost first
  - it is different for record construction: outermost first
  - Left associativity: `1+2+3` means `((1+2)+3)`
  - Right associativity: `a|b|X` means `(a|(b|X))`, so build the first `'|'`, then the second `'|'`

Function Call to Procedure Call

```
X={F Y}        ⇨  {F Y X}
```

Replacing Nested Calls

```
{P {F X Y} Z}
```

⇨

```
local U1 in
   {F X Y U1}
   {P U1 Z}
end
```

Replacing Nested Calls

```
{P {F {G X} Y} Z}
```

⇨

```
local U2 in
   local U1 in
      {G X U1}
      {F U1 Y U2}
   end
   {P U2 Z}
end
```

Replacing Conditionals

```
if X>Y then
   ...
else
   ...
end
```

⇨

```
local B in
   B = (X>Y)
   if B then
      ...
   else
      ...
   end
end
```

Expressions to Statements

```
X = if B then
        ...
    else
        ...
    end
```

```
if B then
   X = ...
else
   X = ...
end
```

Functions to Procedures: Length (0)

```
fun {Length Xs}
    case Xs
    of nil then 0
    [] X|Xr then 1+{Length Xr}
    end
end
```

Functions to Procedures: Length (1)

```
proc {Length Xs N}
   N=case Xs
     of nil then 0
     [] X|Xr then 1+{Length Xr}
     end
end
```

- Make it a procedure

Functions to Procedures: Length (2)

```
proc {Length Xs N}
   case Xs
   of nil then N=0
   [] X|Xr then N=1+{Length Xr}
   end
end
```

- Expressions to statements

Functions to Procedures: Length (3)

```
proc {Length Xs N}
   case Xs
   of nil then N=0
   [] X|Xr then
      local U in
         {Length Xr U}
         N=1+U
      end
   end
end
```

- Replace function call by its corresponding proc call.

Functions to Procedures: Length (4)

```
proc {Length Xs N}
   case Xs
   of nil then N=0
   [] X|Xr then
      local U in
         {Length Xr U}
         {Number.'+' 1 U N}
      end
   end
end
```

- Replace operation (+, dot-access, <, >, …): procedure!

Kernel Language Statement Syntax

$\langle s \rangle$ denotes a statement

$\langle s \rangle$ ::= skip	*empty statement*
| $\langle x \rangle = \langle y \rangle$	*variable-variable binding*
| $\langle x \rangle = \langle v \rangle$	*variable-value binding*
| $\langle s_1 \rangle\ \langle s_2 \rangle$	*sequential composition*
| local $\langle x \rangle$ in $\langle s_1 \rangle$ end	*declaration*
| if $\langle x \rangle$ then $\langle s_1 \rangle$ else $\langle s_2 \rangle$ end	*conditional*
| $\{ \langle x \rangle\ \langle y_1 \rangle \ldots \langle y_n \rangle \}$	*procedure application*
| case $\langle x \rangle$ of $\langle pattern \rangle$ then $\langle s_1 \rangle$ else $\langle s_2 \rangle$ end	*pattern matching*
$\langle v \rangle$ ::= ...	*value expression*

$\langle pattern \rangle$ ::= ...

Abstract Machine

- *Environment* maps variable identifiers to store entities
- *Semantic statement* is a pair of:
  - statement
  - environment
- *Execution state* is a pair of:
  - stack of semantic statements
  - single assignment store
- *Computation* is a sequence of execution states
- An **abstract machine** performs a computation

Single Assignment Store

- Single assignment store σ
  - set of store variables
  - partitioned into
    - sets of variables that are equivalent but unbound
    - variables bound to a value (number, record or procedure)
- Example store $\{x_1,\ x_2=x_3,\ x_4=a|x_2\}$
  - x_1 unbound
  - $x_2,\ x_3$ equal and unbound
  - x_4 bound to partial value $a|x_2$

Environment

- Environment *E*
  - maps variable identifiers to entities in store σ
  - written as set of pairs $X \rightarrow x$
    - identifier X
    - store variable x
- Example of environment: $\{ X \rightarrow x, Y \rightarrow y \}$
  - maps identifier X to store variable x
  - maps identifier Y to store variable y

Environment and Store

- Given: environment E, store σ
- Looking up value for identifier X:
  - find store variable in environment using E(X)
  - take value from σ for E(X)
- Example:

  $\sigma=\{x_1, x_2=x_3, x_4=a|x_2\}$ $E = \{ X \rightarrow x_1, Y \rightarrow x_4 \}$

  - E(X) = x_1 where no information in σ on x_1
  - E(Y) = x_4 where σ binds x_4 to $a|x_2$

Calculating with Environments

- Program execution looks up values
  - assume store σ
  - given identifier $\langle x \rangle$
  - $E(\langle x \rangle)$ is the value of $\langle x \rangle$ in store σ
- Program execution modifies environments
  - for example: declaration
  - add mappings for new identifiers
  - overwrite existing mappings
  - restrict mappings on sets of identifiers

Environment Adjunction

- Given: Environment E

then $E + \{\langle x \rangle_1 \rightarrow x_1, \ldots, \langle x \rangle_n \rightarrow x_n\}$

is a new environment E' with mappings added:
  - always take store entity from new mappings
  - might overwrite (or shadow) old mappings

Environment Projection

- Given: Environment E

  $$E \mid \{\langle x \rangle_1, \ldots, \langle x \rangle_n\}$$

  is a new environment E' where only mappings for $\{\langle x \rangle_1, \ldots, \langle x \rangle_n\}$ are retained from E

Adjunction Example

- $E_0 = \{\langle \mathrm{Y} \rangle \rightarrow 1\ \}$
- $E_1 = E_0 + \{\langle \mathrm{X} \rangle \rightarrow 2\ \}$
  - corresponds to $\{\langle \mathrm{X} \rangle \rightarrow 2,\ \langle \mathrm{Y} \rangle \rightarrow 1\ \}$
  - $E_1(\langle \mathrm{X} \rangle) = 2$
- $E_2 = E_1 + \{\langle \mathrm{X} \rangle \rightarrow 3\ \}$
  - corresponds to $\{\langle \mathrm{X} \rangle \rightarrow 3,\ \langle \mathrm{Y} \rangle \rightarrow 1\ \}$
  - $E_2(\langle \mathrm{X} \rangle) = 3$

Why Adjunction?

E_0

```
local X in
   X = 2
   local X in
      X = 3
      {… Y}
   end
   …
end
```

E_2

E_1

E_0

Semantic Statements

- Semantic statement $(\langle s\rangle, E)$
  - pair of (statement, environment)
- To actually execute statement:
  - environment to map identifiers
    - modified with execution of each statement
    - each statement has its own environment
  - store to find values
    - all statements modify same store
    - single store

Stacks of Statements

- Execution maintains stack of semantic statements **ST =** $[(\langle s\rangle_1, E_1), \ldots, (\langle s\rangle_n, E_n)]$
  - always topmost statement $(\langle s\rangle_1, E_1)$ executes first
    - <s> is statement
    - E denotes the environment mapping
  - rest of stack: remaining work to be done
- Also called: *semantic stack*

Execution State

- *Execution state* $(\ ST, \sigma\ )$
  - pair of (semantic stack, store)
- *Computation*

$$(ST_1, \sigma_1) \Rightarrow (ST_2, \sigma_2) \Rightarrow (ST_3, \sigma_3) \Rightarrow \ldots$$

  - sequence of execution states

Program Execution

- Initial execution state

  $([(\langle s \rangle,\varnothing)] , \varnothing)$

  - empty store $\varnothing$
  - stack with semantic statement $[(\langle s \rangle,\varnothing)]$
    - single statement $\langle s \rangle$, empty environment $\varnothing$
- At each execution step
  - pop topmost element of semantic stack
  - execute according to statement
- If semantic stack is empty, then execution stops

Semantic Stack States

- Semantic stack can be in following states
  - *terminated* — stack is empty
  - *runnable* — can do execution step
  - *suspended* — stack not empty, no execution step possible
- Statements
  - *non-suspending* — can always execute
  - *suspending* — need values from store
    dataflow behavior

Summary up to now

- Single assignment store σ
- Environments E
  - adjunction, projection $E + \{\ldots\}$ $E \mid_{\{\ldots\}}$
- Semantic statements $(\langle s \rangle, E)$
- Semantic stacks $[(\langle s \rangle, E) \ \ldots \ ]$
- Execution state (ST, σ)
- Computation = sequence of execution states
- Program execution
  - runnable, terminated, suspended
- Statements
  - suspending, non-suspending

Statement Execution

- Simple statements
  - skip and sequential composition
  - variable declaration
  - store manipulation
  - Conditional (`if` statement)
- Computing with procedures (next lecture)
  - lexical scoping
  - closures
  - procedures as values
  - procedure call

Simple Statements

$\langle s \rangle$ denotes a statement

$\langle s \rangle$ ::=	skip	*empty statement*
|	$\langle x \rangle = \langle y \rangle$	*variable-variable binding*
|	$\langle x \rangle = \langle v \rangle$	*variable-value binding*
|	$\langle s_1 \rangle \langle s_2 \rangle$	*sequential composition*
|	local $\langle x \rangle$ in $\langle s_1 \rangle$ end	*declaration*
|	if $\langle x \rangle$ then $\langle s_1 \rangle$ else $\langle s_2 \rangle$ end	*conditional*
$\langle v \rangle$ ::=	...	*value expression* (no procedures here)

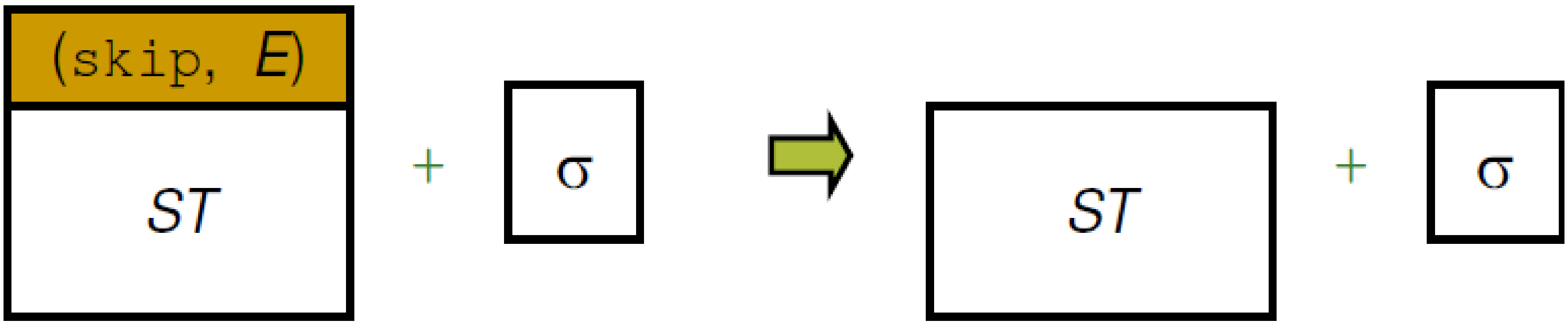
Executing `skip`

- Execution of semantic statement

  (`skip`, E)

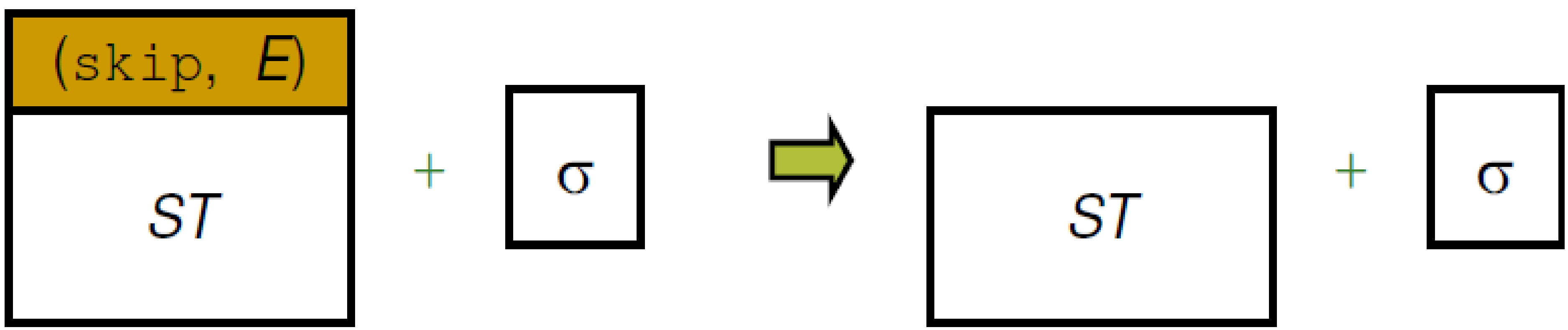
- Do nothing
  - means: continue with next statement
  - non-suspending statement

Executing `skip`



- No effect on store σ
- Non-suspending statement

Executing `skip`

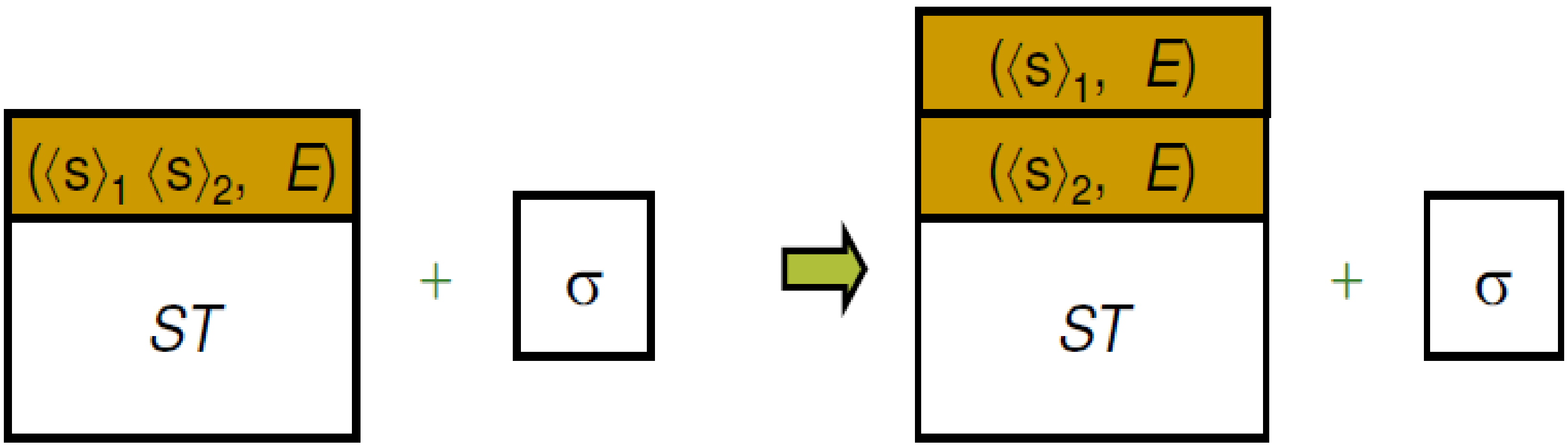


- Remember: topmost statement is always popped!

Executing Sequential Composition

- Semantic statement is
  $(\langle s \rangle_1\ \langle s \rangle_2,\ E)$
- Push in following order
  - $\langle s \rangle_2$ executes after
  - $\langle s \rangle_1$ executes next
- Statement is non-suspending

Sequential Composition



- Decompose statement sequences
  - environment is given to both statements

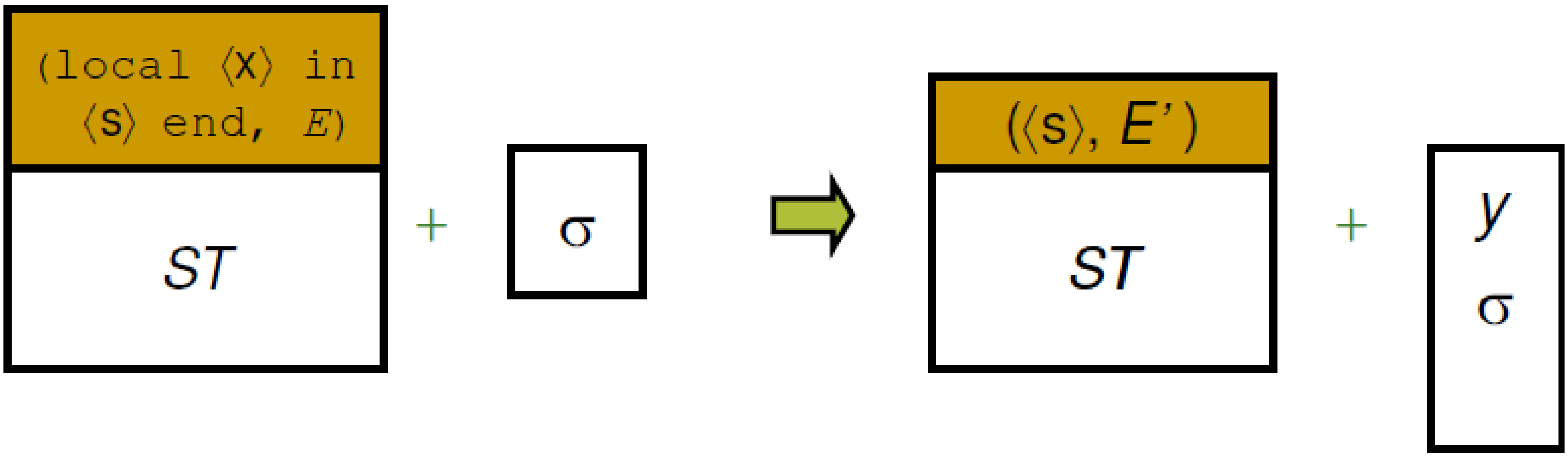
Executing `local`

- Semantic statement is

  (`local` ⟨x⟩ `in` ⟨s⟩ `end`, E)

- Execute as follows:
  - create new variable y in store
  - create new environment $E' = E + \{\langle x\rangle \rightarrow y\}$
  - push (⟨s⟩, E')
- Statement is non-suspending

Executing `local`

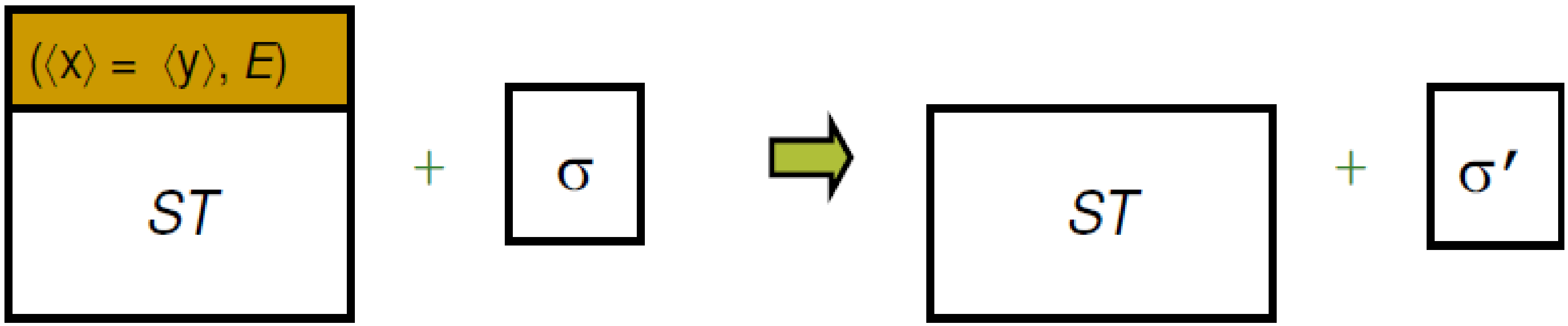


- With $E' = E + \{\langle x\rangle \rightarrow y\}$

Variable-Variable Equality

- Semantic statement is
  $(\langle x \rangle = \langle y \rangle, E)$
- Execute as follows
  - bind $E(\langle x \rangle)$ and $E(\langle y \rangle)$ in store
- Statement is non-suspending

Executing Variable-Variable Equality



- σ’ is obtained from σ by binding *E*(⟨x⟩) and *E*(⟨y⟩) in store

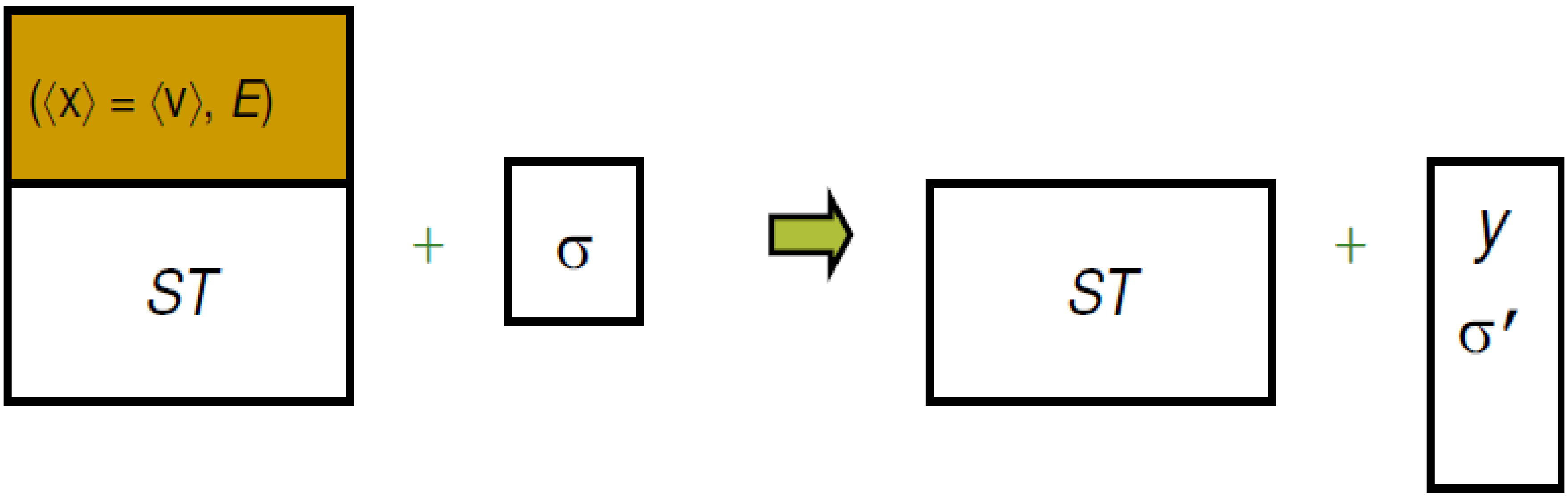
Variable-Value Equality

- Semantic statement is

  $$(\langle x\rangle = \langle v\rangle, E)$$

  where $\langle v\rangle$ is a number or a record (procedures will be discussed later)
- Execute as follows
  - create a variable y in store and let y refers to value $\langle v\rangle$
  - any identifier $\langle z\rangle$ from $\langle v\rangle$ is replaced by $E(\langle z\rangle)$
  - bind $E(\langle x\rangle)$ and y in store
- Statement is non-suspending

Executing Variable-Value Equality



- y refers to value ⟨v⟩
- Store σ is modified into σ' such that:
  - any identifier ⟨z⟩ from ⟨v⟩ is replaced by *E*(⟨z⟩)
  - bind *E*(⟨x⟩) and y in store σ

Suspending Statements

- All statements so far can always execute
  - non-suspending (or immediate)
- Conditional?
  - requires condition ⟨x⟩ to be bound variable
  - *activation condition*: ⟨x⟩ is bound (determined)

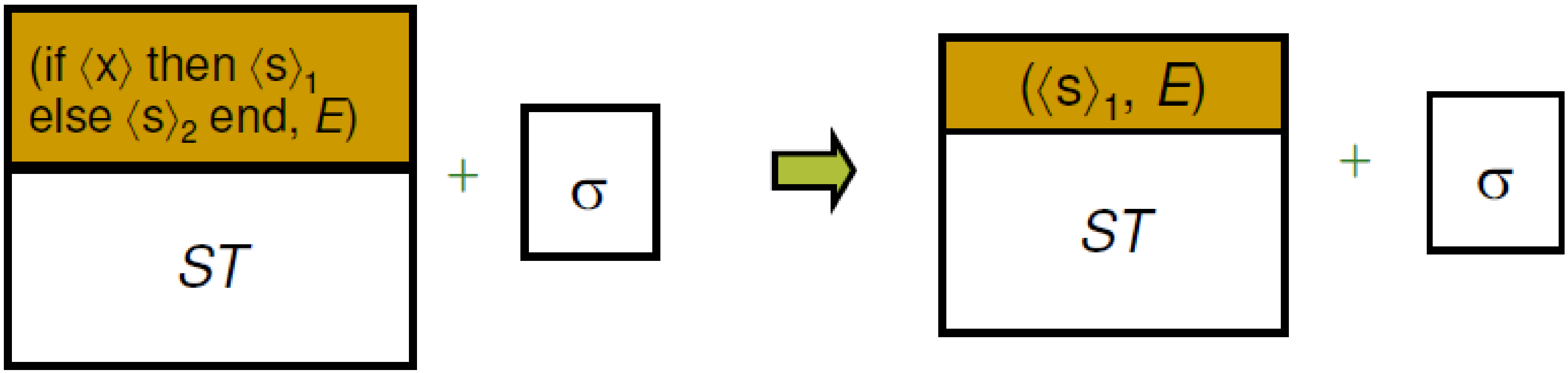
Executing `if`

- Semantic statement is

  $(\texttt{if}\ \langle \mathrm{x} \rangle\ \texttt{then}\ \langle \mathrm{s} \rangle_1\ \texttt{else}\ \langle \mathrm{s} \rangle_2\ \texttt{end},\ E)$
- If the activation condition "bound($\langle$x$\rangle$)" is `true`
  - if $E(\langle \mathrm{x} \rangle)$ bound to `true` push $\langle \mathrm{s} \rangle_1$
  - if $E(\langle \mathrm{x} \rangle)$ bound to `false` push $\langle \mathrm{s} \rangle_2$
  - otherwise, raise error
- Otherwise, suspend the `if` statement…

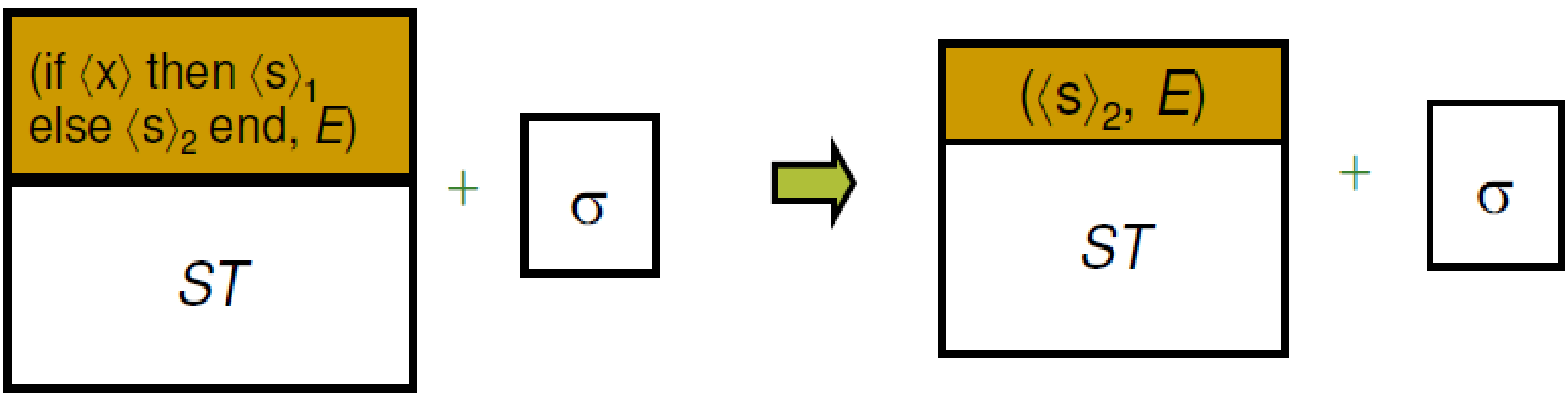
Executing `if`

- If the activation condition “bound(⟨x⟩)” is `true`
  - if E(⟨x⟩) bound to `true`



Executing `if`

- If the activation condition "bound(⟨x⟩)" is `true`
  - if *E*(x⟩) bound to `false`



An Example

```
local X in
   local B in
      B=true
      if B then X=1 else skip end
   end
end
```

- We can reason that `X` will be bound to `1`

Example: Initial State

```
([(local X in
     local B in
        B=true
        if B then X=1 else skip end
     end
  end, ∅)],
 ∅)
```

- Start with empty store and empty environment

Example: `local`

```
([(local B in
       B=true
       if B then X=1 else skip end
     end,
     {X → x})],
 {x})
```

- Create new store variable *x*
- Continue with new environment

Example: `local`

([(`B=true`

`if B then X=1 else skip end`

,

$\{$`B` $\rightarrow b$, `X` $\rightarrow x\}$)] ,

$\{b, x\}$)

- Create new store variable *b*
- Continue with new environment

Example: Sequential Composition

([(B=true, $\{B \rightarrow b, X \rightarrow x\}$),

(if B then X=1

else skip end, $\{B \rightarrow b, X \rightarrow x\}$)],

$\{b,x\}$)

- Decompose to two statements
- Stack has now two semantic statements

Example: Variable-Value Assignment

([(if B then X=1
else skip end, $\{$B $\to$ b, X $\to$ $x\}$)],
$\{b$=true, $x\}$)

- Environment maps B to b
- Bind b to true

Example: `if`

$$([(\texttt{X=1},\ \{\texttt{B} \to b, \texttt{X} \to x\})],$$
$$\{b{=}\texttt{true},\ x\})$$

- Environment maps `B` to *b*
- Bind *b* to `true`
- Because the activation condition "bound(⟨x⟩)" is `true`, continue with `then` branch of `if` statement

Example: Variable-Value Assignment

`([],`
 $\{b$=`true`, x=`1`$\}$`)`

- Environment maps `x` to x
- Binds x to `1`
- Computation terminates as stack is empty

Summary up to now

- Semantic statement execute by

popping itself	always
creating environment	`local`
manipulating store	`local, =`
pushing new statements	`local, if`
	sequential composition

- Semantic statement can suspend
  - activation condition (`if` statement)
  - read store

Pattern Matching

- Semantic statement is

  ($\mathbf{case}\ \langle x \rangle$

  $\mathbf{of}\ \langle \mathrm{lit} \rangle(\langle \mathrm{feat} \rangle_1{:}\langle y \rangle_1\ \ldots\ \langle \mathrm{feat} \rangle_n{:}\langle y \rangle_n)\ \mathbf{then}\ \langle s \rangle_1$

  $\mathbf{else}\ \langle s \rangle_2\ \mathbf{end}$, E)

- It is a suspending statement
- Activation condition is: "bound(⟨x⟩)"
- If activation condition is `false`, then suspend!

Pattern Matching

- Semantic statement is

  $(\texttt{case}\ \langle x\rangle$
  $\texttt{of}\ \langle lit\rangle(\langle feat\rangle_1{:}\langle y\rangle_1 \ldots \langle feat\rangle_n{:}\langle y\rangle_n)\ \texttt{then}\ \langle s\rangle_1$
  $\texttt{else}\ \langle s\rangle_2\ \texttt{end},\ E)$

- If $E(\langle x\rangle)$ matches the pattern, that is,
  - label of $E(\langle x\rangle)$ is $\langle lit\rangle$ and
  - its arity is $[\langle feat\rangle_1 \ldots \langle feat\rangle_n]$),

  then push

$$(\langle s\rangle_1,\ E + \{\langle y\rangle_1 \rightarrow E(\langle x\rangle).\ \langle feat\rangle_1,\ \ldots,\ \langle y\rangle_n \rightarrow E(\langle x\rangle).\ \langle feat\rangle_n\})$$

Pattern Matching

- Semantic statement is

  ($\texttt{case}$ $\langle x \rangle$

  $\texttt{of}$ $\langle \text{lit} \rangle(\langle \text{feat} \rangle_1 : \langle y \rangle_1 \ldots \langle \text{feat} \rangle_n : \langle y \rangle_n)$ $\texttt{then}$ $\langle s \rangle_1$

  $\texttt{else}$ $\langle s \rangle_2$ $\texttt{end}$, E)

- If $E(\langle x \rangle)$ does not match pattern, push

  $(\langle s \rangle_2, E)$

Pattern Matching

- Semantic statement is

  ($\textbf{case}\ \langle x \rangle$

  $\textbf{of}\ \langle \text{lit} \rangle(\langle \text{feat} \rangle_1{:}\langle y \rangle_1\ \ldots\ \langle \text{feat} \rangle_n{:}\langle y \rangle_n)\ \textbf{then}\ \langle s \rangle_1$

  $\textbf{else}\ \langle s \rangle_2\ \textbf{end}$, E)
- It does not introduce new variables in the store
- Identifiers $\langle y \rangle_1 \ldots \langle y \rangle_n$ are visible only in $\langle s \rangle_1$

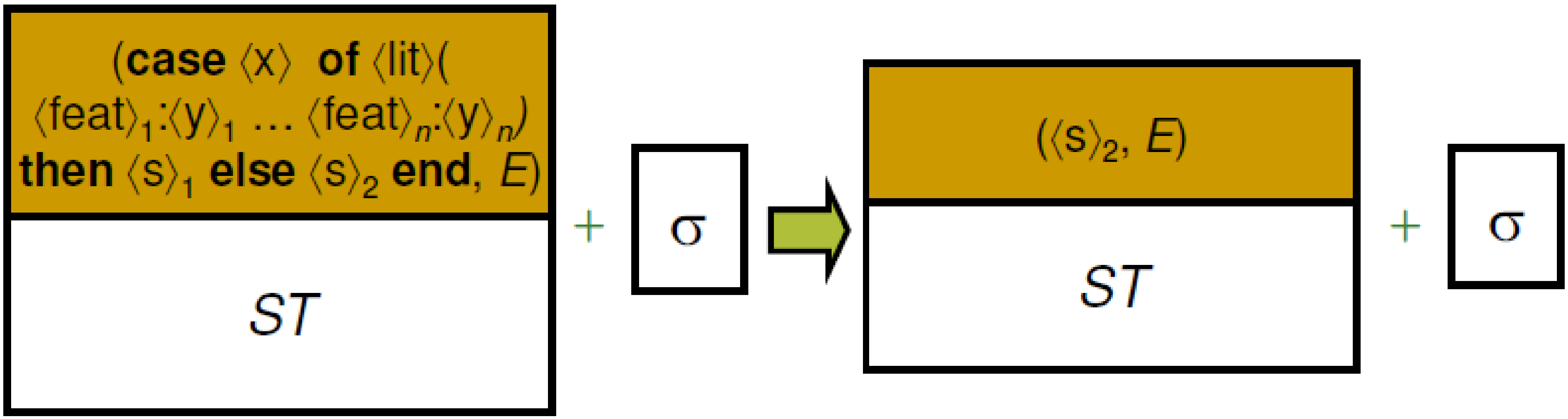
Executing `case`

- If the activation condition "bound(⟨x⟩)" is `true`
  - if $E(\langle x\rangle)$ *matches* the pattern

$$\begin{array}{|c|}\hline (\textbf{case}\ \langle \mathrm{x}\rangle\ \ \textbf{of}\ \langle \mathrm{lit}\rangle(\\ \langle \mathrm{feat}\rangle_1{:}\langle \mathrm{y}\rangle_1 \ \ldots\ \langle \mathrm{feat}\rangle_n{:}\langle \mathrm{y}\rangle_n) \\ \textbf{then}\ \langle \mathrm{s}\rangle_1\ \textbf{else}\ \langle \mathrm{s}\rangle_2\ \textbf{end},\ E) \\ \hline \\ ST \\ \\ \hline \end{array} + \boxed{\sigma} \Rightarrow \begin{array}{|c|}\hline (\langle \mathrm{s}\rangle_1,\ E + \\ \{\langle \mathrm{y}\rangle_1 \rightarrow E(\langle \mathrm{x}\rangle).\ \langle \mathrm{feat}\rangle_1, \\ \ldots, \\ \langle \mathrm{y}\rangle_n \rightarrow E(\langle \mathrm{x}\rangle).\ \langle \mathrm{feat}\rangle_n\ \}) \\ \hline \\ ST \\ \\ \hline \end{array} + \boxed{\sigma}$$

Executing `case`

- If the activation condition "bound(⟨x⟩)" is `true`
  - if $E(\langle x \rangle)$ *does not* match the pattern



Example: `case` Statement

```
([(case X of
      f(X1 X2) then Y = g(X2 X1)
    else Y = c
    end,
  {X →v1, Y →v2})], % Env
  {v1=f(v3 v4), v2, v3=a, v4=b} % Store
)
```

- We declared `X`, `Y`, `X1`, `X2` as local identifiers and `X=f(v3 v4)`, `X1=a` and `X2=b`
- What is the value of `Y` after executing `case`?

Example: `case` Statement

```
([(Y = g(X2 X1),
  {X →v1, Y →v2, X1 →v3, X2 →v4})
  ],
  {v1=f(v3 v4), v2, v3=a, v4=b}
)
```

- The activation condition "bound(⟨x⟩)" is `true`
- Remember that `X1=a, X2=b`

Example: `case` Statement

```
([],
  {v1=f(v3 v4),
   v2=g(v4 v3),v3=a,v4=b}
)
```

- Remember `Y` refers to `v2`, so

  `Y = g(b a)`

Summary

- Kernel language
  - linguistic abstraction
  - data types
  - variables and partial values
  - statements and expressions
- Computing with procedures (next lecture)
  - lexical scoping
  - closures
  - procedures as values
  - procedure call