Homework 1

Andreea Munteanu, group 3E1

Machine Learning, 3rd year, Faculty of Computer Science of Iași

1 Requirement

In order to finish a board game, a player must get an exact 3 on a die. Using *scipy.stats.geom* package, determine how many tries it will take to win the game (on average). What are the best and worst cases?

2 Solution and approach

The experiment at hand consists of rolling a single die. The event of interest is rolling a 3 (what we are going to call a success). The problem deal with geometric distribution, used for determining the probability of the time of first success.

We consider X to be the random variable representing the number of die rolls until a 3 occurs.

Supposing we roll a regular die, the probability of occurrence for a 3 facet is $\frac{1}{6}$ and the probability of failure is 1-$\frac{1}{6} = \frac{5}{6}$. The die has 6 facets (therefore there are 6 possible outcomes), with numbers from 1 to 6, all of the facets being equally likely to occur, and each roll being an independent event.

S	1	2	3	4	5	6
P(S)	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$

We are asked to determine the number of tries on average until the die rolls 3, or the mean (expected value) μ of the distribution:

$$\mu = \frac{1}{p} \tag{1}$$

In our case, p is $\frac{1}{6}$, so μ will be 6.

2.1 Best and worst case

The best possible case is that we roll a 3 on our first try, for which we have a probability of occurrence of $\frac{1}{6}$.

The worst case, however, would be that we don't roll a 3 on our nth try, $n \to \infty$. The probability of success on the nth try is computed using the formula:

$$P(X = n) = p\dot{(}1 - p)^{n-1} \tag{2}$$

We find that:
Probability of success on the 1st try: $P(X = 1) = p\dot{(}1 - p)^0 = \frac{1}{6}$
Probability of success on the 2nd try: $P(X = 2) = p(1 - p)^1 = \frac{1}{6}\frac{5}{6} = \frac{5}{36}$ etc.

The expected value for the worst case then becomes:

$$E(X) = \sum_{n=1}^{\infty} n\dot{p}\dot{(}1 - p)^{n-1} = \frac{1}{p} \tag{3}$$

Finally, the expected value for the event that we don't roll a 3 until the nth die roll, $n \to \infty$ is: $E(X) = \sum_{n=1}^{\infty} n\frac{1}{6}\dot{(}\frac{5}{6})^{n-1}$