Artificial Intelligence UE
Assignment 3

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1 Predicate Logic – Backward Chaining

2 Probabilistic Inference

(A) $P(\text{crying}) =$
$0.02 + 0.07 + 0.01 + 0.03 + 0.10 + 0.04 + 0.05 + 0.01 = 33\%$

(B) $P(\text{toothache}|\neg\text{nightmare}) =$
$\frac{P(\text{toothache}\wedge\neg\text{nightmare})}{P(\neg\text{nightmare})} =$
$\frac{0.01+0.01+0.05+0.03}{0.01+0.03+0.01+0.02+0.05+0.01+0.03+0.40} =$
$\frac{0.10}{0.56} \approx 17.9\%$

(C) $P(\text{excited} \wedge \text{nightmare}) =$
$0.02 + 0.07 + 0.01 + 0.05 = 15\%$

(D) $P((\text{nightmare} \wedge \neg\text{toothache})|\neg\text{crying}) =$
$\frac{P(\text{nightmare}\wedge\neg\text{toothache}\wedge\neg\text{crying})}{P(\neg\text{crying})} =$
$\frac{0.05+0.10}{0.01+0.05+0.01+0.02+0.05+0.10+0.03+0.40} =$
$\frac{0.15}{0.67} \approx 22.4\%$

(E) $P(\neg\text{excited}|(\text{excited} \wedge \text{toothache})) =$
$\frac{P(\neg\text{excited}\wedge\text{excited}\wedge\text{toothache})}{P(\text{excited}\wedge\text{toothache})} = 0$

(F) $P(\neg\text{nightmare}) =$
$0.01 + 0.03 + 0.01 + 0.02 + 0.05 + 0.01 + 0.03 + 0.40 = 56\%$

(G) $P(\text{crying} \vee \neg\text{crying}) = 1$

Variable	Parents	Values
passable tiles		0, 1, 2, 3, 4
marker		T, F
rainbow seed		T, F
control	passable tiles	T, F
interesting	marker, control	T, F
dangerous	passable tiles, rainbow seed	T, F
Move	interesting, dangerous	T, F

Table 1: The variables and their parents.

(H) $P((\text{toothache} \wedge \text{nightmare}) \vee (\text{crying} \wedge \text{excited})) =$
$0.02 + 0.07 + 0.01 + 0.01 + 0.03 + 0.10 + 0.05 = 29\%$

(I) $P(\text{crying}|(\text{toothache} \wedge \text{nightmare})) =$
$\frac{P(\text{crying}\wedge\text{toothache}\wedge\text{nightmare})}{P(\text{toothache}\wedge\text{nightmare})} =$
$\frac{0.02+0.10}{0.02+0.01+0.10+0.05} = \frac{0.12}{0.18} \approx 66.7\%$

(J) $P(\text{toothache} \vee \text{excited}) =$
$0.02 + 0.07 + 0.01 + 0.05 + 0.01 + 0.03 + 0.01 + 0.02 + 0.10 + 0.05 + 0.05 + 0.03 = 45\%$

3 Bayesian Nets – Constructing a Net

(A) We order the set of variables such that causes precede effects. Next, we determine for each variable a minimal set of parents. The variables with their set of parents is given in table 1. Based this table, the network is constructed. The resulting Baysian network for this domain is depicted in figure 1.

(B) The values of each variable are listed in table 1. There are
$5\times2\times2\times2\times2\times2\times2-1 = 319$ independent values in the joint probability distribution. The network tables would contain
$4 + 1 + 1 + 5 \times 1 + 2 \times 2 \times 1 + 5 \times 2 \times 1 + 2 \times 2 \times 1 = 29$ independent values.

(C)
i) $P(\text{interesting}|\text{control}) \neq (\text{interesting}|\text{control}, \text{rainbow seed})$

ii) $P(\text{dangerous}) \neq P(\text{dangerous}|\text{interesting})$

iii) $P(\text{control}|\text{passable tiles}) = P(\text{control}|\text{passable tiles}, \text{marker})$
$P(\text{control}|\text{passable tiles}) \neq P(\text{control}|\text{passable tiles}, \text{Move})$

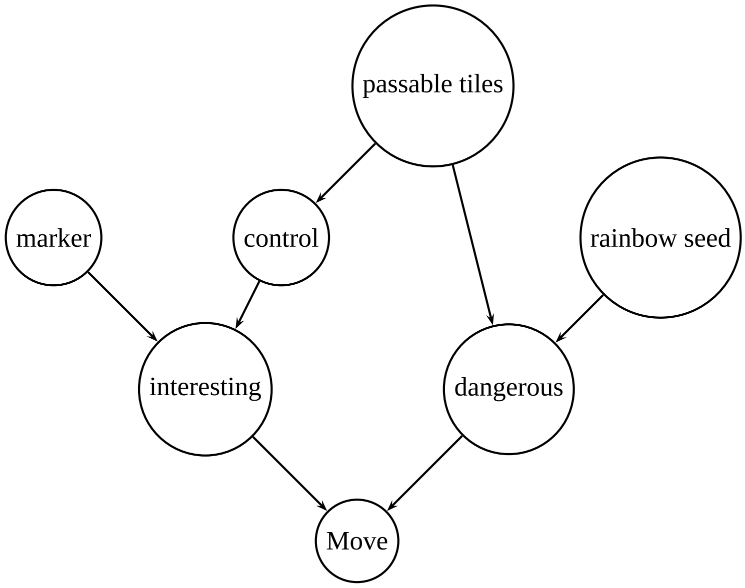

Figure 1: The Baysian network for this domain.

4 Bayesian Nets – Inference by Enumeration

(A) $P(m|s,\neg h) = \alpha \sum_{R\in\{r,\neg r\}} \sum_{P\in\{p,\neg p\}} \sum_{W\in\{w,\neg w\}} P(R,P,s,m,W,\neg h) =$

$$
\begin{array}{lllllll}
\alpha[\, P(\, r, & p, & s, & m, & w, & \neg h &)+ \\
P(\, r, & p, & s, & m, & \neg w, & \neg h &)+ \\
P(\, r, & \neg p, & s, & m, & w, & \neg h &)+ \\
P(\, r, & \neg p, & s, & m, & \neg w, & \neg h &)+ \\
P(\, \neg r, & p, & s, & m, & w, & \neg h &)+ \\
P(\, \neg r, & p, & s, & m, & \neg w, & \neg h &)+ \\
P(\, \neg r, & \neg p, & s, & m, & w, & \neg h &)+ \\
P(\, \neg r, & \neg p, & s, & m, & \neg w, & \neg h &)] =
\end{array}
$$

$$
\begin{array}{llllllll}
\alpha[& P(r) & P(p) & P(s|r,p) & P(m|s) & P(w|m) & P(\neg h|s,w) & + \\
 & P(r) & P(p) & P(s|r,p) & P(m|s) & P(\neg w|m) & P(\neg h|s,\neg w) & + \\
 & P(r) & P(\neg p) & P(s|r,\neg p) & P(m|s) & P(w|m) & P(\neg h|s,w) & + \\
 & P(r) & P(\neg p) & P(s|r,\neg p) & P(m|s) & P(\neg w|m) & P(\neg h|s,\neg w) & + \\
 & P(\neg r) & P(p) & P(s|\neg r,p) & P(m|s) & P(w|m) & P(\neg h|s,w) & + \\
 & P(\neg r) & P(p) & P(s|\neg r,p) & P(m|s) & P(\neg w|m) & P(\neg h|s,\neg w) & + \\
 & P(\neg r) & P(\neg p) & P(s|\neg r,\neg p) & P(m|s) & P(w|m) & P(\neg h|s,w) & + \\
 & P(\neg r) & P(\neg p) & P(s|\neg r,\neg p) & P(m|s) & P(\neg w|m) & P(\neg h|s,\neg w) &] = \\
\alpha[& 0.6\cdot & 0.3\cdot & 0.5\cdot & 0.1\cdot & 0.6\cdot & 0.70 & +
\end{array}
$$

$$
\begin{array}{lllllll}
0.6\cdot & 0.3\cdot & 0.5\cdot & 0.1\cdot & 0.4\cdot & 0.99 & +\\
0.6\cdot & 0.7\cdot & 0.9\cdot & 0.1\cdot & 0.6\cdot & 0.70 & +\\
0.6\cdot & 0.7\cdot & 0.9\cdot & 0.1\cdot & 0.4\cdot & 0.99 & +\\
0.4\cdot & 0.3\cdot & 0.1\cdot & 0.1\cdot & 0.6\cdot & 0.70 & +\\
0.4\cdot & 0.3\cdot & 0.1\cdot & 0.1\cdot & 0.4\cdot & 0.99 & +\\
0.4\cdot & 0.7\cdot & 0.2\cdot & 0.1\cdot & 0.6\cdot & 0.70 & +\\
0.4\cdot & 0.7\cdot & 0.2\cdot & 0.1\cdot & 0.4\cdot & 0.99 &] \approx 0.0437\alpha
\end{array}
$$

$$P(\neg m|s,\neg h) = \alpha \sum_{R\in\{r,\neg r\}} \sum_{P\in\{p,\neg p\}} \sum_{W\in\{w,\neg w\}} P(R,P,s,\neg m,W,\neg h) =$$

$$
\begin{array}{lllllllll}
\alpha[& P(& r, & p, & s, & \neg m, & w, & \neg h &)+\\
& P(& r, & p, & s, & \neg m, & \neg w, & \neg h &)+\\
& P(& r, & \neg p, & s, & \neg m, & w, & \neg h &)+\\
& P(& r, & \neg p, & s, & \neg m, & \neg w, & \neg h &)+\\
& P(& \neg r, & p, & s, & \neg m, & w, & \neg h &)+\\
& P(& \neg r, & p, & s, & \neg m, & \neg w, & \neg h &)+\\
& P(& \neg r, & \neg p, & s, & \neg m, & w, & \neg h &)+\\
& P(& \neg r, & \neg p, & s, & \neg m, & \neg w, & \neg h &)] =
\end{array}
$$

$$
\begin{array}{llllllll}
\alpha[& P(r) & P(p) & P(s|r,p) & P(\neg m|s) & P(w|\neg m) & P(\neg h|s,w) & +\\
& P(r) & P(p) & P(s|r,p) & P(\neg m|s) & P(\neg w|\neg m) & P(\neg h|s,\neg w) & +\\
& P(r) & P(\neg p) & P(s|r,\neg p) & P(\neg m|s) & P(w|\neg m) & P(\neg h|s,w) & +\\
& P(r) & P(\neg p) & P(s|r,\neg p) & P(\neg m|s) & P(\neg w|\neg m) & P(\neg h|s,\neg w) & +\\
& P(\neg r) & P(p) & P(s|\neg r,p) & P(\neg m|s) & P(w|\neg m) & P(\neg h|s,w) & +\\
& P(\neg r) & P(p) & P(s|\neg r,p) & P(\neg m|s) & P(\neg w|\neg m) & P(\neg h|s,\neg w) & +\\
& P(\neg r) & P(\neg p) & P(s|\neg r,\neg p) & P(\neg m|s) & P(w|\neg m) & P(\neg h|s,w) & +\\
& P(\neg r) & P(\neg p) & P(s|\neg r,\neg p) & P(\neg m|s) & P(\neg w|\neg m) & P(\neg h|s,\neg w) &] =
\end{array}
$$

$$
\begin{array}{llllllll}
\alpha[& 0.6\cdot & 0.3\cdot & 0.5\cdot & 0.9\cdot & 0.1\cdot & 0.70 & +\\
& 0.6\cdot & 0.3\cdot & 0.5\cdot & 0.9\cdot & 0.9\cdot & 0.99 & +\\
& 0.6\cdot & 0.7\cdot & 0.9\cdot & 0.9\cdot & 0.1\cdot & 0.70 & +\\
& 0.6\cdot & 0.7\cdot & 0.9\cdot & 0.9\cdot & 0.9\cdot & 0.99 & +\\
& 0.4\cdot & 0.3\cdot & 0.1\cdot & 0.9\cdot & 0.1\cdot & 0.70 & +\\
& 0.4\cdot & 0.3\cdot & 0.1\cdot & 0.9\cdot & 0.9\cdot & 0.99 & +\\
& 0.4\cdot & 0.7\cdot & 0.2\cdot & 0.9\cdot & 0.1\cdot & 0.70 & +\\
& 0.4\cdot & 0.7\cdot & 0.2\cdot & 0.9\cdot & 0.9\cdot & 0.99 &] \approx 0.4636\alpha
\end{array}
$$

$$P(m|s,\neg h) + P(\neg m|s,\neg h) = 1 \Rightarrow \alpha \approx \frac{1}{0.0437+0.4636} \approx 1.971$$

$$P(M|s,\neg h) \approx (0.0437, 0.4636)\alpha \approx (0.086, 0.914)$$

(B) $P(w|\neg p,\neg r,h) = \alpha \sum_{S\in\{s,\neg s\}} \sum_{M\in\{m,\neg m\}} P(\neg r,\neg p,S,M,w,h) =$

$$
\begin{array}{lllllllll}
\alpha[& P(& \neg r, & \neg p, & s, & m, & w, & h &)+\\
& P(& \neg r, & \neg p, & s, & \neg m, & w, & h &)+\\
& P(& \neg r, & \neg p, & \neg s, & m, & w, & h &)+\\
& P(& \neg r, & \neg p, & \neg s, & \neg m, & w, & h &)] =
\end{array}
$$

$$
\begin{array}{llllllll}
\alpha[& P(\neg r) & P(\neg p) & P(s|\neg r,\neg p) & P(m|s) & P(w|m) & P(h|s,w) & +
\end{array}
$$

$$
\begin{array}{lllllllll}
& P(\neg r) & P(\neg p) & P(s|\neg r,\neg p) & P(\neg m|s) & P(w|\neg m) & P(h|s,w) & + \\
& P(\neg r) & P(\neg p) & P(\neg s|\neg r,\neg p) & P(m|\neg s) & P(w|m) & P(h|\neg s,w) & + \\
& P(\neg r) & P(\neg p) & P(\neg s|\neg r,\neg p) & P(\neg m|\neg s) & P(w|\neg m) & P(h|\neg s,w) & = \\
& 0.4\cdot & 0.7\cdot & 0.2\cdot & 0.1\cdot & 0.6\cdot & 0.30 & + \\
& 0.4\cdot & 0.7\cdot & 0.2\cdot & 0.9\cdot & 0.1\cdot & 0.30 & + \\
& 0.4\cdot & 0.7\cdot & 0.8\cdot & 0.5\cdot & 0.6\cdot & 0.90 & + \\
& 0.4\cdot & 0.7\cdot & 0.8\cdot & 0.5\cdot & 0.1\cdot & 0.90 &] \approx 0.0731\alpha
\end{array}
$$

$$P(\neg w|\neg p,\neg r,h) = \alpha \sum_{S\in\{s,\neg s\}} \sum_{M\in\{m,\neg m\}} P(\neg r,\neg p,S,M,\neg w,h) =$$

$$
\begin{array}{lllllllll}
\alpha[\ P(& \neg r, & \neg p, & s, & m, & \neg w, & h &)+ \\
\ \ P(& \neg r, & \neg p, & s, & \neg m, & \neg w, & h &)+ \\
\ \ P(& \neg r, & \neg p, & \neg s, & m, & \neg w, & h &)+ \\
\ \ P(& \neg r, & \neg p, & \neg s, & \neg m, & \neg w, & h &)] =
\end{array}
$$

$$
\begin{array}{lllllllll}
\alpha[& P(\neg r) & P(\neg p) & P(s|\neg r,\neg p) & P(m|s) & P(\neg w|m) & P(h|s,\neg w) & + \\
& P(\neg r) & P(\neg p) & P(s|\neg r,\neg p) & P(\neg m|s) & P(\neg w|\neg m) & P(h|s,\neg w) & + \\
& P(\neg r) & P(\neg p) & P(\neg s|\neg r,\neg p) & P(m|\neg s) & P(\neg w|m) & P(h|\neg s,\neg w) & + \\
& P(\neg r) & P(\neg p) & P(\neg s|\neg r,\neg p) & P(\neg m|\neg s) & P(\neg w|\neg m) & P(h|\neg s,\neg w) & = \\
& 0.4\cdot & 0.7\cdot & 0.2\cdot & 0.1\cdot & 0.4\cdot & 0.01 & + \\
& 0.4\cdot & 0.7\cdot & 0.2\cdot & 0.9\cdot & 0.9\cdot & 0.01 & + \\
& 0.4\cdot & 0.7\cdot & 0.8\cdot & 0.5\cdot & 0.4\cdot & 0.50 & + \\
& 0.4\cdot & 0.7\cdot & 0.8\cdot & 0.5\cdot & 0.9\cdot & 0.50 &] \approx 0.0732\alpha
\end{array}
$$

$$P(w|\neg p,\neg r,h) + P(\neg w|\neg p,\neg r,h) = 1 \Rightarrow \alpha \approx \frac{1}{0.0731+0.0732} \approx 6.833$$

$$P(W|\neg p,\neg r,h) \approx (0.0731, 0.0732)\alpha \approx (0.499, 0.501)$$