Artificial Intelligence UE
Assignment 3

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12th January 2014

1 Predicate Logic – Backward Chaining

The expansion steps for the backward chaining algorthm are listed in table 1. The corresponding trees are depicted in figure 1 to figure 21. There is no substitution that results in ThinksSomethingIsWrongWithOurSociety(x) for any x.

Depth	Applied rules	Substitutions	Tree
0		$\{\}$	figure 1
1	3	$\{\}$	figure 2
2	3, 4	$\{\}$	figure 3
3	3, 4, 1	$\{\}$	figure 4
4	3, 4, 1, 2	$\{\}$	figure 5
5	3, 4, 1, 2, 5	$\{\}$	figure 6
6	3, 4, 1, 2, 5, 6	$\{\}$	figure 7
7	3, 4, 1, 2, 5, 6, 7	$\{\}$	figure 8
8	3, 4, 1, 2, 5, 6, 7, 8	$\{b/\text{CIA}\}$	figure 9
9	3, 4, 1, 2, 5, 6, 7, 8, 9	$\{b/\text{CIA}\}$	figure 10
10	3, 4, 1, 2, 5, 6, 7, 8, 9, 10	$\{b/\text{CIA}\}$	figure 11
11	3, 4, 1, 2, 5, 6, 7, 8, 9, 10, 11	$\{b/\text{CIA}\}$	figure 12
12	3, 4, 1, 2, 5, 6, 7, 8, 9, 10, 11, 12	$\{b/\text{CIA}, x/\text{Me}\}$	figure 13
13	3, 4, 1, 2, 5, 6, 7, 8, 9, 10, 11, 12, 13	$\{b/\text{CIA}, x/\text{Me}\}$	figure 14
14	3, 4, 1, 2, 5, 6, 7, 8, 9, 10, 11, 12, 13, 15	$\{b/\text{CIA}, x/\text{Me}, a/\text{Aliens}\}$	figure 15
15	3, 4, 1, 2, 5, 6, 7, 8, 9, 10, 11, 12, 13, 15, 17	$\{b/\text{CIA}, x/\text{Me}, a/\text{Aliens}\}$	figure 16
12	3, 4, 1, 2, 5, 6, 7, 8, 9, 10, 11, 14	$\{b/\text{CIA}, x/\text{Fox}\}$	figure 17
13	3, 4, 1, 2, 5, 6, 7, 8, 9, 10, 11, 14, 16	$\{b/\text{CIA}, x/\text{Fox}\}$	figure 18
14	3, 4, 1, 2, 5, 6, 7, 8, 9, 10, 11, 14, 16, 19	$\{b/\text{CIA}, x/\text{Fox}\}$	figure 19
15	3, 4, 1, 2, 5, 6, 7, 8, 9, 10, 11, 14, 16, 19, 20	$\{b/\text{CIA}, x/\text{Fox}\}$	figure 20
12	3, 4, 1, 2, 5, 6, 7, 8, 9, 10, 11, 18	$\{b/\text{CIA}, x/\text{You}\}$	figure 21

Table 1: The expansion of the tree using backward chaining.

$\boxed{\text{ThinksSomethingIsWrongWithOurSociety}(x)}$

Figure 1: The inital proof tree.

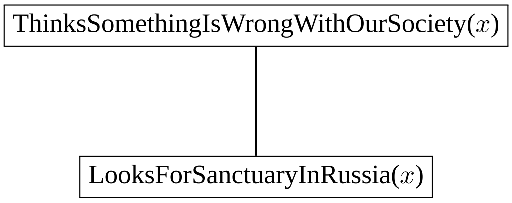

Figure 2: The proof tree at depth 1 (rule 3).

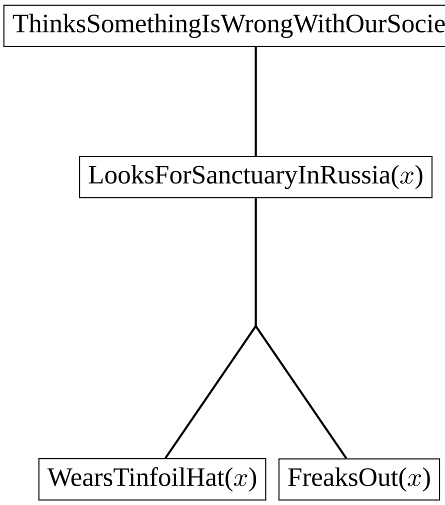

Figure 3: The proof tree at depth 2 (rule 3, 4).

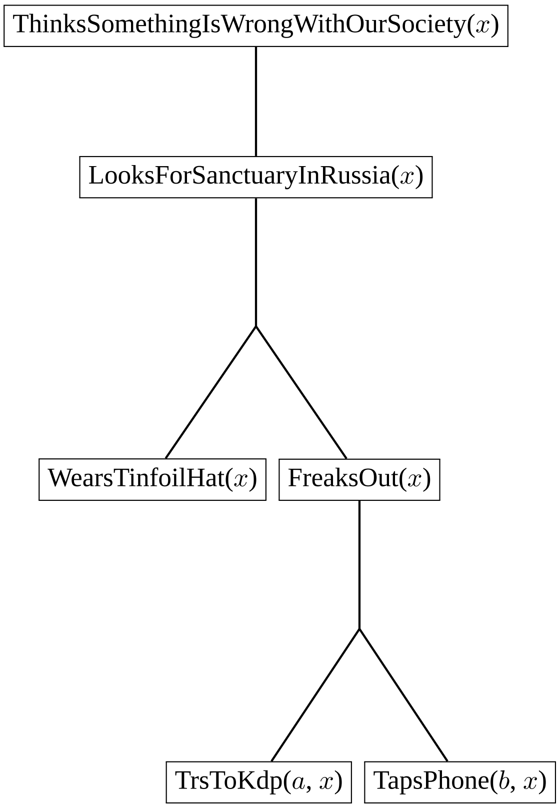

Figure 4: The proof tree at depth 3 (rule 3, 4, 1).

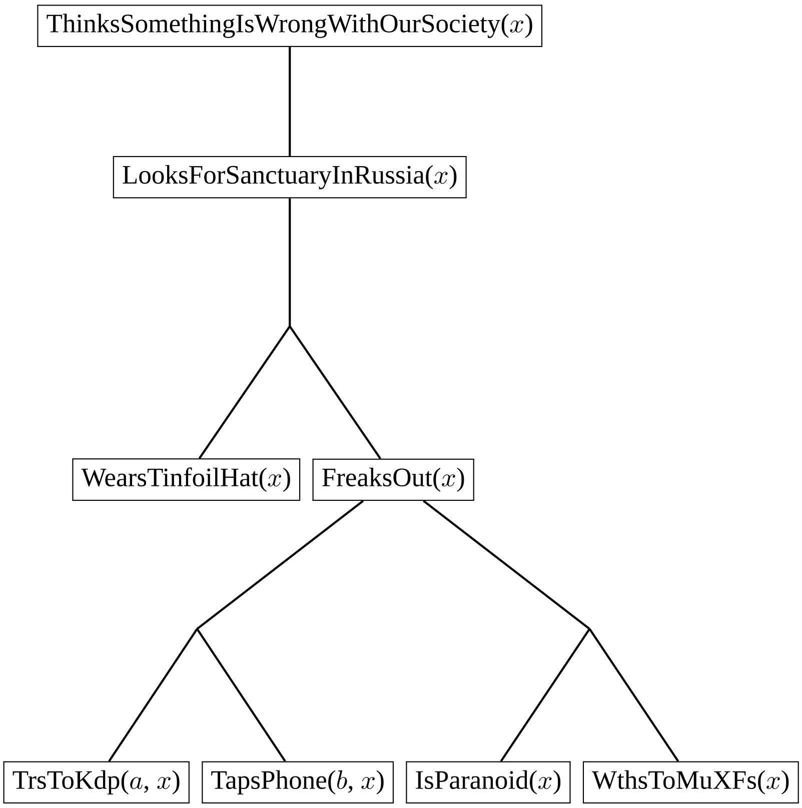

Figure 5: The proof tree at depth 4 (rule 3, 4, 1, 2).

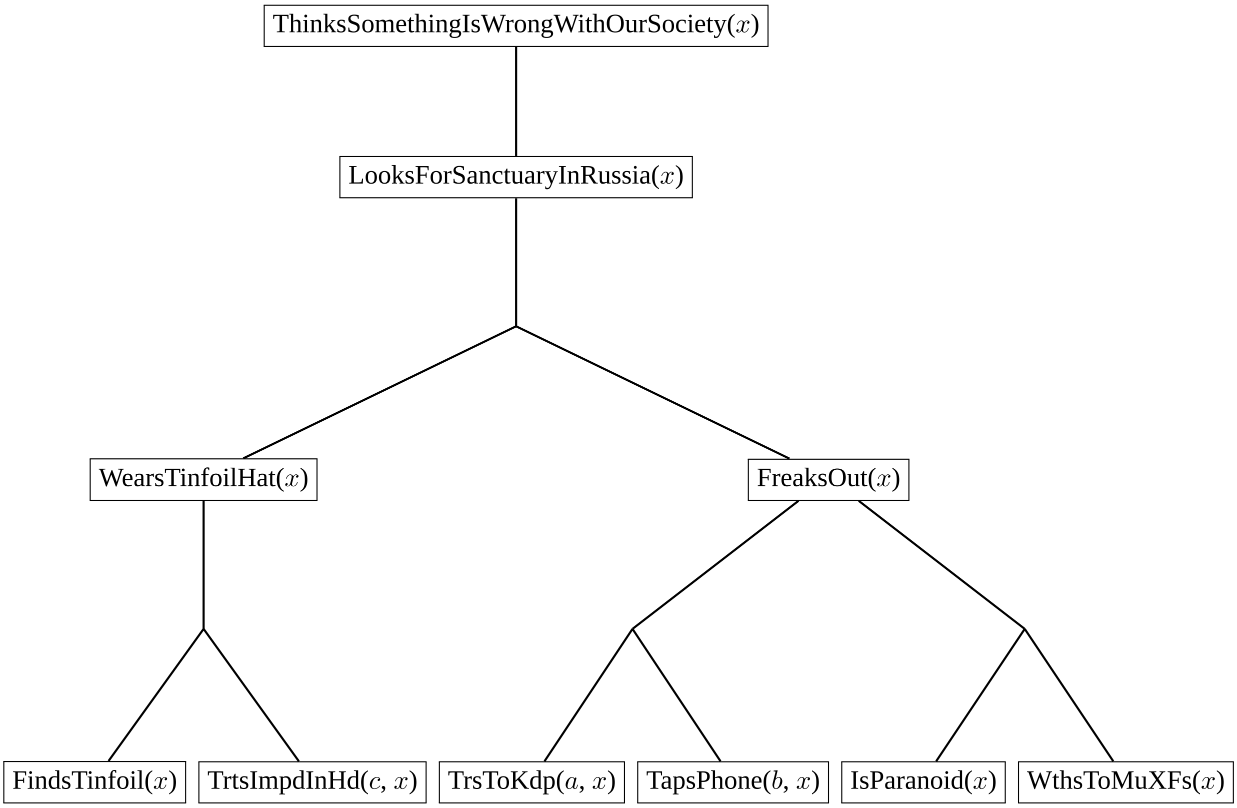

Figure 6: The proof tree at depth 5 (rule 3, 4, 1, 2, 5).

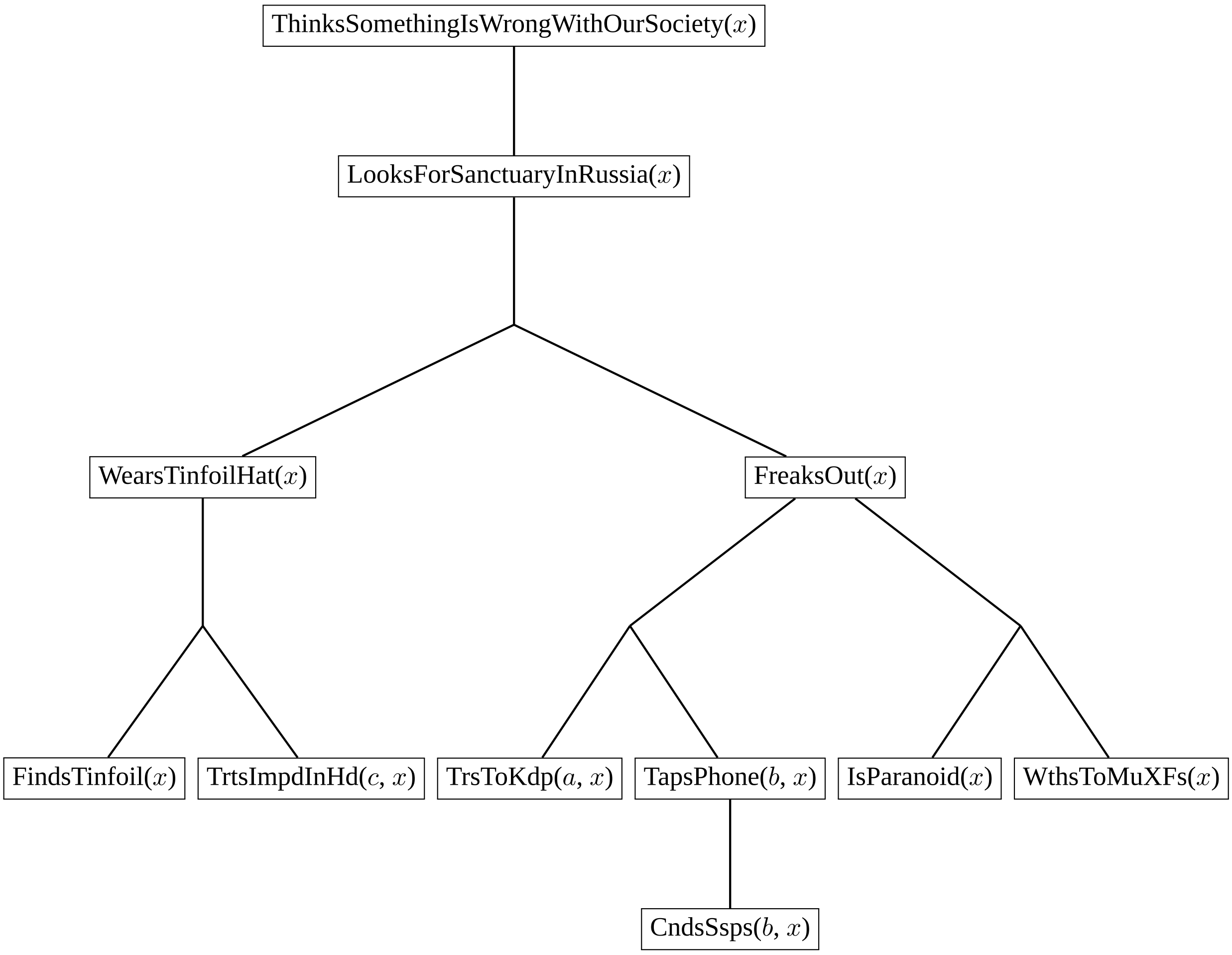

Figure 7: The proof tree at depth 6 (rule 3, 4, 1, 2, 5, 6).

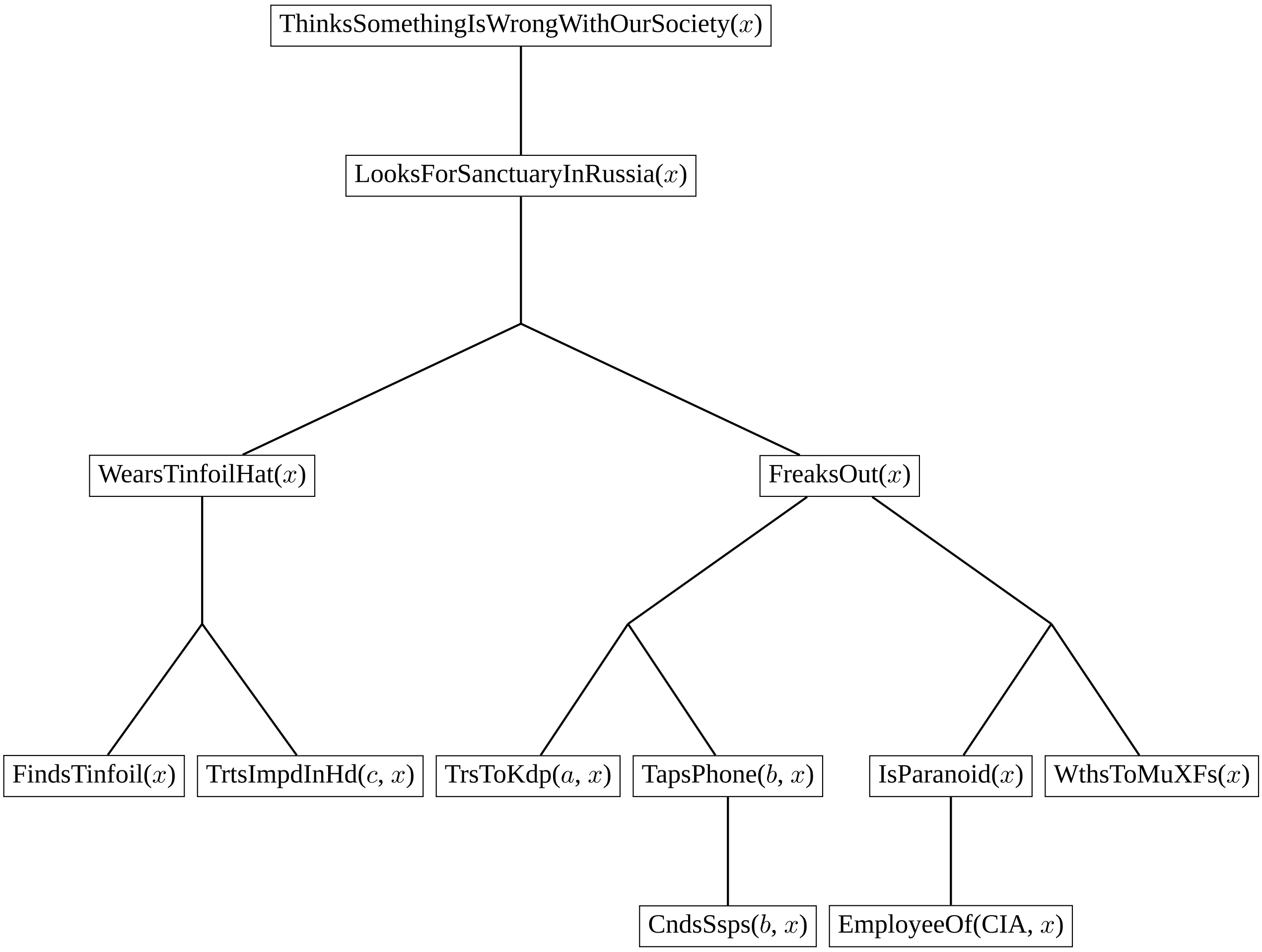

Figure 8: The proof tree at depth 7 (rule 3, 4, 1, 2, 5, 6, 7).

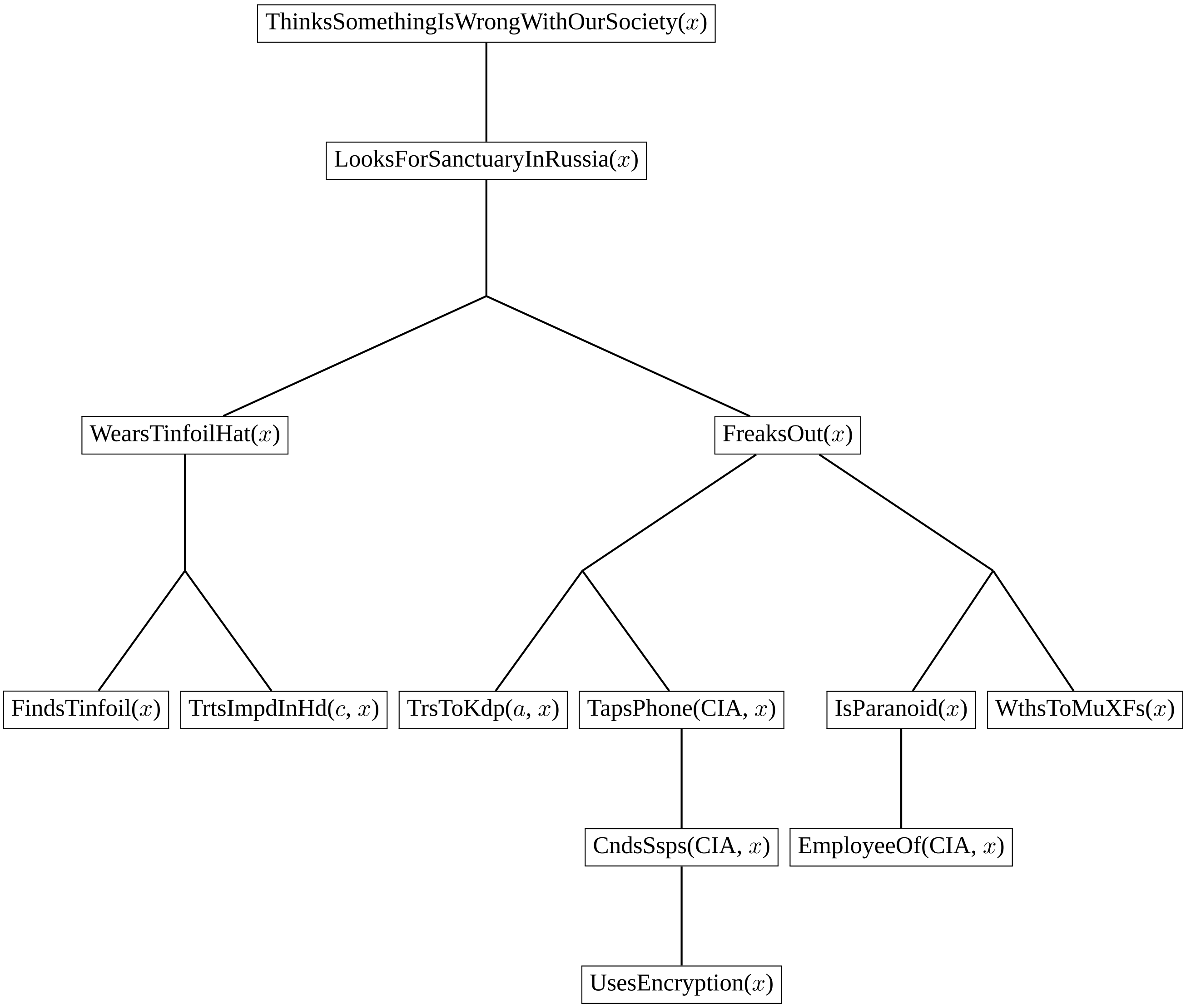

Figure 9: The proof tree at depth 8 (rule 3, 4, 1, 2, 5, 6, 7, 8).

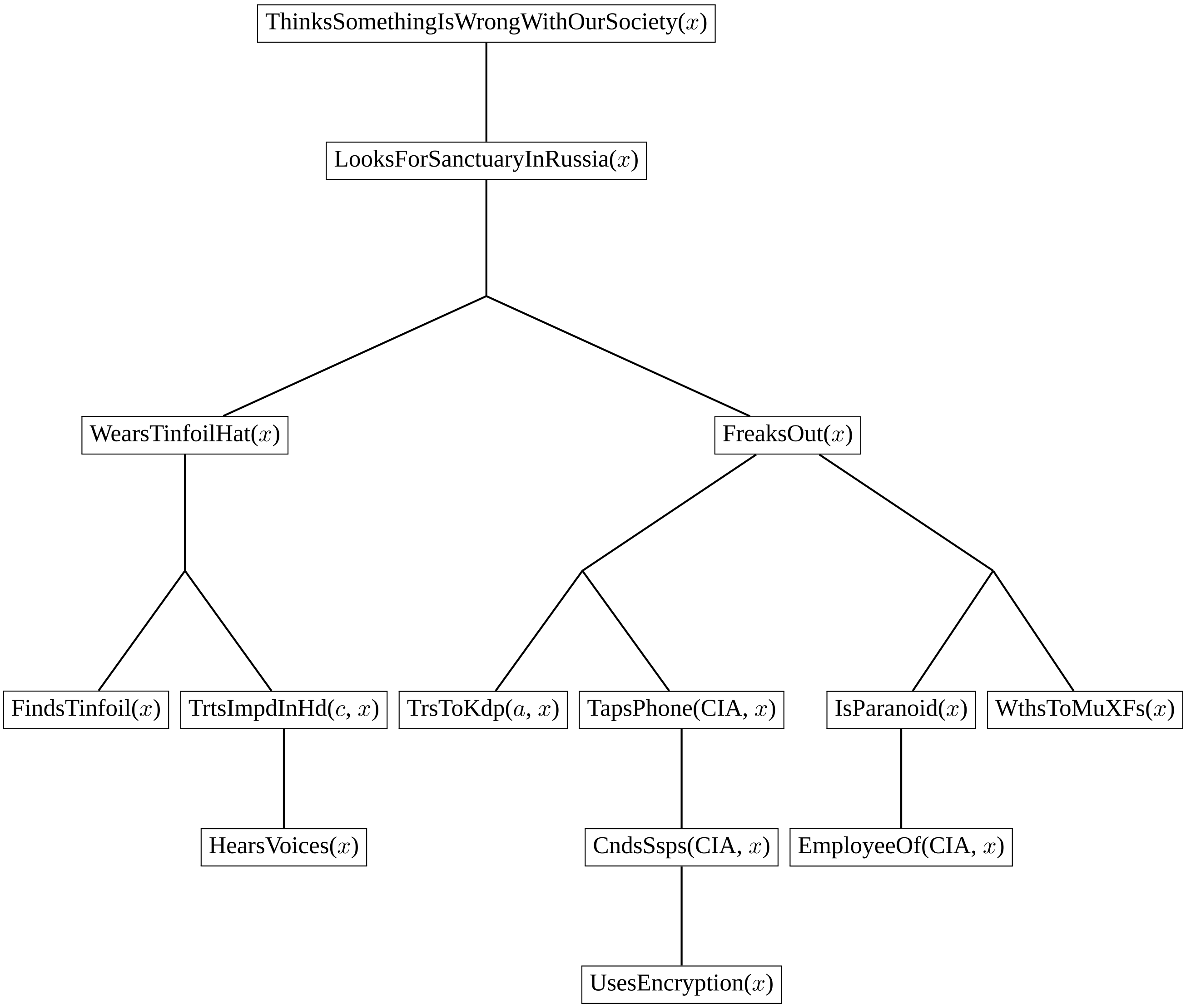

Figure 10: The proof tree at depth 9 (rule 3, 4, 1, 2, 5, 6, 7, 8, 9).

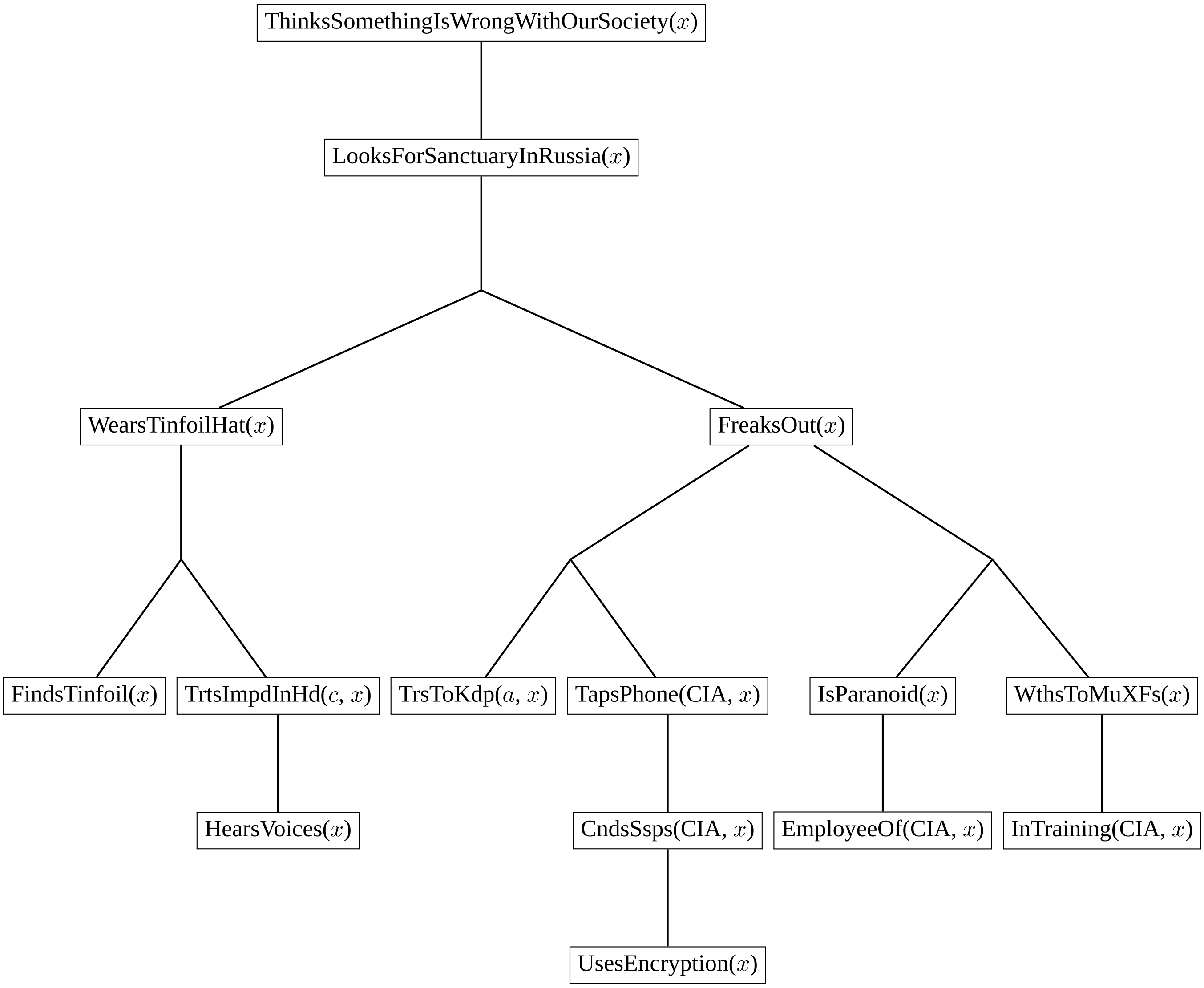

Figure 11: The proof tree at depth 10 (rule 3, 4, 1, 2, 5, 6, 7, 8, 9, 10).

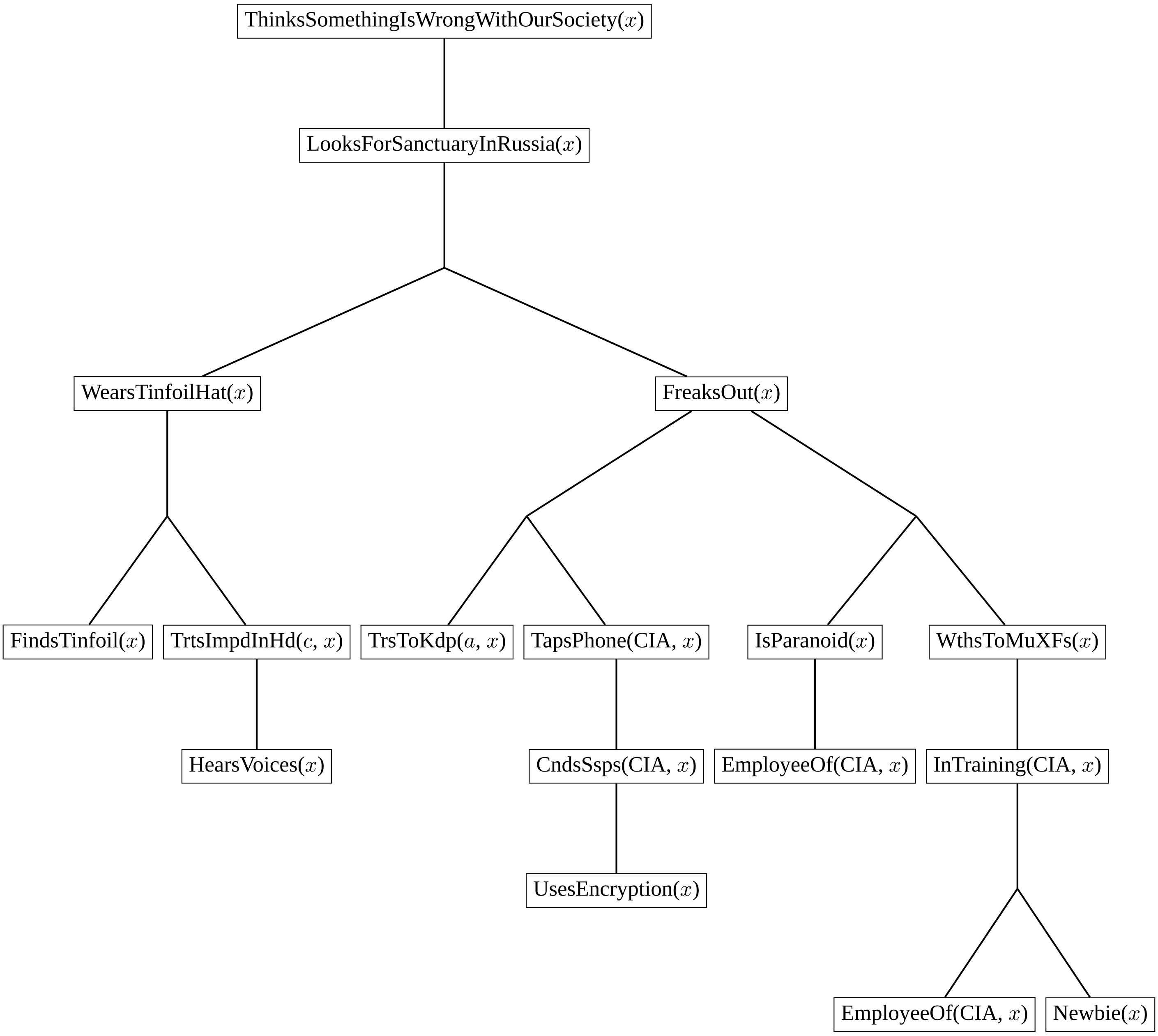

Figure 12: The proof tree at depth 11 (rule 3, 4, 1, 2, 5, 6, 7, 8, 9, 10, 11).

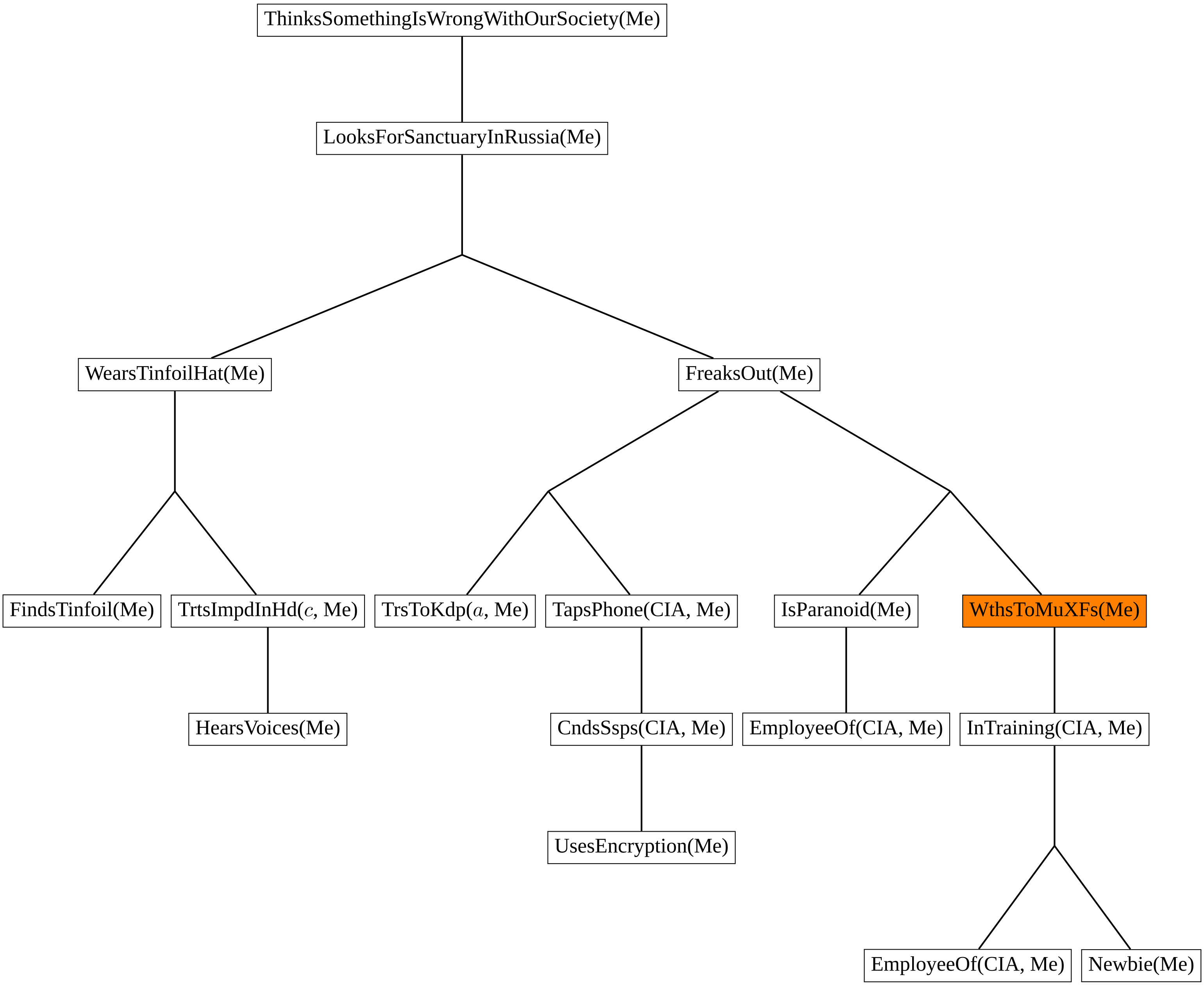

Figure 13: The proof tree at depth 12 (rule 3, 4, 1, 2, 5, 6, 7, 8, 9, 10, 11, 12).

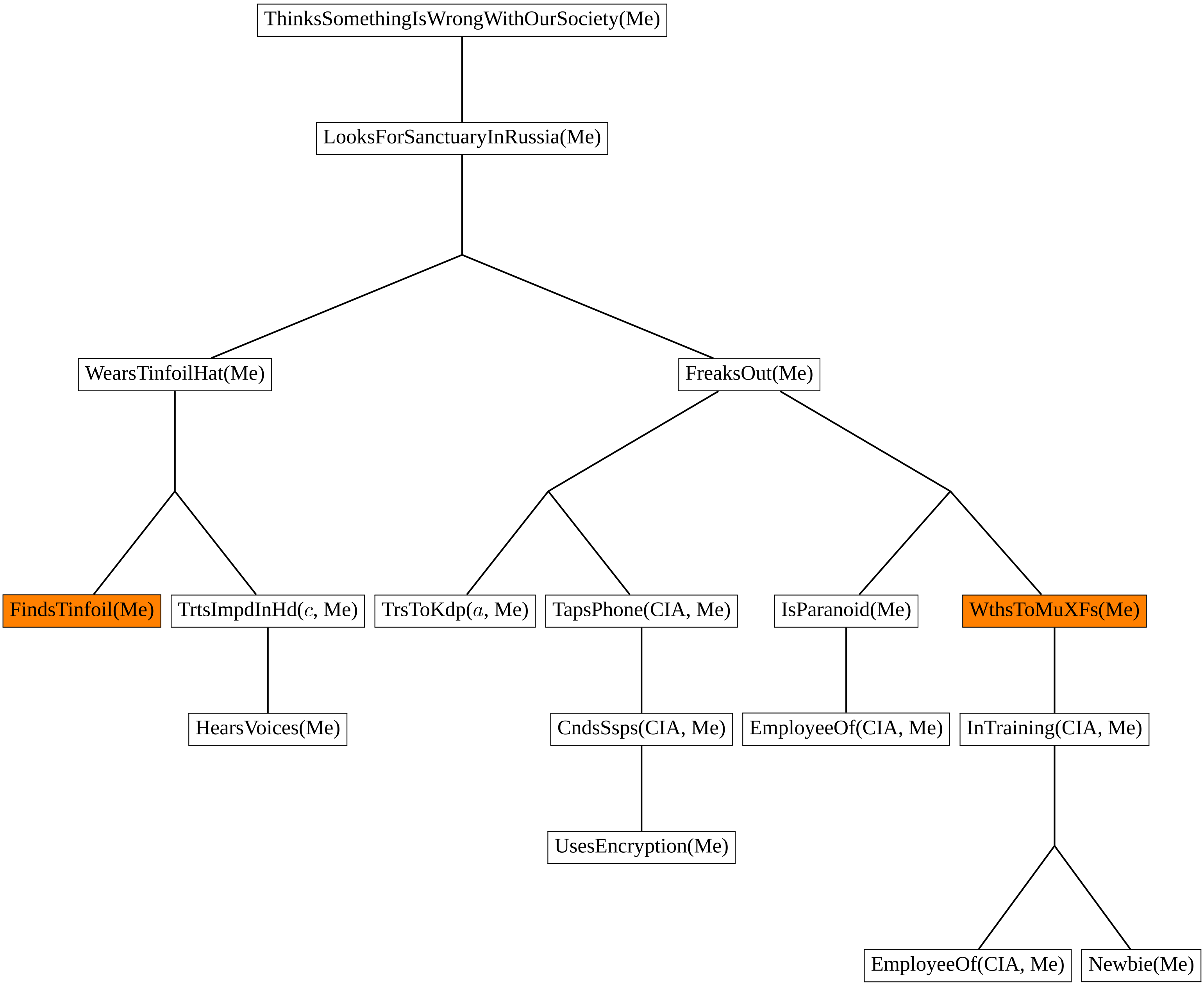

Figure 14: The proof tree at depth 13 (rule 3, 4, 1, 2, 5, 6, 7, 8, 9, 10, 11, 12, 13).

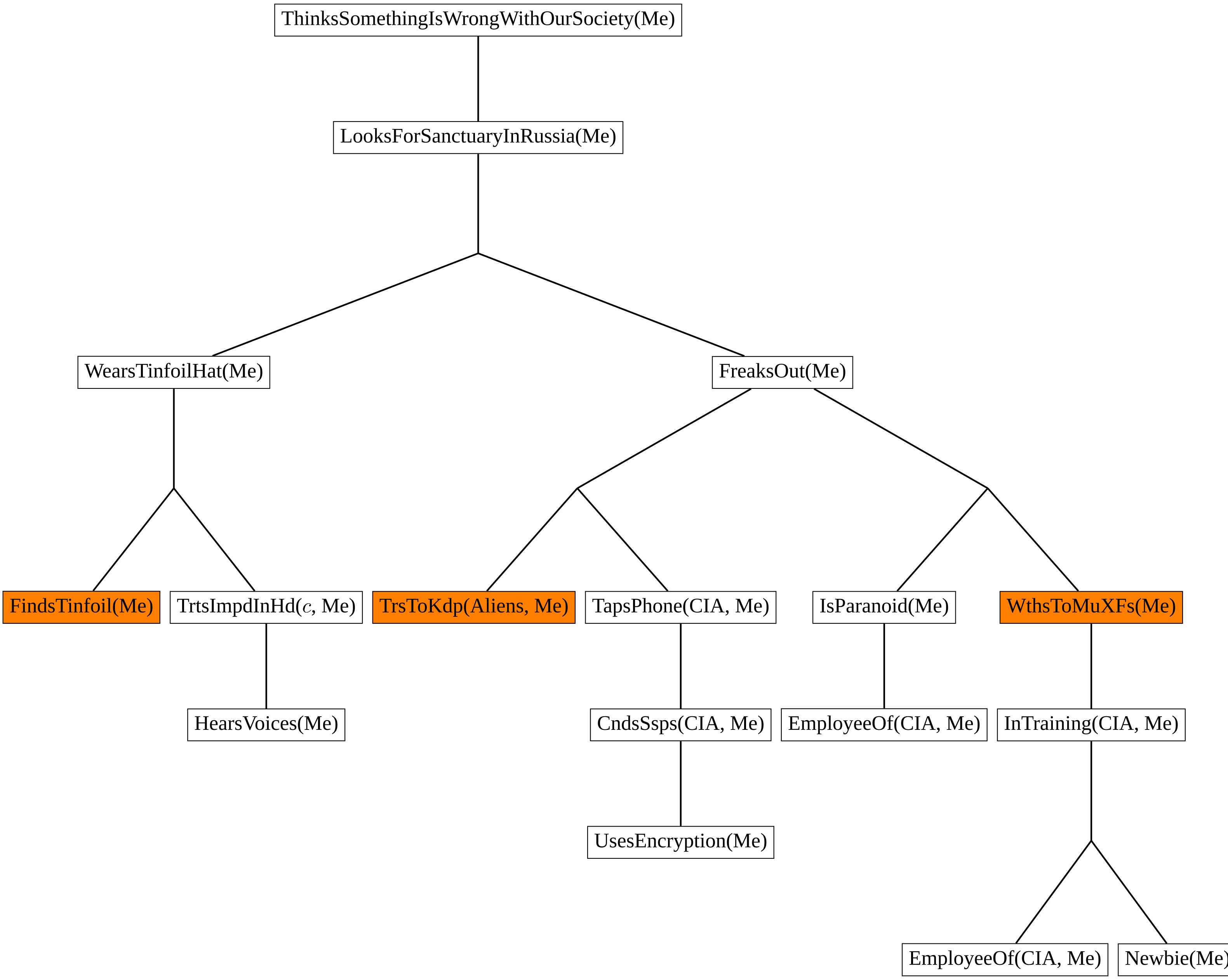

Figure 15: The proof tree at depth 14 (rule 3, 4, 1, 2, 5, 6, 7, 8, 9, 10, 11, 12, 13, 15).

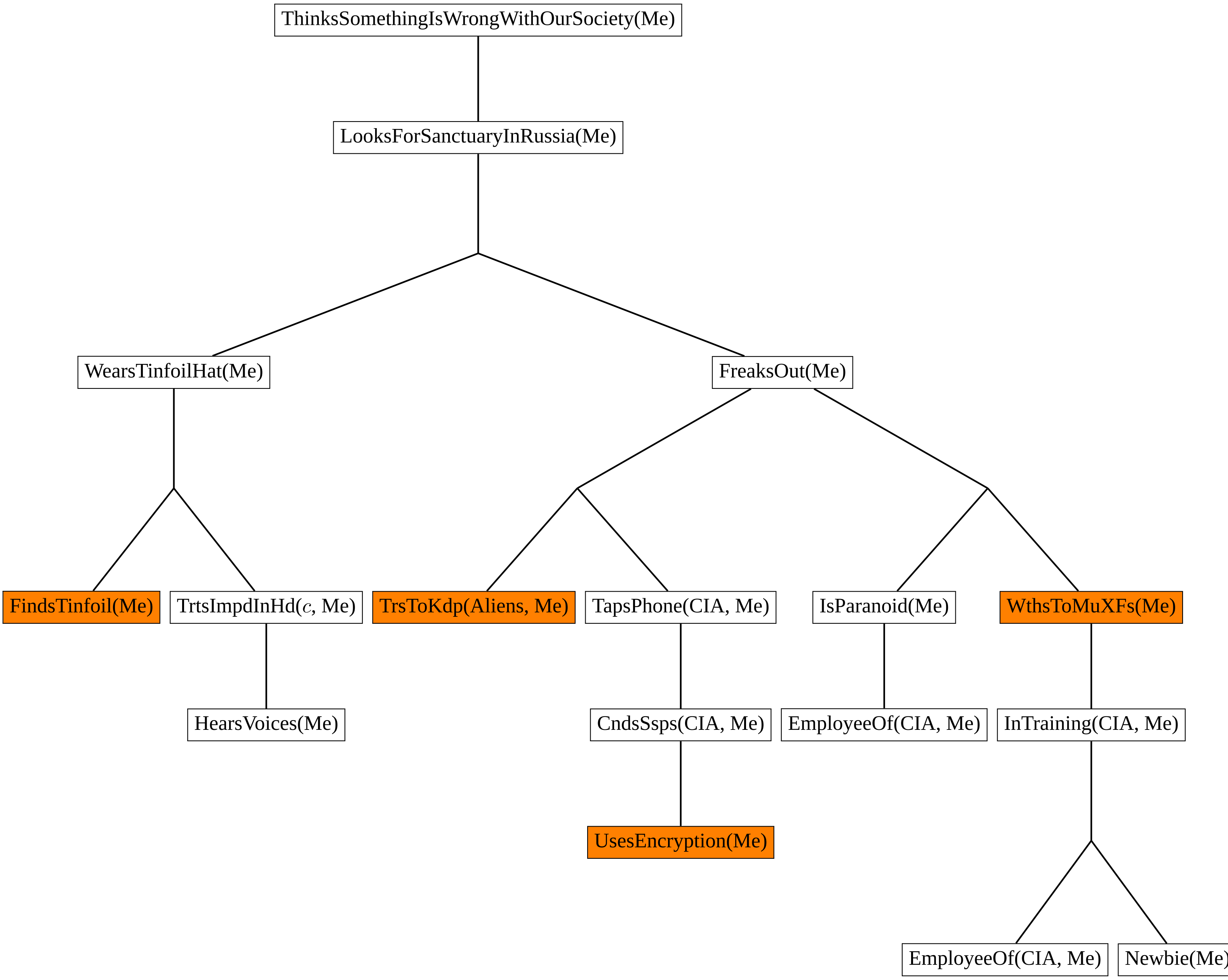

Figure 16: The proof tree at depth 15 (rule 3, 4, 1, 2, 5, 6, 7, 8, 9, 10, 11, 12, 13, 15, 17).

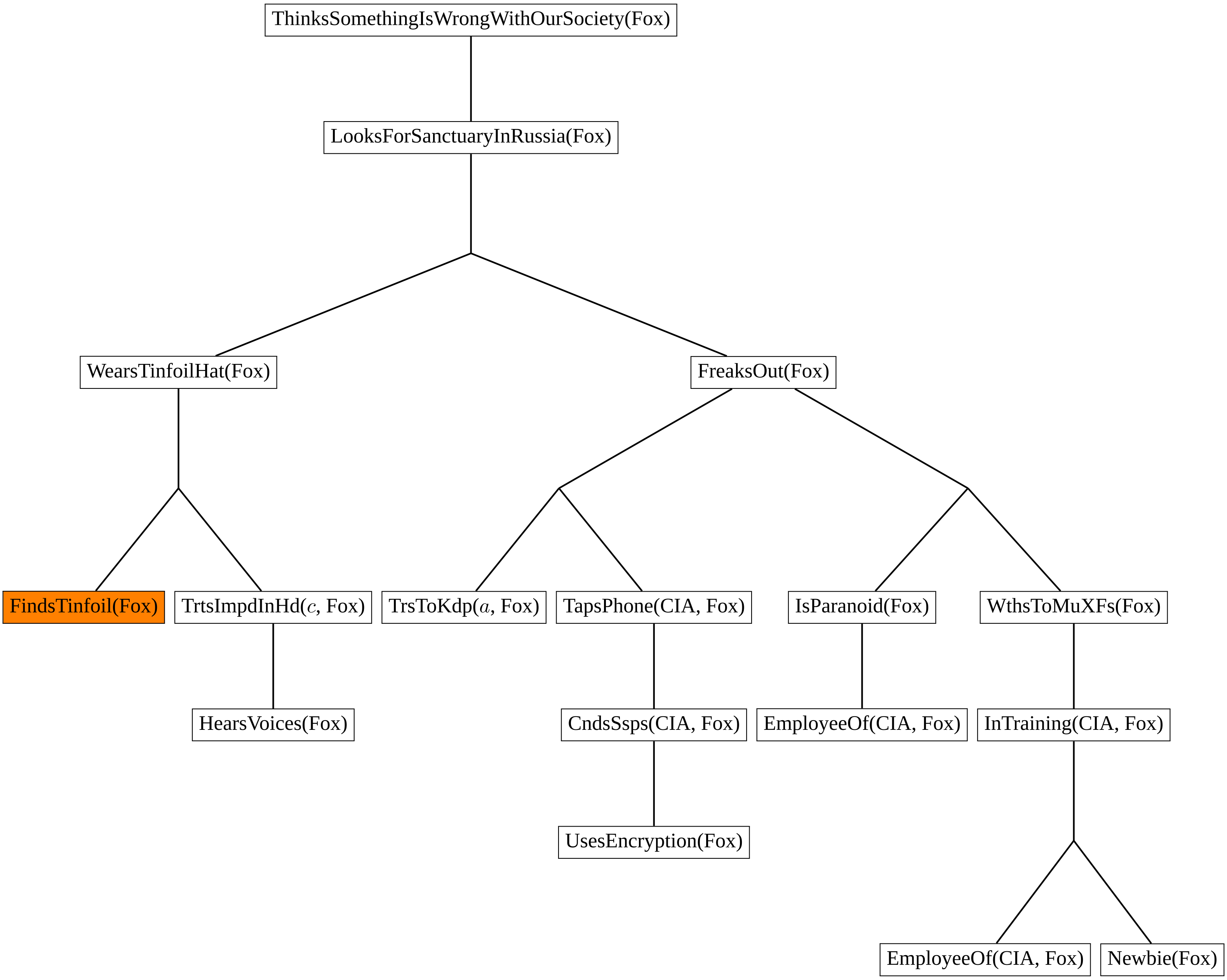

Figure 17: The proof tree at depth 12 (rule 3, 4, 1, 2, 5, 6, 7, 8, 9, 10, 11, 14).

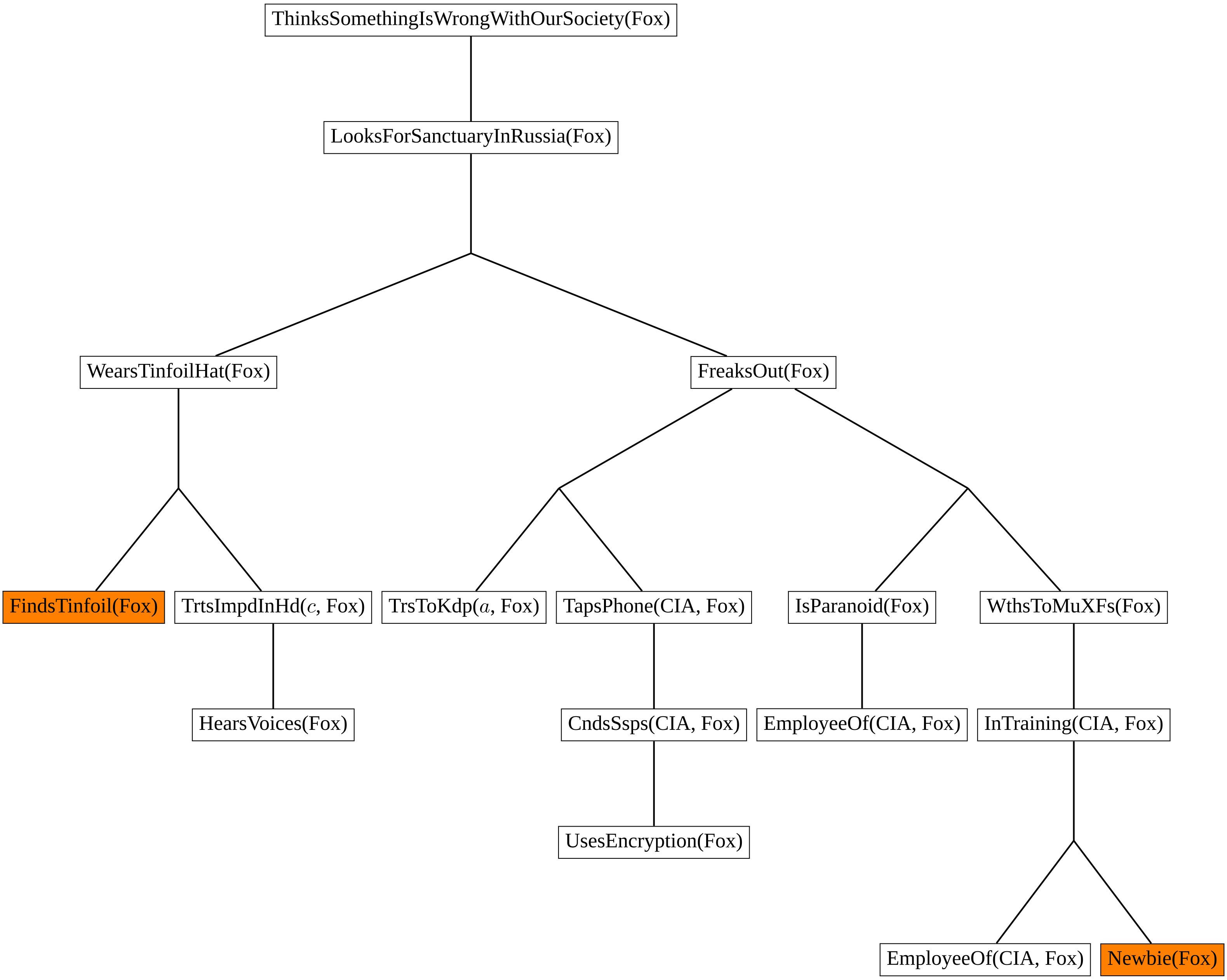

Figure 18: The proof tree at depth 13 (rule 3, 4, 1, 2, 5, 6, 7, 8, 9, 10, 11, 14, 16).

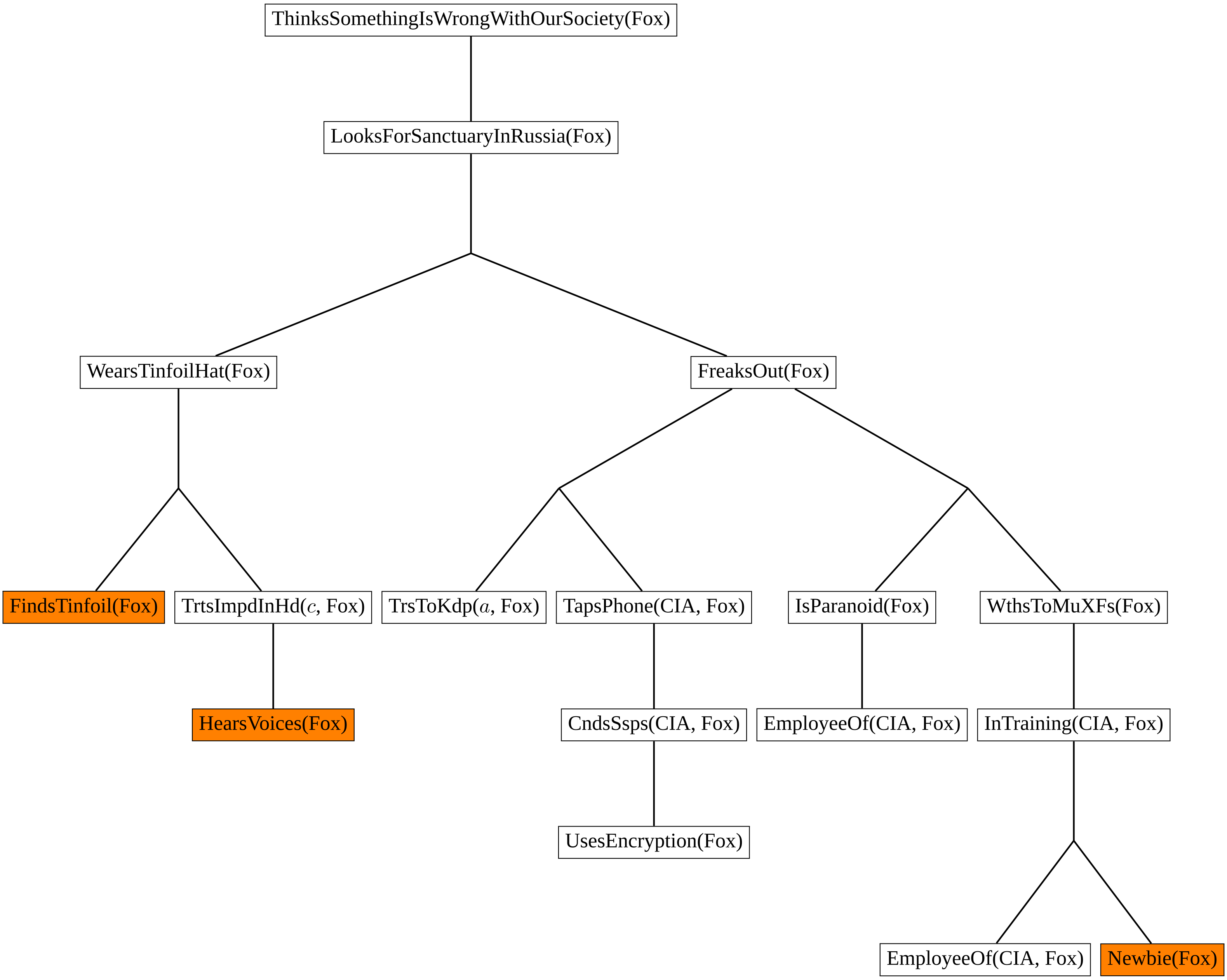

Figure 19: The proof tree at depth 14 (rule 3, 4, 1, 2, 5, 6, 7, 8, 9, 10, 11, 14, 16, 19).

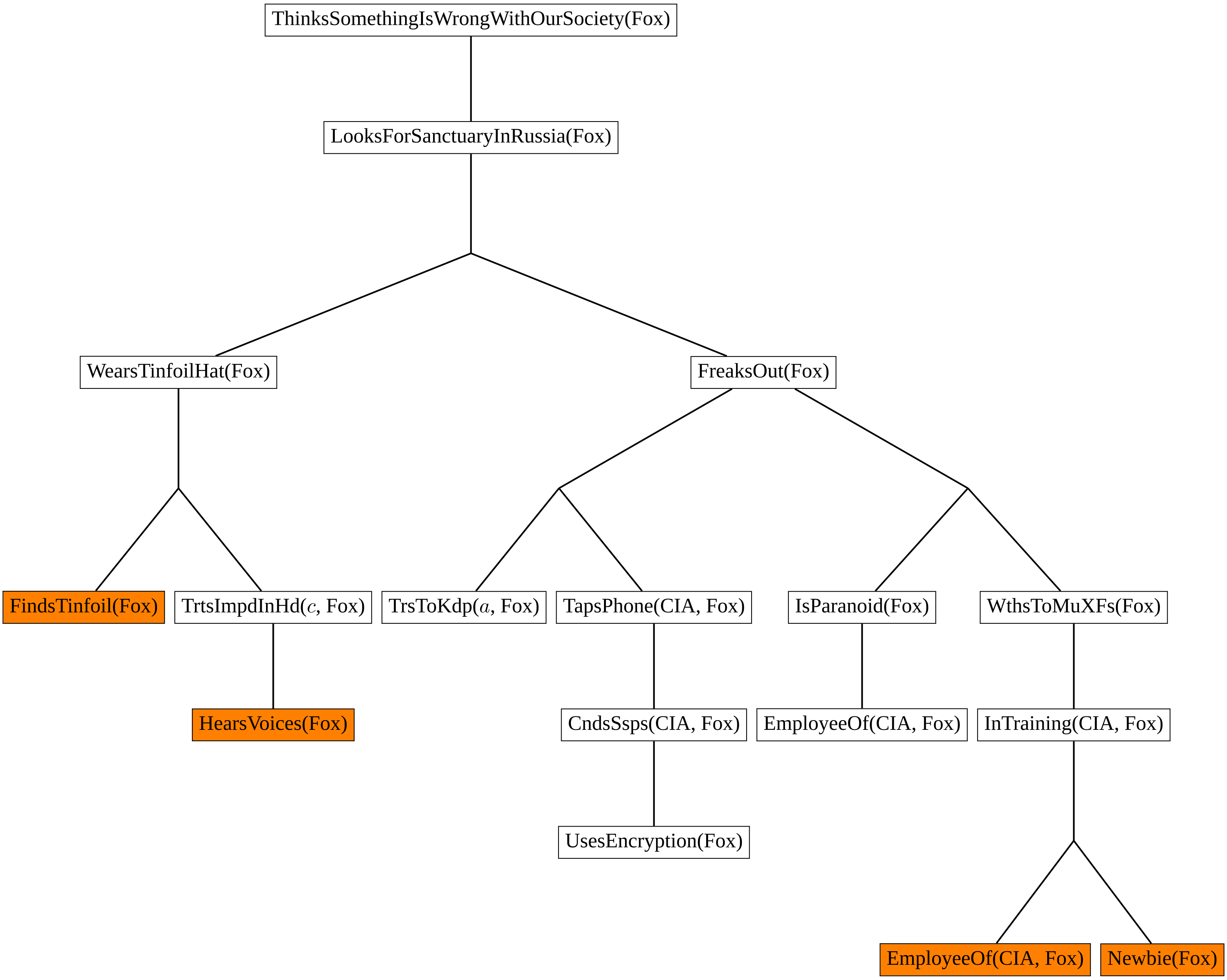

Figure 20: The proof tree at depth 15 (rule 3, 4, 1, 2, 5, 6, 7, 8, 9, 10, 11, 14, 16, 19, 20).

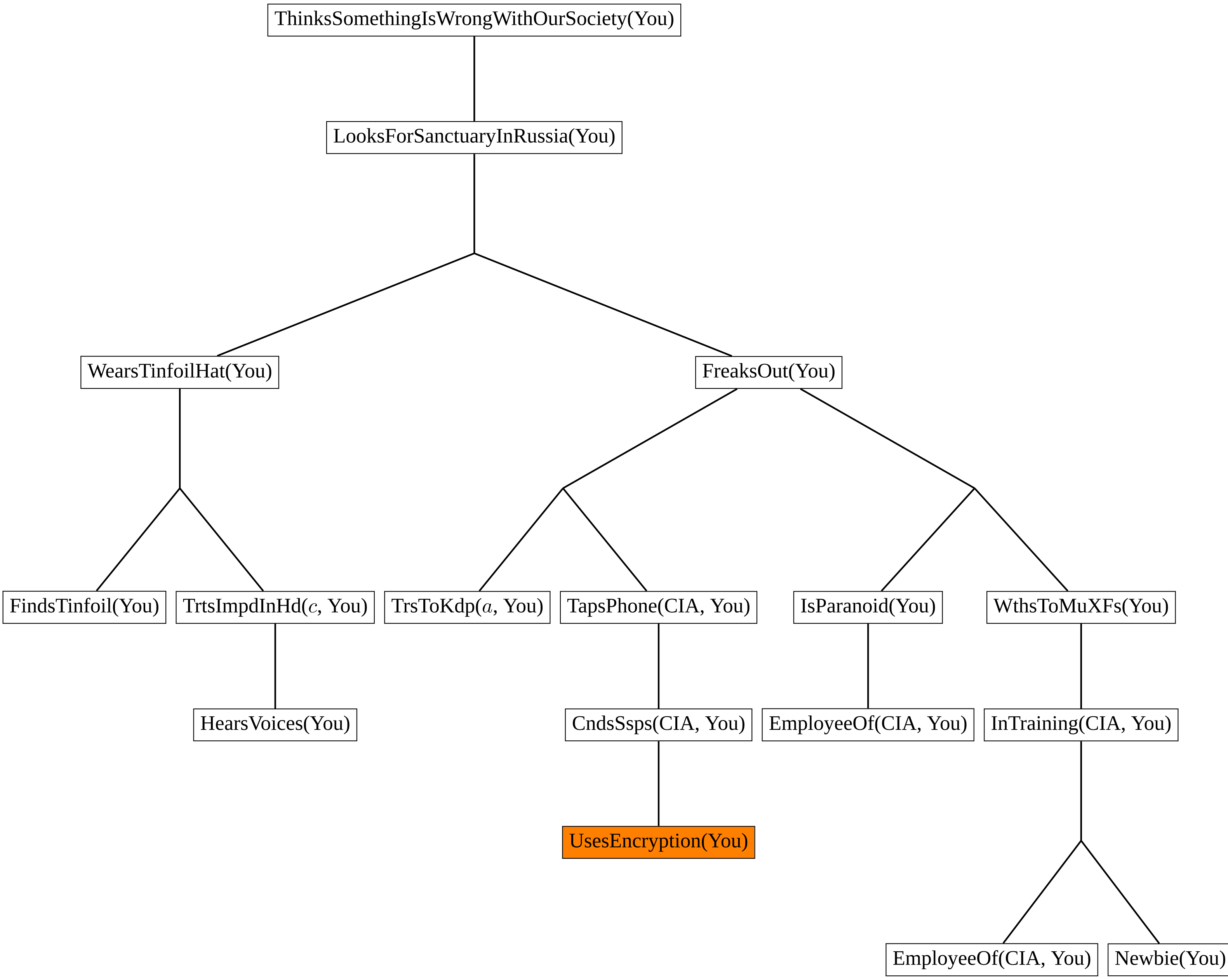

Figure 21: The proof tree at depth 12 (rule 3, 4, 1, 2, 5, 6, 7, 8, 9, 10, 11, 18).

2 Probabilistic Inference

(A) $P(\text{crying}) =$
$0.02 + 0.07 + 0.01 + 0.03 + 0.10 + 0.04 + 0.05 + 0.01 = 33\%$

(B) $P(\text{toothache}|\neg\text{nightmare}) =$
$\frac{P(\text{toothache}\wedge\neg\text{nightmare})}{P(\neg\text{nightmare})} =$
$\frac{0.01+0.01+0.05+0.03}{0.01+0.03+0.01+0.02+0.05+0.01+0.03+0.40} =$
$\frac{0.10}{0.56} \approx 17.9\%$

(C) $P(\text{excited} \wedge \text{nightmare}) =$
$0.02 + 0.07 + 0.01 + 0.05 = 15\%$

(D) $P((\text{nightmare} \wedge \neg\text{toothache})|\neg\text{crying}) =$
$\frac{P(\text{nightmare}\wedge\neg\text{toothache}\wedge\neg\text{crying})}{P(\neg\text{crying})} =$
$\frac{0.05+0.10}{0.01+0.05+0.01+0.02+0.05+0.10+0.03+0.40} =$
$\frac{0.15}{0.67} \approx 22.4\%$

(E) $P(\neg\text{excited}|(\text{excited} \wedge \text{toothache})) =$
$\frac{P(\neg\text{excited}\wedge\text{excited}\wedge\text{toothache})}{P(\text{excited}\wedge\text{toothache})} = 0$

(F) $P(\neg\text{nightmare}) =$
$0.01 + 0.03 + 0.01 + 0.02 + 0.05 + 0.01 + 0.03 + 0.40 = 56\%$

(G) $P(\text{crying} \vee \neg\text{crying}) = 1$

(H) $P((\text{toothache} \wedge \text{nightmare}) \vee (\text{crying} \wedge \text{excited})) =$
$0.02 + 0.07 + 0.01 + 0.01 + 0.03 + 0.10 + 0.05 = 29\%$

(I) $P(\text{crying}|(\text{toothache} \wedge \text{nightmare})) =$
$\frac{P(\text{crying}\wedge\text{toothache}\wedge\text{nightmare})}{P(\text{toothache}\wedge\text{nightmare})} =$
$\frac{0.02+0.10}{0.02+0.01+0.10+0.05} = \frac{0.12}{0.18} \approx 66.7\%$

(J) $P(\text{toothache} \vee \text{excited}) =$
$0.02 + 0.07 + 0.01 + 0.05 + 0.01 + 0.03 + 0.01 + 0.02 + 0.10 + 0.05 +$
$0.05 + 0.03 = 45\%$

3 Bayesian Nets – Constructing a Net

(A) We order the set of variables such that causes precede effects. Next, we determine for each variable a minimal set of parents. The variables with their set of parents is given in table 2. Based this table, the network is constructed. The resulting Baysian network for this domain is depicted in figure 22.

Variable	Parents	Values
passable tiles		0, 1, 2, 3, 4
marker		T, F
rainbow seed		T, F
control	passable tiles	T, F
interesting	marker, control	T, F
dangerous	passable tiles, rainbow seed	T, F
Move	interesting, dangerous	T, F

Table 2: The variables and their parents.

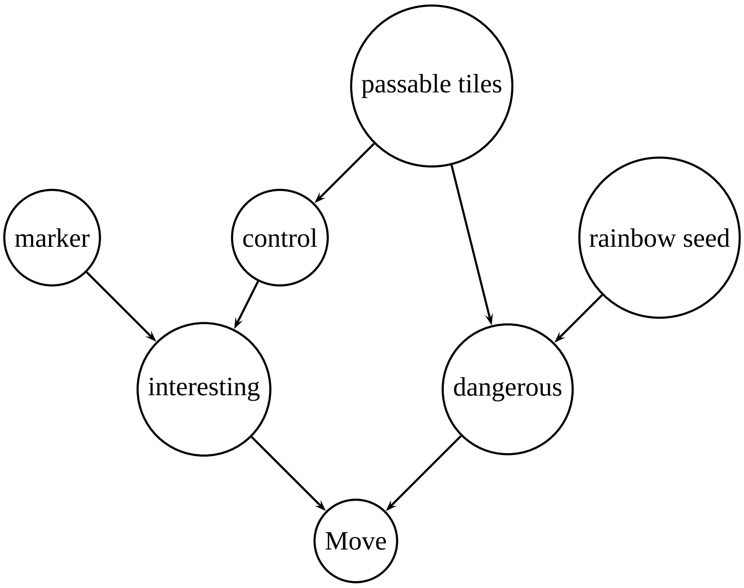

Figure 22: The Baysian network for this domain.

(B) The values of each variable are listed in table 2. There are $5 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 - 1 = 319$ independent values in the joint probability distribution. The network tables would contain $4 + 1 + 1 + 5 \times 1 + 2 \times 2 \times 1 + 5 \times 2 \times 1 + 2 \times 2 \times 1 = 29$ independent values.

(C) i) $P(\text{interesting}|\text{control}) \neq (\text{interesting}|\text{control}, \text{rainbow seed})$

ii) $P(\text{dangerous}) \neq P(\text{dangerous}|\text{interesting})$

iii) $P(\text{control}|\text{passable tiles}) = P(\text{control}|\text{passable tiles}, \text{marker})$
$P(\text{control}|\text{passable tiles}) \neq P(\text{control}|\text{passable tiles}, \text{Move})$

4 Bayesian Nets – Inference by Enumeration

(A) $P(m|s, \neg h) = \alpha \sum_{R\in\{r,\neg r\}} \sum_{P\in\{p,\neg p\}} \sum_{W\in\{w,\neg w\}} P(R, P, s, m, W, \neg h) =$

$$
\begin{array}{llllllll}
\alpha[\, P(r, & p, & s, & m, & w, & \neg h &)+ \\
P(r, & p, & s, & m, & \neg w, & \neg h &)+ \\
P(r, & \neg p, & s, & m, & w, & \neg h &)+ \\
P(r, & \neg p, & s, & m, & \neg w, & \neg h &)+ \\
P(\neg r, & p, & s, & m, & w, & \neg h &)+ \\
P(\neg r, & p, & s, & m, & \neg w, & \neg h &)+ \\
P(\neg r, & \neg p, & s, & m, & w, & \neg h &)+ \\
P(\neg r, & \neg p, & s, & m, & \neg w, & \neg h &)] =
\end{array}
$$

$$
\begin{array}{llllllll}
\alpha[& P(r) & P(p) & P(s|r,p) & P(m|s) & P(w|m) & P(\neg h|s,w) & + \\
 & P(r) & P(p) & P(s|r,p) & P(m|s) & P(\neg w|m) & P(\neg h|s,\neg w) & + \\
 & P(r) & P(\neg p) & P(s|r,\neg p) & P(m|s) & P(w|m) & P(\neg h|s,w) & + \\
 & P(r) & P(\neg p) & P(s|r,\neg p) & P(m|s) & P(\neg w|m) & P(\neg h|s,\neg w) & + \\
 & P(\neg r) & P(p) & P(s|\neg r,p) & P(m|s) & P(w|m) & P(\neg h|s,w) & + \\
 & P(\neg r) & P(p) & P(s|\neg r,p) & P(m|s) & P(\neg w|m) & P(\neg h|s,\neg w) & + \\
 & P(\neg r) & P(\neg p) & P(s|\neg r,\neg p) & P(m|s) & P(w|m) & P(\neg h|s,w) & + \\
 & P(\neg r) & P(\neg p) & P(s|\neg r,\neg p) & P(m|s) & P(\neg w|m) & P(\neg h|s,\neg w) &] =
\end{array}
$$

$$
\begin{array}{llllllll}
\alpha[& 0.6\cdot & 0.3\cdot & 0.5\cdot & 0.1\cdot & 0.6\cdot & 0.70 & + \\
 & 0.6\cdot & 0.3\cdot & 0.5\cdot & 0.1\cdot & 0.4\cdot & 0.99 & + \\
 & 0.6\cdot & 0.7\cdot & 0.9\cdot & 0.1\cdot & 0.6\cdot & 0.70 & + \\
 & 0.6\cdot & 0.7\cdot & 0.9\cdot & 0.1\cdot & 0.4\cdot & 0.99 & + \\
 & 0.4\cdot & 0.3\cdot & 0.1\cdot & 0.1\cdot & 0.6\cdot & 0.70 & + \\
 & 0.4\cdot & 0.3\cdot & 0.1\cdot & 0.1\cdot & 0.4\cdot & 0.99 & + \\
 & 0.4\cdot & 0.7\cdot & 0.2\cdot & 0.1\cdot & 0.6\cdot & 0.70 & + \\
 & 0.4\cdot & 0.7\cdot & 0.2\cdot & 0.1\cdot & 0.4\cdot & 0.99 &] \approx 0.0437\alpha
\end{array}
$$

$P(\neg m|s, \neg h) = \alpha \sum_{R\in\{r,\neg r\}} \sum_{P\in\{p,\neg p\}} \sum_{W\in\{w,\neg w\}} P(R, P, s, \neg m, W, \neg h) =$

$$
\begin{array}{llllllll}
\alpha[\, P(r, & p, & s, & \neg m, & w, & \neg h &)+ \\
 P(r, & p, & s, & \neg m, & \neg w, & \neg h &)+ \\
 P(r, & \neg p, & s, & \neg m, & w, & \neg h &)+ \\
 P(r, & \neg p, & s, & \neg m, & \neg w, & \neg h &)+ \\
 P(\neg r, & p, & s, & \neg m, & w, & \neg h &)+ \\
 P(\neg r, & p, & s, & \neg m, & \neg w, & \neg h &)+ \\
 P(\neg r, & \neg p, & s, & \neg m, & w, & \neg h &)+ \\
 P(\neg r, & \neg p, & s, & \neg m, & \neg w, & \neg h &)] =
\end{array}
$$

$$
\begin{array}{llllllll}
\alpha[& P(r) & P(p) & P(s|r,p) & P(\neg m|s) & P(w|\neg m) & P(\neg h|s,w) & + \\
 & P(r) & P(p) & P(s|r,p) & P(\neg m|s) & P(\neg w|\neg m) & P(\neg h|s,\neg w) & + \\
 & P(r) & P(\neg p) & P(s|r,\neg p) & P(\neg m|s) & P(w|\neg m) & P(\neg h|s,w) & + \\
 & P(r) & P(\neg p) & P(s|r,\neg p) & P(\neg m|s) & P(\neg w|\neg m) & P(\neg h|s,\neg w) & + \\
 & P(\neg r) & P(p) & P(s|\neg r,p) & P(\neg m|s) & P(w|\neg m) & P(\neg h|s,w) & + \\
 & P(\neg r) & P(p) & P(s|\neg r,p) & P(\neg m|s) & P(\neg w|\neg m) & P(\neg h|s,\neg w) & + \\
 & P(\neg r) & P(\neg p) & P(s|\neg r,\neg p) & P(\neg m|s) & P(w|\neg m) & P(\neg h|s,w) & + \\
 & P(\neg r) & P(\neg p) & P(s|\neg r,\neg p) & P(\neg m|s) & P(\neg w|\neg m) & P(\neg h|s,\neg w) &] =
\end{array}
$$

$$
\begin{array}{llllllll}
\alpha[& 0.6\cdot & 0.3\cdot & 0.5\cdot & 0.9\cdot & 0.1\cdot & 0.70 & + \\
 & 0.6\cdot & 0.3\cdot & 0.5\cdot & 0.9\cdot & 0.9\cdot & 0.99 & + \\
 & 0.6\cdot & 0.7\cdot & 0.9\cdot & 0.9\cdot & 0.1\cdot & 0.70 & + \\
 & 0.6\cdot & 0.7\cdot & 0.9\cdot & 0.9\cdot & 0.9\cdot & 0.99 & + \\
 & 0.4\cdot & 0.3\cdot & 0.1\cdot & 0.9\cdot & 0.1\cdot & 0.70 & + \\
 & 0.4\cdot & 0.3\cdot & 0.1\cdot & 0.9\cdot & 0.9\cdot & 0.99 & + \\
 & 0.4\cdot & 0.7\cdot & 0.2\cdot & 0.9\cdot & 0.1\cdot & 0.70 & + \\
 & 0.4\cdot & 0.7\cdot & 0.2\cdot & 0.9\cdot & 0.9\cdot & 0.99 &] \approx 0.4636\alpha
\end{array}
$$

$P(m|s,\neg h) + P(\neg m|s,\neg h) = 1 \Rightarrow \alpha \approx \frac{1}{0.0437+0.4636} \approx 1.971$

$P(M|s,\neg h) \approx (0.0437, 0.4636)\alpha \approx (0.086, 0.914)$

(B) $P(w|\neg p,\neg r,h) = \alpha \sum_{S\in\{s,\neg s\}} \sum_{M\in\{m,\neg m\}} P(\neg r,\neg p,S,M,w,h) =$

$$
\begin{array}{llllllll}
\alpha[\, P(\neg r, & \neg p, & s, & m, & w, & h &)+ \\
 P(\neg r, & \neg p, & s, & \neg m, & w, & h &)+ \\
 P(\neg r, & \neg p, & \neg s, & m, & w, & h &)+ \\
 P(\neg r, & \neg p, & \neg s, & \neg m, & w, & h &)] =
\end{array}
$$

$$
\begin{array}{llllllll}
\alpha[& P(\neg r) & P(\neg p) & P(s|\neg r,\neg p) & P(m|s) & P(w|m) & P(h|s,w) & + \\
 & P(\neg r) & P(\neg p) & P(s|\neg r,\neg p) & P(\neg m|s) & P(w|\neg m) & P(h|s,w) & + \\
 & P(\neg r) & P(\neg p) & P(\neg s|\neg r,\neg p) & P(m|\neg s) & P(w|m) & P(h|\neg s,w) & + \\
 & P(\neg r) & P(\neg p) & P(\neg s|\neg r,\neg p) & P(\neg m|\neg s) & P(w|\neg m) & P(h|\neg s,w) & = \\
 & 0.4\cdot & 0.7\cdot & 0.2\cdot & 0.1\cdot & 0.6\cdot & 0.30 & + \\
 & 0.4\cdot & 0.7\cdot & 0.2\cdot & 0.9\cdot & 0.1\cdot & 0.30 & + \\
 & 0.4\cdot & 0.7\cdot & 0.8\cdot & 0.5\cdot & 0.6\cdot & 0.90 & + \\
 & 0.4\cdot & 0.7\cdot & 0.8\cdot & 0.5\cdot & 0.1\cdot & 0.90 &] \approx 0.0731\alpha
\end{array}
$$

$P(\neg w|\neg p,\neg r,h) = \alpha \sum_{S\in\{s,\neg s\}} \sum_{M\in\{m,\neg m\}} P(\neg r,\neg p,S,M,\neg w,h) =$

$$
\begin{array}{llllllll}
\alpha[\ P(& \neg r, & \neg p, & s, & m, & \neg w, & h &)+ \\
\ P(& \neg r, & \neg p, & s, & \neg m, & \neg w, & h &)+ \\
\ P(& \neg r, & \neg p, & \neg s, & m, & \neg w, & h &)+ \\
\ P(& \neg r, & \neg p, & \neg s, & \neg m, & \neg w, & h &)] = \\
\alpha[& P(\neg r) & P(\neg p) & P(s|\neg r, \neg p) & P(m|s) & P(\neg w|m) & P(h|s, \neg w) & + \\
 & P(\neg r) & P(\neg p) & P(s|\neg r, \neg p) & P(\neg m|s) & P(\neg w|\neg m) & P(h|s, \neg w) & + \\
 & P(\neg r) & P(\neg p) & P(\neg s|\neg r, \neg p) & P(m|\neg s) & P(\neg w|m) & P(h|\neg s, \neg w) & + \\
 & P(\neg r) & P(\neg p) & P(\neg s|\neg r, \neg p) & P(\neg m|\neg s) & P(\neg w|\neg m) & P(h|\neg s, \neg w) & = \\
 & 0.4\cdot & 0.7\cdot & 0.2\cdot & 0.1\cdot & 0.4\cdot & 0.01 & + \\
 & 0.4\cdot & 0.7\cdot & 0.2\cdot & 0.9\cdot & 0.9\cdot & 0.01 & + \\
 & 0.4\cdot & 0.7\cdot & 0.8\cdot & 0.5\cdot & 0.4\cdot & 0.50 & + \\
 & 0.4\cdot & 0.7\cdot & 0.8\cdot & 0.5\cdot & 0.9\cdot & 0.50 &] \approx 0.0732\alpha
\end{array}
$$

$P(w|\neg p, \neg r, h) + P(\neg w|\neg p, \neg r, h) = 1 \Rightarrow \alpha \approx \frac{1}{0.0731+0.0732} \approx 6.833$

$P(W|\neg p, \neg r, h) \approx (0.0731, 0.0732)\alpha \approx (0.499, 0.501)$