Proiect Teoria Sistemelor

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Cuprins

Introducere

Filtrul Twin-T de tip Notch

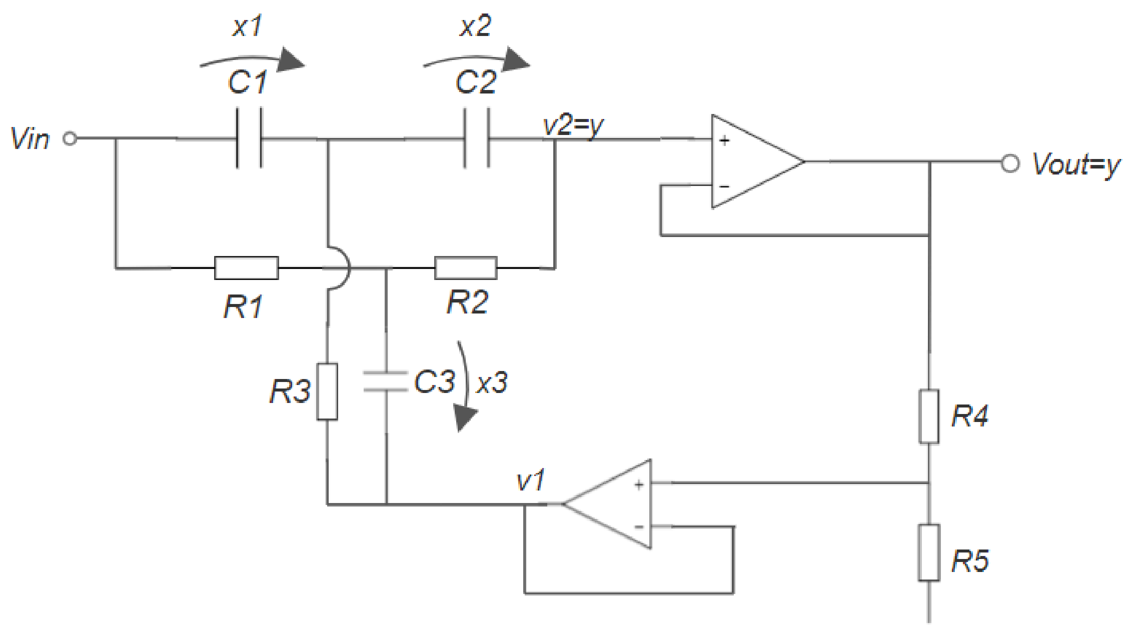


Figure 1. Filtrul Twin-T de tip Notch

Parametrii circuitului sunt:

$$R_1 = 88k\Omega \quad R_2 = 9{,}1k\Omega \quad R_3 = 57k\Omega$$

$$R_4 = 3{,}8k\Omega \quad R_5 = 22k\Omega$$

$$C_1 = 330 \cdot 10^{-7}F \quad C_2 = 47 \cdot 10^{-6}F \quad C_3 = 38 \cdot 10^{-6}F$$

Elemente active : C1, C2, C3 ;

1 . Model matematic intrare-stare-iesire x/u/y

Fie:

$$\begin{matrix} \dot{X} = A \cdot X + B \cdot u \\ Y = C \cdot X + D \cdot u \end{matrix}$$ - reprezentarea in spatiul starilor

Consideram variabilele de stare x1, x2, x3, unde:

$$i_{C_1} = c_1 \cdot \dot{x}_1$$

$$i_{C_2} = c_2 \cdot \dot{x}_2$$

$$i_{C_3} = C_3 \cdot \dot{x}_3$$

$$x = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} u \cdot c_1 \\ u \cdot C_2 \\ u \cdot C_3 \end{pmatrix}$$

Aplicăm Teoremele lui Kirchoff:

TK1: $$\begin{cases} ic_1 = i_{R_3} + i_{c_2} & (1) \\ i_{R_1} = i_{R_2} + ic_3 & (2) \\ ic_2 + i_{R_2} = 0 & (3) \end{cases}$$

TK2: $$\begin{cases} u_{C1} + u_{C2} = u_{R1} + u_{R2} & (4) \\ u_{C2} + u_{C3} = u_{R2} + u_{R3} & (5) \\ u + u_{R2} = u_{C1} + u_{C2} + u_{C3} + u_{R5} & (6) \end{cases}$$

Din (1) rezulta $i_{R_3} = c_2 \cdot \dot{x}_2 - c_2 \cdot \dot{x}_2$

Din (2) rezulta $i_{R_1} = C_3 \cdot \dot{x}_3 - c_2 \cdot \dot{x}_2$

Din (3) rezulta $i_{R_2} = -c_2 \cdot \dot{x}_2$

Înlocuind cu (1), (2), și (3) în (4) obținem

$$x_1 + x_2 = i_{R_1} * R_1 + i_{R_2} * R_2 \Rightarrow x_1 + x_2 = R_1 * (C_3 \cdot \dot{x}_3 - c_2 \cdot \dot{x}_2) - R_2 * c_2 \cdot \dot{x}_2 \tag{7}$$

Înlocuind cu (1), (2), și (3) în (5) obținem

$$x_2 + x_3 = R_3 * c_1 \cdot \dot{x}_1 - (R_2 + R_3) * c_2 \cdot \dot{x}_2 \tag{8}$$

Înlocuind cu (1), (2), și (3) în (6) obținem

$$u = x_1 + x_2 + x_3 + R_2 * c_2 \cdot \dot{x}_2 + u_{R5} \tag{9}$$

Știm că :

$$u_{C3} = u_{R2} + u_{R4} \Leftrightarrow x_3 = i_{R_2} * R_2 + i_{R_4} * R_4 \Rightarrow i_{R_4} = \frac{x_3 - i_{R_2} * R_2}{R_4} \tag{10}$$

Cum

$i_{R_4} = i_{R_5}$, putem inlocui in relatia (9) pentru a afla $\dot{x}_2$

$$u = x_1 + x_2 + x_3 + R_2 * c_2 \cdot \dot{x}_2 + \frac{x_3 - i_{R_2} * R_2}{R_4} * R_5$$

$$\Rightarrow \dot{x}_2 * \left(\frac{R_2 * R_4 * C_2 + R_2 * R_5 * C_2}{R_4}\right) = u - x_1 - x_2 - x_3 - \frac{R_5}{R_4} * x_3$$

$$\Rightarrow \dot{x}_2 = \frac{-\left[x_1 + x_2 + \left(1 + \frac{R_5}{R_4}\right) * x_3\right] + u}{\frac{R_2 * C_2 * (R_4 * + R_5)}{R_4}} \tag{11}$$

⇨ $\dot{x}_2 = -\frac{R_4}{R_2 * C_2(R_4+R_5)} * \ x_1 - \frac{R_4}{R_2 * C_2(R_4+R_5)} * \ x_2 - \frac{1}{R_2 * C_2} * \ x_3 + \frac{R_4}{R_2 * C_2(R_4+R_5)} * u$

Acum că am aflat x2 derivat putem merge mai departe pentru a afla x3 derivat.

Pentru a afla x3 derivat, rezolvăm ecuația (7) .

$$x_1 + x_2 = R_1 * (C_3 \cdot \dot{x}_3 - c_2 \cdot \dot{x}_2) - R_2 * c_2 \cdot \dot{x}_2$$

⇨ $R_1 * (C_3 \cdot \dot{x}_3) = x_1 + x_2 + (R_1 + R_2) * c_2 \cdot \dot{x}_2$

⇨ $\dot{x}_3 = \frac{x_1 + x_2 + (R_1+R_2) * c_2 \cdot \dot{x}_2}{R_1 * C_3}$

Acum în această relație, inlocuim x2 derivat cu relația aflată anterior.

⇨ $\dot{x}_3 = \frac{R_2 * R_5 - R_1 * R_4}{R_1 * R_2 * C_3 * (R_4+R_5)} * x_1 + \frac{R_2 * R_5 - R_1 * R_4}{R_1 * R_2 * C_3 * (R_4+R_5)} * x_2 - \frac{R_1+R_2}{R_1 * R_2 * C_3} * x_3 + \frac{+(R_1+R_2) * R_4}{R_1 * R_2 * C_3 * (R_4+R_5)} * \ u$ (12)

La fel procedăm și pentru x1 derivat.

$$x_2 + x_3 = R_3 * c_1 \cdot \dot{x}_1 - (R_2 + R_3) * c_2 \cdot \dot{x}_2$$

⇨ $R_3 * c_1 \cdot \dot{x}_1 = x_2 + x_3 + \ (R_2 + R_3) * c_2 \cdot \dot{x}_2$

⇨ $\dot{x}_1 = \frac{x_2 + x_3 + (R_2+R_3) * c_2 \cdot \dot{x}_2}{R_3 * c_1}$ (13)

⇨ $\dot{x}_1 = -\frac{(R_2+R_3) * R_4}{R_2 * R_3 * C_1 * (R_4+R_5)} * x_1 + \frac{R_2 * R_5 - R_3 * R_4}{R_2 * R_3 * C_1 * (R_4+R_5)} * \ x_2 - \frac{1}{R_2 * C_1} * x_3 + \frac{(R_2+R_3) * R_4}{R_2 * R_3 * C_1 * (R_4+R_5)} * u$

Ajungem la

$$\begin{cases} \dot{x}_1 = -\frac{(R_2+R_3)*R_4}{R_2*R_3*C_1*(R_4+R_5)} * x_1 + \frac{R_2*R_5-R_3*R_4}{R_2*R_3*C_1*(R_4+R_5)} * x_2 - \frac{1}{R_2*C_1} * x_3 + \frac{(R_2+R_3)*R_4}{R_2*R_3*C_1*(R_4+R_5)} * u \\ \dot{x}_2 = -\frac{R_4}{R_2*C_2(R_4+R_5)} * x_1 - \frac{R_4}{R_2*C_2(R_4+R_5)} * x_2 - \frac{1}{R_2*C_2} * x_3 + \frac{R_4}{R_2*C_2(R_4+R_5)} * u \\ \dot{x}_3 = \frac{R_2*R_5-R_1*R_4}{R_1*R_2*C_3*(R_4+R_5)} * x_1 + \frac{R_2*R_5-R_1*R_4}{R_1*R_2*C_3*(R_4+R_5)} * x_2 - \frac{R_1+R_2}{R_1*R_2*C_3} * x_3 + \frac{(R_1+R_2)*R_4}{R_1*R_2*C_3*(R_4+R_5)} * u \end{cases}$$

y = $-x_1 - x_2 + u$

$$X = \begin{pmatrix} -\frac{(R_2+R_3)*R_4}{R_2*R_3*C_1*(R_4+R_5)} & \frac{R_2*R_5-R_3*R_4}{R_2*R_3*C_1*(R_4+R_5)} & -\frac{1}{R_2*C_1} \\ -\frac{R_4}{R_2*C_2(R_4+R_5)} & -\frac{R_4}{R_2*C_2(R_4+R_5)} & -\frac{1}{R_2*C_2} \\ \frac{R_2*R_5-R_1*R_4}{R_1*R_2*C_3*(R_4+R_5)} & \frac{R_2*R_5-R_1*R_4}{R_1*R_2*C_3*(R_4+R_5)} & -\frac{R_1+R_2}{R_1*R_2*C_3} \end{pmatrix} * X + \begin{pmatrix} \frac{(R_2+R_3)*R_4}{R_2*R_3*C_1*(R_4+R_5)} \\ \frac{R_4}{R_2*C_2(R_4+R_5)} \\ \frac{(R_1+R_2)*R_4}{R_1*R_2*C_3*(R_4+R_5)} \end{pmatrix} * u$$

$$Y = (-1, -1, 0) * X + (1) * u$$

Inlocuim cu valorile circuitului nostru:

$$R_1 = 88k\Omega \quad R_2 = 9{,}1k\Omega \quad R_3 = 57k\Omega$$

$$R_4 = 3{,}8k\Omega \quad R_5 = 22k\Omega$$

$$C_1 = 330 \cdot 10^{-7}F \quad C_2 = 47 \cdot 10^{-6}F \quad C_3 = 38 \cdot 10^{-6}F$$

$$\dot{X} = \begin{pmatrix} -0.57 & -0.04 & -3.33 \\ -0.34 & -0.34 & -2.33 \\ -0.17 & -0.17 & -3.19 \end{pmatrix} * X + \begin{pmatrix} 0.57 \\ 0.34 \\ 0.47 \end{pmatrix} * u$$

$$Y = (-1, -1, 0) * X + (1) * u$$

2. Model intrare-iesire. Funcția de transfer

2.1 Determinarea modelului intrare-iesire

Avand in vedere sistemul aflat anterior:

$$\begin{cases} \dot{x}_1 = -\frac{(R_2+R_3)*R_4}{R_2*R_3*C_1*(R_4+R_5)}*x_1+\frac{R_2*R_5-R_3*R_4}{R_2*R_3*C_1*(R_4+R_5)}*\ x_2-\frac{1}{R_2*C_1}*x_3+\frac{(R_2+R_3)*R_4}{R_2*R_3*C_1*(R_4+R_5)}*u & (1) \\ \dot{x}_2 = -\frac{R_4}{R_2*C_2(R_4+R_5)}*\ x_1-\frac{R_4}{R_2*C_2(R_4+R_5)}*\ x_2-\frac{1}{R_2*C_2}*\ x_3+\frac{R_4}{R_2*C_2(R_4+R_5)}*u & (2) \\ \dot{x}_3 = \frac{R_2*R_5-R_1*R_4}{R_1*R_2*C_3*(R_4+R_5)}*x_1+\frac{R_2*R_5-R_1*R_4}{R_1*R_2*C_3*(R_4+R_5)}*x_2-\frac{R_1+R_2}{R_1*R_2*C_3}*x_3+\frac{(R_1+R_2)*R_4}{R_1*R_2*C_3*(R_4+R_5)}*\ u & (4) \end{cases}$$

y = $-x_1-x_2+u$

Derivăm y = $-x_1-x_2+u$ **și obținem :**

(5)

$$\dot{y} = -\dot{x}_1-\dot{x}_2+\dot{u}$$

⇨ $\dot{y} = \frac{(R_2+R_3)*R_4}{R_2*R_3*C_1*(R_4+R_5)}*x_1-\frac{R_2*R_5-R_3*R_4}{R_2*R_3*C_1*(R_4+R_5)}*\ x_2+\frac{1}{R_2*C_1}*x_3-\frac{(R_2+R_3)*R_4}{R_2*R_3*C_1*(R_4+R_5)}*u+\frac{R_4}{R_2*C_2(R_4+R_5)}*\ x_1+$
$\frac{R_4}{R_2*C_2(R_4+R_5)}*\ x_2+\frac{1}{R_2*C_2}*\ x_3-\frac{R_4}{R_2*C_2(R_4+R_5)}*u+\dot{u}$

Notam $R_2*C_2(R_4+R_5)$ **cu a, si** $R_2*R_3*C_1*(R_4+R_5)$ **cu b.**

⇨ $\dot{y} = \frac{(R_2+R_3)*R_4}{b}*x_1-\frac{R_2*R_5-R_3*R_4}{b}*\ x_2+\frac{1}{R_2*C_1}*x_3-\frac{(R_2+R_3)*R_4}{b}*u+\frac{R_4}{a}*\ x_1+\frac{R_4}{a}*\ x_2+\frac{1}{R_2*C_2}*\ x_3-\frac{R_4}{a}*$
$u+\dot{u}$

⇨ $\dot{y} = \left(\frac{(R_2+R_3)*R_4}{b}+\frac{R_4}{a}\right)*x_1+\left(\frac{R_4}{a}-\frac{R_2*R_5-R_3*R_4}{b}\right)*\ x_2+\left(\frac{1}{R_2*C_2}+\frac{1}{R_2*C_1}\right)*x_3-\left(\frac{(R_2+R_3)*R_4}{b}+\frac{R_4}{a}\right)*u+\dot{u}$

Acum mai derivam inca odata :

$$\ddot{y} = \left(\frac{(R_2+R_3)*R_4}{b}+\frac{R_4}{a}\right)*\dot{x}_1+\left(\frac{R_4}{a}-\frac{R_2*R_5-R_3*R_4}{b}\right)*\ \dot{x}_2+\left(\frac{1}{R_2*C_2}+\frac{1}{R_2*C_1}\right)*\dot{x}_3 \\ -\left(\frac{(R_2+R_3)*R_4}{b}+\frac{R_4}{a}\right)*\dot{u}+\ddot{u}$$

Din :

$\dot{x}_2 = -\frac{R_4}{R_2 * C_2(R_4+R_5)} * \ x_1 - \frac{R_4}{R_2 * C_2(R_4+R_5)} * \ x_2 - \frac{1}{R_2 * C_2} * \ x_3 + \frac{R_4}{R_2 * C_2(R_4+R_5)} * u \quad / * -R_2 * C_2(R_4+R_5)$

⇨ $-R_2 * C_2(R_4+R_5) * \dot{x}_2 = R_4 * \ x_1 + R_4 * \ x_2 + (R_4+R_5) * x_3 - R_4 * u$

⇨ $x_1 = \frac{-R_2 * C_2(R_4+R_5) * \dot{x}_2 - R_4 * x_2 - (R_4+R_5) * x_3 + R_4 * u}{R_4}$

⇨ $La\ final\ ar\ trebui\ sa\ ajungem\ la\ o\ ecuatie\ de\ intrare - iesire\ de\ forma:$

⇨ $\boldsymbol{\dddot{y} + a * \ddot{y} + b * \dot{y} + c * y = \dddot{u} + d * \ddot{u} + e * \dot{u} + f * u}$

⇨ $H(s) = \frac{s^3+3.19 * s^2+1.698 * s+0.3705}{s^3+4.1 * s^2+2.121 * s+0.3649}$

2.2 Determinarea functiei de transfer.

2.2.1 Folosind relatia sa cu spatiul starilor

Descrierea sistemului in planul starilor este:

$$X = \begin{pmatrix} -\frac{(R_2+R_3)R_4}{R_2R_3C_1(R_4+R_5)} & \frac{R_2R_5-R_3R_4}{R_2R_3C_1(R_4+R_5)} & -\frac{1}{R_2C_1} \\ -\frac{R_4}{R_2C_2(R_4+R_5)} & -\frac{R_4}{R_2C_2(R_4+R_5)} & -\frac{1}{R_2C_2} \\ \frac{R_2R_5-R_1R_4}{R_1R_2C_3(R_4+R_5)} & \frac{R_2R_5-R_1R_4}{R_1R_2C_3(R_4+R_5)} & -\frac{R_1+R_2}{R_1R_2C_3} \end{pmatrix} * X + \begin{pmatrix} \frac{(R_2+R_3)R_4}{R_2R_3C_1(R_4+R_5)} \\ \frac{R_4}{R_2C_2(R_4+R_5)} \\ \frac{(R_1+R_2) * R_4}{R_1R_2C_3(R_4+R_5)} \end{pmatrix} * u$$

$$Y = (-1, -1, 0) * X + (1) * u$$

Functia de transfer este definita ca:

H(s) = C ∗ (s ∗ I − A)^(-1)∗ B + D

$$\mathrm{H(s)} = (-1,-1,0) * \frac{\mathrm{adj(sI-A)}}{\mathrm{det(sI-A)}} * \begin{pmatrix} \frac{(R_2+R_3)R_4}{R_2R_3C_1(R_4+R_5)} \\ \frac{R_4}{R_2C_2(R_4+R_5)} \\ \frac{(R_1+R_2)R_4}{R_1R_2C_3(R_4+R_5)} \end{pmatrix} + (1)$$

$$H(s) = \frac{\begin{pmatrix} -1 & -1 & 0 \end{pmatrix} \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} \begin{pmatrix} \frac{(R_2+R_3)R_4}{R_2R_3C_1(R_4+R_5)} \\ \frac{R_4}{R_2C_2(R_4+R_5)} \\ \frac{(R_1+R_2)R_4}{R_1R_2C_3(R_4+R_5)} \end{pmatrix}}{\det(sI-A)} + 1$$

Rezulta ca doar termenii $\begin{matrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{matrix}$ sunt cei care conteaza, ceilalti fiind 0.

$$a_{11} = \det\begin{pmatrix} s + \frac{R_4}{R_2C_2(R_4+R_5)} & \frac{1}{R_2C_2} \\ -\frac{R_2R_5 - R_1R_4}{R_1R_2C_3(R_4+R_5)} & s + \frac{R_1+R_2}{R_1R_2C_3} \end{pmatrix} * (-1)^2$$

$$a_{12} = \det\begin{pmatrix} \frac{R_4}{R_2C_2(R_4+R_5)} & \frac{1}{R_2C_2} \\ -\frac{R_2R_5 - R_1R_4}{R_1R_2C_3(R_4+R_5)} & s + \frac{R_1+R_2}{R_1R_2C_3} \end{pmatrix} * (-1)^3$$

$$a_{13} = \det\begin{pmatrix} \frac{R_4}{R_2C_2(R_4+R_5)} & s + \frac{R_4}{R_2C_2(R_4+R_5)} \\ -\frac{R_2R_5 - R_1R_4}{R_1R_2C_3(R_4+R_5)} & -\frac{R_2R_5 - R_1R_4}{R_1R_2C_3(R_4+R_5)} \end{pmatrix} * (-1)^4$$

$$a_{21} = \det\begin{pmatrix} -\frac{R_2R_5 - R_3R_4}{R_2R_3C_1(R_4+R_5)} & \frac{1}{R_2C_2} \\ -\frac{R_2R_5 - R_1R_4}{R_1R_2C_3(R_4+R_5)} & s + \frac{R_1+R_2}{R_1R_2C_3} \end{pmatrix} * (-1)^3$$

$$a_{22} = \det\begin{pmatrix} s + \frac{(R_2 + R_3)R_4}{R_2R_3C_1(R_4 + R_5)} & \frac{1}{R_2C_2} \\ -\frac{R_2R_5 - R_1R_4}{R_1R_2C_3(R_4 + R_5)} & s + \frac{R_1 + R_2}{R_1R_2C_3} \end{pmatrix} * (-1)^4$$

$$a_{23} = \det\begin{pmatrix} s + \frac{(R_2 + R_3)R_4}{R_2R_3C_1(R_4 + R_5)} & -\frac{R_2R_5 - R_3R_4}{R_2R_3C_1(R_4 + R_5)} \\ -\frac{R_2R_5 - R_1R_4}{R_1R_2C_3(R_4 + R_5)} & -\frac{R_2R_5 - R_1R_4}{R_1R_2C_3(R_4 + R_5)} \end{pmatrix} * (-1)^5$$

Dupa calcule, ajungem la :

$a_{11} = s^2 + 3.53s + 0.68$

$a_{12} = -0.34s + 0.68$

$a_{13} = -0.17s$

$a_{21} = -0.04s + 0.43$

$a_{22} = s^2 + 3.76s + 1.24$

$a_{23} = -0.17s - 0.0892$

$$\Rightarrow (-1 \quad -1 \quad 0)\begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{pmatrix}\begin{pmatrix} \frac{(R_2+R_3)R_4}{R_2R_3C_1(R_4+R_5)} \\ \frac{R_4}{R_2C_2(R_4+R_5)} \\ \frac{(R_1+R_2)R_4}{R_1R_2C_3(R_4+R_5)} \end{pmatrix} = -0.91s^2 - 0.423s + 0.0056$$

$$sI - A = \begin{pmatrix} s + \frac{(R_2+R_3)R_4}{R_2R_3C_1(R_4+R_5)} & -\frac{R_2R_5-R_3R_4}{R_2R_3C_1(R_4+R_5)} & +\frac{1}{R_2C_1} \\ +\frac{R_4}{R_2C_2(R_4+R_5)} & s + \frac{R_4}{R_2C_2(R_4+R_5)} & +\frac{1}{R_2C_2} \\ -\frac{R_2R_5-R_1R_4}{R_1R_2C_3(R_4+R_5)} & -\frac{R_2R_5-R_1R_4}{R_1R_2C_3(R_4+R_5)} & s + \frac{R_1+R_2}{R_1R_2C_3} \end{pmatrix}$$

$$\det(sI - A) = s^3 + s^2\frac{R_1C_2C_3R_4(R_2+R_3) + R_1R_3R_4C_1C_2 + R_3C_1C_2(R_1+R_2)(R_4+R_5)}{R_1R_2R_3C_1C_2C_3(R_4+R_5)} +$$

$$+s\frac{R_4}{R_2^2}\frac{C_2(R_1+R_2)(R_3+R_4)(R_4+R_5)+R_3C_1(R_1-R_2)(R_4+R_5)+R_1R_4C_3(R_2+R_3)}{R_1R_3C_1C_2C_3(R_4+R_5)^2} -$$

$$-\frac{R_2C_2(R_4+R_5)(R_1R_4-R_2R_5)+R_1R_4C_3(R_2R_5-R_3R_4)(R_4+R_5)-R_3C_2(R_4+R_5)(R_2R_5-R_3R_4)}{R_1R_3C_1C_2C_3(R_4+R_5)^2}$$

$$+\frac{C_1(R_2R_5-R_3R_4)(R_2R_5-R_1R_4)-R_4C_1(R_2R_5-R_3R_4)+R_3R_4C_1(R_1R_4-R_2R_5)}{R_1R_2^2R_3{C_1}^2C_2C_3(R_4+R_5)^2}$$

$$-\frac{C_1^2R_4(R_1+R_2)(R_2R_5-R_3R_4)-R_4C_2(R_2+R_3)(R_2R_5-R_3R_4)}{R_1R_2^2R_3{C_1}^2C_2C_3(R_4+R_5)^2}$$

⇨ $\det(sI - A) = s^3 + 4.1s^2 + 2.11s + 0.36$

2.2.2 Folosind Matlab

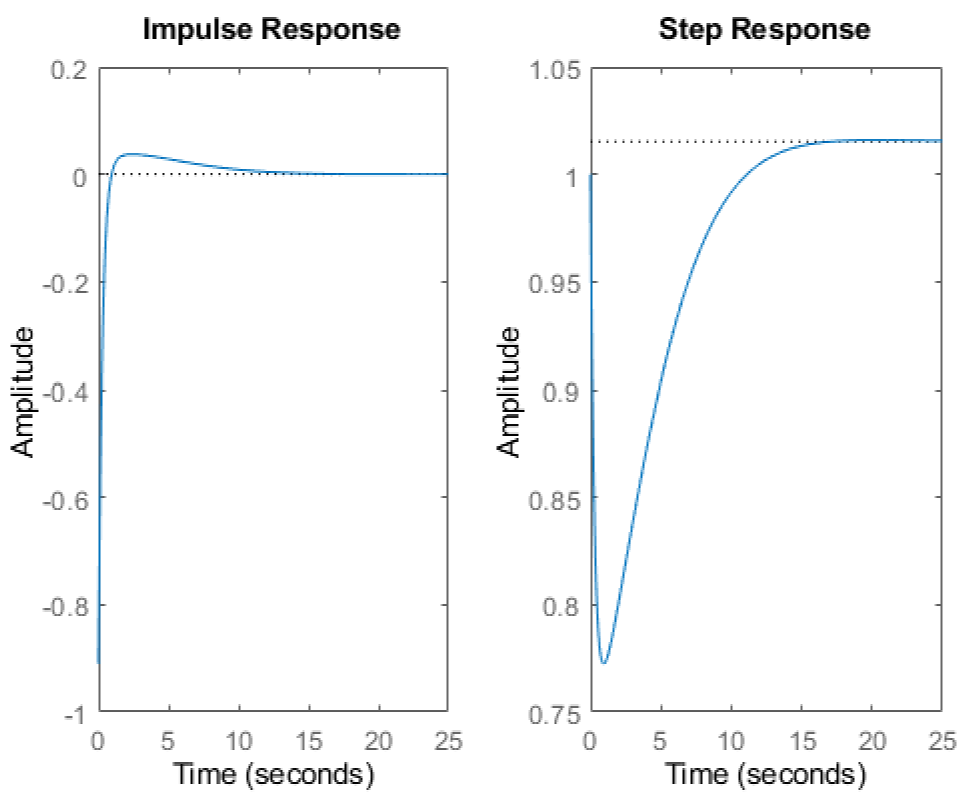
Fie :

```
%% Functia de transfer
A = [-0.57,-0.04,-3.33;
    -0.34,-0.34,-2.33;
    -0.17,-0.17,-3.19];
B = [0.57;0.34;0.47];
C = [-1,-1,0];
D = 1;

[num,den]= ss2tf(A,B,C,D)
sys = ss(A,B,C,D)
```

```
H = tf(num,den)
figure,subplot(121), impulse(A,B,C,D)
subplot(122), step(sys)
```

```
H =

  s^3 + 3.19 s^2 + 1.698 s + 0.3705
  ---------------------------------
  s^3 + 4.1 s^2 + 2.121 s + 0.3649

Continuous-time transfer function.
```

$$H(s) = \frac{s^3 + 3.19 * s^2 + 1.698 * s + 0.3705}{s^3 + 4.1 * s^2 + 2.121 * s + 0.3649}$$



3. Determinare singularitati. Afisare in plan complex.

3.1 Determinare singularitati

Cunoastem functia de transfer:

$$H(s) = \frac{s^3 + 3.19 * s^2 + 1.698 * s + 0.3705}{s^3 + 4.1 * s^2 + 2.121 * s + 0.3649}$$

Pentru a afla zerourile, egalam numaratorul cu 0.

$s^3 + 3.19 * s^2 + 1.698 * s + 0.3705 = 0$

- ⇨ $\boldsymbol{\dot{s}1 = -2.5895 + 0.0000i}$
- ⇨ $\boldsymbol{\dot{s}_2 = -0.3002 + 0.2301i}$
- ⇨ $\boldsymbol{\dot{s}_3 = -0.3002 + 0.2301i}$

Functia folosita in Matlab pentru a afla zerourile :

`z = zero(H);` **unde H este functia de transfer.**

```
>> z

z =

  -2.5895 + 0.0000i
  -0.3002 + 0.2301i
  -0.3002 - 0.2301i
```

Pentru a afla polii, egalam numitorul cu 0.

$s^3 + 4.1 * s^2 + 2.121 * s + 0.3649 = 0$

- ⇨ $\boldsymbol{\widehat{s_1} = -3.5282 + 0.0000i}$
- ⇨ $\boldsymbol{\widehat{s_2} = -0.2859 + 0.1472i}$
- ⇨ $\boldsymbol{\widehat{s_3} = -0.2859 - 0.1472i}$

Functia folosita in Matlab pentru a afla polii :

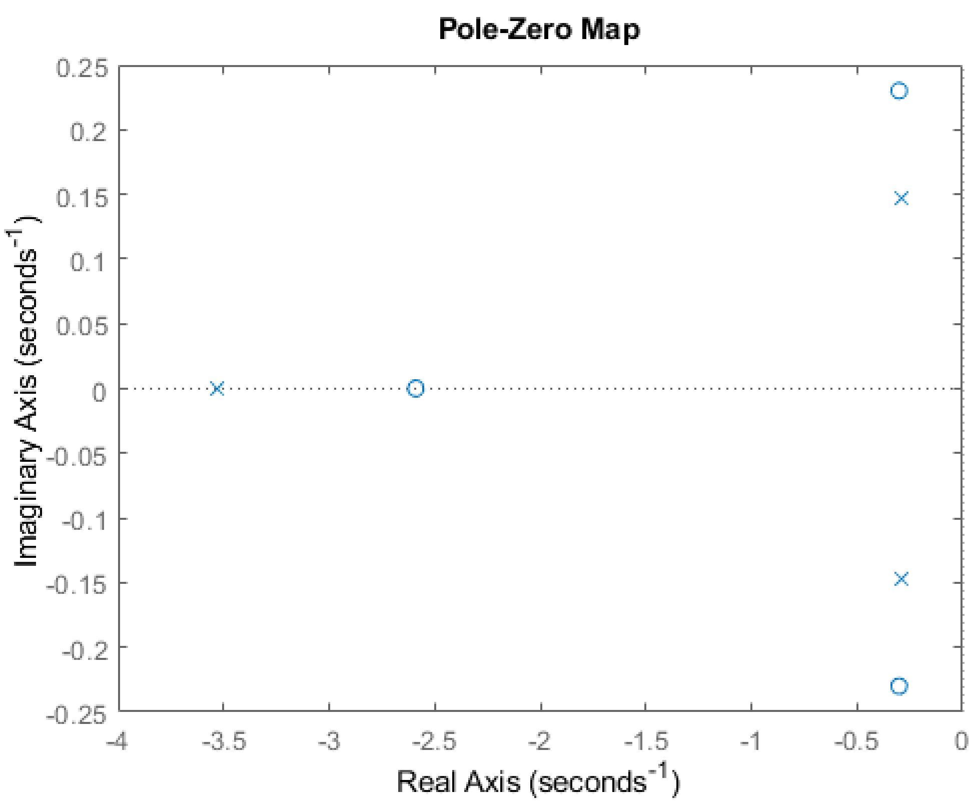
`p = pole(H);` **unde H este functia de transfer**

```
>> p

p =

  -3.5282 + 0.0000i
  -0.2859 + 0.1472i
  -0.2859 - 0.1472i
```

Singularitatile sistemului sunt:

- ⇨ $\dot{s}1 = -2.5895 + 0.0000i$
- ⇨ $\dot{s}_2 = -0.3002 + 0.2301i$
- ⇨ $\dot{s}_3 = -0.3002 - 0.2301i$
- ⇨ $\widehat{s_1} = -3.5282 + 0.0000i$
- ⇨ $\widehat{s_2} = -0.2859 + 0.1472i$
- ⇨ $\widehat{s_3} = -0.2859 - 0.1472i$

Reprezentarea lor in planul complex :



4. Determinare forma canonica de control(FCC) si forma canonica de observare(FCO). Realizare schema simulink

4.1 Determinare forma canonica de control (FCC)

Cunoastem functia de transfer:

$$H(s) = \frac{s^3 + 3.19 * s^2 + 1.698 * s + 0.3705}{s^3 + 4.1 * s^2 + 2.121 * s + 0.3649}$$

Conform definitiei, FCC este :

$$\left(\begin{array}{c|c} \boldsymbol{A_{FCC}} & \boldsymbol{B_{FCC}} \\ \hline \boldsymbol{C_{FCC}} & \boldsymbol{D} \end{array}\right) = \left(\begin{array}{ccc|c} \mathbf{-4.1} & \mathbf{-2.1209} & \mathbf{-0.3649} & \mathbf{1} \\ \mathbf{1} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{1} & \mathbf{0} & \mathbf{0} \\ \hline \mathbf{-0.91} & \mathbf{-0.4229} & \mathbf{0.0056} & \mathbf{1} \end{array}\right)$$

De aici, scoatem relatiile pentru variabilele de stare pentru a le putea implementa in Simulink

$$\begin{cases} \dot{\boldsymbol{x}}_1 = -4.1 * \boldsymbol{x}_1 - 2.1209 * \boldsymbol{x}_2 - 0.3649 * \boldsymbol{x}_3 + \boldsymbol{u} \\ \dot{\boldsymbol{x}}_2 = \boldsymbol{x}_1 \\ \dot{\boldsymbol{x}}_3 = \boldsymbol{x}_2 \\ \boldsymbol{y} = -0.91 * \boldsymbol{x}_1 - 0.4229 * \boldsymbol{x}_2 - 0.0056 * \boldsymbol{x}_3 + \boldsymbol{u} \end{cases}$$

Matlab :

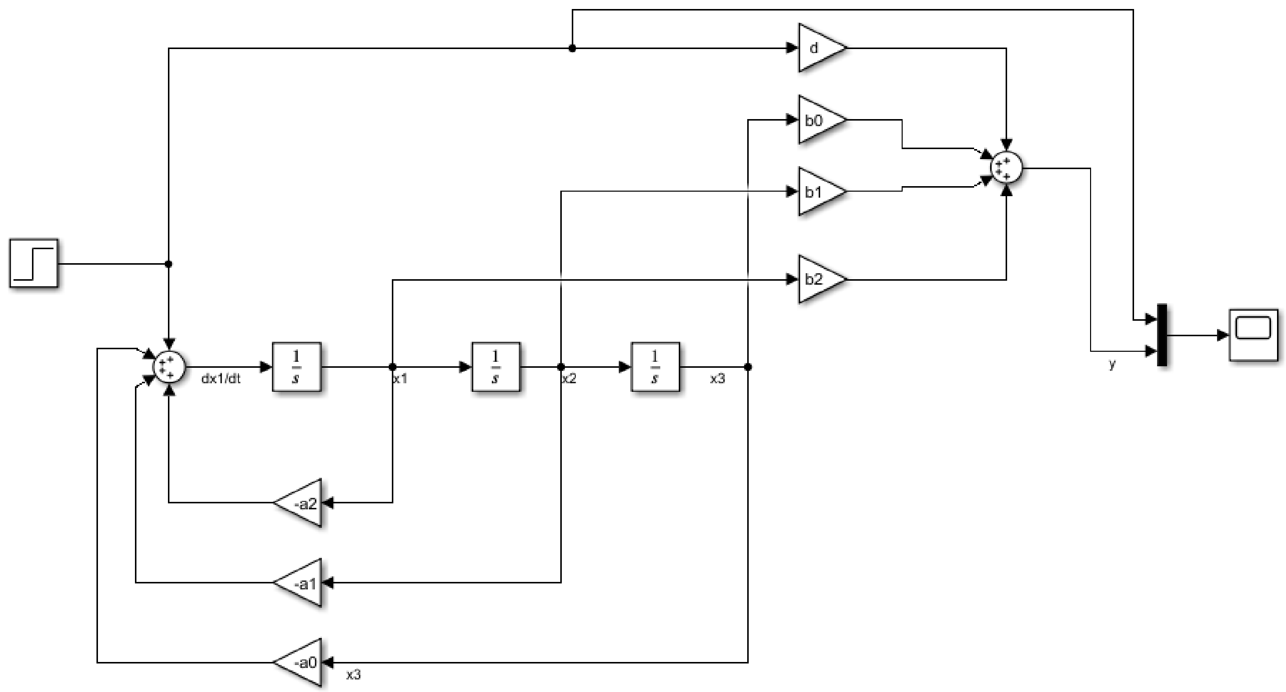
```
[AFCC,BFCC,CFCC,D] = tf2ss(num,den);
```

```
>> AFCC

AFCC =

   -4.1000   -2.1209   -0.3649
    1.0000         0         0
         0    1.0000         0
```

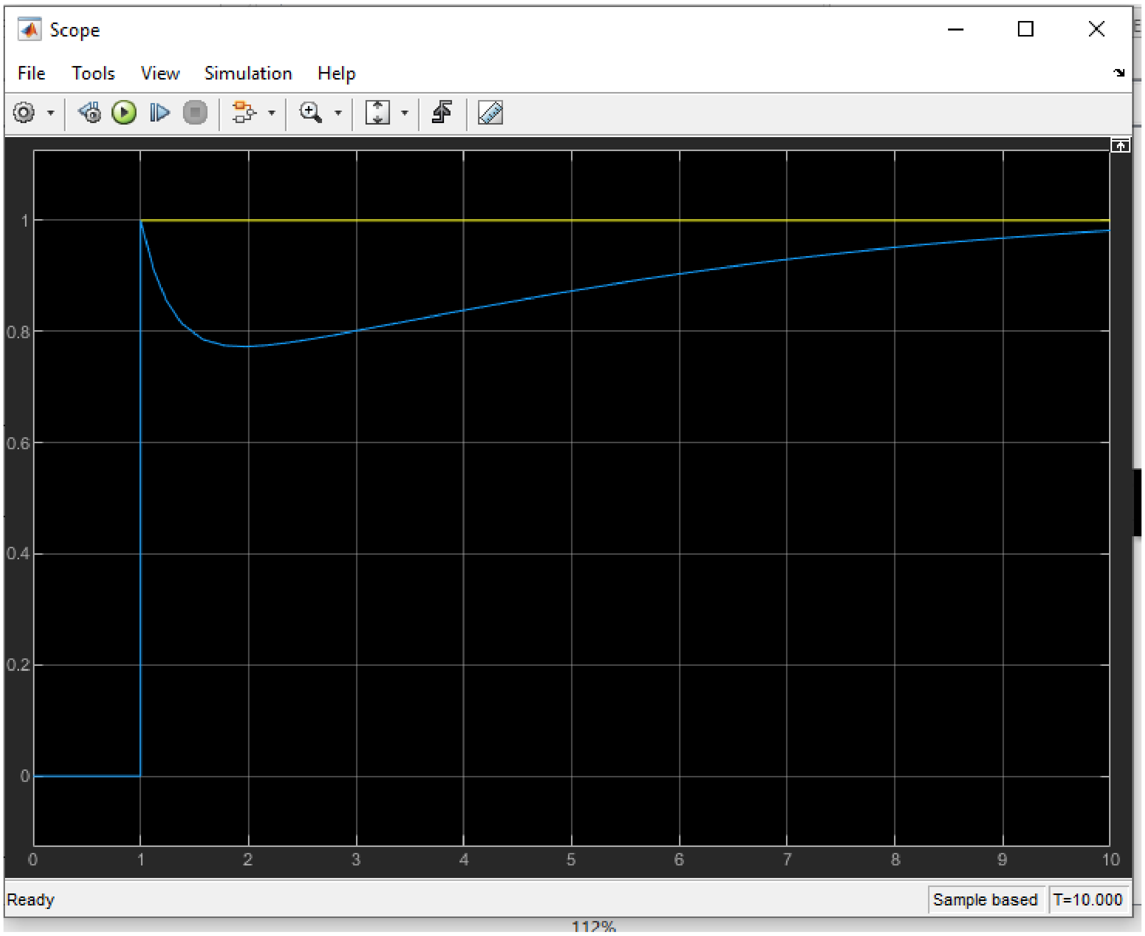
```
>> BFCC

BFCC =

     1
     0
     0
```

```
>> CFCC

CFCC =

   -0.9100   -0.4229    0.0056
```

4.1.1 Realizare FCC simulink.



4.1.2 Simulare simulink.



4.2 Determinare forma canonica de observare (FCO).

Functia de transfer:

$$H(s) = \frac{s^3 + 3.19 * s^2 + 1.698 * s + 0.3705}{s^3 + 4.1 * s^2 + 2.121 * s + 0.3649}$$

Conform definitiei, FCO este:

$$\left(\begin{array}{c|c} \boldsymbol{A_{FCC}} & \boldsymbol{B_{FCC}} \\ \hline \boldsymbol{C_{FCC}} & \boldsymbol{D} \end{array}\right) = \left(\begin{array}{c|c} \boldsymbol{A^T{}_{FCC}} & \boldsymbol{C^T{}_{FCC}} \\ \hline \boldsymbol{B^T{}_{FCC}} & \boldsymbol{D} \end{array}\right) = \left(\begin{array}{ccc|c} \mathbf{-4.1} & \mathbf{1} & \mathbf{0} & \mathbf{-0.91} \\ \mathbf{-2.1209} & \mathbf{0} & \mathbf{1} & \mathbf{-0.4229} \\ \mathbf{-0.3649} & \mathbf{0} & \mathbf{0} & \mathbf{0.0056} \\ \hline \mathbf{1} & \mathbf{0} & \mathbf{0} & \mathbf{1} \end{array}\right)$$

De aici, scoatem relatiile pentru variabilele de stare pentru a le putea implementa in Simulink

$$\begin{cases} \dot{x}_1 = -4.1 * x_1 + x_2 - 0.91 * u \\ \dot{x}_2 = -2.1209 * x_1 + x_3 - 0.4229 * u \\ \dot{x}_3 = -0.3649 * x_1 + 0.0056 * u \\ y = x_1 + u \end{cases}$$

Matlab:

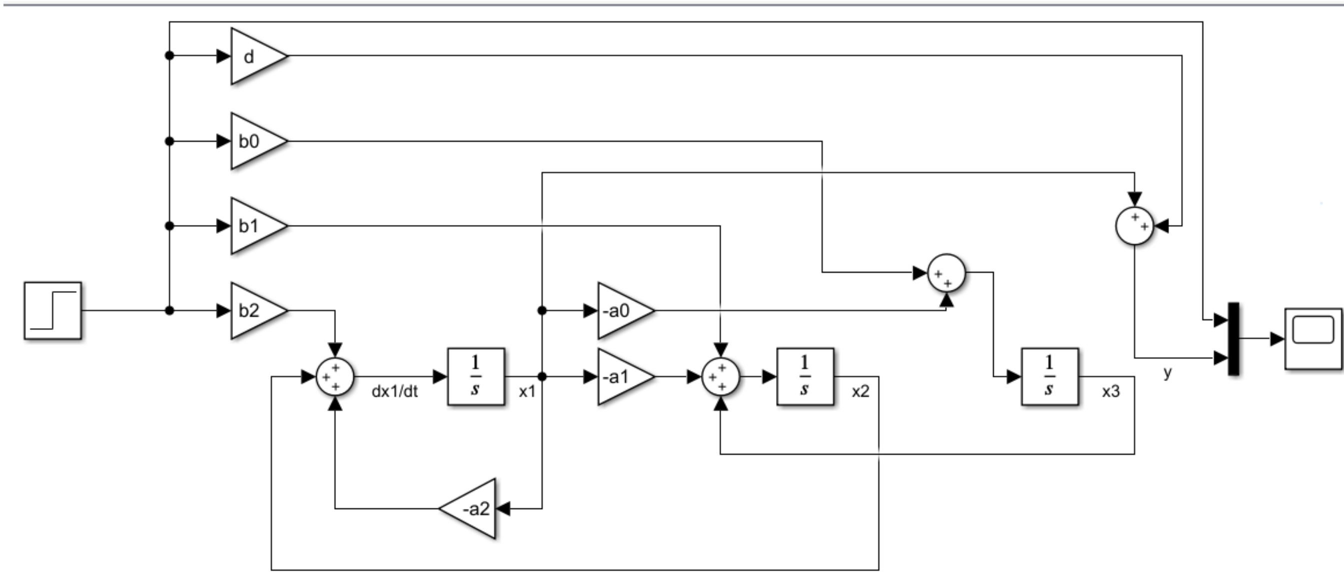
```
AFCO = AFCC';
BFCO = CFCC';
CFCO = BFCC';
```

```
>> AFCO

AFCO =

   -4.1000    1.0000         0
   -2.1209         0    1.0000
   -0.3649         0         0
```

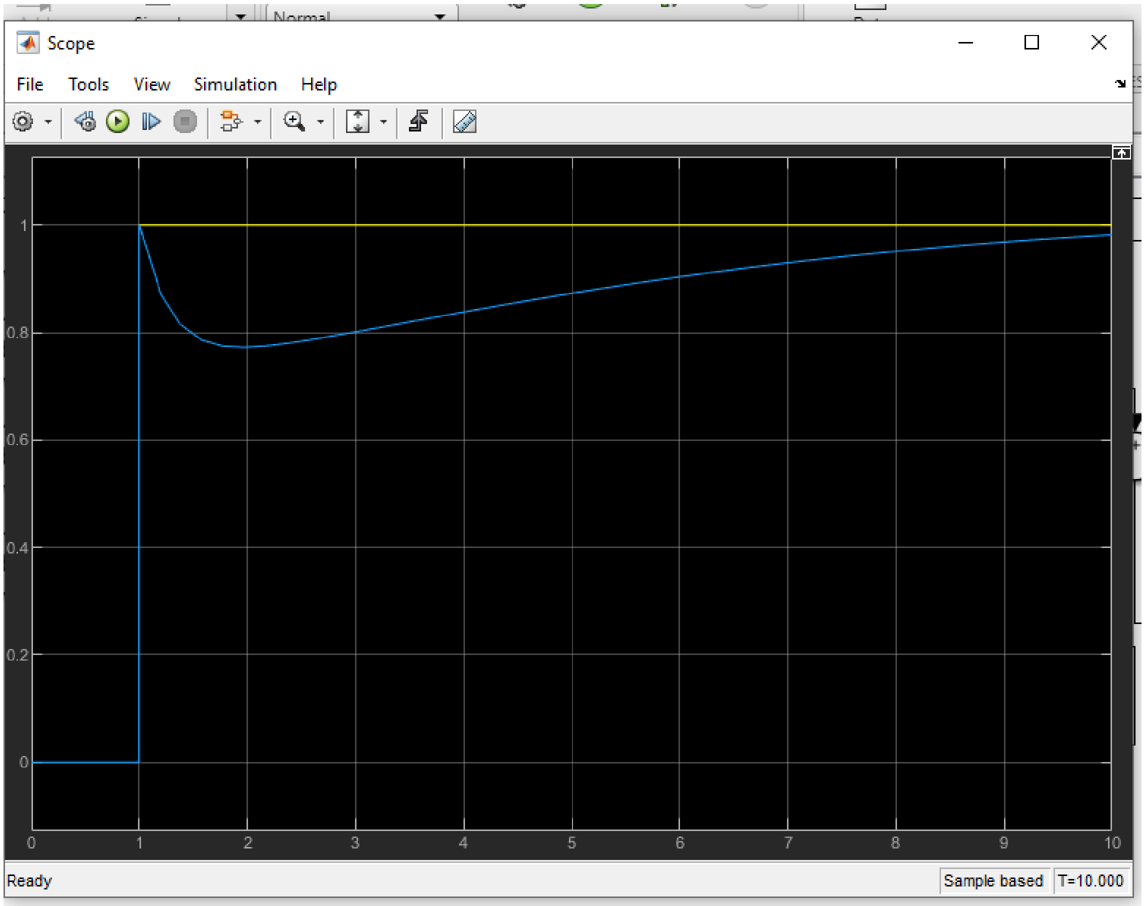
```
>> BFCO

BFCO =

   -0.9100
   -0.4229
    0.0056
```

```
>> CFCO

CFCO =

     1     0     0
```

4.2.1 Realizare FCO simulink



4.2.2 Simulare simulink.



5. Determinare functia de transfer in forma minimala.

Impartim polinoamele din functia de transfer pentru aflarea coeficientiilor Markov:

$$H(s) = \frac{s^3 + 3.19 * s^2 + 1.698 * s + 0.3705}{s^3 + 4.1 * s^2 + 2.121 * s + 0.3649}$$

N=3 => 6 parametrii Markov

$$\left(\begin{array}{c|c} s^3 + 3.19 * s^2 + 1.698 * s + 0.3705 & \\ s^3 + 4.1 * s^2 + 2.121 * s + 0.3649 & \\ -0.91 * s^2 - 0.422 * s + 0.0056 & \\ -0.91 * s^2 - 3.731 * s - 1.93 - 0.33 * s^{-1} & \\ 3.309 * s + 1.9356 + 0.33 * s^{-1} & \\ 3.309 * s + 13.56 + 7.01 * s^{-1} + 1.2 * s^{-2} & s^3 + 4.1 * s^2 + 2.121 * s + 0.3649 \\ -11.604 - 6.68 * s^{-1} - 1.2 * s^{-2} & 1 - 0.91 * s^{-1} + 3.309 * s^{-2} - 11.604 * s^{-3} + 40.89 * s^{-4} - 144.24 * s^{-5} \\ -11.604 - 47.57 * s^{-1} - 24.61 * s^{-2} - 4.23 * s^{-3} & \\ 40.89 * s^{-1} + 23.41 * s^{-2} + 4.23 * s^{-3} & \\ 40.89 * s^{-1} + 167.64 * s^{-2} + 86.72 * s^{-3} + 14.92 * s^{-4} & \\ -144.24 * s^{-2} - 82.49 * s^{-3} - 14.92 * s^{-4} & \\ -144.24 * s^{-2} - 591.38 * s^{-3} - 305.93 * s^{-4} - 52.63 * s^{-5} & \\ 508.89 * s^{-3} + 291.01 * s^{-4} + 52.63 * s^{-5} & \end{array}\right)$$

In urma algoritmului am obtinut urmatorii parametrii Markov :

$\gamma_0 = 1$; $\gamma_1 = -0.91$; $\gamma_2 = 3.309$; $\gamma_3 = -11.604$; $\gamma_4 = 40.89$; $\gamma_5 = -144.24$;

Matricea Hankel este urmatoarea:

$$\mathcal{H}_{3,3} = \begin{pmatrix} \gamma_1 & \gamma_2 & \gamma_3 \\ \gamma_2 & \gamma_3 & \gamma_4 \\ \gamma_3 & \gamma_4 & \gamma_5 \end{pmatrix} = \begin{pmatrix} -0.91 & 3.309 & -11.604 \\ 3.309 & -11.604 & 40.89 \\ -11.604 & 40.89 & -144.24 \end{pmatrix}$$

Calculam rangul matricei Hankel:

$$\begin{vmatrix} -0.91 & 3.309 & -11.604 \\ 3.309 & -11.604 & 40.89 \\ -11.604 & 40.89 & -144.24 \end{vmatrix}$$
$$= -1523.12 - 1570.07 - 1570.07 - (-1562.51 - 1521.51 - 1579.35)$$

= -4663.26+4663.37= 0.11 => **diferit de 0 => Rangul este 3 => Functia de transfer este in forma minimala.**

Verificare Matlab :

```
N = 6;
gv = deconv([num,zeros(1,N)],den);

H33 = [gv(2),gv(3),gv(4);
        gv(3),gv(4),gv(5);
        gv(4),gv(5),gv(6);]
rang=rank(H33);
```

```
>> rank(H33)

H33 =

   -0.9100    3.3081  -11.6276
    3.3081  -11.6276   40.9892
  -11.6276   40.9892 -144.6018
```

```
rang =

     3
```

6. Stabilitatea interna si externa.

<u>Metoda 1</u> : valorile proprii ale matricei A

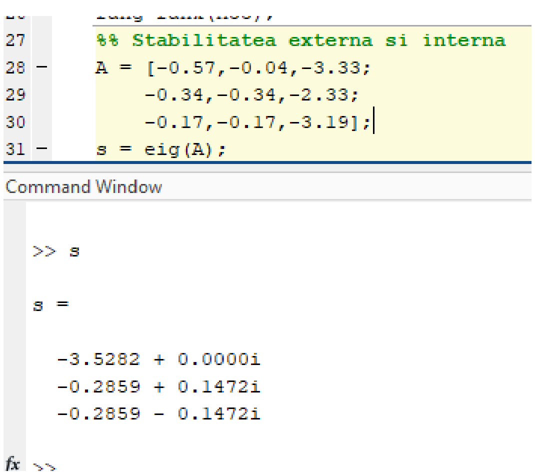
In planul starilor avem :

$$\dot{X} = \begin{pmatrix} -0.57 & -0.04 & -3.33 \\ -0.34 & -0.34 & -2.33 \\ -0.17 & -0.17 & -3.19 \end{pmatrix} * X + \begin{pmatrix} 0.57 \\ 0.34 \\ 0.47 \end{pmatrix} * u$$

$$Y = (-1, -1, 0) * X + (1) * u$$

Cu ajutorul Matlab-ului determinam valorile proprii ale lui A si astfel putem deduce stabilitatea sistemului .

```
%% Stabilitatea externa si interna
A = [-0.57,-0.04,-3.33;
    -0.34,-0.34,-2.33;
    -0.17,-0.17,-3.19];
s = eig(A);
```



In ceea ce priveste valorile proprii ale matricei A, acestea , coiincid cu polii sistemului. Partea reala a valorilor proprii este negativa,acestea aflandu-se in semiplanul stang, ceea ce inseamna ca sistemul este **<u>stabil intern</u>**. Daca sistemul este **<u>stabil intern</u>** => acesta este si **<u>stabil extern</u>**.

<u>Metoda 2</u> : Lyapunov

```
Q = eye(length(A));
P = lyap(A',Q);
```

```
>> eig(P)

ans =

    0.1349
    2.1658
    3.0822
```

<u>Metoda 3 :</u> Criteriul Routh-Hurwitz

$$\det(\lambda I - A) = \det\begin{pmatrix} \lambda + 0.57 & 0.04 & 3.33 \\ 0.34 & \lambda + 0.34 & 2.33 \\ 0.17 & 0.17 & \lambda + 3.19 \end{pmatrix}$$

$$\det(\lambda I - A) = \lambda^3 + 4.1\lambda^2 + 2.11\lambda + 0.36$$

$$\begin{array}{c|cc} \lambda^3 & 1 & 2.11 \\ \lambda^2 & 4.1 & 0.36 \\ \lambda^1 & 2.022 & 0 \\ \lambda^0 & 0.36 & 0 \end{array}$$

Toti termenii de pe prima coloana sunt pozitivi (nu exista schimbare de semn) => sistemul este asimptotic stabil.

7. Determinare stabilitate interna folosind Lyapunov

Pentru a verifica stabilitatea sistemului folosind lyapunov, trebuie ca: $A^T * P + P * A = -Q$. Daca ecuatia are solutii pentru orice $Q = Q^T$ pozitiv definit ales, astfel incat P sa fie pozitiv definita, atunci sistemul este stabil.

$$A = \begin{pmatrix} -0.57 & -0.04 & -3.33 \\ -0.34 & -0.34 & -2.33 \\ -0.17 & -0.17 & -3.19 \end{pmatrix}$$

```
%% Metoda Lyapunov
A = [-0.57,-0.04,-3.33;
    -0.34,-0.34,-2.33;
    -0.17,-0.17,-3.19];
B = [0.57;0.34;0.47];
C = [-1,-1,0];
D = 1;

Q = eye(length(A));
P = lyap(A',Q);
```

```
>> P

P =

    1.4405   -0.4463   -0.9963
   -0.4463    2.0199   -0.9935
   -0.9963   -0.9935    1.9225
```

```
>> Q

Q =

     1     0     0
     0     1     0
     0     0     1
```

$$V(x) = x^t P x$$

$$P = P^t$$

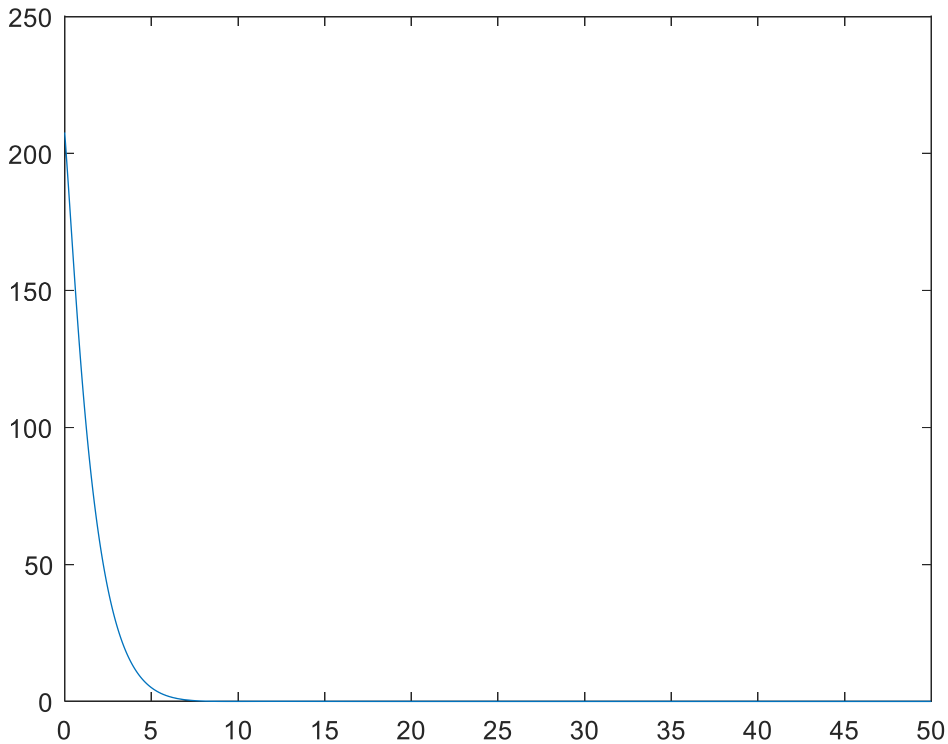
$$\dot{V}(x) = \dot{x}^t P x + x^t P \dot{x} = x^t (A^t P + P A) x$$

$$V(x) = (x_1 \quad x_2 \quad x_3) \begin{pmatrix} 1.4405 & -0.4463 & -0.9963 \\ -0.4463 & 2.0199 & -0.9935 \\ -0.9963 & -0.9935 & 1.9225 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$$

```
>> eig(P)

ans =

    0.1349
    2.1658
    3.0822
```

Valorile proprii ale lui P sunt pozitiv definite => Sistemul este asimptotic stabil

Reprezentarea grafica pentru Lyapunov :



8. Determinarea functiei pondere, raspunsului indicial, raspunsul la rampa si reprezentarea lor.

8.1. Functia pondere

$$\boldsymbol{h}(t) = \boldsymbol{\mathcal{L}^{-1}}\{H(s)\} = \boldsymbol{\mathcal{L}^{-1}}\left\{\frac{s^3 + 3.19 * s^2 + 1.698 * s + 0.3705}{s^3 + 4.1 * s^2 + 2.121 * s + 0.3649}\right\}(\boldsymbol{t})$$

$$=\boldsymbol{\mathcal{L}^{-1}}\left\{\frac{-0.9334}{s+3.5285} + \frac{0.0117-0.0755i}{s+(0.2858+0.1475i)} + \frac{0.0117+0.0755i}{s+(0.2858+0.1475i)} + 1\right\}$$

$$= -0.9334e^{-3.52t} + (0.0117 - 0.0755i)e^{-(0.2858+0.1475i)t} + (0.0117 + 0.0755i)e^{-(0.2858+0.1475i)t}.$$

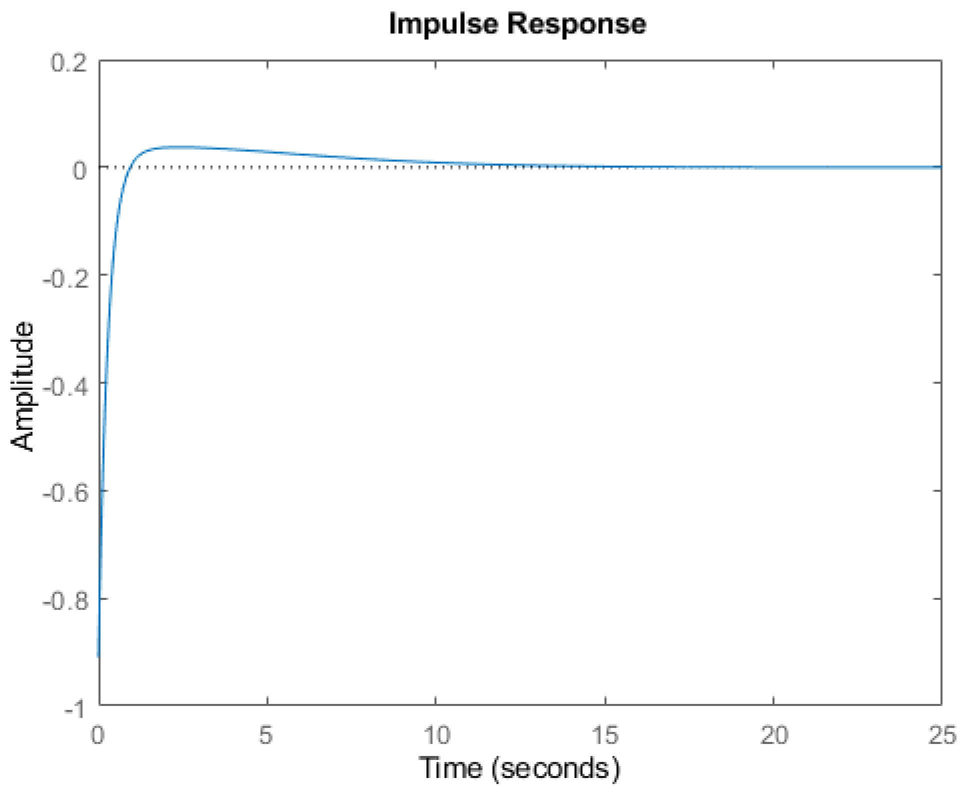
Rezolvarea am realizat-o in Matlab, cu ajutorul functiei **residue.**

```
B =[1 3.19 1.698 0.3705];
A = [1 4.1 2.12 0.3649];
[R,P,K] = residue(B,A);
```

```
>> R

R =

  -0.9334 + 0.0000i
   0.0117 - 0.0755i
   0.0117 + 0.0755i
```

```
>> P

P =

  -3.5285 + 0.0000i
  -0.2858 + 0.1475i
  -0.2858 - 0.1475i
```

```
>> K

K =

     1
```



8.2 Raspunsul Indicial (treapta).

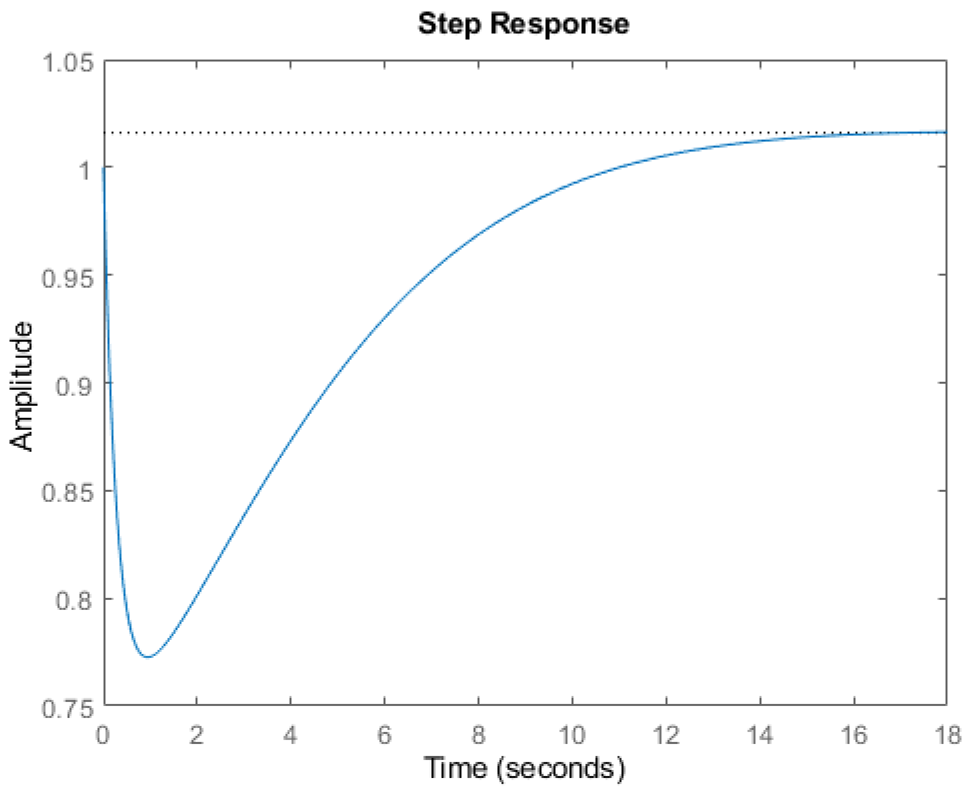
$$y(t) = \mathcal{L}^{-1}\left\{H(s) * \frac{1}{s}\right\} = \mathcal{L}^{-1}\left\{\frac{s^3 + 3.19 * s^2 + 1.698 * s + 0.3705}{s^3 + 4.1 * s^2 + 2.121 * s + 0.3649} * \frac{1}{s}\right\} =$$

$$\mathcal{L}^{-1}\left\{\frac{0.2645}{s + 3.5285} - \frac{0.1399 - 0.19181i}{s - (-0.2858 + 0.1475i)} - \frac{0.1399 + 0.19181i}{s - (-0.2858 - 0.1475i)} + \frac{1.0153}{s}\right\}$$

$$= 0.2645e^{-3.52t} - (0.1399 - 0.19181i)e^{(-0.2858+0.1475i)t} - (0.1399 + 0.19181i)e^{(-0.2858-0.1475i)t} + 1.0153$$

```
>> R

R =

   0.2645 + 0.0000i
  -0.1399 + 0.1918i
  -0.1399 - 0.1918i
   1.0153 + 0.0000i
```

```
>> P

P =

  -3.5285 + 0.0000i
  -0.2858 + 0.1475i
  -0.2858 - 0.1475i
   0.0000 + 0.0000i
```



8.3 Raspunsul la rampa

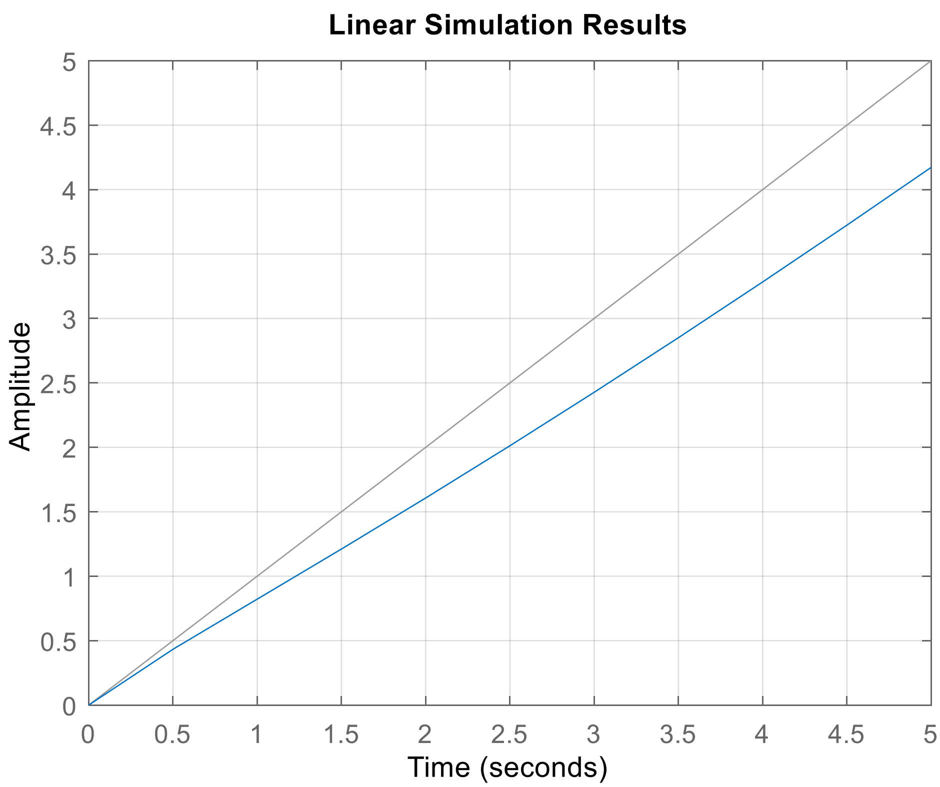
$$y_v(t) = \mathcal{L}^{-1}\left\{H(s) * \frac{1}{s^2}\right\} = \mathcal{L}^{-1}\left\{\frac{s^3 + 3.19 * s^2 + 1.698 * s + 0.3705}{s^3 + 4.1 * s^2 + 2.121 * s + 0.3649} * \frac{1}{s^2}\right\} =$$

$$\mathcal{L}^{-1}\left\{\frac{-0.0750}{s + 3.5285} + \frac{0.6603 - 0.3305i}{s - (-0.2858 + 0.1475i)} + \frac{0.6603 + 0.3305i}{s - (-0.2858 - 0.1475i)} - \frac{1.2456}{s} + \frac{1.0153}{s}\right\}$$

$$= -0.075e^{-3.52t} + (0.6603 - 0.3305i)e^{(-0.2858+0.1475i)t} + (0.6603 + 0.3305i)e^{(-0.2858-0.1475i)t} - 1.2456 + 1.0153$$

```
>> R

R =

  -0.0750 + 0.0000i
   0.6603 - 0.3305i
   0.6603 + 0.3305i
  -1.2456 + 0.0000i
   1.0153 + 0.0000i
```

```
>> P

P =

  -3.5285 + 0.0000i
  -0.2858 + 0.1475i
  -0.2858 - 0.1475i
   0.0000 + 0.0000i
   0.0000 + 0.0000i
```



9. Performante circuit si grafice corespunzatoare

Functia de transfer:

$$H(s) = \frac{s^3 + 3.19 * s^2 + 1.698 * s + 0.3705}{s^3 + 4.1 * s^2 + 2.121 * s + 0.3649}$$

$$H(s) = \frac{(s + 2.59)(s^2 + 0.6005s + 0.1431)}{(s + 3.528)(s^2 + 0.5718s + 0.1034)}$$

Constante de timp:

$$\dot{T}_1 = -\frac{1}{\dot{s}_1} = 0.283$$

$$\hat{T}_1 = -\frac{1}{\hat{s}_1} = 0.386$$

Pulsatia de oscilatie :

Fie : $s^2 + 2\zeta\omega_n s + \omega_n^2$

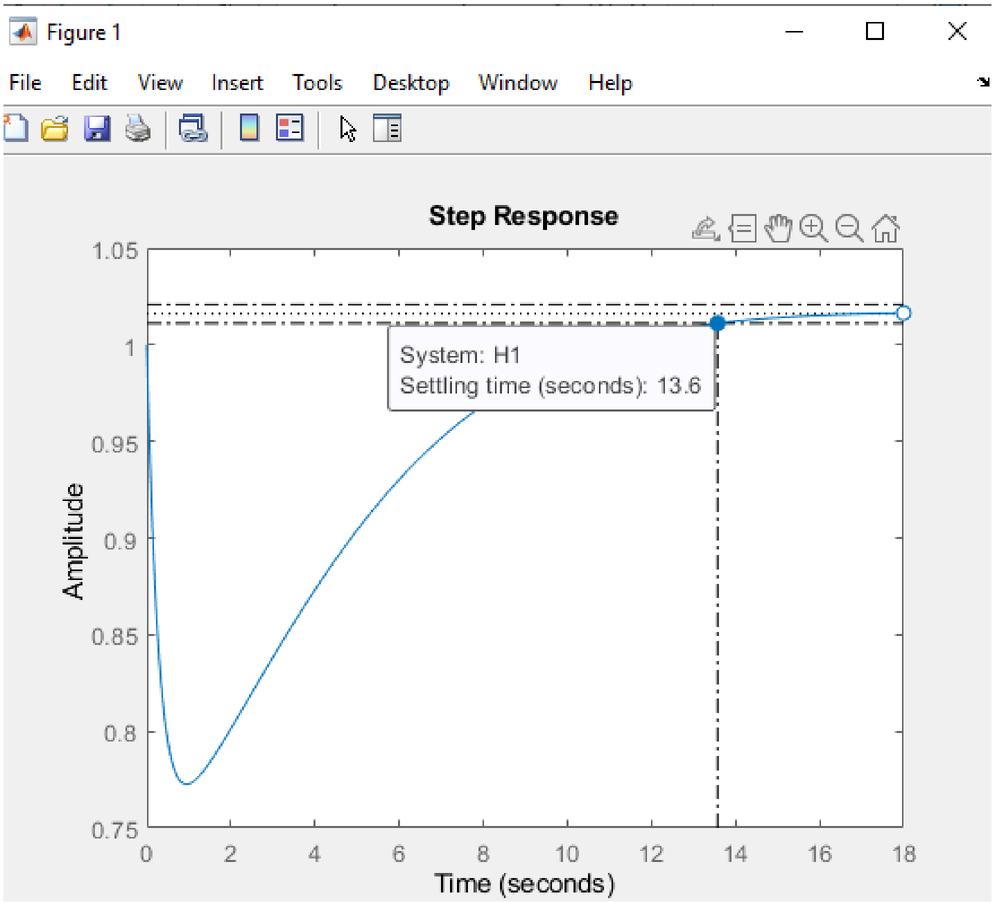
- ⇨ $\widehat{\omega}_n^2 = 0.1034 => \widehat{\omega}_n = \sqrt{0.1034}$ rad/s;
- ⇨ $2\hat{\zeta}\widehat{\omega}_n = 0.5718 => \hat{\zeta} = \frac{0.5718}{2\sqrt{0.1034}}$ < 1
- ⇨ $\dot{\omega}_n^2 = 0.1431 => \dot{\omega}_n = \sqrt{0.1431}$ rad/s ;
- ⇨ $2\dot{\zeta}\dot{\omega}_n = 0.6005 => \dot{\zeta} = \frac{0.6005}{2\sqrt{0.1431}}$ <1

Factorul de proportionalitate :

H(0) = k = $\frac{0.3705}{0.3649} = 1.01$

Timpul de raspuns :

$$t_r \cong \frac{4}{\zeta\omega_n} \cong \frac{4}{\frac{0.5718}{2\sqrt{0.1034}} * \sqrt{0.1034}} \cong \frac{8}{0.5718} \cong 13.99\ s$$



Se poate observa in graficul de mai sus ca timpul de raspuns este 13.6 secunde,aproximativ cat reiese si din calcul, si cum sistemul se stabilizeaza in 1.01, avand un semnal de intrare treapta, se poate observa ca semnalul este amplificat cu ”1.01”, deci factorul de proportionalitate este K=1.01.

Pulsatia oscilatiilor :

$$\omega_{osc} = \omega_n\sqrt{1-\zeta^2}$$

⇨ $\omega_{osc} = \sqrt{0.1034} * \sqrt{1 - {\frac{0.5718}{2\sqrt{0.1034}}}^{2}} = 0.0217\ rad/s$

Suprareglajul :

$$\sigma = e^{-\frac{\pi\zeta}{\sqrt{1-\zeta^2}}}$$

$$\sigma = e^{-\frac{\pi \frac{0.5718}{2\sqrt{0.1034}}}{\sqrt{0.2099}}} = 22.37\%$$

9.1. Performantele regimului stationar:

$$\boldsymbol{\mathcal{E}_{ss} = \lim_{t\to\infty}\big(u(t) - y(t)\big) = \lim_{s\to 0} s\big(U(s) - Y(s)\big)}$$

$$\boldsymbol{Y(s) = H(s) * U(s)}$$

- **Eroarea la pozitie:**

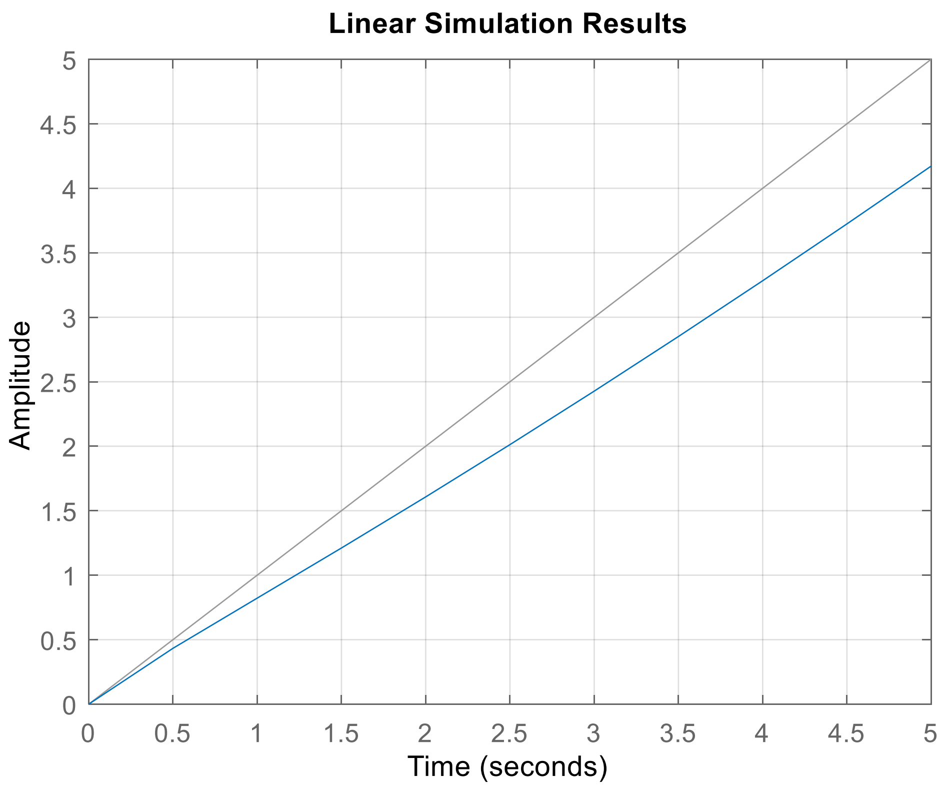
$$\boldsymbol{\varepsilon_{ssp} = 1 - H(0) = 1 - k = 1 - 1.01 = -0.01 \neq 0}$$

- **Eroarea la viteza:**

$$\boldsymbol{\varepsilon_{ssp}} \neq \mathbf{0} \Rightarrow \boldsymbol{\varepsilon_{ssv}} = \pm\infty$$

$$\begin{aligned}
\mathcal{E}_{ssp} &= \lim_{t\to\infty}\big(u(t) - y(t)\big) \\
&= \lim_{s\to 0} s\big(U(s) - Y(s)\big) \\
&= \lim_{s\to 0} s\left(\frac{1}{s^2} - \frac{1}{s^2}H(s)\right) \\
&= \lim_{s\to 0} s\left(\frac{1}{s^2} - \frac{1}{s^2}\,\frac{s^3 + 3.19 * s^2 + 1.698 * s + 0.3705}{s^3 + 4.1 * s^2 + 2.121 * s + 0.3649}\right) \\
&= \lim_{s\to 0} s\ \left(\frac{0.91s^2 + 0.423s^2 - 0.0056}{s^2(s^3 + 4.1 * s^2 + 2.121 * s + 0.3649)}\right) = +\infty
\end{aligned}$$

Se poate observa ca cele doua linii nu sunt paralele, se duc in directii diferite, se separa, deci eroarea la viteza va fi infinita.


Linear Simulation Results
Amplitude
Time (seconds)
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5

10. Structura unui sistem de reglare cu regultor proportional

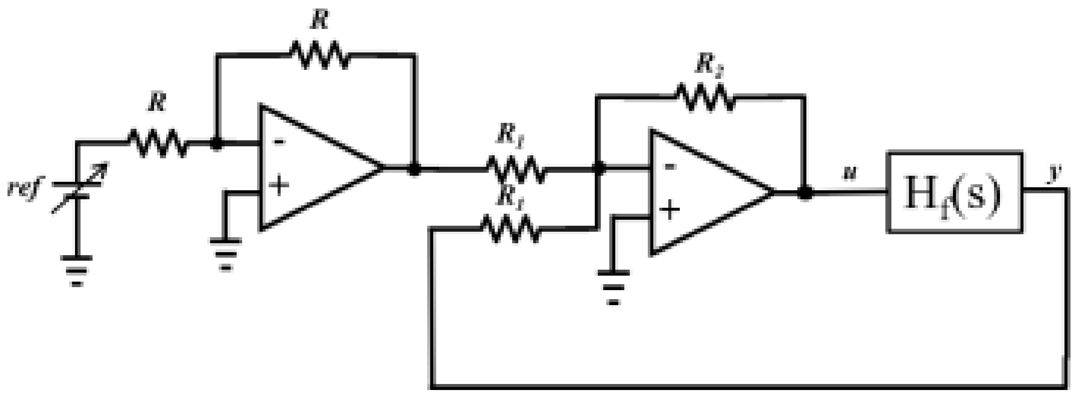
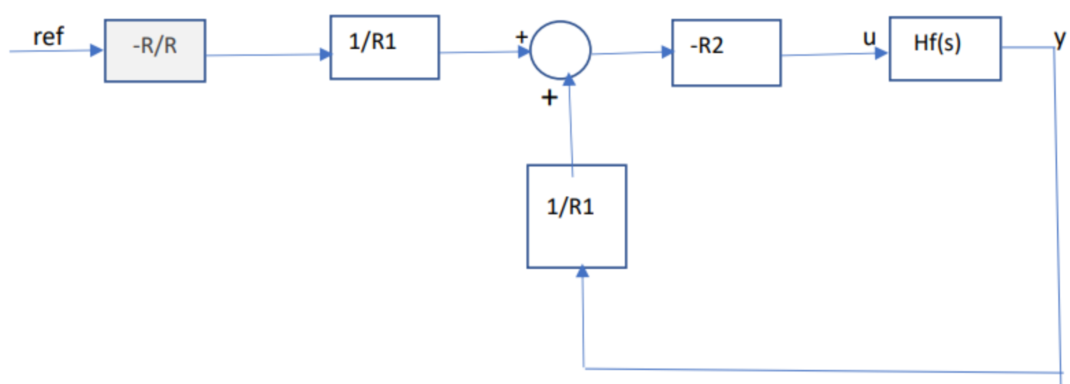


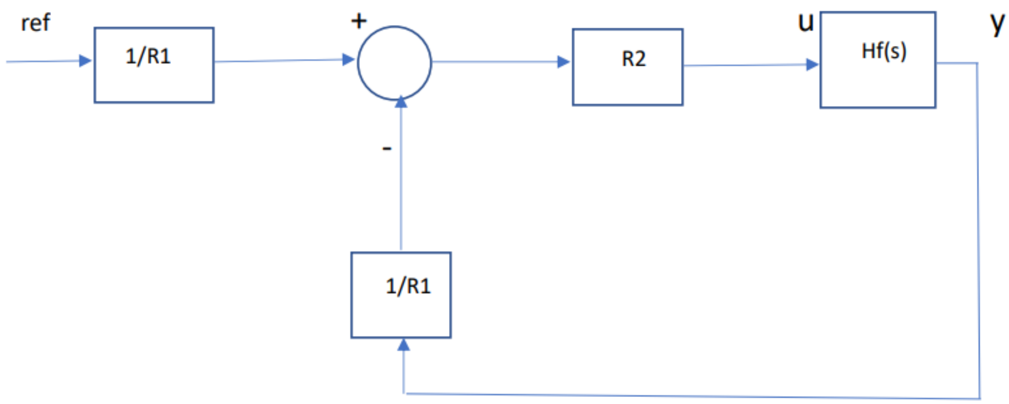
Figure 1: Structura unui sistem de reglare cu regulator proporţional

10.1. Functia de transfer in bucla inchisa

$$H(s) = \frac{s^3 + 3.19 * s^2 + 1.698 * s + 0.3705}{s^3 + 4.1 * s^2 + 2.121 * s + 0.3649}$$

Schema bloc a sistemului de reglare cu regulator proportional, dupa realizarea functiilor de transfer pentru fiecare parte a acestuia este:





In urma calculelor, am ajuns la urmatoarele :

- **Functia de transfer pe calea directa:**

$$H_d(s) = R_2 H_f(s)$$

- **Functia de transfer pe calea de reactie:**

$$H_r(s) = \frac{1}{R_1}$$

- **Functia de transfer in bucla deschisa :**

$$H_{des}(s) = \frac{R_2}{R_1} H_f(s)$$

- **Rezulta ca functia de transfer a sistemului in bucla inchisa este:**

$$H_o(s) = \frac{R_2 H_f(s)}{1 + \frac{R_2}{R_1} H_f(s)}$$

$$H_o(s) = \frac{R_2 \frac{s^3 + 3.19 * s^2 + 1.698 * s + 0.3705}{s^3 + 4.1 * s^2 + 2.121 * s + 0.3649}}{1 + \frac{R_2}{R_1} \frac{s^3 + 3.19 * s^2 + 1.698 * s + 0.3705}{s^3 + 4.1 * s^2 + 2.121 * s + 0.3649}}$$

10.2 *Analiza locului radacinilor in functie de* $\frac{\boldsymbol{R_2}}{\boldsymbol{R_1}}$

Pentru sistemul de mai sus, considerand $k = \frac{R_2}{R_1}$, am putea spune ca :

$$H'_{des} = \frac{s^3 + 3.19 * s^2 + 1.698 * s + 0.3705}{s^3 + 4.1 * s^2 + 2.121 * s + 0.3649}$$

- Se poate observa ca $k' = 1 > 0$
- Singularitatile sistemului :

$\dot{s}1 = -2.5895 + 0.0000i$

$\dot{s}_2 = -0.3002 + 0.2301i$

$\dot{s}_3 = -0.3002 - 0.2301i$

$\widehat{s_1} = -3.5282 + 0.0000i$

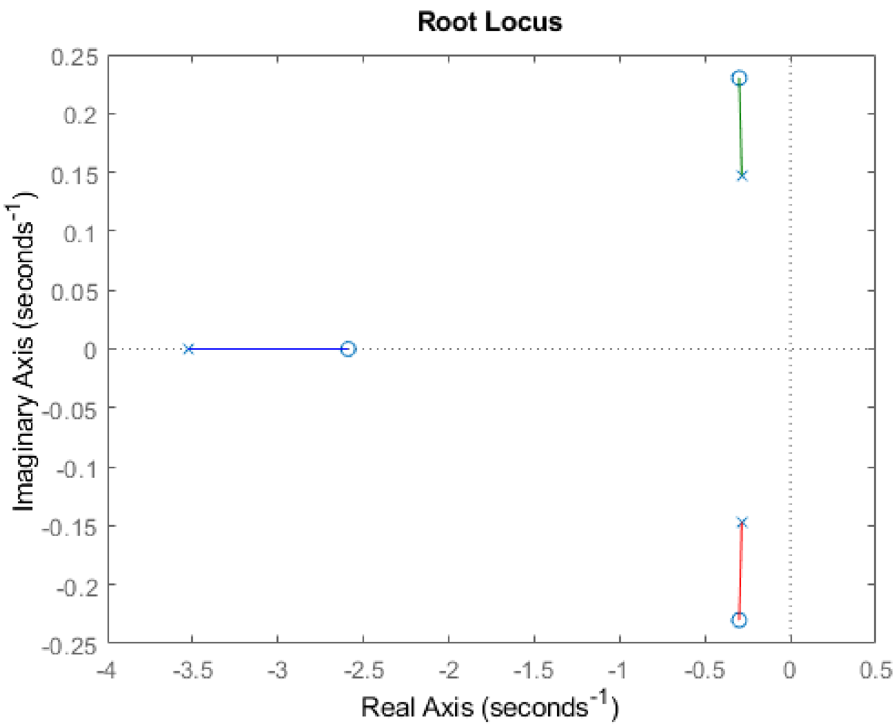
$\widehat{s_2} = -0.2859 + 0.1472i$

$\widehat{s_3} = -0.2859 - 0.1472i$

- Stim ca numarul de poli =3 si numarul de zerouri =3;
- Numarul asimptotelor : $n_a = n - m \Rightarrow n_a = 0$.

  $\frac{R2}{R1} = 0.103$

Datorita faptului ca numarul asimptotelor este 0, nu avem centru de greutate sau unghiuri de plecare .



Polul $\widehat{s_1} = -3.5282 + 0.0000i$ pleaca cu un unghi de 0 pe axa reala si ajunge in zeroul $\dot{s}1 = -2.5895 + 0.0000i$ cu un unghi de 180 de grade tot pe axa reala.

Polii $\widehat{s_2} = -0.2859 + 0.1472i$ si $\widehat{s_3} = -0.2859 - 0.1472i$ complex conjugati, pleaca de pe axa imaginara si ajung in zerourile $\dot{s}_2 = -0.3002 + 0.2301i$ si $\dot{s}_3 = -0.3002 - 0.2301i$, tot complecsi conjugati, situati pe axa imaginara .

Sistemul este extern stabil oricare ar fi R2/R1 pozitiv definit.

- Regimul de functionare :

In cazul de fata, regimul este dat de polul cel mai dominant(polul cel mai din dreapta) si anume de

$\widehat{s_2} = -0.2859 + 0.1472i$ si $\widehat{s_3} = -0.2859 - 0.1472i$, acestia fiind poli complecsi conjugati negativi. Astfel regimul este **<u>oscilant amortizat .</u>**

- Modul de oscilatie :

$$\widehat{s_2}\, si\, \widehat{s_3} \in \mathbb{C}^- \Rightarrow modul\ de\ oscilatie\ este\ e^{Re\{\widehat{s_2}\}t} \sin Im\{\widehat{s_2},\}\, t\ ,$$

$e^{Re\{\widehat{s_3}\}t} \cos Im\{\widehat{s_3}\}\, t$;

10.3. Determinare performante in functie de $\frac{\boldsymbol{R_2}}{\boldsymbol{R_1}}$

Timp de raspuns cu 25% mai mic.

$$t_r{}' = t_r - \frac{25}{100} t_r = 13.99 - 0.25 * 13.99 = 10.49\ s$$

$$\frac{4}{Re\{\hat{s}_1\}} = t_r{}' \Rightarrow Re\{\hat{s}_1\} = \frac{4}{10.49} = 0.381$$

11.A. Analiza sistem de reglare cu regulator de tip Lead/Lag.

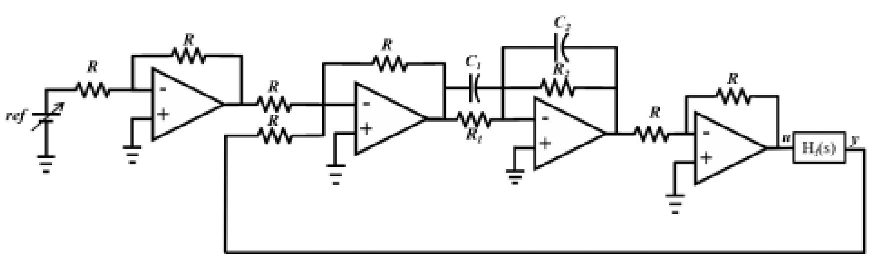
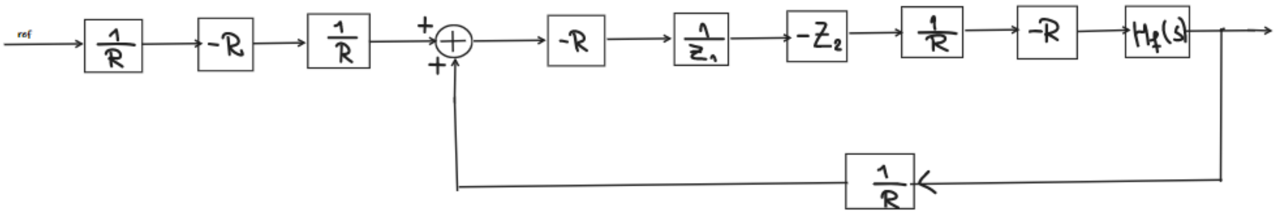
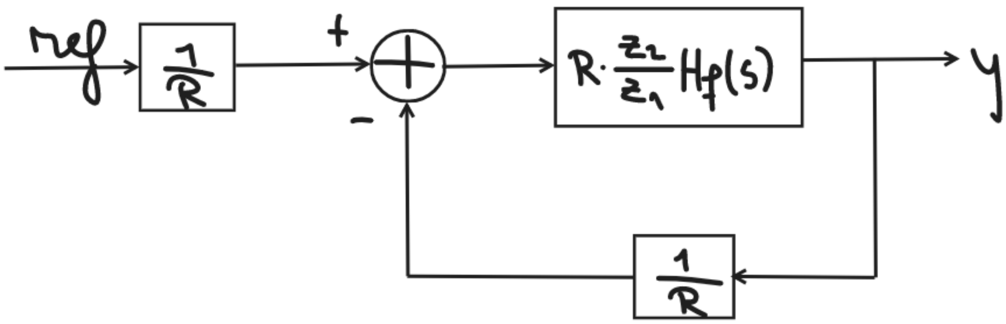


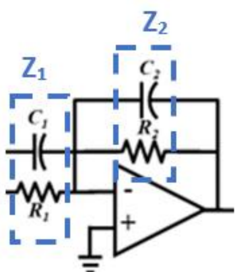
Figure 2: Structura unui sistem de reglare cu regulator de tip *Lead*/*Lag* (cu avans/întârziere de fază)

11.1. Functia de transfer a regulatorului cu avans/intarziere de faza in functie de componentele electrice.

Rezolvam sistemul folosind schema bloc, considerand z1 conexiunea paralela dintre C1 si R1 si z2 conexiunea paralela dintre R2 si C2:







In urma realizarii schemei bloc, se pot observa urmatoarele :

- $H_d = R\frac{Z_2}{Z_1}H_f(s)$
- $H_r = \frac{1}{R}$
- $H_{des} = \frac{Z_2}{Z_1}H_f(s)$.

$$Z_1 = R_1 \parallel C_1 = \frac{R_1 \cdot \frac{1}{sC_1}}{R_1 + \frac{1}{sC_1}} = \frac{R_1}{R_1C_1s + 1}$$

$$Z_2 = R_2 \parallel C_2 = \frac{R_2 \cdot \frac{1}{sC_2}}{R_2 + \frac{1}{sC_2}} = \frac{R_2}{R_2C_2s + 1}$$

Deci vom avea functia de transfer a regulatorului :

$$H_R(s) = K\frac{T_1s+1}{T_2s+1}$$

- $H_R = R \cdot \frac{Z_2}{Z_1} = R \cdot \frac{R_2(R_1C_1s+1)}{R_1(R_2C_2s+1)} = \frac{RR_2}{R_1} \cdot \frac{R_1C_1s+1}{R_2C_2s+1}$

De aici reiese, coeficientul de proportionalitate, $k = \frac{RR_2}{R_1}$, si perioada de timp , $T_1 = R_1C_1$ $si\ T_2 = R_2C_2$

11.2. Functia de transfer a sistemului

Din cele prezentate mai sus, stim urmatoarele :

- $H_d = R \cdot \frac{R_2(R_1C_1s+1)}{R_1(R_2C_2s+1)} \cdot H_f(s) = R \cdot \frac{R_2(R_1C_1s+1)}{R_1(R_2C_2s+1)} \cdot \frac{s^3+3.19*s^2+1.698*s+0.3705}{s^3+4.1*s^2+2.121*s+0.3649}$
- $H_r = \frac{1}{R}$
- $H_{des} = \frac{R_2(R_1C_1s+1)}{R_1(R_2C_2s+1)} \cdot H_f(s) = \frac{R_2(R_1C_1s+1)}{R_1(R_2C_2s+1)} \cdot \frac{s^3+3.19*s^2+1.698*s+0.3705}{s^3+4.1*s^2+2.121*s+0.3649}$

- $H_o = \frac{H_d}{1+H_{des}} = \frac{R \cdot \frac{R_2(R_1C_1s+1)}{R_1(R_2C_2s+1)} \cdot \frac{s^3+3.19*s^2+1.698*s+0.3705}{s^3+4.1*s^2+2.121*s+0.3649}}{1+\frac{R_2(R_1C_1s+1)}{R_1(R_2C_2s+1)} \cdot \frac{s^3+3.19*s^2+1.698*s+0.3705}{s^3+4.1*s^2+2.121*s+0.3649}}$ - functia de transfer a sistemului in bucla inchisa

Functia de transfer a sistemului :

- $H(s) = \frac{1}{R} \cdot H_o = \frac{\frac{R_2(R_1C_1s+1)}{R_1(R_2C_2s+1)} \cdot \frac{s^3+3.19*s^2+1.698*s+0.3705}{s^3+4.1*s^2+2.121*s+0.3649}}{1+\frac{R_2(R_1C_1s+1)}{R_1(R_2C_2s+1)} \cdot \frac{s^3+3.19*s^2+1.698*s+0.3705}{s^3+4.1*s^2+2.121*s+0.3649}}$

11.3. Analiza locului radacinilor in functie de un T

Analiza se va face dupa T2, rescriem functia de transfer in bucla inchisa astfel incat sa punem T2 sub forma de ”k”.

$$H_o = \frac{R \cdot \frac{R_2(R_1C_1s + 1)}{R_1(R_2C_2s + 1)} \cdot \frac{s^3 + 3.19 * s^2 + 1.698 * s + 0.3705}{s^3 + 4.1 * s^2 + 2.121 * s + 0.3649}}{1 + \frac{R_2(R_1C_1s + 1)}{R_1(R_2C_2s + 1)} \cdot \frac{s^3 + 3.19 * s^2 + 1.698 * s + 0.3705}{s^3 + 4.1 * s^2 + 2.121 * s + 0.3649}}$$

$$= \frac{R \cdot \frac{R_2(T_1 s + 1)}{R_1(T_2 s + 1)} \cdot \frac{s^3 + 3.19 * s^2 + 1.698 * s + 0.3705}{s^3 + 4.1 * s^2 + 2.121 * s + 0.3649}}{1 + \frac{R_2(T_1 s + 1)}{R_1(T_2 s + 1)} \cdot \frac{s^3 + 3.19 * s^2 + 1.698 * s + 0.3705}{s^3 + 4.1 * s^2 + 2.121 * s + 0.3649}}$$

$$= \frac{R \cdot R_2(T_1 s + 1) \cdot (s^3 + 3.19 * s^2 + 1.698 * s + 0.3705)}{R_1(T_2 s + 1) \cdot (s^3 + 4.1 * s^2 + 2.121 * s + 0.3649) + R_2(T_1 s + 1) \cdot (s^3 + 3.19 * s^2 + 1.698 * s + 0.3705)}$$

$$= \frac{R \cdot R_2(T_1 s + 1) \cdot (s^3 + 3.19 * s^2 + 1.698 * s + 0.3705)}{T_2 R_1 s \cdot (s^3 + 4.1 * s^2 + 2.121 * s + 0.3649) + R_1 \cdot (s^3 + 4.1 * s^2 + 2.121 * s + 0.3649) + R_2(T_1 s + 1) \cdot (s^3 + 3.19 * s^2 + 1.698 * s + 0.3705)}$$

$$= \frac{\frac{1}{T_2} \cdot \frac{R \cdot R_2(T_1 s + 1) \cdot (s^3 + 3.19 * s^2 + 1.698 * s + 0.3705)}{R_1 s \cdot \left(s^3 + 4.1 * s^2 + 2.121 * s + 0.3649\right)}}{1 + \frac{1}{T_2} \cdot \frac{\left(s^3 + 4.1 * s^2 + 2.121 * s + 0.3649\right) + R_2\left(T_1 s + 1\right) \cdot \left(s^3 + 3.19 * s^2 + 1.698 * s + 0.3705\right)}{R_1 s \cdot \left(s^3 + 4.1 * s^2 + 2.121 * s + 0.3649\right)}}$$

Deci $k = \frac{1}{T_2}$.

Lucram cu urmatoarele valori:

$R = 1\Omega;\ R_1 = 1\Omega; R_2 = 3\Omega; C_1 = 5F$

$$\Rightarrow H_o = \frac{\frac{1}{T_2} \cdot \frac{3 \cdot (5s+1) \cdot \left(s^3+3.19*s^2+1.698*s+0.3705\right)}{s \cdot \left(s^3+4.1*s^2+2.121*s+0.3649\right)}}{1+\frac{1}{T_2} \cdot \frac{\left(s^3+4.1*s^2+2.121*s+0.3649\right)+3(5s+1) \cdot \left(s^3+3.19*s^2+1.698*s+0.3705\right)}{s \cdot \left(s^3+4.1*s^2+2.121*s+0.3649\right)}}$$

De aici se observa faptul ca :

$$H'_{des} = \frac{(s^3 + 4.1 * s^2 + 2.121 * s + 0.3649) + 3(5s + 1) \cdot (s^3 + 3.19 * s^2 + 1.698 * s + 0.3705)}{s \cdot (s^3 + 4.1 * s^2 + 2.121 * s + 0.3649)}$$

$$= \frac{15s^4+49.85s^3+32.76s^2+9.37s+0.7354}{s^4+4.1*s^3+2.121s^2+0.3649s}$$

Singularitatile sistemului sunt :

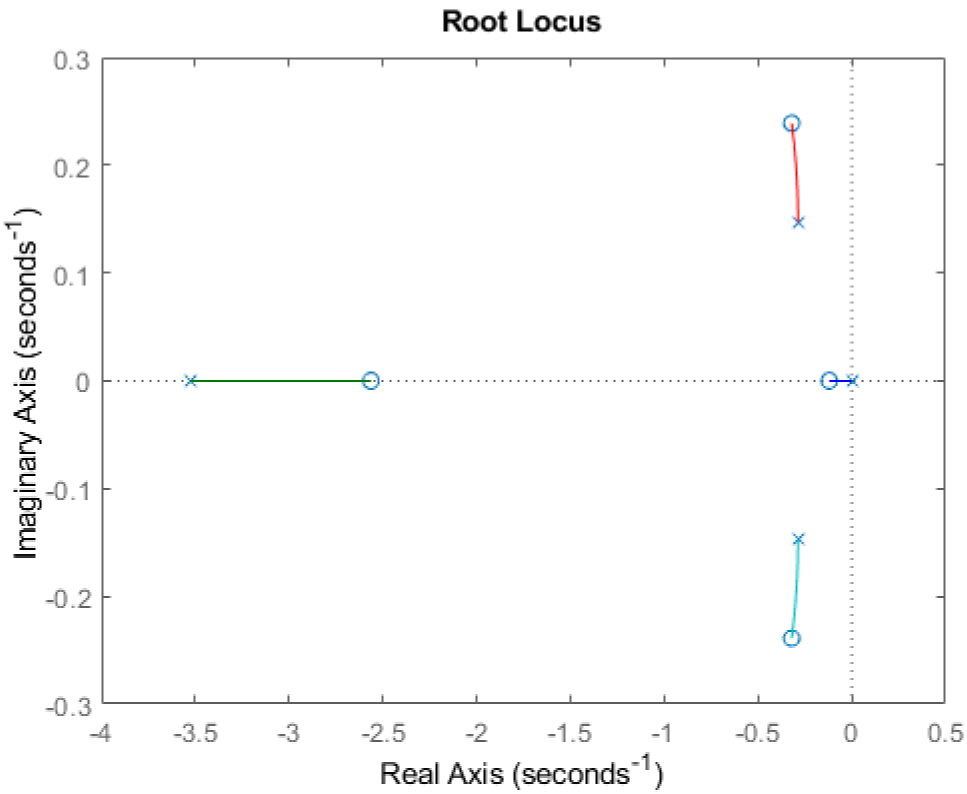
- 4 zerouri , dintre care 2 sunt reale negative si 2 complex conjugate.

$\dot{s}_1 = -2.5635$
$\dot{s}_2 = -0.3199 + 0.2390i$
$\dot{s}_3 = -0.3199 - 0.2390i$
$\dot{s}_4 = -0.1199$

- 4 poli, dintre care 1 real negativ, 2 complex conjugate si unul in 0.

$\hat{s}_1 = 0;$

$\hat{s}_2 = -3.5281;$
$\hat{s}_3 = -0.2859 + 0.1472i;$
$\hat{s}_4 = -0.2859 - 0.1472i;$



- Stim ca numarul de poli =4 si numarul de zerouri =4;
- Numarul asimptotelor : $n_a = n - m \Rightarrow n_a = 0$.

Datorita faptului ca numarul asimptotelor este 0, nu avem centru de greutate sau unghiuri de plecare .

Polul $\widehat{s_1} = 0$ pleaca cu un unghi de 0 pe axa reala si ajunge in zeroul $\dot{s}_4 = -0.1199$ cu un unghi de 180 de grade tot pe axa reala.

Polul $\hat{s}_2 = -3.5281;$ pleaca cu un unghi de 0 pe axa reala si ajunge in zeroul $\dot{s}_1 = -2.5635$de 180 de grade tot pe axa reala.

Polii $\hat{s}_3 = -0.2859 + 0.1472i;$ si $\hat{s}_4 = -0.2859 - 0.1472i,$ complex conjugati, pleaca de pe axa imaginara si ajung in zerourile $\dot{s}_2 = -0.3199 + 0.2390i$ si $\dot{s}_3 = -0.3199 - 0.2390i$, tot complecsi conjugati, situati pe axa imaginara .

- Regimul de functionare :

In cazul de fata, regimul este dat de polul cel mai dominant(polul cel mai din dreapta) si anume de

$\widehat{s_1} = 0$ acesta fiind pol real. Astfel regimul este **aperiodic amortizat .**

Sistemul este stabil pentru orice $T_2 > 0$.

- Modul de oscilatie :

$$\widehat{s_1} = 0 \;=> modul\ de\ oscilatie\ este\ e^{\widehat{s_1}\,t} = e^{0t} = 1\,;$$

11.4. Determinare performante in functie de un T fixat

Pulsatia de oscilatie minima este 0, deoarece pleaca dintr-un pol dominant real, ce nu are oscilatie. Deci am avea $\frac{1}{T_2} = 0$, deci am avea nevoie de un T2$- > \infty$