Seminar 4. Deriving multipliers and calculating the multiplier effects

Consider the following model, which describes the case of a closed economy with government:

$$\begin{cases} Y = C + I + G \\ C = C_0 + c \cdot Y^D \\ Y^D = Y - T + TR \\ T = T_0 + t \cdot Y \end{cases}$$

where we have: output / GDP (or aggregate income/ aggregate expenditure) (Y), private consumption (C), disposable income (Y^D) and the level of taxes (T) as **endogenous** (dependent) variables, and autonomous consumption (C_0), private investment (I), government expenditure (G), transfers to households (TR) and autonomous taxes (T_0) as **exogenous** (independent) variables included in the model. Endogenous variables are determined by the model itself, while the exogenous variables are predetermined- that the model takes as given.

Budgetary and fiscal policy decisions regarding the level of government spending, transfers to households, and taxes, as well as other factors that may affect the level of private investment or even autonomous consumption, will generate changes in output, private consumption, disposable income or even in aggregate taxes or the budget deficit.

A **multiplier** refers to an economic factor that, when changed (increased or decreased), causes changes in many other related economic variables. These changes are known as **multiplier effects**.

Based on the model presented above, we can derive an entire family of multipliers that are useful in quantifying the macroeconomic effects of different changes on the main economic indicators. Possible sets of multipliers include:

a) multipliers that describe the variation in output (Y)

$\alpha_G = \frac{\Delta Y}{\Delta G}$ is the government spending multiplier

$\alpha_I = \frac{\Delta Y}{\Delta I}$ is the investment multiplier

$\alpha_{C0} = \frac{\Delta Y}{\Delta C_0}$ is the autonomous consumption multiplier

$\alpha_{TR} = \frac{\Delta Y}{\Delta TR}$ is the transfers multiplier

$\alpha_{T0} = \frac{\Delta Y}{\Delta T_0}$ is the autonomous tax multiplier

$\alpha_t = \frac{\Delta Y}{\Delta t}$ is the tax rate multiplier

b) multipliers that describe the variation in disposable income (Y^D)

$mdi_G = \frac{\Delta Y^D}{\Delta G}$ indicates the variation in Y^D due to a change in G

$mdi_{C0} = \frac{\Delta Y^D}{\Delta C_0}$ indicates the variation in Y^D due to a change in C_0

$mdi_{TR} = \frac{\Delta Y^D}{\Delta TR}$ indicates the variation in Y^D due to a change in TR

$mdi_{T0} = \frac{\Delta Y^D}{\Delta T_0}$ indicates the variation in Y^D due to a change in T_0

$mdi_t = \frac{\Delta Y^D}{\Delta t}$ indicates the variation in Y^D due to a change in the tax rate

c) multipliers that describe the variation in consumption (C)

$mc_G = \frac{\Delta C}{\Delta G}$ indicates the variation in consumption due to a change in G

$mc_{C0} = \frac{\Delta C}{\Delta C_a}$ indicates the variation in consumption due to a change in C_0

$mc_{TR} = \frac{\Delta C}{\Delta TR}$ indicates the variation in consumption due to a change in TR

$mc_{T0} = \frac{\Delta C}{\Delta T_0}$ indicates the variation in consumption due to a change in T_0

$mc_t = \frac{\Delta C}{\Delta t}$ indicates the variation in consumption due to a change in the tax rate

d) multipliers that describe the variation in taxes (T)

$mt_G = \frac{\Delta T}{\Delta G}$ indicates the variation in taxes due to a change in G

$mt_{TR} = \frac{\Delta T}{\Delta TR}$ indicates the variation in taxes due to a change in TR

$mt_{T0} = \frac{\Delta T}{\Delta T_0}$ indicates the variation in taxes due to a change in T_0

$mt_t = \frac{\Delta T}{\Delta t}$ indicates the variation in taxes due to a change in the tax rate

e) multipliers that describe the variation in the budget deficit (BD)

$mbd_G = \frac{\Delta BD}{\Delta G}$ indicates the variation in budget deficit due to a change in G

$mbd_{TR} = \frac{\Delta BD}{\Delta TR}$ indicates the variation in budget deficit due to a change in TR

$mbd_{T0} = \frac{\Delta BD}{\Delta T_0}$ indicates the variation in budget deficit due to a change in T_0

$mbd_t = \frac{\Delta BD}{\Delta t}$ indicates the variation in budget deficit due to a change in the tax rate

To determine the multipliers, the model must first be rewritten in **reduced form**. This means that Y, an endogenous variable, will be expressed in terms of exogenous variables only. To this purpose, we will substitute T in the equation of Y^D, then Y^D in the equation of C, and finally, C in the equation of Y, as indicated by the arrows:

$$\begin{cases} Y = C_0 + c \cdot (1-t)Y - cT_0 + cTR + I + G + NX \\ C = C_0 + c \cdot (1-t)Y - cT_0 + cTR \\ Y^D = (1-t)Y - T_0 + TR \\ T = T_0 + tY \end{cases}$$

The reduced form of the model becomes:

$$Y = \frac{1}{1-c(1-t)} * [C_0 - cT_0 + cTR + G + I + NX]$$

a) We can therefore determine the multipliers used to calculate the **variation in output** (Y).

In case of a change in the level of government spending (ΔG), we will have (all the other exogenous factors are assumed constant):

$$\Delta Y = \frac{1}{1-c(1-t)} \cdot \Delta G \Rightarrow \Delta Y = \alpha_G \cdot \Delta G$$

Thus, the **government spending multiplier** is:

$$\alpha_G = \frac{\Delta Y}{\Delta G} = \frac{1}{1-c(1-t)},$$

which shows the multiple by which Y increases or decreases in response to an increase or decrease in government spending.

Following the same reasoning, in the case of a change in the level of private investment (ΔI), we will have (ceteris paribus):

$$\Delta Y = \frac{1}{1-c(1-t)} \cdot \Delta I .$$

$$\alpha_I = \frac{\Delta Y}{\Delta I} = \frac{1}{1-c(1-t)} = \alpha_G.$$

It follows that for this economic model, the **investment multiplier** will be equal to that of government spending. Similar conclusions can be drawn in the case of the autonomous consumption multiplier:

$$\alpha_{C0} = \frac{\Delta Y}{\Delta C_0} = \frac{1}{1-c(1-t)} = \alpha_G$$

However, when the level of transfers to households changes, the calculation formula changes as follows:

$$\Delta Y = \frac{1}{1-c(1-t)} \cdot c\,\Delta TR \Rightarrow \Delta Y = c \cdot \alpha_G \cdot \Delta TR$$

The result is the **transfers multiplier**:

$$\alpha_{TR} = \frac{\Delta Y}{\Delta TR} = \frac{1}{1-c(1-t)} \cdot c = c \cdot \alpha_G$$

Similarly, in case of a change in the level of autonomous taxes, ceteris paribus, we have:

$$\Delta Y = \frac{1}{1-c(1-t)} \cdot (-c) \cdot \Delta T_0 \Rightarrow \Delta Y = -c \cdot \alpha_G \cdot \Delta T_0$$

The result is the **autonomous tax multiplier**:

$$\alpha_{T0} = \frac{\Delta Y}{\Delta T_a} = \frac{1}{1-c(1-t)} \cdot (-c) = -c \cdot \alpha_G$$

A special case is the tax rate multiplier. In this situation, a change in the tax rate *t* (from t_0 to t_1) will affect the slope of the line. The calculation of the tax rate multiplier involves the following steps:

We write Y_0 and Y_1 in relation to t_0 and t_1 and compute ΔY.

$$Y_0 = \frac{1}{1-c(1-t_0)} * [C_0 - cT_0 + cTR + G + I + NX]$$

$$Y_1 = \frac{1}{1-c(1-t_1)} * [C_0 - cT_0 + cTR + G + I + NX]$$

$$\Delta Y = Y_1 - Y_0 = \left[\frac{1}{1-c(1-t_1)} - \frac{1}{1-c(1-t_0)}\right] \cdot [C_0 - cT_0 + cTR + G + I + NX]$$

$$\Delta Y = \frac{-c \cdot \Delta t}{1-c(1-t_1)} \cdot Y_0$$

The result is the **tax rate multiplier** (its value is negative):

$$\alpha_t = \frac{\Delta Y}{\Delta t} = \frac{-c \cdot Y_0}{1-c(1-t_1)}$$

Note: In the literature, for practical reasons, the following formula for the tax rate multiplier is also accepted, as the numerical differences are very small:

$$\alpha_t = \frac{\Delta Y}{\Delta t} = \frac{-c \cdot Y_0}{1-c(1-t_0)} = -c \cdot \alpha_G \cdot Y_0.$$

Exercise:

Consider a closed economy, for which we know the following: government spending G = 500 bil.lei, autonomous taxes T_0 = 400 bil.lei, investments I = 200 bil.lei, autonomous consumption C_0 = 100 bil.lei, transfers TR = 0 and the marginal propensity to consume = 0.9

a) Determine the levels of GDP, private consumption, taxes, and budgetary deficit if the government only intends to collect lump-sum taxes (tax rate *t* is 0).
b) Determine the level of GDP and the budgetary deficit if the tax rate *t* is 1/3.