Seminar 4. Deriving multipliers and calculating the multiplier effects (cont.)

Consider the following model, which describes the case of a closed economy with government:

$$\begin{cases} Y = C + I + G \\ C = C_0 + c \cdot Y^D \\ Y^D = Y - T + TR \\ T = T_0 + t \cdot Y \end{cases}$$

b) We follow the procedure explained in the previous seminar to derive the multipliers used to calculate the **variation in disposable income** (Y^D). However, we now must consider the **extended formula of disposable income** and the indirect effects on output.

For example, if there is a change in the level of government spending (ΔG), ceteris paribus, we will have:

$$\Delta Y^D = (1-t)\Delta Y - \Delta T_0 + \Delta TR$$

Since autonomous taxes and transfers are exogenous variables and do not change, we will have $\Delta T_0 = 0$ and $\Delta TR = 0$. So:

$$\Delta Y^D = (1-t)\Delta Y = (1-t) \cdot \alpha_G \cdot \Delta G.$$

Therefore:

$$\text{mdi}_G = \frac{\Delta Y^D}{\Delta G} = (1-t)\frac{\Delta Y}{\Delta G} = (1-t) \cdot \alpha_G$$

will indicate the multiple by which the disposable income increases or decreases in response to an increase or decrease in government spending.

Using the same reasoning we get the following multipliers:

$$\text{mdi}_{C0} = \frac{\Delta Y^D}{\Delta C_0} = (1-t)\frac{\Delta Y}{\Delta C_a} = (1-t) \cdot \alpha_G$$

which will indicate the multiple by which the disposable income increases or decreases in response to an increase or decrease in autonomous consumption,

$$\text{mdi}_{TR} = \frac{\Delta Y}{\Delta TR} = (1-t)\frac{\Delta Y}{\Delta TR} + \frac{\Delta TR}{\Delta TR} = (1-t) \cdot c \cdot \alpha_G + 1 = \frac{c \cdot (1-t) \cdot}{1-c(1-t)} + 1 = \alpha_G$$

which will indicate the multiple by which the disposable income increases or decreases in response to an increase or decrease in in transfers,

$$\text{mdi}_{T0} = \frac{\Delta Y^D}{\Delta T_0} = (1-t)\frac{\Delta Y}{\Delta T_a} - \frac{\Delta T_a}{\Delta T_a} = (1-t)\cdot(-c)\cdot\alpha_G - 1 = \frac{-c\cdot(1-t)\cdot}{1-c(1-t)} - 1 = -\alpha_G$$

which will indicate the multiple by which the disposable income changes in response to a change in autonomous taxes. Note that $\text{mdi}_{T0} < 0$.

If there is a change in the tax rate t (from t_0 to t_1), the calculation of the multiplier will involve the following steps:

We write Y^D_0 and Y^D_1 in relation to t_0 and t_1 and compute ΔY^D:

$$Y^D_0 = (1-t_0)Y_0 - T_0 + TR$$

$$Y^D_1 = (1-t_1)Y_1 - T_0 + TR$$

$$\Delta Y^D = (1-t_1)Y_1 - (1-t_0)Y_0 = \Delta Y - t_1 Y_1 + t_0 Y_0$$

which can be written as:

$$\Delta Y^D = \Delta Y - \Delta t\, Y_1 - t_0\, \Delta Y,$$

where $\Delta Y = \frac{-c\cdot\Delta t}{1-c(1-t_1)}\cdot Y_0$ was previously determined.

Hence:

$$\text{mdi}_t = \frac{\Delta Y^D}{\Delta t} = \frac{\Delta Y - \Delta t\, Y_1 - t_0\, \Delta Y}{\Delta t}$$

c) We now determine the multipliers that are used to calculate the **variation in consumption** (C), considering the **extended consumption formula** and the indirect effects on output.

If we consider a change in the level of government spending (ΔG), ceteris paribus, we can write the variation in consumption as:

$$\Delta C = \Delta C_0 + c\cdot(1-t)\Delta Y - c\Delta T_0 + c\Delta TR$$

Since only ΔG and ΔY are non-zero, it results: $\Delta C = c\cdot(1-t)\Delta Y$.

Thus, we have:

$$\text{mc}_G = \frac{\Delta C}{\Delta G} = c(1-t)\frac{\Delta Y}{\Delta G} = c(1-t)\alpha_G$$

that will indicate the multiple by which the consumption increases or decreases in response to an increase or decrease in government spending,

$$\text{mc}_{C0} = \frac{\Delta C}{\Delta C_a} = 1 + c(1-t)\frac{\Delta Y}{\Delta C_a} = 1 + c(1-t)\alpha_G$$

that will indicate the multiple by which the consumption increases or decreases in response to an increase or decrease in autonomous consumption C_0,

$$mc_{TR} = \frac{\Delta C}{\Delta TR} = c + c(1 - t)\frac{\Delta Y}{\Delta TR} = c + c^2 \cdot (1 - t) \cdot \alpha_G = c \cdot \alpha_G$$

that will indicate the multiple by which the consumption increases or decreases in response to an increase or decrease in transfers.

$$mc_{T0} = \frac{\Delta C}{\Delta T_a} = -c + c(1 - t)\frac{\Delta Y}{\Delta T_a} = -c - c^2 \cdot (1 - t) \cdot \alpha_G = -c \cdot \alpha_G$$

that will indicate the multiple by which the consumption changes in response to a change in autonomous taxes, T_0. Note that $mc_{T0} < 0$.

If there is a change in the tax rate *t* (from t_0 to t_1), the calculation of the multiplier will involve the following steps:

We write C_0 and C_1 in relation to t_0 and t_1 and compute ΔC.

$$C_0 = C_0 + c(1 - t_0)Y_0 - cT_0 + cTR$$

$$C_1 = C_0 + c(1 - t_1)Y_1 - cT_0 + cTR$$

$$\Delta C = c(1 - t_1)Y_1 - c(1 - t_0)Y_0 = c\Delta Y - ct_1Y_1 + ct_0Y_0$$

which can be written as:

$$\Delta C = c\Delta Y - c\,\Delta t\,Y_1 - ct_0\,\Delta Y,$$

where $\Delta Y = \frac{-c \cdot \Delta t}{1 - c(1 - t_1)} \cdot Y_0$, was previously determined.

Therefore:

$$mc_t = \frac{\Delta C}{\Delta t} = \frac{c\Delta Y - c\,\Delta t\,Y_1 - ct_0\Delta Y}{\Delta t}$$

which will indicate the multiple by which the consumption changes in response to a change in the tax rate t.

d) The multipliers that are used to calculate the **variation in taxes** (T) are as follows:

$$mt_G = \frac{\Delta T}{\Delta G}$$

indicates the multiple by which the taxes increase or decrease as a result of an increase or decrease in G,

$$mt_{TR} = \frac{\Delta T}{\Delta TR} = t\frac{\Delta Y}{\Delta TR} = ct\alpha_G$$

indicates the multiple by which the taxes increase or decrease as a result of an increase or decrease in TR,

$$mt_{T0} = \frac{\Delta T}{\Delta T_a} = 1 + t\frac{\Delta Y}{\Delta T_a} = 1 - c \cdot t \cdot \alpha_G$$

indicates the multiple by which the taxes change as a result of a change in autonomous taxes T_0.

If there is a change in the tax rate *t* (from t_0 to t_1), the calculation of the multiplier will involve the following steps:

We write T_0 and T_1 in relation to t_0 and t_1 and compute ΔT.

$$T_0 = T_0 + t_0 Y_0$$

$$T_1 = T_0 + t_1 Y_1$$

$$\Delta T = t_1 Y_1 - t_0 Y_0 = \Delta t Y_0 + t_1 \Delta Y$$

Therefore:

$$mt_t = \frac{\Delta T}{\Delta t} = \frac{\Delta t Y_0 + t_1 \Delta Y}{\Delta t}$$

will be used to calculate the change in taxes as a result of the change in the tax rate.

e) We now derive the multipliers that are used to calculate the **variation in budget deficit** (BD). To this purpose, we will use the budget deficit equation rewritten as follows:

$$\Delta BD = \Delta G + \Delta TR - \Delta T = \Delta G + \Delta TR - \Delta T_0 - t\Delta Y$$

Thus:

$$mbd_G = \frac{\Delta DB}{\Delta G} = 1 - t\frac{\Delta Y}{\Delta G} = 1 - t \cdot \alpha_G$$

will indicate the multiple by which the budget deficit will change as a result of the change in government spending,

$$mbd_{TR} = \frac{\Delta DB}{\Delta TR} = 1 - t\frac{\Delta Y}{\Delta TR} = 1 - c \cdot t \cdot \alpha_G$$

will indicate the multiple by which the budget deficit will change as a result of the change in transfers (TR),

$$mbd_{T0} = \frac{\Delta DB}{\Delta T_a} = -1 - t\frac{\Delta Y}{\Delta T_a} = -1 + c \cdot t \cdot \alpha_G$$

will indicate the multiple by which the budget deficit will change as a result of the change in autonomous taxes T_0.

If there is a change in the tax rate *t* (from t_0 to t_1), the multiplier calculation will involve the following steps:

We write BD_0 and BD_1 in relation to t_0 and t_1 and compute ΔBD.

$$BD_0 = G + TR - T_0 - t_0 Y_0$$

$$BD_1 = G + TR - T_0 - t_1 Y_1$$

$$\Delta BD = -\Delta T = -t_1 Y_1 + t_0 Y_0 = -\Delta t Y_0 - t_1 \Delta Y$$

$$mbd_t = \frac{\Delta DB}{\Delta t} = \frac{-\Delta t Y_0 - t_1 \Delta Y}{\Delta t}$$

will indicate the multiple by which the budget deficit will change as a result of the change in the tax rate.

Exercise 2

Consider a closed economy with government in which the tax rate is 33.33%, and the marginal propensity to consume is 0.9. Determine the change in government spending needed to reduce the budget deficit by 100 million lei.