

```
In [5]: #problema 1a
x=var('x')
y=function('y')(x)
deq=diff(y,x)+y*cot(x)==1/sin(x)
show(desolve(deq,y))
```

$$\frac{C + x}{\sin(x)}$$

```
In [6]: #1b
show(desolve(deq,y,ics=[1,3]))
```

$$\frac{x + 3 \sin(1) - 1}{\sin(x)}$$

```
In [19]: #problema 2a
reset()
t=var('t')
x=function('x')(t)
f=function('s')(t)
f(s)=s*(3-s^2)
eqp=solve(f(s)==0,s)
eqp
```

```
Out[19]: [s == -sqrt(3), s == sqrt(3), s == 0]
```

```
In [20]: s1=eqp[0].rhs()
diff(f,s)(s1) #s1 punct de echilibru local asimptotic stabil
```

```
Out[20]: -6
```

```
In [21]: s2=eqp[1].rhs()
diff(f,s)(s2) #s1 punct de echilibru local asimptotic stabil
```

```
Out[21]: -6
```

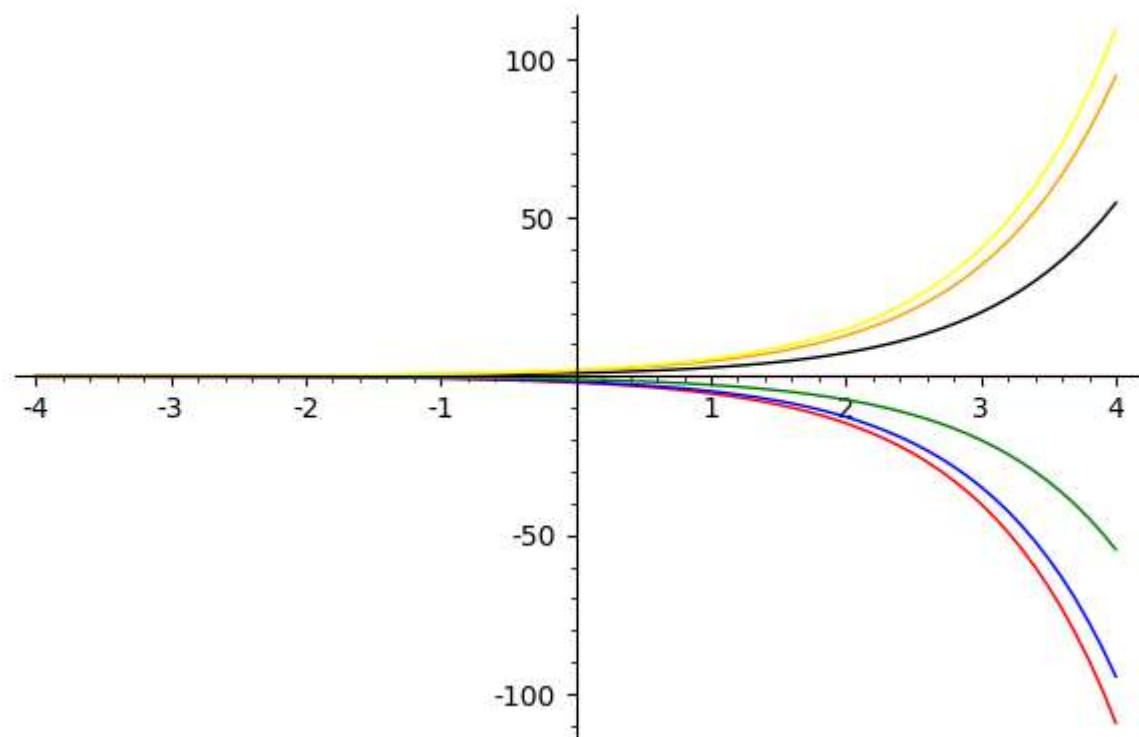
```
In [22]: s3=eqp[2].rhs()
diff(f,s)(s3) #s1 punct de echilibru instabil
```

```
Out[22]: 3
```

```
In [52]: #2b
deq=diff(x,t)==x(t)
g1=desolve_rk4(deq,x,[0,-2], step=0.1, end_points=[-4,4], output='plot', color='r')
g2=desolve_rk4(deq,x,[0,-sqrt(3)], step=0.1, end_points=[-4,4], output='plot', color='b')

g3=desolve_rk4(deq,x,[0,-1], step=0.1, end_points=[-4,4], output='plot', color='g')
g4=desolve_rk4(deq,x,[0,0], step=0.1, end_points=[-4,4], output='plot', color='p')
g5=desolve_rk4(deq,x,[0,1], step=0.1, end_points=[-4,4], output='plot', color='b')

g6=desolve_rk4(deq,x,[0,sqrt(3)], step=0.1, end_points=[-4,4], output='plot', color='r')
g7=desolve_rk4(deq,x,[0,2], step=0.1, end_points=[-4,4], output='plot', color='y')
show(g1+g2+g3+g4+g5+g6+g7)
```



```
In [23]: #problema 3a
reset()
t=var('t')
x=function('x')(t)
y=function('y')(t)
eq1=diff(x,t)==-2*x-4*y
eq2=diff(y,t)==-x+y

syst=[eq1,eq2]
sol=desolve_system(syst,[x,y])
show(sol)
```

$$\left[x(t) = \frac{1}{5} (x(0) - 4)e^{(2t)} + \frac{4}{5} (x(0) + 1)e^{(-3t)}, y(t) = -\frac{1}{5} (x(0) - 4)e^{(2t)} + \frac{1}{5} (x(0) + 1)e^{(-3t)} \right]$$

```
In [24]: #3b
sol1=desolve_system(syst,[x,y],ics=[0,0,1])
show(sol1)
```

$$\left[x(t) = -\frac{4}{5} e^{(2t)} + \frac{4}{5} e^{(-3t)}, y(t) = \frac{4}{5} e^{(2t)} + \frac{1}{5} e^{(-3t)} \right]$$

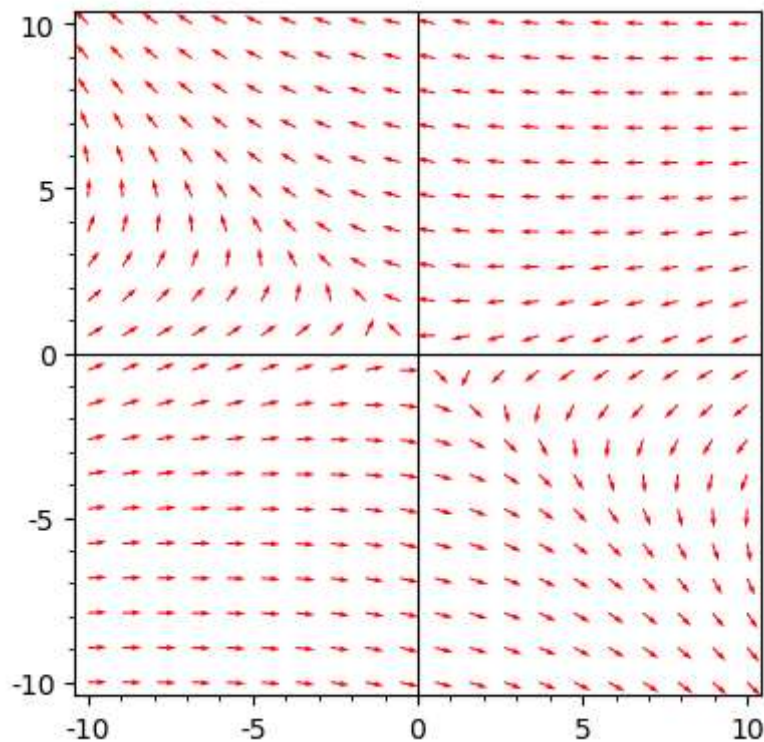
```
In [71]: #3c
reset()
t=var('t')
x=function('x')(t)
y=function('y')(t)
eq1=diff(x,t)==-2*x-4*y
eq2=diff(y,t)==-x+y
syst=[eq1,eq2]

f1(u,v)=-2*u+(-4)*v
f2(u,v)=(-1)*u+v

n=sqrt(f1(u,v)^2+f2(u,v)^2)
g1 = plot_vector_field([f1(u,v)/n,f2(u,v)/n], [u,-10,10], [v,-10,10], color='red'
show(g1)

#putem observa din portretul fazic ca atunci cand t tinde la infinit din x(t)
#   sagetile se indeparteaza de catre origine, deci limita nu este 0, ci tinde

#la fel si in cazul lui y, sagetile se indeparteaza de origine
#   deci limita nu este 0, ci tinde spre infinit
```



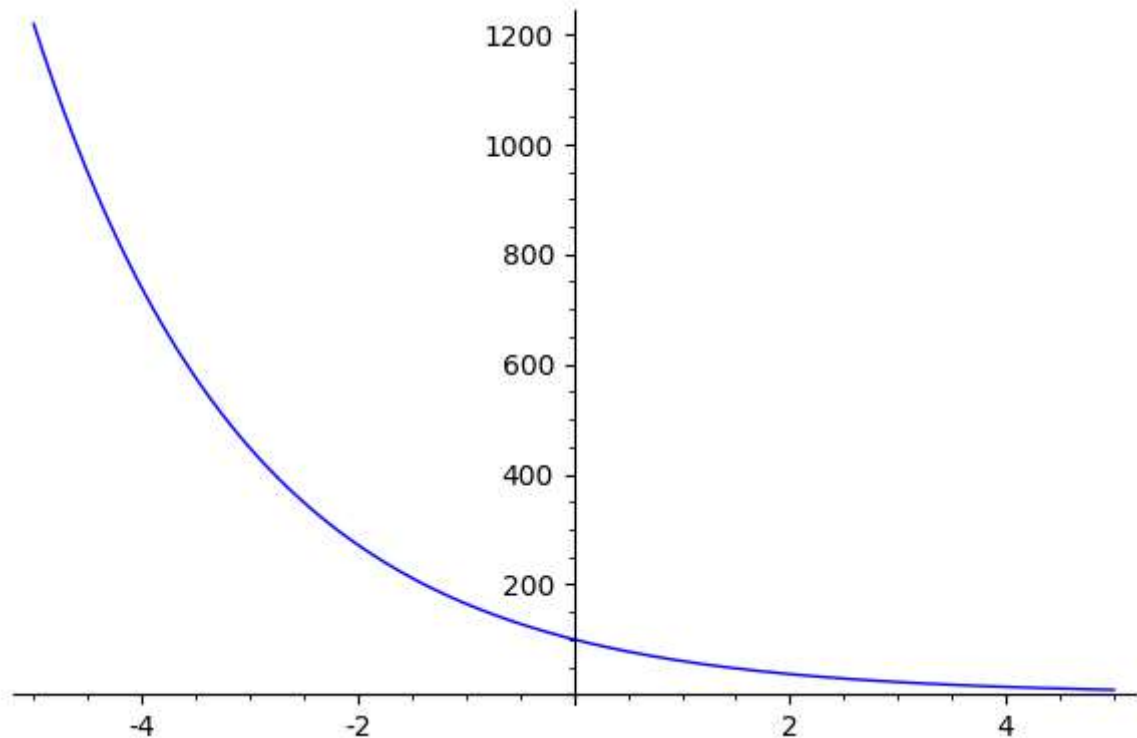
```
In [82]: #problema 4a
reset()
t,r,x0=var('t,r,x0')
x=function('x')(t)
deq=diff(x,t)==r*x
sol(t,x0,r)=desolve(deq,[x,t],[0,x0])
show(sol(t,x0,r))
```

$$x_0 e^{(rt)}$$

```
In [83]: #4b
reset()
t,r,x0=var('t,r,x0')
x=function('x')(t)
r=-0.5
deq=diff(x,t)==r*x
sol(t,x0,r)=desolve(deq,[x,t],ics=[0,100])
show(sol(t,x0,r))
```

$$100 e^{(-\frac{1}{2} t)}$$

```
In [84]: g=plot(sol(t,x0,r),t,-5,5)
show(g)
```



```
In [149]: #4c
reset()
t,r,x0=var('t,r,x0')
x=function('x')(t)
deq=diff(x,t)==r*x
sol(t,x0,r)=desolve(deq,[x,t],ics=[0,x0])
show(sol(t,x0,r))

assume(r,'real')
eq=sol(10,1000,r)==2000
show(solve(eq,r))
```

$$x_0 e^{(rt)}$$

$$\left[r = \frac{1}{10} \log(2) \right]$$

In []: