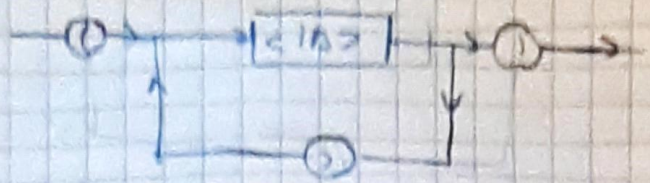
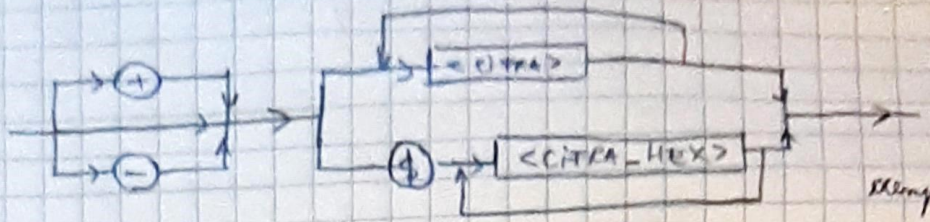
# SEMINAR 1

① **DIAGRAME DE SINTAXĂ** = reprezentări grafice pentru a descrie sintaxa limbajelor formale

1.1




exemple (a, b, c, 123)
(a, b, c, 123)



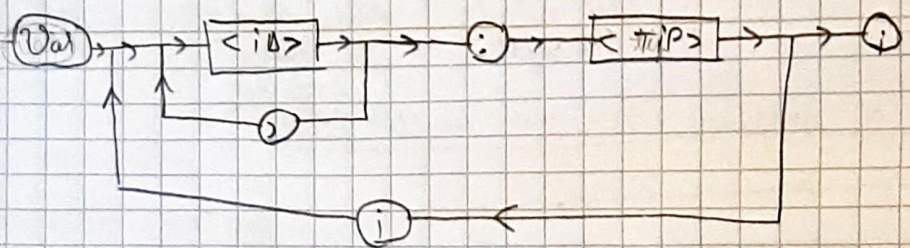

exemple +1234
-$ABCDEF
1

OBS

REGEXP (modificat)
- <ID> = [a-zA-Z] [a-zA-Z0-9]*
- <CIFRA> = [0-9]
- <CIFRA_HEX> = [0-9A-F]

1.2 PASCAL

exemplu → Var a, b, c : integer;
x : real;




② BNF și EBNF

③ ELEMENTE LEXICALE ȘI SINTACTICE ALE UNUI LIMBAJ DE PROGRAMARE

3.1 GRAMATICĂ

<PROGRAM> → begin <LISTA_INSTR> end.
<LISTA_INSTR> → <INSTR> ; <LISTA_INSTR>
<LISTA_INSTR> → <INSTR>
<INSTR> → <ATRIBUIRE>
<INSTR> → <INSTR_IF>
<ATRIBUIRE> → ID = <EXPR>
<EXPR> → <EXPR> + <VARIABILĂ>

<EXPRES> ::= <VARIABILĂ>

<VARIABILĂ> ::= id

<INSTR_IF> ::= if (<EXPRES>) then <INSTR>

a)

TERMINALE: "begin", "end", ".", ":", ":=", "+", "-", "*"
"if", "(", ")", "then"

NETERMINALE: <PROGRAM>, <LISTA_INSTR>, <INSTR>, <ATRIBUIRE>,
<INSTR_IF>, <EXPRES>, <VARIABILĂ>

b) id = a(a|b|c)*

```
begin
    a = aa,
end.
```

```
begin
    a = ab,
    if (ab) then a = abbc + ac;
end.
```

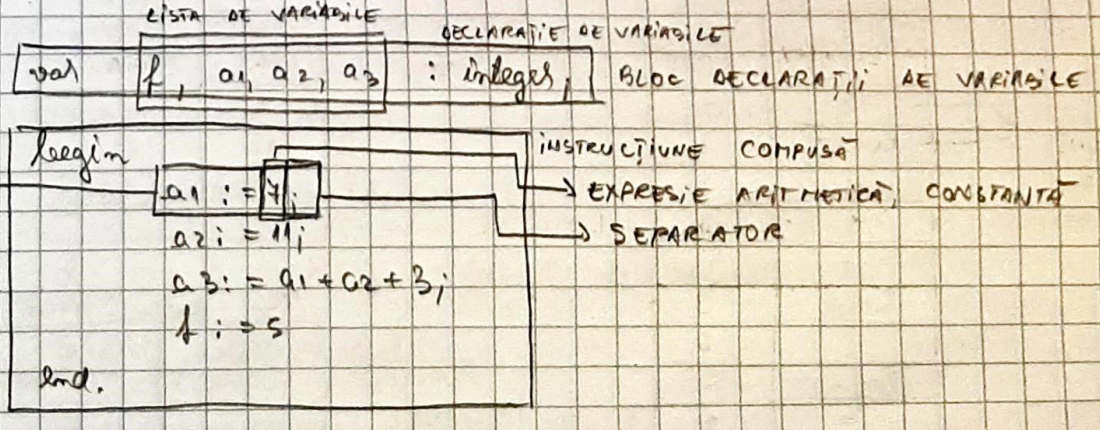
## 3.2 PASCAL

a) ELEMENTE LEXICALE:

- IDENTIFICATORI: f, a1, a2, a3
- CONSTANTE: 7, 11, 5, 3
- OPERATORI: "+", ":=", ":"
- CUV. REZERVATE/CHEIE: real, begin, integer, end
- SEPARATORI/DELIMITATORI: ",", ";", ".", "␣", "/n"

OBS: în PASCAL, „integer" nu este cuvânt cheie

STRUCTURI SINTACTICE:



1.) BNF

<PROGRAM> ::= <BLOC-DECL-VAR> <INSTR-COMP> .

<BLOC-DECL-VAR> ::= Var <LISTA-DECL-VAR>

<LISTA-DECL-VAR> ::= <DECL-VAR> | <DECL-VAR> <LISTA-DECL-VAR>

<DECL-VAR> ::= <LISTA-VAR> : integer ;

<LISTA-VAR> ::= id, <LISTA-VAR> | id

<INSTR-COMP> ::= begin <LISTA-INSTR> end

<LISTA-INSTR> ::= <INSTR> ; <LISTA-INSTR> | <INSTR>

<INSTR> ::= <ATRIBUIRE>

<ATRIBUIRE> ::= id := <EXPR>

<EXPR> ::= <EXPR> + <EXPR> | CONST | id

<EXPR> ::= CONST | id

EBNF

PROGRAM = BLOC_DECL_VAR INSTR_COMP "."

BLOC_DECL_VAR = "Var" <LISTA_DECL_VAR

LISTA_DECL_VAR = DECL_VAR {DECL_VAR}

DECL_VAR = LISTA_VAR ":" "integer" ";"

LISTA_VAR = "id" {"," "id"}

INSTR_COMP = "begin" LISTA_INSTR "end"

LISTA_INSTR = INSTR";" {INSTR";"}

INSTR = ATRIBUIRE

ATRIBUIRE = "id" ":=" EXPR

EXPR = EXPR "+" EXPR | "CONST" | "id"

EXPR = "CONST" | "id"

c)

```
var     a1: integer;
        a2, a3 : integer;
begin
        a1 := a2+a3+7
end.
```

d)

Atomi lexicali speciali

| ATOM LEXICAL | COD ATOM |
|---|---|
| id | 0 |
| CONST | 1 |
| var | 2 |
| $ | 3 |
| : | 4 |
| integer | 5 |
| ; | 6 |
| begin | 7 |
| := | 8 |
| + | 9 |
| end | 10 |
| . | 11 |

TS pentru id

| POZIȚIE ÎN TABEL | SIMBOL (id) | ALTE ATRIBUTE |
|---|---|---|
| 1 | a1 | DACA |
| 2 | a2 | E |
| 3 | a3 | CA-EL |
| 4 | f | |

TS pentru CONST

| POZIȚIE ÎN TABEL | SIMBOL (CONST) | ALTE ATRIBUTE |
|---|---|---|
| 1 | 3 | DACA |
| 2 | 5 | E |
| 3 | 7 | CA-EL |
| 4 | 11 | |

FIP

| COD ATOM | COD TS |
|---|---|
| 2 | |
| 0 | 4 |
| 3 | |
| 0 | 1 |
| 3 | |
| 0 | 2 |
| 3 | |
| 0 | 3 |
| 4 | |
| 5 | |
| 6 | |

| COD ATOM | COD TS |
|---|---|
| 7 | |
| 0 | 1 |
| 8 | |
| 1 | 3 |
| 6 | |
| 0 | 2 |
| 8 | |
| 1 | 4 |
| 6 | |
| 0 | 3 |
| 8 | |

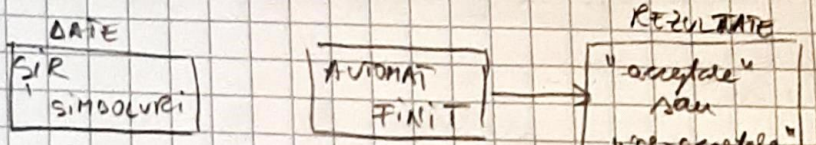
| COD ATOM | COD TS |
|---|---|
| 5 | 1 |
| 9 | |
| 0 | 2 |
| 9 | |
| 1 | 1 |
| 6 | |
| 0 | 4 |
| 8 | |
| 1 | 2 |
| 10 | |
| 11 | |

OBS: TS sunt ordonate lexicografic

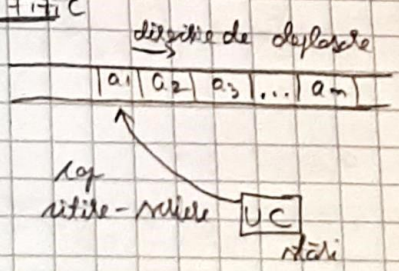
TS
- TABEL SORTAT LEXICOGRAFIC
- ARBORE BINAR DE CĂUTARE
- TABELA DE DISPERSIE

## CURS 2

### AUTOMAT FINIT




MODEL FIZIC




UC = unitate centrală

MODEL MATEMATIC

AUTOMAT FINIT = un ansamblu $M = (Q, \Sigma, \delta, q_0, F)$

$Q$ = alfabetul stărilor

$\Sigma$ = alfabet de intrare

$\delta: Q \times \Sigma \to P(Q)$ = funcția de tranziție

$q_0 \in Q$ = starea inițială

$F \subseteq Q$ = mulțimea stărilor finale

# SEMINAR 2

## 1 MULȚIMI ȘI LIMBAJE

Obs: $NR = \{1,2,3,4,5,6,7,8,9\}$
$NR_0 = NR \cup \{0\}$

A. Limbajul numerelor naturale (scrise lizibil)

$L_A = \{1w \mid w \in \{0,1\}^*\} \cup \{0\}$

B. Limbajul numerelor întregi (scrise lizibil)

$L_B = \{sw \mid s \in \{+,-,\varepsilon\}, w \in \{0,1\}^*\} \cup \{0\}$

C. Limbajul numerelor reale pozitive (scrise lizibil)

$L_C = \{1w1 \mid w \in \{0,1\}^*\} \cup \{0\} \cup \{1w,v \mid w \in \{0,1\}^*, v \in \{0,1\}^+\} \cup \{0,v1 \mid v \in \{0,1\}^+\}$

D. Limbajul numerelor naturale (scrise zecimal)

$L_D = \{w_1 w_2 \mid w_1 \in NR, w_2 \in NR_0\} \cup \{0\}$

E. Limbajul numerelor întregi (scrise zecimal)

$L_E = \{w_1 w_2 w_3 \mid w_1 \in \{+,-,\varepsilon\}, w_2 \in NR, w_3 \in NR_0\} \cup \{0\}$

F. Limbajul numerelor reale pozitive (scrise zecimal)

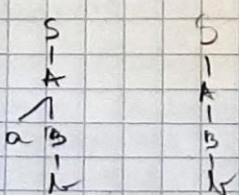
$L_F = \{w_1 w_2 \mid w_1 \in NR, w_2 \in NR_0^*\} \cup \{0\} \cup \{w_1 w_2 w_3 \mid w_1 \in NR, w_2 \in NR_0^*, w_3 \in NR_0^+\}$
$\cup \{0, w_1 \mid w_1 \in NR_0^+\}$

## 2 GRAMATICI INDEPENDENTE DE CONTEXT

1.

a) $G = (N, \Sigma, S, P)$

$N = \{A, B\}$ $\Sigma = \{a, b\}$ $S = A$ $P$: $A \to aB$, $A \to B$, $B \to b$




$L(G) = \{ab, b\} = \{wb \mid w \in \{\varepsilon, a\}\}$

$A \Rightarrow_1 aB \Rightarrow_3 ab$

$A \Rightarrow_2 B \Rightarrow_3 b$

b) $G = (N, \Sigma, S, P)$

OBS: GRAMATICILE NU TREBUIESC SAU !

$N = \{$ <PROPOZIȚIE>, <SUBIECT>, <PREDICAT-NOMINAL>, <VERB-COPULATIV>, <NUME-PREDICATIV>, <SUBSTANTIV>, <ADJECTIV>, <VERB>, <DETERMINANT> $\}$

$\Sigma = \{$ o, ale, funcție, derivabilă, continuă, este $\}$

$S =$ <PROPOZIȚIE>

P:

<PROPOZIȚIE> → <SUBIECT> <PREDICAT-NOMINAL>

<SUBIECT> → <DETERMINANT> <SUBSTANTIV>

<PREDICAT-NOMINAL> → <VERB-COPULATIV> <NUME-PREDICATIV>

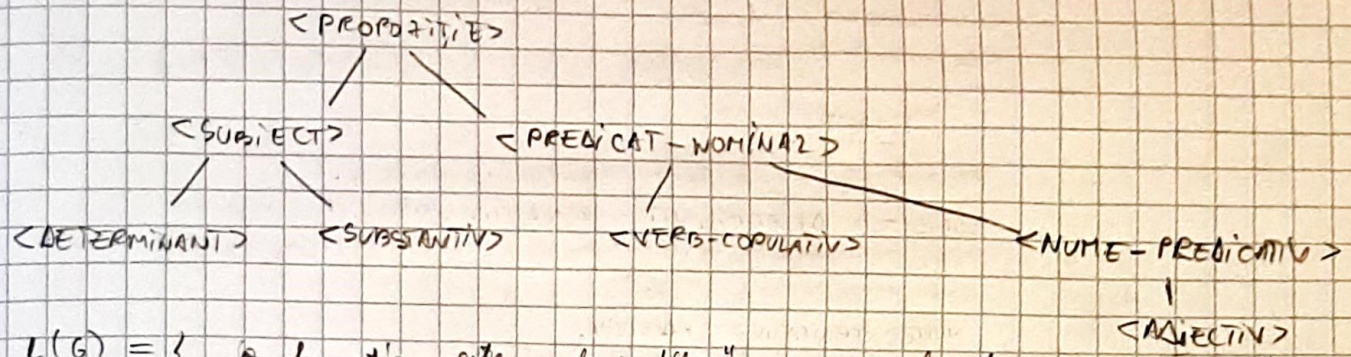
<VERB-COPULATIV> → este

<NUME-PREDICATIV> → <ADJECTIV>

<ADJECTIV> → derivabilă | continuă

<SUBSTANTIV> → funcție

<DETERMINANT> → o/ale




$L(G) = \{$ „o funcție este derivabilă", „o funcție este continuă", „ale funcție este derivabilă", „ale funcție este continuă" $\}$

2.

a) $L(G) = \{a^k, a \geq 1\}$

$N = \{A, B\}$

$\Sigma = \{a, b, \lambda\}$

$S = A$

P: $A \to aB$

$B \to b$

$B \to \lambda$

b) $L(G) = \{a, b, \lambda\}$

$N = \{A, B, C\}$

$\Sigma = \{a, b, c\}$

$S = A$

P: $S \to aB$

$B \to bC$

$C \to \lambda$

3) BNF și EBNF

a)

(1a) BNF

<A> ::= a<B> | c

<B> ::= b

EBNF

A = [{"a"}]"b".

(1b) BNF

<PROPOZIȚIE> ::= <SUBIECT><PREDICAT-NOMINAL>
<SUBIECT> ::= <DETERMINANT><SUBSTANTIV>
<PREDICAT-NOMINAL> ::= <VERB-COPULATIV><NUME-PREDICATIV>
<VERB-COPULATIV> ::= este
<NUME-PREDICATIV> ::= <ADJECTIV>
<ADJECTIV> ::= demodată | continuă
<SUBSTANTIV> ::= funcție
<DETERMINANT> ::= O | olice

EBNF

PROPOZIȚIE = SUBIECT PREDICAT-NOMINAL.
SUBIECT = DETERMINANT SUBSTANTIV.
PREDICAT-NOMINAL = VERB-COPULATIV NUME-PREDICATIV.
VERB-COPULATIV = "este".
NUME-PREDICATIV = ADJECTIV.
ADJECTIV = "demodată" | "continuă".
SUBSTANTIV = "funcție".
DETERMINANT = "o" | "olice".

(2a) BNF

<A> ::= a<B>
<B> ::= b | c

EBNF

A = "a"B.
B = "b" | "c".

(2b) BNF

<A> ::= a<B>
<B> ::= b<C>
<C> ::= c

EBNF

A = "a"B.
B = "b"C.
C = "c".

b)

$L_A$ ; NR = ["-"]"0" | "1"{"0" | "1"}

EBNF

NR = ["-"]"0" | "1"{"0" | "1"}

BNF

<NR> ::= 1 | <CIFRA>
<CIFRA> ::= 0 | 1<CIFRA>

$L_B$ ; NR = ["+" | "-"]"1"{"0" | "1"}

EBNF

NR = ["+" | "-"]"1"{"0" | "1"}

BNF

<NR> ::= <SEMN><NRN> | <NR
<SEMN> ::= 0 | 1
<NRN> ::= 1 | <CIFRA>
<CIFRA> ::= 0 | 1 | <CIFRA

## 4 DESCRIERI DE LIMBAJE FOLOSIND MECANISME GENERATIVE

1. L limbaj peste alfabetul $\{a, b\}$:
   i) $ab \in L$
   ii) $x \in L \Rightarrow axb \in L$
   iii) Niciun alt cuvant nu apartine lui L

a) $L = \{a^n b^n \mid n \in \mathbb{N}^*\}$, $\Sigma = \{a, b\}$, $L \subset \Sigma^*$

b) $G = (N, \Sigma, S, P)$

$N = \{A\}$ $\quad S = A$

$\Sigma = \{a, b\}$ $\quad P: A \to ab$

$A \to aAb$

## 5 GRAMATICI INDEPENDENTE DE CONTEXT ȘI LIMBAJUL GENERAT

1. $L = \{x^n y^n \mid n \in \mathbb{N}\}$

$G = (N, \Sigma, S, P)$

$N = \{A\}$ $\quad S = A$

$\Sigma = \{x, y\}$ $\quad P: A \to \varepsilon$

$A \to xAy$

I $L \in L(G)$

$w = x^n y^n$, n fixat

$A \overset{n}{\Rightarrow} \underbrace{xx\ldots x}_{n} A \underbrace{y\ldots yy}_{n} = x^n A y^n \Rightarrow x^n y^n \in L(G)$

II $L(G) \subseteq L$

$P(m): x^m A y^m$ $\quad P(1): A \to \varepsilon$

$x^{m-1} y^{m-1}$ $\quad A \to xAy$

(de demonstrat...)

2. $L = \{a^{2m} b a \mid m \in \mathbb{N}\}$

$G = (N, \Sigma, S, P)$

$N = \{A, B\}$ $\quad S = A$

$\Sigma = \{a, b, c\}$ $\quad P: A \to Bbc$

$B \to aaB$

$B \to \varepsilon$

I $L \subseteq L(G)$

$w = a^{2m} bc$, m fixat

$A \overset{m}{\Rightarrow} aaaa\ldots aBbc = a^{2m} Bbc \Rightarrow a^{2m} bc \in L(G)$

II $L(G) \subseteq L$

$P(m+1): a^{2m} Bbc$ $\quad P(1): A \to Bbc$

$a^{2m-2} bc$ $\quad P(2): A \to aaBbc$

$A \to bc$

(de demonstrat...)

3. $L = \{a^{2^m+1} \mid m \in \mathbb{N}\}$

$G = (N, \Sigma, S, P)$

$N = \{A, B\}$ $\quad S = A$

$\Sigma = \{a\}$ $\quad P:$

$A \to aB$

$B \to \varepsilon$

$B \to aaB$

- $\equiv^0$ - diferențe: două cuvinte $x \in \Sigma^*$, $|x| < k$ nu se pot diferențiere
- $\equiv$ echivalență ($\equiv$): două cuvinte $x \in \Sigma^*$ nu se pot diferențiere
- $\equiv^k$ - echivalență ($\equiv^k$): două cuvinte $x \in \Sigma^*$, $x$ e de lung $< k$ nu se pot diferențiere

PROPRIETĂȚI ALE RELAȚIEI DE $k$-echivalență ($\equiv^k$)

$q_1 \not\equiv^0 q_2$ dacă $q_1, q_2 \in F$ sau $q_1, q_2 \in Q - F$

$\equiv^0 \supseteq \equiv^1 \supseteq \equiv^2 \supseteq \equiv^3 \supseteq \dots \supseteq \equiv^n \supseteq \dots$

LEMĂ:

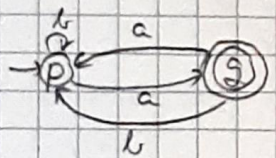
- $\forall$ dacă $\exists n \in \mathbb{N}$ a.î. $q_1 \equiv^n q_2 \Rightarrow q_1 \equiv q_2$

ideea: dacă are nr finit de rel. distincte (maxim $|Q|$)

## SEMINAR 3

### 1. PROBLEME CU AF

a)

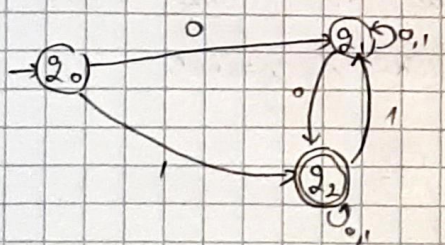



AFD

$M_1 = (\Sigma_1, Q_1, \delta_1, q_{01}, F_1)$

$Q_1 = \{p, q\}$ $q_{01} = p$
$F_1 = \{q\}$ $\Sigma_1 = \{a, b\}$

| $\delta_1$ | a | b |
|---|---|---|
| p | q | p |
| q | p | p |




AFN

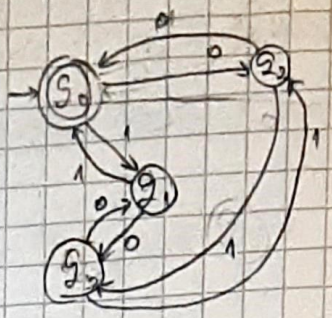
$M_2 = (\Sigma_2, Q_2, \delta_2, q_{02}, F_2)$

$Q_2 = \{q_0, q_1, q_2\}$ $\Sigma_1 = \{0, 1\}$
$F_1 = \{q_2\}$

| $\delta_2$ | 0 | 1 | |
|---|---|---|---|
| $q_0$ | $\{q_1\}$ | $\{q_2\}$ | 0 |
| $q_1$ | $\{q_1, q_2\}$ | $\{q_1\}$ | 0 |
| $q_2$ | $\{q_2\}$ | $\{q_1, q_2\}$ | 1 |

② $M = (Q, \Sigma, \delta, q_0, F)$

| $\delta$ | 0 | 1 | |
|---|---|---|---|
| $q_0$ | $q_2$ | $q_1$ | 1 |
| $q_1$ | $q_3$ | $q_0$ | 0 |
| $q_2$ | $q_0$ | $q_3$ | 0 |
| $q_3$ | $q_1$ | $q_2$ | 0 |



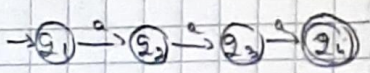

a) $1010 \Rightarrow (q_0, 1010) \vdash (q_1, 010) \vdash (q_3, 10) \vdash (q_2, 0) \vdash (q_0, \varepsilon)$

$1100 \Rightarrow (q_0, 1100) \vdash (q_1, 100) \vdash (q_0, 00) \vdash (q_2, 0) \vdash (q_0, \varepsilon)$

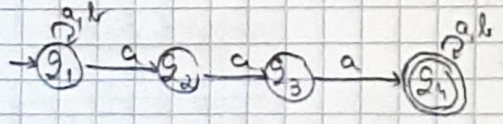
b) $1011 \Rightarrow (q_0, 1011) \vdash (q_1, 011) \vdash (q_3, 11) \vdash (q_2, 1) \vdash (q_3, \varepsilon)$ blocat

③

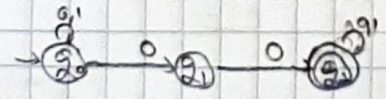
a) $L = \{aaa\}$, $\Sigma = \{a\}$



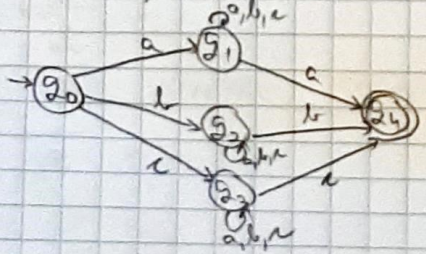

b) $L = \{w_1 aaa w_2 \mid w_1, w_2 \in \{a,b\}^+\}$

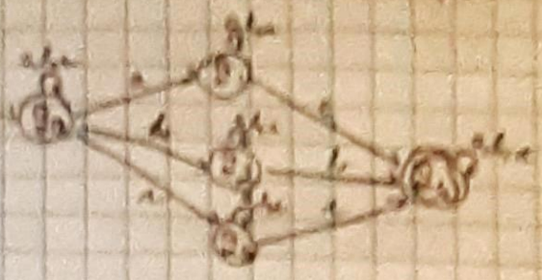



c) cuv. peste alfabetul $\{0,1\}$, care conțin cel puțin două zerouri consecutive



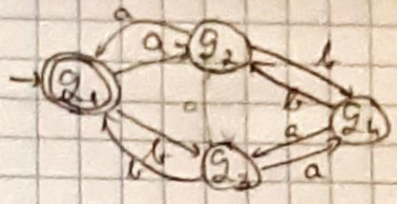

d) cuvinte peste alfabetul $\{0,b,c\}$, primul simbol al cuv. este celălalt nu mai apare și termină cuv.





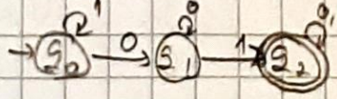
f) $L = \{a^n \mid n \geq 4\}$

g) limbajul ce conține cuvinte peste $\Sigma = \{a, b\}$ cu nr par de simb. a și nr par de simb. b

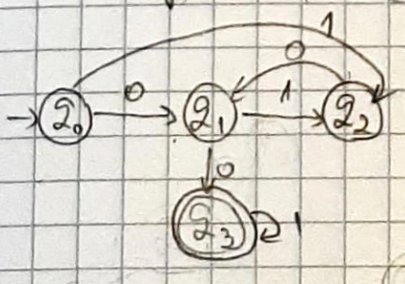


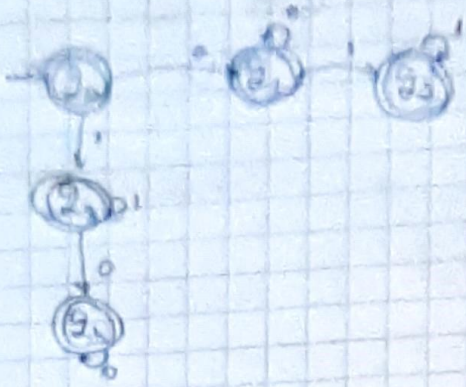
h) limbajul ce conține cuvinte peste $\Sigma = \{a, b\}$ cu nr impar de simb. a și nr. impar de simb. b

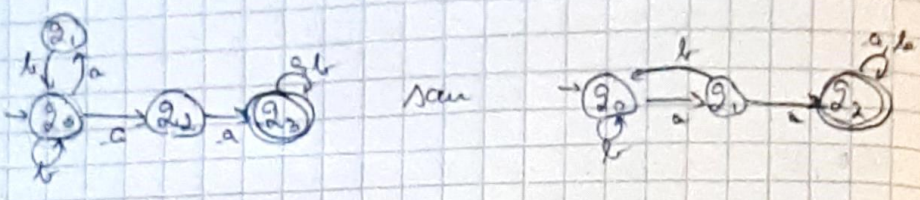
i) $L = \{1^n 0^m 1 u \mid n \geq 0, m \geq 1, u \in \{0,1\}^*\}$



j) $L = \{0(10)^n 01^m \mid n \geq 0, m \geq 0\} \cup \{(10)^n 01^m \mid n \geq 1, m \geq 0\}$ cuvinte cu 4 stări








## 1.2 STRUCTURI DE DATE PENTRU AUTOMATE FINITE

# SEMINAR 4

① a) $L = \{a\}$

$G = (N, \Sigma, P, S)$

$N = \{A\}$

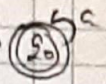
$\Sigma = \{a\}$

$S = A$

P:

$A \to a$

b) $L = \{a^m \mid m \in \mathbb{N}\}$



$G = (N, \Sigma, P, S)$

$N = \{A\}$

$\Sigma = \{a\}$

$S = A$

P:

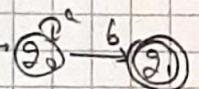
$S \to \varepsilon$

$S \to A$

$A \to aA$

$A \to a$

c) $L = \{a^m b \mid m \in \mathbb{N}\}$



$G = (N, \Sigma, P, S)$

$N = \{A\}$

$\Sigma = \{a, b\}$

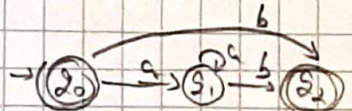
$S = A$

P:

$A \to aA$

$A \to b$

d) $L = \{\varepsilon\} \cup \{a^m b \mid m \in \mathbb{N}\}$



$G = (N, \Sigma, P, S)$

$N = \{A, B\}$

$\Sigma = \{a, b\}$

$S = A$

P:

$S \to \varepsilon$

$S \to A$

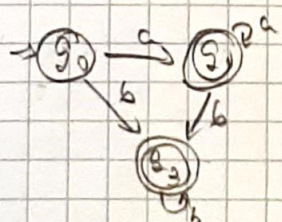
$A \to aB$

$A \to b$

$B \to aB$

$B \to b$

e) $L = \{a^m b^n \mid m, n \in \mathbb{N},\ m+n > 0\}$



$G = (N, \Sigma, P, S)$

$\Sigma = \{a, b\}$

$S = A$

$N = \{A, B\}$

P:

$A \to b$

$A \to bB$

$A \to aA$

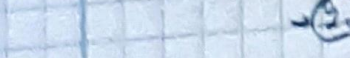
$A \to a$

$A \to aB$

$B \to bB$

$B \to b$

① $L = \{ab^m \mid m \in \mathbb{N}\}$




$G = (N, \Sigma, P, S)$

$N = \{A, B\}$

$\Sigma = \{a, b\}$

$S = A$

P:

$A \to a$

$A \to aB$

$B \to b$

$B \to bB$

② $M = (Q, \Sigma, \delta, q_0, F) \Rightarrow G(N, \Sigma, P, S)$

$S = q_0 \quad \Sigma = \Sigma$

$Q = N$

P: $\begin{cases} A \to aB, & B \in \delta(A,a) \\ A \to a, & B \in \delta(A,a),\ B \in F \end{cases}$

Obs: ou $\Rightarrow$ ambos $A \to aB$

③ $L = \{a^{3m} \mid m \in \mathbb{N}^+\}$

$G = (N, \Sigma, P, S)$

$N = \{A, B, C\}$

$\Sigma = \{a\}$

$S = A$

P:

$A \to aB$

$B \to aC$

$C \to aA$

$C \to a$

④ $L = \{a^{3m} \mid a \in \mathbb{N}^*\}$

$G = (N, \Sigma, P, S)$

$N = \{A, B, C\}$

$\Sigma = \{a\}$

P:

$S \to \varepsilon$

$S \to A$

$A \to aB$

$B \to aC$

$C \to aA$

$C \to c$

c) $L = \{ a^n b^m \mid m, n \in \mathbb{N}^* \}$

$G = (N, \Sigma, P, S)$

$N = \{A, B\}$

$\Sigma = \{a, b\}$

$S = A$

P:

$A \to aA$

$A \to aB$

$B \to bB$

$B \to b$

d) L = toate numerele pare scrise în baza 2

$G = (N, \Sigma, P, S)$

$N = \{A, B\}$

$\Sigma = \{0, 1\}$

$S = A$

P:

$A \to 0$

$A \to 1$

$A \to 1B$

$B \to 0B$

$B \to 1B$

$B \to 0$

$B \to 1$

e) L = nume de identificatori

$G = (N, \Sigma, P, S)$

$N = \{A, B\}$

$\Sigma = \{a \dots z, 0 \dots 9\}$

$S = A$

P:

$A \to a \mid \dots \mid z$

$A \to aB \mid \dots \mid zB$

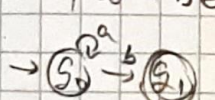
$B \to aB \mid \dots \mid zB \mid 0B \mid \dots \mid 9B$

$B \to a \mid b \mid \dots \mid z \mid 0 \mid 1 \mid \dots \mid 9$

4) a) $A \to aA$

$A \to b$

$L = \{ a^n b \mid n \in \mathbb{N} \}$

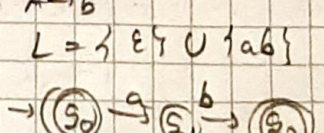



b) $S \to \varepsilon$

$S \to aA$

$A \to b$

$L = \{\varepsilon\} \cup \{ab\}$



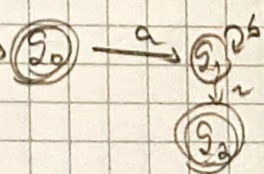

c) $S \to \varepsilon$

$S \to aA$

$A \to bA$

$A \to a$

$L = \{\varepsilon\} \cup \{ ab^n a \mid n \in \mathbb{N} \}$



⑤ $G = (N, \Sigma, P, S)$ → $M(Q, \Sigma, \delta, q_0, F)$

$Q = N \cup \{k\}$, $k \notin T$

$\Sigma = \Sigma$

$q_0 = S$

$F = \{k\}$ dacă $S \to \varepsilon \in P$

$\delta$ $\quad \delta(A, c) = \{B \mid A \to cB \in P\}$

$\cup \{k \mid A \to c \in P\}$

## PROPRIETĂȚI ALE LIMBAJELOR INDEPENDENTE DE CONTEXT

### LEMA DE POMPARE PENTRU LIMBAJE INDEPENDENTE DE CONTEXT

- fie $L$ un limbaj independent de context. Există atunci o constantă $p$ dependentă numai de $L$ astfel încât dacă $z \in L$ și $|z| \geq p$, atunci avem descompunerea $z = uvwxy$, cu proprietățile

a) $|vx| \geq 1$

b) $|vwx| \leq p$

c) $uv^i wx^i y \in L$, $\forall i \in \mathbb{N}$

FORMAL:

$\forall L \in GIC$, $\exists p \in \mathbb{N}^*$ $\forall z \in L$ $|z| \geq p$

$\Rightarrow$

$(\exists u, v, w, x, y \quad z = uvwxy$

$\wedge \quad |vwx| \leq p \wedge |vx| \geq 1$

$\wedge \ (\forall m \in \mathbb{N} : uv^m wx^m y \in L)$

$)$

Teorema

- dacă $L_1$ și $L_2$ sunt limbaje independente de context atunci:

$L_1 \cup L_2$, $L_1 L_2$, $L_1^*$

sunt limbaje independente de context

Obs. $L_1 \cap L_2$ și $\overline{L_1}$ nu sunt neapărat l.i.c.

$L_1 = \{a^m b^n \mid m \in \mathbb{N}\}$

P:
$S \to \varepsilon$
$S \to A$
$A \to aAb$
$A \to ab$

$L_2 = \{c^m \mid m \in \mathbb{N}\}$

P:
$S \to \varepsilon$
$S \to A$
$A \to cA$
$A \to c$

$L_3 = \{a^n b^n c^m \mid m, n \in \mathbb{N}\}$

P: $S \to \varepsilon$
$S \to AC$
$A \to aAb$
$A \to ab$
$C \to cC$
$C \to \varepsilon$

$L_4 = \{a^m b^m c^m \mid m, n \in \mathbb{N}\}$

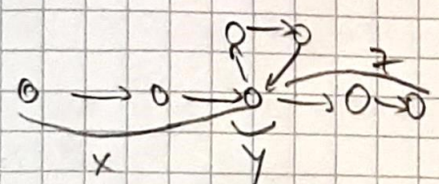
P:
$S \to \varepsilon$
$S \to AB$
$A \to aA$
$A \to \varepsilon$
$B \to bBc$
$B \to \varepsilon$

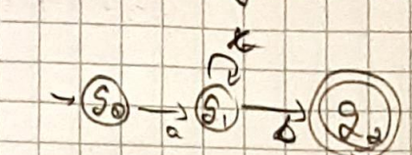
## SEMINAR 5

### Lema de pompare pt. limbaje regulate

L reg. atunci $\exists p \geq 1$ pt. care $\forall w \in L$, $|w| \geq p$

$\exists w = xyz$, $0 < |y| \leq p$ a.î. $w_i = xy^i z$, $w_i \in L$, $\forall i \in \mathbb{N}$




NEGAȚIA

$\forall p \geq 1$, $\exists w \in L$, $\forall w = xyz$, $0 < |y| \leq p$, $|w| \geq p$

$0 < |xy| \leq p$ $\quad w_i = xy^i z$

$\exists i \in \mathbb{N}$ pt. care $w_i \notin L$

1.

a) $L = \{a^n b^{2n} \mid n \in \mathbb{N}\}$

pp $L$ regular

$\exists p \geq 1$ a.?  $\forall w \in L \quad |w| \geq p$

$\exists w = xyz, \quad 0 < |y| \leq p$

$w_i = xy^i z$

$w = a^p b^{2p}$

$|w| = 3p$

$y = a^k$

$x = a^{p-k}$

$z = b^{2p}$

$w_i = a^{p-k} (a^k)^i b^{2p}$

$w_i = a^{p+k(i-1)} b^{2p}$

$w_2 = a^{p+k} b^{2p}$

$2(p+k) \neq 3p \rightarrow w_2 \notin L$

$\Rightarrow L$ nu e reg

b) $L = \{a^k \mid k \text{ prim}\}$

pp. $L$ regulat

$\exists p \geq 1$ a.?  $\forall w \in L \quad |w| \geq p$

$\exists w = xyz, \quad 0 < |y| \leq p$

$w_i = xy^i z$

$w = a^p$

$|w| = p$

$y = a^j, \quad 0 < |y| \leq p$

$w = \underbrace{a^{p-j}}_{x} \underbrace{a^j}_{y} \quad (z = \varepsilon)$

$w_i = a^{p+j(i-1)}$

$i = p+1 \Rightarrow w_{p+1} = a^{p+jp} = a^{p(j+1)}$

$p(j+1)$ nu e prim

$\Rightarrow w_{p+1} \notin L$

c) $L = \{a^{n^2} \mid n \in \mathbb{N}^*\}$

$w = a^{p^2}$

$|w| = p^2 \geq p$

$y = a^j$, $0 < |y| \leq p \Leftrightarrow 0 < j \leq p$

$w = \underbrace{a^{p^2-j}}_{x} \underbrace{a^j}_{y}$

$w_i = a^{p^2 + j(i-1)}$

$(p+q)^2 = p^2 + 2pq + q^2$

$p^2 < p^2 + j(i-1) \leq p^2 + p(i-1)$

$p^2 < |w_2| < (p+1)^2 \Rightarrow w_2$ nu e patrat

$\Rightarrow w_2 \notin L$

d) $L = \{a^{2^n} \mid n \in \mathbb{N}^*\}$

$w = a^{2^p}$

$|w| = 2^p \geq p$

$y = a^j$, $0 < |y| \leq p \Leftrightarrow 0 < j \leq p$

$w = \underbrace{a^{2^p - j}}_{x} \underbrace{a^j}_{y}$

$w_i = a^{2^p + j(i-1)}$

$2^p < 2^p + j(i-1) \leq 2^p + p(i-1) < 2^{p+1}$

$2^p < |w_2| < 2^{p+1}$

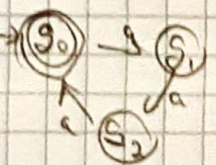
$|w_2| \leq 2^p + p$

$p < 2^p$

e) $L = \{a^{k^n} \mid n \in \mathbb{N}\}$

$k$ fixat

$a^{k^n} \rightarrow$




$S \to \varepsilon$

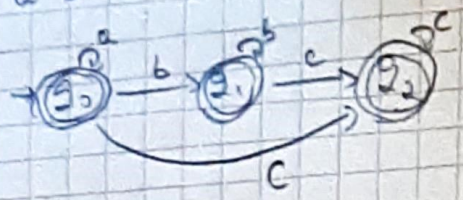
$S \to a^k$

$S \to a^{k-1}A$

$A \to a$

$A \to a^k A$

1) $L = \{ a^n b^m c^p \mid n, m, p \in \mathbb{N} \}$




$\therefore a^* b^* c^*$

P:

$S \to \varepsilon$
$S \to a$
$S \to aS$
$S \to bA$
$S \to b$
$S \to aB$
$S \to c$
$S \to A$
(s/ ε-produções)

$A \to bA$
$A \to b$
$A \to cB$
$B \to c$
$A \to C$
$B \to cB$

2) $L = \{ a^{k!} \mid k \text{ natural} \}$

a) PP L regular $\Rightarrow$ $\bar{L}$ regular

$\bar{L} = \{ a^{k!} \mid k \text{ natural} \}$ é $\to$ L não é regular

b) p - natural $\geq 1$

p = ?

a.i.

$\forall w \in L, \exists w = xyz$ a.i.

$w_i = xy^i z \in L, \forall i \in \mathbb{N}$

p = 3

$|w| = 4$

$w = a^1$

$y = a^2 \Rightarrow w = a^2 a^2$

$w_i = a^2 a^{2i} = a^{2(i+1)}$

$\forall i \in \mathbb{N}$

(1.2)

a) $L = \{a^n b^{n+1} \mid n \in \mathbb{N}\}$

P: $S \to aA$
$A \to b$
$A \to bB$
$B \to bB$
$B \to b$

$N = \{A, B\}$
$\Sigma = \{a, b\}$

P': $S \to abB$
$B \to bB$
$B \to \varepsilon$

b) $L = \{a^n b^{3m} \mid m \in \mathbb{N}\}$

$S \to \varepsilon$
$S \to aSbbb$
$A \to$

(1.3)

1. a) P: $S \to abS$
$S \to ab$

→

P: $S \to aA$
$A \to bS$
$S \to aB$
$B \to b$

b) P: $S \to Sa$
$S \to b$

⇒

P: $S \to bA$
$S \to bA$
$A \to aA$
$A \to a$

# SEMINAR 6

① 

a) 0011011 $(1^*01)^*(11+0)^*$

$(\underset{\varepsilon}{1^*01})(\underset{1}{1^*01})(\underset{11}{11+0})$

b) 111001111 $(1^*0)^* + (0^*11)$

$(\underset{111}{1^*0})(\underset{\varepsilon}{1^*0})(\underset{111}{1^*0}) \Rightarrow$ nu

c) 11100111 $(1^*0)^*(0^*11)$

$(\underset{111}{1^*0})(\underset{\varepsilon}{0^*1})(\underset{\varepsilon}{0^*1})(\underset{11}{0^*1}) \rightarrow$ nu

d) 111001 $(1^*0)^*(0^*11)$

$(\underset{111}{1^*0})(\underset{0}{0^*11})$

e) 0111001 $01^*01^*(11^*0)^*$

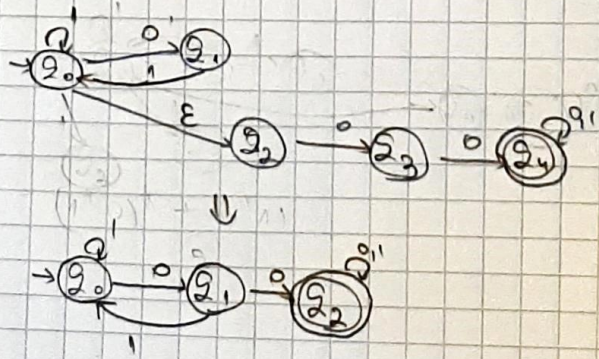
$0\underset{111}{1^*}0 \Rightarrow$ nu

f) 1000011 $(10^* + 11)^*(0^*1)^*$

$10^*(\underset{\varepsilon}{0^*1})(\underset{\varepsilon}{0^*1})$

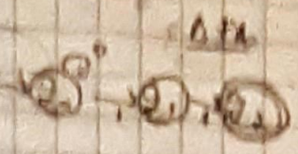
②

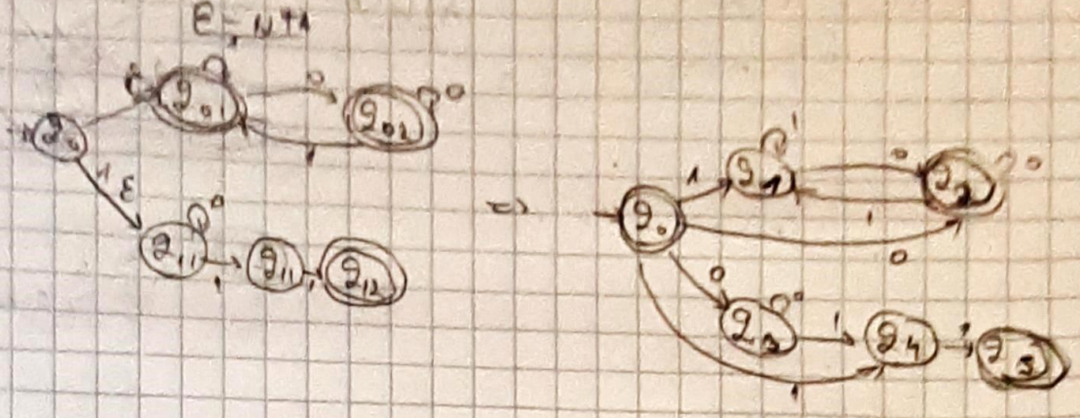
a) $0(1+1)^*00(0+1)^*$



A) $(1^*0)^* + 0^*11$

$(1^*0)^*$ 


$0^*11$ 


| δ | 0 | 1 |
|---|---|---|
| $q_0$ | $q_{23}$ | $q_{14}$ |
| $q_{23}$ | $q_{23}$ | |
| $q_{14}$ | | $q_{14}$ |

| δ | 0 | 1 | |
|---|---|---|---|
| $q_0$ | $\{q_2, q_3\}$ | $\{q_1, q_4\}$ | 1 |
| $q_{01}$ | $q_{02}$ | $q_1$ | 0 |
| $q_{02}$ | $q_2$ | $q_1$ | 1 |
| $q_3$ | $q_3$ | $q_4$ | 0 |
| $q_4$ | $\phi$ | $q_5$ | 0 |
| $q_5$ | $\phi$ | $\phi$ | 1 |

$n=2$

| | $k=0$ | $k=1$ | $k=2$ |
|---|---|---|---|
| $R_{11}^k$ | $\varepsilon+a$ | $a^*$ | $(a+bb)^*$ |
| $R_{12}^k$ | $b$ | $a^*b$ | |
| $R_{21}^k$ | $b$ | $ba^*$ | |
| $R_{22}^k$ | $\varepsilon$ | $\varepsilon+ba^*b$ | |


$(\alpha^*\beta)^*\alpha^* = (\alpha+\beta)^*$

$1^*1 = 1(1+1)$

$(\varepsilon+\alpha)^* = \alpha^*$

$\alpha+\phi = \alpha$

$(\alpha\beta)^*\alpha = \alpha(\beta\alpha)^*$

$$R_{11}^1 = R_{11}^0 + R_{11}^0 (R_{11}^0)^* R_{11}^0 = \underline{\varepsilon+a} + (\varepsilon+a)\underline{(\varepsilon+a)^*(\varepsilon+a)} = (\varepsilon+a)(\varepsilon+(\varepsilon+a)^+) = (\varepsilon+a)(\varepsilon+a)^* = (\varepsilon+a)^+ = a^*$$

$$R_{12}^1 = R_{12}^0 + R_{11}^0 (R_{11}^0)^* R_{12}^0 = b + (\varepsilon+a)(\varepsilon+a)^* b = b + (\varepsilon+(\varepsilon+a)^+)b = (\varepsilon+(\varepsilon+a)^+)b = (\varepsilon+a)^* b = a^*b$$

$$R_{21}^1 = R_{21}^0 + R_{21}^0 (R_{11}^0)^* R_{11}^0 = b + b(\varepsilon+a)^*(\varepsilon+a) = ba^*$$

$$R_{22}^1 = R_{22}^0 + R_{21}^0 (R_{11}^0)^* R_{12}^0 = \varepsilon + b(\varepsilon+a)^* b = \varepsilon + ba^*b$$

$$R_{11}^2 = R_{11}^1 + R_{12}^1 (R_{22}^1)^* R_{21}^1 = a^* + a^*b(\varepsilon+ba^*b)^* ba^*$$

$$= a^*(\varepsilon + b(\varepsilon+ba^*b)^* ba^*)$$

$$= a^*(\varepsilon + b(ba^*b)^* ba^*)$$

$$= a^*(\varepsilon + bba^*(bba^*)^*)$$

$$= a^*(\varepsilon + (bba^*)^+)$$

$$= a^*(bba^*)^* = (bb+a^*)^*$$

$$= (a+bb)^*$$

$$\begin{cases} X_1 = X_1 a + X_2 b \\ X_2 = X_1 b \end{cases}$$

$$X_1 = X_1 a + X_1 bb = X_1(a + bb) + \varepsilon$$

SEMINAR 4

| G reg | G.I.C. | G.D.C |
|---|---|---|
| $S \to \varepsilon$ | $S \to \alpha$ | $\alpha A \beta \to \alpha \gamma \beta$ |
| $S \to aA$ | $\alpha \in N \cup \Sigma$ | |
| $S \to a$ | | |

①

1) $L = \{a^{2n} \mid n \in \mathbb{N}\}$

| G reg | G.I.C | nu e G.I.C |
|---|---|---|
| $S \to \varepsilon$ | $S \to aAa$ | $S \to aSa$ |
| $S \to aA$ | $A \to aa$ | $aSa \to aAa$ |
| $A \to a$ | $S \to \varepsilon$ | $A \to aAa$ |
| $A \to aB$ | | $A \to \varepsilon$ |
| $B \to aA$ | | $S \to \varepsilon$ |

2)

$L = \{wwl \mid w \in \{a,b\}^+\}$

$S \to \varepsilon$
$S \to aSA$
$S \to bSB$
$aS \to aD$
$bS \to bD$
$DB \to Db$
$DA \to Da$
$aA \to aA$
$bA \to Ab$
$aB \to Ba$
$bB \to Bb$
$D \to \varepsilon$

abaaba
VN W

abbabbBABBA
abbabDbABBA
abbabDbABBA
abbabDABBBA
abbabDabBBA
abbabDaBbBA
abbabDBabBA
abbabDbabBA

abbabDabbaab
abbabbabbabb

2) $L = \{wxw \mid w \in \{a,b\}^+, x \in \{a,b\}^*\}$

$S \to aSA$
$S \to bSB$
$aS \to aXA$
$bS \to bXA$
$AB \to Ab$
$AA \to Aa$
$bA \to Ab$
$aA \to Ac$
$bB \to Bb$
$aB \to Ba$
$A \to \varepsilon$

$X \to \varepsilon$
$X \to aX$
$X \to bX$
Dacă $A \to X$

3) $L = \{a^n b^n c^n \mid n \in \mathbb{N}^*\}$

$S \to abc$
$S \to aAbC$
$Ab \to bA$
$AC \to BbcC$
$bB \to Bb$
$aB \to aaA$

4)

$L = \{a^n b^m c^n d^m \mid n \in \mathbb{N}^*\}$

$S \to abcd$
$S \to aAbcd$
$Ab \to bA$
$AC \to cA$
$Ad \to cBdd$
$cB \to Bc$
$bB \to Bb$
$aB \to aab$
$aA \to aaAb$

5. $L = \{a^{2n} \mid n \in \mathbb{N}^*\}$

$S \to B$

$C \to LaR$

$L \to E$

$R \to E$

$L \to LD$

$Da \to aaD$

$DR \to R$

6. $L = \{w \mid w \in \{a,b,c\}^*,\ n_a(w) = n_b(w) = n_c(w)\}$

$S \to AcbC$

$Ac \to cA$
$aA \to Aa$
$bA \to Ab$
$Ab \to bA$
$CA \to Ac$
$Ac \to cA$
$A \to aB$

$aB \to Ba$
$Ba \to aB$
$bB \to Bb$
$Bb \to bB$
$CB \to Bc$
$Bc \to cB$
$B \to bC$

$aC \to Ca$
$Ca \to aC$
$bC \to Cb$
$Cb \to bC$
$C \to c$

7. $L = \{a^n b^m c^m d^m \mid n, m \in \mathbb{N}\}$

$S \to AC$
$A \to aAb$
$A \to E$
$C \to cCd$
$C \to E$

8. $L = \{a^n b^m c^m d^n \mid n, m \in \mathbb{N}\}$

$S \to E$ $\quad$ $S \to B$

$S \to aAd$
$A \to aAd$
$A \to E$
$A \to cBb$
$B \to cBb$
$B \to E$

9. $L = \{a^n b^m c^k \mid n, m, k \in \mathbb{N},\ n = m \text{ sau } m = k\}$

$S \to E$

$S \to AB$
$A \to E$
$A \to aA$
$B \to bBC$
$B \to E$

$S \to CD$
$C \to aCb$
$C \to E$
$D \to cD$
$D \to E$

1A) $L = \{w \mid w \in \{a,b\}^+,$ w inceput si sfarsit cu acelasi ...

$S \to aAa$
$S \to bAb$
$A \to aA$
$A \to bA$
$A \to \varepsilon$

1B) $L = \{a^n b^m c^k \mid n+m=k\}$

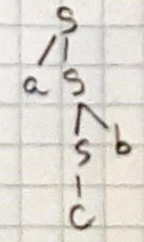
$S \to \varepsilon$
$S \to bDC$
$D \to \varepsilon$
$D \to bDc$
$S \to aEc$
$E \to \varepsilon$
$E \to aEc$

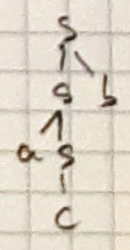
$S \to aAbcc$
$Ab \to bA$
$Ac \to bBcc$
$Bc \to c$
$Bc \to bBcc$
$ba \to ab$
$am \to aac$
$Cb \to bC$
$Cc \to \varepsilon\varepsilon$
$Cc \to Bcc$

2)

1) $S \to aS \mid Sb \mid c$

acb



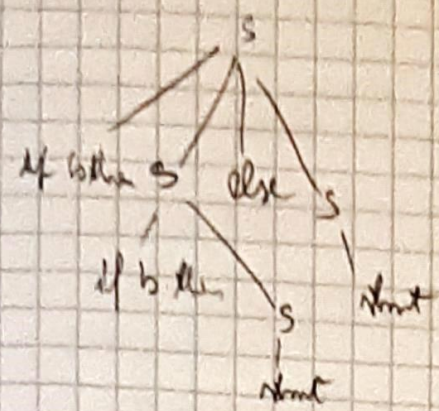



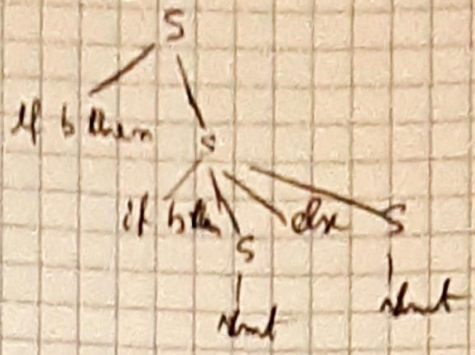

$S \to aS$
$S \to cB$
$B \to \varepsilon$
$B \to bB$

2) S → if b then S

S → if b then S else S

S → stmt





S → if b then S

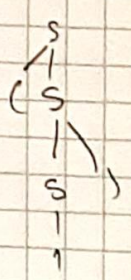
S → stmt

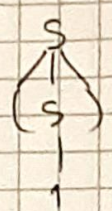
S → if b then A else S

A → stmt

A → if b then A else A

3) S → ( S | S ) | ( S ) | 1





S → ( S

S → 1 B

B → ε

B → ) B

④ $G = (N, \Sigma, P, S)$

CFGGrammar:

startSymbol: Symbol

nonTerminals: Set <Symbol>

terminals: Set <Symbol>

rules: List <Rule>

Symbol:

value: String

isTerminal: Boolean

Rule:

LHS: Symbol

RHS: List<Symbol>

save

multimap <Symbol, List <Symbol>>