UNIVERSITY OF REGINA

CS310-002 DISCRETE COMPUTATIONAL STRUCTURES

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p469

BAYES' THEOREM

Example:

We have two cauldron. The **first** contains **2 frog eyeballs** and **7 crow eyeballs**; the **second** contains **4 frog eyeballs** and **3 crow eyeballs**. Tom Riddle selects an eyeball by first choosing 1 of the 2 cauldrons at random. He then selects 1 of the eyeballs in this cauldron at random. If Voldemort has selected a **crow eyeball**, what is the probability that he selected an eyeball from the **first** cauldron?

Bayes' Theorem

Suppose that E and F are events from a sample space S such that $p(E) \neq 0$ and $p(F) \neq 0$. Then

$$p(F \mid E) = \frac{p(E \mid F)p(F)}{p(E \mid F)p(F) + p(E \mid \overline{F})p(\overline{F})}.$$

Example:

Suppose that one person in 100,000 has a particular rare disease for which there is a fairly accurate diagnostic test. This test is correct 99.0% of the time when given to a person selected at random who has the disease; it is correct 99.5% of the time when given to a person selected at random who does not have the disease. Given this information can we find

1. the probability that a person who tests positive for the disease has the disease?
2. the probability that a person who tests negative for the disease does not have the disease?

GENERALIZED BAYES' THEOREM Suppose that E is an event from a sample space S and that $F_1, F_2, \ldots, F_n$ are mutually exclusive events such that $\bigcup_{i=1}^{n} F_i = S$. Assume that $p(E) \neq 0$ and $p(F_i) \neq 0$ for $i = 1, 2, \ldots, n$. Then

$$p(F_j \mid E) = \frac{p(E \mid F_j)p(F_j)}{\sum_{i=1}^{n} p(E \mid F_i)p(F_i)}.$$

THE PROBABILITY OF "B" BEING TRUE GIVEN THAT "A" IS TRUE

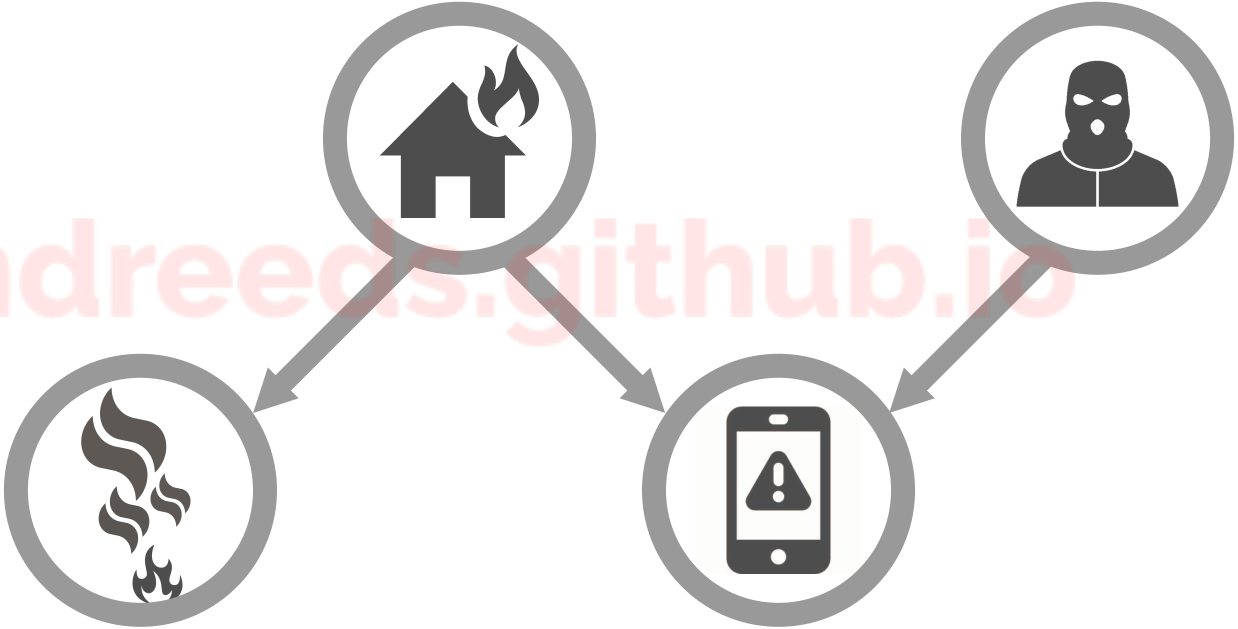
THE PROBABILITY OF "A" BEING TRUE

$$P(A|B) = \frac{P(B|A)\,P(A)}{P(B)}$$

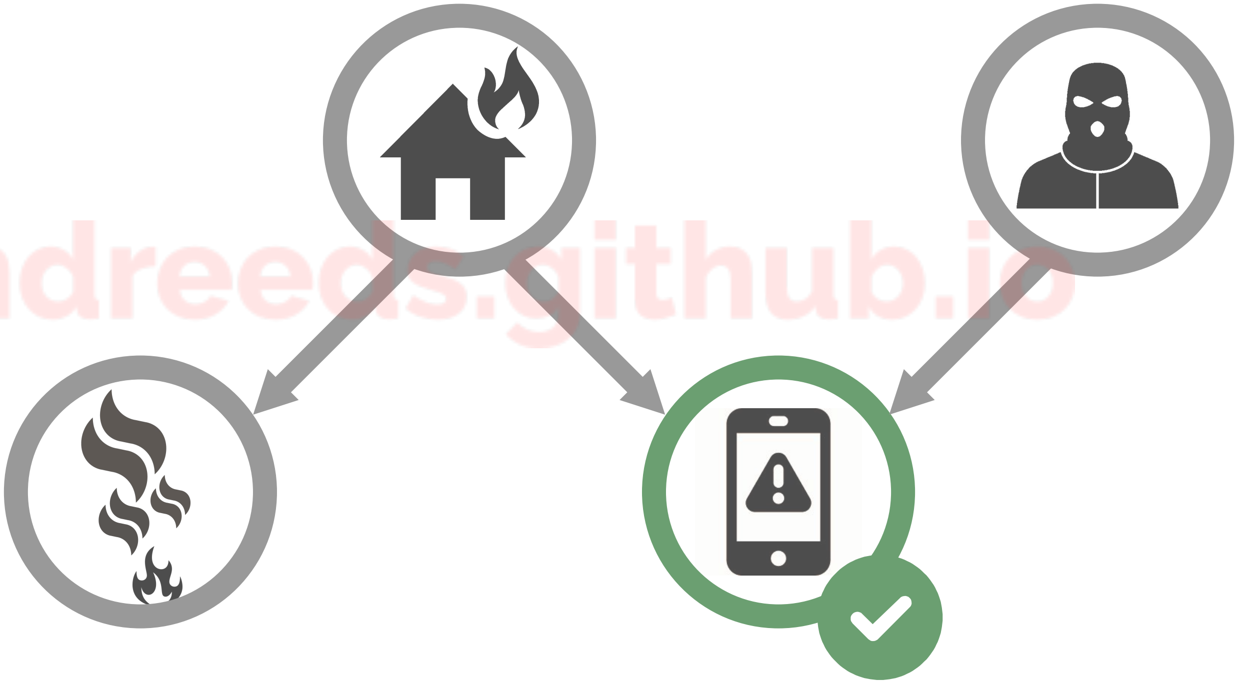
THE PROBABILITY OF "A" BEING TRUE GIVEN THAT "B" IS TRUE

THE PROBABILITY OF "B" BEING TRUE

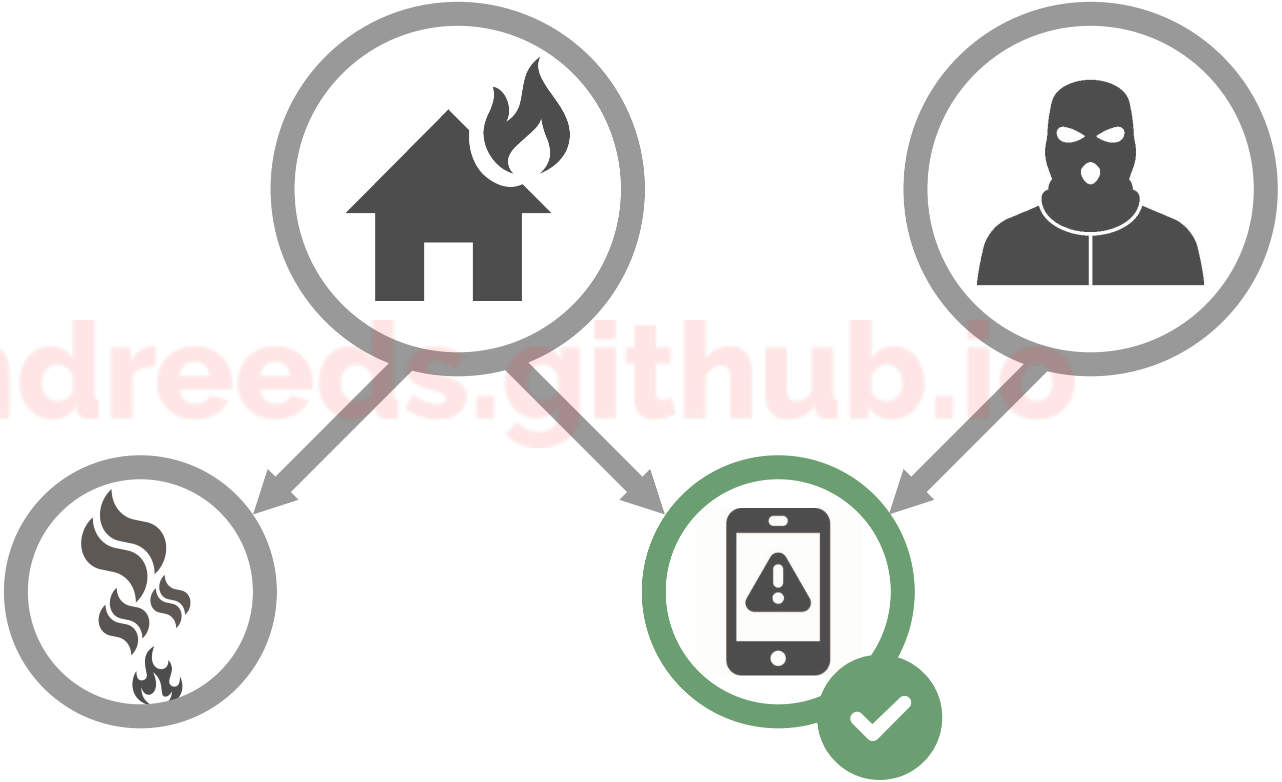
BN EXAMPLE



BN EXAMPLE



BN EXAMPLE



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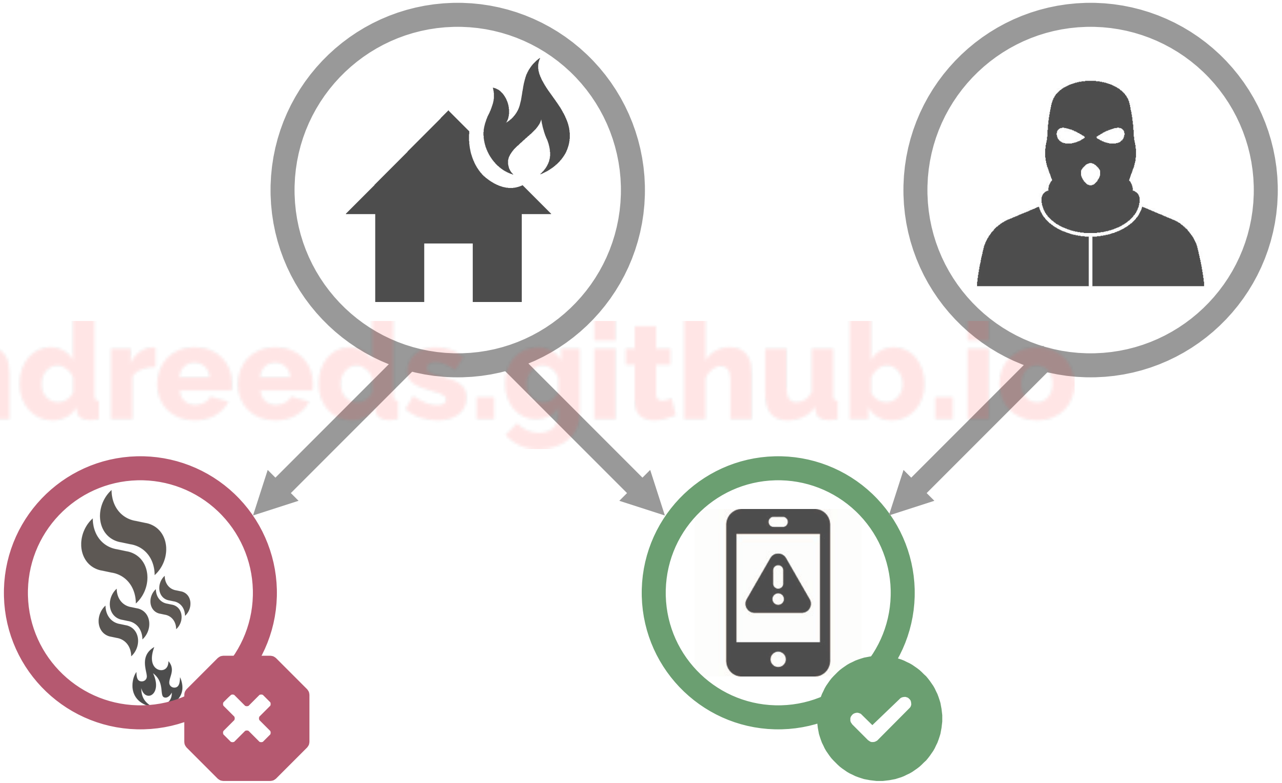
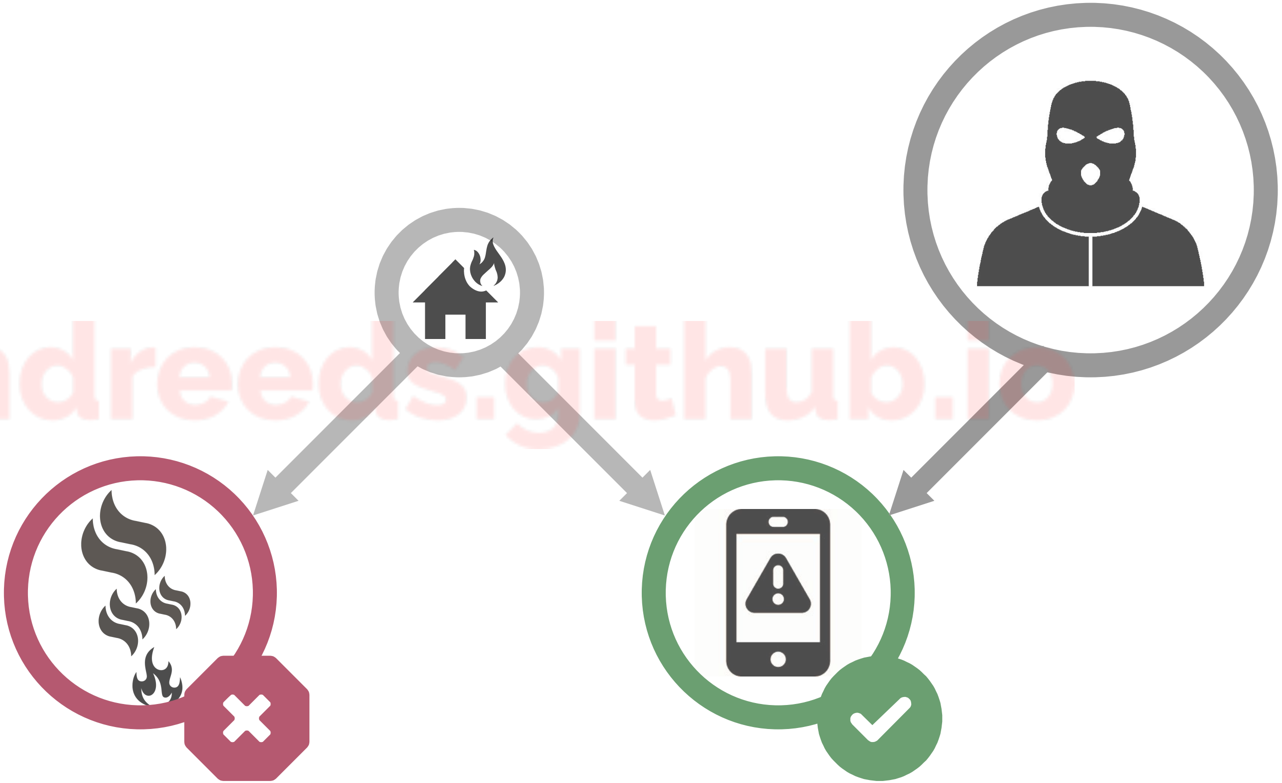
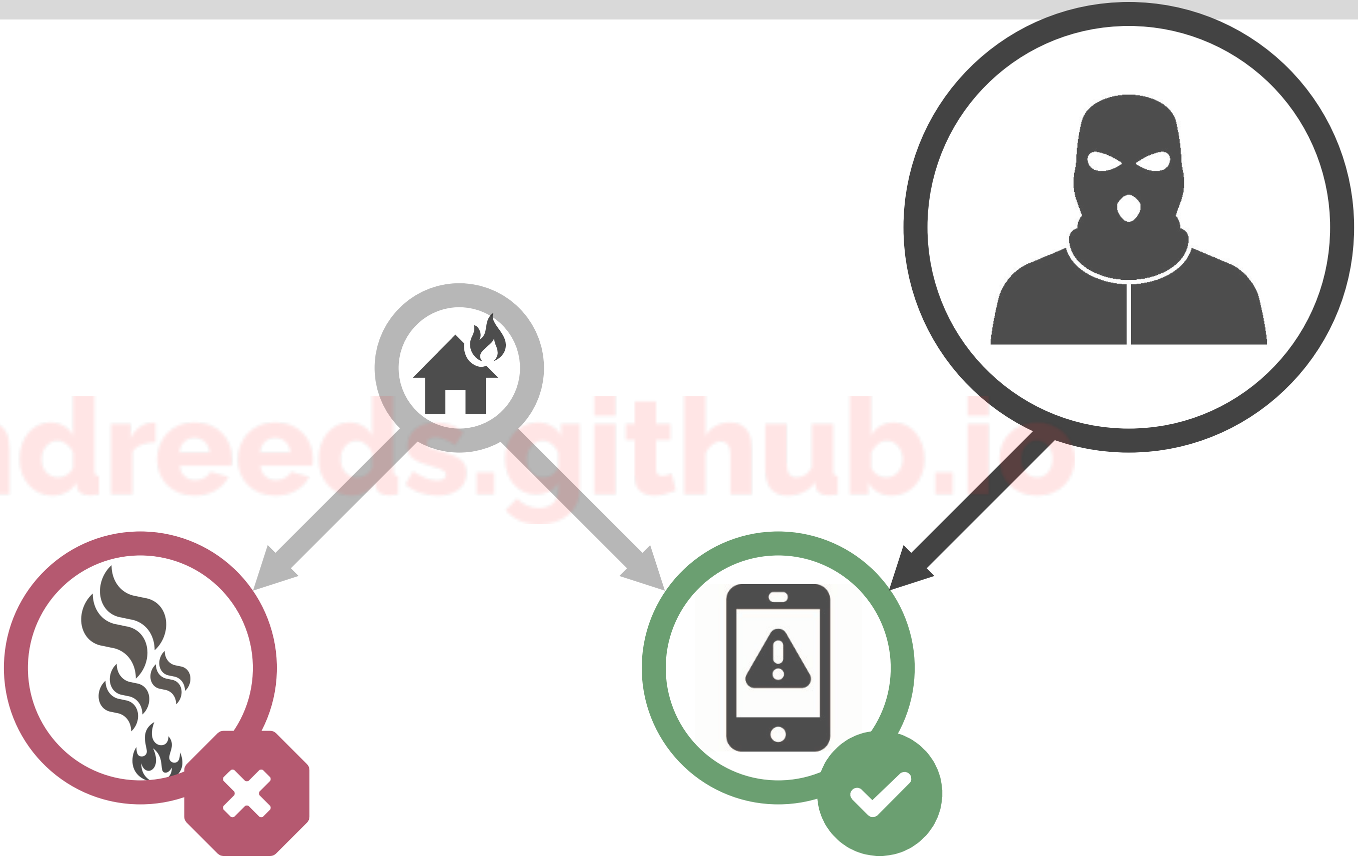


BN EXAMPLE



BN EXAMPLE



EXPECTED VALUE

The **expected value**, also called the **expectation** or **mean**, of the random variable X on the sample space S is equal to

$$E(X) = \sum_{s \in S} p(s)X(s).$$

Examples:

★ Let X be the number that comes up when a fair die is rolled. What is the expected value of X?

★ A fair coin is flipped 3 times. Let S be the sample space of the 8 possible outcomes, and let X be the random variable that assigns to an outcome the number of heads in this outcome. What is the expected value of X?

Theorem

If X is a random variable and $p(X = r)$ is the probability that $X = r$, so that $p(X = r) = \sum_{s \in S, X(s)=r} p(s)$, then

$$E(X) = \sum_{r \in X(S)} p(X = r)r.$$

Examples:

★ What is the expected value of the sum of the numbers that appear when a pair of fair dice is rolled?

AVERAGE-CASE COMPUTATIONAL COMPLEXITY

Computing the average-case computational complexity of an algorithm can be interpreted as computing the expected value of a random variable.

AVERAGE-CASE COMPUTATIONAL COMPLEXITY

Let the *sample space* S of an experiment be the set of possible inputs, and let X be the *random variable* that assigns to each input $s \in S$ the number of operations used by the algorithm when given s as input. Based on our knowledge of the input, we assign a probability $p(s)$ to each possible input $s \in S$. Then the **average-case time complexity of the algorithm** is

$$E(X) = \sum_{s \in S} p(s)X(s).$$

This is the expected value of X.

THE LINEAR SEARCH ALGORITHM

Input: A list $a_1, a_2, \ldots, a_n$ of distinct integers, and an integer x.
Output: The subscript i of the term a_i that equals x, or is 0 if x is not found in the list.

$i \leftarrow 1$;
while $(i \leq n \text{ and } x \neq a_i)$ **do**
  $i \leftarrow i + 1$;
if $i \leq n$ **then**
  location $\leftarrow i$;
else
  location $\leftarrow 0$;
return location

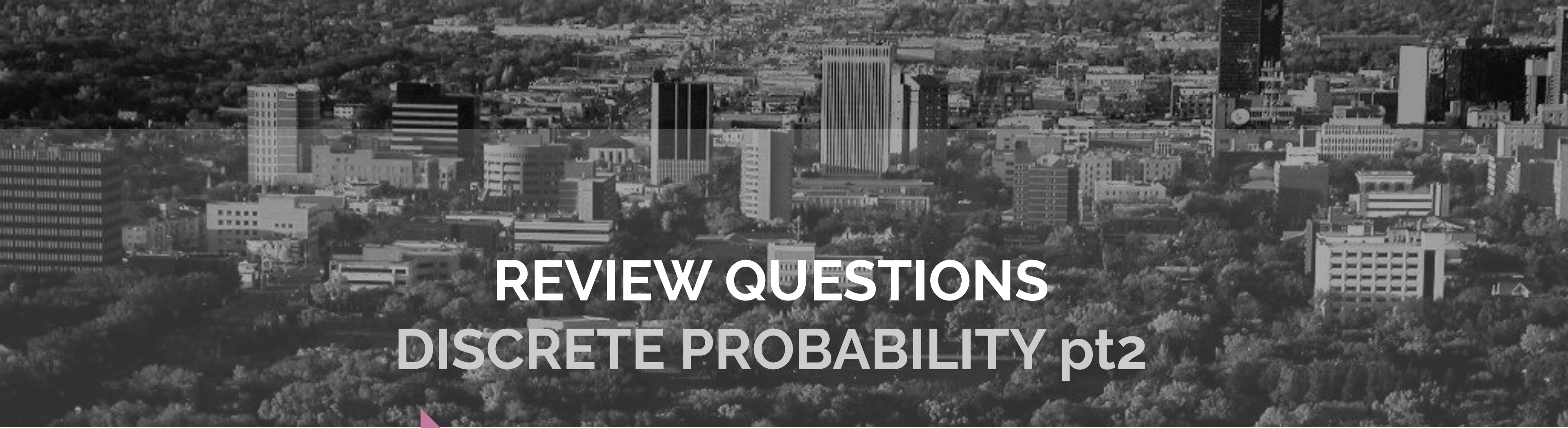
The **worst-case time complexity** of this algorithm is **2n + 2** comparisons.

THE LINEAR SEARCH ALGORITHM

Input: A list $a_1, a_2, \ldots, a_n$ of distinct integers, and an integer x.
Output: The subscript i of the term a_i that equals x, or is 0 if x is not found in the list.

```
i ← 1;
while (i ≤ n and x ≠ a_i) do
    i ← i + 1 ;
if i ≤ n then
    location ← i ;
else
    location ← 0 ;
return location
```


The **average-case time complexity** of the algorithm is **2n + 2 - n*p*** comparisons.



REVIEW QUESTIONS

DISCRETE PROBABILITY pt2

State Bayes' theorem and use it to find $p(F \mid E)$ if $p(E \mid F) = 1/3$, $p(E \mid \overline{F}) = 1/4$, and $p(F) = 2/3$, where E and F are events from a sample space S.

Suppose that Frida selects a ball by first picking one of two boxes at random and then selecting a ball from this box at random. The first box contains two white balls and three blue balls, and the second box contains four white balls and one blue ball. What is the probability that Frida picked a ball from the first box if she has selected a blue ball?

Suppose that we roll a fair die until a 6 comes up or we have rolled it 10 times. What is the expected number of times we roll the die?