UNIVERSITY OF REGINA

CS310-002 DISCRETE COMPUTATIONAL STRUCTURES

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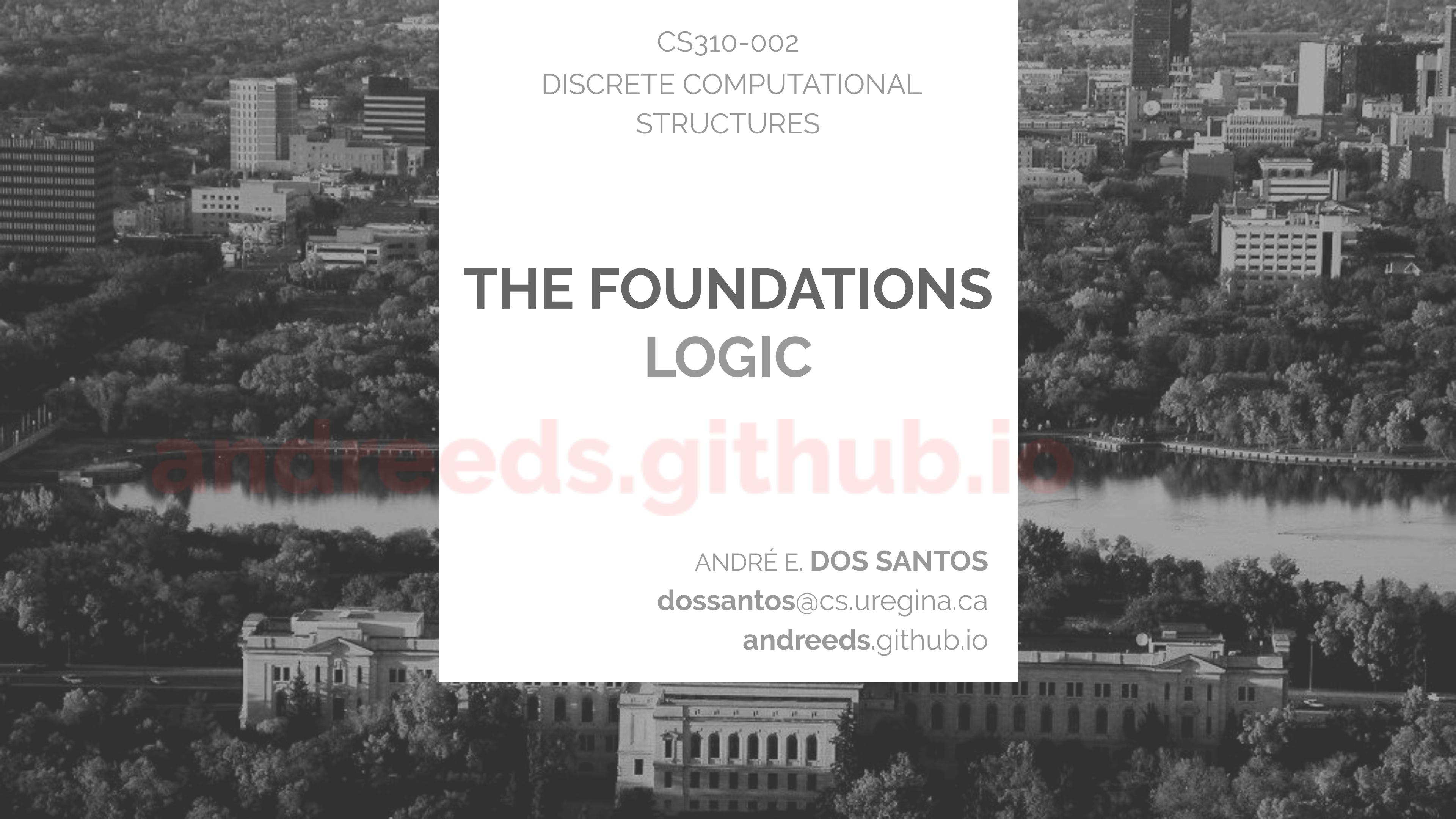

CS310-002

DISCRETE COMPUTATIONAL STRUCTURES

THE FOUNDATIONS
LOGIC

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A **proposition** is a declarative sentence (that is, a sentence that declares a fact) that is either true or false, but not both

1. Regina is the capital of Saskatchewan.
2. 2 + 2 = 3

1. Read this carefully.
2. x + y = z.

Let p and q be propositions.

I.	**negation**	$\neg p$
II.	**conjunction**	$p \land q$
III.	**disjunction**	$p \lor q$
IV.	**exclusive or**	$p \oplus q$
V.	**conditional statement**	$p \rightarrow q$
VI.	**biconditional statement**	$p \leftrightarrow q$

p : It is below freezing.
q : It is snowing.

A. It is below freezing **and** snowing.
B. It is below freezing **but not** snowing.
C. It is **not** below freezing **and** it is **not** snowing.
D. It is **either** snowing **or** below freezing **(or both)**.
E. **If** it is below freezing**,** it is also snowing.
F. **Either** it is below freezing **or** it is snowing, **but** it is **not** snowing **if** it is below freezing.
G. That it is below freezing is **necessary and sufficient** for it to be snowing.

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A. $p \wedge q$
B. $p \wedge \neg q$
C. $\neg p \wedge \neg q$
D. $p \vee q$
E. $p \rightarrow q$
F. $(p \vee q) \wedge (p \rightarrow \neg q)$
G. $q \leftrightarrow p$

truth table & **precedence**

truth table & **precedence**

$$(p \rightarrow q) \land (\neg p \leftrightarrow q)$$

p	q	$\neg p$	$p \rightarrow q$	$\neg p \leftrightarrow q$	$(p \rightarrow q) \land (\neg p \leftrightarrow q)$
T	T				
T	F				
F	T				
F	F				

truth table & **precedence**

$$(p \rightarrow q) \land (\neg p \leftrightarrow q)$$

p	q	$\neg p$	$p \rightarrow q$	$\neg p \leftrightarrow q$	$(p \rightarrow q) \land (\neg p \leftrightarrow q)$
T	T	F			
T	F	F			
F	T	T			
F	F	T			

truth table & precedence

$$(p \to q) \land (\neg p \leftrightarrow q)$$

p	q	$\neg p$	$p \to q$	$\neg p \leftrightarrow q$	$(p \to q) \land (\neg p \leftrightarrow q)$
T	T	F	T	F	
T	F	F	F	T	
F	T	T	T	T	
F	F	T	T	F	

truth table & **precedence**

$$(p \rightarrow q) \land (\neg p \leftrightarrow q)$$

p	q	$\neg p$	$p \rightarrow q$	$\neg p \leftrightarrow q$	$(p \rightarrow q) \land (\neg p \leftrightarrow q)$
T	T	F	T	F	F
T	F	F	F	T	F
F	T	T	T	T	T
F	F	T	T	F	F

A compound proposition that is **always true**, no matter what the truth values of the propositional variables that occur in it, is called a **tautology**

A compound proposition that is **always false** is called a **contradiction**

A compound proposition that is **neither** a tautology nor a contradiction is called a **contingency**

A compound proposition that is **always true**, no matter what the truth values of the propositional variables that occur in it, is called a **tautology**

$p \lor \neg p$

A compound proposition that is **always false** is called a **contradiction**

$p \land \neg p$

A compound proposition that is **neither** a tautology nor a contradiction is called a **contingency**

$\neg p$

Examples

The compound propositions p and q are called **logically equivalent** if $p \leftrightarrow q$ is a *tautology*

Show that $\neg(p \rightarrow q)$ and $p \land \neg q$ are logically equivalent.

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$$\begin{aligned} \neg(p \rightarrow q) &\equiv \neg(\neg p \lor q) \\ &\equiv \neg(\neg p) \land \neg(q) \\ &\equiv p \land \neg q \end{aligned}$$

$\square$

The compound propositions p and q are called **logically equivalent** if $p \leftrightarrow q$ is a *tautology*

Show that $\neg(p \rightarrow q)$ and $p \land \neg q$ are logically equivalent.

$\neg(p \rightarrow q)$

p	q	$p \rightarrow q$	$\neg(p \rightarrow q)$
T	T	T	F
T	F	F	T
F	T	T	F
F	F	T	F

$p \land \neg q$

p	q	$\neg q$	$p \land \neg q$
T	T	F	F
T	F	T	T
F	T	F	F
F	F	T	F

The compound propositions p and q are called **logically equivalent** if $p \leftrightarrow q$ is a *tautology*

Show that $\neg(p \to q)$ and $p \land \neg q$ are logically equivalent.

$\neg(p \to q)$

p	q	$p \to q$	$\neg(p \to q)$
T	T	T	F
T	F	F	T
F	T	T	F
F	F	T	F

=

$p \land \neg q$

p	q	$\neg q$	$p \land \neg q$
T	T	F	F
T	F	T	T
F	T	F	F
F	F	T	F

Quantifiers & **Predicates**

Statement	When True?	When False?
$\forall xP(x)$	P(x) is true for every x.	There is an x for which P(x) is false.
$\exists xP(x)$	There is an x for which P(x) is true.	P(x) is false for every x.

Quantifiers & **Predicates**

Example:

A. No one is perfect
B. Not everyone is perfect.
C. All your friends are perfect
D. At least one of your friends is perfect
E. Everyone is your friend and is perfect
F. Not everybody is your friend or someone is not perfect

Quantifiers & **Predicates**

Example:

Let **P(x)** be “**x is perfect”**;
let **F(x)** be **“x is your friend”**;
and let the **domain** be all **people**.

A. No one is perfect = **∀x ¬P(x)**
B. Not everyone is perfect.= **¬∀x P(x)**
C. All your friends are perfect = **∀x(F(x) → P(x))**
D. At least one of your friends is perfect = **∃x(F(x) ∧ P(x))**
E. Everyone is your friend and is perfect = **∀x(F(x) ∧ P(x))** or **(∀x F(x)) ∧ (∀x P(x))**
F. Not everybody is your friend or someone is not perfect = **(¬∀x F(x)) ∨ (∃x ¬P(x))**

Quantifiers & **Predicates**

Example:

The average of two positive integers is positive.

Quantifiers & **Predicates**

Example:

The average of two positive integers is positive.

$$\forall x \forall y[(x > 0 \land y > 0) \rightarrow (x + y)/2 > 0],$$

where the domain consists of all integers.

Quantifiers & **Predicates**

Example:

There is a student in Gryffindor who has taken all elective classes

Quantifiers & **Predicates**

Example:

There is a student in Gryffindor who has taken all elective classes

$$\exists x \forall y \forall z(H(x, \text{Gryffindor}) \land P(x, y)),$$

where

H(x,z) is **"x is of z house,"**
P(x, y) is **"x has taken y,"**
the domain for **x** consists of all **students in Hogwarts**,
the domain for **y** consists of all **elective classes**,
and the domain for **z** consists of all **Hogwarts houses**.