UNIVERSITY OF REGINA

CS310-002
DISCRETE COMPUTATIONAL STRUCTURES

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BASIC STRUCTURES
SETS & FUNCTIONS

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SETS & FUNCTIONS

set is a collection of distinct **elements**

The set **P** of positive integers less than 100 can be denoted by
P = {1, 2, 3, . . . , 99}

A is a **subset** of **B** iff every element of **A** is also an element of **B**

The set **O = {1, 3, 5, 7, 9}** is a subset of **P**

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a ∈ A denotes
that **a** is an *element* of the set **A**.
Otherwise, **a ∉ A**

P = {1, 2, 3, . . . , 99}

1 ∈ P and **-1 ∉ P**

A ⊆ B indicates that
A is a *subset* of the set **B.**
A is a *proper subset* of the set **B**
if **A ≠B**

O = {1, 3, 5, 7, 9}

O ⊂ P and **P ⊆ P**

Example:

A = **{1, 2, 3}** and **B** = **{1, 2, 3, {1, 2, 3}}**

A ∈ B

A ⊆ B

finite set: a set with **n** elements, where **n** is a nonnegative integer

infinite set: a set that is not finite

|S| (the cardinality of S): the number of elements in **S**

Example:

Let **A** be the set of odd positive integers less than 10. Then **|A| = 5**.

Let **S** be the set of letters in the English alphabet. Then **|S| = 26**.

Because the **null** set has no elements, it follows that **|∅| = 0**.

P(S) (the **power set** of **S**): the set of all subsets of **S**

Example:

What is the power set of the set **{0, 1, 2}**?

$$P(\{0, 1, 2\}) = \{\varnothing,\{0\},\{1\},\{2\},\{0, 1\},\{0, 2\},\{1, 2\},\{0, 1, 2\}\}$$

Note that the **empty set** and the **set itself**
are members of this set of subsets.

A × B denotes the **cartesian product** of **A** and **B**,
that is the set of all ordered pairs **(a, b)**,
where **a** $\in$ **A** and **b** $\in$ **B**

$$A \times B = \{(a, b) \mid a \in A \land b \in B\}.$$

Example:

What is the Cartesian product of **A = {1, 2}** and **B = {a, b, c}**?

Example:

What is the Cartesian product of **A = {1, 2}** and **B = {a, b, c}**?

A × B = {(1, a), (1, b), (1, c), (2, a), (2, b), (2, c)}.

Example:

What is the Cartesian product of **A = {1, 2}** and **B = {a, b, c}**?

A × B = {(1, a), (1, b), (1, c), (2, a), (2, b), (2, c)}.

Note that the Cartesian products **A×B** and **B×A** are not equal, unless **A=∅** or **B=∅** (so that **A × B = ∅**) or **A = B**

Set Operations

$A = \{1, 3, 5\}$

$B = \{1, 2, 3\}$

Universe $U = \{1, 2, 3, 4, 5\}$

$A \cup B = \{1, 2, 3, 5\}$	*union*
$A \cap B = \{1, 3\}$	*intersection*
$A - B = \{5\}$	*difference*
$\bar{A} = \{2, 4\}$	*complement*

SET IDENTITIES

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Identity	*Name*
$A \cap U = A$ $A \cup \emptyset = A$	Identity laws
$A \cup U = U$ $A \cap \emptyset = \emptyset$	Domination laws
$A \cup A = A$ $A \cap A = A$	Idempotent laws
$\overline{(\overline{A})} = A$	Complementation law
$A \cup B = B \cup A$ $A \cap B = B \cap A$	Commutative laws
$A \cup (B \cup C) = (A \cup B) \cup C$ $A \cap (B \cap C) = (A \cap B) \cap C$	Associative laws
$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$ $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$	Distributive laws
$\overline{A \cap B} = \overline{A} \cup \overline{B}$ $\overline{A \cup B} = \overline{A} \cap \overline{B}$	De Morgan's laws
$A \cup (A \cap B) = A$ $A \cap (A \cup B) = A$	Absorption laws
$A \cup \overline{A} = U$ $A \cap \overline{A} = \emptyset$	Complement laws

SETS & FUNCTIONS

function from **A** to **B** : an assignment of exactly one element of **B** to each element of **A**

domain of **f** : the set **A**
codomain of **f** : the set **B**
where **f** is a function from **A** to **B**

b is the **image** of a under **f** : **b = f (a)**
range of **f** : the set of images of **f**

Example:

Consider

$$f(x) = x^2$$

the *domain* of **f** is the set of all integers,
the *codomain* of **f** is the set of all integers,
the *range* of **f** is the set of all integers that are perfect squares, namely, **{0, 1, 4, 9,...}**.

A **onto** function (**surjection**): a function from **A** to **B** such that every element of **B** is the image of some element in **A**

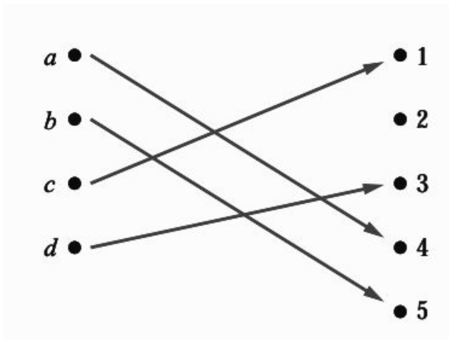
one-to-one function (**injection**): a function such that the images of elements in its domain are distinct

one-to-one correspondence (**bijection**): a function that is both one-to-one and onto

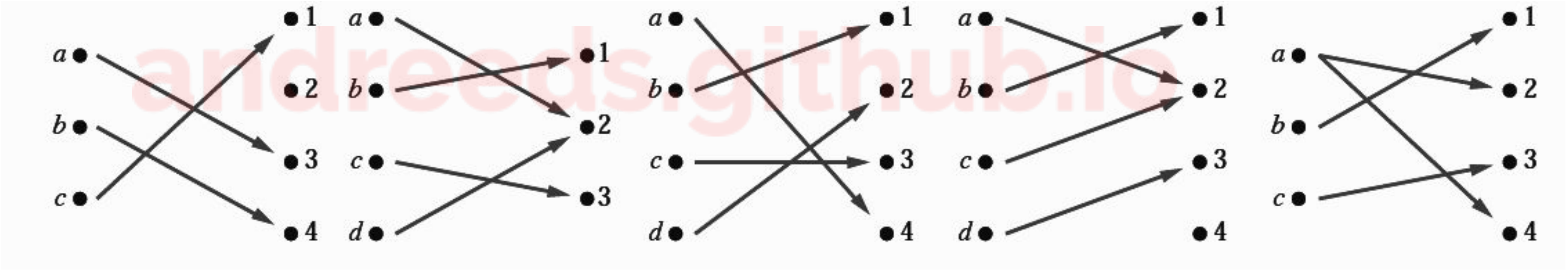
Example:

Determine whether the function **f** from **{a, b, c, d}** to **{1, 2, 3, 4, 5}** with **f (a) = 4**, **f (b) = 5**, **f (c) = 1,** and **f (d) = 3** is one-to-one.

The function f is one-to-one because f takes on different values at the four elements of its domain



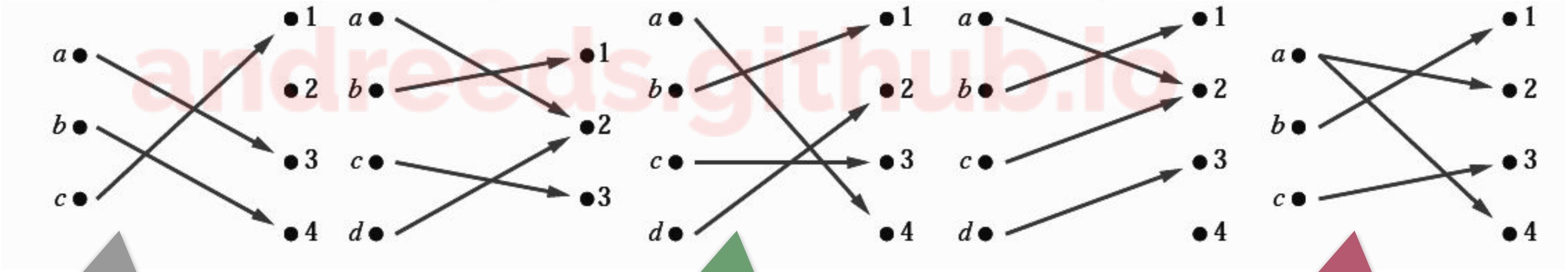
Example:



Example:

Onto,
not one-to-one

Neither
one-to-one
nor onto



One-to-one,
not onto

One-to-one,
and onto

Not a
function

SETS & FUNCTIONS

inverse of **f** : the function that reverses the correspondence given by **f** (when f is a bijection)

$f \circ g$ (**composition** of **f** and **g**): the function that assigns **f (g(x))** to **x**

Example:

Let f be the function from **{a, b, c} to {1, 2, 3}** such that **f (a) = 2**, **f (b) = 3,** and **f (c) = 1.**
Is f *invertible*, and if it is, what is its inverse?

The function **f** is invertible because it is a one-to-one correspondence.

The inverse function $\mathbf{f^{-1}}$ reverses the correspondence given by **f** , so

$$f^{-1}(1) = c,\ f^{-1}(2) = a, \text{ and } f^{-1}(3) = b.$$

Example:

Let **f** be the function from **R** to **R** with **f (x) = x^2.** Is f invertible?

Because **f (−2) = f (2) = 4**, f is not one-to-one. If an inverse function were defined, it would have to assign two elements to **4**. Hence, **f** is not invertible.

Example:

Let **f** and **g** be the functions from the set of integers to the set of integers defined by

$$f(x) = 2x + 3 \text{ and}$$
$$g(x) = 3x + 2.$$

What is the composition of **f** and **g**? What is the composition of **g** and **f** ?

Both the compositions $f \circ g$ and $g \circ f$ are defined.
Moreover,

$$(f \circ g)(x) = f(g(x)) = f(3x + 2) = 2(3x + 2) + 3 = 6x + 7$$

and

$$(g \circ f)(x) = g(f(x)) = g(2x + 3) = 3(2x + 3) + 2 = 6x + 11.$$