UNIVERSITY OF REGINA

CS310-002 DISCRETE COMPUTATIONAL STRUCTURES

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DISCRETE COMPUTATIONAL STRUCTURES

TREES

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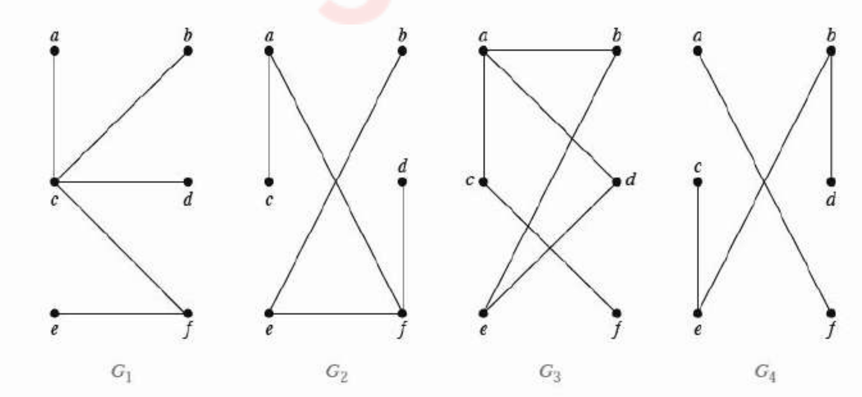
Definition

A **tree** is a connected undirected graph with **no simple circuits**.

Example

Which of the graphs are trees?



Theorem

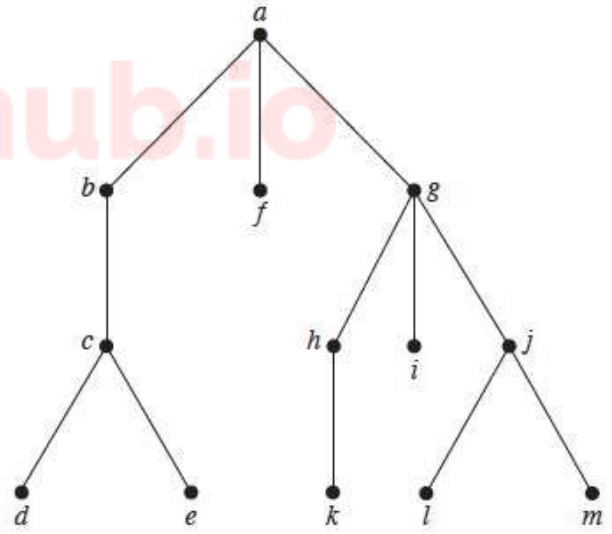
An **undirected graph is a tree** if and only if there is a **unique simple path** between **any two of its vertices**.

Definition

A **rooted tree** is a tree in which one vertex has been designated as the root and every edge is directed away from the root.

Example

Find the **parent** of c, the **children** of g, the **siblings** of h, all **ancestors** of e, all **descendants** of b, all **internal vertices**, and all **leaves**. What is the **subtree** rooted at g?

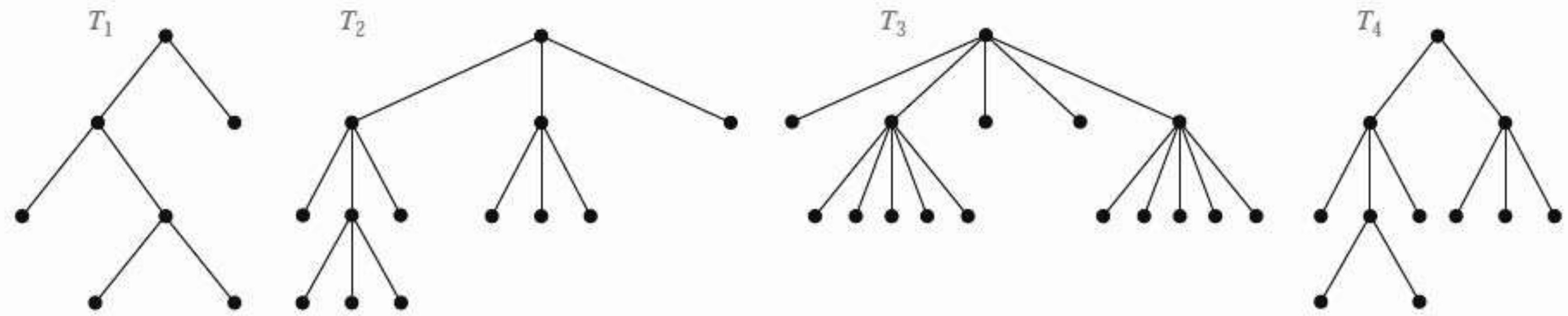


Definition

A rooted tree is called an ***m*-ary tree** if every internal vertex has **no more than *m* children**. The tree is called a **full *m*-ary tree** if **every** internal vertex has exactly m children. An m-ary tree with $\boldsymbol{m = 2}$ is called a ***binary tree***.

Example

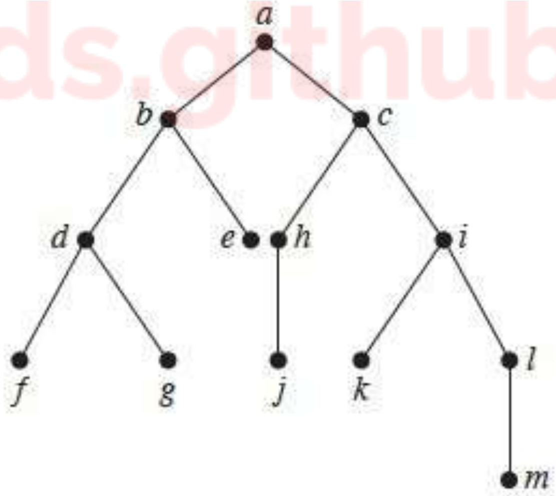
Are the rooted trees full m-ary trees for some positive integer m?

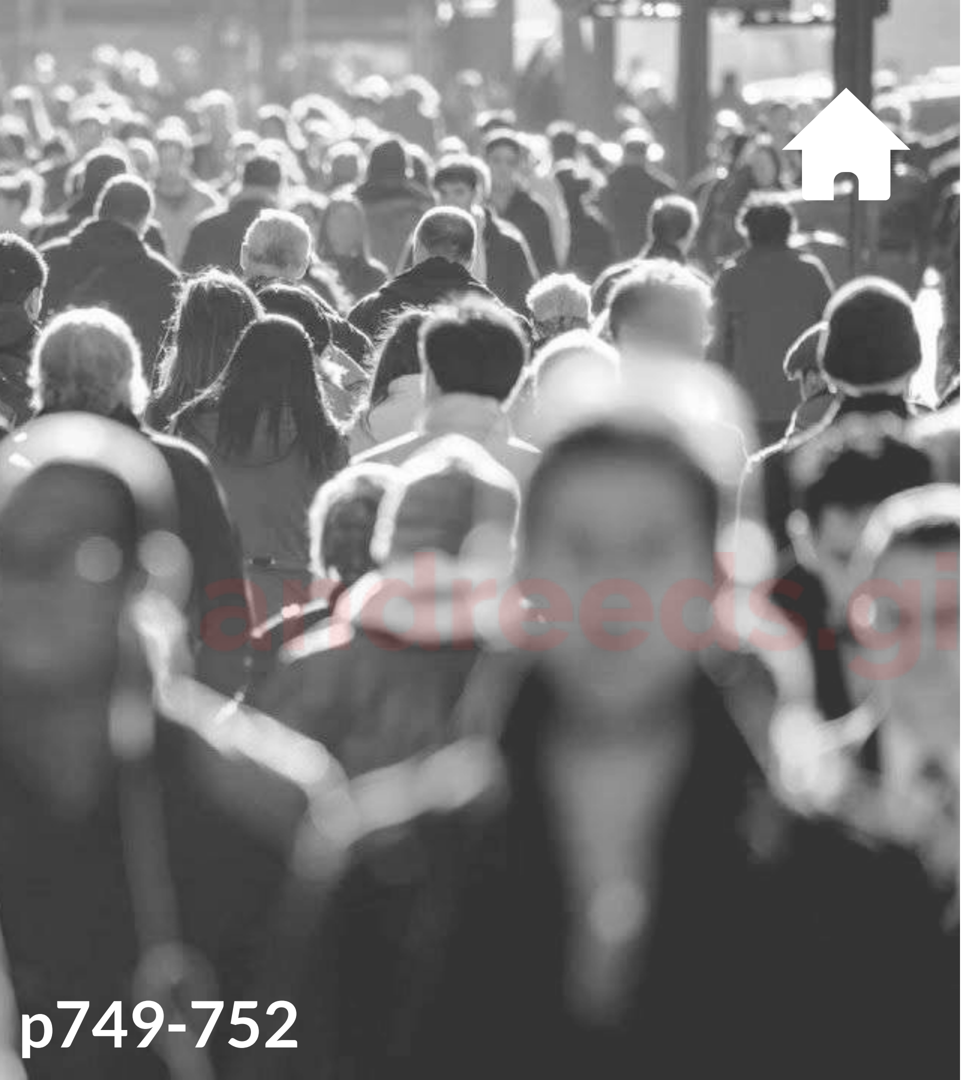


Definition

An **ordered rooted tree** is a rooted tree where the children of each internal vertex are ordered.

Example





APPLICATIONS

CHEMISTRY

Saturated Hydrocarbons andTrees

PSYCHOLOGY

Representing Organizations

COMPUTER SCIENCE

Computer File Systems, Tree-Connected Parallel Processors

~~FUN FACTS~~

JUST SOME PROPERTIES 2

THEOREM A tree with n vertices has $\boldsymbol{n - 1}$ **edges**.

THEOREM A full m-ary tree with $\boldsymbol{i}$ **internal vertices** contains $\boldsymbol{n = mi + 1}$ **vertices**.

A full m-ary tree with

(i) n vertices has $i = (n-1)/m$ internal vertices and $l = [(m-1)n+1]/m$ leaves,
(ii) i internal vertices has $n = mi + 1$ vertices and $l = (m-1)i + 1$ leaves,
(iii) l leaves has $n = (ml - 1)/(m-1)$ vertices and $i = (l-1)/(m-1)$ internal vertices.

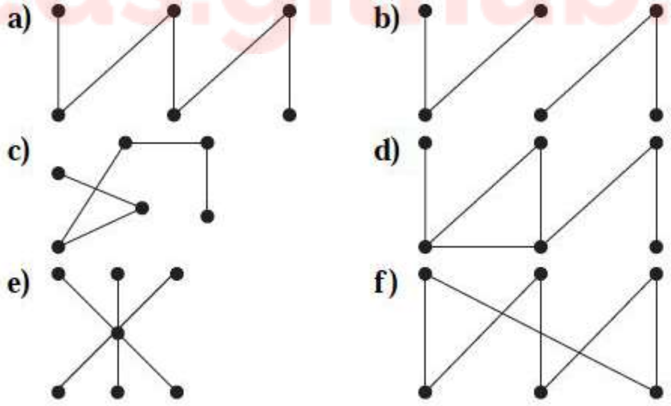
THEOREM There are **at most** $\boldsymbol{m^h}$ **leaves** in an m-ary tree of **height** $\boldsymbol{h}$.

DEFINITION A rooted m-ary tree of height h is **balanced** if all leaves are at levels h or $h - 1$.

COROLLARY If an m-ary tree of height h has l leaves, then $\mathbf{h} \geq \lceil \log_m l \rceil$. If the m-ary tree is full and balanced, then $\mathbf{h} = \lceil \log_m l \rceil$.

REVIEW QUESTIONS
TREES

Which of these graphs are trees?



REVIEW QUESTIONS
TREES

a) Which vertex is the root?
b) Which vertices are internal?
c) Which vertices are leaves?
d) Which vertices are children of j?
e) Which vertex is the parent of h?
f) Which vertices are siblings of o?
g) Which vertices are ancestors of m?
h) Which vertices are descendants of b?