UNIVERSITY OF REGINA

CS310-002 DISCRETE COMPUTATIONAL STRUCTURES

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DISCRETE COMPUTATIONAL STRUCTURES

GRAPHS

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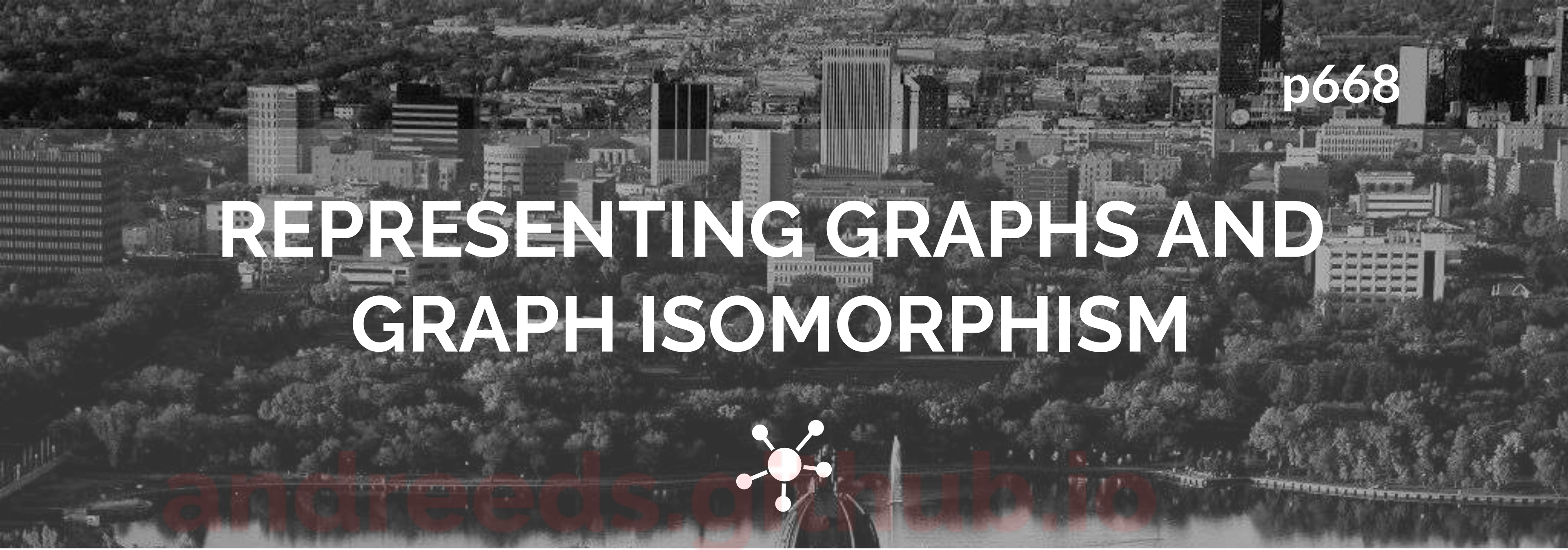
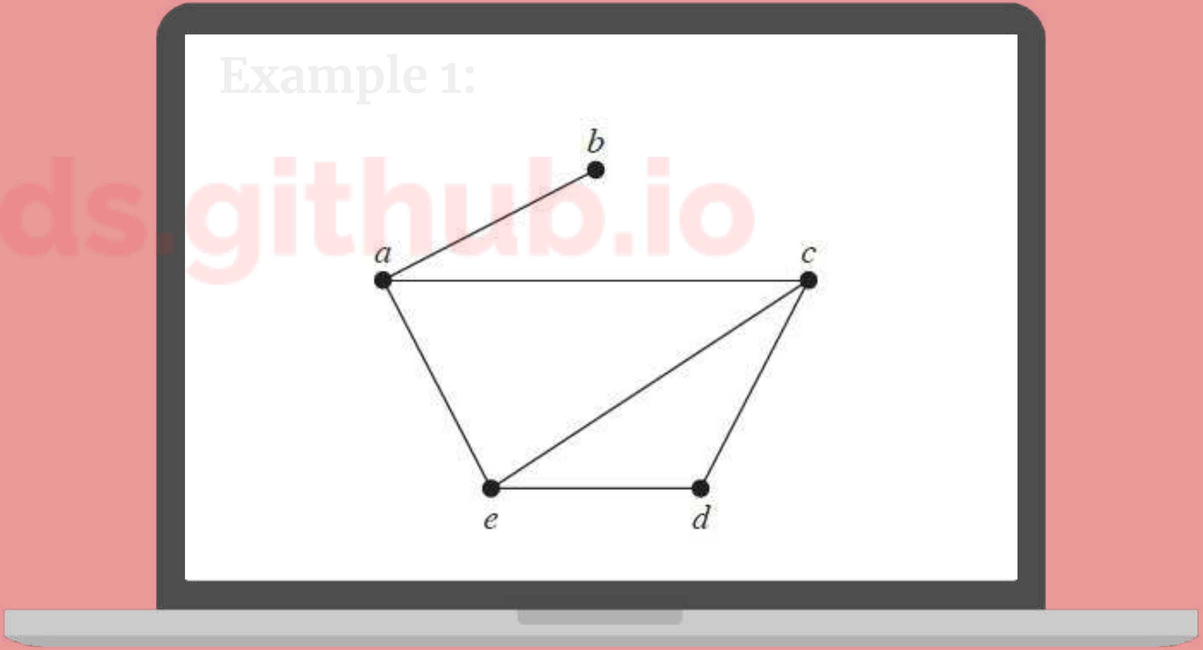
REPRESENTING GRAPHS AND GRAPH ISOMORPHISM



REPRESENTING GRAPHS

- **Adjacency lists**
- Adjacency matrices
- Incidence matrices

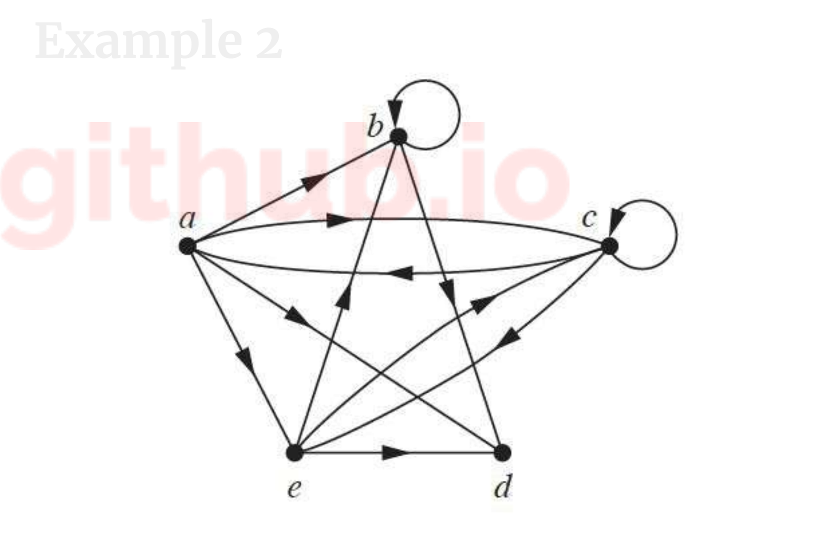


REPRESENTING GRAPHS

Adjacency lists

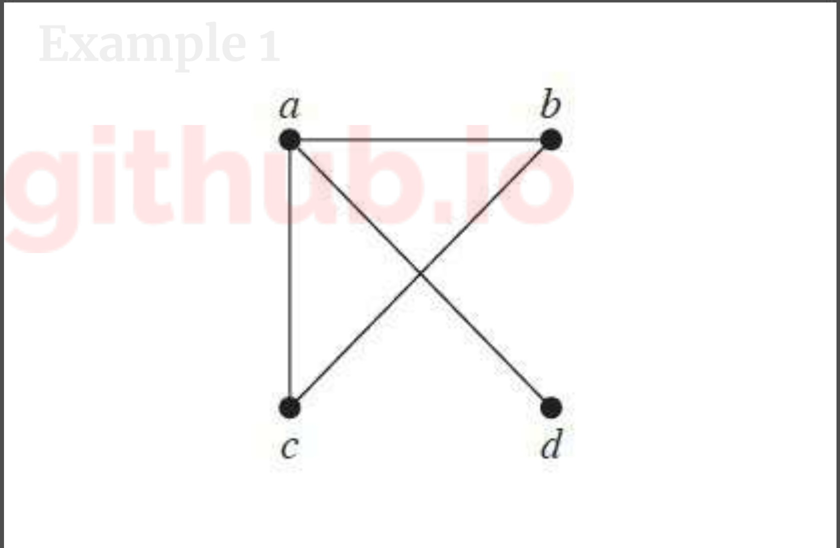
Adjacency matrices

Incidence matrices



REPRESENTING GRAPHS

- Adjacency lists
- **Adjacency matrices**
- Incidence matrices



REPRESENTING GRAPHS

Adjacency lists

Adjacency matrices

Incidence matrices

Example 2

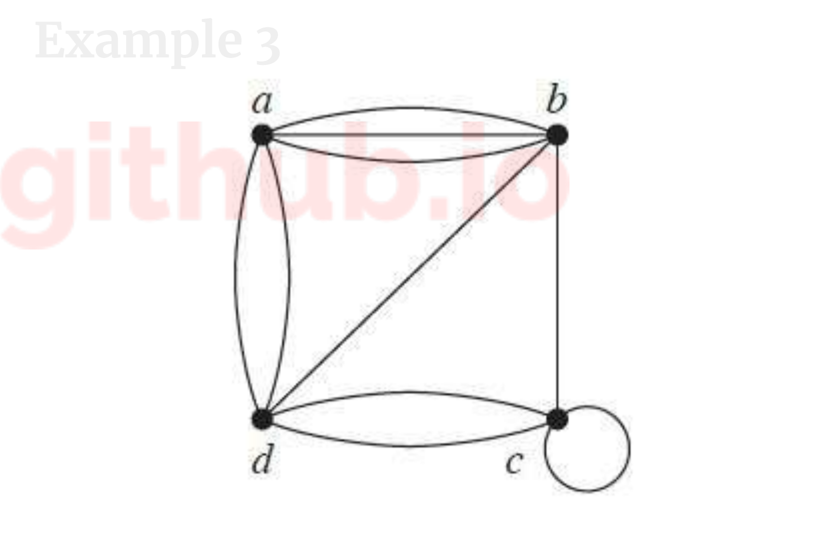
$$\begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix}$$

REPRESENTING GRAPHS

Adjacency lists

Adjacency matrices

Incidence matrices

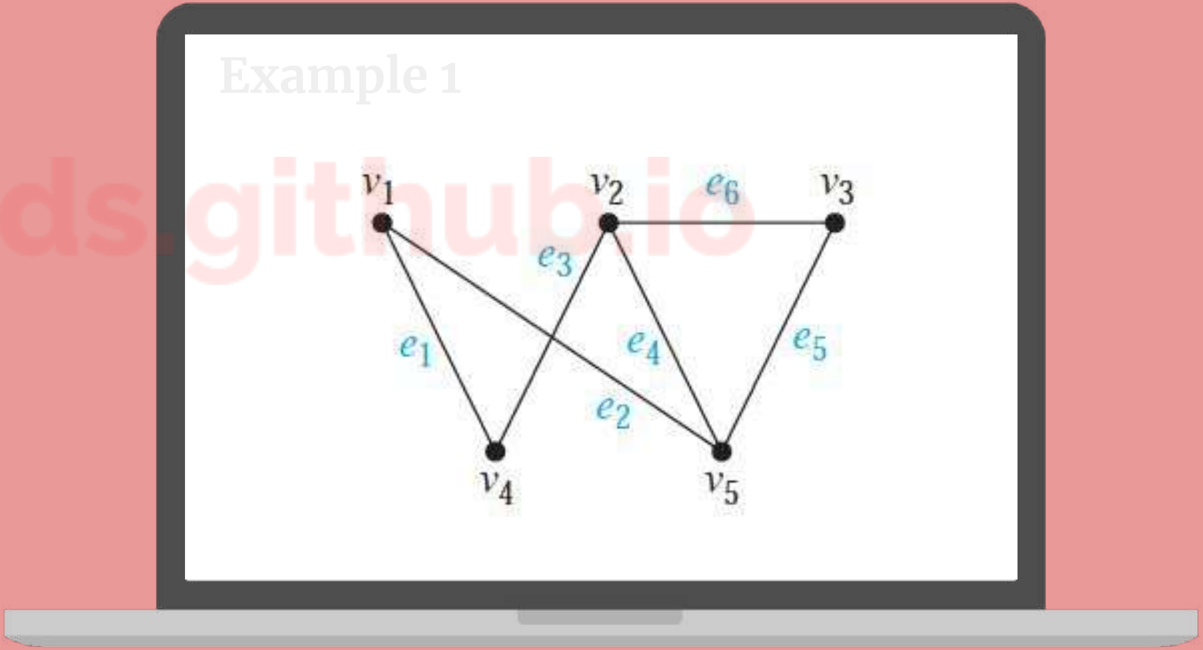


REPRESENTING GRAPHS

Adjacency lists

Adjacency matrices

Incidence matrices



REPRESENTING GRAPHS

Adjacency lists

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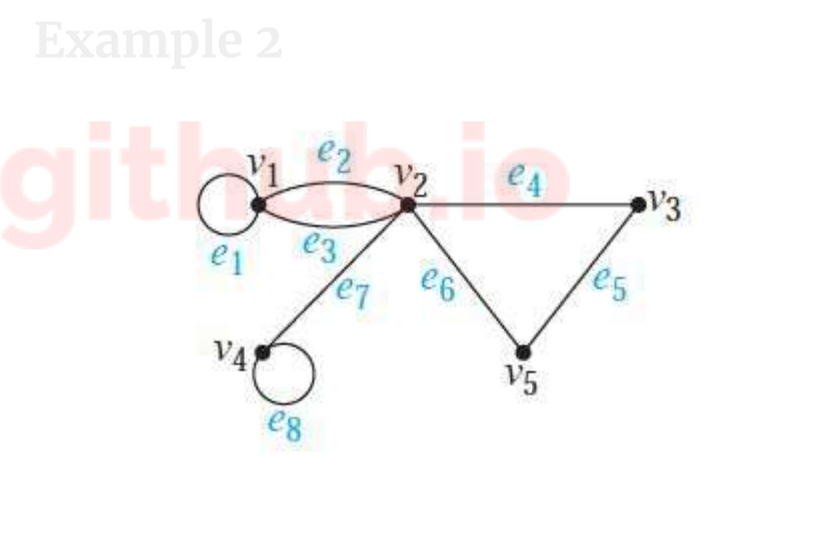


ISOMORPHISM OF GRAPHS

Definition

The simple graphs $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$ are *isomorphic* if there exists a one-to-one and onto function f from V_1 to V_2 with the property that a and b are adjacent in G_1 if and only if $f(a)$ and $f(b)$ are adjacent in G_2, for all a and b in V_1. Such a function f is called an *isomorphism*.* Two simple graphs that are not isomorphic are called *nonisomorphic*.

Example

Show that the graphs $G = (V, E)$ and $H = (W, F)$, displayed in Figure 8, are isomorphic.

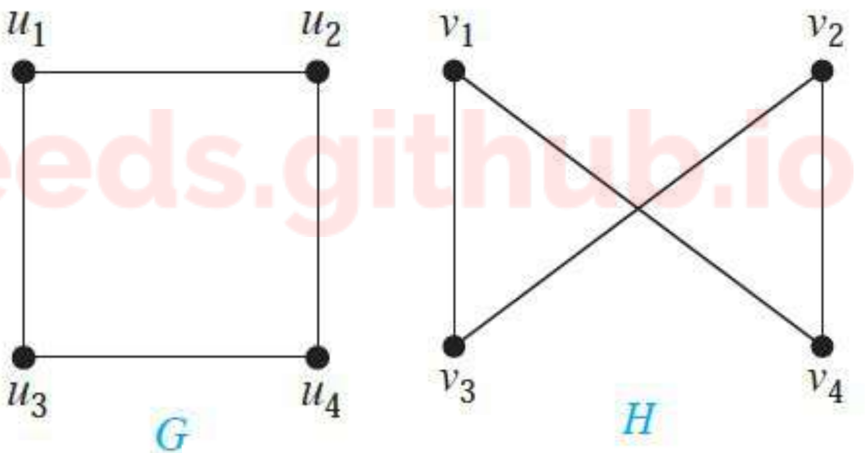


FIGURE 8 The Graphs G and H.

Example

Show that the graphs displayed in Figure 9 are not isomorphic.

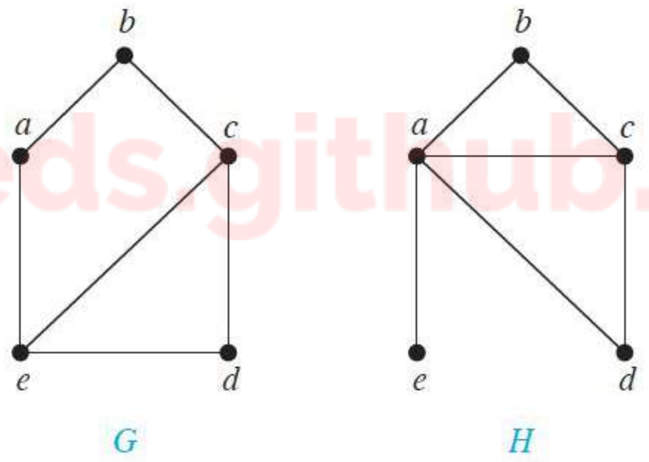


FIGURE 9 **The Graphs G and H.**

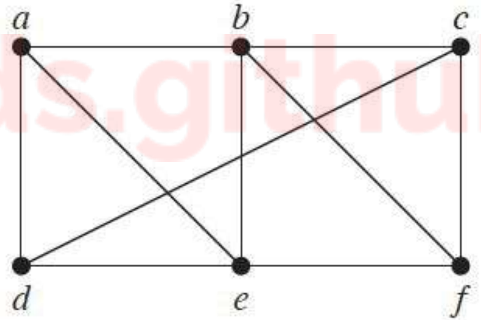
CONNECTIVITY

Definition

Let n be a nonnegative integer and G an undirected graph. A *path* of *length* n from u to v in G is a sequence of n edges $e_1, \ldots, e_n$ of G for which there exists a sequence $x_0 = u, x_1, \ldots, x_{n-1}, x_n = v$ of vertices such that e_i has, for $i = 1, \ldots, n$, the endpoints x_{i-1} and x_i. When the graph is simple, we denote this path by its vertex sequence $x_0, x_1, \ldots, x_n$ (because listing these vertices uniquely determines the path). The path is a *circuit* if it begins and ends at the same vertex, that is, if $u = v$, and has length greater than zero. The path or circuit is said to *pass through* the vertices $x_1, x_2, \ldots, x_{n-1}$ or *traverse* the edges $e_1, e_2, \ldots, e_n$. A path or circuit is *simple* if it does not contain the same edge more than once.

Example

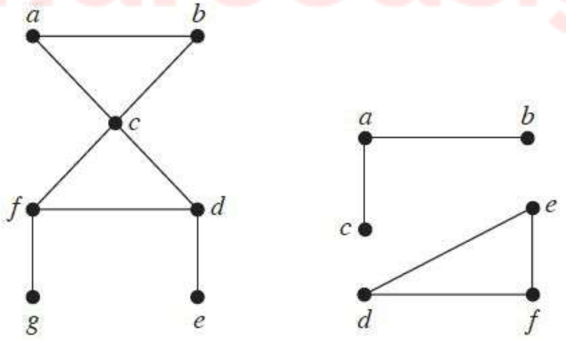
Is **a, d, c, f , e** a simple path? Or **d, e, c, a**? Or **b, c, f , e, b**? Or **a, b, e, d, a, b**?



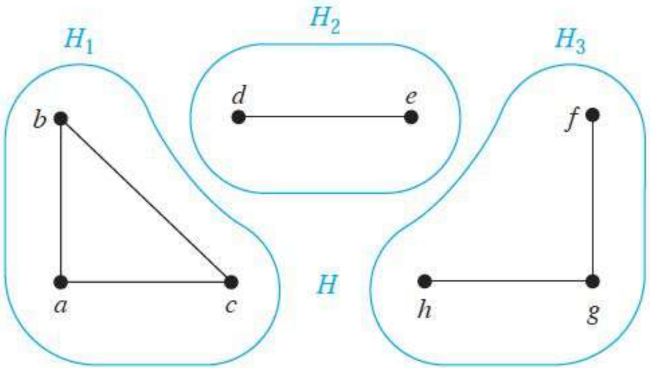
Definition

An undirected graph is called *connected* if there is a path between every pair of distinct vertices of the graph. An undirected graph that is not *connected* is called *disconnected*. We say that we *disconnect* a graph when we remove vertices or edges, or both, to produce a disconnected subgraph.

Examples

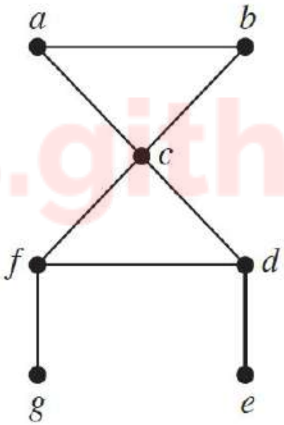


G_1 G_2

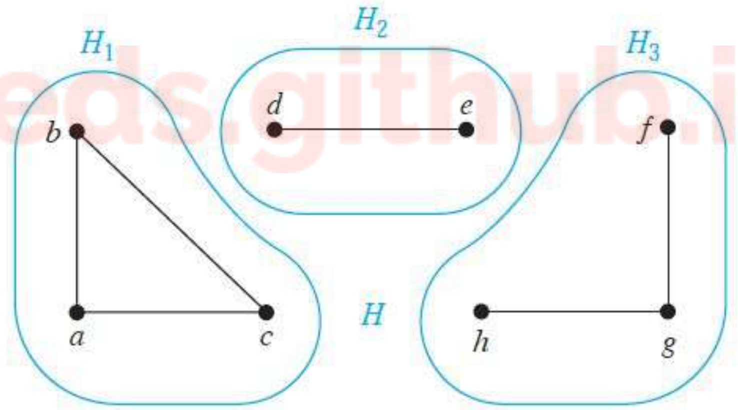


Theorem

There is a simple path between every pair of distinct vertices of a connected undirected graph.



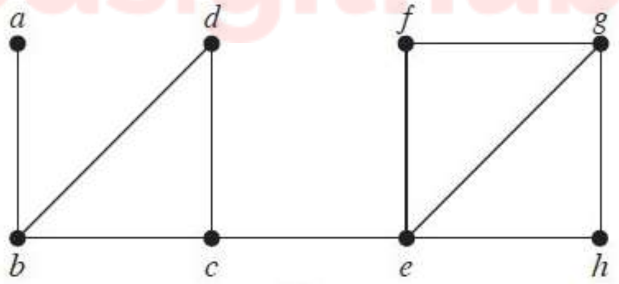
A **connected component** of a graph G is a connected subgraph of G that is not a proper subgraph of another connected subgraph of G. That is, a connected component of a graph G is a maximal connected subgraph of G.



A vertex whose removal produces a graph with more connected components than in the original graph is called a **cut vertex** (or **articulation point**).

Example

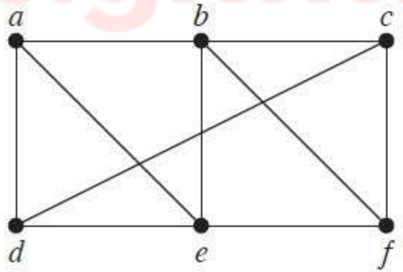
Find the cut vertices



A subset V' of the vertex set V of a graph $G = (V, E)$ is a **vertex cut**, or **separating set**, if $G - V'$ is disconnected.

Examples

Show that the subset $V'_1 = \{b, c, e\}$ is a vertex cut.



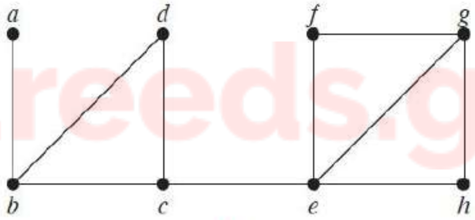
VERTEX CONNECTIVITY

The **vertex connectivity** of a noncomplete graph G, denoted by $k(G)$, is the *minimum number of vertices in a vertex cut.*

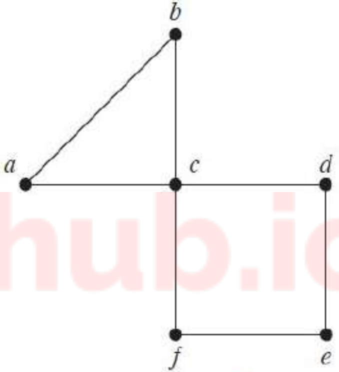
For a complete graph K_n, we denote $k(K_n) = n - 1$.

Example

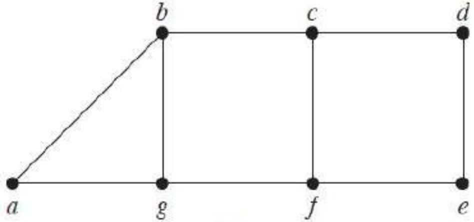
Find the **vertex** connectivity for each of the following graphs



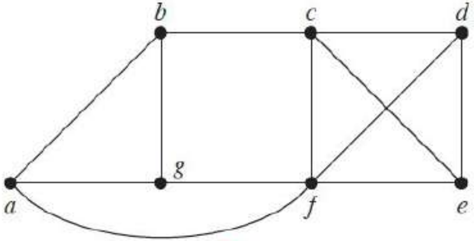
G_1



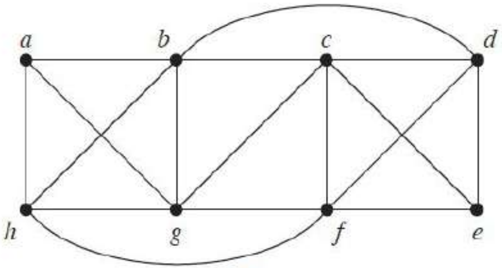
G_2



G_3



G_4



G_5

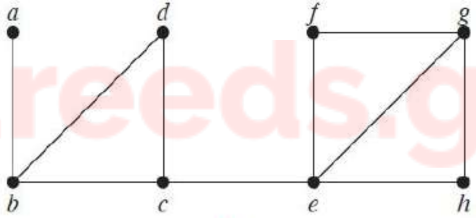
VERTEX CONNECTIVITY

A graph is ***k*-connected** (or ***k*-vertex-connected**), if $k(G) \geq k$.

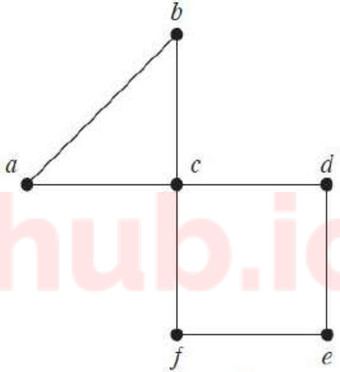
For any graph G with n vertices, we have $0 \leq k(G) \leq \mathrm{n} - 1$.

Example

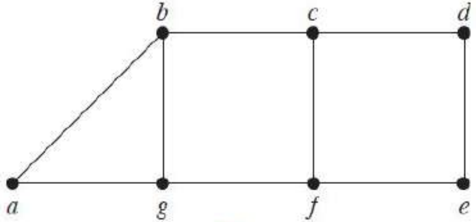
Describe which type of k-connected the following graphs are



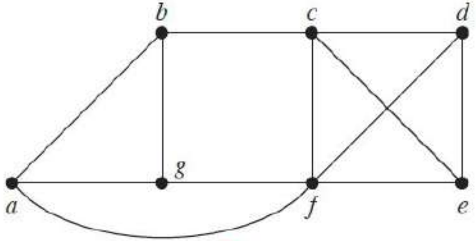
G_1



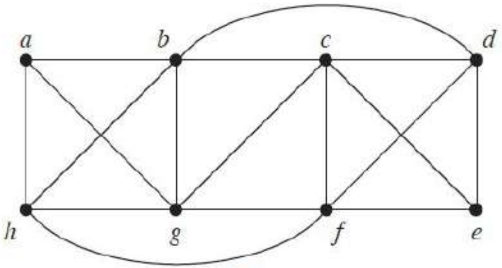
G_2



G_3



G_4



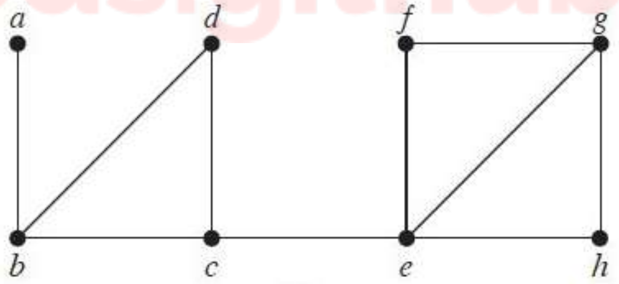
G_5

EDGE CONNECTIVITY

An edge whose removal produces a graph with more connected components than in the original graph is called a **cut edge** (or **bridge**).

Example

Find the cut edges

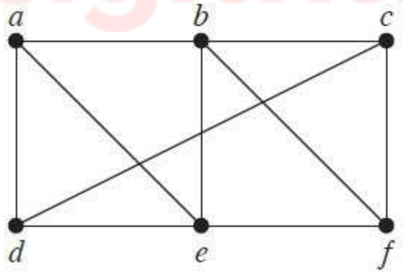


EDGE CONNECTIVITY

A subset E' of the vertex set E of a graph $G = (V, E)$ is a **edge cut** if G - E' is disconnected.

Example

Show that the subset $E'_1 = \{(b, c), (c, d), (b, f), (e, f)\}$ is a edge cut.



EDGE CONNECTIVITY

The **edge connectivity** of a graph G, denoted by $\lambda(G)$, is the *minimum number of edges in an edge cut of G.*

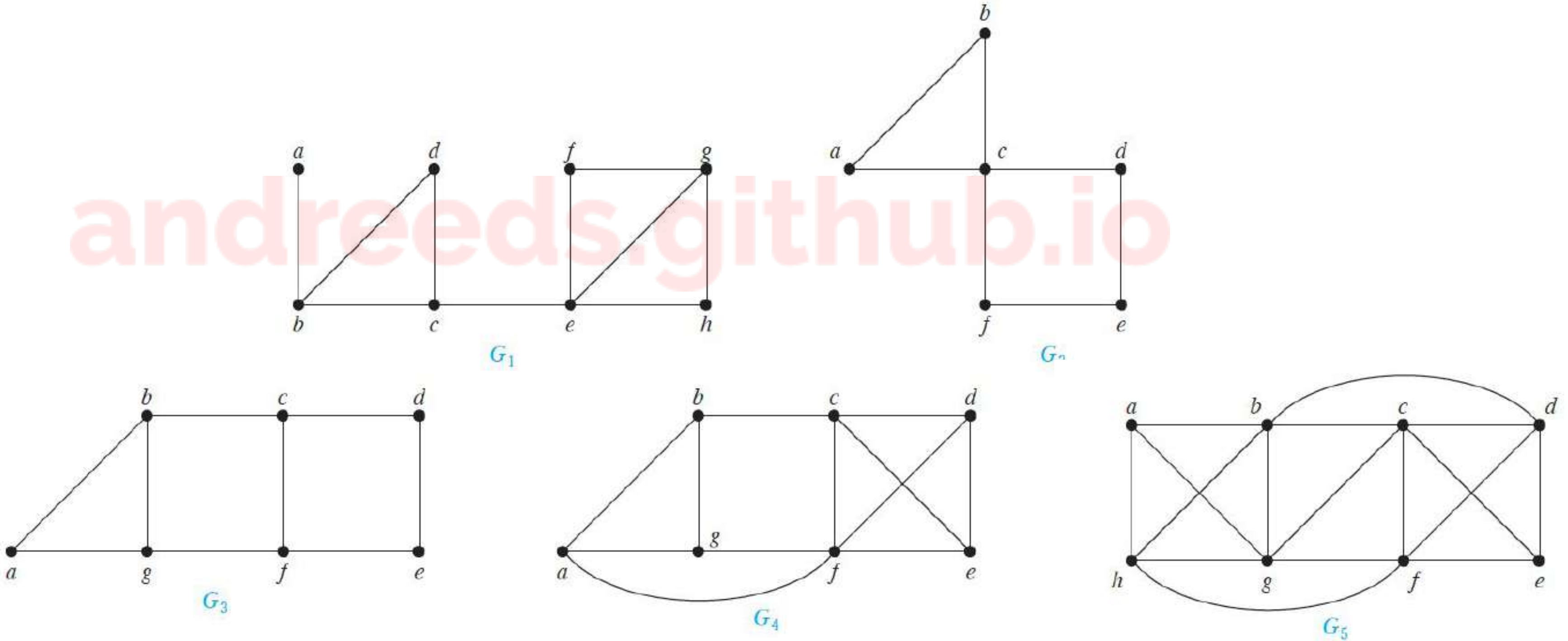
For any graph G with n vertices, $0 \leq \lambda(G) \leq n - 1$.

An Inequality for Vertex Connectivity and Edge Connectivity

$$k(G) \leq \lambda(G) \leq \min_{v \in V} \deg(v)$$

Example

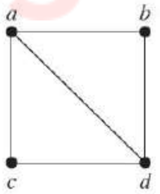
Find the **edge** connectivity for each of the following graphs



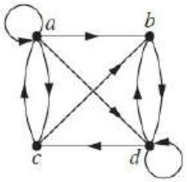
REVIEW QUESTIONS Pt. 3
GRAPHS

Use adjacency lists and adjacency matrices to represent the given graphs.

1.



3.

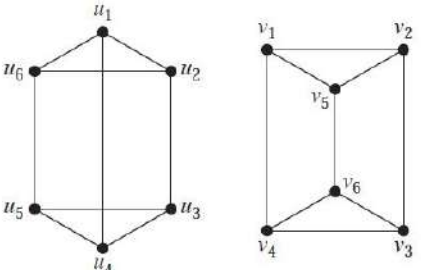


REVIEW QUESTIONS Pt. 3
GRAPHS

- Draw the graph represented by the given adjacency matrix.

$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & 0 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

- Determine whether the given pair of graphs is isomorphic. Exhibit an isomorphism or provide a rigorous argument that none exists.



- Construct a graph G with $\kappa(G) = 1$, $\lambda(G) = 2$, and $\min_{v \in V} \deg(v) = 3$.