

The background of the slide features a black and white aerial photograph of the University of Regina campus. In the foreground, there's a large, multi-story building with classical architectural details. Behind it, a river flows through a park area with many trees. Further back, the city skyline of Regina is visible, featuring several modern office buildings.

UNIVERSITY OF REGINA

CS310-002

DISCRETE

COMPUTATIONAL

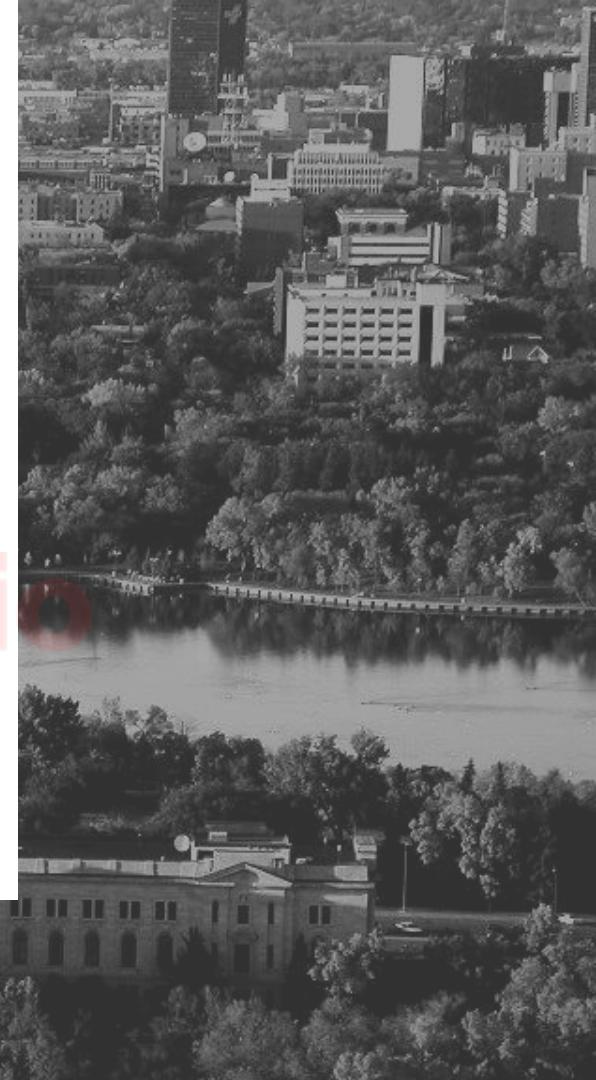
STRUCTURES

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ADVANCED COUNTING TECHNIQUES

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Review solving linear homogeneous recurrence relations

1. **Check** it is linear, homogeneous, and with constant coefficients.
2. Identify the **degree k** .
3. Find the **characteristic equation**
4. Find the **roots** of the equation in **3**.
5. The solution will be of the form $a_n = a_1 r_1^n + a_2 r_2^n + \dots + a_m r_m^n$
6. Substitute the **initial conditions** to find the a 's value

Review solving linear homogeneous recurrence relations of degree 2

1. **Check** it is linear, homogeneous, and with constant coefficients.
2. The **degree** is 2. (Duh!)
3. Find the **characteristic equation**
4. Find the **roots** of the equation in 3.
5. The solution will be of the form $a_n = a_1 r_1^n + a_2 r_2^n$
6. Substitute the **initial conditions** to find the a 's value

ADVANCED COUNTING TECHNIQUES

Theorem

- Suppose
- $r^k - c_1 r^{k-1} + c_2 r^{k-2} + \dots + c_k = 0$ has t **distinct** roots r_1, r_2, \dots, r_t with multiplicities m_1, m_2, \dots, m_t , respectively,
- So,

$$\begin{aligned} a_n = & (\alpha_{1,0} + \alpha_{1,1} n^1 + \dots + \alpha_{1,m_1-1} n^{m_1-1}) r_1^n + \\ & (\alpha_{2,0} + \alpha_{2,1} n^1 + \dots + \alpha_{2,m_2-1} n^{m_2-1}) r_2^n + \dots + \\ & (\alpha_{t,0} + \alpha_{t,1} n^1 + \dots + \alpha_{t,m_t-1} n^{m_t-1}) r_t^n \end{aligned}$$

satisfies the recurrence for $n = 0, 1, 2, \dots$,

where $\alpha_{i,j}$ are constants for $1 \leq i \leq t$ and $0 \leq j \leq m_i - 1$.

Solving linear homogeneous recurrence relations of degree 2 with only one root

1. **Check** it is linear, homogeneous, and with constant coefficients.
2. The **degree** is 2.
3. Find the **characteristic equation**
4. Find the **root** of the equation in **3**.
5. The solution will be of the form $a_n = a_1 r_0^n + a_2 n r_0^n$
6. Substitute the **initial conditions** to find the **a**'s value

Example

What is the solution of the recurrence relation

$$a_n = 6a_{n-1} - 9a_{n-2}$$

with $a_0 = 1$ and $a_1 = 6$?

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Example

What is the solution of the recurrence relation

$$a_n = -3a_{n-1} - 3a_{n-2} - a_{n-3}$$

with $a_0 = 1$, $a_1 = -2$, and $a_2 = -1$?

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A black and white aerial photograph of a city skyline, likely Boise, Idaho. In the foreground, there's a large body of water with a bridge spanning it. The city is built on a hillside, with numerous buildings of varying heights and architectural styles. The sky is clear and blue.

p515

SOLVING LINEAR NON-HOMOGENEOUS RECURRENCES



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linear **nonhomogeneous** recurrence relation
with constant coefficients

is a recurrence relation of the form

$$a_n = c_1 a_{n-1} + c_2 a_{n-2} + \cdots + c_k a_{n-k} + F(n),$$

where c_1, c_2, \dots, c_k are real numbers, and $F(n)$ is a function not identically zero depending only on n .

→ $a_n = c_1 a_{n-1} + c_2 a_{n-2} + \cdots + c_k a_{n-k}$ is called the **associated homogeneous recurrence relation**.

linear **nonhomogeneous** recurrence relation
with constant coefficients

$$a_n = c_1 a_{n-1} + c_2 a_{n-2} + \cdots + c_k a_{n-k} + F(n),$$

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function depending only on n

linear **nonhomogeneous** recurrence relation
with constant coefficients

$$a_n = c_1 a_{n-1} + c_2 a_{n-2} + \cdots + c_k a_{n-k} + F(n),$$

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associated homogeneous recurrence relation

linear **nonhomogeneous** recurrence relation
with constant coefficients

$$a_n = c_1 a_{n-1} + c_2 a_{n-2} + \cdots + c_k a_{n-k} + F(n),$$

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Examples:

★ $a_n = 2a_{n-1} + 1$

★ $a_n = a_{n-1} + a_{n-2} + n$

★ $a_n = a_{n-1} + 2^n$

Theorem

If $\{a_n^{(p)}\}$ is a particular solution of the nonhomogeneous linear recurrence relation with constant coefficients,

$$a_n = c_1 a_{n-1} + c_2 a_{n-2} + \dots + c_k a_{n-k} + F(n),$$

then every solution is of the form $\{a_n^{(p)} + a_n^{(h)}\}$, where $\{a_n^{(h)}\}$ is a solution of the associated homogeneous recurrence relation

$$a_n = c_1 a_{n-1} + c_2 a_{n-2} + \dots + c_k a_{n-k}.$$

Note that there is **no general method** for finding a particular solution that works for every function $F(n)$.

Example

What is the solution of the recurrence relation

$$a_n = 3a_{n-1} + 2n$$

with $a_1 = 3$?

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REVIEW QUESTIONS

ADVANCED COUNTING TECHNIQUES

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What is the solution of the recurrence relation
 $a_n = 8a_{n-2} - 16a_{n-4}$
for $n \geq 4$, with $a_0 = 1$, $a_1 = 4$, $a_2 = 28$, and $a_3 = 32$?

Find all solutions of the recurrence relation

$$a_n = 5a_{n-1} - 6a_{n-2} + 7^n.$$