# PRECEPT 2: Heat and mass balances of a steam cycle

Rossi Andrea 875272

The aim of the precept is to analyse the thermodynamical cycle of a sub critical natural gas fired steam plant. In particular are required the:

A Calculation of the thermodynamic conditions of each mass stream of the steam cycle;

B Calculation of the mass flow rates of each mass stream of the steam cycle;

C Evaluation of cycle performances;

D Representation of T-Q diagram of the surface feed water preheater RB.

Figure 1: Process flow diagram of a steam power plant.
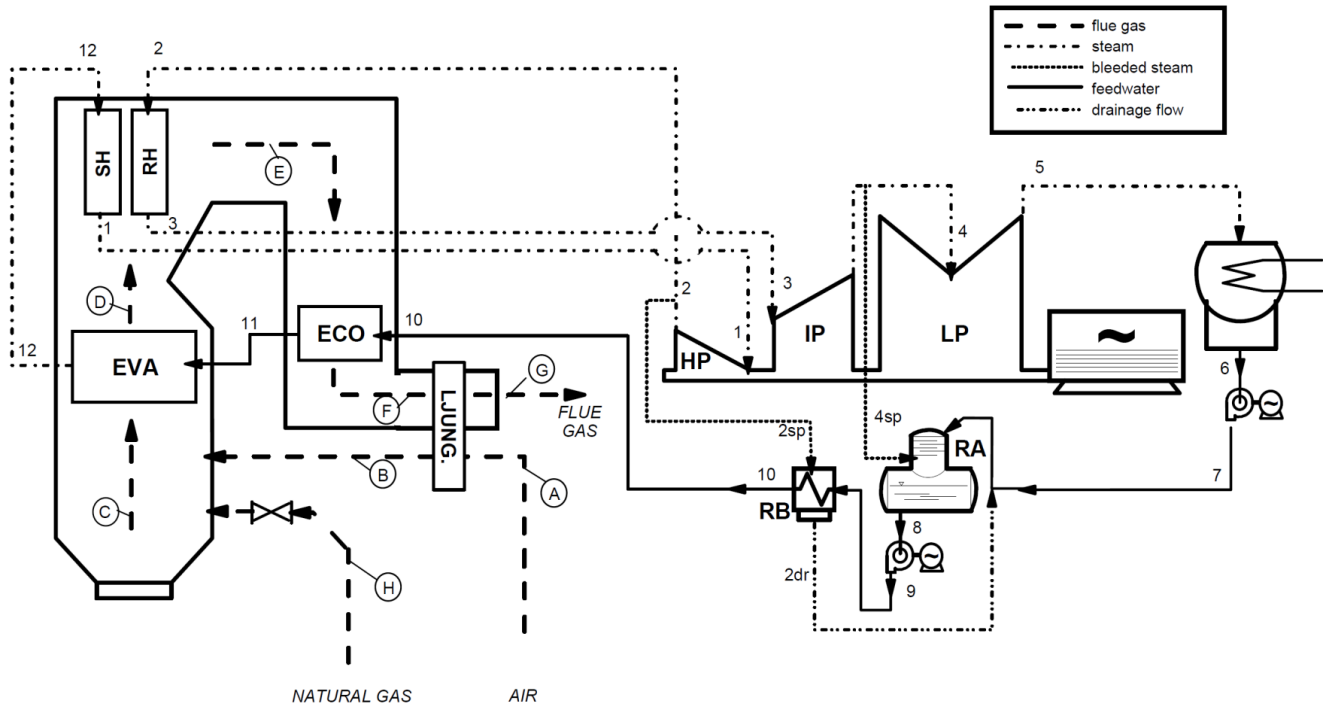
1   Solution

1.A   Calculation of the thermodynamic conditions of each mass stream of the steam cycle

Before to evaluate every mass flow rate, it is needed to know the main thermodynamic conditions in each point of the plant. We start from point 1 and than proceed straight to all points. From the knowledge of two of the main thermodynamic properties like T and p or p and h it is possible to get with Mollier diagram or some computer implementation all the others missing parameters.
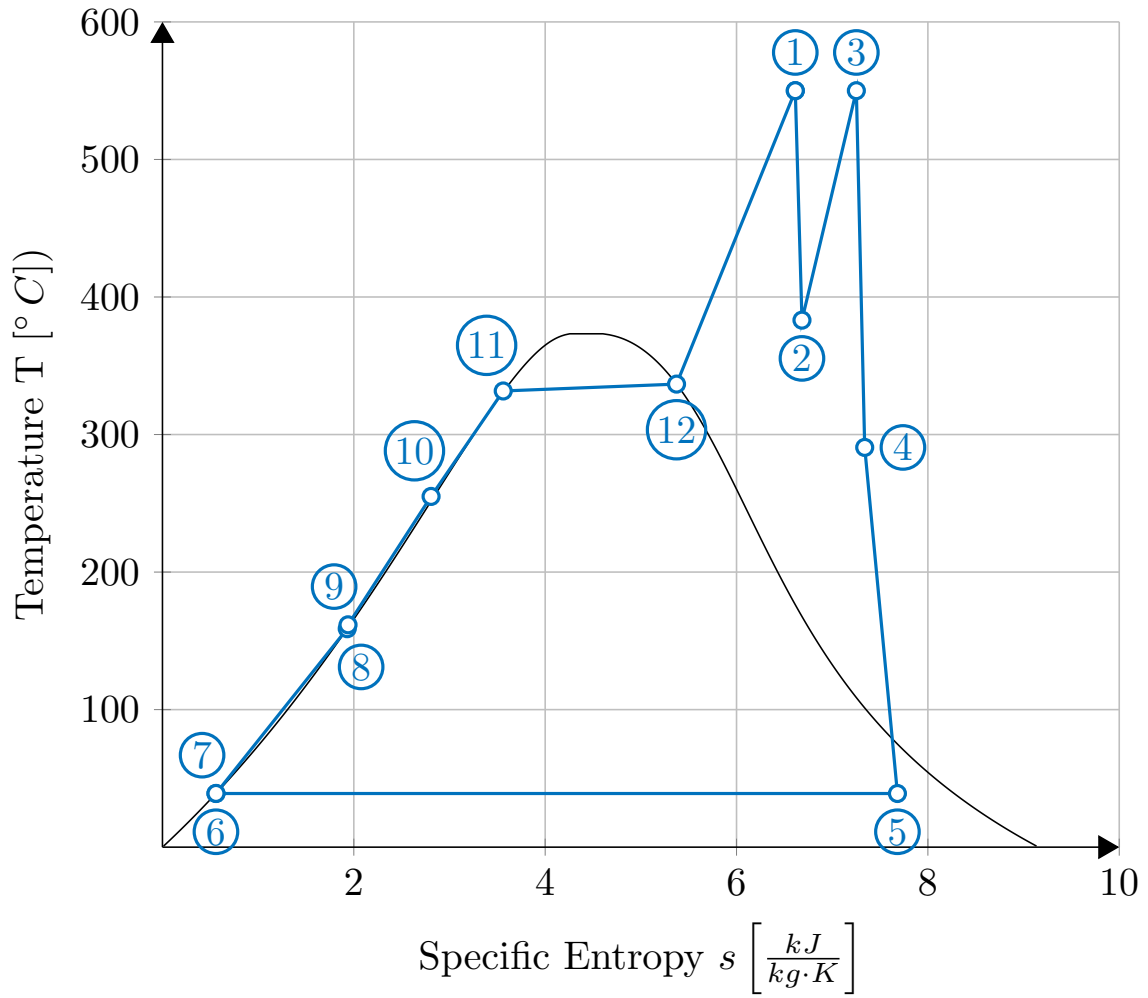
Point 1)

Point 1 is at *high pressure turbine inlet*. We know its temperature $T_1 = 550\,°C$. We do not directly have the pressure p_1 but we can obtain it from the pressure drop at the super heater(SH) and evaporation pressure:

$$p_1 = p_{evap} \cdot \left(1 - \left.\frac{\Delta p}{p}\right|_{SH}\right) = 128.8\,bar \tag{1}$$

Now with the two parameters of temperature and pressure we can evaluate with Mollier diagram also entropy and enthalpy functions of temperature and pressure:

T_1	p_1	h_1	s_1
$550\,°C$	$128.8\,bar$	$3472.6\,\frac{kJ}{kg}$	$6.6142\,\frac{kJ}{kg \cdot K}$

Point 2)

Point 2 is at *high pressure turbine outlet*. We know its pressure $p_2 = 42\,bar$. We do not directly have the temperature T_2 but we can obtain it from the definition of efficiency of the turbine:

$$\eta_T^{isoentropic} = \frac{h_1 - h_2}{h_1 - h_2^{IS}} = 87\% \tag{2}$$

We don't know h_2^{IS} but we can compute it from point 1 to point 2^{IS} considering an *isoentropic process*:

$$\begin{cases} p_2^{IS} = p_2 = 42\,bar \\ s_2^{IS} = s_1 = 6.61\,\frac{kJ}{kg \cdot K} \end{cases}$$

Now we can obtain from Mollier diagram $h_2^{IS} = h(p_2^{IS},\ s_2^{IS}) = 3125.3\,\frac{kJ}{kg}$.
Reversing equation 2 we can get h_2:

$$h_2 = h_1 - \eta_T^{iso} \cdot \left(h_1 - h_2^{IS}\right) = 3170.5\,\frac{kJ}{kg} \tag{3}$$

With Mollier diagram we finally evaluate $s_2 = s(p_2,\ h_2)$ and $T_2 = T(p_2,\ h_2)$ of point 2:

T_2	p_2	h_2	s_2
$383.165\,°C$	$42\,bar$	$3170.5\,\frac{kJ}{kg}$	$6.6839\,\frac{kJ}{kg \cdot K}$

Point 3)

Point 3 is at *intermediate pressure turbine inlet*. We know its temperature $T_3 = T_1 = 550\,°C$. We do not directly have the pressure p_2 but we can obtain it from the pressure drop at the reheater(RH) and pressure at point 2:

$$p_3 = p_2 \cdot \left(1 - \left.\frac{\Delta p}{p}\right|_{RH}\right) = 38.64\,bar \tag{4}$$

Now with the two parameters of Temperature and pressure we can evaluate with Mollier diagram also entropy and enthalpy functions of temperature and pressure:
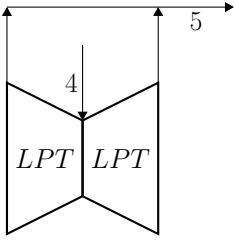
T_3	p_3	h_3	s_3
$550\,°C$	$38.64\,bar$	$3561.5\,\frac{kJ}{kg}$	$7.2524\,\frac{kJ}{kg \cdot K}$

Point 4)

Point 4 is at *intermediate pressure turbine outlet* and *low pressure turbine inlet*. We know its pressure $p_4 = 6\,bar$. We do not directly have the temperature T_2 but we can obtain it from the definition of efficiency of the turbine:

$$\eta_T^{isoentropic} = \frac{h_3 - h_4}{h_3 - h_4^{IS}} = 91.5\% \tag{5}$$

We don't know h_4^{IS} but we can compute it from point 3 to point 4^{IS} considering an *isoentropic process*:

$$\begin{cases} p_4^{IS} = p_4 = 6\,bar \\ s_4^{IS} = s_3 = 7.2524\,\frac{kJ}{kg \cdot K} \end{cases}$$

Now we can obtain from Mollier diagram $h_4^{IS} = h(p_4^{IS},\ s_4^{IS}) = 2994.4\,\frac{kJ}{kg}$.
Reversing equation 5 we can get h_4:

$$h_4 = h_3 - \eta_T^{iso} \cdot \left(h_3 - h_4^{IS}\right) = 3042.6\,\frac{kJ}{kg} \tag{6}$$

With Mollier diagram we finally evaluate $s_4 = s(p_4,\ h_4)$ and $T_4 = T(p_4,\ h4)$ of point 4:

T_4	p_4	h_4	s_4
$290.63\,°C$	$6\,bar$	$3042.6\,\frac{kJ}{kg}$	$7.3397\,\frac{kJ}{kg \cdot K}$

Point 5)

Point 5 is at *low pressure turbine outlet*. We know its pressure because is the condensation pressure $p_5 = p_{COND} = 0.07\,bar$. We do not directly have the temperature T_5 but we can obtain it like point 2 and 4 from the definition of efficiency of the turbine:

$$\eta_T^{isoentropic} = \frac{h_4 - h_5}{h_4 - h_5^{IS}} = 86\% \tag{7}$$

We don't know h_5^{IS} but we can compute it from point 4 to point 5^{IS} considering an *isoentropic process*:

$$\begin{cases} p_5^{IS} = p_5 = 0.07\,bar \\ s_5^{IS} = s_4 = 7.3397\,\frac{kJ}{kg\cdot K} \end{cases}$$

Now we can obtain from Mollier diagram $h_5^{IS} = h(p_5^{IS},\ s_5^{IS}) = 2279.9\,\frac{kJ}{kg}$.
Reversing equation 7 we can get h_5:

$$h_5 = h_4 - \eta_T^{iso} \cdot \left(h_4 - h_5^{IS} \right) = 2.3867\,\frac{kJ}{kg} \tag{8}$$

Point 5 has a very low pressure and it fall in liquid-vapour region. So it is significant also to calculate the title of vapour. With Mollier diagram we finally evaluate $s_5 = s(p_5,\ h_5)$, $T_5 = T(p_5,\ h_5)$ and $\chi_5 = \chi(p_5,\ h_5)$ of point 5:

T_5	p_5	h_5	s_5	χ_5
$39.0009\,^\circ C$	$0.07\,bar$	$2386.7\,\frac{kJ}{kg}$	$7.6817\,\frac{kJ}{kg\cdot K}$	0.9232

Point 6)

Point 6 is at *condenser outlet*. We know that when the flow goes out from the condenser is is a saturated liquid, and that the condenser is approximately isobaric, so we have $p_6 = p_5 = 0.07\,bar$. We can get all missing properties from saturated liquid table: $T_6 = T_{sat}^{liq}(p_6)$, $h_6 = h_{sat}^{liq}(p_6)$, $s_6 = s_{sat}^{liq}(p_6)$ and $\rho_6 = \rho_{sat}^{liq}(p_6)$.

T_6	p_6	h_6	s_6	ρ_6
$39.0009\,^\circ C$	$128.8\,bar$	$163.3655\,\frac{kJ}{kg}$	$0.5591\,\frac{kJ}{kg\cdot K}$	$992.55\,\frac{kg}{m^3}$

Point 7)

Point 7 is at *the outlet of the first pump* and *deaerator inlet*. We know its pressure because point 7 is in contact with point 4^{extr} through the isobaric deaerator, so $p_7 = p_4^{extr} = p_4 = 6\,bar$. We do not directly have the enthalpy h_7 but we can obtain it from the definition of efficiency of the pump:

$$\eta_{p_1}^{hydraulic} = \frac{h_7^{IS} - h_6}{h_7 - h_6} = 84\% \tag{9}$$

We don't know h_7^{IS} and we can get it in two ways:

- with subcooled liquid tables or tools considering an *isoentropic process* between 6 and 7^{IS} considering that

$$\begin{cases} p_7^{IS} = p_7 = 6\,bar \\ s_7^{IS} = s_6 = 0.5591\,\frac{kJ}{kg\cdot K} \end{cases}$$

- estimating $h_7^{IS} - h_6 \approx \frac{p_7 - p_6}{\rho_6}$ with hypothesis of incompressible flow and density constant with temperature and pressure.

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Reverting equation 9 with the second stated approach we get:

$$h_7 = h_6 + \frac{p_7 - p_6}{\rho_6 \cdot \eta_{hyd}} = 164.08 \frac{kJ}{kg} \tag{10}$$

With the same hypothesis we evaluate T_7 and s_7:

$$du = dh - \frac{dp}{\rho} = c_L \cdot dT, \tag{11}$$

$$\Delta T = \frac{\Delta u}{c_L} = T_7 - T_6 \tag{12}$$

$$\Delta u = \Delta h - \frac{\Delta p}{\rho} = \frac{\Delta h^{IS}}{\eta_{hyd}} - \frac{\Delta p}{\rho} \tag{13}$$

$$\approx \frac{\Delta p}{\rho \cdot \eta_{hyd}} - \frac{\Delta p}{\rho} = \frac{\Delta p}{\rho} \cdot \left(\frac{1}{\eta_{hyd}} - 1 \right) \tag{14}$$

$$T_7 = T_6 + \frac{\Delta u}{c_L} = T_6 + \frac{\Delta p}{\rho \cdot c_L} \cdot \left(\frac{1}{\eta_{hyd}} - 1 \right) \tag{15}$$

$$Tds = du + p\not{dv} = c_L \cdot DT \tag{16}$$

$$ds = c_L \cdot \frac{dT}{T} \tag{17}$$

$$\Delta s = s_7 - s_6 = c_L \cdot \log \left(\frac{dT}{T} \right) \tag{18}$$

$$s_7 = s_6 + c_L \cdot \log \left(\frac{dT}{T} \right) \tag{19}$$

Finally we find:

T_7	p_7	h_7	s_7
$39.0607\,°C$	$6\,bar$	$164.08\,\frac{kJ}{kg}$	$0.5597\,\frac{kJ}{kg \cdot K}$

Point 8)

Point 8 is at *outlet of deaerator*. We know that when the flow goes out from the deaerator is is a saturated liquid, and that deareator is approximately isobaric, so we have $p_8 = 6\,bar$. We can get all missing properties from saturated liquid table: $T_8 = T_{sat}^{liq}(p_8)$, $h_8 = h_{sat}^{liq}(p_8)$, $s_8 = s_{sat}^{liq}(p_8)$ and $\rho_8 = \rho_{sat}^{liq}(p_8)$.

T_8	p_8	h_8	s_8	ρ_8
$158.83\,°C$	$128.8\,bar$	$670.50\,\frac{kJ}{kg}$	$1.9311\,\frac{kJ}{kg \cdot K}$	$908.5657\,\frac{kg}{m^3}$

Point 9)

Point 9 is at *the outlet of the second pump*. We can simply evaluate its pressure because we know the pressure drop in economizer and that RB is approximately isobaric. So:

$$p_9 = \frac{p_{EVAP}}{1 - \frac{\Delta p}{p}\Big|_{ECO}} = 175\,bar \tag{20}$$

We do not directly have the enthalpy h_9 but we can obtain it from the definition of efficiency of the pump and with the same hypothesis of equation 10

$$h_9 = h_8 + \frac{p_9 - p_8}{\rho_8 \cdot \eta_{hyd}} = 692.645 \, \frac{kJ}{kg} \tag{21}$$

And in the same way of point 7 we can estimate T_9 and s_9 from equations 15 and 19:

$$T_9 = T_8 + \frac{\Delta u}{c_L} \tag{22}$$

$$s_9 = s_8 + c_L \cdot \log\left(\frac{dT}{T}\right) \tag{23}$$

Finally we find:

T_9	p_9	h_9	s_9
$161.6521\,^\circ C$	$175\,bar$	$692.6450\,\frac{kJ}{kg}$	$1.9396\,\frac{kJ}{kg\cdot K}$

Point 10)

Point 10 is at *regenerator outlet* and *economizer inlet*. We have from data $T_{10} = 255\,^\circ C$ and moreover $P_9 = P_{10} = 175\,bar$ assuming the regenerator approximately isobaric. From the regenerator the flow is a subcooled liquid. We have the two main properties so we are able with Mollier diagram for subcooled liquid to find all the missing properties needed. Alternatively we may use the following approximations:

$$\begin{cases} h_{10} \approx h_{sat}^{liq}(T_{10}) + v_{sat}^{liq}(T_{10}) \cdot (p_{10} - p_{sat}(T_{10})) & (24) \\ s_{10} \approx s_{sat}^{liq}(T_{10}) & (25) \end{cases}$$

T_{10}	p_{10}	h_{10}	s_{10}
$255\,^\circ C$	$175\,bar$	$1109.9\,\frac{kJ}{kg}$	$2.8074\,\frac{kJ}{kg\cdot K}$

Point 11)

Point 11 is at *economizer outlet*. We have from data $p_{11} = p_{EVAP} = 140\,bar$ and the temperature is $5\,^\circ C$ lower than evaporation temperature so $T_{11} = T_{EVAP} - 5\,^\circ C = T_{sat}(p_{EVAP}) - 5\,^\circ C = 331.67\,^\circ C$. As point 10, point 11's flow is subcooled liquid than we can use approximation of equations 24 and 25 to compute all properties:

T_{11}	p_{11}	h_{11}	s_{11}
$331.67\,^\circ C$	$140\,bar$	$1533.6\,\frac{kJ}{kg}$	$3.5616\,\frac{kJ}{kg\cdot K}$

Point 12)

Point 12 is at *evaporator outlet* and *superheat inlet*. We know that when the flow goes out from the evaporator it is saturated vapour, so we have $p_{12} = p_{EVAP} = 140\,bar$. We can get all missing properties from saturated vapour table: $T_{12} = T_{sat}^{vap}(p_{12})$, $h_{12} = h_{sat}^{vap}(p_{12})$, $s_{12} = s_{sat}^{vap}(p_{12})$ and $\rho_{12} = \rho_{sat}^{vap}(p_{12})$.

T_{12}	p_{12}	h_{12}	s_{12}	ρ_{12}
$336.67\,^\circ C$	$140\,bar$	$2638.1\,\frac{kJ}{kg}$	$5.3730\,\frac{kJ}{kg\cdot K}$	$87.03\,\frac{kg}{m^3}$

Figure 2: Regenerator flow Scheme.
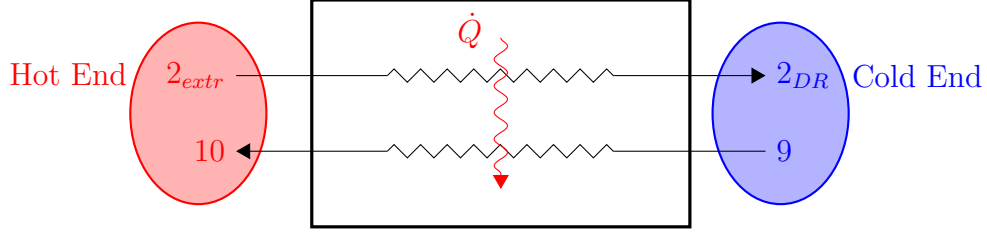
Point 2_{DR})

Point 2_{DR} is at *regenerator outlet* and *deaerator inlet*. We have from data $T_{2_{DR}} = T_9 - 5\,°C = 166.65\,°C$. Assuming that the regenerator is isobaric we have that $p_{2_{DR}} = p_{4_{extr}} = p_4 = 42\,bar$. From the regenerator the flow is a subcooled liquid. We have the two main properties so we are able with Mollier diagram for subcooled liquid to find all the missing properties needed. Alternatively we may use the following approximations like point 7 and 10:

$$
\begin{cases}
h_{10} \approx h_{sat}^{liq}(T_{10}) + v_{sat}^{liq}(T_{10}) \cdot (p_{10} - p_{sat}(T_{10})) & (26) \\
s_{10} \approx s_{sat}^{liq}(T_{10}) & (27)
\end{cases}
$$

$T_{2_{DR}}$	$p_{2_{DR}}$	$h_{2_{DR}}$	$s_{2_{DR}}$
$166.65\,°C$	$42\,bar$	$706.51\,\frac{kJ}{kg}$	$2.0046\,\frac{kJ}{kg\cdot K}$

Table 1: Thermodynamical properties of all points.

Point	T [°C]	p [bar]	h $\left[\frac{kJ}{kg}\right]$	s $\left[\frac{kJ}{kg\cdot K}\right]$	χ
1	550	128.80	3472.63	6.61	1
2	383.16	42	3170.47	6.68	1
3	550	38.64	3561.50	7.25	1
4	290.63	6	3042.56	7.34	1
5	39.00	0.07	2386.71	7.68	0.92
6	39.00	0.07	163.37	0.56	0
7	39.06	6	164.08	0.56	0
8	158.83	6	670.50	1.93	0
9	161.65	175	692.64	1.94	0
10	255	175	1109.92	2.81	0
11	331.67	140	1533.59	3.56	0
12	336.67	140	2638.09	5.37	1
2_{DR}	166.65	42	706.51	2.00	0

1.B   Calculation of the mass flow rates of each mass stream of the steam cycle

In order to evaluate the mass flow rates $\dot{m}_1$ and $\dot{m}_{extr}$, we need to apply the mass conservation law in combination with the energy balance to some section (control volume) of the steam cycle (remembering that we deal with an open system in steady state process).

- With energy balance at the boiler and energy balance at the regenerator we can write:

$$\begin{cases} \dot{H}_{SH} + \dot{H}_{EVAP} + \dot{H}_{ECO} + \dot{H}_{RH} = \dot{Q}^{IN}_{cycle} & (28) \\ \dot{H}_{2 \to 2_{DR}} = \dot{H}_{9 \to 10} & (29) \end{cases}$$

$$\begin{cases} \dot{m}_1(\overbrace{h_1 - h_{12}}^{SH} + \overbrace{h_{12} - h_{11}}^{EVAP} + \overbrace{h_{11} - h_{10}}^{ECO}) + \dot{m}_2(\overbrace{h_3 - h_2}^{RH}) = \dot{Q}^{IN}_{cycle} & (30) \\ \dot{m}_{2_{extr}}(h_{2_{extr}} - h_{2DR}) = \dot{m}_1(h_{10} - h_9) & (31) \\ \dot{m}_1 = \dot{m}_2 + \dot{m}_{2_{extr}} & (32) \end{cases}$$

We have computed all the enthalpy before so we can solve the linear system and we are able to find all the 3 unknown mass flow rates:

$\dot{m}_1$	$\dot{m}_2$	$\dot{m}_{2_{extr}}$
$372.09 \frac{kg}{s}$	$309.08 \frac{kg}{s}$	$63.01 \frac{kg}{s}$

- With energy balance at the reaerator and mass flow rates balance at the reaerator we can write:
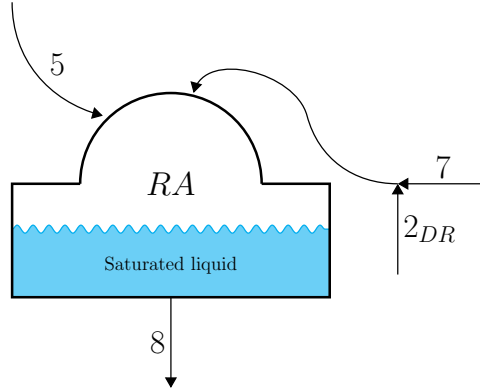

Figure 3: Reaerator flow scheme.

$$\begin{cases} \dot{m}_8 h_8 = \dot{m}_{4_{extr}} h_{4_{extr}} + \dot{m}_7 h_7 + \dot{m}_{2_{DR}} h_{2_{DR}} & (33) \\ \dot{m}_7 = \dot{m}_1 - \dot{m}_{2_{DR}} - \dot{m}_{4_{extr}} & (34) \end{cases}$$

Where the only unknowns are $\dot{m}_{4_{extr}}$ and $\dot{m}_7$. We have already computed enthalpy of every point so we can solve the linear system and we are able to find all the 2 unknown mass flow rates:

$\dot{m}_7$	$\dot{m}_{4_{extr}}$
$255.49 \frac{kg}{s}$	$53.59 \frac{kg}{s}$

1.C   Evaluation of cycle performances

We can compute all the performance parameters for the steam cycle:

- Blade power output of the turbine (that is the inner power output, or equivalently the turbine power without mechanical losses):

$$\dot{W}_T = \dot{m}_1(h_1 - h_2) + \dot{m}_2(h_3 - h_4) + \dot{m}_4(h_4 - h_5) = 440.39 \, kW \qquad (35)$$

- Blade power output of the condensate extraction pump:

$$\dot{W}_{P1} = \dot{m}_6(h_7 - h_6) = 181.715 \, kW \tag{36}$$

- Blade power output of the feedwater pump:

$$\dot{W}_{P2} = \dot{m}_8(h_9 - h_8) = 8.240 \, kW \tag{37}$$

- Thermal power rejected from the condenser:

$$\dot{Q}_{condenser} = \dot{m}_6(h_5 - h_6) = 568.04 \, kW \tag{38}$$

- Gross electrical power output (Steam Turbine), that is the electric power at the electric generator, accounting for the mechanical and electric conversion losses

$$\dot{W}_T^{gross} = \dot{W}_T \cdot \eta_T^M \cdot \eta_T^E = 425.10 \, kW \tag{39}$$

- Net electrical power output of the cycle, which accounts for the electric consumption not only in the steam cycle (condenser and feedwater pumps) but also plant auxiliaries electric consumption W_{aux} (fans supplying air and evacuating flue gases to the stack, heat rejection, etc.):

$$\dot{W}_{cycle}^{net} = \dot{W}_T^{gross} - \frac{\dot{W}_{P1}}{\eta_{P1}^M} - \frac{\dot{W}_{P2}}{\eta_{P2}^M} = 415.75 \, kW \tag{40}$$

- Thermodynamic efficiency of the cycle, which is defined as the ratio between the net fluid power of the cycles before any mechanical losses and the thermal power input into the cycle:

$$\eta_{cycle} = \frac{\dot{W}_T - \dot{W}_{P1} - \dot{W}_{P1}}{\dot{Q}_{cycle}^{IN}} = 43.20\% \tag{41}$$

- Net electrical power output of the power plant:

$$\dot{W}_{plant}^{net} = \dot{W}_{cycle}^{net} - 3.2\% \cdot \dot{W}_T^{gross} = 402.14 \, kW \tag{42}$$

- Net electrical efficiency of the power plant, which is defined referred to the thermal input to the plant (associated to the fuel) as:

$$\eta_{plant} = \eta_{boiler} \cdot \frac{\dot{W}_{plant}^{net}}{\dot{Q}_{cycle}^{IN}} = 38.16\% \tag{43}$$

1.D Representation of T-Q diagram of the surface feedwater pre-heater RB

At this point, only the inlet and outlet thermodynamic conditions of both of the streams are known. In order to evaluate the T-Q profiles (temperature-heat exchanged) of hot ($2_{extr} - 2_{DR}$) and cold (9-10) streams, three assumptions can be outlined:

1. The heat exchanger is supposed to be arranged in a *countercurrent* configuration;

2. *Three different sections* can be identified along the feedwater preheater:

    (a) De-superheating of the bled steam (RB_I);

(b) Condensation of the bled stream (RB_{II});

(c) Sub-cooling of the bled stream (RB_{II});

3. In each section, we suppose that the streams have a *constant thermal capacity*.

$$\dot{Q}^I_{RB} = \dot{m}_9 \cdot (h_{10} - h_{9b}) = \dot{m}_9 \cdot \overline{c_p} \cdot (T_{10} - T_{9b}) = \dot{m}_{2_{extr}} \cdot (h_{2_{extr}} - h^{vap}_{sat}(p_2)) = 23.35\,kW \quad (44)$$

But with the assumption of c_p approximately constant we can evaluate a medium value between 9 and 10:

$$\dot{Q}_{RB} = \dot{m}_9 \cdot (h_{10} - h_9) = \dot{m}_9 \cdot \overline{c_p} \cdot (T_{10} - T_9) = 155.27\,kW \quad (45)$$

$$\dot{m}_9 \cdot \overline{c_p} = \frac{\dot{Q}_{RB}}{T_{10} - T_9} = 1663.3\,\frac{kJ}{kg} \quad (46)$$

$$T_{9b} = T_{10} - \frac{\dot{Q}^I_{RB}}{\dot{m}_9 \cdot \overline{c_p}} = T_{10} - \frac{\dot{m}_{2_{extr}} \cdot (h_{2_{extr}} - h^{vap}_{sat}(p_2))}{\dot{m}_9 \cdot \overline{c_p}} = 240.96\,°C \quad (47)$$

We can similarly proceed from point b to point a:

$$\dot{Q}^{II}_{RB} = \dot{m}_9 \cdot (h_{9b} - h_{9a}) = \dot{m}_9 \cdot \overline{c_p} \cdot (T_{9b} - T_{9a}) = \dot{m}_{2_{extr}} \cdot (h^{vap}_{sat}(p_2) - h^{liq}_{sat}(p_2)) = 10.70\,kW \quad (48)$$

$$T_{9a} = T_{9b} - \frac{\dot{Q}^{II}_{RB}}{\dot{m}_9 \cdot \overline{c_p}} = T_{10} - \frac{\dot{m}_{2_{extr}} \cdot (h^{vap}_{sat}(p_2) - h^{liq}_{sat}(p_2))}{\dot{m}_9 \cdot \overline{c_p}} = 176.62\,°C \quad (49)$$

The only missing data is the temperature of saturation at p_2; with Mollier diagram we can find it: $T_{sat} = 253.27\,°C$

Now we can plot Q-T diagram: