

PRECEPT 1: Energy balance of a steam power plant

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The aim of the precept is to analyse, from the energy and environmental points of view, the use of two different fuels in a boiler.

There are 2 different fuel possibilities:

1. Natural Gas
2. Coal

1 Problem

Natural Gas Data

- Excess Air = 16%
- Losses (unburned fuel and radiation losses in boiler) = 0.7% of LHV
- $\Delta p_{fan} = 250 \text{ mm}_{H_2O}$
- T_{FG} at the stack = $110^\circ C$
- T_{FG} at the boiler output = $300^\circ C$
- Molar fraction of every molecule

Coal Data

- Excess Air = 29%
- Losses (unburned fuel and radiation losses in boiler) = 0.7% of LHV
- $\Delta p_{fan} = 350 \text{ mm}_{H_2O}$
- $C_{ash} = 2 \frac{KJ}{KgK}$
- T_{ash} at the outlet of the boiler = $400^\circ C$
- T_{FG} at the boiler output must be = $25^\circ C$ higher than the acid dew temperature ¹ that must be evaluated considering that 2.5% of SO_2 oxidises to SO_3
- Mass fraction of every element
- $LHV = 24.826 \frac{MJ}{Kg}$

$$\begin{aligned} \frac{1}{T_{H_2SO_4}} \frac{1000}{K} &= 2.276 - 0.0294 \log(p_{H_2O}) + 0.0858 \log(p_{SO_3}) - 0.00062 \log(p_{H_2O}) \log(p_{SO_3}) \\ \frac{1}{T_{H_2SO_3}} \frac{1000}{K} &= 3.9526 - 0.1863 \log(p_{H_2O}) + 0.000867 \log(p_{SO_2}) - 0.000913 \log(p_{H_2O}) \log(p_{SO_2}) \end{aligned}$$

Other Useful Common Data

- Output power $\dot{Q}_u = 1000 MW$
- Air composition
- Fan efficiency $\eta_{fan} = 0.8$
- Electric efficiency $\eta_e = 0.9$
- $T_{ambient} = 15^\circ C$
- $p_{ambient} = 1.013 \text{ bar}$

Requests

1. Natural Gas

- (a) LHV
- (b) Flue gas composition at the outlet of the boiler
- (c) Efficiency of boiler considering ambient conditions
- (d) Fan power consumption
- (e) T_{air} at Ljungstrom outlet
- (f) Flame adiabatic temperature

2. Coal

- (a) Flue gas composition at the outlet of the boiler
- (b) Efficiency of boiler considering ambient conditions
- (c) Fan power consumption

2 Solution

1.a LHV of natural gas

Enthalpy for each reaction is defined as:

$$\Delta \tilde{H}_{reaction}^\circ = \sum_{j=1}^m \nu_{P_j} \Delta \tilde{H}_{f,P_j} - \sum_{i=1}^n \nu_{R_i} \Delta \tilde{H}_{f,R_i} \quad (1)$$

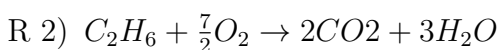
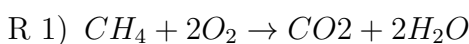
Lower heating value is:

$$LHV = -\frac{\Delta \tilde{H}_{reaction}^\circ}{MM_{fuel}} \quad (2)$$

Where $MM_{fuel} = \sum_{i=1}^{n_{species \text{ in fuel}}} \chi_i \times MM_i = 17.36 \frac{Kg}{Kmol}$

$$MM_{air} = \sum_{i=1}^{n_{species \text{ in air}}} \chi_i \times MM_i = 28.85 \frac{Kg}{Kmol}.$$

In natural gas we have just 2 reactants species, CH_4 and C_2H_6 and the corresponding reactions and $\Delta \tilde{H}_{reaction}^\circ$ are



$$R\ 1) \ \Delta \tilde{H}_{reaction}^{\circ} \cong -802.308 \frac{KJ}{Kg}$$

$$R\ 2) \ \Delta \tilde{H}_{reaction}^{\circ} \cong -1427.863 \frac{KJ}{Kg}$$

Then calculating a weighted sum respect of χ_i we get $\Delta \tilde{H}_{reaction}^{\circ}$ of the fuel

$$\Delta \tilde{H}_{reaction}^{\circ} = \sum_{i=1}^{species} \chi_i \times \Delta \tilde{H}_{reaction_i} = -807.749 \frac{MJ}{Kg} \quad (3)$$

Finally LHV of the fuel equals to

$$LHV = -\frac{\Delta \tilde{H}_{reaction}^{\circ}}{MM_{fuel}} = 46.52 \frac{MJ}{Kg} \quad (4)$$

1.b Flue gas composition at the outlet of the boiler

To know flue gas composition first we need to compute $\beta_{O_2}^{Stoichiometric}$ of each reaction that is the required moles of O_2 to have a stoichiometric oxidation reaction.

$$\beta_{O_2}^{Stoichiometric} = N_C + \frac{N_H}{4} + N_S - N_{O_2} \quad (5)$$

Where N_i is the number of atoms of i in the fuel.

	N_C	N_H	N_S	N_{O_2}
CH_4	1	4	0	0
C_2H_6	2	6	0	0

The total $\beta_{O_2}^{Stoichiometric}$ is the averaged balance between the two hydrocarbon molecules in the fuel.

$$\beta_{O_2}^{Stoichiometric} = \beta_{O_2}^{CH_4} \times \chi_{CH_4} + \beta_{O_2}^{C_2H_6} \times \chi_{C_2H_6} = 2.01 \quad (6)$$

Now we can evaluate the real β_{O_2} with excess air:

$$\beta_{O_2} = (1 + \varepsilon) \times \beta_{O_2}^{ST} = 2.332 \quad (7)$$

We can write the complete oxidation equation:

$$0.9CH_4 + 0.06C_2H_6 + 0.04N_2 + \beta_{O_2} \times (O_2 + \frac{\chi_{N_2}}{\chi_{O_2}}N_2 + \frac{\chi_{Ar}}{\chi_{O_2}}Ar + \frac{\chi_{H_2O}}{\chi_{O_2}}H_2O) = aCO_2 + bH_2O + cN_2 + dAr + eO_2 \quad (8)$$

Where a, b, c, d and e are the unknowns. Balancing the reaction we can get them:

	$a (CO_2)$	$b (H_2O)$	$c (N_2)$	$d (Ar)$	$e (O_2)$
n	1.02	2.096	8.747	0.109	0.322

Finally we obtain χ and y of each species in the flue gases remembering that $\chi_i = \frac{n_i}{\sum n_i}$ and

$$y_i = \chi_i \times \frac{MM_i}{MM_{FG}} \text{ where } MM_{FG} = \sum_{i=1}^{n_{species \text{ in flue gases}}} \chi_i \times MM_i = 27.8461 \frac{Kg}{Kmol}.$$

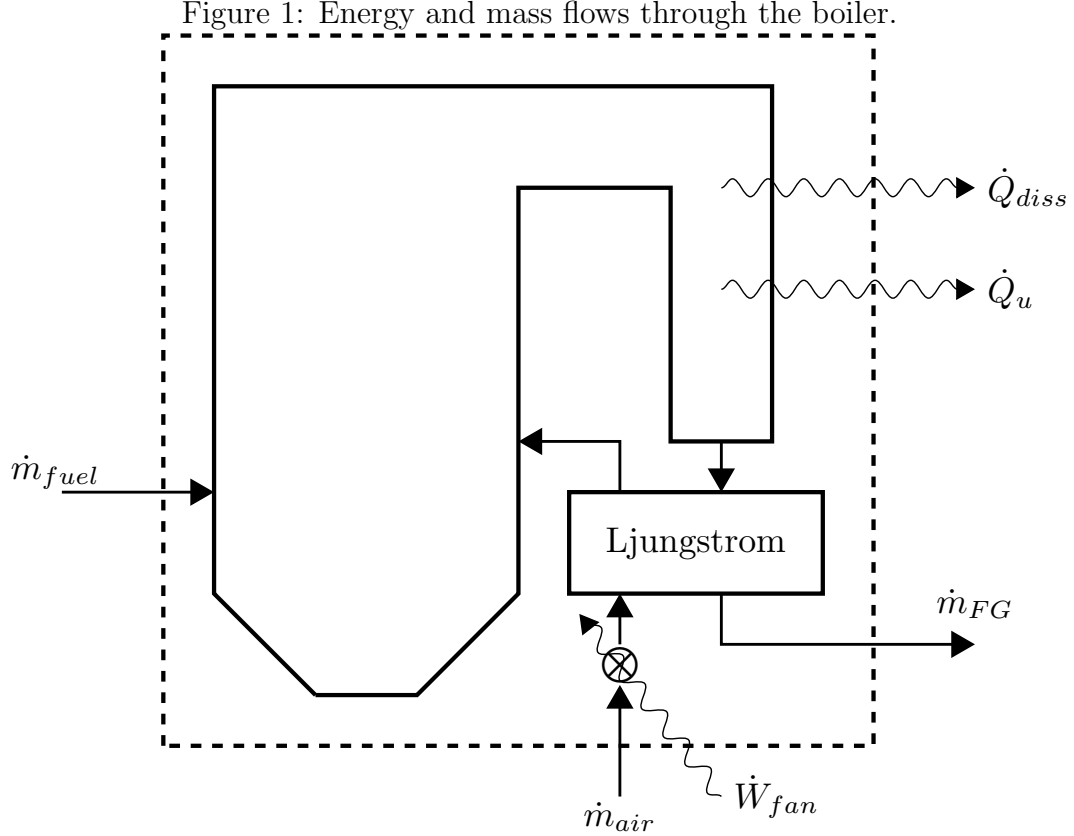
	CO_2	H_2O	N_2	Ar	O_2
χ	8.30%	17.05%	71.15%	0.89%	2.62%
y	13.11%	11.03%	71.58%	1.27%	3.01%

1.c Efficiency of boiler

The efficiency is defined as the useful output over the total input, or in formulas it is:

$$\eta_{Boiler} = \frac{\dot{Q}_u}{\dot{m}_{Fuel} \times LHV} \quad (9)$$

But if \dot{Q}_u is a data of the problem and it equals to 1 GW \dot{m}_{Fuel} is unknown. To evaluate it is necessary to compute an energy balance over the total boiler.



Writing the total energy balance equation neglecting potential and kinetic energies contribution we get:

$$\dot{H}_{air} + \dot{H}_{fuel} + \dot{m}_{Fuel} \cdot LHV + \dot{W}_{fan} = \dot{H}_{FG} + \dot{Q}_u + \dot{Q}_{diss} \quad (10)$$

To simplify the expression we define $\alpha = \dot{m}_{AIR}/\dot{m}_{fuel}$ so that we can write \dot{m}_{AIR} , \dot{m}_{fuel} and \dot{m}_{FG} function of α and \dot{m}_{fuel} .

\dot{m}	Air	Fuel	Flue gas
\dot{m}_{fuel}	α	1	$1 + \alpha$

$$\alpha \cdot \dot{m}_{fuel} \cdot h_{air} \Big|_{T_{ref}}^{T_{air}^{IN}} + \dot{m}_{fuel} \cdot h_{fuel} \Big|_{T_{ref}}^{T_{fuel}^{IN}} + (1 - \eta_{loss}) \cdot \dot{m}_{Fuel} \cdot LHV + \dot{W}_{fan} = (1 + \alpha) \cdot \dot{m}_{fuel} \cdot h_{FG} \Big|_{T_{ref}}^{T_{FG}^{ST}} + \dot{Q}_u \quad (11)$$

First of all we can get α directly from β_{O_2} with the following formula:

$$\alpha = \frac{\dot{m}_{AIR}}{\dot{m}_{fuel}} = \beta_{O_2} \cdot \frac{1}{\chi_{O_2}^{air}} \cdot \frac{MM_{air}}{MM_{fuel}} = 18.7157 \quad (12)$$

We know the enthalpy of each of the species in air, fuel and flue gases, but we need to know the total enthalpy of the mixtures.

$$h_{species_i} = \int C_{p_i} dT = \int (A_i + B_i \cdot T) dT \quad (13)$$

$$h_{mix} = \int C_{p_{mix}} dT = \int (A_{mix} + B_{mix} \cdot T) dT \quad (14)$$

Where

$$A_{mix} = \sum_{i=1}^{species} A_i \cdot y_i$$

$$B_{mix} = \sum_{i=1}^{species} B_i \cdot y_i$$

	<i>Air</i>	<i>Fuel</i>	<i>Flue gas</i>
$A[\frac{J}{KgK}]$	952.2	1052.22	1012.39
$B[\frac{J}{Kg}]$	0.1874	0.1723	0.2756

Before proceeding to resolve the energy equation we have to choose if to consider \dot{W}_{fan} or to neglect it and then in case we neglect it we will need to verify if this hypothesis is close to reality. We consider *NOT* to neglect \dot{W}_{fan} . So we have to evaluate it, knowing that it depends also on \dot{m}_{fuel} that until now is unknown

$$\dot{W}_{fan} = \frac{\Delta p \cdot \dot{m}_{fuel}}{\eta_e \cdot \eta_{fan}} \cdot \left(\frac{\alpha}{\rho_{air}^{IN}} + \frac{1 + \alpha}{\rho_{FG}^{ST}} \right) \quad (15)$$

But we have Δp in mm_{H_2O} and we have to convert in Pa. Moreover we have to evaluate the density of flue gas and air:

$$\Delta p^{(Pa)} = g \cdot 250 mm_{H_2O} = 2452.5 Pa$$

$$\rho_{air} = \frac{MM_{air} \cdot p}{R \cdot T} = 1.2199 \frac{Kg}{m^3}$$

$$\rho_{FG} = \frac{MM_{FG} \cdot p}{R \cdot T} = 0.8640 \frac{Kg}{m^3}$$

Now the only unknown in \dot{W}_{fan} is the same as energy balance. Finally we solve equation 11 computing \dot{m}_{fuel} and therefore \dot{m}_{air} and \dot{m}_{FG}

	<i>Air</i>	<i>Fuel</i>	<i>Flue gas</i>
$\dot{m}[\frac{Kg}{s}]$	422.75	22.59	445.34

We have obtained the value of all our unknown so now we can easy evaluate η_{boiler} from equation 9 :

$$\eta_{Boiler} = \frac{\dot{Q}_u}{\dot{m}_{fuel} \times LHV} = 95.165\% \quad (16)$$

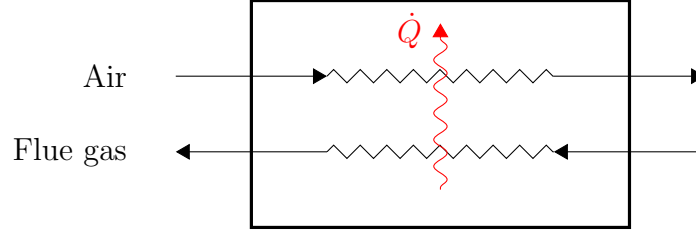
1.d Fan power consumption

In equation 15 we have obtained \dot{m}_{fuel} function of \dot{m}_{fuel} that before was unknown. Now instead we can correctly evaluate the power of fan:

$$\dot{W}_{fan} = \frac{\Delta p \cdot \dot{m}_{fuel}}{\eta_e \cdot \eta_{fan}} \cdot \left(\frac{\alpha}{\rho_{air}^{IN}} + \frac{1 + \alpha}{\rho_{FG}^{ST}} \right) = 2.94 MW \quad (17)$$

1.e T_{air} at Ljungstrom outlet

Figure 2: Ljungstrom flow Scheme.



We know T_{air}^{IN} , T_{FG}^{IN} , T_{FG}^{OUT} and we miss only T_{air}^{OUT} . We can get T_{air}^{OUT} with an energy balance over the Ljungstrom:

$$\dot{H}_{air} = \dot{m}_{air} \cdot \int_{T_{air}^{IN}}^{T_{air}^{OUT}} C_{p_{air}} dT = \dot{m}_{FG} \cdot \int_{T_{FG}^{IN}}^{T_{FG}^{OUT}} C_{p_{FG}} dT = \dot{H}_{FG} \quad (18)$$

Where IN and OUT refer to the inlet and the outlet of the Ljungstrom.

To solve the equation respect of our unknown we can or evaluate the integral and than solve the second order algebraic equation, or in term of generality we can simply us a numeric method like Secant one. We Matlab support we have considered to proceed with the second opportunity. Therefore we have obtained the following result:

$$T_{air}^{OUT} = 237.96^{\circ}C \quad (19)$$

So finally we can draw the temperature-heat plot

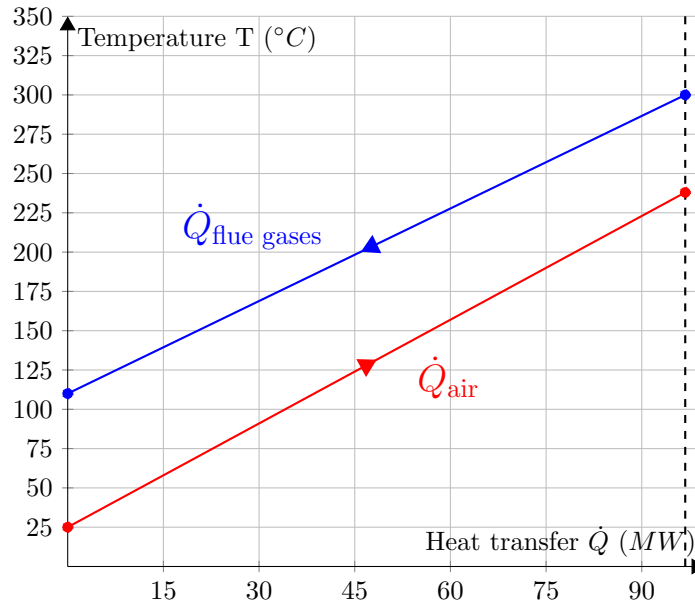


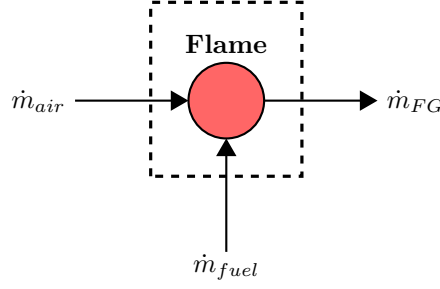
Figure 3: Ljungstrom temperature-heat exchanger plot.

In this analysis we have neglected the fact that the fan increase a bit the temperature of air.

1.f Flame adiabatic temperature

To compute the flame adiabatic temperature we need to make an energy balance around the flame: Because the request is about *adiabatic* temperature we do not consider losses.

Figure 4: Flame flow scheme.



$$\dot{H}_{air} + \dot{H}_{fuel} + \dot{m}_{fuel} \cdot LHV = \dot{H}_{FG} \quad (20)$$

$$\dot{m}_{air} \cdot h_{air} \Big|_{T_{ref}}^{T_{air}^{PH}} + \dot{m}_{fuel} \cdot \left(LHV + h_{fuel} \Big|_{T_{ref}}^{T_{fuel}^{IN}} \right) = \dot{m}_{FG} \cdot h_{fuel} \Big|_{T_{ref}}^{T_{flame}} \quad (21)$$

We can solve this equation numerically like equation 18. So we can get T_{flame} with secants method:

$$T_{flame} = 1924.54^\circ C \quad (22)$$

Solution for Coal firing

2.a Flue gas composition at the outlet of the boiler

In this case we have a solid fuel; we do not have the molecular composition of our fuel but from a *chemical analysis* we got the mass fraction of each atoms in coal. First of all it is necessary to evaluate for each atom the molar fraction respect to carbon. We take carbon as reference because we are sure to have it in every fossil fuel.

$$n_i = \left[\frac{Kmol s_i}{Kmol s_C} \right] = \frac{y_i}{y_C} \cdot \frac{MM_c}{MM_i} \quad (23)$$

Where y_i is the mass fraction of i .

Moreover the molar fraction respect of fuel of each atom (χ_i) is computed as:

$$\chi_i = y_i \cdot \frac{MM_{fuel}}{MM_i} \quad (24)$$

Where $MM_{fuel} = \sum_{i=1}^{n_{species \text{ in fuel}}} \chi_i \cdot MM_i = 12.22 \frac{Kg}{Kmol}$

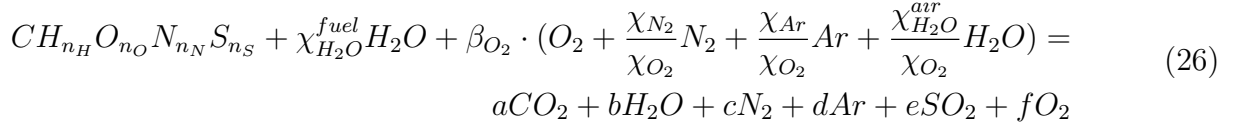
The results are reported in the following table.

	<i>C</i>	<i>H</i>	<i>O</i>	<i>N</i>	<i>S</i>	<i>Moisture</i>
<i>n</i>	1	0.912	0.108	0.0154	0.0208	0.131
χ	62.38%	56.90%	6.75%	0.96%	1.30%	8.145%

Before writing the complete equation of reaction we need to evaluate β_{O_2} , the required moles of O_2 to have a stoichiometric oxidation reaction:

$$\beta_{O_2} = (1 + \varepsilon) \cdot \beta_{O_2}^{Stoichiometric} = (1 + \varepsilon) \cdot \left(N_C + \frac{N_H}{4} + N_S - N_{O_2} \right) = \dots \quad (25)$$

Now we can write the complete equation of reaction; we have neglected SO_3 which has low molar fraction and it is significant only as regarding acids formation that corrode tubes:



Where a, b, c, d and e are the unknowns. Balancing the reaction we can get them:

	$a (CO_2)$	$b (H_2O)$	$c (N_2)$	$d (Ar)$	$e (SO_2)$	$f (O_2)$
n	1	0.667	5.771	0.0723	0.0209	0.347

Finally we can get χ and y of each species in the flue gases remembering that $\chi_i = \frac{n_i}{\sum n_i}$ and $y_i = \chi_i \times \frac{MM_i}{MM_{FG}}$

	CO_2	H_2O	N_2	Ar	SO_2	O_2
χ	12.69%	8.46%	73.26%	0.918%	0.265%	4.40%

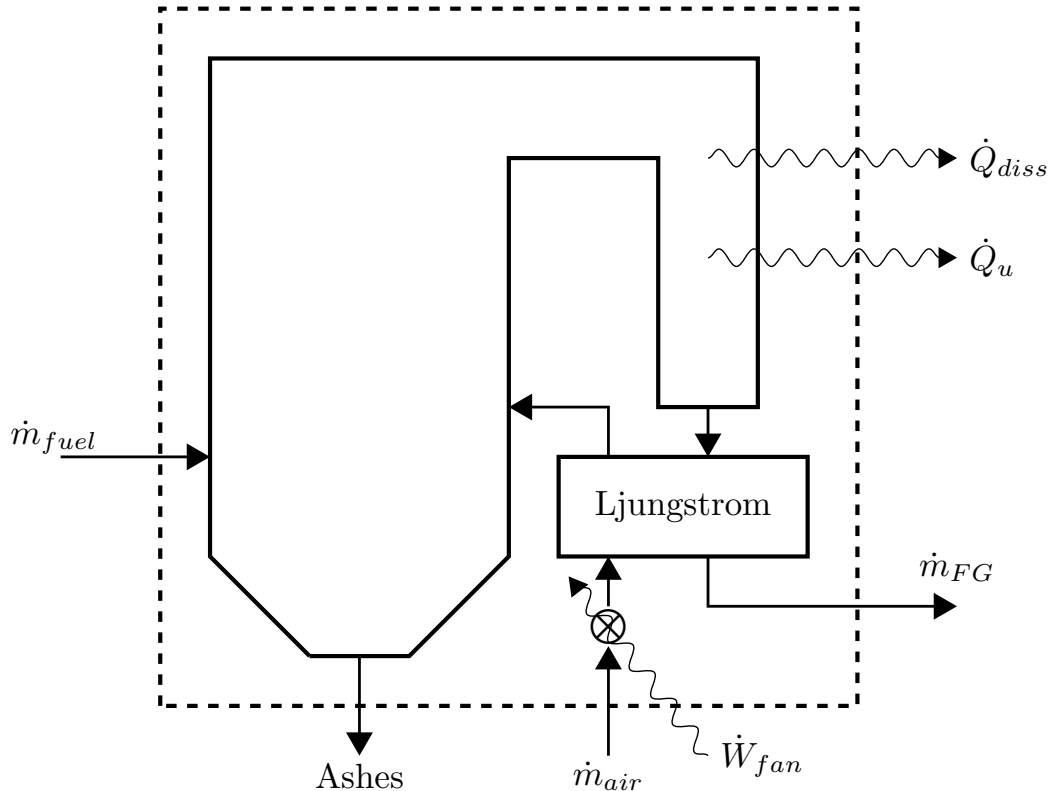
2.b Efficiency of boiler considering

The efficiency is defined as the useful output over the total input, or in formulas it is:

$$\eta_{Boiler} = \frac{\dot{Q}_u}{\dot{m}_{Fuel} \times LHV} \quad (27)$$

But if \dot{Q}_u is a data of the problem \dot{m}_{Fuel} is unknown. To evaluate it is necessary to compute an energy balance over the total boiler.

Figure 5: Energy and mass flows through the boiler.



Writing the total energy balance equation we get:

$$\dot{H}_{air} + \dot{H}_{fuel} + \dot{m}_{Fuel} \cdot LHV + \dot{W}_{fan} = \dot{H}_{FG} + \dot{H}_{ash} + \dot{Q}_u + \dot{Q}_{diss} \quad (28)$$

The ashes do not react with other species in the chemical reaction, but instead they are significant from the energy point of view because ashes absorb energy generated in the combustion process.

$$\begin{aligned} \alpha \cdot \dot{m}_{Fuel} \cdot h_{air} \Big|_{T_{ref}}^{T_{air}^{IN}} + \cancel{\dot{m}_{fuel} \cdot h_{fuel} \Big|_{T_{ref}}^{T_{fuel}^{IN}}} + (1 - \eta_{loss}) \cdot \dot{m}_{Fuel} \cdot LHV + \dot{W}_{fan} = \\ (1 + \alpha) \cdot \dot{m}_{fuel} \cdot h_{FG} \Big|_{T_{ref}}^{T_{FG}^{ST}} + y_{ash} \cdot \dot{m}_{fuel} \cdot Cp_{ash}(T_{ash}^{OUT} - T_{ref}) + \dot{Q}_u \end{aligned} \quad (29)$$

h_{fuel} contribute is negligible respect of LHV. In this case we miss both the \dot{m}_i and the

First of all we can get α directly from β_{O_2} with the following formula:

$$\alpha = \beta_{O_2} \cdot \frac{1}{\chi_{O_2}^{air}} \cdot \frac{MM_{air}}{MM_C} \cdot y_C = 10.96 \quad (30)$$

We know the enthalpy of each of the species in air, fuel and flue gases, but we need to know the total enthalpy of the mixtures.

$$h_{species_i} = \int C_{p_i} dT = \int (A_i + B_i \cdot T) dT \quad (31)$$

$$h_{mix} = \int C_{p_{mix}} dT = \int (A_{mix} + B_{mix} \cdot T) dT \quad (32)$$

Where

$$A_{mix} = \sum_{i=1}^{species} A_i \cdot y_i$$

$$B_{mix} = \sum_{i=1}^{species} B_i \cdot y_i$$

	<i>Air</i>	<i>Ash</i>	<i>Flue gas</i>
$A[\frac{J}{KgK}]$	952.2	2000	947.23
$B[\frac{J}{Kg}]$	0.1874	0	0.2790

Unlike the previous case with natural gas we do not know the temperature at which the flue gas goes out from our control volume. We can obtain it considering that it must be at least $25^\circ C$ more than dew point temperature of SO_2 and SO_3 . So we proceed to evaluate that temperature.

$$T_{H_2SO_4} = \frac{1000}{2.276 - 0.0294 \cdot \log(p_{H_2O}) + 0.0858 \cdot \log(p_{SO_3}) - 0.00062 \cdot \log(p_{H_2O}) \log(p_{SO_3})}$$

$$T_{H_2SO_3} = \frac{1000}{3.9526 - 0.1863 \cdot \log(p_{H_2O}) + 0.000867 \cdot \log(p_{SO_2}) - 0.000913 \cdot \log(p_{H_2O}) \log(p_{SO_2})}$$

But we do not know p_{SO_2} and p_{SO_3} . We can calculate them with the assumption that they are ideal gases. So the partial pressure is just the total pressure multiplied by the molar fraction.

$$p_{SO_2} = \left(1 - \frac{2.5}{100}\right) \cdot \chi_{SO_2} \cdot p_{ATM} = 1.980 \text{ bar} \quad (33)$$

$$p_{SO_3} = \frac{2.5}{100} \cdot \chi_{SO_2} \cdot p_{ATM} = 0.0508 \text{ bar} \quad (34)$$

Finally we obtain that $T_{H_2SO_3} = 315.06 K$

$$T_{H_2SO_4} = 416.41 K$$

. We must impose an outlet temperature at the stack $25^\circ C$ greater than the maximum dew point temperature so:

$$T_{FG}^{ST} = (25 K + T_{H_2SO_4}) = 441.41 K = 168.26^\circ C \quad (35)$$

Before proceeding to resolve the energy equation 29 we have to choose if to consider \dot{W}_{fan} or to neglect it and then verify if this hypothesis is close to reality. We consider *NOT* to neglect \dot{W}_{fan} . So we have to evaluate it, knowing that it depends also on \dot{m}_{fuel} . We proceed in the same way as equation 15 considering that Δp now is $350 mm_{H_2O}$.

$$\dot{W}_{fan} = \frac{\Delta p \cdot \dot{m}_{fuel}}{\eta_e \cdot \eta_{fan}} \cdot \left(\frac{\alpha}{\rho_{air}^{IN}} + \frac{1 + \alpha}{\rho_{FG}^{ST}} \right) \quad (36)$$

Now we can solve equation 29 where the only unknown is \dot{m}_{fuel} , because also \dot{W}_{fan} depends only on \dot{W}_{fan} . Finally we can compute \dot{m}_{fuel} and therefore \dot{m}_{air} and \dot{m}_{FG} .

	<i>Air</i>	<i>Fuel</i>	<i>Flue gas</i>
$\dot{m} \left[\frac{kg}{s} \right]$	481.49	43.85	525.35

Table 1: Mass flow rates

We have obtained the value of all our unknown so now we can easy evaluate η_{boiler} from equation 9 :

$$\eta_{Boiler} = \frac{\dot{Q}_u}{\dot{m}_{fuel} \times LHV} = 91.86\% \quad (37)$$

2.c Fan power consumption

In equation 36 we have obtained \dot{W}_{fan} function of \dot{m}_{fuel} , that before was unknown, and \dot{m}_{fuel} from table 1. Now instead we can correctly evaluate the power of fan:

$$\dot{W}_{fan} = \frac{\Delta p \cdot \dot{m}_{fuel}}{\eta_e \cdot \eta_{fan}} \cdot \left(\frac{\alpha}{\rho_{air}^{IN}} + \frac{1 + \alpha}{\rho_{FG}^{ST}} \right) = 5.06 MW \quad (38)$$