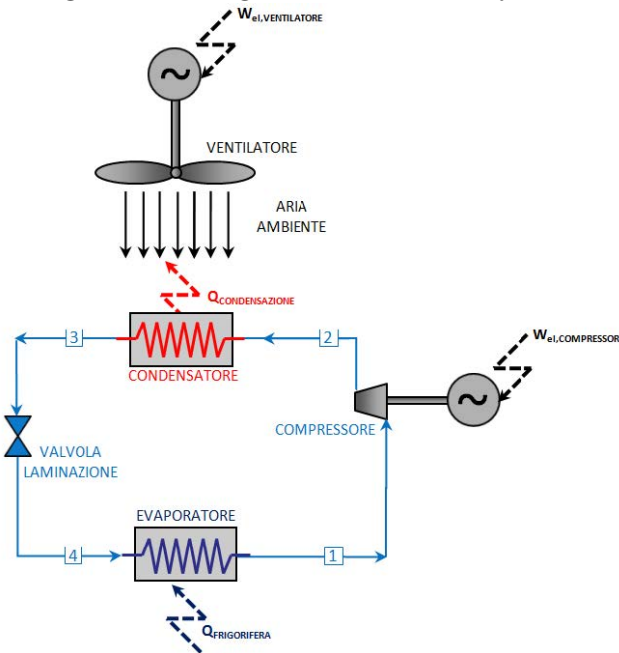


PRECEPT 5: Refrigeration cycle

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The refrigeration circuit of a refrigerator for fresh food storage is based on a vapour-compression refrigeration cycle with a single evaporator (with a single lamination valve). The plant has the layout as shown in figure 1

Figure 1: Refrigeration Circuit Layout.



The volumetric compressor is a reciprocating type with single action. The condenser consists of a compact tube and fin heat exchanger.

In particular for the refrigerator it is required to:

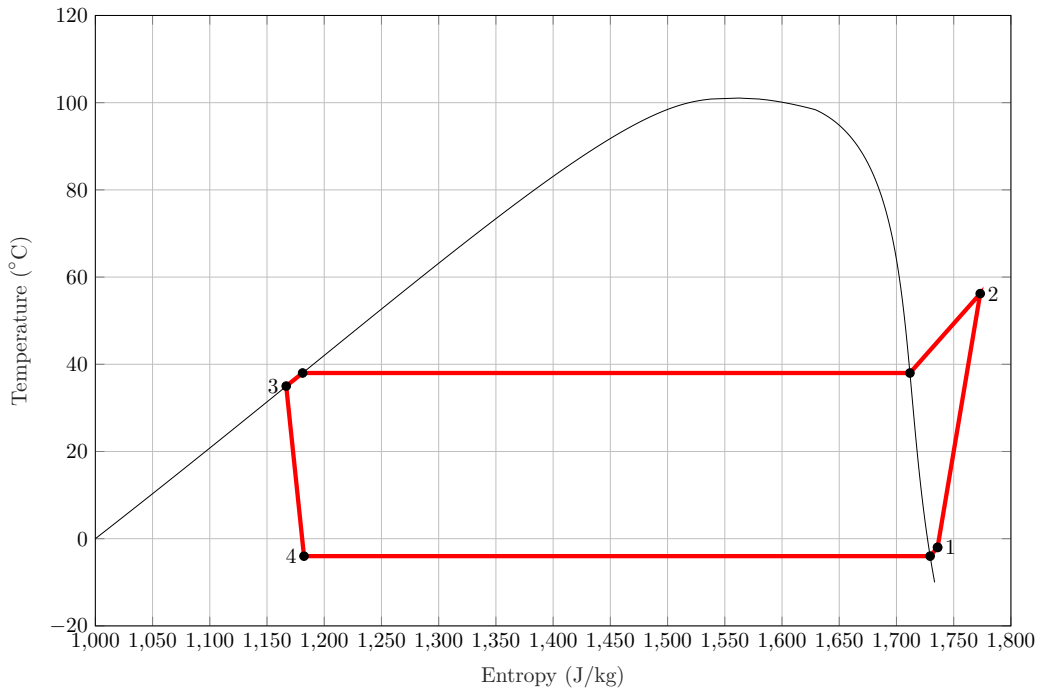
- evaluate the thermodynamic conditions for each point of the cycle for the following condensation temperatures: 30°C , 34°C , 38°C ;
- draw the plant scheme and the corresponding thermodynamic cycle on (P-h) and (T-s) diagrams, reporting the most relevant thermodynamic points;
- calculate the volumetric flow rate at the compressor inlet;
- calculate the compressor displacement (only for the design condition $T_{\text{cond}} = 30^{\circ}\text{C}$);
- evaluate the thermal power rejected by the condenser;
- design the tubes length L_{tubes} for each section of the condenser (de-superheat, condensation and sub-cooling);
- calculate the electrical power of the compressor and of the fan;
- calculate the coefficient of performance (C.O.P.) of the cycle and of the overall plant;

- how do the C.O.P. of the cycle varies adopting a double-throttling refrigeration cycle configuration (to be evaluated only for $T_{\text{cond}} = 34^\circ\text{C}$) assuming as pressure of the vapor-liquid separator (outlet pressure of the first throttling valve) 3,5,7 bar? Draw the plant scheme and the corresponding thermodynamic cycle on (P-h) and (T-s) diagrams, reporting the most relevant thermodynamic points.

1 Thermodynamic points

We need to evaluate the thermodynamic conditions for three different values of T_{cond} . In the following paragraphs are shown the result for $T_{\text{cond}} = 34^\circ\text{C}$, but you can do the same calculations starting with any other values of T_{cond} .

Figure 2: T-s diagram.



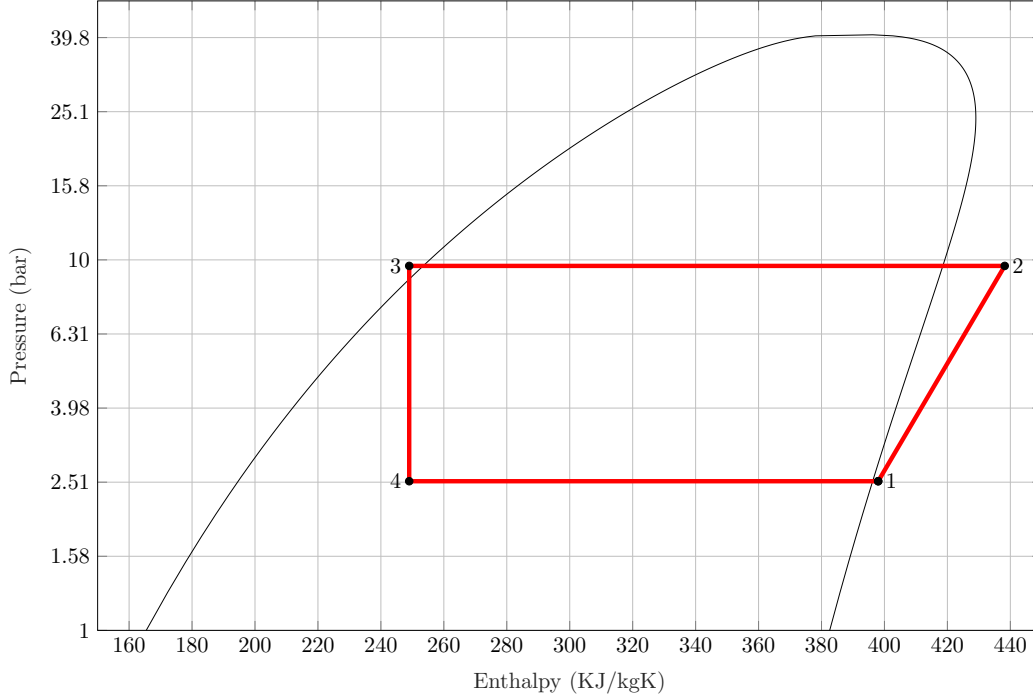
Point 1)

Point 1 is at *compressor inlet*. We know its temperature $T_1 = T_{\text{evap}} + \Delta T_{SH}$ and its pressure exactly the evaporation pressure $p_1 = p_{SAT}(T_{\text{evap}})$. With the two parameters of temperature and pressure we can evaluate with Mollier diagram of R134a also entropy and enthalpy functions of temperature and pressure:

$$\begin{cases} T_1 = T_{\text{evap}} + \Delta T_{SH} = -4^\circ\text{C} + 2^\circ\text{C} = -2^\circ\text{C} \\ p_1 = p_{SAT}(T_{\text{evap}}) = 2.53 \text{ bar} \end{cases}$$

T_1	p_1	h_1	s_1
-2°C	2.53 bar	$398.01 \frac{\text{kJ}}{\text{kg}}$	$1.7359 \frac{\text{kJ}}{\text{kg}\cdot\text{K}}$

Figure 3: p-h diagram.



Point 2)

Point 2 is at *compressor outlet*. We know its pressure $p_2 = p_{\text{cond}} \text{ bar}$. We do not directly have the temperature T_2 but we can obtain it from the definition of efficiency of the compressor:

$$\eta_T^{\text{isoentropic}} = \frac{h_2^{IS} - h_1}{h_2 - h_1} = 70\% \quad (1)$$

We don't know h_2^{IS} but we can compute it from point 1 to point 2^{IS} considering an *isoentropic process*:

$$\begin{cases} p_2^{IS} = p_2 = 8.63 \text{ bar} \\ s_2^{IS} = s_1 = 1.74 \frac{\text{kJ}}{\text{kg} \cdot \text{K}} \end{cases}$$

Now we can obtain from Mollier diagram $h_2^{IS} = h(p_2^{IS}, s_2^{IS}) = 423.80 \frac{\text{kJ}}{\text{kg}}$.

Reversing equation 1 we can get h_2 :

$$h_2 = h_1 - \frac{h_1 - h_2^{IS}}{\eta_T^{\text{iso}}} = 434.85 \frac{\text{kJ}}{\text{kg}} \quad (2)$$

With Mollier diagram we finally evaluate $s_2 = s(p_2, h_2)$ and $T_2 = T(p_2, h_2)$ of point 2:

T_2	p_2	h_2	s_2
51.17°C	8.63 bar	$434.85 \frac{\text{kJ}}{\text{kg}}$	$1.7706 \frac{\text{kJ}}{\text{kg} \cdot \text{K}}$

Point 3)

Point 3 is at *condenser outlet*. We know its temperature $T_3 = T_{\text{cond}} - \Delta T_{SC}$ and its pressure exactly the condensation pressure $p_3 = p_{SAT}(T_{\text{cond}})$. With the two parameters of temperature and pressure we can evaluate with Mollier diagram of R134a also entropy and enthalpy functions of temperature and pressure:

$$\begin{cases} T_3 = T_{\text{cond}} - \Delta T_{SC} = 31^\circ\text{C} \\ p_3 = p_{SAT}(T_{\text{cond}}) = 8.63 \text{ bar} \end{cases}$$

T_3	p_3	h_3	s_3
$31\text{ }^\circ\text{C}$	8.63 bar	$243.17\text{ }\frac{\text{kJ}}{\text{kg}}$	$1.1480\text{ }\frac{\text{kJ}}{\text{kg}\cdot\text{K}}$

Point 4)

Point 4 is at *valve outlet*. We can model the valve as *isoenthalpic* so we know enthalpy of point 4 as it is equal to point 3. Moreover we can consider that point 4 is also at evaporator inlet so it is at evaporation pressure. With the two parameters of enthalpy and pressure we can evaluate with Mollier diagram of R134a also entropy and temperature functions of enthalpy and pressure:

$$\begin{cases} h_4 = h_3 = 243.17\text{ }\frac{\text{kJ}}{\text{kg}} \\ p_3 = p_{\text{evap}} = 2.53\text{ bar} \end{cases}$$

T_4	p_4	h_4	s_4
$-4\text{ }^\circ\text{C}$	2.53 bar	$243.17\text{ }\frac{\text{kJ}}{\text{kg}}$	$1.1606\text{ }\frac{\text{kJ}}{\text{kg}\cdot\text{K}}$

Similarly changing p_{evap} we get the thermodynamical condition of each point of the plant.

Table 1: Condensation temperature $30\text{ }^\circ\text{C}$

Point	T [K]	p [bar]	h [$\frac{\text{kJ}}{\text{kg}}$]	s [$\frac{\text{kJ}}{\text{kg}\cdot\text{K}}$]	$\rho[\text{kg}/\text{m}^3]$
1	-2	2.53	398.01	1.74	12.39
2	46.04	7.70	431.40	1.77	34.19
3	27	7.70	237.40	1.13	1199.50
4	-4.00	2.53	237.40	1.14	57.01

Table 2: Condensation temperature $34\text{ }^\circ\text{C}$

Point	T [K]	p [bar]	h [$\frac{\text{kJ}}{\text{kg}}$]	s [$\frac{\text{kJ}}{\text{kg}\cdot\text{K}}$]	$\rho[\text{kg}/\text{m}^3]$
1	-2	2.53	398.01	1.74	12.39
2	51.17	8.63	434.85	1.77	38.06
3	31	8.63	243.17	1.15	1184.01
4	-4.00	2.53	243.17	1.16	50.50

Table 3: Condensation temperature $38\text{ }^\circ\text{C}$

Point	T [K]	p [bar]	h [$\frac{\text{kJ}}{\text{kg}}$]	s [$\frac{\text{kJ}}{\text{kg}\cdot\text{K}}$]	$\rho[\text{kg}/\text{m}^3]$
1	-2	2.53	398.01	1.74	12.39
2	56.21	9.63	438.21	1.77	42.28
3	35	9.63	249.00	1.17	1168.07
4	-4.00	2.53	249.00	1.18	45.27

2 Volumetric flow rate at the compressor inlet

The volumetric flow rate of a specific point is the mass flow rate of that point divided by its density:

$$\dot{V}_1 = \frac{\dot{m}_{\text{refrigerant}}}{\rho_1} \quad (3)$$

We have already computed the density of the fluid at the inlet of the compressor, but we miss the mass flow rate; we can evaluate it with an energy balance about the condenser, because we have the useful heat exchanged by the cycle.

$$\dot{Q}_{evap} = \dot{m}_{refrigerant} \cdot (h_1 - h_4) = 18 \text{ kW} \quad \Rightarrow \quad \dot{m}_{refrigerant} = \frac{\dot{Q}_{evap}}{h_1 - h_4} = 0.11 \frac{\text{kg}}{\text{s}} \quad (4)$$

Finally we obtain the volumetric flow rate $\dot{V}_1 = \frac{\dot{m}_{refrigerant}}{\rho_1} = 9.04 \text{ l/s}$

3 Compressor displacement

We can obtain the compressor displacement from the definition of the volumetric efficiency of the compressor; in fact:

$$\eta_{vol} = \frac{\dot{V}_1}{n \cdot V_{cil}} \quad \Rightarrow \quad V_{cil} = \frac{\dot{V}_1}{n \cdot \eta_{vol}} = 202.97 \text{ cm}^3 \quad (5)$$

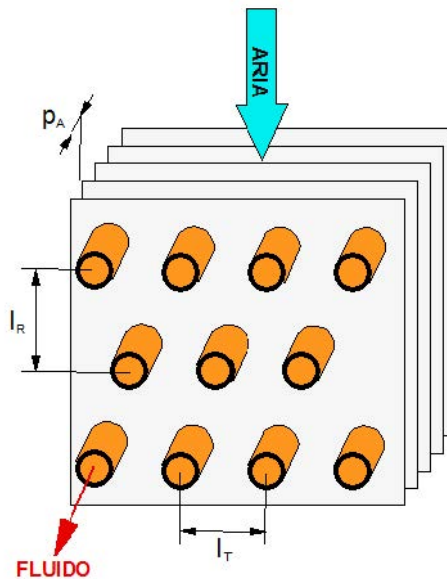
4 Thermal power rejected by the condenser

The thermal power rejected by the refrigeration cycle, \dot{Q}_{cond} , which has to be discharged in the hot heat sink (the environment) by means of the condenser, results from the application of the *first law of thermodynamics* to the hot side (refrigerant stream) of the condenser. This is a simple task since we already have all the thermodynamical properties for each point, in particular at the inlet and at the outlet of the condenser;

$$\dot{Q}_{cond} = \dot{m}_{refrigerant} \cdot (h_2 - h_3) = 21.74 \text{ kW} \quad (6)$$

5 Design of the tubes length L_{tubes} for each section of the condenser

Figure 4: Condenser Layout.



Since the geometry of the heat exchanger is not given, the $\Delta T_{\text{mean log}}$ method is easier. In order to use this method the heat exchanger has to be divided based on the heat transfer coefficient

values. Depending on which type of fluid we use to transfer heat, sub-cooled water, two phase mixture or super-heated vapour, we have different values of heat transfer coefficient. Even if the coefficient could be different the approach would be almost the same for the three cases. For this reason we will develop the entire explanation for sub-cooled part and condensation temperature $T_{\text{cond}} = 34^\circ\text{C}$ and then we will explain the differences for the others. The scheme of the condenser is represented in figure 4

Sub-cooling condenser

The heat exchanged in condenser can be written in two different ways:

$$\dot{Q}_{\text{cond}} = \dot{m}_{\text{refrigerant}} \cdot \Delta h = U_{\text{int}} A_{\text{int}} \cdot \Delta T_{\text{ml}} \cdot F \quad (7)$$

where:

U = Heat transfer coefficient;

A = Surface of heat transfer;

ΔT_{ml} = Mean log delta temperature;

F = Correction factor for an heat exchanger it is not counter current, and it is function of two parameters P ¹ and R ;

The surface A_{int} is the lateral surface of a cylinder for each tube so it is function of L_{tubes} :

$$A_{\text{int}} = \pi D_{\text{int}} \cdot L_{\text{tubes}} \cdot N_{\text{rows}} \cdot N_{\text{tubes per row}} \quad (8)$$

We can also consider the heat exchanged in the air side:

$$\dot{Q}_{\text{cond}} = \dot{m}_{\text{air}} \cdot c_{p\text{air}} (T_{\text{in}} - T_{\text{out}}) = \rho_{\text{air}} \cdot v_{\text{air}} \cdot A_{\text{fin}} \cdot c_{p\text{air}} (T_{\text{in}}^{\text{air}} - T_{\text{out}}^{\text{air}}) \quad (9)$$

A_{fin} is the frontal surface of the condenser, and it takes into account the thickness of the fins and it is also function of the unknown tube length.

$$A_{\text{fin}} = L_{\text{tubes}} \cdot I_{\text{tubes}} \cdot N_{\text{tubes per row}} - s_A \left[\frac{L_{\text{tubes}}}{p_A} \right] \cdot I_{\text{tubes}} \cdot N_{\text{tubes per row}} \quad (10)$$

where: L_{tubes} = tubes length;

I_{tubes} = tubes pitch;

$N_{\text{tubes per row}}$ = number of tubes per row;

p_A = fin pitch;

To evaluate the heat transfer coefficient we apply the series rules of resistance:

$$R_{\text{TOT}} = \frac{1}{U_{\text{int}} A_{\text{int}}} = R_{\text{ext}} + R_{\text{int}} + \cancel{R_{\text{wall}}} + R_{\text{fin}} \quad (11)$$

The resistance of the wall is small compared to the others so we can neglect it.

$$R_{\text{TOT}} = \frac{1}{U_{\text{int}} \cancel{A_{\text{int}}}} = \frac{1}{h_{\text{int}} \cancel{A_{\text{int}}}} + \frac{1}{h_{\text{ext}} (\cancel{A_{\text{bare tube}}} + A_{\text{fin}}) \cdot \eta_0} \quad (12)$$

1

$$P = \frac{T_{\text{cold}}^{\text{out}} - T_{\text{cold}}^{\text{in}}}{T_{\text{hot}}^{\text{in}} - T_{\text{cold}}^{\text{in}}}$$

$$R = \frac{T_{\text{hot}}^{\text{in}} - T_{\text{hot}}^{\text{out}}}{T_{\text{cold}}^{\text{out}} - T_{\text{cold}}^{\text{in}}}$$

We cancel A_{int} with $A_{bare\ tube}$ because they are comparable.

$$U_{int} = \left(\frac{1}{h_{int} + h_{ext} \cdot \eta_0} \right)^{-1} = 623.58 \frac{W}{m^2 \cdot K} \quad (13)$$

Reverting equation 9 we obtain T_{out}^{air} :

$$T_{out}^{air} = T_{in}^{air} + \frac{\dot{Q}_{cond}}{\dot{m}_{air} \cdot c_{p_{air}}} = f(L_{tubes}) \quad (14)$$

We can evaluate the heat exchanged in the sub-cooled part with equation 7:

$$\dot{Q}_{cond} = \dot{m}_{refrigerant} \cdot (h_{vap}^{sat}(p_{cond}) - h_3) = 508.41 W \quad (15)$$

In this way T_{out}^{air} is just function of L_{tubes} . The two parameters R and P are function just of L_{tubes} :

$$P = \frac{T_{out}^{air}(L_{tubes}) - T_{in}^{air}}{T_{cond} - T_3} = f(L_{tubes}) \quad (16)$$

$$R = \frac{T_{cond} - T_3}{T_{out}^{air}(L_{tubes}) - T_{in}^{air}} = f(L_{tubes}) \quad (17)$$

$$(18)$$

With the temperature of the air at the outlet of the condenser we can compute the mean log temperature:

$$\Delta T_{ml} = \frac{\Delta T_{IN} - \Delta T_{OUT}}{\ln \left(\frac{\Delta T_{IN}}{\Delta T_{OUT}} \right)} = f(L_{tubes}) \quad (19)$$

where: $\Delta T_{IN} = T_{cond} - T_{out}^{air}(L_{tubes})$;

$$\Delta T_{OUT} = T_3 - T_{in}^{air};$$

We have two different ways to write the A_{int} :

$$\begin{cases} A_{int} = \pi D_{int} \cdot L_{tubes} \cdot N_{rows} \cdot N_{tubes\ per\ row} = f(L_{tubes}) \\ A_{int} = \frac{\dot{Q}_{cond}}{U_{int} \cdot \Delta T_{ml}(L_{tubes}) \cdot F(R, P)} = f(L_{tubes}) \end{cases}$$

Joining the two equations we finally get one equation in the single unknown L_{tubes}

$$\pi D_{int} \cdot L_{tubes} \cdot N_{rows} \cdot N_{tubes\ per\ row} - \frac{\dot{Q}_{cond}}{U_{int} \cdot \Delta T_{ml}(L_{tubes}) \cdot F(R(L_{tubes}), P(L_{tubes}))} = 0 \quad (20)$$

We can solve equation number 20 with a numeric method like secant one and we obtain for the sub-cooling part of the condenser $L_{tubes} = 2.89\ cm$ With L_{tubes} we can evaluate all the quantities of the condenser.

Condensation condenser

In this case the heat exchanged is the latent heat of evaporation by the mass flow rate of refrigerant:

$$\dot{Q}_{cond} = \dot{m}_{refrigerant} \cdot (h_{vap}^{sat}(p_{cond}) - h_{liq}^{sat}(p_{cond})) = 19\,667.34 W \quad (21)$$

The other difference with the sub-cooling part is that in this case the correction factor is equal to 1 because the parameter R is null.

De-super-heating condenser

In this case the heat exchanged is:

$$\dot{Q}_{cond} = \dot{m}_{refrigerant} \cdot (h_2 - h_{vap}^{sat}(p_{cond})) = 2107.10 \text{ W} \quad (22)$$

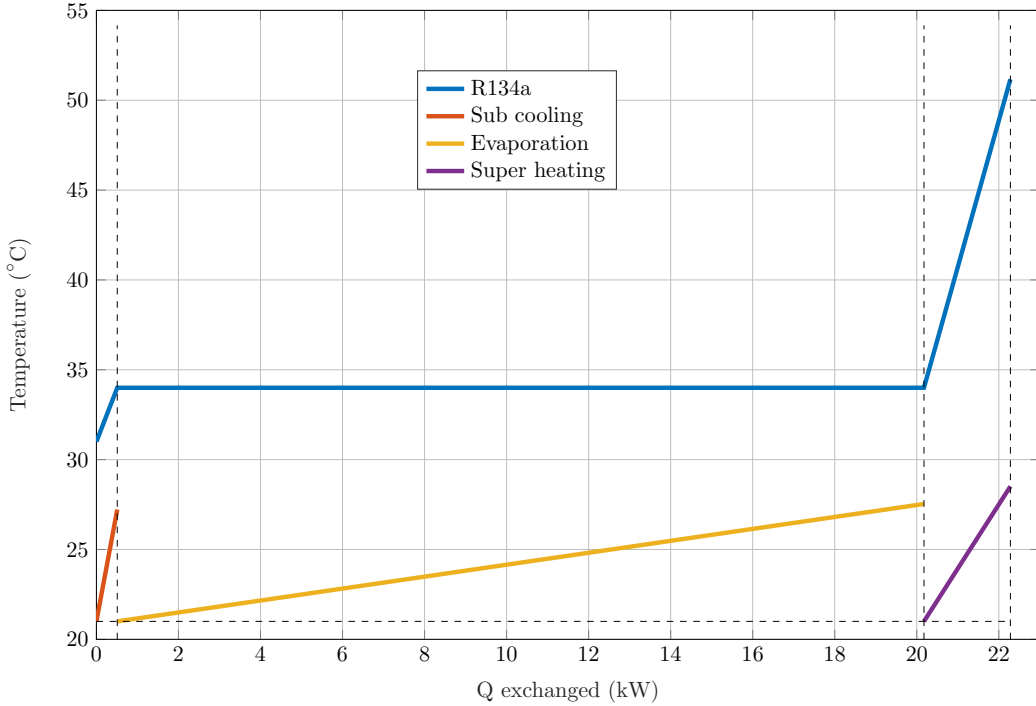
Other than no more significant differences exists.

Results

We can summarize the condenser results in table 5.

	$L_{tubes} (cm)$	$\dot{m}_{air} (kg/s)$	U	Q (Heat transfered W)	$T_{out}^{air} (^\circ C)$
Subcooling	2.89	0.08	623.58	508.41	27.22
Condensation	106.81	2.93	555.37	19 667.34	27.53
De-superheating	9.95	0.27	358.02	2107.10	28.51
TOT	119.65	3.28	—	22 282.85	—

Figure 5: T-Q diagram of the entire condenser.



6 Electrical power of the compressor and of the fan

The electrical power of the compressor \dot{W}_c is:

$$\dot{W}_{compressor} = \frac{\dot{m}_{refrigerant} \cdot (h_2 - h_1)}{\eta_{m/e}} = 4766.86 \text{ W} \quad (23)$$

The fan power \dot{W}_{fan} is:

$$\dot{W}_{fan} = \frac{\dot{V}_{air} \cdot \Delta p}{\eta_{fan}} = \frac{\dot{m}_{air} \cdot \Delta p}{\rho_{air} \cdot \eta_{fan}} = 218.67 \text{ W} \quad (24)$$

Where the pressure variation of the air side for each row is given by an experimental formula: $\Delta p_{each \text{ row}} = 0.208 \cdot v_{air}^{1.87} = 1.62 \text{ mm of } H_2O$. The total $\Delta p (Pa) = g \cdot \Delta p_{each \text{ row}} \cdot N_{rows} = 47.76 \text{ Pa}$

7 Coefficient of performance (C.O.P.) of the cycle and of the overall plant

The C.O.P. of the cycle is defined as the ratio between the *useful refrigeration effect* and the *compression power* (power transferred to the fluid by the compressor)

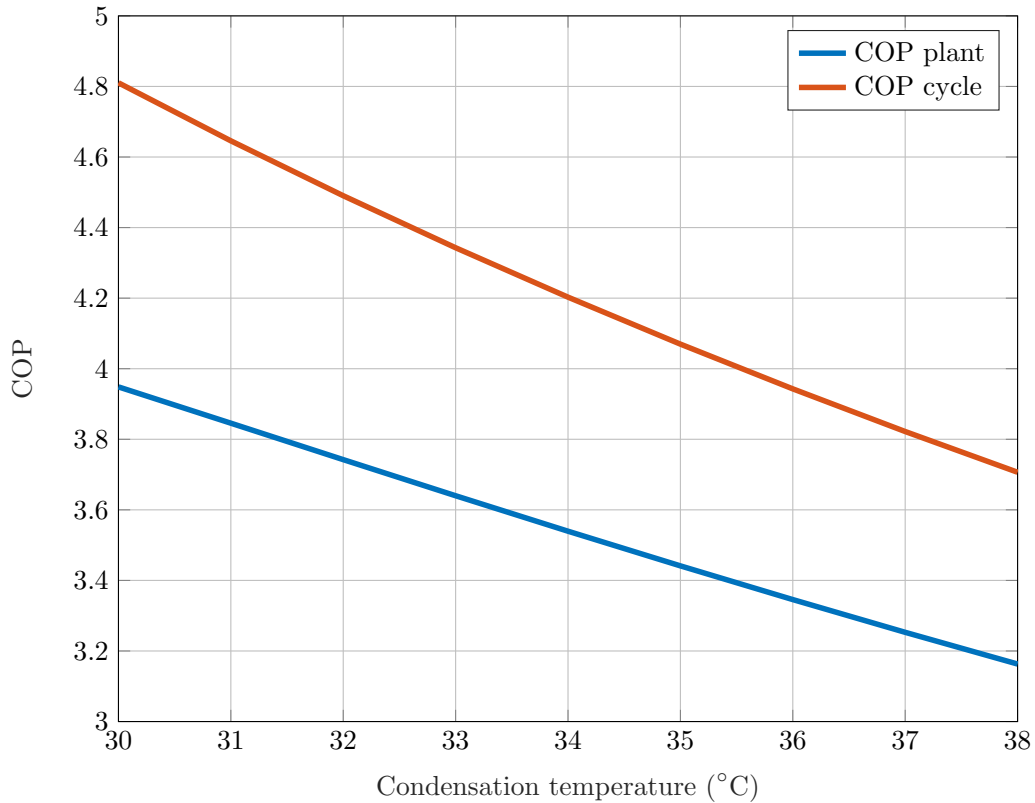
$$C.O.P. \text{ cycle} = \frac{\dot{Q}_u}{\dot{m}_{ref} \cdot (h_2 - h_1)} = 4.20 \quad (25)$$

The overall (or total) C.O.P. of the refrigerator is defined as the ratio between the *useful refrigeration effect* and the *overall electrical consumption* of the refrigeration system (electrical power of the compressor and electrical power of the fan):

$$C.O.P. \text{ plant} = \frac{\dot{Q}_u}{\dot{m}_{ref} \cdot (h_2 - h_1) / \eta_{m/e} + \dot{W}_{fan}} = 3.54 \quad (26)$$

Varying the condensation temperature T_{cond} we can plot the *C.O.P.* of the system, reported in figure 6.

Figure 6: *C.O.P.* function of the condensation temperature.

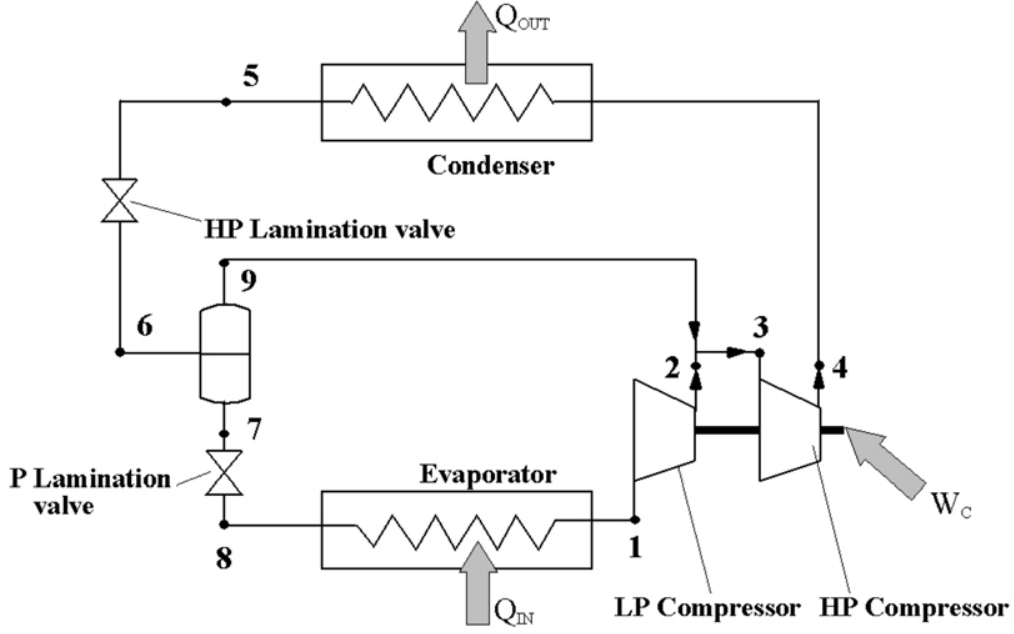


8 Double-throttling refrigeration cycle

We make a change in the previous plant, in fact to improve the total work produced we make a double-throttling refrigeration cycle, that requires an additional valve, a separator of liquid and vapour and a two stage compressor. This is particularly feasible in case the system uses a multi-stage compressor instead of a volumetric one. The new scheme is drawn in figure 7

To evaluate the new *C.O.P.* is necessary to evaluate the thermodynamical conditions of the main point. We must apply the same calculations for three different value of the separation

Figure 7: Double-throttling refrigeration Circuit Layout.



pressure $p_{\text{sep}} = 3 \text{ bar}, 5 \text{ bar}, 7 \text{ bar}$. Since the method is exactly the same we will explain it step by step just for $p_{\text{sep}} = 7 \text{ bar}$ and then show simply the results for the others.

Point 1)

Point 1 is at *compressor inlet*. We know its temperature $T_1 = T_{\text{evap}} + \Delta T_{SH}$ and its pressure exactly the evaporation pressure $p_1 = p_{SAT}(T_{\text{evap}})$. With the two parameters of temperature and pressure we can evaluate with Mollier diagram of R134a also entropy and enthalpy functions of temperature and pressure:

$$\begin{cases} T_1 = T_{\text{evap}} + \Delta T_{SH} = -4^\circ\text{C} + 2^\circ\text{C} = -2^\circ\text{C} \\ p_1 = p_{SAT}(T_{\text{evap}}) = 2.53 \text{ bar} \end{cases}$$

T_1	p_1	h_1	s_1
-2°C	2.53 bar	$398.01 \frac{\text{kJ}}{\text{kg}}$	$1.7359 \frac{\text{kJ}}{\text{kg}\cdot\text{K}}$

Point 2)

Point 2 is at *high pressure compressor outlet*. We know its pressure $p_2 = p_{\text{sep}}$. We do not directly have the temperature T_2 but we can obtain it from the definition of efficiency of the compressor:

$$\eta_T^{\text{isoentropic}} = \frac{h_2^{IS} - h_1}{h_2 - h_1} = 70\% \quad (27)$$

We don't know h_2^{IS} but we can compute it from point 1 to point 2^{IS} considering an *isoentropic process*:

$$\begin{cases} p_2^{IS} = p_2 = 7 \text{ bar} \\ s_2^{IS} = s_1 = 1.74 \frac{\text{kJ}}{\text{kg}\cdot\text{K}} \end{cases}$$

Now we can obtain from Mollier diagram $h_2^{IS} = h(p_2^{IS}, s_2^{IS}) = 419.34 \frac{\text{kJ}}{\text{kg}}$.

Reverting equation 27 we can get h_2 :

$$h_2 = h_1 - \frac{h_1 - h_2^{IS}}{\eta_T^{\text{iso}}} = 428.48 \frac{\text{kJ}}{\text{kg}} \quad (28)$$

With Mollier diagram we finally evaluate $s_2 = s(p_2, h_2)$ and $T_2 = T(p_2, h_2)$ of point 2:

T_2	p_2	h_2	s_2
$41.76\text{ }^\circ\text{C}$	7 bar	$428.48\text{ }\frac{\text{kJ}}{\text{kg}}$	$1.7654\text{ }\frac{\text{kJ}}{\text{kg}\cdot\text{K}}$

Point 5)

Point 5 is at *condenser outlet*. We know its temperature $T_5 = T_{\text{cond}} - \Delta T_{SC}$ and its pressure exactly the condensation pressure $p_5 = p_{SAT}(T_{\text{cond}})$. With the two parameters of temperature and pressure we can evaluate with Mollier diagram of R134a also entropy and enthalpy functions of temperature and pressure:

T_5	p_5	h_5	s_5
$31\text{ }^\circ\text{C}$	8.63 bar	$243.17\text{ }\frac{\text{kJ}}{\text{kg}}$	$1.1480\text{ }\frac{\text{kJ}}{\text{kg}\cdot\text{K}}$

Point 6)

Point 6 is at *valve outlet*. We can model the valve as *isoenthalpic* so we know enthalpy of point 6 as it is equal to point 5. Moreover we can consider that point 6 is also at separator inlet so it is at separation pressure p_{sep} . With the two parameters of enthalpy and pressure we can evaluate with Mollier diagram of R134a also entropy and temperature functions of enthalpy and pressure:

T_6	p_6	h_6	s_6	χ_6
$26.71\text{ }^\circ\text{C}$	7 bar	$243.17\text{ }\frac{\text{kJ}}{\text{kg}}$	$1.1486\text{ }\frac{\text{kJ}}{\text{kg}\cdot\text{K}}$	$3.50\text{ }\%$

Point 7)

Point 7 is at *separator outlet* on the *saturated liquid* side. The pressure is exactly separation pressure p_{sep} . From these considerations we can easily get all the thermodynamical properties from the saturated liquid table of R134a.

T_7	p_7	h_7	s_7
$26.71\text{ }^\circ\text{C}$	7 bar	$236.99\text{ }\frac{\text{kJ}}{\text{kg}}$	$1.1280\text{ }\frac{\text{kJ}}{\text{kg}\cdot\text{K}}$

Point 8)

Point 9 is at *separation valve outlet*. We can model the valve as *isoenthalpic* so we know enthalpy of point 9 as it is equal to point 8. Moreover we can consider that point 9 is also at evaporator inlet so it is at evaporation pressure p_{evap} . With the two parameters of enthalpy and pressure we can evaluate with Mollier diagram of R134a also entropy and temperature functions of enthalpy and pressure:

T_8	p_8	h_8	s_8	χ_8
$-4\text{ }^\circ\text{C}$	2.53 bar	$236.99\text{ }\frac{\text{kJ}}{\text{kg}}$	$1.1377\text{ }\frac{\text{kJ}}{\text{kg}\cdot\text{K}}$	$21.00\text{ }\%$

Point 9)

Point 9 is at *separator outlet* on the *saturated vapour* side. The pressure is exactly separation pressure p_{sep} . From these considerations we can easily get all the thermodynamical properties from the saturated liquid table of R134a.

T_9	p_9	h_9	s_9
$26.71\text{ }^\circ\text{C}$	7 bar	$413.20\text{ }\frac{\text{kJ}}{\text{kg}}$	$1.7156\text{ }\frac{\text{kJ}}{\text{kg}\cdot\text{K}}$

Point 3)

Point 3 is at *low pressure turbine inlet*. Before entering into the turbine we have a mixing before flows 2 coming from the high pressure turbine and 9 that it is saturated vapour from the separator. So we can set up energy and mass balance for the mixing:

$$\begin{cases} \dot{m}_2 \cdot h_2 + \dot{m}_9 \cdot h_9 = \dot{m}_3 \cdot h_3 & (29) \\ \dot{m}_2 + \dot{m}_9 = \dot{m}_3 & (30) \end{cases}$$

We already know enthalpy of points 2 and 9, and we want to obtain h_3 . From mass balance at the separator knowing the quality of point 6 χ_6 we can get the relationship between the mass flow rates.

$$\begin{cases} \dot{m}_2 = \dot{m}_3 \cdot (1 - \chi_6) = 112.18 \cdot 10^{-3} \frac{kg}{s} & (31) \\ \dot{m}_9 = \dot{m}_3 \cdot \chi_6 = 4.07 \cdot 10^{-3} \frac{kg}{s} & (32) \end{cases}$$

Replacing the value of mass flow rates \dot{m}_2 and \dot{m}_9 into equation 29 we can find h_3 . Since the mixing is considerable *isobar* we know pressure of point 3 p_3 equal to separation pressure.

$$\begin{cases} p_3 = p_{\text{sep}} = 7 \text{ bar} \\ h_3 = 427.95 \frac{kJ}{kg} \end{cases}$$

From enthalpy and pressure we can get all the thermodynamical properties from Mollier diagram of R134a.

T_3	p_3	h_3	s_3
41.23 °C	7 bar	427.95 $\frac{kJ}{kg}$	1.7637 $\frac{kJ}{kg \cdot K}$

We summarize all the thermodynamical conditions in the following table.

Table 4: Separation pressure $p_{\text{sep}} = 7 \text{ bar}$

Point	T [K]	p [bar]	h [$\frac{kJ}{kg}$]	s [$\frac{kJ}{kg \cdot K}$]	χ [%]
1	-2	2.53	398.01	1.74	100
2	41.76	7	428.48	1.77	100
3	41.23	7	427.95	1.76	100
4	50.95	8.63	434.63	1.77	100
5	31	8.63	243.17	1.15	0
6	26.71	7	243.17	1.15	3.50
7	26.71	7	236.99	1.13	0
8	-4.00	2.53	236.99	1.14	21.00
9	26.71	7	413.20	1.72	100

Now we can plot T-s and p-h diagrams.

Figure 8: T-s diagram.

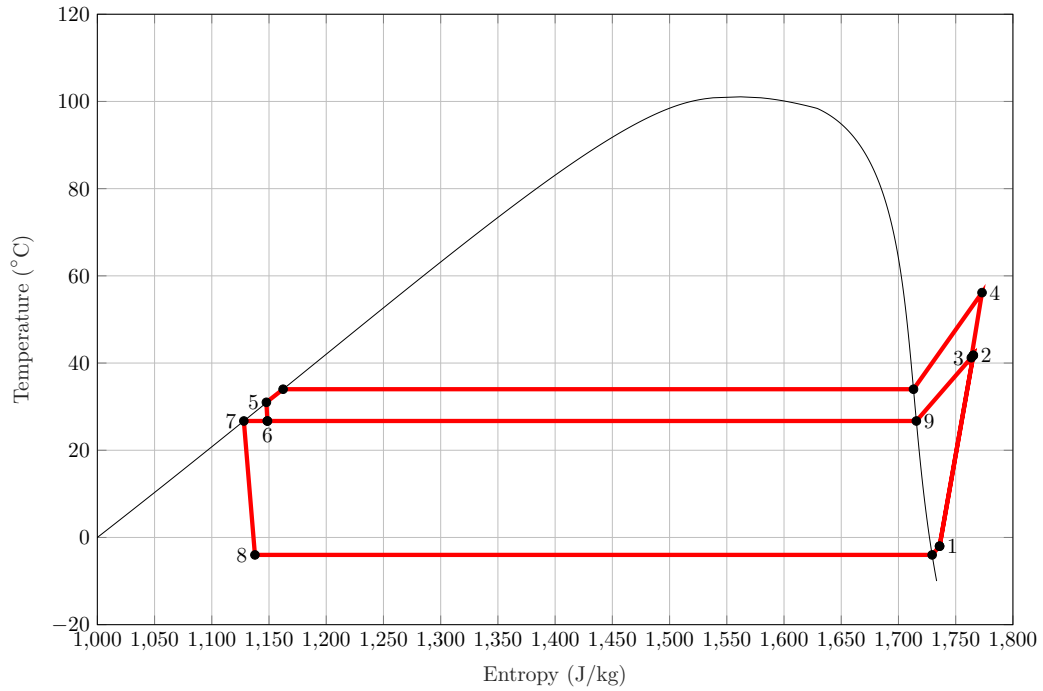
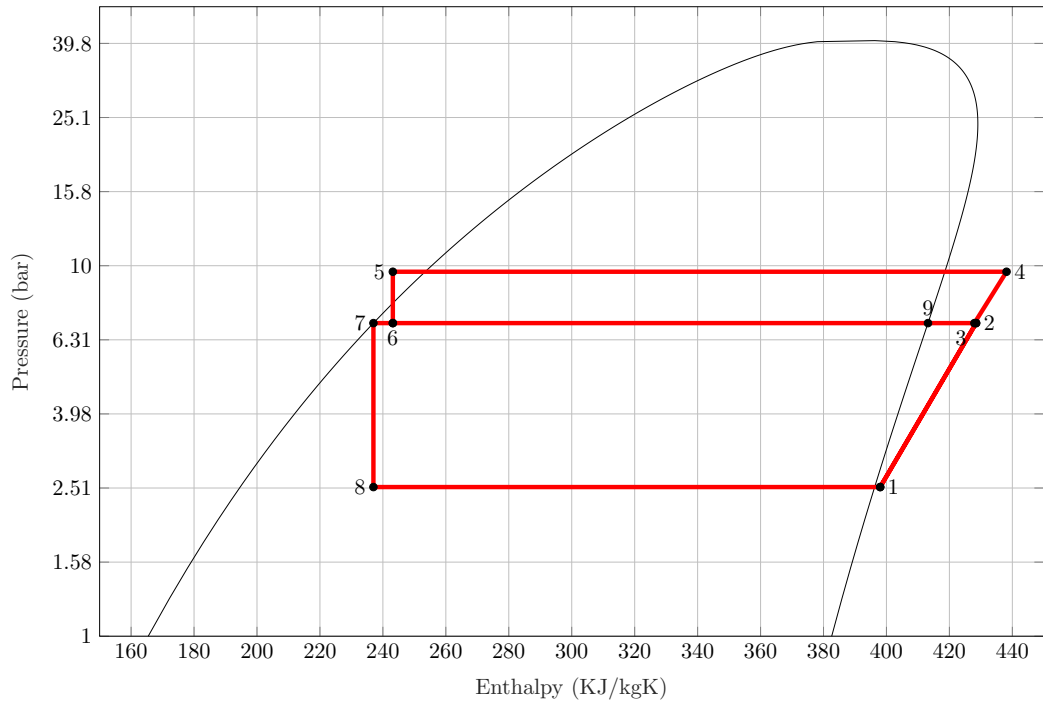


Figure 9: p-h diagram.



9 Coefficient of performance (C.O.P.) of the double-throttling cycle

With the thermodynamical conditions of each point of the plant we can evaluate the performance. The C.O.P. of the cycle is defined as the ratio between the *useful refrigeration effect* and the

compression power (power transferred to the fluid by the compressor)

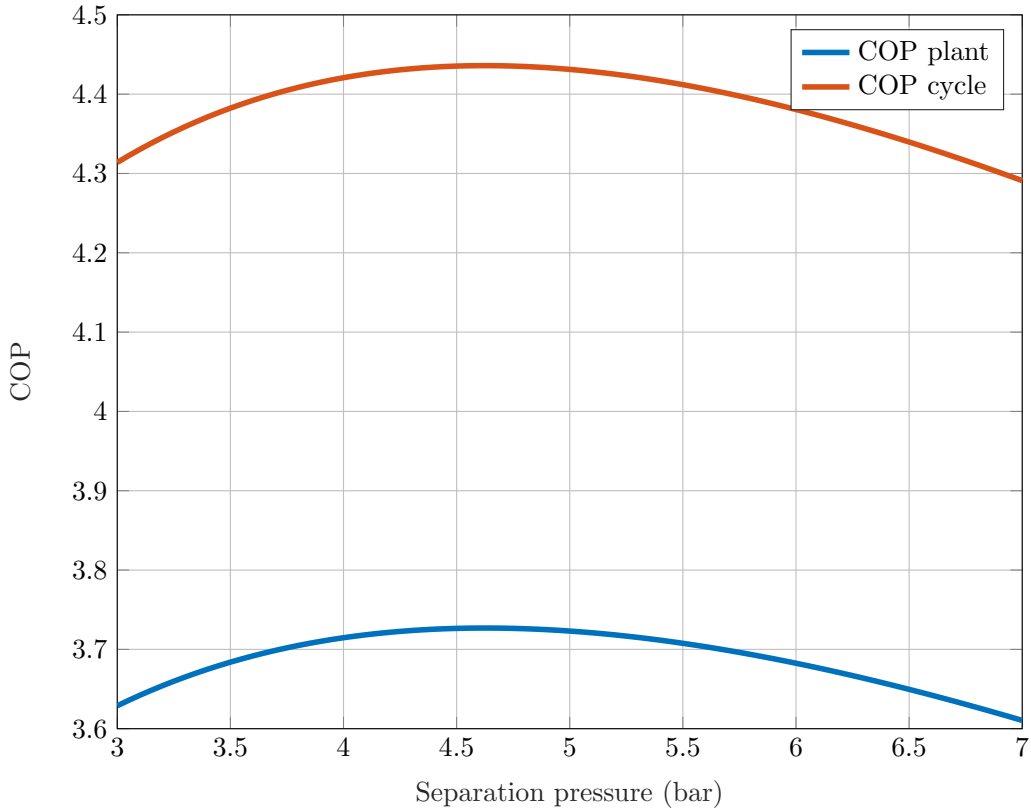
$$C.O.P. cycle = \frac{\dot{Q}_u}{\dot{m}_{ref} \cdot (h_2 - h_1)} = 4.29 \quad (33)$$

The overall (or total) C.O.P. of the refrigerator is defined as the ratio between the *useful refrigeration effect* and the *overall electrical consumption* of the refrigeration system (electrical power of the compressor and electrical power of the fan):

$$C.O.P. plant = \frac{\dot{Q}_u}{\dot{m}_{ref} \cdot (h_2 - h_1) / \eta_{m/e} + \dot{W}_{fan}} = 3.61 \quad (34)$$

Varying the separation pressure p_{sep} we can plot the *C.O.P.* of the system, reported in figure 10.

Figure 10: *C.O.P.* function of the condensation temperature.



In figure 10 is interesting to notice that the *C.O.P.* has a maximum value in correspondence of $p_{sep} \approx 4.5 \text{ bar}$ for both the plant and the cycle. In case of design of the plant can be interesting to choose the right p_{sep} to maximize the performance of the cycle.