Preliminary design of a High Pressure Steam Turbine



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The design specifications

The project consist in the preliminary design of the *high pressure* part of a steam turbine.

The design process requires some initial data other than many other assumptions.

Inlet flow characteristics, in terms of total quantities, are:

- $p_T = 150 \, \text{bar}$
- $T_T = 700^{\circ} \text{C}$

The stages must provide a pressure ratio β of 2.5.

Since the first stage of the design is the first stage of the whole machine, the **inlet** direction is assumed to be purely **axial**.

The model of the fluid

The text leave the choice of the **model of the fluid** to our responsibility. The two main alternatives are:

- perfect gas with $\gamma = 1.33$ and constant c_p ;
- steam tables;

Since steam tables are available as an additional library in Matlab, to guarantee a better precision they are used.

In particular the library **XSteam** is applied in any calculation of steam properties.

The design process

Most of the choices related to the machine are taken based **not only** on common techniques like *loading criteria*, but also on the comparison of the result from the implementation of the model in computer code.

The machine will be implemented in a power plant, so the first goal of the design is

highest efficiency as possible

The model of the machine - The inputs

The data available from the costumer are necessary but not sufficient. So as additional and more specific input for our model we take:

- the reaction degree in the middle diameter χ ,
- ullet the rotational speed n in rpm,
- the number of stages,
- the flow coefficient $\phi = v_a/U$,
- the minimum value of blade height over the diameter b/D_{mid} ,
- the partial admission coefficient ε ,
- the solidity $\sigma = \text{chord/pitch}$
- the ratio of the chord over the blade height

The model of the machine - The inputs

- the blade thickness over the chord
- the absolute clearance
- the number of seals per rotor
- an initial guess of the efficiency ($\eta = 1$ could be a guess)
- the number of section in which divide the span
- other numerical and computational parameters
 - the maximum number of iterations;
 - the tolerances $(1 \cdot 10^{-6})$.

The model of the machine - The inputs

Some of the input are trivial and mandatory, but other are chosen.

For example the **number of stages** is taken as input but it could also be the result of the calculation.

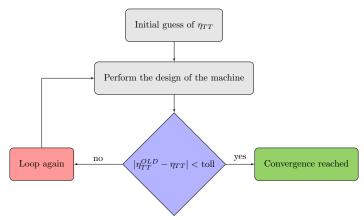
In fact the alternative was to choose the *work coefficient* $\lambda = l/U^2$, and in that case the number of stages would have been the result.

Since the number of stages is a very significant parameter respect to the work coefficient this choice has been made.

Another parameter is the flow coefficient that can be replaced with $k_p = U/v_1\,$

The model of the machine - The main loop

The whole process requires the total efficiency of the machine. The efficiency is also an output of the design: \Rightarrow **iterative loop**.



The model of the machine - The main loop

At the beginning we only have the total enthalpy at the inlet.

$$h_T^0 = h(p_{T_0}, \mathsf{T}_{T_0}) \tag{1}$$

To compute the isoentropic outlet enthalpy we have simply to consider that the initial final entropy are the same by definition.

$$s_0 = s_{T_0} = s(p_{T_0}, \mathsf{T}_{T_0}) \tag{2}$$

$$h_{T_{\text{isoentropic}}}^{\text{end}} = h\left(\frac{p_{T_0}}{\beta_{TT}}, s_0\right)$$
 (3)

Finally we calculate the outlet enthalpy from the efficiency:

$$h_T^{\sf end} = h_T^0 - \eta_{TT} \cdot (h_{T_0} - h_{T_{\sf is}}^{\sf end})$$
 (4)

The model of the machine - The main loop

Dividing the enthalpy drop by the stages number we find the enthalpy drop of the single stage that is also the *Euler work* of the stage.

$$l_{eu} = u \cdot (v_{1T} - v_{2T}) = \frac{h_T^0 - h_T^{\text{end}}}{\mathsf{N}_{\text{stages}}} \tag{5}$$

The first quantity in which we are interested is the peripheral speed U. We can obtain it from the mass flow rate:

$$\dot{m} = \rho_0^{\ 1} \cdot v_A \cdot S = \rho_0 \cdot v_A \cdot \pi \left(\frac{b}{D_{\mathsf{mid}}}\right) D_{\mathsf{mid}}^2 \tag{6}$$

Having defined both $\phi = v_A/{\rm U}$ and $D_{\rm mid} = 60/(2 \pi \, n) \, {\rm U}$ the mass flow rate is only function of the the cube of the peripheral speed.

 $^{^1}$ The static density is unknown but since we are iterating we keep the total one as first guess ad we update it at every cycle

The model of the machine - The velocity triangle

To design the velocity triangle we have to make some choices:

- repeated stage, so $v_0 = v_2$
- constant axial velocity $v_{0A} = v_{1A} = v_{2A} = v_A$
- the first stator has an axial inlet so it can break the repeated stage rule. In this case its velocity will be $v_0=v_A$

Now what we need are:

- U previously found from equation 6;
- $v_A = \phi \cdot \mathsf{U}$
- $v_{1T} v_{2T} = \Delta v_T = -\frac{l}{\mathsf{U}}$
- $v_{1T} + v_{2T} = 2 \, \mathsf{U} \cdot (1 \chi_{\mathsf{mid}})$

The model of the machine - The velocity triangle

As we can see in figure 1 the outlet velocity of the stage is almost but not exactly axial.

The **first stage** will have a reaction degree at midspan slightly different from the desired.

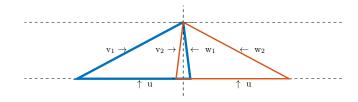


Figure: Example of result of a velocity triangle

The model of the machine - The inlet of the machine

After the definition of the velocity triangle the next step is the thermodynamical analysis, and we start from point 0. From the definition of total entropy we can get the static one:

$$h_T^0 = h_0 + \frac{v_1^2}{2} \quad \Rightarrow \quad h_0 = h_T^0 - \frac{v_A^2}{2}$$
 (7)

To get the static pressure and temperature we have to remember that:

• the static entropy is equal to the total entropy $s_0 = s_T^0 = s(p_T^0, \mathsf{T}_T^0)$.

Than we can obtain the static pressure remembering that the total pressure is obtained stopping the flow in an isoentropic way and measuring the pressure.

$$p_0 = p(s_0, h_0) (8)$$

From the two thermodynamical quantities we can obtain all the others. $^{13\ of\ 62}$

The model of the machine - Losses definition

In point 0 we know everything so we can calculate the blade height:

$$b_0 = \frac{\dot{m}}{\rho_0 \,\pi \, D_{\mathsf{mid}} \, v_A \,\varepsilon} \tag{9}$$

To perform the thermodynamical analysis of the stator we have to introduce the calculation of the losses. In particular according to the definition of the total pressure loss.

$$Y_{\text{stator}} = \frac{p_{T0} - p_{T1}}{p_{T1} - p_1} \tag{10}$$

$$Y_{\text{rotor}} = \frac{p_{T1}^{\text{rel}} - p_{T2}^{\text{rel}}}{p_{T1}^{\text{rel}} - p_2} \tag{11}$$

Where p_T^{rel} is the total pressure in the relative frame.

The model of the machine - Solution of the stator

If we obtain the loss coefficient we are able to find all the thermodynamical quantities at the outlet of the stator (Point 1). The solution is a bit trickier than the previous since it is iterative. Both p_{T1} and p_1 shares the same entropy that is unknown. In both cases we know enthalpy but as known we miss another quantity. Substituting p_{T1} and p_1 into equation 10 leave that equation with just the entropy as unknown.

$$\mathsf{Y}_{\mathsf{stator}} - \frac{p_{T0} - p_{T1}(h_{T1}, s)}{p_{T1}(h_{T1}, s) - p_{1}(h_{1}, s)} = 0 \tag{12}$$

where: $h_1 = h_{T1} - v_1^2/2 = h_{T0} - v_1^2/2$.

Equation 12 is implicit and non-linear and have to be solved numerically since requires the computation of the steam table. We apply the secants method to get a solution in terms of entropy.

The model of the machine - Solution of the rotor

The same procedure can be applied to the rotor with the difference that in this case we use relative quantities.

$$p_T^{\mathsf{rel}} = p\left(h + \frac{w^2}{2}, s\right) \tag{13}$$

$$h_2 = h_{T2}^{\mathsf{rel}} - \frac{w_2^2}{2} = h_{T1}^{\mathsf{rel}} - \frac{w_2^2}{2} \tag{14}$$

Total relative enthalpy conserves along the rotor in an axial machine.

$$Y_{\text{rotor}} - \frac{p_{T1}^{\text{rel}} - p_{T2}^{\text{rel}}(h_{T2}^{\text{rel}}, s)}{p_{T2}^{\text{rel}}(h_{T2}^{\text{rel}}, s) - p_2(h_2, s)} = 0$$
 (15)

We solve equation 15 numerically as the stator case, and from the entropy the solve completely the outlet of the rotor.

The model of the machine - Span evolution

Up to now we have only considered each section uniform along the span. Clearly this is an approximation since the peripheral speed u changes with the radius.

To improve the quality of the result we consider that the velocity triangles evolve along the span with a specific rule.

The approach chosen is the **constant angle**. It cannot guarantee that the work is constant along the blade height, but since the is only few centimetre height, this approach is good enough.

ADVANTAGE ⇒ Lower manufacturing cost.

We can solve the velocities along the span considering:

- the radial equilibrium,
- $v_T/v_A = \tan(\alpha_1) = \text{constant}.$

The model of the machine - Span evolution

Applying the previous relation into the radial equilibrium we obtain:

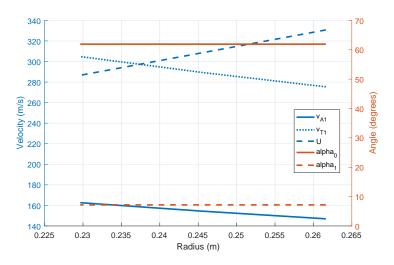
$$v_A \frac{\partial v_A}{\partial r} + \frac{v_T}{r} \frac{\partial (r v_T)}{\partial r} = 0$$
 and $\frac{v_T}{v_A} = \tan(\alpha_1)$ (16)

The solution to equation 16 is:

$$\begin{cases} v_A(r) = v_A^{\mathsf{mid}} \cdot \left(\frac{r_{\mathsf{mid}}}{r}\right)^{\sin^2(\alpha_1)} \\ v_T(r) = v_A(r) \cdot \tan(\alpha_1) \\ l(r) = u \, \Delta v_T = \omega \, r \, \left(v_{T1}(r) - v_{T2}(r)\right) \neq \mathsf{constant} \end{cases}$$

For the rotor is the same except that we substitute the absolute velocity with the relative one

Figure: Sketch of velocity and angle evolution in a stator.



The model of the machine - Section solution

The span velocity is required to compute a better value for the blade height from the mass flow rate.

$$\dot{m} = \int_{r_{\mathsf{hub}}}^{r_{\mathsf{tip}}} \rho(r) \cdot 2 \,\pi \, r \cdot v_A(r) dr \tag{17}$$

Equation 17 cannot be solved analytically to find $b=r_{\rm tip}-r_{\rm hub}$ since the density depends on the steam table.

The solution in to approximate the integral with a summation for some points along the span.

The number of point an input, but their location is chosen smartly.

The solution is in the form:

$$\dot{m} \simeq \sum_{i}^{\text{N points}} w_{i} \cdot \rho_{i} \, v_{A_{i}} \, \pi D_{\text{mid}} \, \varepsilon \, b = b \cdot \left(\pi D_{\text{mid}} \, \varepsilon \cdot \sum_{i}^{\text{N points}} w_{i} \cdot \rho(r_{i}) \, v_{A}(r_{i}) \right)$$
(18)

20 of 62

The model of the machine - Section solution

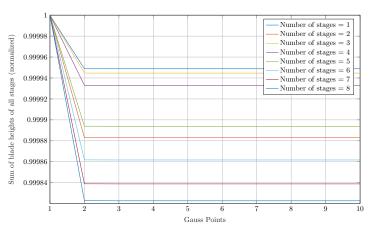
The radius at which we compute the solution are chosen according to the following rule:

$$r_i = \frac{D_{\mathsf{mid}} - b}{2} + x_i \cdot b \tag{19}$$

Where the first guess of b is taken from the midspan solution. The coefficients x_i and w_i are taken form the **Legendre-Gauss** quadrature technique. For the case of 4 points for example the coefficients result:

x_i	w_i
0.9306	0.1739
0.67	0.3261
0.33	0.3261
0.0694	0.1739

Figure: Influence of the choise of the number of Gauss points



We can notice that the difference is absolutely negligible for a number of Gauss points greater than 2. For safety region we will use 4 points to be sure not to lose precision and to have a better span evolution

The model of the machine - Section solution

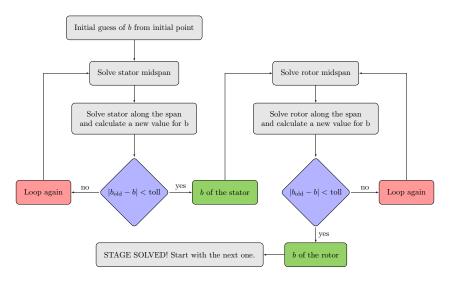
So we start a new iterative process to find the **blade height**. In this process we need the density along the span.

 \Rightarrow We have to pass from the loss coefficient.

Loss coefficient accounts for 3 contributions:

- profile losses ← local property
- clearance losses ← global property
- secondary losses \leftarrow global property

So along the span we compute the specific profile losses that depends on the velocity triangle, but we keep the others from the midspan. So we have two other iterative loops inside the main one.



The model of the machine - The end of the loop

Repeating the loop sketched in the previous slide for all the stages we completely solve the thermodynamics of all the points of interest. From all the data we collect we can verify that the total pressure ratio is that desired and we can compute the efficiency of each stage.

Finally we can calculate the most important parameter, the **total to total efficiency** that we have guess at the beginning.

$$h_{T_{\text{end}}}^{\text{isoentropic}} = h(p_T^{\text{end}}, s_0) \tag{20}$$

$$\eta_{\rm TT} = \frac{h_{T0} - h_T^{\rm end}}{h_{T0} - h_{T_{\rm end}}^{\rm isoentropic}} \tag{21}$$

If convergence is not reached the main loop start again.

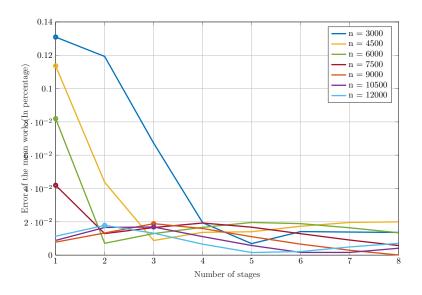
The model of the machine - Effective work

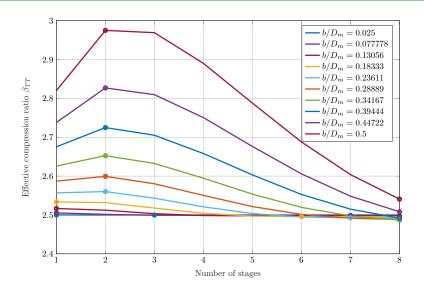
Applying constant angle methodology the work is not constant along the span, so per each stage we can compute the effective work and compare to the real one.

If the difference is significant we have to redesign the velocity triangles to guarantee that the expansion work is that desired. We can take a mean along the span weighting on the mass flow or on the Gauss weight.

$$l \simeq \frac{\sum_{i}^{\mathsf{N points}} w_{i} \cdot u_{i} \cdot (w_{1T_{i}} - w_{2T_{i}}) \cdot \pi \, D_{\mathsf{mid}} \, b \cdot \rho_{i} \, v_{Ai}}{\dot{m}} \tag{22}$$

The difference due to small blade height in high pressure part of the turbine is negligible.





In mid or low pressure turbine the assumption could be not true.

The model of the machine - The losses

Up to now we have simply spoken of rotor and stator losses in a generic way without explaining how to evaluate them. The losses are divided in 3 groups:

- Profile losses:
- Secondary losses;
- Clearance losses.

As previously stated the last two are evaluated by the model as mean losses along the section.

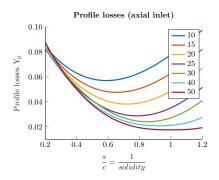
The calculation of profile losses is performed according to the **Ainley-Mathieson** correlation.

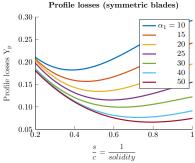
The calculation of others instead is performed according to the **Dunham-Came** correlation.

The model of the machine - Profile losses

Profile losses are calculated in a general case with a linear interpolation between:

- axial inlet with $\chi = 0.5$ ($\alpha_0 = 0$)
- symmetrical blades ($\alpha_0 = -\alpha_1$)





The model of the machine - Profile losses

We compute the total pressure losses in both cases and than we apply the following interpolation:

$$Y_{p_{\text{reference}}} = Y_A + m_\alpha^2 \left(Y_B - Y_A \right) \tag{23}$$

Where $m_{\alpha} = -\alpha_0/\alpha_1$. Then we apply further corrections:

$$\begin{split} \bullet \text{ Max thickness: } Y_p &= Y_{p_{\text{ref}}} \cdot \left(\frac{t_{\text{max}}/c}{t_{\text{max}}/c|_{\text{ref}}}\right)^{m_{\alpha}} \\ \bullet \text{ Reynolds: } Y_p &= Y_{p_{\text{ref}}} \cdot \left(\frac{\text{Re}_{\text{ref}}}{\text{Re}}\right)^{0.2} \\ \bullet \text{ Trailing edge: } Y_p &= Y_{p_{\text{ref}}} \cdot \left(1 + 7\left.\left(\frac{t}{s} - \frac{t}{s}\right|_{\text{ref}}\right)\right) \end{split}$$

Where the Reynolds number is that referred to the chord and values:

$$Re = \frac{\rho_0 \, \overline{v} \, c}{\mu_0} \tag{24}$$

The model of the machine - The solidity

The main parameter that influences profile losses is the **solidity** Even if it can be chosen arbitrarily it must be subjected to the number of blades that is instead an *integer*. So the solidity is computed passing through a rounding operation.

$$N_{\text{blades}} = \text{round}\left(\frac{\pi \, D_{\text{mid}}}{c} \cdot \sigma\right) \tag{25}$$

And then we recalculate the solidity.

$$\sigma = \frac{c}{s} = \frac{c}{\pi D_{\text{mid}}} \cdot N_{\text{blades}}$$
 (26)

Than when we calculate profile losses along the span we keep that number of blades and recalculate the solidity in the same way as equation 26 with the new value of the diameter.

The model of the machine - Secondary losses

Secondary losses are accounted as follow:

$$Y_{\text{secondary}} = 0.0334 \frac{\cos(\alpha_1)}{\cos(\alpha_0)} \cdot H \tag{27}$$

$$H = 4\frac{c}{b}(\tan\alpha_0 - \tan\alpha_1)^2 \cdot \frac{\cos^2(\alpha_1)}{\cos(\alpha_m)}$$
 (28)

$$\tan \alpha_m = \frac{\tan \alpha_0 + \tan \alpha_1}{2} \tag{29}$$

Every loss equation in the way it is written holds for a stator, for a rotor instead we must replace

•
$$\alpha_0 \Leftrightarrow \beta_1$$

•
$$\alpha_1 \Leftrightarrow \beta_2$$

The model of the machine - Clearance losses

Secondary losses are accounted as follow:

$$Y_{\text{clearance}} = B \left(\frac{k}{c}\right)^{0.78} \cdot H \tag{30}$$

Where H has been previously defined in equation 28 The term k is the clearance normalized respect to the number of seals.

$$k = \left(\frac{\text{clearance}}{\text{seals number}}\right)^{0.42} \tag{31}$$

The coefficient B distinguish between the stator and the rotor and for the type of casing.

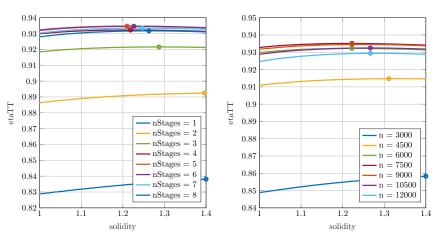
$$B = \begin{cases} 0 & \text{stator} \\ \mathbf{0.37} & \text{shrouded rotor} \\ 0.47 & \text{unshrouded rotor} \end{cases}$$

The choice of parameters - Reference condition

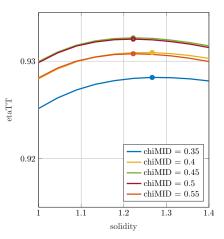
When testing many configurations, we keep for parameters which are note analysed the following values:

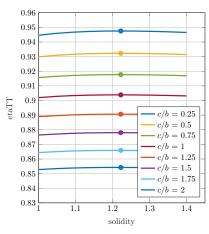
- solidity $\sigma = 1.225$
- number of stages $n_{\text{stages}} = 4$
- $\bullet \ \ {\rm rotational \ speed} \ n=6000\, {\rm rpm} \\$
- reaction degree at midspan $\chi=0.5$
- flow coefficient $\phi = 0.5$
- partial admission $\varepsilon = 1$
- blade height over mean diameter $b/D_{\sf mid} = 0.05$
- blade height over chord b/c = 0.5
- seals number = 2
- clearance = $0.0006 \, \text{m}$

The choice of parameters - The solidity



The choice of parameters - The solidity

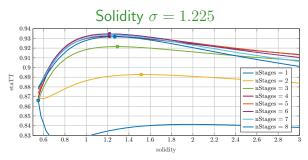




The choice of parameters - The solidity

Even if the solidity is a key parameter for the performances of the machine, keeping into a reasonable range suggested by the theory, it not influences too much the efficiency.

From the analysis of the previous graphs seems reasonable to pick a value of the solidity of 1.225;



The choice of parameters - The solidity

Comparison of the solidity with loading criterion of Ainley.

As we see in figure 4 the value that we take is very similar to that predicted by the Ainley loading criterion.

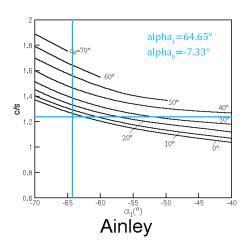


Figure: Ainley criterion

The choice of parameters - The reaction degree

The reaction degree at midspan is also a relevant parameter.

If too low, the stator is too loaded.

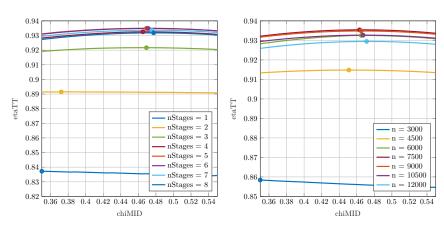
If too high the load is mainly on the rotor.

From the theory we know that a mean between this two extreme condition let to have a good result.

So we check what is the reaction degree that gives the better performances around $\chi=0.5.$

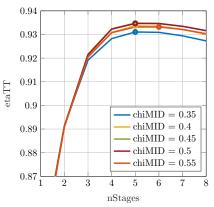
The choice of parameters - The reaction degree

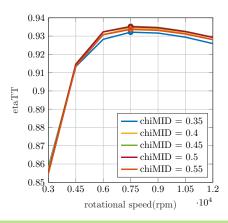
A good value for χ seams to be 0.47, just a bit less than 0.5.



The choice of parameters - The reaction degree

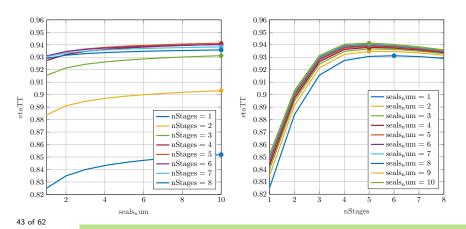
It is also interesting to analyse the graphs in a reverse way, to start to check the best value for the *number of stages* and the *rotational speed*.





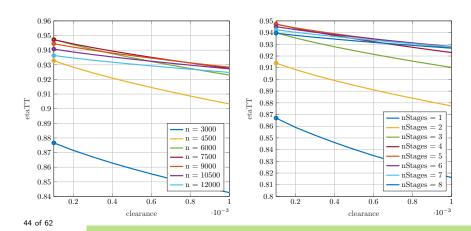
The choice of parameters - Clearance and seals

From 1 to 2 seals we have a good increase in efficiency that is decreases so we take 2 seals per rotor.



The choice of parameters - Clearance and seals

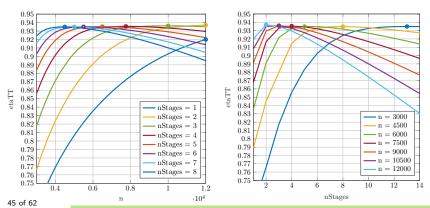
Decreasing the clearance the efficiency increases. We take as clearance $k \approx 0.02 \cdot b \approx 0.5$ mm.



The choice of parameters - Stages and speed

For any number of stages exists a proper value of the rotational speed that let to reach almost the same efficiency. Critical decision:

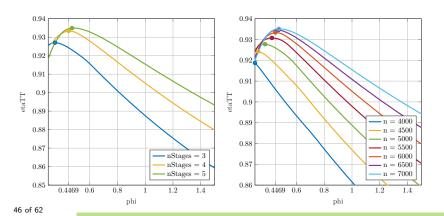
- \Rightarrow at 3000rpm, good efficiency for over 10 stages.
- \Rightarrow for 2 stages, good efficiency for n above 10000 rpm.



The choice of parameters - Stages and speed and phi

For $\phi = 0.4469$ we have the optimal efficiency at:

 \Rightarrow 4 stages; \Rightarrow n = 6000 rpm;



The design of the blades

The blades are designed according two different rules:

- NACA 4-digits for the mean line;
- A_3K_7 for the thickness.

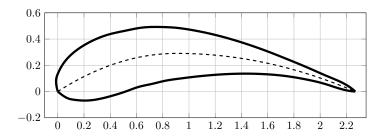


Figure: Example of a rotor blade with max thickness over chord of 0.2

The design of the blades - The mean line

Calling m the maximum of the mean line and p the position between 0 and 1 at which the maximum is present, the equation of the mean line are represented by a parabola.

$$B = \begin{cases} \frac{m}{p^2} (2 p x - x^2) & \text{for } x$$

The advantage of this approach is that we can easily calculate everything we want of the mean line like the slope or the osculating circle radius.

It is reasonable to take p = 0.4.

The design of the blades - The mean line

We miss the parameter m.

We can obtain it from the camber angle required and from p.

$$\tan \theta = \tan(\alpha_0 + \alpha_1) = \frac{\tan \alpha_0 + \tan \alpha_1}{1 - \tan \alpha_0 \tan \alpha_1}$$
(32)

Where $\tan \alpha_0 = \frac{2m}{p}$ and $\tan \alpha_1 = \frac{2m}{1-p}$.

Finally m is the solution of the following equation.

$$4 \tan(\theta) m^2 + 2m + p(p-1) \cdot \tan \theta = 0$$
 (33)

The design of the blades - The thickness

Since the points are quite rough we apply a spline interpolation to get a smoother profile.

To find the top and bottom points of the blade we project the thickness perpendicular to the mean line

$$x_{\text{up}} = x - t \cdot \sin(\tan^{-1}(y'_c))$$
 $x_{\text{down}} = x + t \cdot \sin(\tan^{-1}(y'_c))$ (34)
 $y_{\text{up}} = x + t \cdot \sin(\tan^{-1}(y'_c))$ $y_{\text{down}} = x - t \cdot \cos(\tan^{-1}(y'_c))$ (35)

x/c(%)	0	1.25	2.5	5	10	15	20	25	30	35	40	45
t/c	0	3.469	4.972	6.916	9.007	9.827	10	9.699	9.613	9.106	8.594	7.913
x/c(%)	50	55	60	65	70	75	80	85	90	95	100	
t/c	7.152	6.339	5.5	4.661	3.846	3.067	2.406	1.83	1.367	1.101	0	

Table: Thickness along the mean line.

The design of the blades - The deviation

Even if the blades drive the flow, at the trailing edge we have just one side that keep it in the desired direction.

 \Rightarrow The flow deviates.

Theroretical 1D approach can provide the result, but the semi-experimental correlations are more reliable.

Ainley and Mathieson provide a correlation for the deviation given:

• the geometrical angle;

51 of 62

- the curvature of the suction side e^2 ;
- the mach number if greater than 0.5.

²We approximate it by computing the radius of the circumference passing through the last 3 points of the mean line.

The design of the blades - The deviation

We know how to calculate the real flow angle from the geometrical angle but we are interested in the *opposite process*.

We need a specific flow angle that provides the work required by the compression.

 \Rightarrow We want to know the geometric angle of the blade.

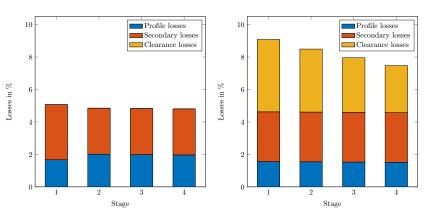
$$\alpha_{\text{flow}} = f(\alpha_{\text{geom}}) - 4 \frac{s}{e(\alpha_{\text{geom}})}$$
 (36)

In the case ${\cal M}$ is greater than 0.5 we must apply a further correction. We solve equation 36 by applying the secants method.

From that solution \Rightarrow design the blade that provides the required work also accounting for the deviation angle.

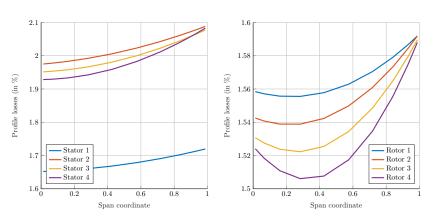
The results - Losses

Losses of the four stages for the stators and the rotors.



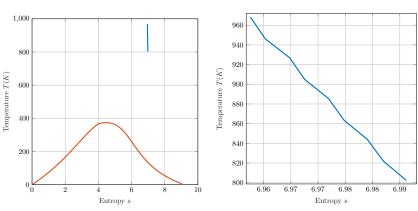
The results - Profile losses along the span

Losses of the four stages for the stators and the rotors.

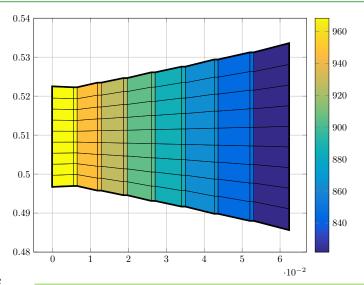


The results - TS transformation

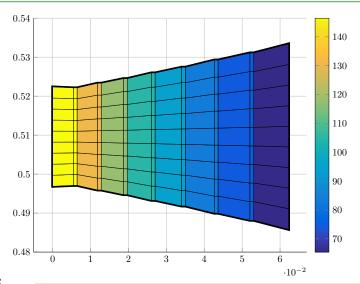
Representation of the stages in the T-S plane and the comparison with the liquid vapour curve.



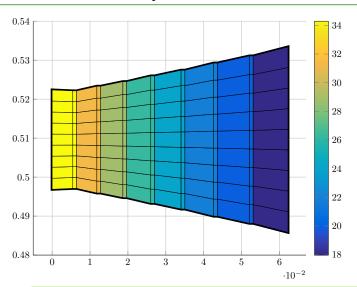
The results - The temperature



The results - The pressure



The results - The density



The results - The cascade

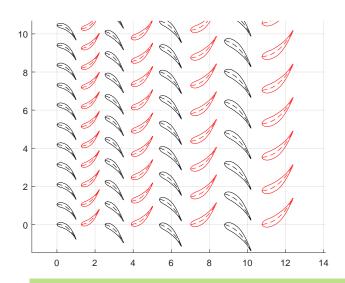


Table: Stator properties

Table: Grate: properties							
Stage	Blades	Solidity	β	Blade $h.(m)$	Inlet M	C_p	
1	156	1.219	1.121	0.025	0.194	2524.1	
2	131	1.217	1.124	0.027	0.463	2480.5	
3	110	1.223	1.130	0.033	0.473	2437.8	
4	91	1.220	1.136	0.039	0.484	2395.9	

Table: Rotor properties

Stage	Blades	Solidity	β	Blade $h.(m)$	Inlet M_w	C_p
1	143	1.223	1.110	0.025	0.198	2501.2
2	119	1.214	1.114	0.030	0.202	2458.2
3	99	1.214	1.119	0.036	0.207	2415.8
4	82	1.218	1.125	0.043	0.212	2374.3

Table: Stage properties

Stage	χ	η_{TT}	β	β_{TT}			
1	0.461	0.927	1.244	1.240			
2	0.465	0.930	1.252	1.251			
3	0.465	0.933	1.264	1.263			
4	0.465	0.936	1.278	1.276			
Total	-	0.936	2.518	2.500			

Further develops

- Compare constant angle with other methodologies;
- Iterate to guarantee the required work along the span;
- Different solidity for the rotor and the stator;
- Variable χ with stages (lower in the first stages);
- Off-design performances;
- Mechanical analysis
 - Optimize rotor internal diameter.
 - Vibration analysis and natural frequencies.
- Apply more refined losses correlation.
- · Clearance design;
- Cost/performance comparison between 3000 and 6000 rpm.