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DEVIATION IN AXIAL TURBINES AT SUBSONIC CONDITIONS

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ABSTRACT

An improved correlation is presented for the deviation in axial turbines at subsonic flow conditions. Development of the correlation began with the assumption that the deviation is primarily determined by the blade loading (pressure difference) towards the trailing edge. The blade row parameters which influence this pressure difference are identified. A functional form for the correlation was then assumed and the unknown coefficients in the function were determined based on a database of known values of the deviation. This database includes in-house experimental results, estimates of deviation obtained from Navier-Stokes simulations, as well as cases from the literature for which sufficient geometric data are provided. The evaluation of the empirical coefficients of the correlation was treated as an optimization problem. The optimization was performed using a genetic algorithm which proved very effective and robust. The new deviation correlation appears to be significantly more successful than existing correlations.

NOMENCLATURE

AVDR $= \rho_2 V_{x2} / \rho_1 V_{x1}$ = axial velocity density ratio

c = blade chord

$C_p = \left(P - P_{CL}\right) / \frac{1}{2}\rho V_{CL}^2$ = static-pressure coefficient

C_x = blade axial chord length
e = suction-side radius of curvature from throat to trailing edge
i $= \alpha_1 - \alpha_{1des}$ = incidence
o = throat opening
P = pressure
s = blade pitch or spacing
t_m = blade maximum thickness
V = velocity
α = flow angle, measured from the axial direction
β = blade metal angle, measured from the axial direction
ξ = blade stagger angle, measured from the axial direction
δ = flow deviation
ρ = fluid density

Subscripts

CL = centerline value at inlet
ref = reference value
x = axial direction
1,2 = cascade inlet and outlet respectively

INTRODUCTION

The prediction of blade outlet flow angle is an important step in the preliminary design phase of a turbine. The estimation of optimum work output and inlet flow angle of successive stages of a turbine are intrinsically related to the accurate prediction of the deviation, the difference between the metal angle and the flow angle at the trailing edge. Thus, empirical deviation correlations continue to play an important role in the early stages of blade design and there is an ongoing need to review and improve these correlations. Unfortunately, relatively little attention seems to have been paid to this aspect of turbine blade design.

The lack of agreement among several published correlations for turbine blade deviation was noted by Riess et al. (1986). The authors concluded that most of the correlations were valid only for particular profile forms and narrow ranges of cascade geometry and operating conditions. The most reasonable among the alternatives and perhaps the most widely used is that due to Ainley & Mathieson (1951) which is expressed in terms of flow outlet angle rather than the deviation as such. At the choking condition, for "straight-backed" blades, Ainley & Mathieson assume that the outlet flow angle is equal to the gauge angle, $\cos^{-1}(o/s)$, where o is the throat opening. For subsonic conditions and for blades with unguided turning, the outlet angle is given as a function of the gauge angle, Mach number and s/e, where e is the radius of curvature of the blade suction side from the throat to the trailing edge. Another well-established subsonic deviation rule is that due to Carter & Hughes (1950). This correlation was originally developed for compressors, but the form recommended for compressor inlet guide vanes has also been used in turbine analysis. Carter & Hughes correlate the deviation with the blade stagger angle, blade camber, and the solidity. The correlations mentioned were developed using data for blade designs of the 1940's and 50's and therefore do not account for recent improvements in airfoil design.

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In view of the shortcomings of the existing correlations, an investigation was made to re-examine the factors that influence subsonic deviation and, if possible, to develop an improved correlation.

The paper begins with a discussion of the factors that influence deviation. It is concluded that additional parameters have significant effects on deviation compared with those used in the traditional correlations. Based on this, a new functional relationship between deviation and the various parameters is proposed. A database has been compiled of the deviation values for turbine blades of recent design. This database includes in-house experimental measurements as well as measurements taken from the literature. The experimental results are supplemented with computations using a Navier-Stokes solver in order to have as wide a range as possible of the important geometric parameters. The values of the empirical coefficients in the new correlation function have been determined using this database.

FACTORS INFLUENCING DEVIATION

The investigation began with the premise that subsonic deviation is largely determined by the pressure difference towards the trailing edge of the blade. As the suction and pressure surface flows meet at the trailing edge and adjust to their common downstream static pressure, there will be a tendency for the pressure side flow, as it expands, to turn towards the suction side. The connection between the magnitude of the deviation and the loading towards the aft of the blade has already been noted by Denton & Xu (1990) and Roberts & Denton (1996). Implicitly it is also assumed that viscous effects have little direct influence on the deviation. There is some support for this assumption in the measurements of Kind et al. (1998). The authors measured the losses and deviation for the flow through a turbine cascade in which patches of roughness of varying extent and severity had been applied to the surfaces. The roughness resulted in a wide range of boundary layer thicknesses at the trailing edge while the blade loading was essentially constant. The variation in the boundary layer thickness had very little effect on the measured deviation. Of course, if the blade loading is such that the suction surface boundary layer separates near the trailing edge then deviation will be largely determined by viscous effects. The present discussion is intended to apply only for fully-attached blade-surface boundary layers.

It follows from the initial premise that the deviation should correlate with the geometric parameters which determine the blade loading towards the trailing edge. This was the basis for identifying the correlating parameters.

The blade loading towards the trailing edge will vary according to the total blade loading (the total lift generated) and how that loading is distributed.

The total blade loading is determined primarily by two parameters: the total amount of flow turning and the spacing between blades in the row (or more precisely by the spacing-to-chord ratio, s/c). The closer the blade spacing the smaller the loading since the total tangential force is carried by a larger number of blades. The total flow turning in the blade passage is the sum of the inlet and outlet flow angles, measured from the axial direction. In the deviation correlation of Carter & Hughes (1950) the flow turning is approximated by the blade camber angle. This neglects the effect of both incidence and deviation on the flow turning. Since the inlet flow direction will be known, there seems no reason not to use the inlet flow angle, as opposed to the metal angle, in specifying the flow turning. Also, this makes the correlation potentially valid for off-design values of incidence. The outlet flow angle is itself a function of the deviation. Thus, it would require iteration to apply a deviation correlation which uses the true flow turning. In the present work, the total flow turning is approximated by the sum of the inlet flow angle and the outlet metal angle. Since the deviation is generally small compared with the total flow turning in turbines, this approximation introduces relatively little error. It would be less satisfactory in the case of compressor flows.

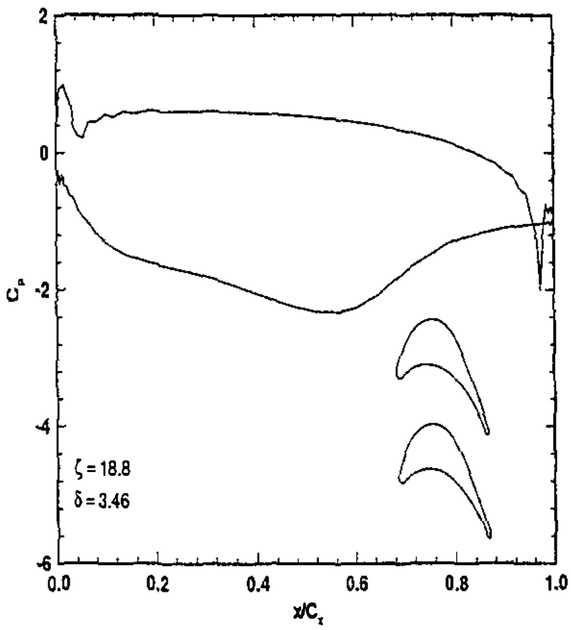
The chordwise distribution of the loading is determined by other geometric parameters. Korakianitis & Papagiannidis (1993) have noted the effect of stagger angle and suction-side curvature on the loading distribution. The effect of stagger angle is considered first.

Figure 1 shows the surface pressure distributions for two cascades with stagger angles of 18.8 and 28.8 degrees but same inlet and outlet blade metal angles. The pressure distributions were calculated using the Navier-Stokes code of Dawes (1988). As stagger angle is increased the thickness of the blade has to be reduced to maintain the same throat opening. More importantly, to obtain the same metal angles at inlet and outlet, the higher stagger blade must produce more of its turning in the forward part of the passage. As a result, as stagger angle increases the blade tends to become more forward loaded, as confirmed by the predicted pressure distributions. The resulting reduced pressure differences towards the trailing edge should tend to reduce the deviation. This is what was predicted by the code, as indicated on the figures.

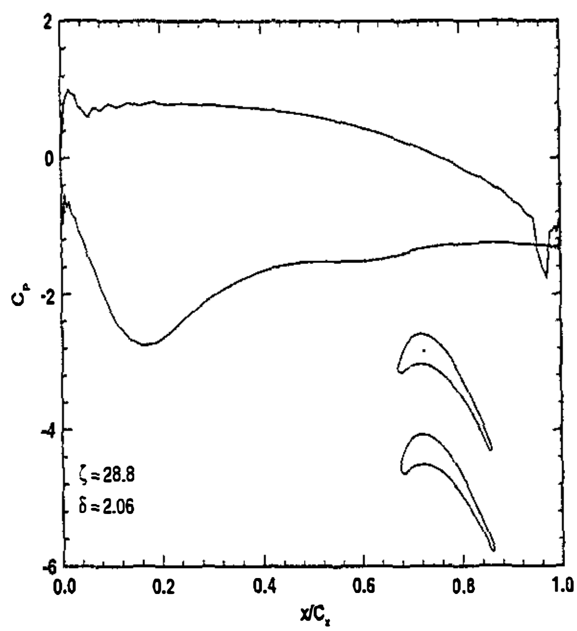
The effect of suction side curvature is illustrated in Figure 2. The suction side curvature would be an inconvenient parameter to use in a correlation since it will not be known in the early stages of design. However, the increased curvature tends to be obtained by increasing the maximum thickness of the blade. Therefore, in the present work the maximum-thickness-to-chord ratio is used as an indirect measure of the suction-side curvature. As shown in the figure, increasing the suction-side curvature has the effect of increasing the strength of the suction peak on the suction side and thus making the blade apparently more forward loaded. However, it is seen that this has less effect on the pressure difference towards the trailing edge than might have been expected, due to the adjustment of the pressure on the pressure side of the blade. This suggests that suction-side curvature (or t_m/c) may have a somewhat weak influence on the deviation. This was confirmed by the final deviation correlation but t_m/c is retained as a correlating parameter.

The previous discussion leads to the conclusion that the geometric and flow parameters that influence deviation most are the total flow turning angle, the solidity, the thickness-to-chord ratio and the stagger angle. An initial attempt was made to correlate the deviation on these parameters. While the correlation appeared somewhat better than the existing ones, there was sufficient scatter to suggest that one or more significant parameters were missing. It was subsequently found that including the blade inlet metal angle as a correlating parameter substantially improved the quality of the correlation.

The effect of inlet metal angle on deviation is not immediately obvious. However, it is noteworthy that Ainley & Mathieson (1951) correlated profile losses in turbines with the blade inlet angle because of its effect on the blade loading. For a given total flow turning, two extremes exist: an inlet angle of zero (a nozzle row) gives maximum flow acceleration while an impulse row gives essentially zero acceleration. Ainley & Mathieson found that the resulting differences in the pressure distributions led to different profile losses. In their loss system, the profile losses for a given blade row are obtained as a weighted average of the values at the two extremes of nozzle and impulse rows with the same flow turning. Since the inlet metal angle affects the pressure distribution around the blade, it is plausible that it would affect the deviation at the trailing edge as well as the losses. It could be argued the inlet flow angle, rather than the metal angle, should be used to correlate



(a) Stagger Angle of 18.8 Degrees



(b) Stagger Angle of 28.8 Degrees

Figure 1 Surface Pressure Distributions of Cascades with Stagger

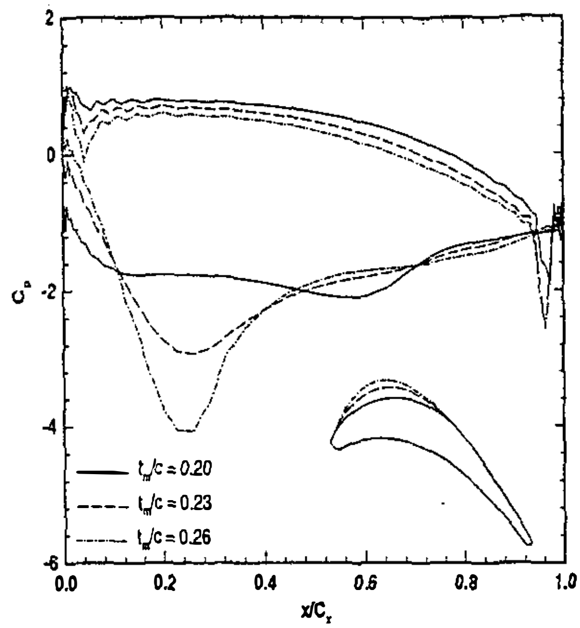


Figure 2 Surface Pressure Distributions of Blades with Varying Maximum Thickness

this effect. However, the inlet metal angle is convenient and the effect of incidence is already included in the total flow turning.

One final parameter was included in the deviation correlation: the axial velocity density ratio (AVDR). AVDR imposes on the flow in the passage an overall acceleration or deceleration which alters the blade pressure distribution in an effect somewhat analogous to the influence of the inlet metal angle. In an in-house experimental study, Rodger (1992) observed that for turbine blades an increase in the AVDR increased the pressure differential across the trailing edge and resulted in a higher deviation. Rodger suggested a linear trend in deviation with AVDR. An inverse linear variation in deviation with AVDR had been observed by Pollard & Gostelow (1967) for compressors. In developing the present correlation, the power on the term involving AVDR was left open and it was found that in fact an approximately cubic variation gave the best fit. This was based on the data of Rodger as well as a few other experimental cases from the literature. Figure 3 compares the trends in deviation as observed by Rodger with linear and cubic functional relationships given by:

$$\delta = \delta_0 + 14(AVDR - 1) \tag{1}$$

and

$$\delta = (AVDR^3)\delta_0 \tag{2}$$

where δ_0 is the deviation at AVDR = 1.0. It can be seen from the figure that the two curves are difficult to distinguish from each other because the AVDR varies over a very narrow range centred on 1.0. However, the cubic dependence was retained since it gave the best fit for all of the available data.

SOURCES OF DATA

The deviation correlation was developed using a database of 45 cases, as summarized in Table 1. The sources of the data are indicated in the second column.

A total of 16 experimental cases were identified. A number of these were obtained in-house during low-speed cascade studies using three different blade geometries. A number of other cases were located in the open literature for which the required geometric parameters were

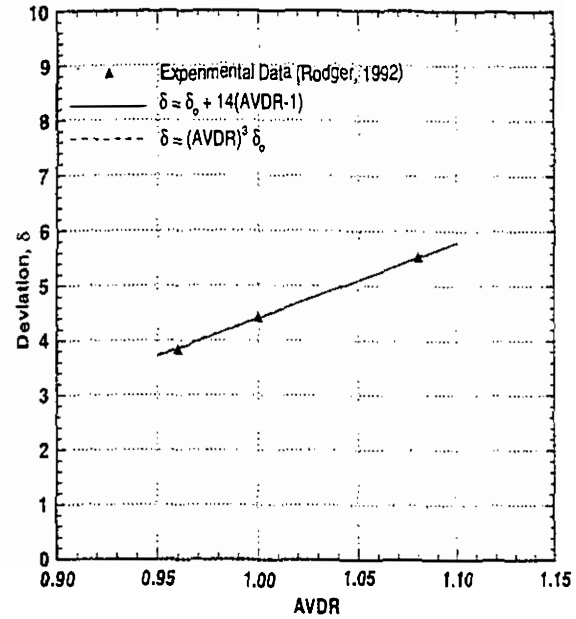


Figure 3 Effect of AVDR on Deviation

available, or could be estimated with acceptable accuracy. It should be noted that the experimental cases include several measurements made at two different times for the identical cascades: cases (3)-(4), (13)-(15) and (14)-(16). The results are seen to agree within about ±0.5 degrees. This is indicative of the uncertainty present in measured values of deviation.

Unfortunately, the experimental data did not cover wide ranges of all six parameters which had been identified as likely to influence the deviation. Additional cases were therefore generated computationally. To verify the validity of this approach, the computations were performed for three of the geometries for which in-house experimental results were available: in Table 1 the companion cases are (1)-(34), (2)-(17), (3)-(18), (15)-(44) and (16)-(45). The computations were obtained using the three-dimensional Navier-Stokes solver of Dawes (1988, 1992). It is seen that the measured and computed values of deviation agree to within, at worst, 1.0 degree, and are usually considerably closer. In view of the estimated ±0.5 degree uncertainty in the measured values, it was felt that the computed values were sufficiently accurate to use as an additional basis for developing the correlation.

The Navier-Stokes solver was then used to calculate the flow through a series of hypothetical turbine blade rows which broadened the range of the parameters for which deviation values were available. The geometry of each hypothetical blade row was defined uniquely by specifying 17 geometric parameters. Leading- and trailing-edge circles were used and the remainder of the pressure and suction surfaces were defined by Bezier cubics. A very similar procedure has been used by Trigg et al. (1997) to generate steam-turbine blade rows. The present procedure differs slightly in that continuous surface curvature was enforced on both sides of the blade from the leading- to the trailing-edge blend points. The additional hypothetical blade rows are given as cases (19) through (45) in Table 1. The resulting ranges of the geometric and flow parameters covered by the cases in Table 1 are summarized in Table 2. Note that all the cases shown in Table 1 were used in the optimization process

OPTIMIZING CORRELATION USING GENETIC ALGORITHM

The genetic algorithm is a powerful and broadly-applicable stochastic search and optimization technique based on principles derived from genetics and evolution. Genetic algorithms have been successfully applied to optimization in the physical sciences, social sciences and engineering (e.g. see Goldberg, 1989, Trigg et al., 1997). Due to their evolutionary nature, genetic algorithms will search for solutions without regard to the specific inner workings of the problem. Particularly in discontinuous and multimodal search spaces they are very effective at rejecting local maxima (or minima) and identifying the true global optimum. These characteristics seemed to make the genetic algorithm well suited for optimizing the coefficients in empirical correlations such as the present one for deviation in axial turbines.

The genetic algorithm used in the optimization process was originally adapted from Goldberg (1989). It was modified to suit the requirements of the present problem but the simple structure of the various operators and functions was retained. Suppose that the deviation at the trailing edge of a turbine blade row is assumed to vary as follows:

$$\delta = A^x B^y C^z D^v E^w$$

where δ is the deviation, A, B, ... E are geometric parameters of the blade row (such as the stagger angle, solidity, etc.) and x, y, ... w are the (unknown) coefficients of the correlation. The goal then is to determine the optimum set of the correlation parameters which gives the best overall agreement between the predicted deviation and that observed experimentally, or predicted computationally, for all the cases in the database. After some experimentation, it was found that repeatable and consistent results could be obtained using a population size of 700, a crossover probability of 0.70 and a mutation probability of 0.002. The population was typically allowed to evolve through 200 generations, by which point few further changes were occurring to the predicted optimum.

NEW CORRELATION

As mentioned earlier, the two most widely used correlations for subsonic deviation in axial turbines appear to be those due to Ainley & Mathieson (1951) and Carter & Hughes (1950).

As noted, the Ainley & Mathieson correlation requires a knowledge of the throat opening and the radius of curvature of the suction side aft of the throat. The throat opening is not available for many of the experimental cases in Table 1, although it is known for the hypothetical blade rows. The suction-side radius of curvature is not known for any of the experimental cases. While it might be possible to estimate an average value of the radius of curvature from the dimensions of the blade and the unguided turning (where known), this value would be approximate at best. For these reasons, no attempt was made to compare the predictions of the Ainley & Mathieson correlation with the deviation values for the blade rows in the database.

The Carter & Hughes correlation is usually expressed as (e.g. see Horlock, 1973):

$$\delta = m\theta\left(\frac{s}{c}\right)^n \qquad (3)$$

where, δ = deviation
θ = blade camber
s/c = blade spacing-to-chord ratio

The multiplier m depends on the stagger angle (ξ) and the shape of the blade camber line: namely whether it is circular-arc or parabolic-arc. Modern turbine blade profiles have mean lines which are neither circular nor parabolic. However, they are probably closer to parabolic than circular. Therefore, the correlation for m for parabolic-arc camber lines has been used for the comparisons here. As noted, the Carter & Hughes correlation was originally developed for compressors. The authors recommended a value of 1/2 for the power n for general compressor blades and a value of 1 for inlet guide vanes. Dixon (1989) has suggested also using n=1 when applying the correlation to turbine blade rows since they, like compressor inlet guide vanes, have accelerating flows. This suggestion has been adopted in the present work.

Figure 4 compares the values of deviation predicted by the Carter & Hughes correlation with the reference values for all the cases in Table 1. The solid diagonal line represents perfect agreement between the predicted and reference deviations. It is seen that for modern turbine profiles the Carter & Hughes correlation almost invariably overpredicts the deviation, and often by a substantial amount. It should be mentioned that the results are even poorer if n= 1/2 is used in Equation (3). It will be noted that the Carter & Hughes correlation uses several of the parameters which were identified earlier as influencing deviation. The poor agreement shown in Figure 4 can be attributed partly to the fact that the correlation was based on old cascade data. However, a more important reason is probably the fact that it omits one or more important parameters.

The final form of the new deviation correlation proposed here is given in Equation (4):

$$\delta = \frac{(AVDR)^3\left(\dfrac{s}{c}\right)^{1.1}(\alpha_t + \beta_2)^{2.25}}{\xi^{1.45}\left(\dfrac{t_m}{c}\right)^{0.3}\left(22 + 0.22\beta_1^{\,1.64}\right)} \qquad (4)$$

where,
δ = flow deviation (in degrees)
α_t = inlet flow angle (in degrees)
β_1 = inlet metal angle (in degrees)
β_2 = outlet metal angle (in degrees)
s/c = blade space-to-chord ratio
t_m/c = blade maximum thickness-to-chord ratio
ξ = stagger angle (in degrees)

The empirical coefficients in (4) are the values obtained using the genetic-algorithm optimization procedure described earlier.

In Equation (4), the effect of the inlet metal angle β_1 is expressed in the form $a+b\beta_1^{\,n}$. The additive constant a was included to allow the correlation to apply for $\beta_1 = 0$; that is, for nozzle guide vanes. The stagger angle might also have been expressed in this way. However, impulse turbines ($\xi = 0$) have rarely been used for gas turbines and therefore no attempt was made to include them in the range of validity of the correlation. Finally, it will be noted that the influence of t_m/c (and thus of suction-side curvature) is relatively weak. This had been anticipated earlier from Figure 2.

Figure 5 compares the deviations predicted by Equation (4) with the reference values for all the cases in Table 1. Bearing in mind the estimated uncertainty of ±0.5 degrees in both the measured and computed values of deviation, the agreement is seen to be very satisfactory. The improvement over the Carter & Hughes correlation (Figure 4) is evident.

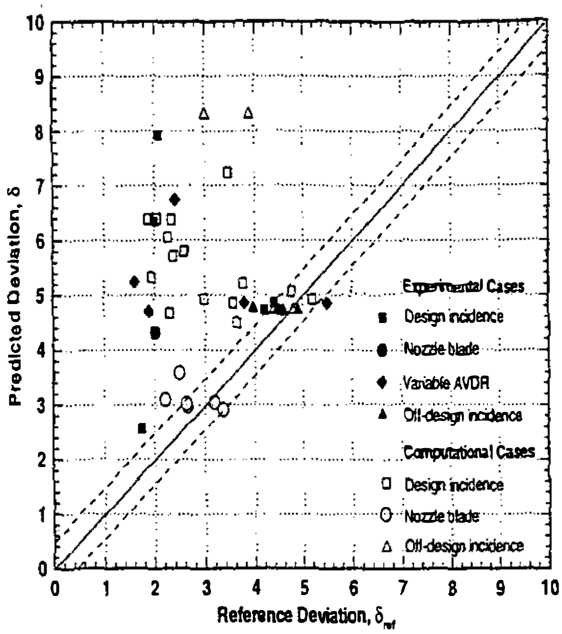


Figure 4 Evaluation of Deviation Correlation of Carter & Hughes

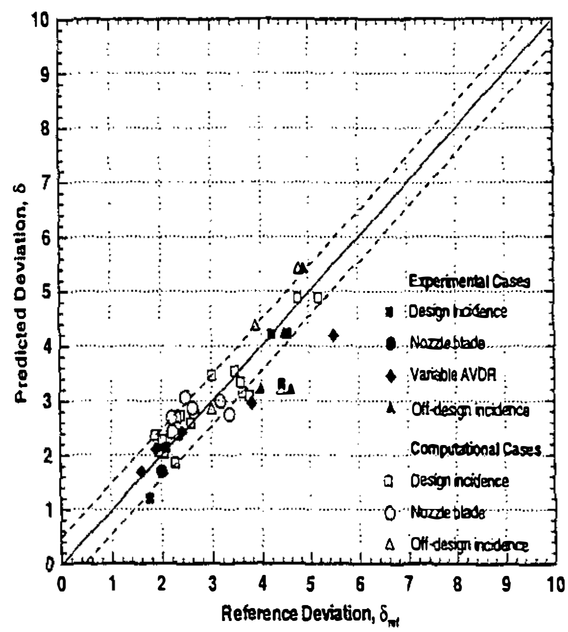


Figure 5 Evaluation of Present Deviation Correlation

The largest discrepancies occur mainly for cases with positive incidence or values of AVDR significantly different from 1.0. In the former, viscous effects in the form of trailing-edge separation may be starting to influence the deviation. The influence of AVDR has been determined from rather sparse data and may therefore be somewhat inaccurately resolved.

It should be noted that the new correlation is based on cascade experiments and computations for which the outlet Mach number was less than about 0.7. For outlet Mach numbers of 1.0 or higher the deviation is dominated by compressibility effects and the outlet flow angle is determined by the gauge angle, the unguided turning and the downstream Mach number. It is anticipated that Equation (4) would be used up to some outlet Mach number limit, such as 0.5 or 0.7. Between this limit and $M_2 = 1$, the deviation might be assumed to vary linearly. The correlations of both Ainley & Mathieson and Carter & Hughes have often been used in this way to estimate the deviation in the high-subsonic Mach number range.

CONCLUSIONS

Starting from the assumption that subsonic deviation is largely determined by the blade pressure difference towards the blade trailing edge, the factors influencing deviation in axial turbines were identified These include parameters which do not appear in earlier correlations for deviation. Based on a database of measured and computed values of deviation for turbine blades of modern design, a revised correlation for deviation was developed. The values of the empirical coefficients in the correlation were optimized using a genetic algorithm. The new correlation appears to be substantially better than existing ones.

ACKNOWLEDGMENTS

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Table 1

Experimental and Computed Deviation for Axial Turbines

Case	Source	Incidence (i)	Experimental Cases							Deviation		Symbol
			ζ	t_m/c	s/c	α_1	β_2	β_1	AVDR	Measured	Present Correlation	
1	De Cecco (1995)	0.0	37.20	0.090	0.600	0.00	49.60	11.10	1.00	1.8	1.21	
2	Rodger (1992)	0.0	23.06	0.182	0.682	28.4	57.50	29.29	1.00	4.4	3.33	
3	Isaacs (1992)	0.0	21.60	0.196	0.682	28.4	57.50	25.35	1.00	4.5	4.22	■
4	Serjak (1995)	0.0	21.60	0.196	0.682	28.4	57.50	25.35	1.00	4.2	4.22	
5	Mee & Baines (1992)	0.0	42.00	0.250	0.840	42.8	70.00	45.00	1.00	2.0	1.70	
6	Haller & Camus (1984)	0.0	29.60	0.375	0.842	56.0	64.50	56.00	1.00	2.1	2.14	
7	Moustapha et al. (1993)	0.0	62.10	0.250	0.840	0.00	76.00	10.00	1.00	2.0	1.70	●
8	Kiock et al. (1986)	0.0	33.30	0.240	0.710	30.0	67.03	33.50	0.929	1.6	1.68	
9	Hodson & Dominy (1990)	0.0	19.60	0.125	0.564	38.8	53.60	42.80	0.91	1.9	2.09	
10	Hoheisel (1990)	0.0	30.72	0.100	0.799	37.7	63.20	48.80	1.014	2.4	2.41	◆
11	Rodger (1992)	0.0	23.06	0.182	0.682	28.4	57.50	29.29	0.96	3.8	2.94	
12	Rodger (1992)	0.0	23.06	0.182	0.682	28.4	57.50	29.29	1.08	5.5	4.20	
		Off-Des										
13	Abbott (1993)	-10.0	21.60	0.196	0.682	18.4	57.50	25.35	1.00	4.0	3.19	
14	Abbott (1993)	+10.0	21.60	0.196	0.682	38.4	57.50	25.35	1.00	4.5	5.40	▲
15	Serjak (1995)	-10.0	21.60	0.196	0.682	18.4	57.50	25.35	1.00	4.6	3.19	
16	Serjak (1995)	+10.0	21.60	0.196	0.682	38.4	57.50	25.35	1.00	4.9	5.40	
		Incidence	Computational Cases							Computed	Present Correlation	
17	3D Navier-Stokes Solver	0.0	23.06	0.182	0.682	28.4	57.50	29.29	1.00	3.57	3.33	
18		0.0	21.60	0.196	0.682	28.4	57.50	25.35	1.00	4.53	4.22	
19		0.0	25.10	0.203	0.728	46.0	59.00	50.50	1.00	1.88	2.37	
20		0.0	25.10	0.200	0.728	46.0	59.00	50.50	1.00	2.34	2.37	
21		0.0	18.80	0.250	0.760	46.0	59.00	50.50	1.00	3.46	3.54	
22		0.0	18.80	0.250	0.600	46.0	59.00	50.50	1.00	2.38	2.67	□
23		0.0	25.10	0.231	0.728	46.0	59.00	50.50	1.00	2.03	2.27	
24		0.0	25.10	0.195	0.728	36.0	59.00	40.50	1.00	2.58	2.59	
25		0.0	25.10	0.202	0.728	25.5	59.00	25.50	1.00	3.00	3.46	
26		0.0	25.10	0.221	0.682	26.9	57.50	25.35	1.00	3.62	3.14	
27		0.0	18.80	0.235	0.682	28.4	57.50	25.35	1.00	5.20	4.90	
28		0.0	18.80	0.260	0.700	28.4	57.50	25.35	1.00	4.76	4.90	

Table 1 (continued)

Case	Source	Incidence (i)	Computational Cases							Deviation		Symbol
			ζ	t_m/C	S/C	α_1	β_2	β_1	AVDR	Computed	Present Correlation	
29	3D Navier-Stokes Solver	0.0	21.6	0.240	0.689	30.0	57.50	33.00	1.00	3.76	3.10	□
30		0.0	28.8	0.200	0.760	46.0	59.00	50.50	1.00	2.06	2.04	
31		0.0	30.1	0.190	0.728	46.0	59.00	50.50	1.00	2.27	1.85	
32		0.0	30.1	0.191	0.728	33.0	59.00	37.50	1.00	1.95	2.06	
33		0.0	30.1	0.195	0.728	25.5	59.00	25.50	1.00	2.31	2.70	
34		0.0	37.2	0.090	0.600	0.00	49.60	11.10	1.00	1.00	1.21	
35		0.0	27.6	0.200	0.780	10.0	49.00	10.00	1.00	2.48	3.06	○
36		0.0	32.1	0.186	0.746	5.0	51.00	5.00	1.00	2.20	2.66	
37		0.0	32.1	0.186	0.746	2.5	51.00	5.00	1.00	2.21	2.40	
38		0.0	27.6	0.226	0.780	0.00	49.00	0.00	1.00	2.64	2.78	
39		0.0	26.9	0.242	0.786	0.00	49.00	0.00	1.00	2.62	2.85	
40		0.0	29.5	0.200	0.749	0.00	51.00	0.00	1.00	3.35	2.75	
41		0.0	26.9	0.235	0.780	0.00	49.90	0.00	1.00	3.18	3.00	
		Off-Des										
42		-10	18.8	0.250	0.760	36.0	59.00	50.50	1.00	3.00	2.82	Δ
43		+10	18.8	0.250	0.760	56.0	59.00	50.50	1.00	3.89	4.34	
44		-10	21.6	0.196	0.682	18.4	57.50	25.35	1.00	4.40	3.19	
45		+10	21.6	0.196	0.682	38.4	57.50	25.35	1.00	4.80	5.40	

Table 2
Range of Blade Parameters

Blade Parameters	Maximum	Minimum
Stagger (ζ)	62.10	18.80
Maximum Thickness-to-Chord Ratio (t_m/C)	0.375	0.09
Space-to-Chord Ratio (S/C)	0.842	0.564
Inlet Metal Angle (β_1)	56.00	0.00
Inlet Flow Angle (α_1)	56.00	0.00
Outlet Metal Angle (β_2)	76.00	49.00
Axial Velocity Density Ratio (AVDR)	1.08	0.91
Deviation (δ)	5.50	1.60