MODELING TECHNIQUES FOR FLUID MACHINES

Steps of a CFD simulation

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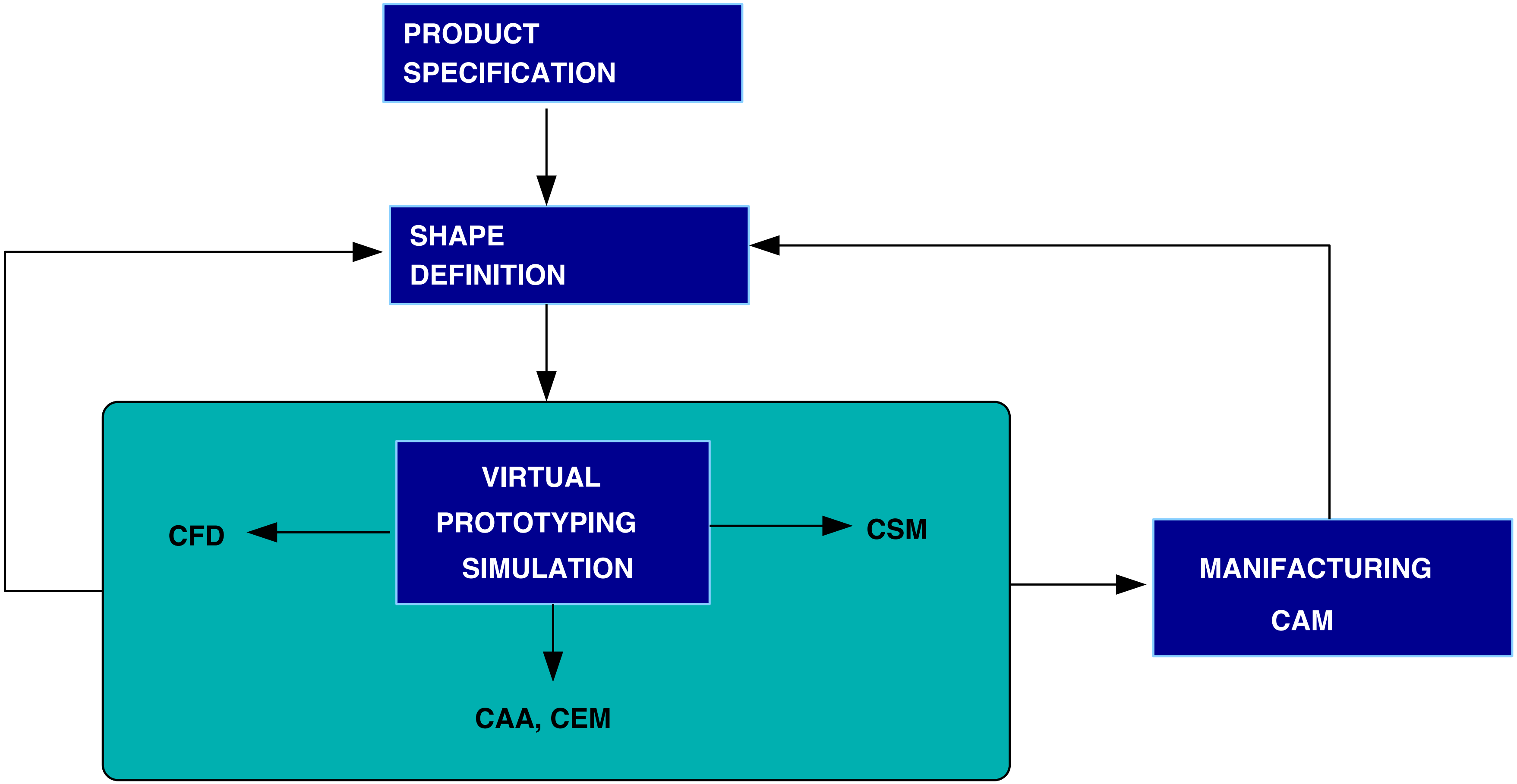
CFD today

- CFD is part of the CAE (Computer assisted Engineering) process



Steps of a CFD simulation

- Level of approximation, defined by the mathematical model chosen (Navier-Stokes).

- Discretization phase:
  - ✓ Domain discretization (calculation grid)
  - ✓ Equation discretization (definition of the numerical method)

- Analysis of the chosen numerical scheme: stability, accuracy.

- Solution of the problem: time integration, matrix solution method.

Level of approximation

- The set of Navier-Stokes equations is extremely complicated.
- System of nonlinear partial differential equations due to the presence of:
  - turbulence
  - shock waves (discontinuous fields)
  - spontaneous unsteadiness of flows (for instance the vortex shedding behind a cylinder)
  - non uniqueness of the solution
- Any modeling assumption will be associated with a generally undefined level of error when compared to real world. There is always a level of **empiricism** in the models we use to describe reality.
- Good understanding of the physical properties and limitations of the accepted models is very important.
- If you see discrepancies between measurements and calculation do not always blame the measurements or calculations, maybe the model describing the real world is just not adequate.

Discretization phase

- The translation of geometrical and mathematical models in to numbers is called **discretization.**

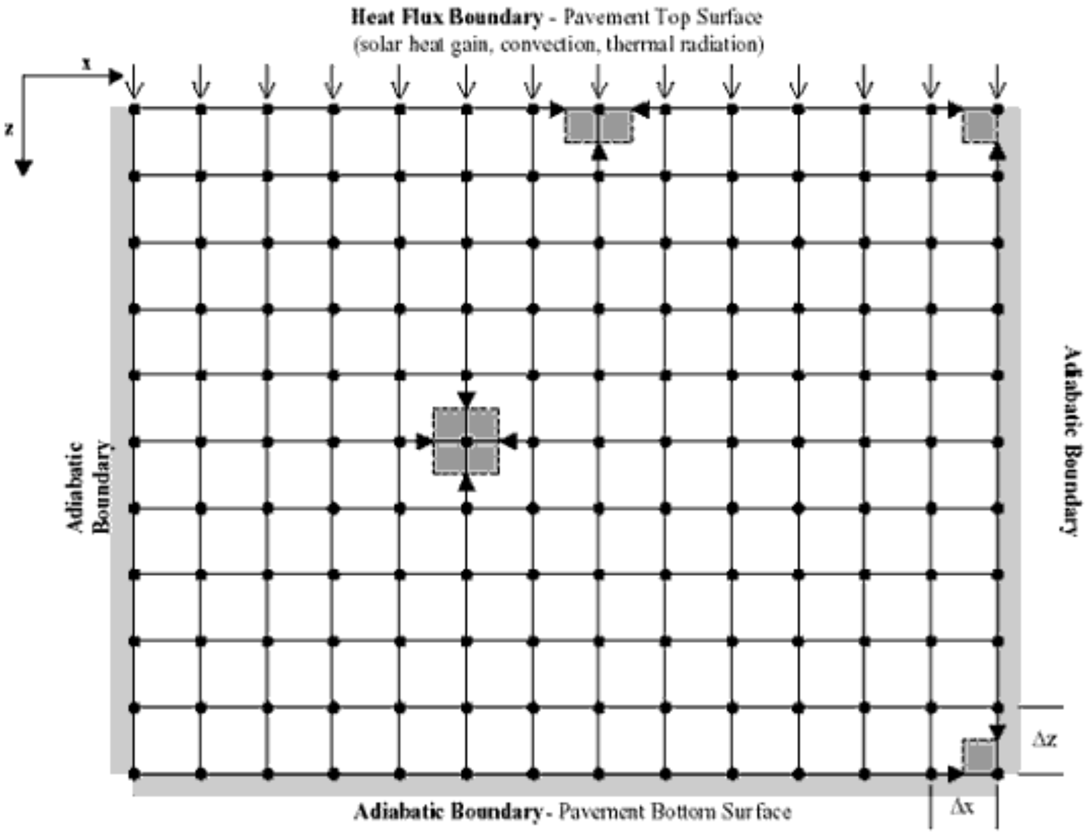
- The set of points or elements which replace the continuity of the real space is called **grid** or **mesh.**

- The shape of the mesh has issues related to the numerics, hence the grid generation process is a delicate and time consuming step.

- Once the grid is generated, all the mathematical operators, such as partial derivatives of the quantities will have to be transformed into arithmetic operations on the mesh point values.

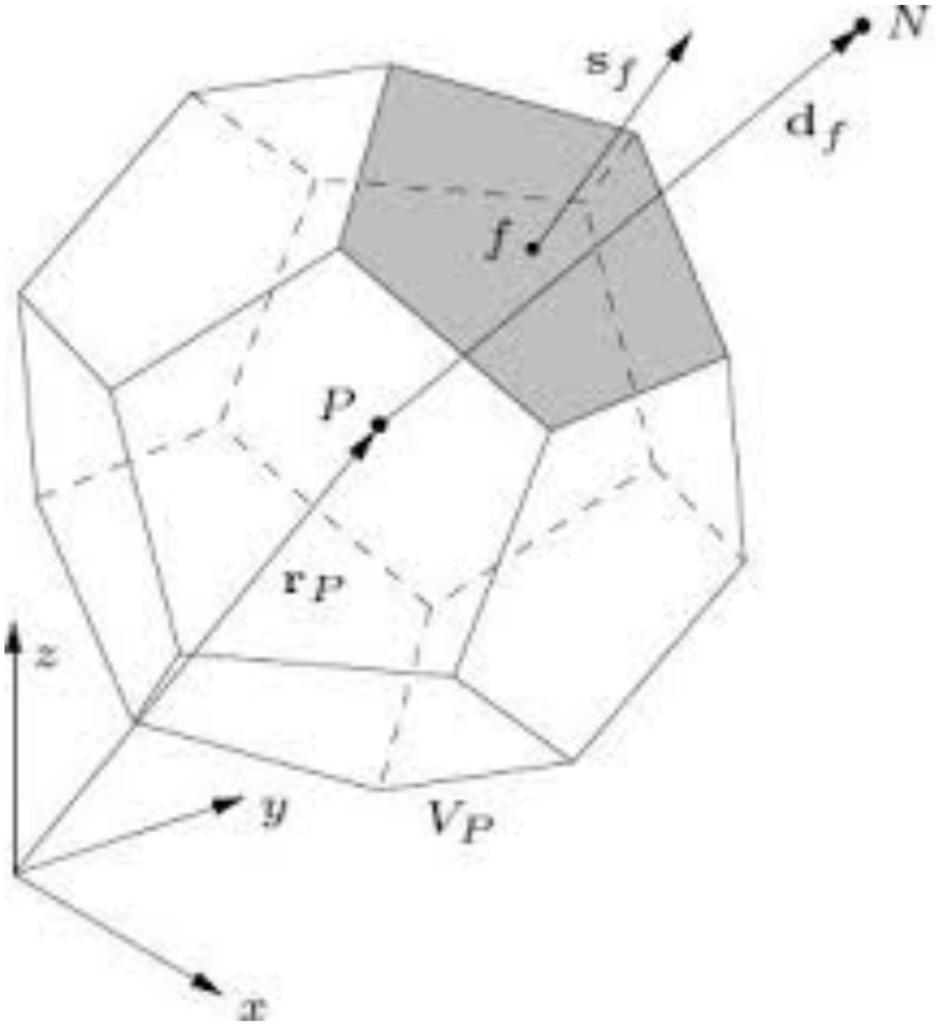
- Grid generation, or the choice of the meshing strategy is a major step in the CFD analysis: the outcome of CFD calculations is extremely dependent of the grid quality

Finite difference discretization



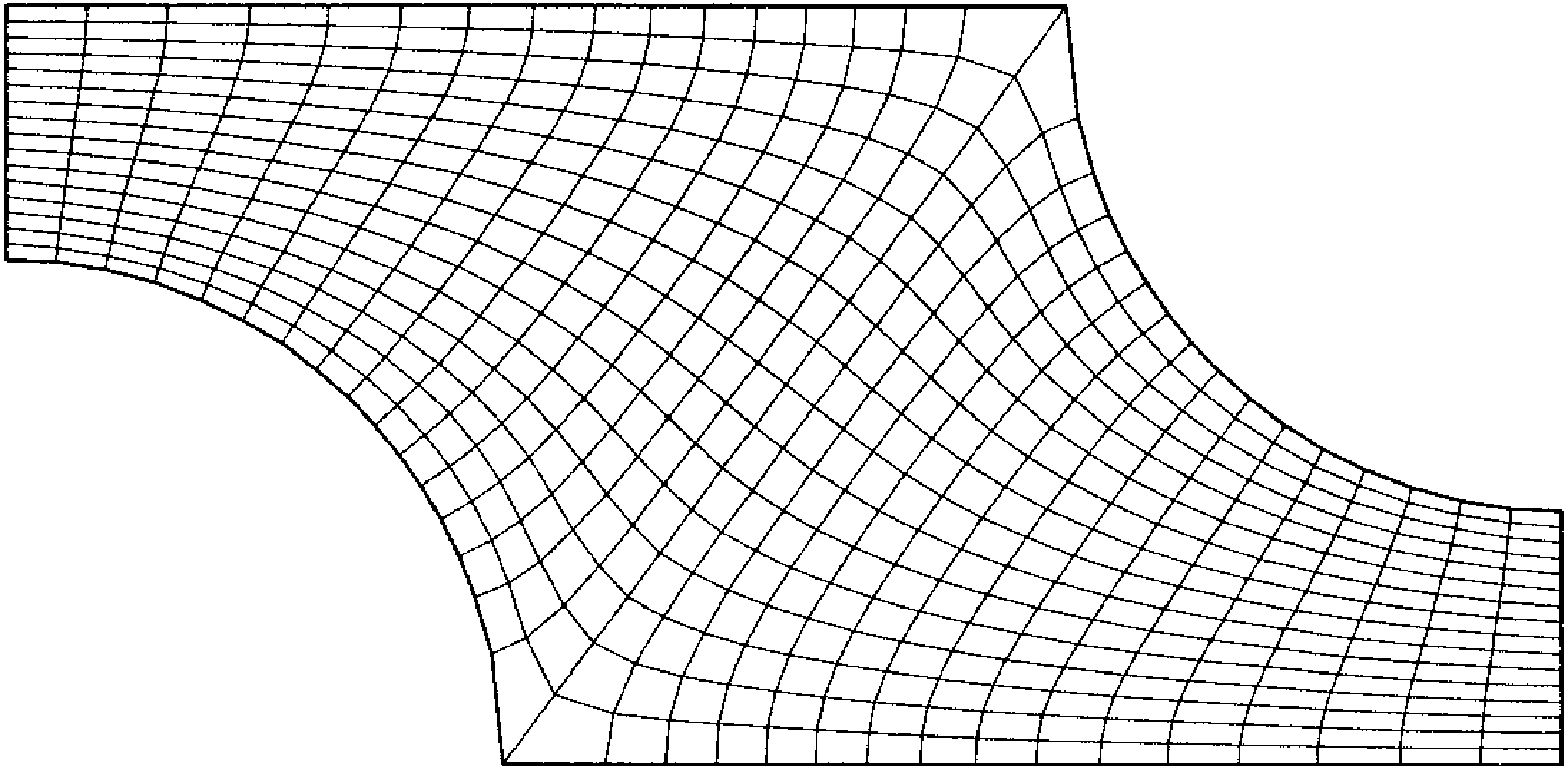
- Quantities are defined over the points.
- Indexing of calculation point is straightforward.
- Solution method can be optimized thanks to the indexing.

Finite volume discretization



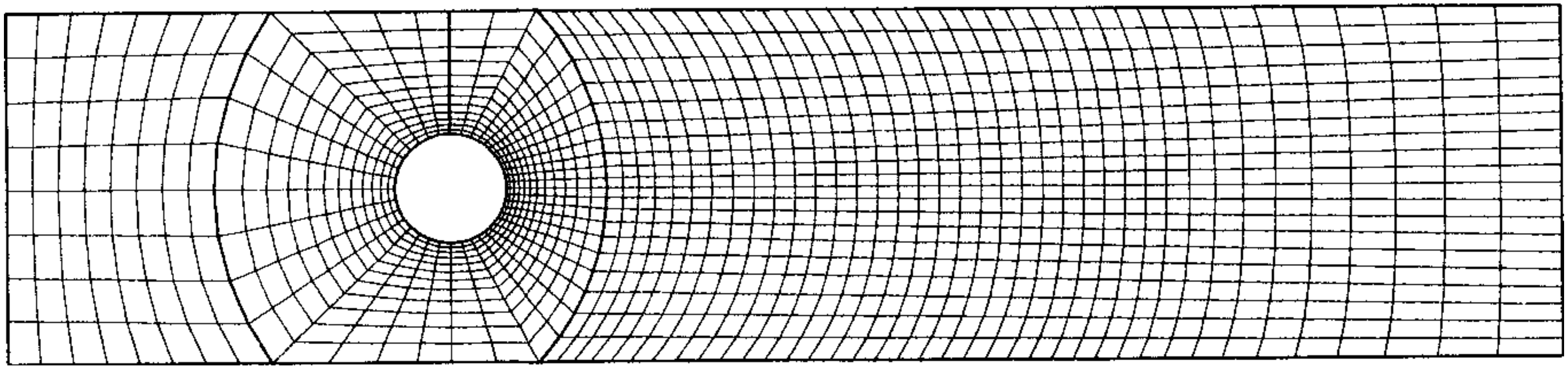
- Quantities are defined over the volume, or referred to its centroid.
- Indexing of calculation point is variable, depending on the shape of each cell.
- Solution method must be adequate to any type of cell shape.

Structured grid

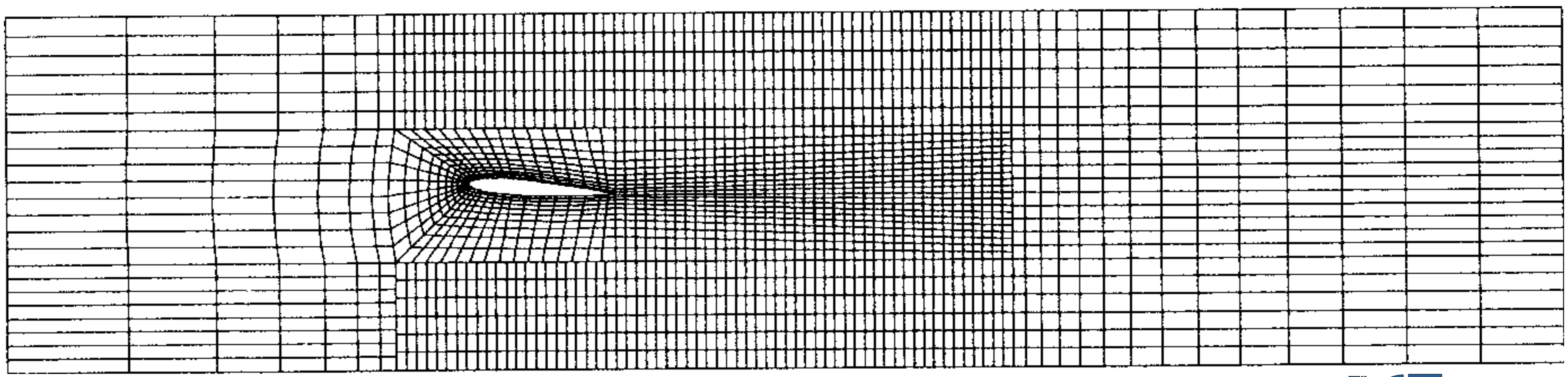


- Each cell has four neighbors, this allows to have a regular structure of the solution matrix.
- Specific solvers can be developed.
- They can be applied to simple geometries.

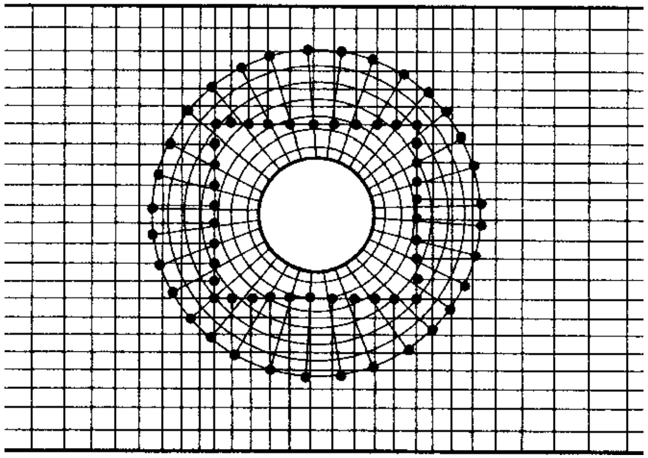
Block structured grid

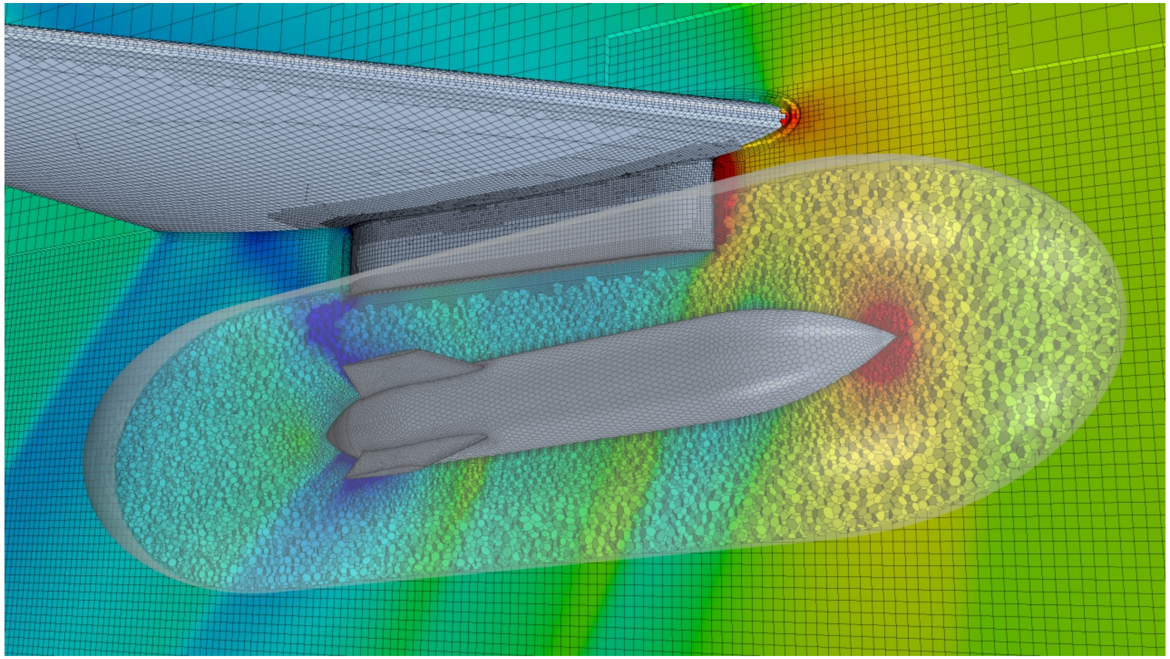


- Sub level subdivision of solution domain
- Non matching interface is possible but requires special treatment.



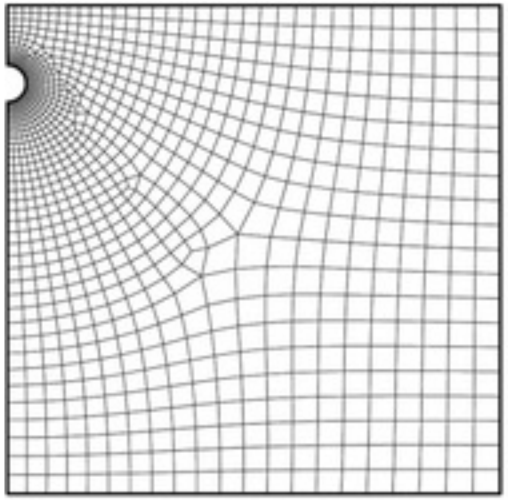
Composite or chimera grids

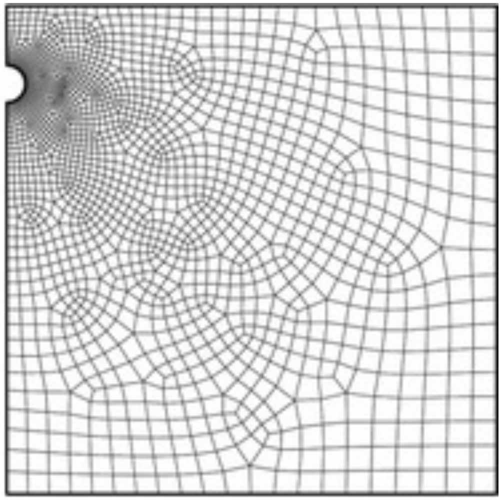


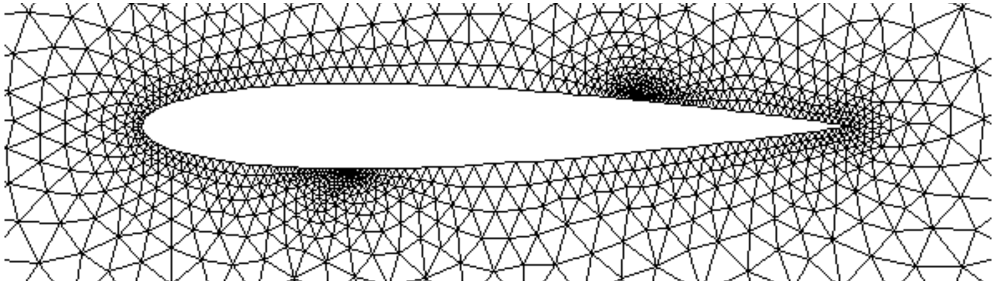


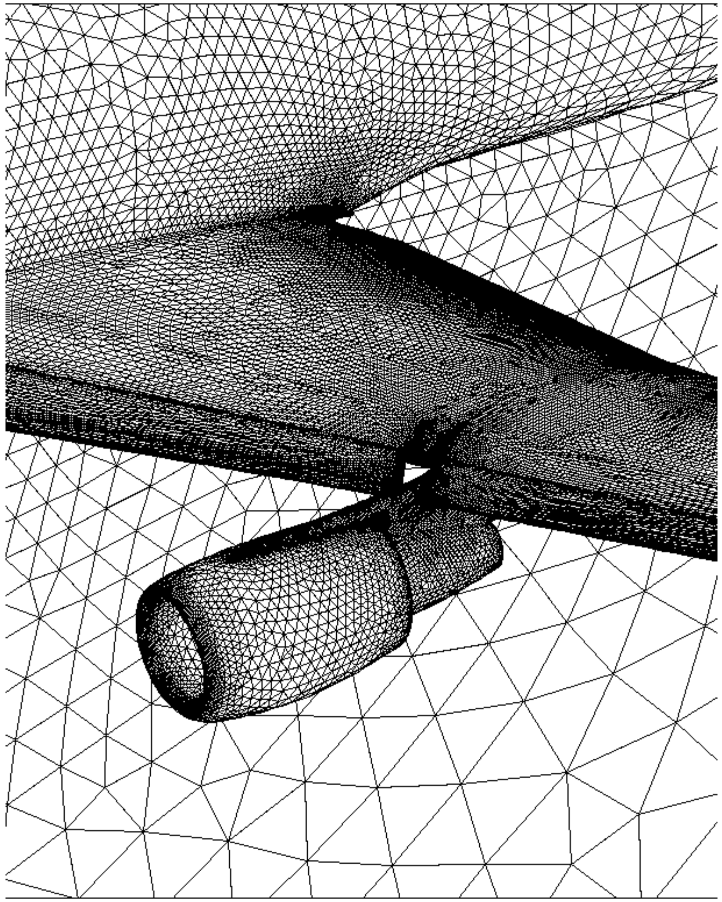
- Particular type of block structured matrix
- Boundary condition for one block are interpolated from the solution field of the moving block.

Unstructured grid





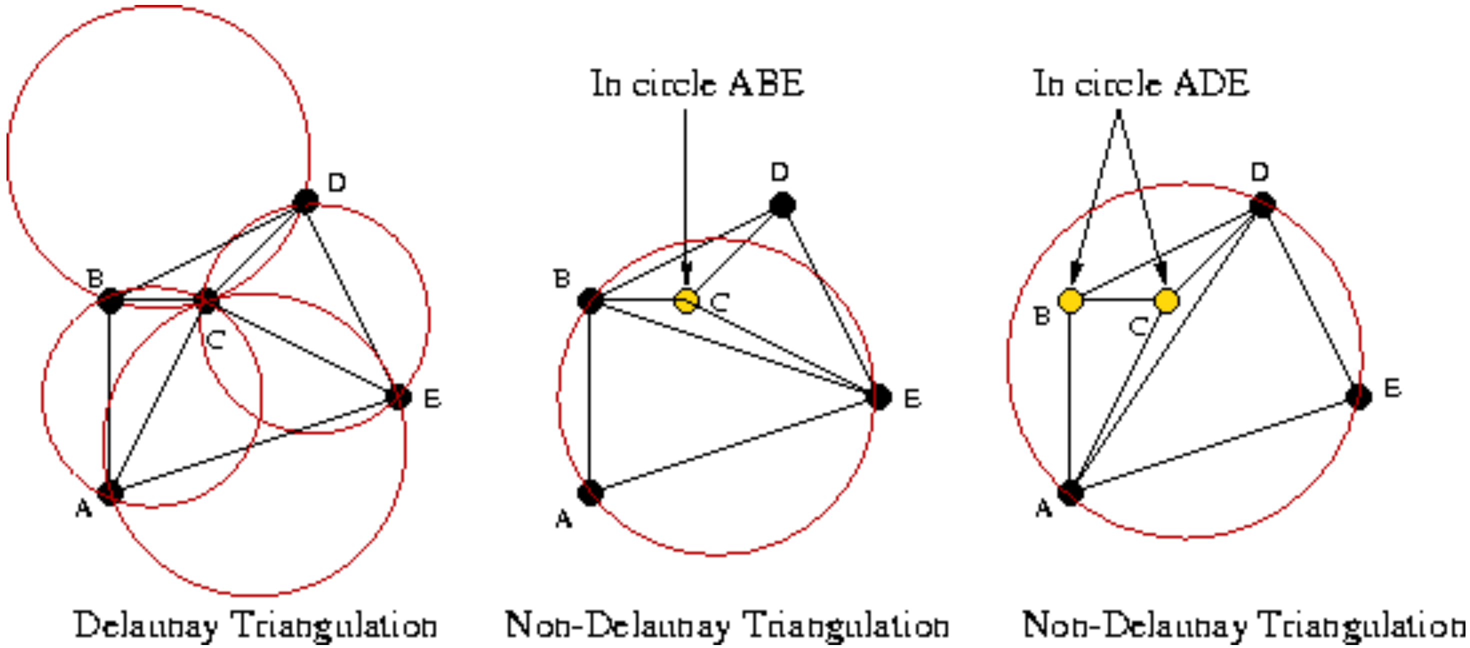




- The solution matrix is no longer regular
- The solution procedure may be slower that the one for ordered meshes
- High flexibility in shape reconstruction

Tetrahedral meshes: Delauney

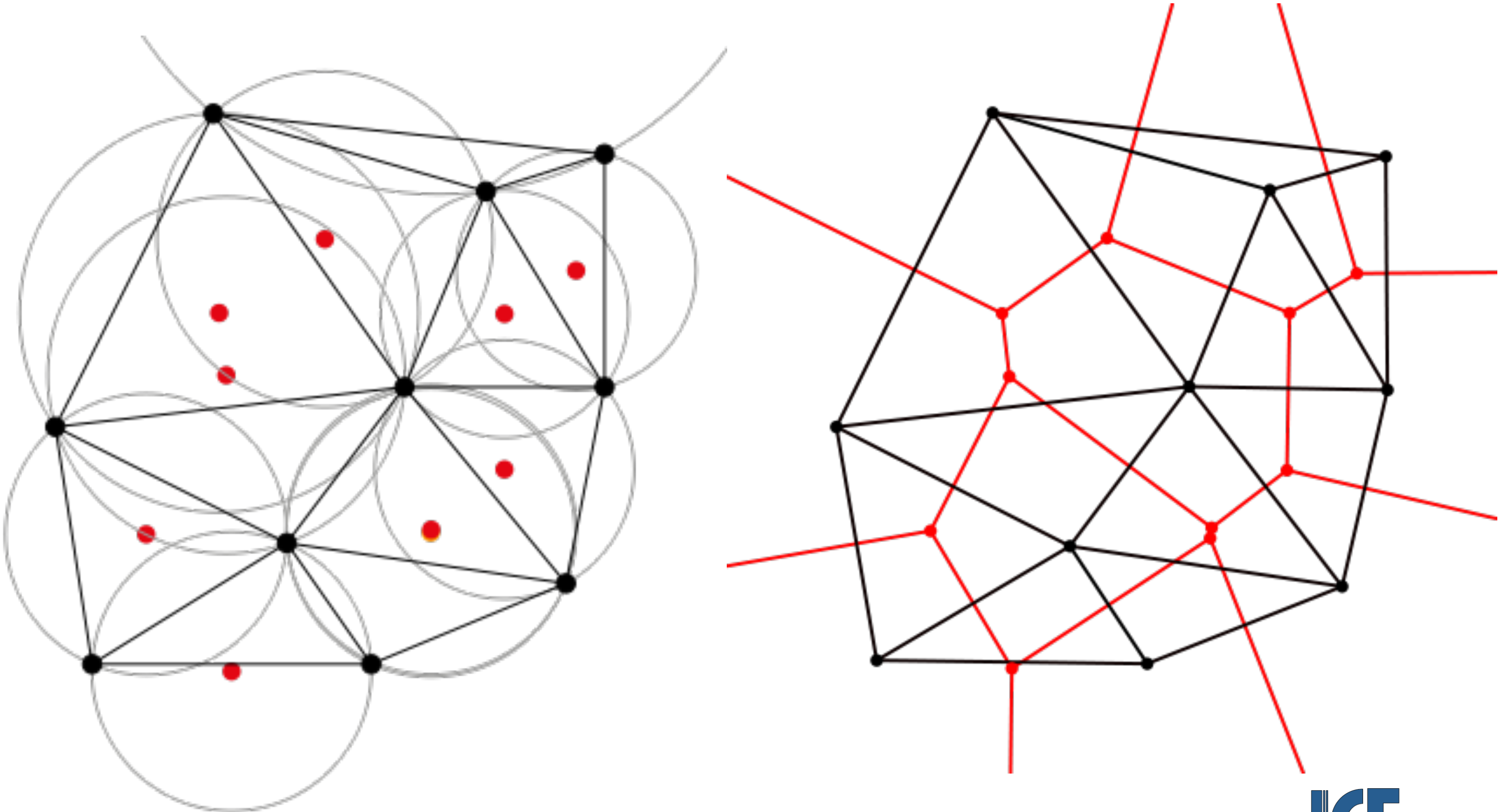
- There are particular meshing strategies for tetrahedral meshes that are used to produce other type of meshes:



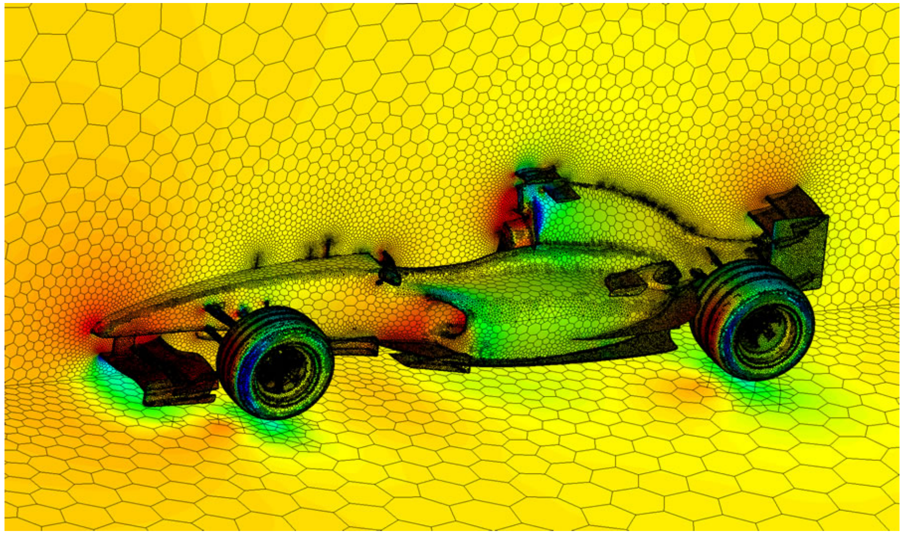
- Delaunay triangulation for a set P of points in a plane is a triangulation such that no point in P is inside the circumcircle of any triangle

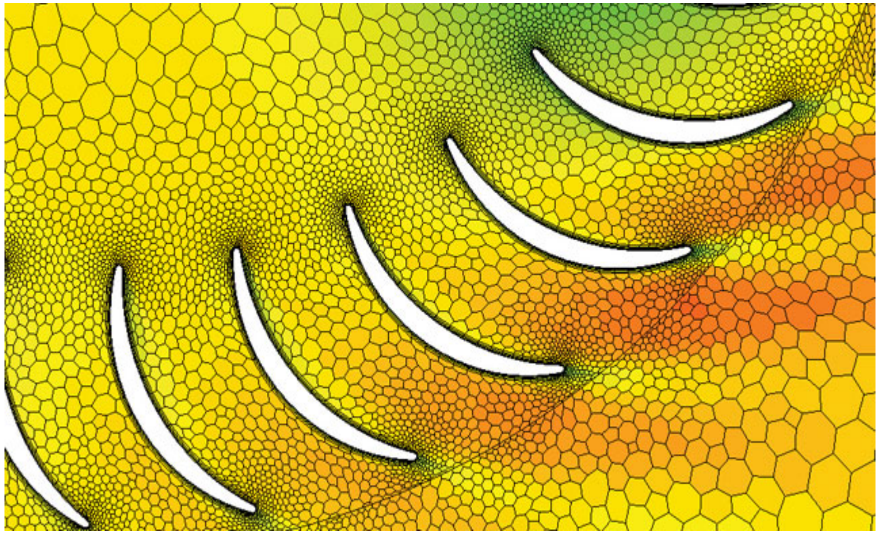
Polyhedral meshes: dual Voronoi

- Connecting the centers of the circumcircles produces the Voronoi diagram



Polyhedral grid





- Reduction of the bad aspect ratios of cells (deformation, distortion)
- Reduced number of calculation cells with respect to tet meshes
- High flexibility in shape reconstruction

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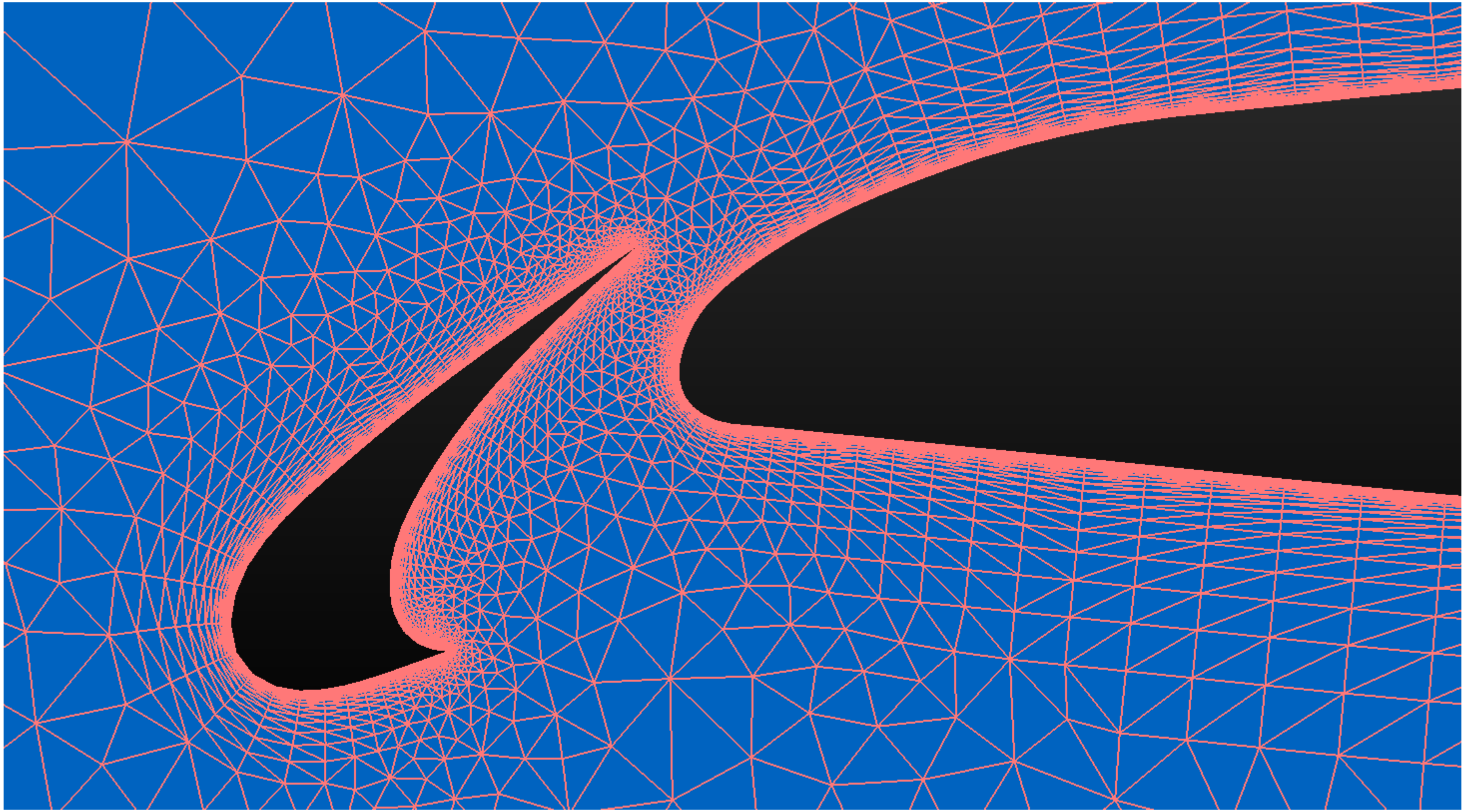
Hybrid grid



Analysis of the numerics

- Numerical schemes establish a relation between neighboring mesh points.

- Any discretization will generate errors, this is a direct consequence of the replacement of the continuum model by its discrete representation (truncation error).

- The numerical scheme must satisfy a certain number of conditions:
  - stability
  - convergence
  - consistency

Analysis of the numerics

- We shall make a distinction between the partial differential equation (PDE) and its numerical translation.

$D(U) =$ differential equation

$N(U) =$ discretized equation

$\tilde{u}$ exact solution of the differential equation

$\tilde{u}_i$ exact solution of the discretized equation

- An example can be done using the linear convection equation:

$$\frac{\partial u}{\partial t} + a\frac{\partial u}{\partial x} = 0$$

$D(U) =$ differential equation

$$\frac{u_i^{n+1} - u_i^n}{\Delta t} + a\frac{u_{i+1}^n - u_{i-1}^n}{2\Delta x} = 0$$

$N(U) =$ discretized equation

Questions

- Is there a condition that we have to impose on a numerical scheme to obtain an acceptable approximation to the differential problem?
- Different numerical schemes behave in a different way, some will be stable and some unstable. Is there a condition or a criteria to verify stability?
- If a numerical scheme is stable, how can we quantify the accuracy of the simulation?

CONVERGENCE **STABILITY**

CONVERGENCE

Analysis of the numerics

- **Consistency** is a condition of the numerical scheme (N(U)) and states that the numerical scheme must tend to the differential equation, when the space and time discretization tend to zero.

- **Stability** is a condition on the numerical scheme and imposes that the round-off error due to the finite algebra, remains bounded iteration after iteration.

$$\bar{\varepsilon}_i^n = u_i^n - \tilde{u}_i^n \qquad \lim_{n \to +\infty} |\bar{\varepsilon}_i^n| < K$$

- **Convergence** is a condition on the numerical solution and requires that the output of the simulation is a correct representation of the model we are solving. The numerical solution must tend to the exact solution of the mathematical problem when space and time discretization tend to zero. that the round-off error due to the finite algebra, remains bounded iteration after iteration.

$$\tilde{\varepsilon}_i^n = \tilde{u}_i^n - \tilde{u}^n \qquad \lim_{\Delta x, \Delta t \to 0} |\tilde{\varepsilon}_i^n| = 0$$

For a well-posed initial value problem and a consistent discretization scheme, stability is the necessary and sufficient condition for stability. (**Equivalence theorem of Lax**)

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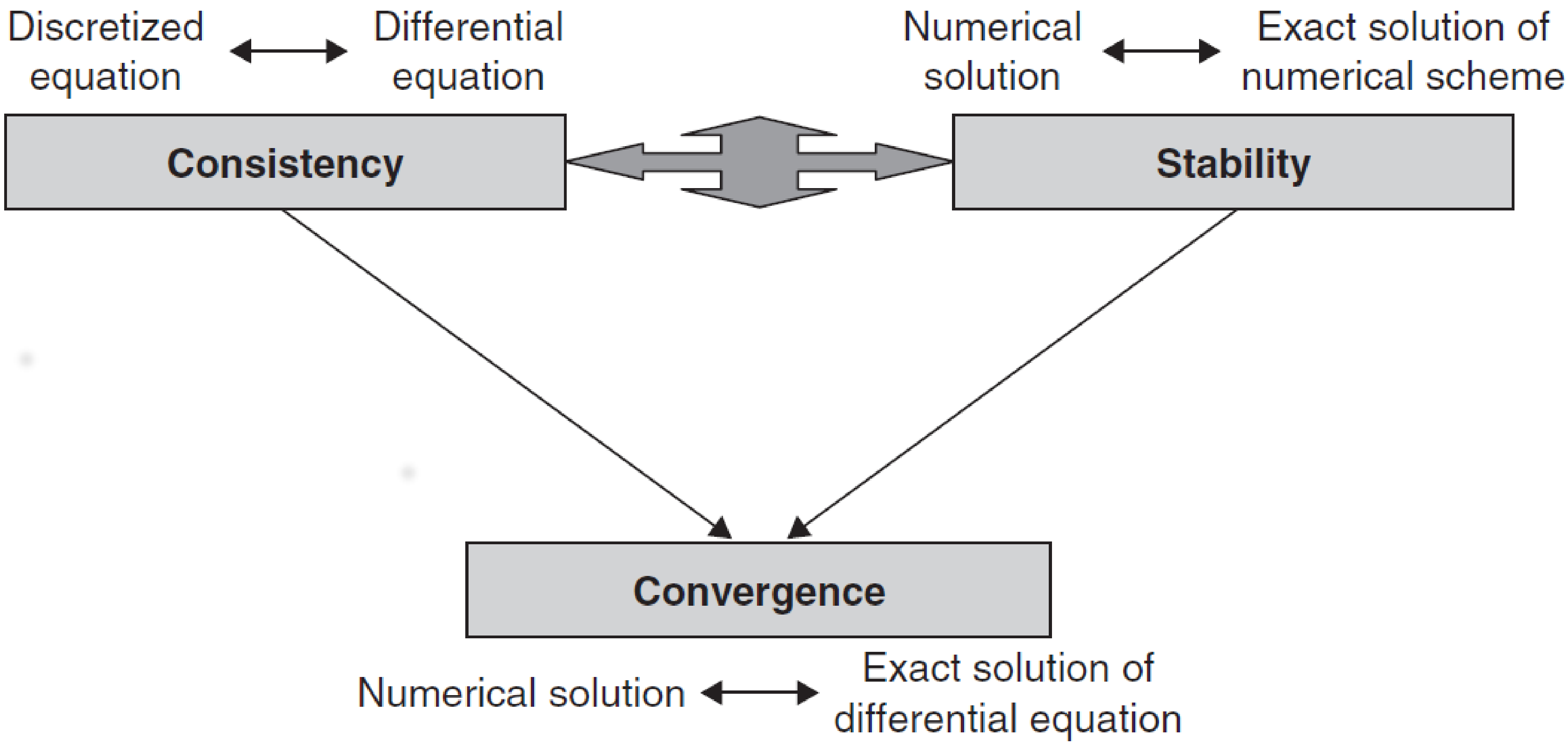
Analysis of the numerics



In order to analyze a time dependent or initial value problem, two tasks have to be performed:

- Analyze the consistency condition; this leads to the determination of the order of **accuracy** of the scheme and its **truncation error**.
- Analyze the stability properties

Resolution phase

- The solution algorithm depends on the type of the problem we are simulating
    - time-dependent
    - steady state flow

- Solution of a set of ordinary differential equations or an algebraic system

- At the end of the discretization process all the numerical schemes finally result in an algebraic system of equations with as many equations as unknowns.