MACROECONOMETRICS
PROBLEM SET 2

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Question 1

$$Y_t = 0,5 + 0,1Z_t + 0,3Y_{t-1} + 0,3Z_{t-1} + \varepsilon_{Yt}$$

$$Z_t = 1 - 0,2Y_t + 0,1Y_{t-1} + 0,4Z_{t-1} + \varepsilon_{Zt}$$

(a) In matrix representation, we have the following:

$$\underbrace{\begin{bmatrix} 1 & -0,1 \\ 0,2 & 1 \end{bmatrix}}_{B} \times \underbrace{\begin{bmatrix} Y_t \\ Z_t \end{bmatrix}}_{x_t} = \underbrace{\begin{bmatrix} 0,5 \\ 1 \end{bmatrix}}_{\Gamma_0} + \underbrace{\begin{bmatrix} 0,3 & 0,3 \\ 0,1 & 0,4 \end{bmatrix}}_{\Gamma_1} \times \underbrace{\begin{bmatrix} Y_{t-1} \\ Z_{t-1} \end{bmatrix}}_{x_{t-1}} + \underbrace{\begin{bmatrix} \varepsilon_{Yt} \\ \varepsilon_{Zt} \end{bmatrix}}_{\varepsilon_t}$$

$$\boxed{Bx_t = \Gamma_0 + \Gamma_1 x_{t-1} + \varepsilon_t} \text{ SVAR}$$

Now, we want the reduced form. In order to do so, we need to multiply each side of the previous equation by B^{-1}.

$$\det(B) = 1 + 0,02 = 1,02$$

$$B^{-1} = \frac{1}{\det(B)} \cdot \text{Adj}(B)$$

$$B^{-1} = \frac{1}{1,02} \cdot \begin{bmatrix} 1 & 0,1 \\ -0,2 & 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{1,02} & 0,98 \\ -0,196 & \frac{1}{1,02} \end{bmatrix}$$

$$B^{-1}\Gamma_0 = A_0 \Leftrightarrow \underset{2\times 2}{\begin{bmatrix} \frac{1}{1,02} & 0,98 \\ -0,196 & \frac{1}{1,02} \end{bmatrix}} \cdot \underset{2\times 1}{\begin{bmatrix} 0,5 \\ 1 \end{bmatrix}} = \underset{2\times 1}{\begin{bmatrix} 1,47 \\ 0,88 \end{bmatrix}}$$

$$B^{-1}\Gamma_1 = A_1 \Leftrightarrow \begin{bmatrix} \frac{1}{1,02} & 0,98 \\ -0,196 & \frac{1}{1,02} \end{bmatrix} \begin{bmatrix} 0,3 & 0,3 \\ 0,1 & 0,4 \end{bmatrix} = \begin{bmatrix} 0,39 & 0,69 \\ 0,039 & 0,33 \end{bmatrix}$$

Now, let's see if the model is stationary

If each eigenvalue λ of matrix A, satisfies $|\lambda|<1$, then

$\lim_{n \to \infty} A^n x = 0$ → And that's the condition we need for stationarity.

The characteristic polynomial:

$$\lambda^2 - \lambda(a_{22}+a_{11}) + a_{11}a_{22} - a_{12}a_{21} = 0$$

$$\lambda^2 - \lambda(0{,}33+0{,}39) + 0{,}33 \cdot 0{,}39 - 0{,}69 \cdot 0{,}039 = 0$$

$$\lambda^2 - 0{,}72\lambda + 0{,}10179 = 0$$

$$\lambda = \frac{0{,}72 \pm \sqrt{0{,}72^2 - 4 \cdot 0{,}10179}}{2} \Leftrightarrow \lambda_1 = 0{,}5268 \vee \lambda_2 = 0{,}1932$$

As both $|\lambda_1|$ and $|\lambda_2|$ are lower than one, we can conclude that the series is stationary.