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Problem Set 2 – Exercise 3

I) $y_t = 0.7y_{t-1} + \varepsilon_t$
II) $y_t = 2 + y_{t-1} + 0.5t + \varepsilon_t$

The initial condition is given and it is y_0

Line a)

Given y_0, there exists a forward iterating solution for I):

$$y_1 = 0.7y_0 + \varepsilon_1$$

$$y_2 = 0.7y_1 + \varepsilon_2 \Leftrightarrow y_2 = 0.7[0.7y_0 + \varepsilon_1] + \varepsilon_2 \Leftrightarrow y_2 = 0.7^2y_0 + 0.7\varepsilon_1 + \varepsilon_2$$

$$y_3 = 0.7y_2 + \varepsilon_3 \Leftrightarrow y_3 = 0.7[0.7y_1 + \varepsilon_2] + \varepsilon_3 \Leftrightarrow y_3 = 0.7(0.7[0.7y_0 + \varepsilon_1] + \varepsilon_2) \Leftrightarrow y_3 = 0.7^3y_0 + 0.7^2\varepsilon_1 + 0.7\varepsilon_2 + \varepsilon_3$$

From here, it follows that:

$$y_t = 0.7^t y_0 + \sum_{i=0}^{t} 0.7^i \varepsilon_i$$

Given y_0, there exists a forward iterating solution for II):

$$y_1 = 2 + y_0 + 0.5 + \varepsilon_1$$

$$y_2 = 2 + y_1 + 0.5 * 2 + \varepsilon_2 \Leftrightarrow y_2 = 2 + (2 + y_0 + 0.5 + \varepsilon_1) + 0.5 * 2 + \varepsilon_2 \Leftrightarrow y_2 = 2 + 2 + y_0 + 0.5 + 0.5 * 2 + \varepsilon_1 + \varepsilon_2$$

$$y_3 = 2 + y_2 + 0.5 * 3 + \varepsilon_3 \Leftrightarrow y_3 = 2 + (2 + 2 + y_0 + 0.5 + \varepsilon_1 + 0.5 * 2 + \varepsilon_2) + 0.5 * 3 + \varepsilon_3 \Leftrightarrow y_3 = 2 + 2 + 2 + y_0 + 0.5 + 0.5 * 2 + 0.5 * 3 + \varepsilon_1 + \varepsilon_2 + \varepsilon_3$$

From here, it follows that:

$$y_t = 2t + y_0 + \sum_{i=1}^{t} 0.5i + \sum_{i=1}^{t} \varepsilon_i$$

Line b)

The difference equation "I" is already stationary because its root lies within the unit root circle: $|0.7| < 1$

The difference equation "II" is not stationary. In order to make it stationary we have to take first-differences. By taking FD we get rid of the deterministic component and of the stochastic component.

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Line c)

Taking FD from equation II yields:

$$\begin{aligned} y_t - y_{t-1} &= (2 + y_{t-1} + 0.5t + \varepsilon_t) - (2 + y_{t-2} + 0.5(t-1) + \varepsilon_{t-1}) \\ &= y_{t-1} - y_{t-2} + 0.5 + \varepsilon_t - \varepsilon_{t-1} \end{aligned}$$

Now, to see whether this process is stationary, we solve for the homogeneous solution, using forward iteration:

Given y_0, we have that:

$$y_1 = y_0 + 0.5 + \varepsilon_0$$

$$\begin{aligned} y_2 &= y_1 - y_0 + 0.5 + \varepsilon_2 - \varepsilon_1 = y_0 + 0.5 + \varepsilon_0 - y_0 + 0.5 + \varepsilon_2 - \varepsilon_1 \\ &= 0.5 + 0.5 + \varepsilon_0 - \varepsilon_1 + \varepsilon_2 \end{aligned}$$

$$\begin{aligned} y_3 &= y_2 - y_1 + 0.5 + \varepsilon_3 - \varepsilon_2 \\ &= (0.5 + 0.5 + \varepsilon_0 - \varepsilon_1 + \varepsilon_2) - (y_0 + 0.5 + \varepsilon_0) + 0.5 + \varepsilon_3 - \varepsilon_2 \\ &= 0.5 + 0.5 + \varepsilon_0 - \varepsilon_1 + \varepsilon_2 - y_0 - 0.5 - \varepsilon_0 + 0.5 + \varepsilon_3 - \varepsilon_2 \\ &= 0.5 + 0.5 - y_0 - \varepsilon_1 + \varepsilon_3 \end{aligned}$$

$$\begin{aligned} y_4 &= y_3 - y_2 + 0.5 + \varepsilon_4 - \varepsilon_3 \\ &= (0.5 + 0.5 - y_0 - \varepsilon_1 + \varepsilon_3) - (0.5 + 0.5 + \varepsilon_0 - \varepsilon_1 + \varepsilon_2) + 0.5 + \varepsilon_4 \\ &- \varepsilon_3 = -y_0 + 0.5 - \varepsilon_0 - \varepsilon_2 + \varepsilon_4 \end{aligned}$$

While we can't compute a general homogenous solution, due to the different results we get from further iterations, it seems clear so far that the process is stationary as what only affects it is the moving average component and there is clear deterministic trend.

As we took FD, the AR component which lied outside the unit root, now lies inside the unit root. As the FD also took the time subscript out, we no longer are in the presence of a deterministic trend. The only component which seems to add any variability is the MA component. However, from our successive iterations, we can see that through time, this component offsets itself, thus allowing the series to be stationary around the mean.