

Problem Set 2
VAR and Nonstationary Models

Problem 1 Suppose we have the following model:

$$\begin{aligned}y_t &= 0.5 + 0.1z_t + 0.3y_{t-1} + 0.3z_{t-1} + \varepsilon_{yt} \\z_t &= 1 - 0.2y_t + 0.1y_{t-1} + 0.4z_{t-1} + \varepsilon_{zt}\end{aligned}$$

a) Show the model in a matrix representation whereby you clearly show the elements of each matrix (Γ_0 , Γ_1 and B). Show the reduced form model and find its coefficients (A_0 and A_1). Is the model stationary (you can use R or do it by hand following the lecture notes)?

b) Do we need to identify the model? Compute the responses of y_t and y_{t+1} and to a unit shock in z_t . Interpret them.

c) Suppose you assume a recursive identification where y_t is ordered first. What are the reduced form parameters now? Recalculate the responses of y_t and y_{t+1} to a shock in z_t . How different are they from b)? Explain.

d) Let $x_t = \begin{bmatrix} y_t \\ z_t \end{bmatrix}$. Use R to save matrix B^{-1} and A_1 . We know the IRF for step j is given by $\frac{\partial x_{t+j}}{\partial \varepsilon_t} = A_1^j B^{-1}$. Compute the IRFs for 30 steps using this formula in R and plot them (note that in R matrix multiplication of say A with B is done by `A%*%B`). Do they converge to zero? Explain.

Problem 2

- a) Can we apply OLS to the structural VAR? Why or why not?
- b) Can we interpret the reduced form errors of the VAR? Explain.
- c) Can we use VAR models to infer causal relationships? Explain.

Problem 3 Given an initial condition y_0 , consider the two following processes:

$$\begin{aligned} I) \quad & y_t = 0.7y_{t-1} + \varepsilon_t \\ II) \quad & y_t = 2 + y_{t-1} + 0.5t + \varepsilon_t \end{aligned}$$

- a) Find the solution of y_t iterating forward for both I) and II).
- b) What transformations of the series should you apply to I) and II) to make them stationary?
- c) Suppose you take the first differences in II). Is the resulting series stationary? Explain.

Problem 4 For this problem, we will make use of the data set included in the *ps_2_data.xls*. The data set consists of two series: U.S monthly unemployment rate (unrate) and Average Hourly Earnings (wage) from first month of 1964 to February of 2020. The data are from the Federal Reserve Bank of St. Louis.

- a) Test each series for stationarity using the ADF test.
- b) Estimate VAR model of the two series in levels selecting the number of lags using the AIC criteria (remember that you should only use the columns that have data and not the column with the dates). Check the residuals of the VAR, are they stationary?
- c) Compute and plot the IRF. You will notice that the dimension of the wage raw data does not help with interpretation. Lets transform that series into log so that the IRF are in percentage. Take log of wage and multiply it by 100. Re-estimate the VAR and plot the IRF. Interpret your results.
- d) Change the order of the variables. Estimate a VAR model by ordering wage first and plot the IRF. Does your conclusions in b) change?
- e) Perform a forecast error variance decomposition. Explain your findings.
- f) Does unemployment rate Granger cause wages?