Macroeconometrics - 2168 Joao B. Duarte
Nova SBE, Spring 2020

**Problem Set 1**

**Basic Concepts in Time Series Analysis - Each Question is Worth 25 points**

**Problem 1** Consider the stochastic processes: i) $y_t = 0.5 + 1.2y_{t-1} + \varepsilon_t$, and ii) $y_t = 2.5y_{t-1} - 0.7y_{t-2} + \varepsilon_t$.

**a)** Find the homogeneous solution for both processes. Are they stable?

**b)** Since i) is unstable, there is no backward looking solution without an initial condition. However, there exists a forward looking solution. Find the particular solution using the method of undetermined coefficients for difference equation i), whereby the challenge solution is given by: $y_t = b_0 + b_1 t - \alpha_0\varepsilon_{t+1} - \alpha_1\varepsilon_{t+2} - \alpha_2\varepsilon_{t+3} - ...$. What is the general solution to i)?

**c)** Find the particular solution for both difference equations using lag operators. For the particular solution of i) apply the following property of lag operators: for $|a| > 1$, $\frac{\varepsilon_t}{1-aL} = -(aL)^{-1}\sum_{i=0}^{\infty}(aL)^{-i}\varepsilon_t$.

**d)** Use the initial condition $y_0 = 2$ for i) to solve for the constant $A$ of the general forward looking solution.

**e)** With an initial condition there exists a backward looking solution even for unstable difference equations. Use the iteration method in i) starting from the same initial condition $y_0 = 2$ to find a solution in terms of past values of $\varepsilon$.

**Problem 2** AR and MA models

**a)** For the following MA model, $y_t = \varepsilon_t - \theta_1\varepsilon_{t-1} - \theta_2\varepsilon_{t-3}$, derive the unconditional mean and variance, the autocovariance $\gamma_1$, $\gamma_2$, $\gamma_3$ and $\gamma_4$ as well as the associated autocorrelation coefficients. Interpret your results.

**b)** For the AR(2) model given by $y_t = 0.6y_{t-1} - 0.06y_{t-2} + \varepsilon_t$, find the roots of the characteristic equation, and then sketch the ACF for the first 4 steps. Is it stationary? Using the lag operator, show that the roots of the lag polynomial are the inverse of the roots of the characteristic equation.

**Problem 3** Let $y_t = 0.6 + 0.4y_{t-2} + \varepsilon_t - 0.1\varepsilon_{t-1}$

**a)** Find $E[y_t]$, $var(y_t)$ and $cov(y_t, y_{t-1})$. Is it stationary? Is it invertible?

**b)** Find the autocorrelation $\rho_i$ for $i = 1, 2, 3, 4, ....$

**c)** Find the forecast $E_t[y_{t+1}]$, $E_t[y_{t+2}]$. How does the MA component affect both steps-ahead forecasts?

**Problem 4** In this problem set, you will make a forecast of the real GDP annual growth rate for Portugal in 2020. Download the real GDP annual growth rate for Portugal from FRED. The series code is: "NAEXKP01PTA657S".

**a)** Does the time series look stationary? Why or why not?

**b)** How do you identify the ARMA model for this series? Identify the model.

**c)** Estimate the model you have identified in b). Print the output of the estimates and discuss your results. Is the model you estimated stable? Why or why not?

**d)** Check the model. Is it a good model? Why or why not?

**e)** Forecast the next two years: 2019 and 2020. Plot your forecasts. How do your predictions compare with the official forecasts of the Bank of Portugal (https://www.bportugal.pt/en/page/projecoes-economicas)?