

b) For us to be able to calculate the impulse Response Functions, we need to rewrite the model as a VAR( $\infty$ )!

Reduced form model in a VAR( $\infty$ ) form:

$$x_t = \mu + e_t + A_1 e_{t-1} + A_1^2 e_{t-2} + \dots$$

in matrix form:

$$\begin{bmatrix} y_t \\ z_t \end{bmatrix} = \begin{bmatrix} \bar{y} \\ \bar{z} \end{bmatrix} + \sum_{i=0}^{\infty} \begin{bmatrix} 0,39 & 0,69 \\ 0,039 & 0,33 \end{bmatrix} \begin{bmatrix} e_{1t-i} \\ e_{2t-i} \end{bmatrix}$$

As  $e_{1t}$  and  $e_{2t}$  are the linear combination of the structural errors, directly, we cannot infer anything from them. If our goal with the impulse Response Functions is to understand how a specific shock impacts a variable, we should rewrite the model in terms of the original shocks. (Recall that  $B^{-1} \varepsilon_t = e_t$ )

So,

$$\begin{bmatrix} e_{1t} \\ e_{2t} \end{bmatrix} = \frac{1}{1 - b_{12}b_{21}} \begin{bmatrix} 1 & -b_{12} \\ -b_{21} & 1 \end{bmatrix} \begin{bmatrix} \varepsilon_{1t} \\ \varepsilon_{2t} \end{bmatrix}$$

$$\begin{bmatrix} y_t \\ z_t \end{bmatrix} = \begin{bmatrix} \bar{y} \\ \bar{z} \end{bmatrix} + \frac{1}{1 - b_{12}b_{21}} \sum_{i=0}^{\infty} \begin{bmatrix} 0,39 & 0,69 \\ 0,039 & 0,33 \end{bmatrix} \begin{bmatrix} 1 & -b_{12} \\ -b_{21} & 1 \end{bmatrix} \begin{bmatrix} \varepsilon_{1t-i} \\ \varepsilon_{2t-i} \end{bmatrix}$$

$$\text{if, } \phi_i = \frac{A^i}{1 - b_{12}b_{21}} \begin{bmatrix} 1 & -b_{12} \\ -b_{21} & 1 \end{bmatrix}$$

Then,

$$\begin{bmatrix} y_t \\ z_t \end{bmatrix} = \begin{bmatrix} \bar{y} \\ \bar{z} \end{bmatrix} + \sum_{i=0}^{\infty} \begin{bmatrix} \phi_{11}(i) & \phi_{12}(i) \\ \phi_{21}(i) & \phi_{22}(i) \end{bmatrix} \begin{bmatrix} \varepsilon_{1t-i} \\ \varepsilon_{2t-i} \end{bmatrix}$$

The response of  $y_t$  to a unit shock of  $z_t$ , at impact, is represented by  $\phi_{12}(0)$  and the effect of  $z_t$  on  $y_{t+1}$  is represented by  $\phi_{12}(1)$ . In theory, we could calculate the values of all impulse response functions  $\phi_{11}(i)$ ,  $\phi_{12}(i)$ ,  $\phi_{21}(i)$  and  $\phi_{22}(i)$ . However, since VAR is underidentified, we cannot retrieve all the coefficients of the structural form. Thus, we must make an assumption regarding how  $y_t$  and  $z_t$  relate contemporaneously. Without that assumption, it is not possible to calculate the numerical response of  $y_t$  and  $y_{t+1}$  to a shock of  $z_t$ .

Hence, answering the question of the need to identify the model, the answer is yes if you want to understand the impact of exogenous shocks or if you want to understand how  $y_t$  and  $z_t$  relate contemporaneously. However, if your goal is to forecast, or only to understand how past lags of  $z_t$  impact  $y_t$ , you don't need to identify the model.