Math 206

Fall 2015

8/26

Definitions

A *norm* on a vector space X (over $\mathbb{F}$) is a function $\|\| : X \to \mathbb{R}^+$ such that
$\|x\| = 0$ iff $x = 0$
$\|\alpha x\| = |\alpha|\|x\|$ (for $\alpha \in F$)
$\|x + y\| \leq \|x\| + \|y\|$
An *algebra* $\mathcal{A}$ over $\mathbb{F}$ is a vector space with distributive $\cdot$ satisfying
$cx \cdot y = c(x \cdot y)$
$x \cdot cy = c(x \cdot y)$ for all $c \in F$
A *normed algebra* over $\mathbb{R}$ or $\mathbb{C}$ is an algebra $\mathcal{A}$ equipped with (vector space) norm satisfying
$\|ab\| \leq \|a\|\|b\|$ for all $a, b \in \mathcal{A}$
A norm on $\mathcal{A}$ induces a metric
$d(a, b) = \|a - b\|$ on $\mathcal{A}$ and therefore a topology
if $\mathcal{A}$ is complete for this norm, it is a *Banach algebra*

To figure out (use https://www.math.ksu.edu/ nagy/real-an/2-05-b-alg.pdf)

Supposing $\mathcal{A}$ is not necessarily complete
$\|ab\| \leq \|a\|\|b\|$ gives uniform continuity on the product
hence the norm can be extended to the completion $\bar{\mathcal{A}}$ to form a Banach algebra
A metric space M is complete if all Cauchy sequences converge to an element of M
The completion M is all equivalence classes of Cauchy sequences where
$\{a_n\} \sim \{b_n\}$ iff $\lim_{x\to\infty} d(a_n - b_n) = 0$