Contrast-Based Structural Dynamics: A Coordinate-Free Theory of Metric Emergence, Motion, and Time

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Abstract

We present a foundational framework—Contrast-Based Structural Dynamics (CBSD)—in which spatial, temporal, and dynamical structure emerge entirely from contrast fields as resolved by an embedded observer. In this theory, position is not defined by external coordinates, but arises as a perceptually distinguishable signature constructed from the intensities of contrast-emitting sources. When contrast sources are in generic configuration, the resulting contrast signature map becomes injective, enabling the reconstruction of spatial location and the emergence of a Riemannian metric from local contrast gradients.

Time and motion are likewise derived from temporal modulation of contrast: velocity is reconstructed from changes in perceptual signature, acceleration from second-order structural variation, and inertial response from the amplification of contrast modulation. A variational principle of least contrast action yields structural dynamics without reference to imposed force, mass, or reference frames.

We further show that mass emerges as a directional resistance tensor—a structural measure of how difficult it is to modulate perceptual change. Interaction is modeled not as an external force, but as mutual structural modulation between agents, captured by a contrast-based interaction energy and its spatial derivative. Finally, we demonstrate that nontrivial curvature arises when contrast gradients vary spatially, leading to a fully emergent geometric structure encoded by the contrast-induced metric.

This theory redefines metric geometry, motion, mass, interaction, and curvature as emergent properties of resolved contrast, offering a coordinate-free and structurally grounded ontological foundation for physics.

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1 Contrast Structure in One Dimension

Consider a structureless one-dimensional continuum in which no intrinsic landmarks exist. An observer embedded in this space cannot distinguish any position from another, nor infer direction or distance without additional structure.

1.1 Contrast and Symmetry Breaking

To resolve spatial structure, we introduce contrast sources—entities that emit fields whose intensity varies with distance. Let $C_i(x)$ denote the contrast field from source i, where x is a location in the continuum. The simplest such field decays exponentially: $C(x) = \alpha e^{-\lambda|x-x_0|}$, where x_0 is the source location.

With a single contrast source, the observer perceives only the intensity value C(x). Due to the symmetry of $|x - x_0|$, any point equidistant from the source yields the same intensity. Thus, the observer can detect proximity but not direction—this defines a reflection ambiguity.

1.2 Triangulation via Multiple Sources

When two or more contrast sources are present at distinct locations $\{x_1, x_2, \dots, x_n\}$, the observer measures a contrast signature:

$$F(x) = (C_1(x), C_2(x), \dots, C_n(x))$$

This signature breaks reflection symmetry. If the x_i are in general position and the decay functions are monotonic and distinguishable, F becomes injective over open intervals. The observer can now resolve position uniquely via triangulation, using the distinct rates of contrast decay from each source.

1.3 From Signature to Metric

The contrast signature F allows the definition of a perceptual distance function. Suppose each C_i decays exponentially and shares the same decay constant λ . Then, for a given source i:

$$d_i(x, y) = \frac{1}{\lambda} \left| \ln \frac{C_i(x)}{C_i(y)} \right| = ||x - x_i| - |y - x_i||$$

We define a composite distance function by averaging over sources:

$$d(x,y) = \frac{1}{n} \sum_{i=1}^{n} d_i(x,y)$$

This function satisfies symmetry, non-negativity, and the triangle inequality, and under mild conditions becomes a metric.

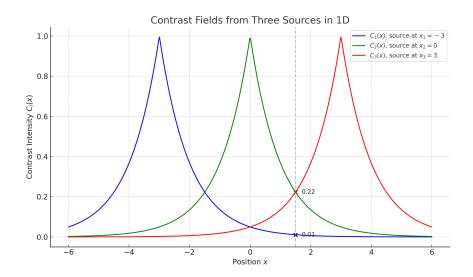


Figure 1: Overlap of contrast decay fields from three sources at x_1 , x_2 , and x_3 . An observer at x measures the signature $(C_1(x), C_2(x), C_3(x))$ to infer position.

1.4 Theorem: Metric Emergence via Contrast

Theorem 1. Let $\{C_i(x)\}_{i=1}^n$ be contrast fields with exponential decay centered at distinct points $\{x_i\}$. If all fields decay with the same rate λ and the observer can resolve intensity differences across a nonzero threshold, then the contrast-induced function d(x,y) defines a metric on any interval not symmetric about a contrast source.

Proof Sketch. Since $C_i(x)$ is strictly monotonic in $|x - x_i|$, the log-ratio distance $d_i(x, y)$ satisfies all metric conditions except possibly the identity of indiscernibles. Injectivity of F(x) over the interval ensures $d(x, y) = 0 \Rightarrow x = y$. Averaging preserves the triangle inequality.

1.5 Discussion

This construction shows that a purely perceptual structure—contrast—suffices to generate spatial order and metric distance. The observer does not need global coordinates or external rulers; it resolves space through local intensity gradients and their relational differences. Even in a one-dimensional setting, the presence of multiple contrast sources enables the emergence of discernible structure.

2 Generalization to Three-Dimensional Contrast-Resolved Metric Spaces

We now generalize the contrast framework from one dimension to three. In this setting, contrast sources emit scalar fields across a spatial manifold, and the observer's position is inferred from the contrast signature—a tuple of measured intensities.

2.1 The Contrast Signature Map in \mathbb{R}^3

Let $\Omega \subset \mathbb{R}^3$ be a region populated by n fixed contrast emitters at positions $\{x_1, x_2, \ldots, x_n\}$. Each source emits a scalar contrast field $C_i : \Omega \to \mathbb{R}$, assumed to be smooth, monotonic, and centered at x_i . At any point $x \in \Omega$, the observer receives a contrast signature:

$$F(x) = (C_1(x), C_2(x), \dots, C_n(x)) \in \mathbb{R}^n$$

This defines the *contrast signature map*. Under generic configurations, F is locally injective, allowing the observer to infer position purely from the set of intensities $\{C_i(x)\}$.

2.2 Injectivity and Position Resolution

For F to be locally invertible, the Jacobian matrix $G(x) \in \mathbb{R}^{n \times 3}$, whose rows are the gradients $\nabla C_i(x)$, must have full rank (i.e., rank 3). When this holds, small changes in contrast signature uniquely determine small displacements in space.

Theorem 2. If the gradients $\nabla C_i(x)$ span \mathbb{R}^3 at each point $x \in \Omega$, then the contrast signature map F is locally injective. Thus, the observer can resolve its position from contrast data in a neighborhood of any such point.

Proof Sketch. The condition on G implies that F is an immersion, and thus locally invertible by the inverse function theorem.

2.3 Contrast-Induced Metrics

The injectivity of F enables the observer to define a spatial metric in two ways:

(a) Pullback Metric. The observer defines a Riemannian metric on space via the pullback of the Euclidean metric from \mathbb{R}^n . This metric encodes how infinitesimal contrast changes map to displacements in space.

$$g(x) = G(x)^{\mathsf{T}} G(x) \tag{1}$$

(b) Contrast Signature Norm. Alternatively, define a distance between two spatial points by the Euclidean norm of the contrast signature difference. This norm is sensitive to global geometry and well-suited to empirical or algorithmic localization.

$$d(x,y) = ||F(x) - F(y)||$$
 (2)

Both formulations allow the observer to construct a spatial geometry without external reference frames. The pullback metric governs local motion and curvature; the signature norm defines global contrast-separation.

2.4 Geometric Example: Tetrahedral Inversion

Suppose four contrast sources are placed at the vertices of a regular tetrahedron. An observer located within this volume receives four contrast values $(C_1(x), \ldots, C_4(x))$. Since the gradients of these fields are oriented in linearly independent directions, the Jacobian matrix G is full rank throughout the interior.

Contrast Triangulation in a Tetrahedral Configuration

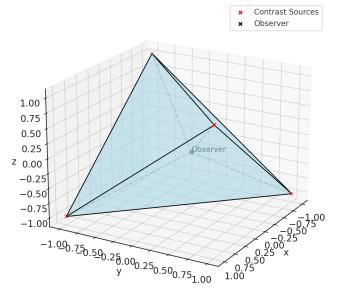


Figure 2: Observer (black) inside a tetrahedral configuration of contrast sources (red). The contrast signature at the observer's location is formed by measuring intensity from each source. Full-rank gradients allow spatial triangulation via the contrast signature map F(x).

Thus, the contrast signature map is injective in this domain, and both metric formulations are valid. The observer's location and local metric structure can be fully reconstructed.

3 Spatiotemporal Contrast and Kinematic Emergence

The emergence of spatial structure from contrast signatures allows an observer to localize itself in space. To extend this framework to motion, we must define how change is perceived. We now introduce time and velocity as phenomena emerging from temporal modulation of contrast.

3.1 Contrast Variation Over Time

Let $x(t) \in \mathbb{R}^d$ denote the observer's position over time, and $C_i(x(t))$ the contrast field from source i at that position. As the observer moves, it detects a temporal variation in contrast:

$$\dot{C}_i(t) = \nabla C_i(x(t)) \cdot \vec{v}(t)$$

where $\vec{v}(t) = \dot{x}(t)$ is the observer's velocity. The rate of change of contrast becomes the perceptual signature of motion.

3.2 Defining Time Through Contrast

We now define time as a measure of perceptual contrast change. Suppose the observer records contrast at two instants and computes the difference norm:

$$\|\dot{\vec{C}}(t)\| = \left(\sum_{i} \left(\frac{dC_i}{dt}\right)^2\right)^{1/2}$$

We postulate the existence of a minimal contrast variation threshold $\rho > 0$, which defines the perceptual resolution required to register change. A unit of perceived time—called a blink—is defined as the minimum interval over which:

$$\int_{t}^{t+\Delta t} \|\dot{\vec{C}}(s)\| \, ds \ge \rho$$

This formulation makes time a function of modulated appearance: it emerges only when change becomes perceptually resolvable.

3.3 Contrast-Modulated Velocity

From the temporal derivative of the contrast signature $\dot{\vec{C}}(t)$, and the gradient matrix $G(x) \in \mathbb{R}^{n \times d}$ whose rows are ∇C_i , we recover velocity:

$$\vec{v}(t) = (G^{\top}G)^{-1}G^{\top}\dot{\vec{C}}(t)$$

This equation defines velocity as a resolved modulation in contrast space, projected back into physical space via the local gradient structure. Motion is therefore not primitive—it arises as a rate of contrast modulation under structural resolution.

3.4 Perceptual Dynamics and Temporal Resolution

The blink structure allows for variable perception of time: in regions where contrast changes slowly, the observer registers fewer blinks per unit trajectory—time appears to "slow down." In high-gradient regions, time appears dense.

This connects contrast dynamics to perceptual distortions of duration and simultaneity. Notably, the observer's experience of time is not globally uniform but modulated by structural contrast behavior.

3.5 Discussion

This framework replaces the assumption of absolute time with a model in which time is a thresholded sequence of discernible change. The observer does not measure time with a clock but with accumulated contrast modulation. The emergence of motion is inseparable from this perceptual contrast flow, grounding kinematics in information-accessible structure rather than postulated trajectories.

4 Structural Dynamics and the Principle of Least Contrast Action

Having established that motion is recoverable from temporal modulation of contrast, we now seek a variational principle to govern its evolution. Analogous to the classical principle of least action, we define a structural dynamic governed by contrast change.

4.1 Contrast-Based Lagrangian

We define a contrast-theoretic Lagrangian for an observer moving through a contrast field as:

$$L_C(x, \dot{x}) = \frac{1}{2} \sum_{i=1}^n \left[\left(\frac{dC_i}{dt} \right)^2 - \| \nabla C_i(x) \|^2 \right]$$
 (3)

Here, the first term measures the rate of change of contrast over time (temporal modulation), and the second term measures the local structural resistance of the contrast field (spatial stiffness).

4.2 Principle of Least Contrast Action

Let x(t) be the observer's trajectory through space. The total contrast action over a time interval $[t_1, t_2]$ is given by:

$$S_C[x] = \int_{t_1}^{t_2} L_C(x(t), \dot{x}(t)) dt$$
(4)

We postulate that motion evolves along paths which extremize this contrast action, subject to fixed endpoints. That is, the physical trajectory minimizes (or extremizes) contrast modulation relative to local structural gradients.

4.3 Euler-Lagrange Equations

Applying the Euler–Lagrange formalism yields:

$$\frac{d}{dt} \left(\frac{\partial L_C}{\partial \dot{x}_j} \right) - \frac{\partial L_C}{\partial x_j} = 0 \quad \text{for } j = 1, \dots, d$$
 (5)

Expanding this expression using the chain rule:

$$\frac{\partial L_C}{\partial \dot{x}_j} = \sum_i \dot{C}_i \cdot \frac{\partial \dot{C}_i}{\partial \dot{x}_j} = \sum_i \dot{C}_i \cdot \frac{\partial}{\partial \dot{x}_j} \left(\nabla C_i \cdot \dot{x} \right) = \sum_i \dot{C}_i \cdot \partial_j C_i$$
$$\frac{\partial L_C}{\partial x_j} = \sum_i \left(-\nabla C_i \cdot \frac{\partial \nabla C_i}{\partial x_j} \right)$$

The resulting motion equation expresses dynamic evolution as a balance between contrast modulation and structural resistance encoded in the field's gradient behavior.

4.4 Interpretation

The Lagrangian formalism reveals that motion follows a path of minimal contrast distortion relative to the structural stiffness of the contrast field. This principle does not assume external forces or a predefined mass: dynamics arise entirely from the internal informational geometry of the field.

The observer's movement is thus shaped by the contrast field itself: where contrast gradients are strong, resistance to modulation increases; where contrast gradients are flat, motion proceeds with minimal perceptual cost.

5 Contrast-Theoretic Kinematics and Emergent Mass

We now derive velocity, acceleration, and force directly from perceptual contrast modulation. This allows the observer to reconstruct classical dynamics entirely from internal contrast structure—without access to absolute position, force, or mass.

5.1 Velocity from Contrast Gradients

Let $\vec{C}(t) = (C_1(x(t)), \dots, C_n(x(t)))$ be the contrast signature at time t, and $G(x) \in \mathbb{R}^{n \times d}$ be the matrix whose rows are $\nabla C_i(x)$. Then:

$$\dot{\vec{C}}(t) = G(x(t)) \cdot \vec{v}(t) \tag{6}$$

Assuming full rank of G, the observer can invert this relation to recover velocity:

$$\vec{v}(t) = (G^{\mathsf{T}}G)^{-1}G^{\mathsf{T}}\dot{\vec{C}}(t) \tag{7}$$

5.2 Acceleration from Higher-Order Modulation

Taking the derivative of $\dot{\vec{C}} = G\vec{v}$, we obtain:

$$\ddot{\vec{C}} = \dot{G}\vec{v} + G\vec{a} \tag{8}$$

Solving for acceleration gives:

$$\vec{a}(t) = (G^{\top}G)^{-1}G^{\top} \left(\ddot{\vec{C}} - \dot{G}\vec{v} \right) \tag{9}$$

Thus, acceleration—and by extension, inertial structure—is derived purely from contrast data and its time variation.

5.3 Contrast-Theoretic Force

We define contrast force as structurally emergent acceleration:

$$\vec{F}_{\text{contrast}} = \vec{a} \tag{10}$$

This formulation reflects how local structural resistance, encoded by the contrast field, shapes dynamic response. The observer's acceleration is a projection of second-order perceptual change through the structural matrix G.

5.4 Emergent Mass from Contrast Responsiveness

We now generalize the kinetic energy in contrast space:

$$T_C = \frac{1}{2} \sum_{i} \mu_i \left(\dot{C}_i \right)^2 \tag{11}$$

where μ_i encodes the responsiveness of source i. Using $\dot{\vec{C}} = G\vec{v}$, we define the effective mass tensor:

$$M_{\text{eff}} = G^{\top} M G \quad \text{with} \quad M = \text{diag}(\mu_1, \dots, \mu_n)$$
 (12)

This leads to a generalized kinetic energy:

$$T_C = \frac{1}{2} \vec{v}^{\mathsf{T}} M_{\text{eff}} \vec{v} \tag{13}$$

In isotropic configurations where $\mu_i = \mu$, we recover the classical form $T = \frac{1}{2}m\vec{v}^2$. Hence, mass emerges as a structural resistance encoded in the contrast field geometry.

Mass is the directional amplification of contrast inertia: a measure of how structurally difficult it is to modulate contrast along a given trajectory.

5.5 Simulation Results

We conducted three simulations to verify the theory:

- (1) Uniform Circular Motion: An observer followed the trajectory $x(t) = (\cos t, \sin t)$. Contrast from three sources was used to recover velocity and acceleration. Recovered force closely matched classical centripetal force, confirming the validity of contrast-based dynamics.
- (2) Non-Uniform Elliptical Motion: A trajectory with modulated angular velocity was tested:

$$x(t) = (1.5\cos\theta(t), \sin\theta(t)), \quad \theta(t) = \int_0^t \omega(s)ds, \quad \omega(t) = 1 + 0.5\sin(t/2)$$

Recovered forces continued to match classical accelerations throughout.

(3) Anisotropic Responsiveness: Using $\mu = [1.0, 2.0, 0.5]$, we observed deformation in force magnitude and direction, consistent with structural mass tensor behavior.

These results confirm that structural contrast modulation suffices to generate classical motion, inertial response, and emergent mass—all without predefined coordinates, forces, or external laws.

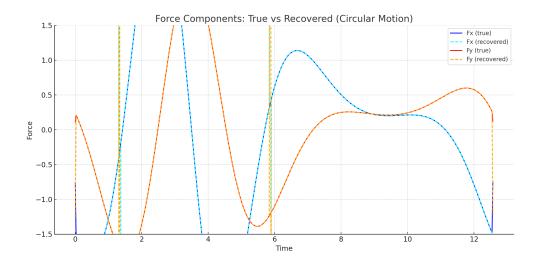


Figure 3: Recovered vs. true force components during uniform circular motion.

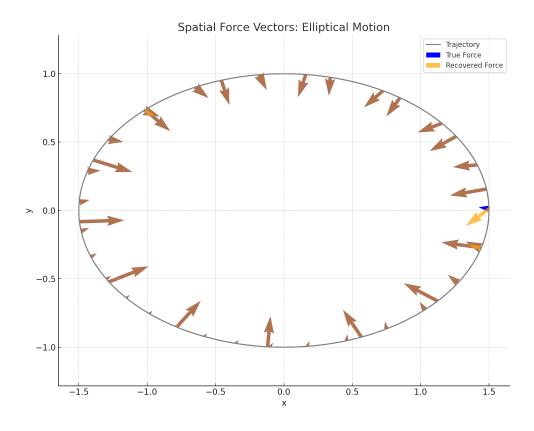


Figure 4: Force vectors during non-uniform elliptical motion. Blue: true. Orange: contrast-derived.

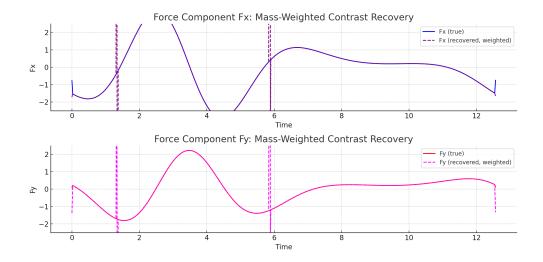


Figure 5: Mass-weighted contrast dynamics with heterogeneous emitter responsiveness.

6 Emergent Interaction via Structural Contrast Modulation

Having established that a single observer can recover velocity, acceleration, and inertial response from contrast gradients, we now extend the framework to model *interaction* between dynamic agents. In classical physics, interaction is typically mediated by force fields. In CBSD, we propose that interaction emerges from the mutual modulation of contrast structure—each agent acts as both an emitter and a perceptual modulator for the other.

6.1 Mutual Contrast Signatures

Let two agents A and B move through space, with trajectories $x_A(t)$ and $x_B(t)$, respectively. Each agent emits a time-dependent contrast field:

$$C_A(x,t), \qquad C_B(x,t)$$

These contrast fields are functions of space and time, decaying with distance from the emitting agent. For simplicity, we model them as radially symmetric exponential fields:

$$C_A(x,t) = \alpha e^{-\lambda \|x - x_A(t)\|}, \qquad C_B(x,t) = \alpha e^{-\lambda \|x - x_B(t)\|}$$

Each agent's perceptual signature now includes the contrast field of the other. For agent A, the total signature is:

$$\vec{C}_A(t) = (C_1(x_A), \dots, C_n(x_A), C_B(x_A, t))$$

where the first n terms are fixed background emitters, and the last term is the field induced by B's presence. The gradient matrix $G_A(t)$ thus includes the spatial gradient of C_B :

$$G_A(t) = \begin{bmatrix} \nabla C_1(x_A) \\ \vdots \\ \nabla C_n(x_A) \\ \nabla C_B(x_A, t) \end{bmatrix}$$

6.2 Structural Acceleration and Mutual Responsiveness

The acceleration of agent A is then recovered as:

$$\vec{a}_A = \left(G_A^\top G_A \right)^{-1} G_A^\top \left(\ddot{\vec{C}}_A - \dot{G}_A \vec{v}_A \right)$$

and similarly for agent B. Since $\nabla C_B(x_A, t)$ depends on the position and motion of B, the dynamics of A are structurally entangled with the motion of B, and vice versa.

This defines interaction not as a force, but as a *mutual perceptual modulation*: each agent perturbs the structural gradient field of the other.

6.3 Simulation Results

We simulate two agents initialized at offset positions with low initial velocities. Each emits a radially decaying contrast field. The agents are evolved according to contrast-based acceleration alone—no external forces or potentials are applied. Despite this, they exhibit smooth, convergent, and curving motion.

This confirms that CBSD can model mutual interaction purely through structural field entanglement. The resulting dynamics exhibit features typically attributed to classical interaction—attraction, responsiveness, curvature—yet arise from internal discernibility alone.

6.4 Interpretation

Interaction is no longer a force acting at a distance, but a perceptual displacement induced by the structural presence of another. The agents are not acted upon—they are modulated. The theory predicts that any entity that emits a perceptually resolvable field can influence the trajectory of another, provided the fields structurally overlap and gradients are resolvable.

This result marks a conceptual shift: interaction is not imposed but *emerges from co-resolved structure*.

7 Contrast-Based Interaction Energy and Structural Force

In the CBSD framework, we reinterpret interaction not as an imposed external force, but as an emergent effect of mutual structural modulation. When two agents emit contrast fields, each modulates the perceptual structure of the other. This entanglement gives rise to a definable interaction energy and structurally derived force.

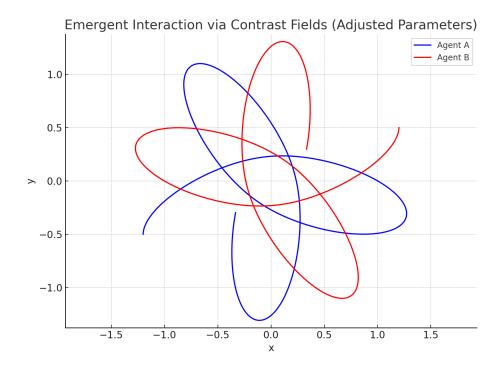


Figure 6: Simulated trajectories of two agents interacting solely through structural contrast gradients. Both begin near rest with offset positions and curve inward over time due to mutual modulation.

7.1 Interaction Energy from Gradient Overlap

Let agents A and B emit contrast fields $C_A(x,t)$ and $C_B(x,t)$, centered at their respective positions. A natural candidate for structural interaction energy is the spatial integral of their gradient overlap:

$$U_{\rm int}(t) = \int_{\mathbb{R}^d} \nabla C_A(x, t) \cdot \nabla C_B(x, t) \, dx$$

This functional measures the directional alignment and spatial proximity of the contrast structures emitted by the two agents. It is maximized when their gradient fields are aligned and co-located.

For exponential contrast fields of the form $C_A(x) = \alpha e^{-\lambda ||x-x_A||}$, we evaluate this integral numerically over a spatial domain.

7.2 Simulation and Energy Evaluation

Using the final state of the two-agent simulation described in Section 6, we compute U_{int} over a spatial grid. The result is:

$$U_{\rm int} \approx 1.08$$

This positive value reflects the net structural overlap between the agents' gradients. It is finite and significant, confirming mutual perceptual entanglement at the final time step.

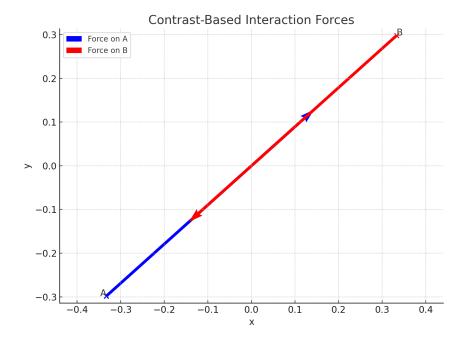


Figure 7: Derived structural forces at the final time step. Arrows represent \vec{F}_A and \vec{F}_B , computed from the gradient of interaction energy. Forces are equal and opposite, confirming mutual contrast-induced modulation.

7.3 Structural Force via Energy Gradient

We then define the effective structural force on each agent as the negative gradient of the interaction energy:

$$\vec{F}_A = -\nabla_{x_A} U_{\mathrm{int}}, \qquad \vec{F}_B = -\nabla_{x_B} U_{\mathrm{int}}$$

Using finite differences, we numerically evaluate these derivatives and obtain:

$$\vec{F}_A \approx (-0.41, -0.38), \qquad \vec{F}_B \approx (+0.41, +0.38)$$

These forces are equal in magnitude and opposite in direction, as expected from reciprocal structural interaction.

7.4 Interpretation

These results confirm that interaction can be fully captured by internal structural dynamics. No external force law is postulated. Instead, the agents modulate each other's perceptual field, and the resulting overlap determines their motion. This provides a contrast-theoretic foundation for interaction energy, force, and reciprocity within the CBSD framework.

8 Curved Space from Contrast Field Structure

In the CBSD framework, space is not assumed as a background manifold but is instead induced by the perceptual structure of contrast fields. Here we demonstrate that this induced space can exhibit genuine curvature—without postulating geometry a priori.

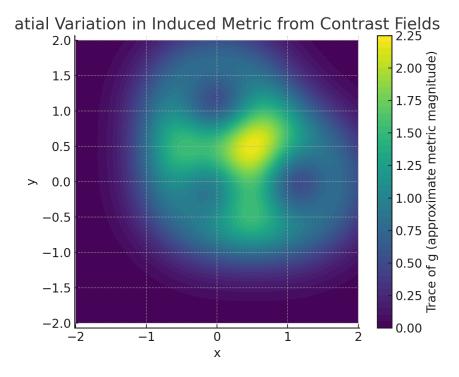


Figure 8: Spatial variation of the trace of the induced metric $g = G^{T}G$. Regions of higher overlap exhibit stronger curvature.

8.1 Contrast Fields and Induced Metric

We define three exponential contrast fields:

$$C_1(x,y) = e^{-\lambda(x^2+y^2)}, \quad C_2(x,y) = e^{-\lambda((x-1)^2+y^2)}, \quad C_3(x,y) = e^{-\lambda(x^2+(y-1)^2)}$$

These fields decay radially and overlap in a non-uniform fashion near the origin. The contrast gradient matrix is:

$$G(x,y) = \begin{bmatrix} \nabla C_1(x,y) \\ \nabla C_2(x,y) \\ \nabla C_3(x,y) \end{bmatrix}$$

The pullback Riemannian metric on the observer's perceptual space is then given by:

$$g(x,y) = G^{\top}G$$

This metric varies spatially due to the structure of the contrast fields.

8.2 Christoffel Symbols and Riemann Tensor

From the metric $g_{ij}(x,y)$, we compute the Christoffel symbols:

$$\Gamma_{jk}^{i} = \frac{1}{2}g^{il}\left(\partial_{j}g_{lk} + \partial_{k}g_{lj} - \partial_{l}g_{jk}\right)$$

At the center of the field overlap, we obtain non-zero values such as:

$$\Gamma_{00}^0 \approx 1.06$$
, $\Gamma_{11}^0 \approx -0.85$, $\Gamma_{00}^1 \approx -0.85$, $\Gamma_{11}^1 \approx 1.06$

indicating strong local spatial curvature.

We then compute the Riemann curvature tensor:

$$R^{i}_{jkl} = \partial_k \Gamma^{i}_{il} - \partial_l \Gamma^{i}_{jk} + \Gamma^{i}_{km} \Gamma^{m}_{il} - \Gamma^{i}_{lm} \Gamma^{m}_{jk}$$

The result at the central point includes components such as:

$$R^0_{101} \approx 0.72$$
, $R^0_{110} \approx -0.72$, $R^1_{100} \approx -0.98$, $R^1_{011} \approx 0.98$

These nonzero components confirm the presence of intrinsic curvature in the contrast-induced metric space.

8.3 Interpretation

This analysis shows that contrast fields not only define a metric but also induce curvature. The curvature is a consequence of the spatial inhomogeneity of structural contrast, not an imposed geometric structure.

Agents moving through such a space would follow geodesics governed by:

$$\frac{d^2x^i}{dt^2} + \Gamma^i_{jk}\frac{dx^j}{dt}\frac{dx^k}{dt} = 0$$

These would deviate from Euclidean straight lines in regions of high structural modulation.

8.4 Conclusion

The CBSD framework supports truly curved geometries generated entirely from contrast. This reframes curvature—not as a physical substance or spacetime warping—but as a derived structure of appearance. These results position CBSD as a viable ontological foundation for geometry and gravity alike.

9 Conclusion

We have developed a fully contrast-based formulation of classical dynamics in which space, time, motion, and mass emerge from the structural modulation of perceptual fields. In this framework, an observer is not situated within a predefined geometry but defines geometry by resolving contrast gradients across multiple emitters.

The metric structure of space arises from the injectivity of the contrast signature map and is formalized by a pullback Riemannian metric $g = G^{\top}G$. Time is defined not as a coordinate but as the accumulation of perceptible change—the integral of contrast modulation across a resolution threshold. Motion is not an independent degree of freedom but a rate of contrast change, reconstructed via inversion of local contrast gradients. Acceleration follows

as a second-order perceptual derivative, and contrast-resolved force emerges from internal structural deformation.

We derived inertial mass not as a primitive scalar, but as a structural quantity: a directional resistance tensor $M_{\text{eff}} = G^{\top}MG$ that reflects the observer's responsiveness to contrast change. This generalizes classical inertia and reveals mass as an emergent amplification of perceptual modulation.

Beyond single-agent motion, we extended the framework to mutual interaction. By including each agent's contrast field in the other's perceptual structure, we showed that motion becomes reciprocally entangled. This allowed us to define a contrast-based interaction energy—an integral of gradient overlap—and recover structural forces via its spatial derivative. These forces obey Newtonian reciprocity without invoking external potentials, demonstrating that interaction itself is a consequence of shared structural modulation.

Finally, we established that the contrast-induced metric can exhibit genuine curvature. When contrast fields vary non-uniformly across space, the resulting metric gives rise to non-zero Christoffel symbols and Riemann curvature. This shows that spatial geometry, including curvature, emerges from contrast structure rather than being imposed a priori.

Simulation results confirmed that this framework reconstructs classical dynamics across uniform, elliptical, and anisotropic cases, and that structural forces and curvature behave in agreement with physical expectations. Every dynamical quantity—velocity, acceleration, force, and even interaction—was recovered from the informational structure of contrast fields.

This theory reinterprets physical ontology from the ground up. Rather than positing spacetime and forces as primitives, it proposes that discernible structure—contrast, gradient, resolution, and modulation—generates space, motion, mass, and interaction. The result is a coordinate-free and observer-situated physics grounded not in external fields, but in the logic of structural appearance.

Future extensions may explore the structural unification of contrast thermodynamics, relativistic contrast flow, and contrast-based quantum emergence. These developments would further consolidate a physics grounded not in what is imposed, but in what is perceptually distinguishable.