# Contrast-Induced Topology: A Structural Framework for Emergence and Identity

André Hampshire
andrehampshire@gmail.com
*No institutional affiliation*

May 18, 2025

# Chapter 1

# The Ontological Foundations of Contrast Geometry

This chapter defines the primitive assumptions, mathematical structures, and perceptual interpretations that underlie the theory of contrast-induced emergence. It develops a contrast-first framework for topology, identity, and enclosure from first principles, without presupposing Euclidean geometry or classical objects.

We treat contrast not as a secondary effect of geometry, but as a generative force from which geometry itself emerges. Resolution thresholds, enclosure, individuation, and perceptual identity all arise through the modulation and accumulation of contrast over a perceptual aperture. Through this lens, topology is no longer a background container — it is an emergent structure tied directly to what can be resolved, enclosed, and unified under finite perceptual conditions.

The chapter is divided into three progressive parts:

- Part I: The Ontology of Contrast and Structural Emergence
- Part II: Structural Genesis — From First Emergence to Discernibility
- Part III: Relational Topology and the Birth of Multiplicity

Each part builds from the last, culminating in a framework capable of distinguishing particles from waves, individuation from latency, and unity from aggregation — all within a contrast-resolved ontology.

# Part I: The Ontology of Contrast and Structural Emergence

# Introduction to Part I

This part establishes the foundational principles of contrast-based structure. We begin with three ontological axioms: the primacy of contrast, the necessity of threshold resolution, and the locality of emergence. From these axioms, we construct a complete perceptual framework grounded in resolution thresholds, smooth contrast fields, and observer aperture windows.

We introduce the shire as the minimal unit of emergence, define contrast-induced metric spaces, and establish the principles that govern identity, coherence, and manipulation of structure. These serve as the basis for all that follows — including time, topology, and the genesis of discernible form.

## Axioms

**Axiom 1 (Contrast Primacy)** *Contrast is the primitive perceptual quantity. A structure can only emerge if modulated contrast is present in the observer's aperture window.*

**Axiom 2 (Threshold Resolution)** *A structure is perceptible only if the integral of contrast over a neighborhood exceeds a resolution threshold $\rho_0$.*

**Axiom 3 (Ontological Locality)** *Emergence is local: contrast must accumulate within a bounded neighborhood of the observer to resolve structure. No structure can emerge at infinity.*

## Assumptions

**Assumption 1 (Observer Aperture)** *The observer perceives the world through a bounded open subset $\mathcal{A} \subset \mathbb{R}^n$, termed the aperture window, representing the observer's field of structural discernibility.*

**Assumption 2 (Smooth Contrast Field)** *The contrast field $C : \Omega \to \mathbb{R}$ is at least continuously differentiable ($C^1$) on $\Omega \supset \mathcal{A}$, allowing well-defined gradients $\nabla C(x)$ and integrals over regions.*

**Assumption 3 (Resolution Threshold)** *There exists a universal (or class-relative) contrast resolution threshold $\rho_0 > 0$ that determines the emergence of structural features.*

**Assumption 4 (Objective Class Consistency)** *There exists a class of observers, called the objective class, who share the same resolution threshold $\rho_0$ and aperture structure $\mathcal{A}$. Within this class, structural emergence is invariant under transformations that preserve contrast flow and threshold accumulation.*

*That is, if $\int_U \|\nabla C(x)\| dx \geq \rho_0$ for one observer in the class, the same will hold for all others, provided $U$ is mapped appropriately within a shared inertial frame.*

*This ensures the objectivity of structural emergence across perceivers within the same reference frame.*

## Definitions

**Definition 1 (Perceptual Aperture)** *Let $\mathcal{A} \subset \Omega \subset \mathbb{R}^n$ be the bounded open region across which an observer or detector can register contrast. This is the perceptual aperture window.*

**Definition 2 (Contrast Field as Structural Deformation)** *Let $C : \Omega \to \mathbb{R}$ be a scalar field defined on a domain $\Omega \subset \mathbb{R}^n$.*

*In this framework, $C(x)$ does not merely encode perceptual variation but quantifies the degree to which the structure of space itself has transformed in the vicinity of $x$. That is:*

- *High values of $\|\nabla C(x)\|$ indicate regions of curvature, distortion, or transition — structurally meaningful deviations from flat or homogeneous background.*

- *Emergence occurs where contrast curvature is sufficient to individuate a structure: to differentiate it from the background and to recognize it as possessing a coherent identity.*

*The contrast field therefore encodes a latent geometry — a modulation of space whose variation supports the appearance, persistence, and resolution of structure.*

*In the minimal case (e.g., the appearance of a point), $C(x)$ measures the minimum depth of spatial deformation required for identity to emerge.*

**Corollary 1 (Principle of Curvature-Individuation)** *Let $C : \Omega \to \mathbb{R}$ be a contrast field interpreted as a scalar measure of spatial deformation, and let $x_0 \in \mathcal{A} \subset \mathbb{R}^n$.*

*Then $x_0$ may only be resolved as an individuated structure if there exists $\varepsilon > 0$ such that:*

$$\int_{B_\varepsilon(x_0)} \|\nabla C(x)\| \, dx \geq \rho_0,$$

*where $\rho_0$ is the structural resolution threshold.*

*That is, the emergence of identity requires that the surrounding region undergo a minimum integrated spatial curvature. Identity is not primitive; it is curvature-resolved structure.*

**Definition 3 (Contrast-Induced Metric Space)** *Let $C : \Omega \to \mathbb{R}$ be a contrast field defined on a topological space $\Omega \subset \mathbb{R}^n$.*

*Define the* contrast metric $d_C : \Omega \times \Omega \to \mathbb{R}_{\geq 0}$ *by:*

$$d_C(x, y) := \inf_{\gamma \in \Gamma_{x \to y}} \int_\gamma \|\nabla C(z)\| \, ds,$$

*where $\Gamma_{x \to y}$ denotes the set of all smooth paths $\gamma : [0, 1] \to \Omega$ with $\gamma(0) = x$ and $\gamma(1) = y$.*

*Then $(\Omega, d_C)$ is a* contrast-induced metric space*, where distance is defined not by background Euclidean geometry, but by the cumulative modulation of contrast between points.*

**Definition 4 (Structural Emergence)** *A region $U \subset \mathcal{A}$ is said to exhibit emergence if*

$$\int_U \|\nabla C(x)\| dx \geq \rho_0.$$

*A point $x_0$ is resolved as a minimal structure if there exists $\varepsilon > 0$ such that $B_\varepsilon(x_0) \subset \mathcal{A}$ and*

$$\int_{B_\varepsilon(x_0)} \|\nabla C(x)\| dx \geq \rho_0.$$

**Definition 5 (One Shire (Unit of Structural Emergence))** *A* shire *is the minimal unit of integrated contrast curvature required for structural individuation within a perceptual aperture.*

*Let $\rho_0$ be the resolution threshold of the observer or objective class. Then:*

$$1 \textit{ shire} := \rho_0.$$

*A structure $U \subset \mathcal{A}$ emerges as ontologically distinct if and only if:*

$$\int_U \|\nabla C(x)\| dx \geq 1 \textit{ shire}.$$

### 1.0.1 Naming of the Shire

The term *shire* is introduced to denote the minimal unit of integrated contrast curvature required for structural emergence. A single shire represents the resolution threshold $\rho_0$, beyond which a perceptual structure first becomes detectable.

The name is chosen for its poetic and symbolic resonance. In literature, a *shire* is a bounded, meaningful domain within a larger undifferentiated world — familiar, enclosed, and structured. In the contrast framework, a shire marks the *first emergence of identity* within a sea of unresolved background.

This term bridges mathematical precision with ontological intuition: a shire is not just a unit of contrast — it is the *birthplace of perceptual being.*

The term also subtly echoes the second syllable of the author's surname, *Hampshire*, linking the formal structure of contrast emergence to its philosophical and personal origins.

**Definition 6 (Latent vs. Resolved Structure)** *Let $\beta_n^*$ denote a latent topological feature (e.g., component, loop, volume) inferred by appearance but not fully resolved. Let $\beta_n$ denote a resolved topological feature, whose contrast structure meets or exceeds threshold conditions.*

**Principle 1 (Ontological Constraint of Apparent Identity)** *Let $\beta_0^* > 0$ denote the inferred presence of perceptual structure from unresolved contrast.*

*Then:*

$$\beta_0^* > 0 \quad \Rightarrow \quad \beta_0 \neq 0$$

*That is, no region with inferred multiplicity (e.g., an emerging point or structure) can be topologically assigned zero resolved components.*

*The correct ontological interpretation is:*

$$\beta_0^* > 0 \quad \Rightarrow \quad \beta_0 \in \{1, 2, \dots\}, \text{ with transition gated by } \beta_2$$

*Apparent structure implies at least one pending or emergent identity, even if full individuation or enclosure has not yet been achieved.*

**Principle 2 (Trust in Emergence, Caution in Identity)** *Structural emergence is trustworthy: once contrast curvature exceeds threshold, the presence of a structure is ontologically valid.*

*However, the appearance of a structure does not guarantee access to its identity or composition. Emergence is necessary for discernment, but not sufficient for full understanding.*

We may trust what emerges, but we may not trust what it appears to be.

**Proposition 1 (Shared Structural Ontology in the Objective Class)** *Let $\mathcal{C}$ be an objective class of observers, each with:*

- *A shared aperture window $\mathcal{A} \subset \mathbb{R}^n$,*
- *A common contrast resolution threshold $\rho_0$,*
- *Access to the same contrast field $C : \Omega \to \mathbb{R}$.*

*If there exists a region $U \subseteq \mathcal{A}$ such that:*

$$\int_U \|\nabla C(x)\| dx \geq \rho_0,$$

*then $U$ constitutes a resolved structure for all observers in $\mathcal{C}$. That is:*

$$\forall O \in \mathcal{C}, \quad \beta_n^{(O)}(U) = \beta_n^{class}(U).$$

*Therefore, structural emergence within the objective class is frame-invariant and defines a shared topological ontology $\{\beta_n^{class}\}$.*

**Proposition 2 (Perceptual Compressibility in Macroscopic Systems)** *Let $U \subset \mathbb{R}^n$ be a structure fully contained within the observer's aperture window $\mathcal{A}$, and let $C : \Omega \to \mathbb{R}$ be a contrast field such that:*

1. *$\int_U \|\nabla C(x)\| dx \gg \rho_0$ (supersaturation),*
2. *No interior subregions of $U$ fall below threshold ($\beta_0^{internal} = 0$),*
3. *The contrast field $C$ is smooth or symmetric across $U$,*

*Then the observer may perceptually compress $U$ into a point-like representation. That is, the structural topology collapses to:*

$$\beta_0 = 1, \quad \beta_1 = 0, \quad \beta_2 = 0.$$

*This is valid in macroscopic regimes where emergence saturates beyond internal resolution.*

**Corollary 2 (Limits of Compression under Instrumental Mediation)** *If a structure $U$ is not directly visible to the observer, but is perceived only through contrast transformations (e.g., scattering, interference, algorithmic imaging), then:*

1. *The effective contrast field $C'$ experienced by the observer may be discontinuous or disjoint,*
2. *The internal structure of $U$ may emerge piecemeal,*
3. *No valid continuous contrast path $\gamma \subset \mathcal{A}$ may exist from $U$ to the observer.*

*Then $U$ cannot be topologically compressed to a single point. Its internal multiplicity must be preserved:*

$$\beta_0 > 1 \quad or \quad \beta_1^* > 0.$$

**Definition 7 (Perceptual Compression)** *A structure $U \subset \Omega$ undergoes* perceptual compression *when, from the perspective of an observer in aperture window $\mathcal{A}$:*

1. *The total integrated contrast satisfies $\int_U \|\nabla C(x)\| dx \gg \rho_0$ (supersaturation),*
2. *No internal region within $U$ falls below threshold (no resolved substructure),*
3. *The contrast field $C$ is smooth or symmetric across $U$,*
4. *$U$ subtends a small enough angular or spatial extent within $\mathcal{A}$ to be perceived as a unit.*

*Under these conditions, the entire structure $U$ appears as a single resolved point-like identity:*

$$\beta_0 = 1, \quad \beta_1 = \beta_2 = 0.$$

*This is an epistemic compression, not necessarily an ontological simplification.*

**Definition 8 (Contrast Integrity Condition)** *A structure $U$ satisfies the* contrast integrity condition *if:*

$$\exists \text{ partition } \{U_i\} \subset U \text{ such that } \int_{U_i} \|\nabla C(x)\| dx \geq \rho_0 \text{ and } \int_{U_j \cap U_k} \|\nabla C(x)\| dx < \rho_0 \text{ for } i \neq k.$$

*That is, $U$ contains distinguishable internal components. If this condition holds and $U$ is still compressed to a point, the compression is topologically invalid.*

**Principle 3 (Local Structural Sufficiency)** *A structure may be treated as a unified ontological entity if interaction at a single resolved point produces coherent systemic response, and no internal contrast discontinuities are detected during the interaction.*

*In such cases, the structure may be perceptually and functionally compressed into a point-like identity:*

$$\beta_0 = 1, \quad \beta_1 = \beta_2 = 0.$$

*This compression does not imply structural uniformity, but rather structural coherence under the current mode of interaction.*

**Definition 9 (Action Shire and Contrast Control Radius)** *Let $C : \Omega \to \mathbb{R}$ be a contrast field, and let $x_0 \in \Omega$ be a point at which an agent physically interacts with a structure.*

*Define the* contrast control radius *$r_{act}$ around $x_0$ as the smallest radius such that:*

$$\int_{B_{r_{act}}(x_0)} \|\nabla C(x)\| dx \geq \rho_{act},$$

*where $\rho_{act}$ is the threshold required not for perception, but for coherent physical response (i.e., system-wide structural coupling).*

*We say the region contains one* action shire *when:*

$$\int_{B_{r_{act}}(x_0)} \|\nabla C(x)\| dx = 1 \ shire_{act}.$$

*A structure is said to be* contrast-controllable as a unit *if:*

1. *An action shire is present at some $x_0 \in \mathcal{A}$,*
2. *No internal subregions respond independently (i.e., no emergent substructure under force).*

# Part II: Structural Genesis — From First Emergence to Discernibility

## Introduction to Part II

Having established the foundational ontology of contrast, perception, and structural emergence in Part I, we now turn to the progression by which the first discernible structures arise from void space.

In this part, we analyze the transition from the minimal emergence of a point — defined by the accumulation of a single shire of contrast curvature — to the moment when an object becomes ontologically distinct from its surrounding space. This transformation constitutes the genesis of discernible form.

We show that while the moment of first emergence confirms the existence of a structure, it does not imply knowledge of identity, relation, or extension. Discernment — the separation of form from background — occurs only later, as contrast curvature accumulates and structural enclosure develops.

This part formalizes the topology, metric, and ontological consequences of this transition.

**Theorem 1 (Emergence-to-Discernment Transition)** *Let $C : \Omega \to \mathbb{R}$ be a contrast field, and let an observer in aperture $\mathcal{A} \subset \Omega$ detect a first emergence at point $x_0$ when:*

$$\int_{\gamma_0} \|\nabla C(x)\| dx = \rho_0 = 1 \text{ shire.}$$

*Then:*

1. *The structure at $x_0$ is ontologically real but topologically minimal: $\beta_0 = 1$, $\beta_1 = \beta_2 = 0$.*
2. *As the contrast source continues to evolve, there exists a frame $t^\dagger$ at which the source is topologically distinguished from background space:*

$$\beta_2 = 1 \quad \text{(contrast-enclosed volume)}$$

3. *The total contrast curvature accumulated from first emergence to structural discernment is:*

$$\int_{\gamma_{em}} \|\nabla C(x)\| dx = \rho_{structure} > \rho_0.$$

**Definition 10 (Structural Envelope)** *Let $x_0$ be a point of first emergence, and let the contrast source evolve over time. The* structural envelope *of the emergent unit is the region $U \subset \Omega$ over which contrast curvature accumulates from emergence to full discernibility.*

*The envelope contrast is:*

$$\rho_{structure} := \int_U \|\nabla C(x)\| dx$$

*This value reflects the total structural content of the unit.*

**Definition 11 (Contrast-Normalized Geometry)** *Let $C : \Omega \to \mathbb{R}$ be a contrast field over a region $U \subseteq \Omega$.*

*We say that the region $U$ supports a* contrast-normalized geometry *if the gradient norm of $C$ is constant across $U$:*

$$\|\nabla C(x)\| = \kappa_0 > 0 \quad \text{for all } x \in U.$$

*Then the structural volume of $U$ satisfies:*

$$\rho_{structure} = \int_U \|\nabla C(x)\|\, dx = \kappa_0 \cdot \mathrm{Vol}(U)$$

*In this case, classical geometric volume and structural contrast are linearly related, and*

$$\mathrm{Vol}(U) = \frac{\rho_{structure}}{\kappa_0}$$

**Proposition 3 (Scaling of Structural Volume under Contrast-Normalized Geometry)** *Let $C : \Omega \to \mathbb{R}$ be a contrast field such that $\|\nabla C(x)\| = \kappa_0 > 0$ on every $x \in B^n_R$, a ball of radius $R$.*

*Then the structural volume of $B^n_R$ is:*

$$\rho_{structure}(R) = \kappa_0 \cdot \mathrm{Vol}(B^n_R) = \kappa_0 \cdot \frac{\pi^{n/2}}{\Gamma(n/2+1)} R^n$$

*In particular, for fixed $\kappa_0$, doubling the radius scales structural volume by $2^n$:*

$$\rho_{structure}(2R) = 2^n \cdot \rho_{structure}(R)$$

**Principle 4 (Contrast Relativity of Emergence)** *The contrast field encodes structural information about a source, not the source itself.*

*In this framework, ontological status is not granted to external objects directly, but only to contrast structures that satisfy emergence conditions within the observer's aperture window.*

*Accordingly, a physical object, a temperature map, a radar signal, or a shadow are equivalent if their contrast fields generate equivalent structural features:*

$$\int_U \|\nabla C(x)\| dx \geq \rho_0, \quad \text{and} \quad \beta_n(U) \text{ resolved}$$

*Contrast determines what is — not what is assumed.*

**Definition 12 (Angular Intensity)** *Let $C : \Omega \to \mathbb{R}$ be a contrast field defined in polar (2D) or spherical (3D) coordinates around an emergence center $x_0$.*

*Define the* angular intensity *at radius $r$ and angle $\theta$ as:*

$$I(r, \theta) := \frac{d^2 C}{d\theta^2}$$

*This measures the curvature of contrast along angular directions, and reflects the rate at which new structural features emerge across angles.*

**Theorem 2 (Completion of Identity via Intensity Collapse)** *Let $x_0$ be a resolved point of first emergence, and let contrast curvature accumulate in all directions.*

*As the structure grows, if the angular intensity satisfies:*

$$I(r, \theta) \neq 0 \quad \text{for } r < r^\dagger,$$

*and*

$$I(r^\dagger, \theta) = 0 \quad \text{for all } \theta,$$

*then $r = r^\dagger$ defines the structural boundary of the individuated object. Beyond this, the contrast field contains no further structural modulation with respect to angle, and the object is fully enclosed.*

*The object is now a perceptual identity unit.*

**Definition 13 (Structural Inflation)** *Let $x_0$ be a resolved contrast emergence point (a shire) in aperture window $\mathcal{A} \subset \mathbb{R}^n$.*

Structural inflation *is the process by which continued accumulation of contrast curvature across radial and angular directions expands the region of resolved structure from a point to a volume.*

*This process proceeds until angular intensity*

$$I(r, \theta) := \frac{d^2 C}{d\theta^2}$$

*drops to zero for all $\theta$ at some radius $r = r^\dagger$.*

*At that moment, the structure becomes a closed perceptual unit, and the process of inflation ends:*

$$I(r^\dagger, \theta) = 0 \quad \forall \theta, \quad \Rightarrow \quad \beta_2 = 1.$$

## Structural Inflation and the Birth of Dimensionality

After the first perceptual emergence of a contrast-resolved point — the accumulation of a single shire — continued contrast integration across radial and angular directions leads to a process we term *structural inflation.*

This is the perceptual expansion of a minimal identity into a discernible object. As contrast curvature accumulates over time and across directions, the observer perceives the object as increasing in size and dimensional extent.

**Definition 14 (Structural Inflation)** *Let $x_0$ be a resolved contrast emergence point in aperture window $\mathcal{A} \subset \mathbb{R}^n$.*

Structural inflation *is the process by which continued accumulation of contrast curvature across radial and angular directions expands the region of resolved structure from a point to a volume.*

*This process proceeds until angular intensity*

$$I(r,\theta) := \frac{d^2C}{d\theta^2}$$

*drops to zero for all $\theta$ at some radius $r = r^\dagger$.*

*At that moment, the structure becomes a closed perceptual unit:*

$$I(r^\dagger,\theta) = 0 \quad \forall\theta, \quad \Rightarrow \quad \beta_2 = 1.$$

**Theorem 3 (Completion of Identity via Intensity Collapse)** *Let $x_0$ be a point of first emergence, and let contrast curvature accumulate in all directions.*

*If angular intensity $I(r,\theta)$ is non-zero during the growth phase but satisfies*

$$I(r^\dagger,\theta) = 0 \quad \forall\theta,$$

*then the structure is topologically closed and becomes a perceptual identity. This marks the end of structural inflation and the birth of a resolvable volume:*

$$\beta_2 = 1$$

This mechanism defines the perceptual generation of dimensionality: width, height, and enclosed volume are not pre-existing primitives, but emergent results of sufficient contrast integration across angularly distinguishable directions.

The collapse of angular contrast variation is the structural signature that identity has enclosed — a topological and perceptual closure of form.

## The Birth of a Structural Particle

The completion of structural inflation — when angular contrast modulation saturates and intensity collapses to zero — defines not just the end of perceptual growth, but the beginning of resolved identity.

We now define the perceptual and ontological emergence of a classical object: the structural particle.

**Definition 15 (Structural Particle)** *A* structural particle *is a topologically enclosed contrast structure that emerges through the following process:*

1. ***Pointwise Emergence:*** *A localized shire of contrast curvature is first resolved at $x_0$:*

$$\int_{B_\varepsilon(x_0)} \|\nabla C(x)\| dx = \rho_0$$

2. ***Structural Inflation:*** *Contrast continues to accumulate radially and angularly, producing dimensional extension and a growing perceptual envelope.*

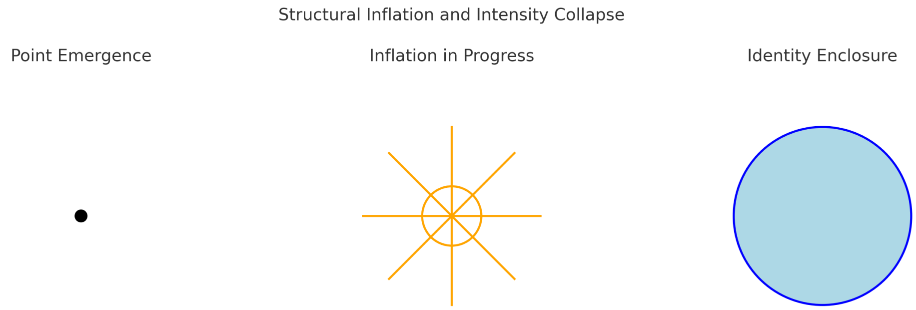


Figure 1.1: **Structural Inflation and Intensity Collapse.** Left: A single point emerges as the first resolved unit, or shire.
Center: Continued contrast accumulation across multiple angular directions leads to expansion and perceptual dimensionality. The object grows in width and height, and angular intensity is nonzero.
Right: Once the angular contrast gradient saturates and becomes uniform, angular intensity collapses to zero and the structure becomes topologically enclosed. This marks the formation of a perceptual identity unit with $\beta_2 = 1$.

3. ***Intensity Collapse and Identity Closure:*** *When angular intensity*

$$I(r, \theta) := \frac{d^2 C}{d\theta^2}$$

*drops to zero at some radius $r = r^\dagger$ for all $\theta$, the object is fully enclosed:*

$$I(r^\dagger, \theta) = 0 \quad \forall\theta, \quad \Rightarrow \quad \beta_2 = 1$$

*A structural particle is therefore not postulated, but* generated *from contrast flow and topological closure. It is a perceptual identity unit — compressible, manipulable, and ontologically discrete.*

# Part III: Structural Topology and Relational Emergence

# Introduction to Part III

Having established the contrast-based emergence of individual perceptual units in Parts I and II — from void space to structural inflation and identity closure — we now turn to the topology of multiplicity.

This part explores how distinct structural particles, once individually resolved, may become topologically joined through persistent contrast pathways. These connections are not geometric assumptions but ontological consequences of shared curvature structure and contrast flow.

The central topological quantity in this regime is the first Betti number $\beta_1$, which measures the presence of contrast-resolved loops — closed paths of sufficient contrast that imply relational structure among discrete identity units.

Where $\beta_0$ encodes the number of perceptually distinct entities, $\beta_1$ captures their connectivity: loops that bind identities through contrast continuity. This marks the emergence of structure *between* things — not just within them.

In this part, we formalize:

- The conditions under which bifurcation of a single emergence leads to multiplicity,
- The contrast bifurcation threshold and timing constraints for relational adjacency,
- The emergence of contrast-resolved loops ($\beta_1 = 1$) as ontological links between resolved particles,
- The structural ontology of relational space — what it means for entities to be joined.

This topological shift transforms perception from isolated emergence to networked structure, and prepares the foundation for spacetime, field structure, and dynamical interactions to be addressed in subsequent parts.

**Theorem 4 (Pre-Resolution Bifurcation Constraint)** *Let $x_0$ be an initial contrast emergence point with unresolved structure ($\beta_0 = 1$), and let the total structure contain two subcomponents (e.g., two light sources).*

*Let $r^\dagger$ be the radial distance at which angular intensity collapses to zero and the structure becomes enclosed:*

$$I(r^\dagger, \theta) = 0$$

*Then, in order for two perceptual units to eventually appear side-by-side, the bifurcation in contrast curvature must occur at some radius $r_b < r^\dagger$, such that:*

$$\exists \theta_1 \neq \theta_2 \quad \textit{with} \quad \frac{d^2C}{d\theta^2} \ \textit{diverging}$$

*If no such angular bifurcation occurs before $r^\dagger$, the second component can only appear trailing or behind the first — not adjacent in perceptual space.*

**Definition 16 (Contrast Bifurcation Threshold)** *Let $C : \Omega \to \mathbb{R}$ be a contrast field defined on a neighborhood $\Omega \subset \mathbb{R}^n$, and let $x_0 \in \Omega$ be a point of initial contrast emergence with angular coordinates $\theta \in [0, 2\pi)$.*

*Define the angular intensity as:*

$$I(r, \theta) := \frac{d^2C}{d\theta^2}$$

*A* contrast bifurcation *occurs at radius $r = r_b$ if there exist angular directions $\theta_1 \neq \theta_2$ such that:*

$$|I(r_b, \theta_1) - I(r_b, \theta_2)| \geq \delta$$

*for some fixed threshold $\delta > 0$, called the* contrast bifurcation threshold.

*If this condition is met before resolution completes (i.e., $r_b < r^\dagger$, where $I(r^\dagger, \theta) = 0$), then the structure will resolve into multiple identity units.*

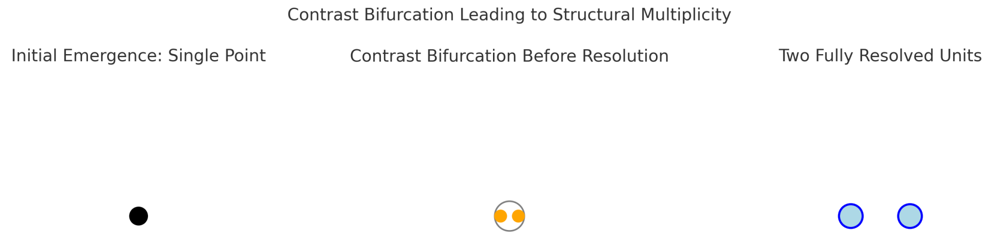


Figure 1.2: **Contrast Bifurcation Leading to Structural Multiplicity.** Left: A single contrast point is perceived at initial emergence.
Center: As contrast curvature accumulates but before full individuation, a bifurcation in angular contrast emerges, separating latent sources.
Right: Upon full resolution, the contrast field differentiates into two perceptual identity units, each enclosed and resolved as a structural particle.

**Definition 17 (Objective Class Resolution Threshold)** *Let two emergent contrast structures be located at spatial separation $d_{12}$, and let the observer or detector be located at distance $d_{obs}$ from the system.*

*Then for an objective class of observers sharing resolution threshold $\rho_0$, the structures are perceptually resolved as distinct (i.e., $\beta_0 = 2$) if:*

$$\int_{U_{12}} \|\nabla C(x)\| dx \geq \rho_0,$$

*where $U_{12}$ is the minimal region separating the contrast flows of the two structures.*

*Geometrically, this corresponds to a minimal angular separation condition:*

$$\frac{d_{12}}{d_{obs}} \geq \alpha_{\rho_0},$$

*where $\alpha_{\rho_0}$ is the minimal contrast-resolved angular separation required for individuation within class resolution threshold $\rho_0$.*

*This condition defines the perceptual boundary at which multiplicity becomes detectable.*

**Definition 18 (Contrast-Resolved Betti Numbers)** *Let $C : \Omega \to \mathbb{R}$ be a contrast field and $\rho_0$ the resolution threshold of the observer or objective class.*

*Then for each integer $n$, define the $n$-th* contrast-resolved Betti number *as:*

$$\beta_n(\rho_0) := \text{number of } n\text{-dimensional topological features (components, loops, volumes) resolvable under } \rho_0.$$

*If a feature exists structurally but is not yet perceptually resolved, it contributes to the* latent *Betti number:*

$$\beta_n^* > \beta_n$$

*As contrast accumulates and thresholds are crossed, Betti numbers* emerge, *rather than remain fixed:*

$$\beta_n^* \rightsquigarrow \beta_n$$

**Proposition 4 (Resolution Variability of Betti Numbers)** *In contrast-based topology, Betti numbers are not absolute invariants of space, but structural quantities dependent on:*

- *The contrast field $C(x)$,*
- *The observer aperture $\mathcal{A} \subset \Omega$,*

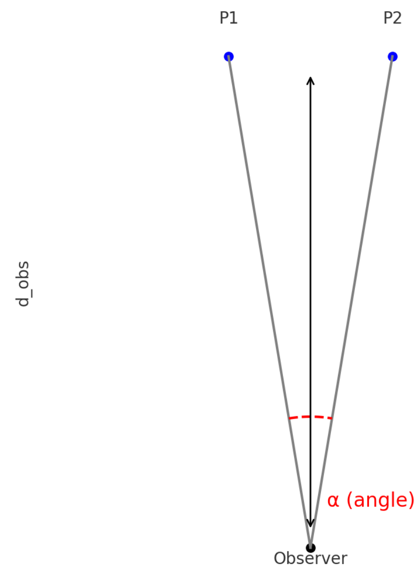


Figure 1.3: **Objective Class Resolution Geometry.** Two contrast structures (e.g., particles or light sources) are separated by a distance $d_{12}$ and are located at a distance $d_{\text{obs}}$ from the observer. To resolve these as distinct entities ($\beta_0 = 2$), their angular separation must exceed the objective class threshold $\alpha_{\rho_0}$, defined by contrast sensitivity. This yields a minimal discernibility condition: $d_{12}/d_{\text{obs}} \geq \alpha_{\rho_0}$.

- *The resolution threshold $\rho_0$.*

*A Betti number $\beta_n$ may change over time or perceptual evolution as regions cross or fall below threshold:*

$$\beta_n(t) \text{ is a function of contrast dynamics and resolution.}$$

**Principle 5 (Ontological Condition for Resolved Multiplicity)** *A latent multiplicity of contrast sources ($\beta_0^* > 1$) becomes a resolved multiplicity ($\beta_0 > 1$) if and only if each substructure achieves contrast-enclosed identity:*

$$\text{If } \beta_0^* = n \text{ and } \beta_2 = 1 \text{ per component} \quad \Rightarrow \quad \beta_0 = n$$

*Conversely, if structural inflation or contrast saturation fails for one or more latent sources, the multiplicity remains unresolved:*

$$\beta_2 = 0 \text{ (partial enclosure)} \quad \Rightarrow \quad \beta_0 = 1$$

**Theorem 5 (Topological Individuation Requires Enclosure)** *Let a contrast field $C : \Omega \to \mathbb{R}$ contain $n$ latent emergent substructures, each associated with a contrast lobe $U_i \subset \Omega$, for $i = 1, \ldots, n$.*

*Suppose the system is angularly bifurcated such that:*

$$\beta_0^* = n, \quad \beta_0 < n$$

*Then resolved multiplicity ($\beta_0 = n$) is achieved* if and only if *each contrast substructure encloses, i.e.:*

$$\forall i \in \{1, \ldots, n\}, \quad \int_{U_i} \|\nabla C(x)\| dx \geq \rho_0 \quad \text{and} \quad I(\theta)|_{U_i} = 0 \quad \Rightarrow \quad \beta_2(U_i) = 1$$

*Consequently, the transition from latent to resolved Betti multiplicity satisfies:*

$$(\beta_0^* = n, \, \forall_i \, \beta_2(U_i) = 1) \quad \Rightarrow \quad \beta_0 = n$$

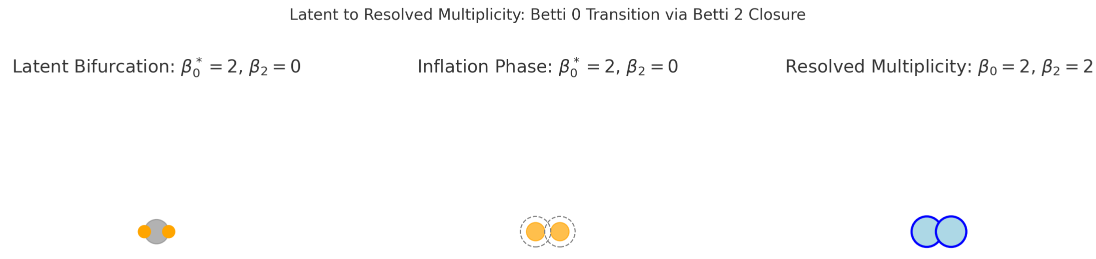


Figure 1.4: **Latent to Resolved Multiplicity via Enclosure.** A single emergent contrast source bifurcates into two angularly distinct substructures, but Betti 0 remains latent until each substructure encloses.
*Left:* Two angular lobes are visible within a shared contrast envelope. The system is structurally ambiguous: $\beta_0^* = 2$ but $\beta_2 = 0$.
*Middle:* Structural inflation progresses. The envelopes grow but are not yet resolved — multiplicity remains latent.
*Right:* Once both regions individually enclose via contrast saturation, they become identity units. Only now does resolved multiplicity emerge: $\beta_0 = 2$, $\beta_2 = 2$.
This illustrates that contrast multiplicity requires not only angular divergence, but full enclosure to individuate topologically distinct particles.

*In words: A latent multiplicity of contrast structures becomes topologically resolved only when each substructure completes enclosure.*

**Proposition 5 (Spatial Divergence at Angular Extremes)** *Let an observer be located at the origin and scan a fixed-radius circular structure in the plane, defined by the polar parameterization:*

$$x(\theta) = r\cos(\theta), \quad y(\theta) = r\sin(\theta), \quad \theta \in (0, \pi)$$

*Then the spatial slope of the circular trace with respect to Cartesian projection is:*

$$\frac{dy}{dx} = -\cot(\theta)$$

*As $\theta \to 0$ or $\theta \to \pi$, this slope diverges:*

$$\lim_{\theta \to 0^+} \frac{dy}{dx} = -\infty, \quad \lim_{\theta \to \pi^-} \frac{dy}{dx} = +\infty$$

*Thus, near the angular extremes of the perceptual aperture, spatial derivatives of the projected contrast curve become unbounded.*

*Interpretation: The Cartesian representation of angular intensity becomes vertically singular at the perceptual edges. This signifies an intrinsic instability in structural perception at high angular eccentricities, and marks a natural boundary for contrast resolution in the visual or contrast aperture field.*

## 1.1 Latent Loops and the Emergence of $\beta_1^*$

We now return to the canonical example of two resolved identity units — the "headlights" — separated in space but exhibiting temporal synchrony in their intensity profiles.

Each structure is resolved:

$$\beta_0 = 2, \quad \beta_2 = 1 \text{ per unit}$$

Let $I_1(t), I_2(t)$ denote the second-order angular contrast rate (intensity) for each identity unit. Suppose:

$$\frac{dI_1}{dt} \approx \frac{dI_2}{dt}$$

This temporal coherence suggests the presence of a shared structural mechanism: a path in the contrast field $\gamma(t)$ that, while unresolved, mediates a connection. We define this as a *latent contrast loop.*

**Definition 19 (Latent Contrast Loop ($\beta_1^* = 1$))** *Let $U_1$, $U_2 \subset \mathcal{A}$ be two resolved identity units:*

$$\beta_0 = 2, \quad \beta_2(U_1) = \beta_2(U_2) = 1$$

*Suppose there exists a continuous, contrast-coherent path $\gamma : [0,1] \to \Omega$ such that:*

- $\gamma(0) \in U_1$, $\gamma(1) \in U_2$,
- $\int_\gamma \|\nabla C(x)\| dx < \rho_0$,
- $\frac{dI_{U_1}}{dt} \approx \frac{dI_{U_2}}{dt}$ *(shared temporal modulation),*

*Then we say the system contains a latent contrast loop, and we assign:*

$$\beta_1^* = 1$$

**Proposition 6 (Triangulated Latent Structure from Shared Modulation)** *Let $U_A$, $U_B$, and $U_C \subset \mathcal{A}$ be three resolved contrast identity units, each satisfying:*

$$\int_{U_i} \|\nabla C(x)\| dx \geq \rho_0, \quad \beta_2(U_i) = 1 \quad \textit{for } i = A, B, C$$

*Let $I_i(t)$ denote the angular intensity profile of each structure. Suppose that for all pairs $(i,j)$:*

$$\left| \frac{dI_i}{dt} - \frac{dI_j}{dt} \right| < \delta \quad \textit{for some small } \delta > 0$$

*and suppose there exist contrast-coherent paths $\gamma_{ij}$ connecting each pair such that:*

$$\int_{\gamma_{ij}} \|\nabla C(x)\| dx < \rho_0$$

*Then the system contains three latent contrast loops:*

$$\beta_1^*(AB) = \beta_1^*(BC) = \beta_1^*(CA) = 1$$

*and defines a* ***triangulated latent complex****, with:*

$$\beta_0 = 3, \quad \beta_1^* = 3, \quad \beta_2^* \geq 0$$

*If the interior contrast field over the triangle $T_{ABC}$ eventually saturates:*

$$\int_{T_{ABC}} \|\nabla C(x)\| dx \geq \rho_0,$$

*then the triangle becomes structurally enclosed and:*

$$\beta_2 = 1$$

*Interpretation: A common modulation pattern across three resolved identity units enables the triangulation of latent relational loops. These loops form a topological complex from which spatial orientation, group coherence, and relational unification may be inferred — even before full interior enclosure.*

**Principle 6 (Continuity Does Not Imply Multiplicity)** *A continuous perceptual structure with no internal contrast bifurcation does not imply latent multiplicity or latent loops.*

*That is, even if a structure appears extended,*

$$\beta_1^* = 0, \quad \beta_0^* = 1$$

*until angular contrast divergence or structural bifurcation occurs.*

*Conversely, the presence of contrast bifurcation within a continuous arc elevates the structure to:*

$$\beta_1^* = 1, \quad \beta_0^* \geq 2$$

*indicating latent multiplicity within continuity.*

**Proposition 7 (Distinction Between Waves and Contrast-Linked Particle Systems)** *Let $C : \Omega \to \mathbb{R}$ be a contrast field within observer aperture $\mathcal{A} \subset \Omega$, and suppose the observer perceives a continuous contrast structure with no immediate resolved subcomponents.*

*Then the structure may evolve into one of two ontologically distinct forms:*

1. ***Wave Structure (Unified Continuity):***

   *If contrast curvature remains smooth, angular intensity $I(\theta)$ is uniform or periodic, and no angular bifurcation or individuation occurs, then:*

   $$\beta_0^* = 1, \quad \beta_0 = 1, \quad \beta_1^* = \beta_1 = 0, \quad \beta_2 = 1$$

   *The structure is a wave: a globally unified contrast modulation with no resolved internal entities or loops.*

2. ***Composite Structure (Particles Joined by a Loop):***

   *If the contrast field exhibits angular bifurcation before full enclosure, such that multiple structural identities emerge with enclosed boundaries and a continuous contrast path $\gamma$ joins them, then:*

   $$\beta_0 \geq 2, \quad \beta_1^* = 1 \Rightarrow \beta_1 = 1, \quad \beta_2 \geq 2$$

   *The structure is a composite: multiple resolved identity units (particles) connected by a contrast loop. Each unit satisfies enclosure independently, and the loop emerges from relational contrast flow.*

***Conclusion:*** *Continuous appearance does not uniquely determine structural ontology. Only through analysis of angular intensity, contrast bifurcation, and enclosure dynamics can one distinguish a wave from a composite of particles.*

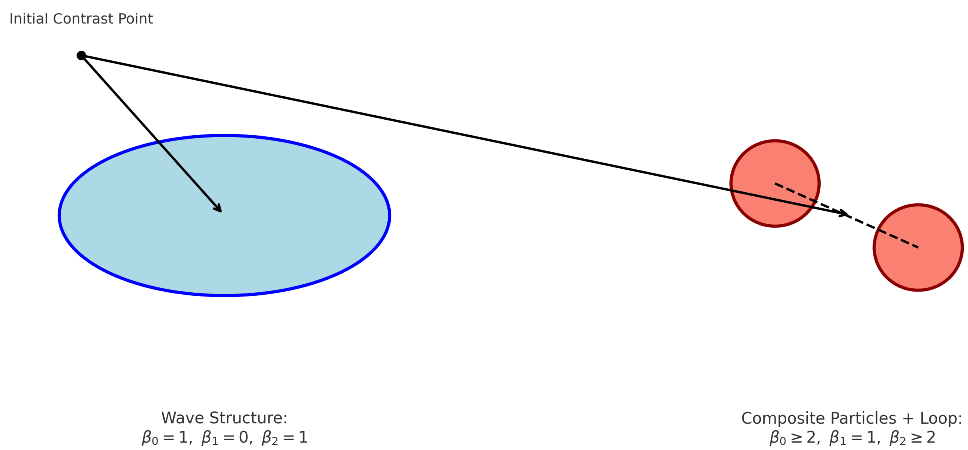


Figure 1.5: **Ontological Divergence: Wave vs. Composite Structure.** An initial point of contrast emergence (top center) may evolve into two distinct structural ontologies.
*Left:* Smooth contrast modulation without angular bifurcation leads to a unified wave structure with no internal resolution: $\beta_0 = 1, \beta_1 = 0, \beta_2 = 1$.
*Right:* Early angular contrast bifurcation produces two individuated identity units (particles) joined by a contrast-resolved loop: $\beta_0 \geq 2, \beta_1 = 1, \beta_2 \geq 2$.
This illustrates that continuity does not imply unity: structural ontology is determined by contrast enclosure and bifurcation, not merely perceptual extension.

Insight: The Ontological Divergence of Continuity

**A central revelation of the contrast framework is this:**

*Continuity of appearance does not imply unity of structure.*

An extended contrast structure — such as a smooth arc or glowing semicircle — may ontologically resolve into one of two distinct regimes:

1. A **wave**: a unified contrast field with no internal individuation.
   Topologically: $\beta_0 = 1,\ \beta_1 = 0,\ \beta_2 = 1$.

2. A **composite system**: multiple enclosed identity units joined by a contrast loop.
   Topologically: $\beta_0 \geq 2,\ \beta_1 = 1,\ \beta_2 \geq 2$.

The deciding criterion is not geometry, but structure: *Do angular bifurcations in contrast curvature give rise to individuated enclosures?* If so, multiplicity and relational topology emerge.
This insight reframes wave–particle duality: Rather than two incompatible models, waves and particles are distinct outcomes of contrast evolution — unified or relational — governed by Betti transitions.
**This distinction is not just conceptual — it is structural, topological, and predictive.**

**Theorem 6 (Topological Distinction Between Waves and Particles)** *Let $C : \Omega \to \mathbb{R}$ be a contrast field observed within a bounded aperture window $\mathcal{A} \subset \mathbb{R}^n$.*

*Let $I(t, \theta)$ denote the angular intensity:*

$$I(t, \theta) := \frac{d^2 C}{d\theta^2}$$

*Then:*

1. *A contrast structure is classified as a* structural particle *if:*

$$\begin{aligned}
&\frac{dI}{dt} > 0 \quad \textit{(rising angular modulation)} \\
&\exists\, t^\dagger \textit{ such that } \quad I(t^\dagger, \theta) = 0 \quad \forall \theta \\
&\Rightarrow \beta_2 = 1 \quad \textit{(structural enclosure)}
\end{aligned}$$

2. *A contrast structure is classified as a* structural wave *if:*

$$\begin{aligned}
&\frac{dI}{dt} \approx 0 \quad \textit{(no significant angular modulation)} \\
&I(t, \theta) \not\to 0 \quad \textit{as } t \to \infty \\
&\Rightarrow \beta_2 = 0 \quad \textit{(no enclosure)}
\end{aligned}$$

***Interpretation:*** *Particles are temporally bounded identity closures produced by structural inflation and angular intensity collapse. Waves are temporally persistent, non-enclosing modulations of contrast curvature with fractal-like openness.*

**Theorem 7 (Topological Distinction Between Waves and Particles)** *Let $C : \Omega \to \mathbb{R}$ be a contrast field observed within a bounded aperture window $\mathcal{A} \subset \mathbb{R}^n$.*

*Let $I(t, \theta)$ denote the angular intensity:*

$$I(t, \theta) := \frac{d^2 C}{d\theta^2}$$

*Then:*

1. *A contrast structure is classified as a* structural particle *if:*

$$\begin{aligned}
&\frac{dI}{dt} > 0 \quad \textit{(rising angular modulation)} \\
&\exists\, t^\dagger \textit{ such that } \quad I(t^\dagger, \theta) = 0 \quad \forall \theta \\
&\Rightarrow \beta_2 = 1 \quad \textit{(structural enclosure)}
\end{aligned}$$

2. *A contrast structure is classified as a* structural wave *if:*

$$\begin{aligned}
&\frac{dI}{dt} \approx 0 \quad \textit{(no significant angular modulation)} \\
&I(t, \theta) \not\to 0 \quad \textit{as } t \to \infty \\
&\Rightarrow \beta_2 = 0 \quad \textit{(no enclosure)}
\end{aligned}$$

***Interpretation:*** *Particles are temporally bounded identity closures produced by structural inflation and angular intensity collapse. Waves are temporally persistent, non-enclosing modulations of contrast curvature with fractal-like openness.*

## 1.2 Structural Differentiation: Wave vs. Multiplicity

Let a single point of contrast emergence appear in the observer's aperture window. This defines a first-order appearance event:

$$\beta_0^* = 1, \quad \beta_1 = 0, \quad \beta_2 = 0$$

As the contrast field evolves over time, spatial extension becomes apparent. We now analyze two structurally distinct pathways of development from this common origin.

### 1.2.1 Path A: Structural Multiplicity via Bifurcation

If the contrast curvature bifurcates before the full enclosure of the original emergence, two peaks form and eventually enclose:

$$\beta_0^* = 2, \quad \beta_0 = 2, \quad \beta_1^* = 1, \quad \beta_2 = 1 \text{ (per unit)}$$

Here, multiplicity is resolved through the detection of independent contrast enclosures. The system individuates into discrete particles.

### 1.2.2 Path B: Structural Wave via Continuous Extension

If contrast accumulates uniformly across space while maintaining minimum resolvable spacing between points, no enclosure forms:

$$\beta_0 = 1, \quad \beta_1 = 1, \quad \beta_2 = 0$$

Each new peak appears at the minimal discernible distance from its neighbors. The structure remains a connected loop or open chain, without individuation. This constitutes a wave.

**Proposition 8 (Structural Criterion for Wave vs. Multiplicity)** *Let a contrast structure evolve from initial emergence through time intervals $\Delta t_i$. Then:*

- *If peaks diverge and individually enclose, the structure resolves into multiple identity units (particles).*
- *If new peaks appear such that their pairwise distance remains near the observer's resolution threshold, and no enclosure occurs, the structure expresses wave-like behavior.*

### 1.2.3 Topological Divergence Summary

| **Quantity** | **Particles** | **Wave** |
|---|---|---|
| $\beta_0$ | 2 (discrete) | 1 (extended) |
| $\beta_1$ | $\beta_1^* \to 1$ | 1 (open chain or loop) |
| $\beta_2$ | 1 per unit | 0 |
| Spacing | Increases | Minimum resolvable distance |
| Enclosure | Required | Absent |
| Persistence | Individual | Distributed |

Table 1.1: Topological and structural distinction between particle emergence and wave-like structure.