Example:

E_1 $x_1 - 2x_2 + x_3 = 0$

E_2 $2x_2 - 8x_3 = 8$

E_3 $-4x_1 + 5x_2 + 9x_3 = 9$

$E_3 \leftarrow E_3 + 4E_1$

$$R_1 \begin{bmatrix} 1 & -2 & 1 & | & 0 \\ 0 & 2 & -8 & | & 8 \\ -4 & 5 & 9 & | & -9 \end{bmatrix} \begin{matrix} R_2 \\ R_3 \end{matrix}$$

no. of colums of the
original matrix: no. of variables

no. of rows: no. of equations

$m \times n$

no. of rows   no. of columns

$R_3 \leftarrow R_3 + 4R_1$

$$\begin{bmatrix} 1 & -2 & 1 & 0 \\ 0 & 2 & -8 & 8 \\ 0 & -3 & 13 & -9 \end{bmatrix}$$

$R_2 \leftarrow \frac{1}{2} R_2 \begin{bmatrix} 1 & -2 & 1 & 0 \\ 0 & 1 & -4 & 4 \\ 0 & -3 & 13 & -9 \end{bmatrix}$

$R_3 \leftarrow 3R_2 + R_3 \begin{bmatrix} 1 & -2 & 1 & 0 \\ 0 & 1 & -4 & 4 \\ 0 & 0 & 1 & 3 \end{bmatrix}$

$R_1 \leftarrow R_1 - R_3$

$R_2 \leftarrow R_2 + 4R_2 \begin{bmatrix} 1 & -2 & 0 & -3 \\ 0 & 1 & 0 & 16 \\ 0 & 0 & 1 & 3 \end{bmatrix}$

$R_1 \leftarrow R_1 + 2R_2$

$$\Rightarrow \begin{bmatrix} 1 & 0 & 0 & 29 \\ 0 & 1 & 0 & 16 \\ 0 & 0 & 1 & 3 \end{bmatrix}$$

$x_1 + 0x_2 + 0x_3 = 29$

$0x_1 + x_2 + 0x_3 = 16 \Rightarrow \begin{cases} x_1 = 29 \\ x_2 = 16 \\ x_3 = 3 \end{cases}$

$0x_1 + 0x_2 + x_3 = 3$