MMSE-Based Optimal Design of Full-Duplex Relay System

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***Abstract*—This paper studies minimum mean square error (MMSE)-based full-duplex relay processing matrices and source/destination beamforming vectors under perfect channel state information by imposing constraints on the transmit power of source, relay, both, separately, and in various combinations. The main contribution of this paper is the derivation of a set of relay processing and source/destination beamforming vectors under diverse conditions of transmit power constraints on the source and relay. By comparing the bit error rate (BER) performance of each case, an efficient design of a full-duplex amplify-and-forward relay system is presented.**

***Index Terms*—Multiple-input-multiple-output (MIMO), amplify-and-forward (AF), full-duplex, minimum mean square error (MMSE), beamforming.**

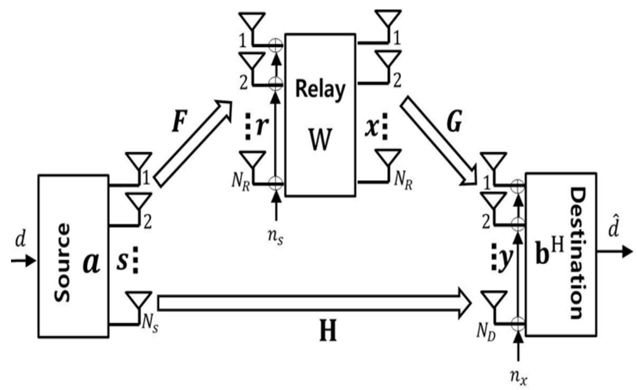

Fig. 1. AF FD MIMO relay system.

I. Introduction

The increasing number of communication users and demand for high-quality data communication are expected to result in a shortage of frequency resources. For this reason, a beamforming technique is highly regarded among the many ways for employing a multiple-input-multiple-output (MIMO) system. The key to beamforming is to improve the signal-to-noise ratio (SNR) by forming the beam to the receiver at a specific angle.

It is known that a full-duplex (FD) system performs better than a half-duplex (HD) system, in a situation that is prenulled at the relay. This is caused by the noise effect of the HD being larger than that of the FD [1], [2]. Also, the amplify-and-forward (AF) relay is widely known to be suited to a low-SNR environment [3].

This paper considers the FD AF relay system with beamforming. It is assumed that perfect channel state information is available at each node. This paper is to find minimum mean square error (MMSE)-based FD relay processing matrices and beamforming vectors of the source-destination by imposing c-onstraints on the transmit power of the source and the relay, separately and in various combinations. Using the Monte Carlo simulation, the performance of the system under different conditions is evaluated and compared in terms of BER.

The remainder of this paper is organized into four sections. Section II describes the AF FD relay system model applied. Section III derives a set of optimal relay processing matrices and source-destination beamforming vectors under the following four transmit power-constraint conditions: 1) source only, 2) relay only, 3) both source and relay simultaneously, and 4) aggregate power constraint on source and relay. Section IV compares BER simulation results under different situations. Section V concludes the paper.

Notation: The following notations are used in this paper: Matrices and vectors are denoted, respectively, by uppercase and lowercase boldface characters (e.g., $\mathbf{A}$ and $\mathbf{a}$). The inverse, pseudo-inverse, complex conjugate, and Hermitian of $\mathbf{A}$ are denoted, respectively, by $\mathbf{A}^{-1}$, $\mathbf{A}^{\dagger}$, $\mathbf{A}^{*}$, and $\mathbf{A}^{H}$. An $N \times N$ identity matrix is denoted by $\mathbf{I}_N$. Notations $|a|$, $||\mathbf{a}||$, and $||\mathbf{A}||_F$ denote the absolute value of a for any scalar, 2-norm of $\mathbf{a}$, and Frobenius-norm of $\mathbf{A}$, respectively. The real operator and the expectation operator are denoted by $\text{Re}\{\mathbf{A}\} = 1/2(\mathbf{A} + \mathbf{A}^*)$ and $E[\,\cdot\,]$, respectively.

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II. SYSTEM MODEL

In this paper, an AF FD MIMO relay system is considered. Figure 1 illustrates this relay system, which consists of a source, relay, and destination that have N_S, N_R, and N_D antennas, respectively. The subscripts $_S$, $_R$, and $_D$ refer to source, relay, and destination, respectively. The relay channel matrix between source and relay is represented by $\mathbf{F} \in \mathbf{C}^{N_R \times N_S}$, and another relay channel matrix between relay and destination is denoted by $\mathbf{G} \in \mathbf{C}^{N_D \times N_R}$. And the channel matrix for the source-destination is represented by $\mathbf{H} \in \mathbf{C}^{N_D \times N_S}$. Data symbols, channel elements, and noises are independent and identically distributed. And the elements of $\mathbf{F}$, $\mathbf{G}$, and $\mathbf{H}$ matrices are zero-mean complex Gaussian random variables with variances of $\sigma^2_{_H}$, $\sigma^2_{_F}$, and $\sigma^2_{_G}$, respectively. And every channel is invariant during a data frame transmission. Noises in the relay and receiver are additive white Gaussian noise (AWGN).

The symbol vector $\mathbf{s} \in \mathbf{C}^{N_S \times 1}$ from the source is transmitted as

$$\mathbf{s} = \mathbf{a}d \tag{1}$$

where the transmit beamforming vector and data symbol are denoted by $\mathbf{a} \in \mathbf{C}^{N_S \times 1}$ and $d \in \mathbf{C}^{1 \times 1}$, respectively. The received signal vector $\mathbf{r} \in \mathbf{C}^{N_R \times 1}$ at the relay can be written as

$$\mathbf{r} = \mathbf{F}\mathbf{s} + \mathbf{n}_s \tag{2}$$

where $\mathbf{n}_s \in \mathbf{C}^{N_R \times 1}$ is an AWGN vector with zero mean and covariance $\sigma^2_{n_s}\mathbf{I}_{N_R}$. By multiplying the relay processing matrix $\mathbf{W} \in \mathbf{C}^{N_R \times N_R}$ to the received relay signal vector, the transmitted signal vector at the relay is given by

$$\mathbf{x} = \mathbf{W}\mathbf{r}. \tag{3}$$

The received signal vector $\mathbf{y} \in \mathbf{C}^{N_D \times 1}$ at the destination is written as

$$\mathbf{y} = \mathbf{G}\mathbf{x} + \mathbf{H}\mathbf{s} + \mathbf{n}_x \tag{4}$$

where $\mathbf{n}_x$ is an AWGN vector with zero mean and covariance $\sigma^2_{n_x}\mathbf{I}_{N_D}$. The detected data symbol $\hat{d} \in \mathbf{C}^{1 \times 1}$ applying a receiver beamforming vector $\mathbf{b}$ can be expressed as

$$\hat{d} = \mathbf{b}^H\mathbf{y}. \tag{5}$$

By substituting equations (3) and (4) into equation (5), the expected data symbol $\hat{d}$ can be rewritten as

$$\hat{d} = \mathbf{b}^H\mathbf{G}\mathbf{W}\mathbf{F}\mathbf{a}d + \mathbf{b}^H\mathbf{G}\mathbf{W}\mathbf{n}_s + \mathbf{b}^H\mathbf{H}\mathbf{a}d + \mathbf{b}^H\mathbf{n}_x. \tag{6}$$

III. AF FD MMSE STRATEGIES

The goal of the work outlined in this section is to design optimal beamforming vectors and a relay processing matrix based on MMSE by controlling the transmit power at the source and relay. This section is divided into four subsections. In detail, in Subsection A and Subsection B, it is assumed that the transmitted power is limited only at the source and only at the relay, respectively. In Subsection C, beamforming vectors and relay processing matrices are derived in the case of two individual transmit power constraints on the source and relay. Subsection D presents the case of an aggregate transmit power constraint on both source and relay.

A. Transmit Power Constraint on Source Only

In this subsection, the optimum beamforming vectors and relay processing matrices for the FD relay scheme based on the MMSE criterion are found. Minimizing the mean square error between the transmit signal d and detected signal $\hat{d}$ at the destination node under the transmitted power constraint at the source can be written as

$$\begin{aligned}(\mathbf{W}^\star, \mathbf{a}^\star, \mathbf{b}^\star) &= \arg\min_{\mathbf{W},\mathbf{a},\mathbf{b}} J(\mathbf{W}, \mathbf{a}, \mathbf{b}) \\ \text{s.t.}\quad & E\left[||\mathbf{s}||^2\right] = \mathrm{P}_s\end{aligned} \tag{7}$$

where the superscript star * means the optimum and P_s is the transmitted power at the source. Using Lagrange multipliers λ_s from the work of Boyd and Vandenberghe [4], this optimization problem can be written as

$$L(\mathbf{W}, \mathbf{a}, \mathbf{b}, \lambda_s) = J(\mathbf{W}, \mathbf{a}, \mathbf{b}) + \lambda_s(E[||\mathbf{s}||^2] - \mathrm{P}_s) \tag{8}$$

where the objective function $J(\mathbf{W}, \mathbf{a}, \mathbf{b}) \triangleq E[|\hat{d} - d|^2]$ with $E[|d|^2] = 1$ can be written as

$$\begin{aligned}J(\mathbf{W}, \mathbf{a}, \mathbf{b}) = {}& \mathbf{b}^H\mathbf{H}\mathbf{a}\mathbf{a}^H\mathbf{H}^H\mathbf{b} + \mathbf{b}^H\mathbf{G}\mathbf{W}\mathbf{F}\mathbf{a}\mathbf{a}^H\mathbf{F}^H\mathbf{W}^H\mathbf{G}^H\mathbf{b} \\ & + \sigma^2_{n_x}\mathbf{b}^H\mathbf{b} + \sigma^2_{n_s}\mathbf{b}^H\mathbf{G}\mathbf{W}\mathbf{W}^H\mathbf{G}^H\mathbf{b} - \mathbf{b}^H\mathbf{G}\mathbf{W}\mathbf{F}\mathbf{a} \\ & - \mathbf{a}^H\mathbf{H}^H\mathbf{b} + \mathbf{b}^H\mathbf{H}\mathbf{a}\mathbf{a}^H\mathbf{F}^H\mathbf{W}^H\mathbf{G}^H\mathbf{b} - \mathbf{b}^H\mathbf{H}\mathbf{a} \\ & + \mathbf{b}^H\mathbf{G}\mathbf{W}\mathbf{F}\mathbf{a}\mathbf{a}^H\mathbf{H}^H\mathbf{b} - \mathbf{a}^H\mathbf{F}^H\mathbf{W}^H\mathbf{G}^H\mathbf{b} + 1.\end{aligned} \tag{9}$$

Differentiating $L(\mathbf{W}, \mathbf{a}, \mathbf{b}, \lambda_s)$ with respect to $\{\mathbf{W}^*$ (i.e., the complex conjugate of $\mathbf{W}$), $\mathbf{a}, \mathbf{b}, \lambda_s\}$, respectively, with the properties of the complex derivative matrix and vector in the work of Hjørungnes and Gesbert [5], the optimal relay processing matrix $\mathbf{W}^\star$, the optimal beamforming vectors $\mathbf{a}^\star$ and $\mathbf{b}^\star$, and the Lagrange multiplier $\lambda_s^\star$ can be obtained as

$$\mathbf{W}^\star = \frac{(1 - \mathbf{b}^H\mathbf{H}\mathbf{a})\mathbf{G}^H\mathbf{b}\mathbf{a}^H\mathbf{F}^H}{(\sigma^2_{n_s} + ||\mathbf{F}\mathbf{a}||^2)||\mathbf{G}^H\mathbf{b}||^2} \tag{10}$$

$$\mathbf{a}^\star = \frac{(\mathbf{F}^H\mathbf{W}^H\mathbf{G}^H + \mathbf{H}^H)\mathbf{b}}{\lambda_s + ||(\mathbf{F}^H\mathbf{W}^H\mathbf{G}^H + \mathbf{H}^H)\mathbf{b}||^2} \tag{11}$$

$$\mathbf{b}^{\star} = \frac{(\mathbf{I}_{N} + ||\mathbf{GW}||_F^2\mathbf{I}_{N} - \mathbf{GWW}^H\mathbf{G}^H)(\mathbf{GWF}+\mathbf{H})\mathbf{a}}{\sigma_{n_s}^2 + \sigma_{n_s}^2||\mathbf{GW}||_F^2 + ||(\mathbf{GWF}+\mathbf{H})\mathbf{a}||^2 + \delta} \tag{12}$$

and

$$\lambda_s^{\star} = \frac{||(\mathbf{F}^H\mathbf{W}^H\mathbf{G}^H + \mathbf{H}^H)\mathbf{b}||}{\sqrt{\mathrm{P}_s}} - ||(\mathbf{F}^H\mathbf{W}^H\mathbf{G}^H + \mathbf{H}^H)\mathbf{b}||^2 \tag{13}$$

where $\delta = ||\mathbf{GW}||_F^2||(\mathbf{GWF}+\mathbf{H})\mathbf{a}||^2 - ||\mathbf{W}^H\mathbf{G}^H(\mathbf{GWF}+\mathbf{H})\mathbf{a}||^2$, the matrix inversion lemma [6], i.e., $(\mathbf{A}+\mathbf{uv}^H)^{-1} = \mathbf{A}^{-1} - (1+\mathbf{v}^H\mathbf{A}^{-1}\mathbf{u})^{-1}\mathbf{A}^{-1}\mathbf{uv}^H\mathbf{A}^{-1}$, and the pseudo-inverse property [7], i.e., $(\mathbf{A}^H\mathbf{uu}^H\mathbf{A})^{\dagger}\mathbf{A}^H\mathbf{u} = \mathbf{A}^H\mathbf{u}||\mathbf{A}^H\mathbf{u}||^{-2}$, are used. Here, $\mathbf{u}$ and $\mathbf{v}$ are $N \times 1$ row vectors, respectively. Regardless of the sign of λ_s, the same BER is yielded. Therefore, only the positive sign of λ_s is expressed in equation (13). The optimal values $\mathbf{W}^{\star}$, $\mathbf{a}^{\star}$, $\mathbf{b}^{\star}$, and $\lambda_s^{\star}$ are interrelated. An iterative method used to solve these numerically will be discussed in Section IV.

B. Transmit Power Constraint on Relay Only

Similarly, applying the same method as in the case under transmit power constraint on source only, when the transmit power is limited to P_R in the relay, this problem can be transformed as

$$(\mathbf{W}^{\star}, \mathbf{a}^{\star}, \mathbf{b}^{\star}) = \arg\min_{\mathbf{W},\mathbf{a},\mathbf{b}} J(\mathbf{W},\mathbf{a},\mathbf{b}) \quad \text{s.t.} \quad E\big[||\mathbf{x}||^2\big] = \mathrm{P}_R \tag{14}$$

where P_R is the transmitted power at the relay. By using Lagrange multipliers λ_R and $\mathbf{x}$ in (3), this optimization problem can be written as

$$\begin{aligned} L(\mathbf{W},\mathbf{a},\mathbf{b},\lambda_R) = {} & 1 - 2\mathrm{Re}[\mathbf{b}^H(\mathbf{H}+\mathbf{GWF})\mathbf{a}] \\ & + |\mathbf{b}^H(\mathbf{H}+\mathbf{GWF})\mathbf{a}|^2 + \sigma_{n_s}^2||\mathbf{W}^H\mathbf{G}^H\mathbf{b}||^2 \\ & + \sigma_{n_x}^2||\mathbf{b}||^2 + 2\mathrm{Re}[\mathbf{b}^H\mathbf{GWFaa}^H\mathbf{H}^H\mathbf{b}] \\ & + \lambda_R(||\mathbf{WFa}||^2 + \sigma_{n_s}^2||\mathbf{W}||_F^2 - \mathrm{P}_R). \end{aligned} \tag{15}$$

As in the previous procedures, differentiating $L(\mathbf{W},\mathbf{a},\mathbf{b},\lambda_R)$ with respect to $\{\mathbf{W}^*, \mathbf{a}, \mathbf{b}, \lambda_R\}$, respectively, and applying the matrix inversion lemma [6] and the pseudo inverse property [7], the optimal relay processing matrix $\mathbf{W}^{\star}$, the optimal beamforming vectors $\mathbf{a}^{\star}$ and $\mathbf{b}^{\star}$, and the optimal Lagrange multiplier $\lambda_R^{\star}$ can be obtained as

$$\mathbf{W}^{\star} = \frac{\mathbf{G}^H\mathbf{b}(1-\mathbf{b}^H\mathbf{Ha})\mathbf{a}^H\mathbf{F}^H}{(\sigma_{n_s}^2 + ||\mathbf{Fa}||^2)(||\mathbf{G}^H\mathbf{b}||^2 + \lambda_R)} \tag{16}$$

$$\mathbf{a}^{\star} = \frac{(\mathbf{F}^H\mathbf{W}^H\mathbf{WF})^{\dagger}(\mathbf{F}^H\mathbf{W}^H\mathbf{G}^H + \mathbf{H}^H)\mathbf{b}}{\lambda_R + \mathbf{b}^H(\mathbf{GWF}+\mathbf{H})(\mathbf{F}^H\mathbf{W}^H\mathbf{WF})^{\dagger}(\mathbf{GWF}+\mathbf{H})^H\mathbf{b}} \tag{17}$$

$$\mathbf{b}^{\star} = \frac{(\mathbf{I}_{N} + ||\mathbf{GW}||_F^2\mathbf{I}_{N} - \mathbf{GWW}^H\mathbf{G}^H)(\mathbf{GWF}+\mathbf{H})\mathbf{a}}{\sigma_{n_s}^2 + \sigma_{n_s}^2||\mathbf{GW}||_F^2 + ||(\mathbf{GWF}+\mathbf{H})\mathbf{a}||^2 + \delta} \tag{18}$$

and

$$\lambda_R^{\star} = \frac{||\mathbf{G}^H\mathbf{ba}^H\mathbf{F}^H||^2(||\mathbf{Fa}||^2 + \sigma_{n_s}^2)}{\sqrt{\mathrm{P}_R}(\sigma_{n_s}^2 + ||\mathbf{Fa}||^2)(1-\mathbf{b}^H\mathbf{Ha})^{-2}} - ||\mathbf{G}^H\mathbf{b}||^2. \tag{19}$$

C. Transmit Power Constraints on Both Source and Relay

Similar to procedures for the previous two cases, the case of two simultaneous transmit power constraints on both the source and relay is considered here. This optimization problem can be stated as

$$(\mathbf{W}^{\star}, \mathbf{a}^{\star}, \mathbf{b}^{\star}) = \arg\min_{\mathbf{W},\mathbf{a},\mathbf{b}} J(\mathbf{W},\mathbf{a},\mathbf{b}) \quad \text{s.t.} \quad E\big[||\mathbf{s}||^2\big] = \mathrm{P}_s \text{ and } E\big[||\mathbf{x}||^2\big] = \mathrm{P}_R \tag{20}$$

where P_s and P_R are denoted by the transmitted power at the source and relay, respectively. Therefore, the constrained optimization Lagrange problem can be written as

$$\begin{aligned} L(\mathbf{W},\mathbf{a},\mathbf{b},\lambda_s,\lambda_R) = {} & \sigma_{n_s}^2||\mathbf{W}^H\mathbf{G}^H\mathbf{b}||^2 - 2\mathrm{Re}[\mathbf{b}^H(\mathbf{H}+\mathbf{GWF})\mathbf{a}] \\ & + |\mathbf{b}^H(\mathbf{H}+\mathbf{GWF})\mathbf{a}|^2 + \lambda_s(||\mathbf{a}||^2 - \mathrm{P}_s) + 1 \\ & + \sigma_{n_x}^2||\mathbf{b}||^2 + 2\mathrm{Re}[\mathbf{b}^H\mathbf{GWFaa}^H\mathbf{H}^H\mathbf{b}] \\ & + \lambda_R(||\mathbf{WFa}||^2 + \sigma_{n_s}^2||\mathbf{W}||_F^2 - \mathrm{P}_R). \end{aligned} \tag{21}$$

Similarly, differentiating (21) with respect to $\{\mathbf{W}^*, \mathbf{a}, \mathbf{b}, \lambda_s, \lambda_R\}$, respectively, the optimal relay processing matrix, beamforming vectors, and Lagrange multiplier can be obtained as

$$\mathbf{W}^{\star} = \frac{\mathbf{G}^H\mathbf{b}(1-\mathbf{b}^H\mathbf{Ha})\mathbf{a}^H\mathbf{F}^H}{(\sigma_{n_s}^2 + ||\mathbf{Fa}||^2)(||\mathbf{G}^H\mathbf{b}||^2 + \lambda_R)} \tag{22}$$

$$\mathbf{a}^{\star} = \frac{(\lambda_s\mathbf{I}_{N} + \lambda_R||\mathbf{WF}||_F^2\mathbf{I}_{N} - \mathbf{F}^H\mathbf{W}^H\mathbf{WF})(\mathbf{GWF}+\mathbf{H})^H\mathbf{b}}{\lambda_s^2 + \lambda_s\lambda_R||\mathbf{WF}||_F^2 + \lambda_s||(\mathbf{GWF}+\mathbf{H})^H\mathbf{b}||^2 + \vartheta} \tag{23}$$

$$\mathbf{b}^{\star} = \frac{(\mathbf{I}_{N} + ||\mathbf{GW}||_F^2\mathbf{I}_{N} - \mathbf{GWW}^H\mathbf{G}^H)(\mathbf{GWF}+\mathbf{H})\mathbf{a}}{\sigma_{n_s}^2 + \sigma_{n_s}^2||\mathbf{GW}||_F^2 + ||(\mathbf{GWF}+\mathbf{H})\mathbf{a}||^2 + \delta} \tag{24}$$

$$\lambda_R^{\star} = \frac{||\mathbf{G}^H\mathbf{b}||^2||\mathbf{a}^H\mathbf{F}^H||^4 + \sigma_{n_s}^2||\mathbf{G}^H\mathbf{ba}^H\mathbf{F}^H||_F^2}{\sqrt{\mathrm{P}_R}(\sigma_{n_s}^2 + ||\mathbf{Fa}||^2)(1-\mathbf{b}^H\mathbf{Ha})^{-2}} - ||\mathbf{G}^H\mathbf{b}||^2 \tag{25}$$

and

$$\lambda_s^{\star} = \frac{-\varepsilon + \sqrt{\varepsilon^2 - 4\vartheta}}{2} \tag{26}$$

where

$$\vartheta = \lambda_R(||\mathbf{WF}||_F^2||(\mathbf{GWF}+\mathbf{H})^H\mathbf{b}||^2 - ||\mathbf{WF}(\mathbf{GWF}+\mathbf{H})^H\mathbf{b}||^2) \tag{27}$$

$$\begin{aligned} \varepsilon = {} & \lambda_R||\mathbf{WF}||_F^2 + ||(\mathbf{F}^H\mathbf{W}^H\mathbf{G}^H + \mathbf{H}^H)\mathbf{b}||^2 \\ & - ||\mathbf{WF}(\mathbf{F}^H\mathbf{W}^H\mathbf{G}^H + \mathbf{H}^H)\mathbf{b}||^2(\mathrm{P}_R - \sigma_{n_s}^2||\mathbf{WF}||_F^2)^{-1/2}. \end{aligned} \tag{28}$$

D. Aggregate Transmit Power Constraint on Source and Relay

In this section, a set of MMSE-based beamforming vectors and relay processing matrix is derived, when it is assumed that an aggregate transmit power constraint is imposed on the source and relay. This optimization problem can be defined as

$$(\mathbf{W}^\star, \mathbf{a}^\star, \mathbf{b}^\star) = \arg\min_{\mathbf{W},\mathbf{a},\mathbf{b}} J(\mathbf{W},\mathbf{a},\mathbf{b}) \quad \text{s.t.} \quad E\big[||\mathbf{s}||^2\big] + E\big[||\mathbf{x}||^2\big] = \mathrm{P}_T \tag{29}$$

where P_T indicates the aggregate transmitted power at the source and relay. Hence, the optimization problem can be written as

$$\begin{aligned} L(\mathbf{W},\mathbf{a},\mathbf{b},\lambda_T) = {} & \sigma_{n_x}^2||\mathbf{b}||^2 - 2\mathrm{Re}[\mathbf{b}^H(\mathbf{H}+\mathbf{GWF})\mathbf{a}] - 1 \\ & + |\mathbf{b}(\mathbf{H}+\mathbf{GWF})\mathbf{a}|^2 + \sigma_{n_s}^2||\mathbf{W}^H\mathbf{G}^H\mathbf{b}||^2 \\ & + 2\mathrm{Re}[\mathbf{b}^H\mathbf{GWFaa}^H\mathbf{H}^H\mathbf{b}] + \lambda_T||\mathbf{a}||^2 \\ & + \lambda_T||\mathbf{WFa}||^2 + \lambda_T\sigma_{n_s}^2||\mathbf{W}||_F^2 - \lambda_T\mathrm{P}_T. \end{aligned} \tag{30}$$

Using the same previous method, the optimal relay processing matrix $\mathbf{W}^\star$, the optimal beamforming vectors $\mathbf{a}^\star$ and $\mathbf{b}^\star$, and the optimal Lagrange multiplier $\lambda_T^\star$, can be obtained, respectively, as

$$\mathbf{W}^\star = \frac{\mathbf{G}^H\mathbf{b}(1-\mathbf{b}^H\mathbf{Ha})\mathbf{a}^H\mathbf{F}^H}{(\sigma_{n_s}^2 + ||\mathbf{Fa}||^2)(||\mathbf{G}^H\mathbf{b}||^2+\lambda_T)} \tag{31}$$

$$\mathbf{a}^\star = \frac{(\mathbf{I}_N + ||\mathbf{WF}||_F^2\mathbf{I}_N - \mathbf{F}^H\mathbf{W}^H\mathbf{WF})(\mathbf{GWF}+\mathbf{H})^H\mathbf{b}}{\lambda_T + \lambda_T||\mathbf{WF}||_F^2 + ||(\mathbf{GWF}+\mathbf{H})^H\mathbf{b}||^2 + \vartheta} \tag{32}$$

$$\mathbf{b}^\star = \frac{(\mathbf{I}_N + ||\mathbf{GW}||_F^2\mathbf{I}_N - \mathbf{GWW}^H\mathbf{G}^H)(\mathbf{GWF}+\mathbf{H})\mathbf{a}}{\sigma_{n_s}^2 + \sigma_{n_s}^2||\mathbf{GW}||_F^2 + ||(\mathbf{GWF}+\mathbf{H})\mathbf{a}||^2} \tag{33}$$

and

$$\lambda_T^\star = \left(\sqrt{\frac{\omega}{\mathrm{P}_T - \sigma_{n_s}^2||\mathbf{W}||_F^2}} + ||(\mathbf{GWF}+\mathbf{H})^H\mathbf{b}||^2 + \vartheta\right)\big(1+||\mathbf{WF}||_F^2\big)^{-1} \tag{34}$$

where

$$\begin{aligned} \omega = {} & ||(\mathbf{I}_N + ||\mathbf{WF}||_F^2\mathbf{I}_N - \mathbf{F}^H\mathbf{W}^H\mathbf{WF})(\mathbf{GWF}+\mathbf{H})^H\mathbf{b}||^2 \\ & + ||\mathbf{WF}(\mathbf{GWF}+\mathbf{H})^H\mathbf{b}||^2. \end{aligned} \tag{35}$$

IV. Simulation Results

A. Assumptions

This section presents the Monte-Carlo BER simulation results under various combinations of transmit power constraints. Here, it is assumed that the perfectly known channel state information is implemented and is considered in the cases of $N_S = N_R = N_D = 2, 3, 4,$ and 6. The transmitted signals from the sources are modulated by quadrature phase shift keying. The MIMO channel matrixes **F**, **G**, and **H** are generated from independent Gaussian random variables. During data transmission, the channels are invariant. It is also assumed that $\mathrm{P}_s = \mathrm{P}_R = 1$ and $\mathrm{P}_T = 2$ and that all nodes have the same thermal noise power, i.e., $\sigma_{n_s}^2 = \sigma_{n_x}^2$.

TABLE I
Iterative Algorithm Procedure

Step 1	Initialization: $k = 0$ $\mathbf{W}_0 = \mathbf{I}_{N_R},\ \mathbf{b}_0 = [1, \cdots, 1],\ \lambda_s = \lambda_R = 1,\ L_0 = 10$
Step 2	Iteration: $k \leftarrow k+1$ $\mathbf{a}_k = f_{\mathbf{a}}(\mathbf{W}_{k-1}, \mathbf{b}_{k-1}, \lambda_s, \lambda_R)$ $\mathbf{W}_k = f_{\mathbf{W}}(\mathbf{a}_k, \mathbf{b}_{k-1}, \lambda_R)$ $\mathbf{b}_k = f_{\mathbf{b}}(\mathbf{W}_k, \mathbf{a}_k)$ $\lambda_R = f_{\lambda_R}(\mathbf{a}_k, \mathbf{b}_k)$ $\lambda_s = f_{\lambda_s}(\mathbf{W}_k, \mathbf{b}_k, \lambda_R)$ $L_k = f_{L_k}(\mathbf{W}_k, \mathbf{a}_k, \mathbf{b}_k, \lambda_s, \lambda_R)$ ※ Applying normalization with respect to $\mathbf{W}_k, \mathbf{a}_k, \mathbf{b}_k$
Step 3	If $0 \le L_{k-1} - L_k \le \eta$ go to Step 4 and stop, otherwise go back to Step 2
Step 4	$\mathbf{a} = \mathbf{a}_k;\ \mathbf{b} = \mathbf{b}_k;\ \mathbf{W} = \mathbf{W}_k$

B. Iterative Algorithm

When it comes to the derived solutions in Section III, these equations are related to each other. In this condition, an iterative algorithm can provide a way of solving this problem. This algorithm calculates one variable at a time while fixing all others. Initial values are $k = 0, \mathbf{W}_0 = \mathbf{I}_{N_R}$, and $\mathbf{b} = [1, \cdots, 1]$, $\lambda_s = \lambda_R = 1,\ L_0 = 10$, where $L \triangleq L(\mathbf{W}, \mathbf{a}, \mathbf{b}, \lambda_s, \lambda_R)$. As k increases one by one, L_k is gradually decreased. Since L_k always has positive and convergent properties, the difference between L_{k-1} and L_k can be used as a criteria to stop the iterative algorithm. For example, Table 1 can be used for optimizing the case of transmit power constraints on both the source and the relay.

C. Analysis of Simulation

Figure 2 shows BER performance comparisons between only the source transmit power constraint and only the relay transmit power constraint when $N = 3, 6, 12$, with $\eta = 0.0001$, where η is the stopping criteria for the difference $L_{k-1} - L_k$. It is observed that the relay-only case shows better BER performance than the source-only case. In addition, it is also found that the BER performance gets about 0.4 dB better as the number of relay nodes increases.

Figure 3 shows comparisons among two transmit power constraints on the source and the relay, individually, and power constraints on the source and relay for $N = 2, 3, 4, 6$, with $\eta = 0.0001$. It can be seen that the aggregate power constraint case shows slightly better BER than the individual power constraint case. Similar to Fig. 2, it can be seen that increasing the number of relay nodes shows a better BER performance.

Figure 4 shows BER performance for $N = 2, 6$, with $\eta = 0.0001$ for all conditions treated in this paper. It can be

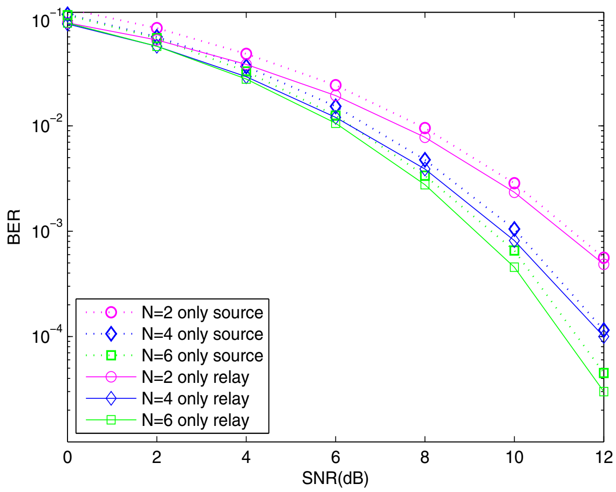

Fig. 2. BER performance comparisons for $N = 2, 4, 6$ between source-only and relay-only transmit power constraint cases.

observed that they perform similar to each other, which means that this case is not affected by the method of imposing power constraints on the source and the relay. However, according to Fig. 4, the BER is distinguishable between the source-only and relay-only cases. The case of imposing power constraints on both source and relay performs better than that of imposing one power constraint on only the source and only the relay.

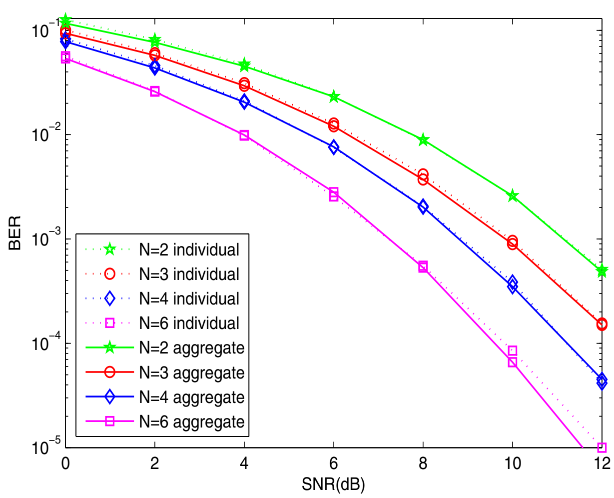

Fig. 3. BER performance comparisons for $N = 2, 4, 6$ between individual and aggregate power constraints on source and relay.

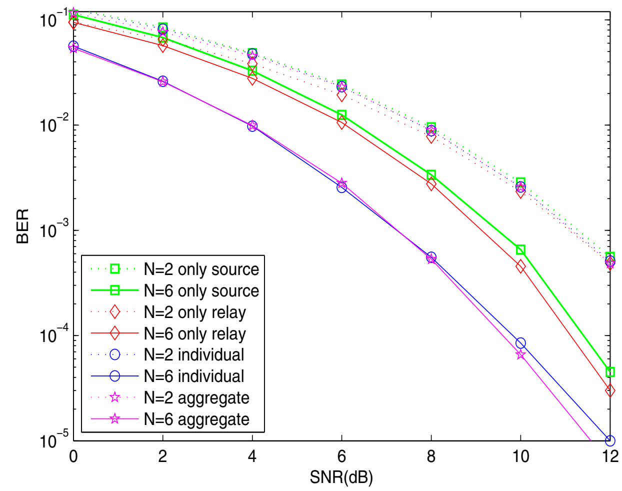

Fig. 4. BER performance comparisons for $N = 2, 6$ with $\eta = 0.0001$ for all conditions treated in this paper.

V. Conclusion

This paper derived a set of optimum relay processing metrics and source/destination beamforming vectors under diverse transmit power constraints at the source and the relay using the MMSE criteria. This paper also presented BER simulation results using an iterative algorithm. It was observed that the relay-only case performs better than the source-only case. In addition, even though there was no significant difference between individual and aggregate constraints on both the source and relay, the aggregate power constraint cases performed slightly better than the individual constraint case for $N = 6$. Therefore, it is suggested that algorithms be used for the case of imposing an aggregate transmit power constraint on the source and the relay. Finally, it is observed that the BER performs better as N increases. The results in this paper can be useful for designing an energy efficient AF FD relay network system.

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