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# A Spatial Self-interference Nullification Method for Full Duplex Amplify-and-Forward MIMO Relays

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***Abstract*—In full duplex amplify-and-forward (FD-AF) relays, self-interference from the transmit side causes a major problem hindering its commercial spread. To alleviate the front-end saturation problem, we propose a spatial self-interference nullification method employing extra transmit antennas in FD-AF multiple-input-multiple-output (MIMO) relay systems, and analyze characteristics of the proposed system when there exist channel estimation errors. In particular, the probability of relay stability is derived, and the capacity of the relay is analyzed with zero delay assumption. The results indicate that excessive channel estimation errors drive FD-AF relay into unstable modes and cause capacity reduction.**

## I. Introduction

Although full duplexing (FD) relays have an obvious advantage over half duplex (HD) relays that the channel usage can be doubled [1] [2] [3], it suffers from the self-interference from the transmit side of the relay. The interference reduces the channel capacity, and the capacity can even fall below that of the HD relay as the interference increases [4]. If a FD relay operates in the amplify-and-forward (AF) mode (called FD-AF), it has an additional problem of the system stability due to its regenerative nature. In order to prevent the instability, some methods [5] [8] may be used to cancel out the interference based on the interference channel estimation. Nevertheless, inevitable channel estimation error makes the system potentially unstable. Thus, two major issues emerge when we deal with the FD-AF relays: one is how to nullify the interference, and the other is how to quantify the stability and the capacity degradation of the system if the nullification of the interference is not perfect.

A survey of related works is as follows. In the case of the single-input-single-output (SISO) relay, a conventional method for interference nullification was a temporal processing after the relay front-end [5]. However, this method still has a front-end saturation problem by the interference because the interference remains unsuppressed yet when it arrives at the front-end. To avoid the problem, efforts such as securing larger isolation (∼100 dB) between the two sides of the relay or employing a front-end with larger dynamic range had to be made.

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Recently, some spatial domain solutions have been proposed so that the FD relays can eliminate or avoid the interference [6] [7] [8]. To this end, space-time equalization [6], precoding/decoding [7], or pre-nulling [8] techniques were applied based on the assumption of perfect interference channel information. However, from the practical point of view, perfect interference suppression is hard to achieve due to ever-existing channel estimation error. More seriously, the regenerative nature of the FD-AF relay makes the system prone to oscillate because of the channel estimation error.

In this paper, we propose a new FD-AF MIMO relay system employing extra transmit antennas for the spatial interference nullification and discuss the stability problem which is intrinsically generated in any FD-AF relay. Although the proposed scheme deals mainly with the AF type relay, it can be naturally applied to the DF type relay as well. We start with the conventional FD-AF MIMO relay system in Section II, and propose a FD-AF MIMO relay system in Section III. Then, the probability of stability due to the channel estimation error is analyzed in Section IV. Capacity comparison with the AF type half-duplex (HD-AF) MIMO relay system is made in Section V, and computer simulations and some concluding remarks are given in Section VI and VII, respectively.

The following notations are used in this paper. The matrices and vectors are denoted as the bold capital and lowercase letters, respectively; $(\cdot)^T$ is the transpose; $(\cdot)^H$ is the conjugate transpose; $\mathcal{C}^{M\times N}$ is the set of the complex $M$-by-$N$ matrices; $\mathcal{CN}(m, v)$ means the complex Gaussian distribution with mean $m$ and variance $v$; $E_z[\cdot]$ means the expectation with respect to r. v. $z$; $\mathbf{I}_{N\times N}$ is the $N$-by-$N$ identity matrix; $\mathbf{O}_{M\times N}$ is the $M$-by-$N$ zero matrix; $Pr\{E\}$ is the probability that the event $E$ occurs; $diag(a_1, \cdots, a_n)$ is the diagonal matrix with diagonal components $a_1, \cdots, a_n$; $\log(\cdot)$ is the logarithm with base-2; $\det(\cdot)$ is the determinant; $tr(\cdot)$ is the trace.

## II. Conventional FD-AF MIMO Relay System

Consider a conventional FD-AF MIMO relay system [9] shown in Fig. 1. The system comprises a source (e.g., base station) with $m$ transmit antennas, a relay with $m$ receive antennas and $m$ transmit antennas, and a destination (e.g., mobile) with $m$ receive antennas, respectively. The numbers of the antennas are taken identically for the purpose of simplicity.

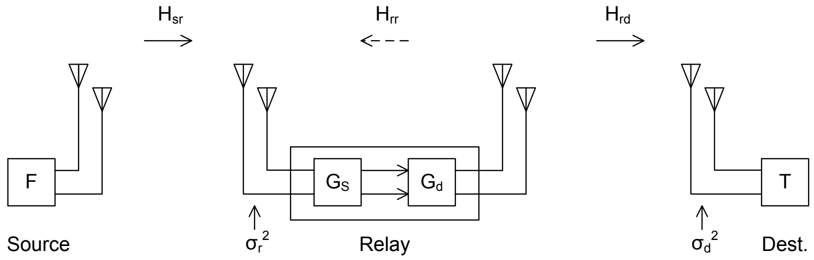


Fig. 1. A conventional FD-AF MIMO relay system. The interference channel $\mathbf{H}_{rr}$ is ignored. For convenience, the figure is drawn for $m = 2$.

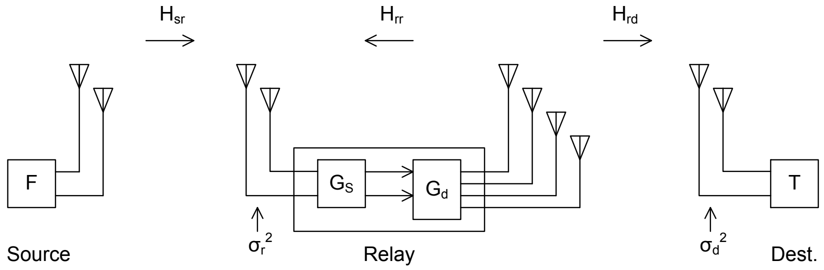


Fig. 2. The proposed FD-AF MIMO relay system. The interference channel $\mathbf{H}_{rr}$ is taken into account. For convenience, the figure is drawn for $m = 2$ and $n = 4$.

The source-to-relay channel $\mathbf{H}_{sr} \in \mathcal{C}^{m\times m}$ and the relay-to-destination channel $\mathbf{H}_{rd} \in \mathcal{C}^{m\times m}$ are assumed all frequency-flat Rayleigh fading MIMO channels. The channel matrices are assumed to have full ranks. The interference channel $\mathbf{H}_{rr} \in \mathcal{C}^{m\times m}$ from the transmit antennas to the receive antennas of the relay is assumed negligible due to perfect isolation between the two sides. The front-ends of the relay and the destination are assumed to be corrupted by the additive white Gaussian noise (AGWN) $\mathbf{n}_r \in \mathcal{C}^{m\times 1}$ and $\mathbf{n}_d \in \mathcal{C}^{m\times 1}$ with each element being i.i.d. $\mathcal{CN}(0, \sigma_r^2)$ and $\mathcal{CN}(0, \sigma_d^2)$, respectively. The direct channel from the source to the destination is assumed not existent.

All components of the system are assumed to have capability of processing signals based on full or partial channel state information (CSI). Let $\mathbf{F} \in \mathcal{C}^{m\times m}$, $\mathbf{G} \in \mathcal{C}^{m\times m}$, $\mathbf{T} \in \mathcal{C}^{m\times m}$ denote the precoding matrices of the source, the relay, and the destination, respectively. The relay precoding matrix is further divided into $\mathbf{G}_s \in \mathcal{C}^{m\times m}$, the precoding matrix towards the source, and $\mathbf{G}_d \in \mathcal{C}^{m\times m}$, the precoding matrix towards the destination, respectively (i.e., $\mathbf{G} = \mathbf{G}_d\mathbf{G}_s$).

The measure of the system performance in this paper is the channel capacity from the source to the destination expressed as

$$C = \log\det\left\{\mathbf{I} + \mathbf{T}\mathbf{H}_{rd}\mathbf{G}\mathbf{H}_{sr}\mathbf{F}\mathbf{\Lambda}\mathbf{F}^H\mathbf{H}_{sr}^H\mathbf{G}^H\mathbf{H}_{rd}^H\mathbf{T}^H \times \left(\sigma_d^2\mathbf{I} + \sigma_r^2\mathbf{T}\mathbf{H}_{rd}\mathbf{G}\mathbf{G}^H\mathbf{H}_{rd}^H\mathbf{T}^H\right)^{-1}\right\}. \quad (1)$$

Here, $\mathbf{\Lambda} \in \mathcal{C}^{m\times m}$ is the covariance of the source signal vector and assumed $\mathbf{\Lambda} = \mathbf{I}_{m\times m}$. Many methods have been reported to maximize the capacity under given CSI conditions [9] [10]. They can be summarized as follows. If full CSI on $\mathbf{H}_{sr}$ and $\mathbf{H}_{rd}$ are available to the corresponding sides, they can convert $\mathbf{H}_{sr}$ and $\mathbf{H}_{rd}$ to $m$ independent parallel subchannels by singular value decomposition (SVD) [12] and by applying the left and the right unitary matrices to the received and the transmission signal vectors, respectively. Then, each parallel channel is weighted according to a certain power allocation scheme.

In order to convey the above argument to mathematical expressions, let us SVD-decompose the channel matrices as

$$\mathbf{H}_{sr} = \mathbf{U}_{sr}\mathbf{\Sigma}_{sr}\mathbf{V}_{sr}^H \quad (2)$$
$$\mathbf{H}_{rd} = \mathbf{U}_{rd}\mathbf{\Sigma}_{rd}\mathbf{V}_{rd}^H \quad (3)$$

where $(\mathbf{U}_{sr}, \mathbf{\Sigma}_{sr}, \mathbf{V}_{sr})$ and $(\mathbf{U}_{rd}, \mathbf{\Sigma}_{rd}, \mathbf{V}_{rd})$ are the sets of the left unitary matrix, the diagonal singular value matrix, and the right unitary matrix of $\mathbf{H}_{sr}$ and $\mathbf{H}_{rd}$, respectively. Here, they all belong to $\mathcal{C}^{m\times m}$.

Then, the precoding matrices are determined as

$$\mathbf{F} = \mathbf{V}_{sr}\mathbf{\Gamma}_s \quad (4)$$
$$\mathbf{G}_s = \mathbf{U}_{sr}^H \quad (5)$$
$$\mathbf{G}_d = \mathbf{V}_{rd}\mathbf{\Gamma}_r \quad (6)$$
$$\mathbf{T} = \mathbf{U}_{rd}^H \quad (7)$$

so that $\mathbf{H}_{sr}$ and $\mathbf{H}_{rd}$ are orthogonalized into

$$\tilde{\mathbf{H}}_{sr} = \mathbf{G}_s\mathbf{H}_{sr}\mathbf{F} = \mathbf{\Sigma}_{sr}\mathbf{\Gamma}_s \quad (8)$$
$$\tilde{\mathbf{H}}_{rd} = \mathbf{T}\mathbf{H}_{rd}\mathbf{G}_d = \mathbf{\Sigma}_{rd}\mathbf{\Gamma}_r, \quad (9)$$

respectively. Here, $\mathbf{\Gamma}_s \in \mathcal{C}^{m\times m}$ and $\mathbf{\Gamma}_r \in \mathcal{C}^{m\times m}$ are diagonal matrices in charge of weighting the orthogonalized subchannels according to a specific optimization criterion. For example, an alternate optimization of the power amplifiers at the source and the relay can be made under individual power constraint [9]. That is, assuming power allocations $\{P_{s,i}\}_{i=1}^m$ and $\{P_{r,i}\}_{i=1}^m$ of the source and relay power amplifiers in the previous iteration step, they are updated as

$$P_{s,i} = \frac{1}{a_i}\left[\frac{P_{r,i}b_i}{2}\left(\sqrt{1+\frac{4a_i}{P_{r,i}b_i\nu_s}}-1\right)-1\right]^+ \quad (10)$$

$$P_{r,i} = \frac{1}{b_i}\left[\frac{P_{s,i}a_i}{2}\left(\sqrt{1+\frac{4b_i}{P_{s,i}a_i\nu_r}}-1\right)-1\right]^+, \quad (11)$$

respectively. Here, $a_i \equiv \frac{\lambda_{sr,i}}{\sigma_r^2}$, $b_i \equiv \frac{\lambda_{rd,i}}{\sigma_d^2}$, and $\lambda_{sr,i}$, $\lambda_{rd,i}$ are the $i$-th eigenvalues (i.e., squares of the singular values) of $\mathbf{H}_{sr}$ and $\mathbf{H}_{rd}$, respectively, and $\nu_s$, $\nu_r$ are chosen so that $\sum_{i=1}^m P_{s,i} = P_s$ and $\sum_{i=1}^m P_{r,i} = P_r$ with $P_s$ and $P_r$ being the total transmission power of the source and the relay, respectively, and $[z]^+ \equiv \max\{0,z\}$. Then, $\mathbf{\Gamma}_s$ and $\mathbf{\Gamma}_r$ are determined as

$$\mathbf{\Gamma}_s = diag(\sqrt{P_{s,1}},\cdots,\sqrt{P_{s,m}}) \quad (12)$$

$$\mathbf{\Gamma}_r = diag(\gamma_1,\cdots,\gamma_m) \quad (13)$$

with relay gains

$$\gamma_i \equiv \sqrt{\frac{P_{r,i}}{P_{s,i}\lambda_{sr,i}+\sigma_r^2}}, \quad i=1,\cdots,m. \quad (14)$$

If the relay-to-destination CSI is not fully but partially available to the relay, some method presented in [10] can be applied to fix $\mathbf{\Gamma}_s$ and $\mathbf{\Gamma}_r$.

Now, let us consider a situation where $\mathbf{H}_{rr}$ is not negligible. Then, an excessive interference from the output of the relay may cause the front-end saturation problem as well as the oscillation problem due to the regenerative nature of the FD-AF relay. These problems can be greatly alleviated by a spatial self-interference nullification method as will be explained in the next section.

## III. Proposed FD-AF MIMO Relay System

The proposed FD-AF MIMO relay system is shown in Fig. 2. Differently from the conventional MIMO relay system, the proposed system assumes the interference channel due to the nonnegligible feedback from the transmit side of the relay. The structure of the proposed MIMO relay is the same as that of the conventional one in Section II except for the increased number $n$ ($n > m$) of the transmit antennas of the relay and associated matrix size of $\mathbf{H}_{rr} \in \mathcal{C}^{m\times n}$, $\mathbf{H}_{rd} \in \mathcal{C}^{m\times n}$, and $\mathbf{G}_d \in \mathcal{C}^{n\times m}$.

The purpose of increasing the number of the transmit antennas is to nullify the interference using increased degree of freedom (d.o.f.) of $\mathbf{G}_d$ while maintaining the conventional precoding scheme discussed in Section II. Then, $\mathbf{H}_{rd}$ in (3) should be replaced by the new SVD

$$\underbrace{\mathbf{H}_{rd}}_{\mathcal{C}^{m\times n}} = \underbrace{\mathbf{U}_{rd}}_{\mathcal{C}^{m\times m}}\underbrace{\mathbf{\Sigma}_{rd}}_{\mathcal{C}^{m\times n}}\underbrace{\mathbf{V}_{rd}}_{\mathcal{C}^{n\times n}}{}^H = \underbrace{\mathbf{U}_{rd}}_{\mathcal{C}^{m\times m}}\underbrace{\mathbf{\Sigma}_{rd}^{(1)}}_{\mathcal{C}^{m\times m}}\underbrace{\mathbf{V}_{rd}^{(1)}}_{\mathcal{C}^{n\times m}}{}^H \quad (15)$$

where $\mathbf{\Sigma}_{rd} \equiv \left[\mathbf{\Sigma}_{rd}^{(1)}, \mathbf{O}_{m\times(n-m)}\right]$ and $\mathbf{V}_{rd} \equiv \left[\mathbf{V}_{rd}^{(1)}, \mathbf{V}_{rd}^{(2)}\right]$ with $\mathbf{V}_{rd}^{(1)} \in \mathcal{C}^{n\times m}$ and $\mathbf{V}_{rd}^{(2)} \in \mathcal{C}^{n\times(n-m)}$ being associated with $\mathbf{\Sigma}_{rd}^{(1)}$ and $\mathbf{O}_{m\times(n-m)}$, respectively. We can observe that the singular values of $\mathbf{\Sigma}_{rd}^{(1)}$ (equivalently, eigenvalues of $\mathbf{H}_{rd}\mathbf{H}_{rd}^H$) in (15) are greater than corresponding ones of $\mathbf{\Sigma}_{rd}$ in (3) due to additional channel matrix elements.

The two aforementioned objectives can be fulfilled at the same time by designing $\mathbf{G}_d$ satisfying

$$\mathbf{H}_{rr}\mathbf{G}_d = \mathbf{O}_{m\times m} \quad (16)$$

for nullification of the interference, and

$$\mathbf{V}_{rd}^{(1)H}\mathbf{G}_d = \mathbf{\Gamma}_r \quad (17)$$

for the relay precoding with other precoding matrices kept the same as (4), (5), (7), respectively. The above equations can be rewritten as

$$\mathbf{A}\mathbf{G}_d = \mathbf{B} \quad (18)$$

with

$$\mathbf{A} \equiv \begin{bmatrix}\mathbf{H}_{rr}\\ \mathbf{V}_{rd}^{(1)H}\end{bmatrix} \in \mathcal{C}^{2m\times n}$$

$$\mathbf{B} \equiv \begin{bmatrix}\mathbf{O}_{m\times m}\\ \mathbf{\Gamma}_r\end{bmatrix} \in \mathcal{C}^{2m\times m}.$$

Since $\mathbf{G}_d$ satisfying (18) exists only when $n \geq 2m$, the minimum $n$ for the proposed precoding scheme is $2m$. If $n = 2m$, $\mathbf{G}_d$ is fixed as

$$\mathbf{G}_d = \mathbf{A}^{-1}\mathbf{B}. \quad (19)$$

## IV. Stability Analysis

For any FD-AF relay, system stability is of a great concern due to the regenerative nature of it and imperfect isolation between the transmit and receive sides. Define the loop matrix, which is the matrix operator through the loop, as $\mathbf{Z} \equiv \mathbf{H}_{rr}\mathbf{G}_d \in \mathcal{C}^{m\times m}$.[1] The necessary and sufficient condition for the stability is that the maximum eigenvalue of $\mathbf{\Theta} \equiv \mathbf{Z}^H\mathbf{Z} \in \mathcal{C}^{m\times m}$ is less than one.

Obviously, the proposed FD-AF system is advantageous over the conventional one in terms of stability because $\mathbf{\Theta} = \mathbf{O}$ from (16) (i.e., always stable) while $\mathbf{\Theta}$ for the conventional FD-AF system remains as it is. In reality, however, the proposed scheme still suffers from the stability problem, although much less serious. This is because the nullification of the interference is made based on an estimate of $\mathbf{H}_{rr}$ (say $\hat{\mathbf{H}}_{rr}$) so that $\hat{\mathbf{H}}_{rr}\mathbf{G}_d = \mathbf{O}_{m\times m}$, but the inevitable random error $\Delta\mathbf{H}_{rr} = \mathbf{H}_{rr} - \hat{\mathbf{H}}_{rr} \in \mathcal{C}^{m\times n}$ still remains not nullified. It is reasonable to assume the elements of $\Delta\mathbf{H}_{rr}$ are i.i.d. with $\mathcal{CN}(0,\sigma_e^2)$.[2]

Noting $\mathbf{Z} = \Delta\mathbf{H}_{rr}\mathbf{G}_d$ for the proposed scheme, $\mathbf{\Theta}$ can be rewritten as

$$\mathbf{\Theta} = \mathbf{Z}'\mathbf{Z}'^H \quad (20)$$

where $\mathbf{Z}' \equiv \mathbf{Z}^H = \mathbf{G}_d^H\Delta\mathbf{H}_{rr}^H \in \mathcal{C}^{m\times m}$. Since the columns of $\Delta\mathbf{H}_{rr}^H$ are statistically independent, so are the columns of

[1] Here, the unitary $\mathbf{G}_s$ was dropped from $\mathbf{Z}$ for convenience because it does not have any effect on the norm variation of $\mathbf{Z}$.

[2] For example, if $\hat{\mathbf{H}}_{rr}$ was obtained through the linear MMSE method, $\Delta\mathbf{H}_{rr}$ becomes uncorrelated with $\hat{\mathbf{H}}_{rr}$.

$\mathbf{Z}'$. Thus, $\mathbf{\Theta}$ follows the central Wishart distribution with the covariance of each column of $\mathbf{Z}'$ being equally calculated as

$$\mathbf{\Sigma} = \sigma_e^2 \mathbf{G}_d^H \mathbf{G}_d. \tag{21}$$

The cumulative distribution function (cdf) of the maximum eigenvalue ($\lambda_{\max}$) of the central Wishart-distributed random matrix $\mathbf{\Theta}$ can be obtained by [11]

$$\begin{aligned} F_{\lambda_{\max}}(u) &= Pr\{\lambda_{\max} \leq u\} \\ &= K_{cc} \left| \left\{ \sigma_i^j \Gamma\left(j, \frac{u}{\sigma_i}\right) \right\}_{i,j} \right|, \quad u > 0 \end{aligned} \tag{22}$$

where $\{\cdot\}_{i,j}$ means the matrix whose $(i,j)$-th element is what is inside the parentheses, $|\mathbf{A}|$ means the determinant of any matrix $\mathbf{A}$, $K_{cc} \equiv K_{uc} \cdot \Pi_{j=1}^m (j-1)! \cdot \frac{|\mathbf{\Sigma}|^{-m}}{|\mathbf{V}_2(\sigma)|}$, $K_{uc} \equiv \{\Pi_{i=1}^m (m-i)!\}^{-2}$, $\mathbf{V}_2(\sigma) \equiv \left\{(-\sigma_j)^{1-i}\right\}_{i,j}$, $\sigma \equiv \{\sigma_1, \cdots, \sigma_m\}$ with $\sigma_1 \geq \cdots \geq \sigma_m$ is the set of the ordered eigenvalues of $\mathbf{\Sigma}$, $\Gamma(k,u)$ is the incomplete Gamma function defined as $\int_0^u x^{k-1} e^{-x} dx$, respectively.

Finally, the stability probability of the proposed system is given by

$$P^{(stability)} = Pr\{\lambda_{\max} < 1\} = F_{\lambda_{\max}}(1). \tag{23}$$

## V. Capacity Comparison with HD-AF Relay

In this section, we derive a lower bound of the capacity of the proposed FD-AF MIMO relay, and compare it with that of the HD-AF MIMO relay. The configuration of the HD-AF MIMO relay system is the same as that in Fig. 1 except the half duplexing.

Assuming an ideal relay without any delay[3] and reflecting $\mathbf{H}_{rr}$, the effective closed-loop relay gain can be expressed as

$$\begin{aligned} \mathbf{G}' &= \mathbf{G}(\mathbf{I} - \mathbf{H}_{rr}\mathbf{G})^{-1} \\ &= \mathbf{G}_d(\mathbf{I} - \mathbf{G}_s\mathbf{H}_{rr}\mathbf{G}_d)^{-1}\mathbf{G}_s. \end{aligned} \tag{24}$$

Then, the capacity of the FD-AF MIMO relay is given by

$$\begin{aligned} C^{(FD-AF)} &= \\ &\log\det\{\mathbf{I} + \mathbf{T}\mathbf{H}_{rd}\mathbf{G}'\mathbf{H}_{sr}\mathbf{F}\mathbf{F}^H\mathbf{H}_{sr}^H\mathbf{G}'^H\mathbf{H}_{rd}^H\mathbf{T}^H \\ &\times \left(\sigma_d^2\mathbf{I} + \sigma_r^2\mathbf{T}\mathbf{H}_{rd}\mathbf{G}'\mathbf{G}'^H\mathbf{H}_{rd}^H\mathbf{T}^H\right)^{-1}\}. \end{aligned} \tag{25}$$

Inserting (4), (5), (7), (17), and (24) into (25), and using an identity $\det\left(\mathbf{I} + \mathbf{X}\mathbf{Y}^{-1}\right) = \det\left\{\mathbf{I} + \left(\mathbf{P}\mathbf{X}\mathbf{P}^{-1}\right)\left(\mathbf{P}\mathbf{Y}\mathbf{P}^{-1}\right)^{-1}\right\}$ for any invertible $\mathbf{P}$, and using $\mathbf{H}_{rr}\mathbf{G}_d = \Delta\mathbf{H}_{rr}\mathbf{G}_d$, we can rewrite the capacity as

$$\begin{aligned} C^{(FD-AF)} &= \\ &\log\det\Big[\mathbf{I} + \mathbf{\Sigma}_{sr}^2\mathbf{\Gamma}_s^2 \times \{\sigma_d^2(\mathbf{I} - \mathbf{G}_s\Delta\mathbf{H}_{rr}\mathbf{G}_d) \\ &\cdot \mathbf{\Sigma}_{rd}^{(1)-2}\mathbf{\Gamma}_r^{-2}(\mathbf{I} - \mathbf{G}_s\Delta\mathbf{H}_{rr}\mathbf{G}_d)^H + \sigma_r^2\mathbf{I}\}^{-1}\Big]. \end{aligned} \tag{26}$$

Since $\Delta\mathbf{H}_{rr}$ is random, we take the average of the capacity over $\Delta\mathbf{H}_{rr}$. Then, we get the average channel capacity of the proposed system, which is lower-bounded as

$$\begin{aligned} \bar{C}^{(FD-AF)} &\geq \bar{C}^{(FD-AF,lb)} = \\ &\log\det\Big[\mathbf{I} + \mathbf{\Sigma}_{sr}^2\mathbf{\Gamma}_s^2 \times \Big\{\sigma_d^2\mathbf{\Sigma}_{rd}^{(1)-2}\mathbf{\Gamma}_r^{-2} + \sigma_r^2\mathbf{I} \\ &+ \sigma_e^2\sigma_d^2 \cdot tr\left(\mathbf{\Sigma}_{rd}^{(1)-2}\mathbf{\Gamma}_r^{-2}\mathbf{G}_d^H\mathbf{G}_d\right)\mathbf{I}\Big\}^{-1}\Big]. \end{aligned} \tag{27}$$

Proof: (see Appendix).

On the other hand, the capacity of the HD-AF MIMO relay is given by[4]

$$\begin{aligned} C^{(HD-AF)} &= \\ &\frac{1}{2} \cdot \log\det\Big[\mathbf{I} + \mathbf{\Sigma}_{sr}^2\mathbf{\Gamma}_s^2 \times \{\sigma_d^2\mathbf{\Sigma}_{rd}^{-2}\mathbf{\Gamma}_r^{-2} + \sigma_r^2\mathbf{I}\}^{-1}\Big]. \end{aligned} \tag{28}$$

Here, the half duplexing brought about the constant $\frac{1}{2}$ due to the half usage of the channel, and made the system interference free.

We want to compare $\bar{C}^{(FD-AF,lb)}$ in (27) and $C^{(HD-AF)}$ in (28) taking a conservative perspective on $\bar{C}^{(FD-AF)}$. Because the singular values of $\mathbf{\Sigma}_{rd}^{(1)}$ in (27) are greater than those of $\mathbf{\Sigma}_{rd}$ in (28) as mentioned in Section III, it is expected that, when $\sigma_e^2 = 0$, $\bar{C}^{(FD-AF,lb)}$ is greater than $2 \cdot C^{(HD-AF)}$ rather than equal to it. Then, $\bar{C}^{(FD-AF,lb)}$ decreases monotonically as $\sigma_e^2$ increases. Finally, $C^{(HD-AF)}$ will exceed $\bar{C}^{(FD-AF,lb)}$ at some $\sigma_e^2$ value called $\sigma_e^{*2}$. In conclusion, the FD-AF has always larger capacity than the HD-AF for $\sigma_e^2 < \sigma_e^{*2}$, and the FD-AF has potentially smaller capacity than the HD-AF for $\sigma_e^2 > \sigma_e^{*2}$.

## VI. Computer Simulations

Two main performances, stability and capacity, of the proposed FD-AF MIMO relay are shown by computer simulations in comparison with the conventional FD-AF and HD-AF schemes.

In Fig. 3, the probability of instability, $P^{(instability)} = 1 - P^{(stability)}$, according to (23) is plotted as a function of $\sigma_e^2$. Simulation conditions are $m = 2$, $n = 4$, $\sigma_r^2 = \sigma_d^2 = 1$, $P_s = P_r = 10$, and each element of $H_{sr}$, $H_{rr}$, $H_{rd}$ is generated according to i.i.d. $\mathcal{CN}(0,1)$ and fixed, and $\mathbf{\Gamma}_s$ and $\mathbf{\Gamma}_r$ are determined according to the alternate optimization method as described in (10)-(14). As is expected, $P^{(instability)}$ of the proposed FD-AF system is very low when $\sigma_e^2$ is small,

[3]Practical relays have a finite delay so that the output of the relay may be considered as a pure interference to the receive side of the relay. However, we just assumed the zero delay for the convenience of analysis. This results in a capacity expression slightly different from that of the previous work [4].

[4]Although the same notations are used for some variables in (27) and (28), they should be distinguished from each other according to the context.

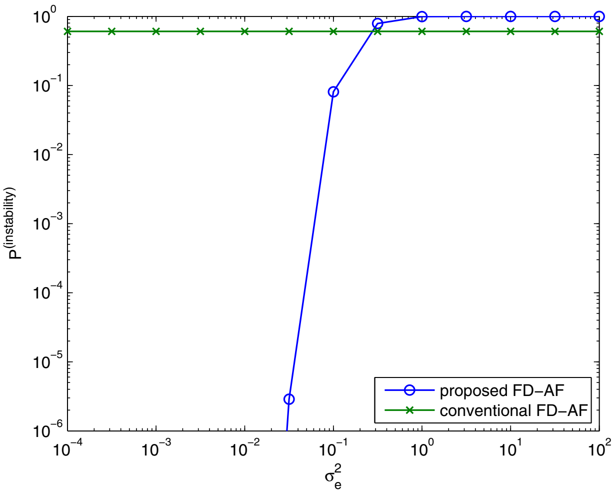


Fig. 3. $P^{(instability)}$ v.s. $\sigma_e^2$ with $P_r = 10$.

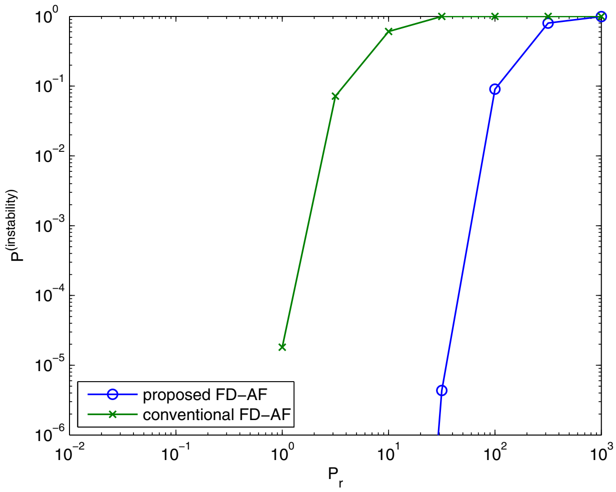


Fig. 5. $P^{(instability)}$ v.s. $P_r$ with $\sigma_e^2 = 10^{-2}$.

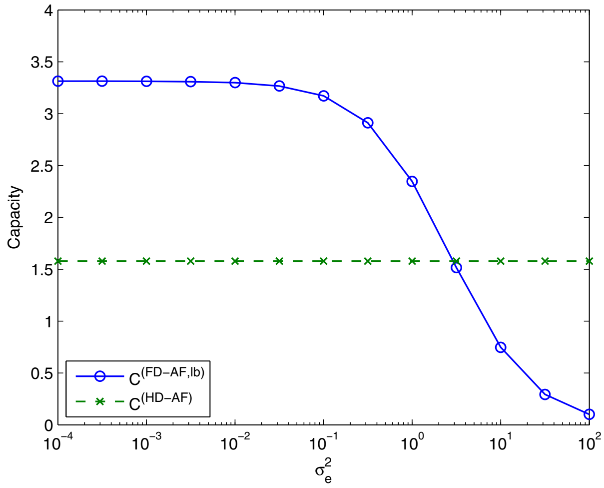


Fig. 4. $\bar{C}^{(FD-AF,lb)}$ and $C^{(HD-AF)}$ v.s. $\sigma_e^2$ with $P_r = 10$.

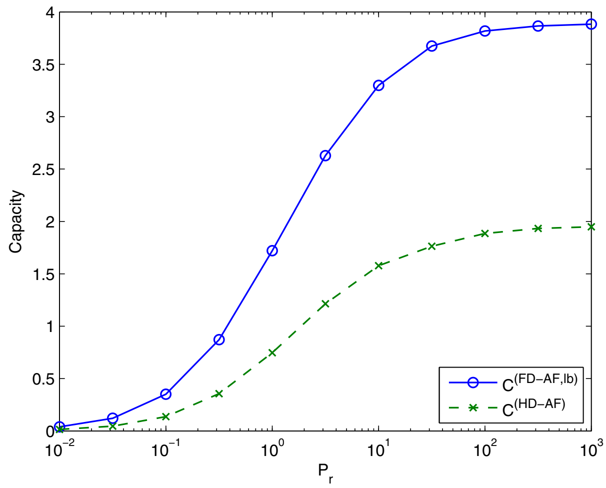


Fig. 6. $\bar{C}^{(FD-AF,lb)}$ and $C^{(HD-AF)}$ v.s. $P_r$ with $\sigma_e^2 = 10^{-2}$.

and increases as $\sigma_e^2$ increases. However, $P^{(instability)}$ of the conventional FD-AF system keeps a high value regardless of $\sigma_e^2$ due to lack of the interference nullification means.

In Fig. 4, $\bar{C}^{(FD-AF,lb)}$ and $C^{(HD-AF)}$ according to (27) and (28) are plotted as a function of $\sigma_e^2$. Simulation conditions are the same as Fig. 3. As is expected, $\bar{C}^{(FD-AF,lb)}$ is more than double the $C^{(HD-AF)}$ in the low $\sigma_e^2$ region, and decreases as $\sigma_e^2$ increases, and finally gets smaller than $C^{(HD-AF)}$.

From Fig. 3 and Fig. 4, we can see that both the stability and the capacity get worse as $\sigma_e^2$ increases, and $\sigma_e^2$ should be less than $10^{-2}$ in order to operate the proposed FD-AF system with negligible chance of oscillation and double the capacity of the HD-AF scheme.

In Fig. 5, $P^{(instability)}$ is plotted as a function of $P_r$ under simulation conditions $\sigma_e^2 = 10^{-2}$. Other conditions are the same as Fig. 3. As is shown, $P^{(instability)}$ increases as $P_r$ increases. This is because the relay gain (thus, loop gain) increases as $P_r$ increases, as is implied in (14). We can see that $P^{(instability)}$ of the proposed scheme is much lower than that of the conventional one thanks to the interference nullification. This curve can be a guideline telling us how much $P_r$ can be increased while keeping the instability probability below a target value.

In Fig. 6, $\bar{C}^{(FD-AF,lb)}$ and $C^{(HD-AF)}$ are plotted as a function of $P_r$. Simulation conditions are the same as Fig. 5. As is shown, both $\bar{C}^{(FD-AF,lb)}$ and $C^{(HD-AF)}$ increase as $P_r$ increases with $\bar{C}^{(FD-AF,lb)}$ maintaining double the $C^{(HD-AF)}$ value, and saturate after around $P_r = 10$. The monotonic increase of $\bar{C}^{(FD-AF,lb)}$ is a result of the zero delay assumption of the relay. If a nonzero delay were assumed instead so that the output of the relay acted as a pure interference to the receive side, $\bar{C}^{(FD-AF,lb)}$ would decrease as $P_r$ increases further after a peak as implied in [4].

From Fig. 5 and Fig. 6, we can see that the stability and the capacity have an opposite trend with respect to $P_r$. In this simulation, we can see that $1 < P_r < 10$ is a suitable operating region for the FD-AF relay from the perspective of both the stability and capacity. The capacity curves should be recognized as valid as long as the system is stable.

## VII. Conclusions

A spatial self-interference nullification method of the FD-AF MIMO relay was proposed, and associated issues such as system stability and channel capacity were analyzed taking into account the interference channel estimation error. Then, its performance was compared with the conventional FD-AF and the HD-AF relays. The proposed FD-AF relay has advantages that the front-end saturation problem and the stability problem can be greatly alleviated, and it can double the capacity of the HD-AF relay. Nevertheless, inevitable channel estimation errors may cause potential instability of the system and channel capacity degradation as the errors increase. The analysis exposed in this paper can be a guideline to predict a suitable operation region and the associated channel capacity.

## Appendix

For notational convenience, define $\mathbf{Q} \equiv \mathbf{\Sigma}_{rd}^{(1)-2}\mathbf{\Gamma}_r^{-2}$ and $\mathbf{X} \equiv (\mathbf{I}-\mathbf{G}_s\Delta\mathbf{H}_{rr}\mathbf{G}_d)\mathbf{Q}(\mathbf{I}-\mathbf{G}_s\Delta\mathbf{H}_{rr}\mathbf{G}_d)^H$ in (26). We can show that (26) is convex with respect to $\mathbf{X}$ using the method presented in [13] p. 74. Then, exploiting Jensen's inequality [13] p. 77, we have

$$\begin{aligned}\bar{C}^{(FD-AF)} &= E_{\mathbf{X}}\left[C^{(FD-AF)}(\mathbf{X})\right]\\ &\geq C^{(FD-AF)}(E\left[\mathbf{X}\right]) \quad (=\bar{C}^{(FD-AF,lb)})\\ &= \log\det\Big[\mathbf{I}+\mathbf{\Sigma}_{sr}^2\mathbf{\Gamma}_s^2\\ &\qquad \times\left\{\sigma_d^2 E_{\Delta\mathbf{H}_{rr}}\left[\mathbf{X}\right]+\sigma_r^2\mathbf{I}\right\}^{-1}\Big]. \qquad \text{(A.29)}\end{aligned}$$

Here, $E_{\Delta\mathbf{H}_{rr}}[\mathbf{X}]$ can be evaluated as

$$\begin{aligned}E_{\Delta\mathbf{H}_{rr}}[\mathbf{X}] &= \mathbf{Q}+E_{\Delta\mathbf{H}_{rr}}\left[\mathbf{G}_s\Delta\mathbf{H}_{rr}\mathbf{G}_d\mathbf{Q}\mathbf{G}_d^H\Delta\mathbf{H}_{rr}^H\mathbf{G}_s^H\right]\\ &= \mathbf{Q}+E_{\Delta\mathbf{H}_{rr}}\Big[\underbrace{\mathbf{G}_s\Delta\mathbf{H}_{rr}\mathbf{V}_{rd}}\mathbf{V}_{rd}^H\mathbf{G}_d\mathbf{Q}\\ &\qquad\qquad \mathbf{G}_d^H\mathbf{V}_{rd}\underbrace{\mathbf{V}_{rd}^H\Delta\mathbf{H}_{rr}^H\mathbf{G}_s^H}\Big].\end{aligned}$$

Here, $\Delta\mathbf{H}_{rr}' \equiv \mathbf{G}_s\Delta\mathbf{H}_{rr}\mathbf{V}_{rd}$ has the same statistics as $\Delta\mathbf{H}_{rr}$ since $\mathbf{G}_s$ and $\mathbf{V}_{rd}$ are unitary. That is, all elements of $\Delta\mathbf{H}_{rr}'$ are i.i.d. with $\mathcal{CN}(0,\sigma_e^2)$. Then, denoting $\mathbf{W} \equiv \mathbf{V}_{rd}^H\mathbf{G}_d\mathbf{Q}\mathbf{G}_d^H\mathbf{V}_{rd}$, we have

$$\begin{aligned}E_{\Delta\mathbf{H}_{rr}}[\mathbf{X}] &= \mathbf{Q}+E_{\Delta\mathbf{H}_{rr}'}\left[\Delta\mathbf{H}'_{rr}\mathbf{W}\Delta\mathbf{H}'^H_{rr}\right]\\ &= \mathbf{Q}+\sigma_e^2\cdot tr(\mathbf{W})\mathbf{I}\\ &= \mathbf{Q}+\sigma_e^2\cdot tr\left(\mathbf{Q}\mathbf{G}_d^H\mathbf{G}_d\right)\mathbf{I}. \qquad \text{(A.30)}\end{aligned}$$

Inserting (A.30) into (A.29), we obtain (27). □

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