A Spatial-Domain Joint-Nulling Method of Self-Interference in Full-Duplex Relays

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***Abstract*—In this letter, a spatial-domain joint-nulling method is proposed as a new means of suppressing the self-interference (or echo) occurring in full-duplex relays. Both amplify-and-forward (AF) and decode-and-forward (DF) type relays are considered. While the conventional method searches for the relay processing matrix from a discrete set of basis vectors obtained from the singular value decomposition (SVD) of the echo channel, the proposed method directly solves the optimal relay processing matrix over a continuous domain. As a result, it is shown that the proposed approach gives better performance than the conventional one in terms of the achievable rate.**

***Index Terms*—Full-duplex relay, self-interference, spatial-domain interference nulling, joint-nulling.**

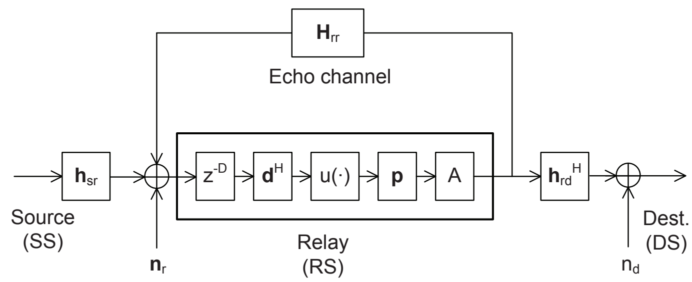

Fig. 1. Block diagram of the full-duplex relay system with the joint-nulling scheme. Precoder $\mathbf{p}$ and decoder $\mathbf{d}^H$ are employed to nullify the echo channel jointly (i.e., $\mathbf{d}^H\mathbf{H}_{rr}\mathbf{p} = 0$). Internal processor $u(\cdot)$ means 1 for AF, and a decoding & re-encoding functionality for DF.

I. Introduction

It is well-known that a full-duplex relay can achieve up to double the capacity of a half-duplex relay [1]. However, in reality, doubling the capacity is difficult to achieve due to the self-interference caused by an extremely large amount of tramsmission power from the relay and the imperfect isolation separating the transmit and receive sides of the relay. To remedy this problem, several spatial domain interference nulling techniques have been proposed [1]-[3]. These are based on the fact that spatial domain interference nulling is possible when there are enough degrees of freedom (d.o.f.) (i.e., enough antennas vs. transmitted data streams). Again, these solutions exploit the SVD in order to find the null space. The tradeoff between the numbers of antennas and data streams in SVD-based joint-nulling is presented by Riihonen *et al.* [3]. The SVD-based scheme, though, is just one example of joint-nulling. In this letter, we will present a more general method of joint-nulling based on a continuous domain, which brings about better performance than the previous methods.

In the other research, an attempt was made to increase the achievable rate by maximizing the signal-to-interference ratio (SIR) at the relay input and output [4]. This scheme may be effective when the d.o.f. are not enough for perfect interference nulling. In this letter, instead, we focus on deriving a new interference nulling algorithm, assuming that sufficient d.o.f. are provided and the self-interference dominates the noise due to a great amplification of the relay.

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II. System Model and Problem Setting

Consider the relay system shown in Fig. 1. The system comprises a source station (SS), a destination station (DS) with a single antenna for each, and a relay station (RS) with N antennas on both the receive and transmit sides.[1] Therefore, the end-to-end link from SS to DS becomes a single-input single-output (SISO) system, while the RS has multiple antennas on both sides to enable spatial-domain interference nulling. The SS-to-RS and RS-to-DS channels are denoted by $\mathbf{h}_{sr} \in \mathbb{C}^{N\times 1}$ and $\mathbf{h}_{rd}^H \in \mathbb{C}^{1\times N}$, and the RS-to-RS echo channel from the transmit to receive sides of the relay by $\mathbf{H}_{rr} \in \mathbb{C}^{N\times N}$, where $(\cdot)^H$ is the Hermitian, and $\mathbb{C}^{m\times n}$ stands for the set of m-by-n complex matrices. Each element of $\mathbf{h}_{sr}$, $\mathbf{h}_{rd}^H$, and $\mathbf{H}_{rr}$ is identically and independently distributed (i.i.d.) as $\mathcal{CN}(0, \sigma_{sr}^2)$, $\mathcal{CN}(0, \sigma_{rd}^2)$, and $\mathcal{CN}(0, \sigma_{rr}^2)$, respectively, where $\mathcal{CN}(m, v)$ means the complex Gaussian distribution with mean m and variance v. The direct channel from SS to DS is neglected assuming sufficiently large path loss between them. The receive sides of the RS and DS are assumed to be corrupted by additive noises $\mathbf{n}_r \in \mathbb{C}^{N\times 1}$ and $n_d \in \mathbb{C}$, respectively, whose elements are i.i.d. with $\mathcal{CN}(0, \sigma_r^2)$ and $\mathcal{CN}(0, \sigma_d^2)$.

The relay is composed of the relay processing delay D, the receive side linear processing vector $\mathbf{d}^H \in \mathbb{C}^{1\times N}$ with $\|\mathbf{d}\| = 1$, an internal processor $u(\cdot)$, the transmit side linear processing vector $\mathbf{p} \in \mathbb{C}^{N\times 1}$ with $\|\mathbf{p}\| = 1$, and the relay gain $A > 0$. D is assumed sufficiently large so that the echo signal from the transmitter to the receiver of the relay is a pure interference with respect to the received signal from SS. $u(\cdot)$ is defined as 1 for AF, and a decoding and re-encoding functionality for DF.

[1]The numbers of the receive and transmit antennas may be different, but we assume an identical number just for convenience.

In this letter, we will design $(\mathbf{d}, \mathbf{p})$ jointly so as to maximize the end-to-end achievable rate (or mutual information) from SS to DS while achieving interference nulling. Assuming interference nulling has been achieved in some way, the achievable rate can be expressed as [5]

$$C(\mathbf{d}, \mathbf{p}) = \begin{cases} \log_2\left[1 + \frac{X(\mathbf{d})Y(\mathbf{p})}{X(\mathbf{d})+Y(\mathbf{p})+1}\right] & \text{for AF,} \\ \log_2\left[1 + \min\left(X(\mathbf{d}), Y(\mathbf{p})\right)\right] & \text{for DF,} \end{cases} \tag{1}$$

where

$$X(\mathbf{d}) = \frac{P_s}{\sigma_r^2}|\mathbf{d}^H\mathbf{h}_{sr}|^2 \quad \text{and} \quad Y(\mathbf{p}) = \frac{P_r}{\sigma_d^2}|\mathbf{h}_{rd}^H\mathbf{p}|^2$$

are the signal-to-noise ratios (SNR) at the RS and DS receive sides, and (P_s, P_r) are the transmission powers from the SS and RS[2], respectively. Since $\log_2(1+w)$ is a monotonically increasing function of w, the rate can be maximized by maximizing $\frac{X(\mathbf{d})Y(\mathbf{p})}{X(\mathbf{d})+Y(\mathbf{p})+1}$ for AF and $\min\left(X(\mathbf{d}), Y(\mathbf{p})\right)$ for DF. Therefore, the problem can be stated as follows:

$$\max_{\mathbf{d},\mathbf{p}} f(\mathbf{d}, \mathbf{p}) = \begin{cases} \frac{X(\mathbf{d})Y(\mathbf{p})}{X(\mathbf{d})+Y(\mathbf{p})+1} & \text{for AF,} \\ \min\left(X(\mathbf{d}), Y(\mathbf{p})\right) & \text{for DF,} \end{cases} \tag{2}$$

$$\text{s.t.} \quad \mathbf{d}^H\mathbf{H}_{rr}\mathbf{p} = 0, \tag{3}$$

$$\|\mathbf{d}\| = \|\mathbf{p}\| = 1, \tag{4}$$

where (3) is the joint-nulling condition.

In the next sections, two approaches which solve the above problem are presented. The first method in Section III is based on the SVD of $\mathbf{H}_{rr}$. This was partly suggested in [1]-[3], but we add it here for clarification of the method in more detail. The second method, shown in Section IV, solves the problem in a more general and efficient way, and is proposed in this letter for the first time.

III. SVD-Based Joint-Nulling (Conventional)

Let us express $\mathbf{H}_{rr}$ in terms of the SVD as

$$\begin{aligned} \mathbf{H}_{rr} &= \mathbf{U}_{rr}\mathbf{\Sigma}_{rr}\mathbf{V}_{rr}^H \\ &= \left[\mathbf{U}_{rr}^{(1)} \; \mathbf{U}_{rr}^{(2)}\right] \begin{bmatrix} \mathbf{\Sigma}_{rr}^{(1)} & \mathbf{O} \\ \mathbf{O} & \mathbf{\Sigma}_{rr}^{(2)} \end{bmatrix} \left[\mathbf{V}_{rr}^{(1)} \; \mathbf{V}_{rr}^{(2)}\right]^H \\ &= \mathbf{U}_{rr}^{(1)}\mathbf{\Sigma}_{rr}^{(1)}\mathbf{V}_{rr}^{(1)H} + \mathbf{U}_{rr}^{(2)}\mathbf{\Sigma}_{rr}^{(2)}\mathbf{V}_{rr}^{(2)H} \end{aligned} \tag{5}$$

where $(\mathbf{U}_{rr}^{(1)}, \mathbf{U}_{rr}^{(2)})$ and $(\mathbf{V}_{rr}^{(1)}, \mathbf{V}_{rr}^{(2)})$ are the column-wise partitions of $\mathbf{U}_{rr}$ and $\mathbf{V}_{rr}$ associated with $(\mathbf{\Sigma}_{rr}^{(1)}, \mathbf{\Sigma}_{rr}^{(2)})$, respectively. Here, $\mathbf{U}_{rr}^{(1)} \perp \mathbf{U}_{rr}^{(2)}$ and $\mathbf{V}_{rr}^{(1)} \perp \mathbf{V}_{rr}^{(2)}$. Note that there exist in total $\sum_{K=1}^{N-1}\binom{N}{K}$ combinations in partitioning $\mathbf{U}_{rr}$ (also $\mathbf{V}_{rr}$). Assuming one instance of the combinations, (say $\mathbf{U}_{rr}^{(1)} \in \mathbb{C}^{N\times K}$, $\mathbf{U}_{rr}^{(2)} \in \mathbb{C}^{N\times(N-K)}$, $1 \le K \le N-1$), the joint-nulling condition in (3) can be satisfied by letting $\mathbf{d} = \mathbf{U}_{rr}^{(1)}\mathbf{b}$ and $\mathbf{p} = \mathbf{V}_{rr}^{(2)}\mathbf{c}$ for any $\mathbf{b} \in \mathbb{C}^{K\times 1}$ and $\mathbf{c} \in \mathbb{C}^{(N-K)\times 1}$ with $\|\mathbf{b}\| = \|\mathbf{c}\| = 1$. Then, $\mathbf{b}$ and $\mathbf{c}$ are determined so as to maximize $X(\mathbf{d})$ and $Y(\mathbf{p})$ below (1), respectively, under the condition (4). This brings about $\mathbf{b} = \frac{\mathbf{U}_{rr}^{(1)H}\mathbf{h}_{sr}}{\|\mathbf{U}_{rr}^{(1)H}\mathbf{h}_{sr}\|}$, $\mathbf{c} = \frac{\mathbf{V}_{rr}^{(2)H}\mathbf{h}_{rd}}{\|\mathbf{V}_{rr}^{(2)H}\mathbf{h}_{rd}\|}$, and maximization of $f(\mathbf{d}, \mathbf{p})$ in (2) for both AF and DF since $f(\mathbf{d}, \mathbf{p})$ is a monotonically increasing function with respect to $X(\mathbf{d})$ and $Y(\mathbf{p})$. The pair $(\mathbf{d}, \mathbf{p})$ which gives the largest $f(\mathbf{d}, \mathbf{p})$ over all partitioning combinations is chosen as the desired solution.

[2]In the later development, we will express the performance of the system in terms of P_r, instead of A.

IV. Continuous Domain-Based Joint-Nulling

The proposed method in this section solves the problem stated previously in (2)-(4) directly over the continuous domain of $\mathbf{d}$ and $\mathbf{p}$. Note that $\mathbf{d}$ and $\mathbf{p}$ are coupled through (3), but once either one (say $\mathbf{d}$) is fixed, $\mathbf{p}$ can be optimized to maximize $Y(\mathbf{p})$ conditioned on $\mathbf{d}$ (say $\tilde{\mathbf{p}}$), then $\mathbf{d}$ can be optimized to maximize $f(\mathbf{d}, \tilde{\mathbf{p}})$ in (2) unconditionally.

For convenience, let us define

$$g(\mathbf{p}) = |\mathbf{h}_{rd}^H\mathbf{p}|^2. \tag{6}$$

Denoting

$$\mathbf{z} = \mathbf{H}_{rr}^H\mathbf{d} \in \mathbb{C}^{N\times 1}, \tag{7}$$

(3) can be rewritten as $\mathbf{z}^H\mathbf{p} = 0$. Then, $\mathbf{p}$ can be expressed as

$$\mathbf{p} = \mathbf{P}_{\mathbf{z}}^{\perp}\mathbf{k} \tag{8}$$

for some $\mathbf{k} \in \mathbb{C}^{N\times 1}$ with $\|\mathbf{p}\|^2 = \|\mathbf{P}_{\mathbf{z}}^{\perp}\mathbf{k}\|^2 = 1$. Here,

$$\mathbf{P}_{\mathbf{z}}^{\perp} = \mathbf{I}_N - \frac{\mathbf{z}\mathbf{z}^H}{\mathbf{z}^H\mathbf{z}} \in \mathbb{C}^{N\times N} \tag{9}$$

means the projection matrix onto the null space of $\mathbf{z}$ with $\mathbf{I}_N$ being the N-by-N identity matrix. Inserting (8) into (6), (6) can be rewritten as $g(\mathbf{p}) = |\mathbf{h}_{rd}^H\mathbf{P}_{\mathbf{z}}^{\perp}\mathbf{k}|^2$. This is maximized as

$$g_{\max} = \|\mathbf{h}_{rd}^H\mathbf{P}_{\mathbf{z}}^{\perp}\|^2 = \mathbf{h}_{rd}^H\mathbf{P}_{\mathbf{z}}^{\perp}\mathbf{h}_{rd} \tag{10}$$

if

$$\mathbf{k} = \frac{\left(\mathbf{h}_{rd}^H\mathbf{P}_{\mathbf{z}}^{\perp}\right)^H}{\|\mathbf{h}_{rd}^H\mathbf{P}_{\mathbf{z}}^{\perp}\|}. \tag{11}$$

In (10), we used the fact that $\mathbf{P}_{\mathbf{z}}^{\perp}$ is idempotent (i.e., $\mathbf{P}_{\mathbf{z}}^{\perp}\mathbf{P}_{\mathbf{z}}^{\perp} = \mathbf{P}_{\mathbf{z}}^{\perp}$). Inserting (7) and (9) into (10), (10) is expressed as

$$g_{\max}(\mathbf{d}) = \mathbf{h}_{rd}^H\left(\mathbf{I}_N - \frac{\mathbf{H}_{rr}^H\mathbf{d}\mathbf{d}^H\mathbf{H}_{rr}}{\mathbf{d}^H\mathbf{H}_{rr}\mathbf{H}_{rr}^H\mathbf{d}}\right)\mathbf{h}_{rd}. \tag{12}$$

Then, inserting $X(\mathbf{d})$ and $Y(\mathbf{p})$ defined below (1) into (2) with $|\mathbf{h}_{rd}^H\mathbf{p}|^2$ in $Y(\mathbf{p})$ being replaced by $g(\mathbf{p})$ in (6), so $g_{\max}(\mathbf{d})$ in (12), the problem (2)-(4) is reduced to a single variable optimization problem as

$$\max_{\mathbf{d}} f(\mathbf{d}) = \begin{cases} \frac{X(\mathbf{d})Y(\mathbf{d})}{X(\mathbf{d})+Y(\mathbf{d})+1} & \text{for AF,} \\ \min\left(X(\mathbf{d}), Y(\mathbf{d})\right) & \text{for DF,} \end{cases} \tag{13}$$

$$\text{s.t.} \quad \|\mathbf{d}\| = 1,$$

where

$$X(\mathbf{d}) = \frac{P_s}{\sigma_r^2}\mathbf{d}^H\mathbf{h}_{sr}\mathbf{h}_{sr}^H\mathbf{d},$$

$$Y(\mathbf{d}) = \frac{P_r}{\sigma_d^2}\left(\mathbf{h}_{rd}^H\mathbf{h}_{rd} - \frac{\mathbf{d}^H\mathbf{H}_{rr}\mathbf{h}_{rd}\mathbf{h}_{rd}^H\mathbf{H}_{rr}^H\mathbf{d}}{\mathbf{d}^H\mathbf{H}_{rr}\mathbf{H}_{rr}^H\mathbf{d}}\right).$$

A local maximum of (13) can be obtained by using a gradient method (e.g., the gradient projection method [6]) as follows:

- Step 1: Initialize $\mathbf{d}_1$.
- Step 2: Update $\mathbf{d}_{k+1} = \left[\mathbf{d}_k + \mu_k\frac{\partial f(\mathbf{d}_k)}{\partial \mathbf{d}^*}\right]^+$ where μ_k is determined by Armijo's rule along the projection arc.
- Step 3: Repeat Step 2 until $\mathbf{d}_k$ converges.

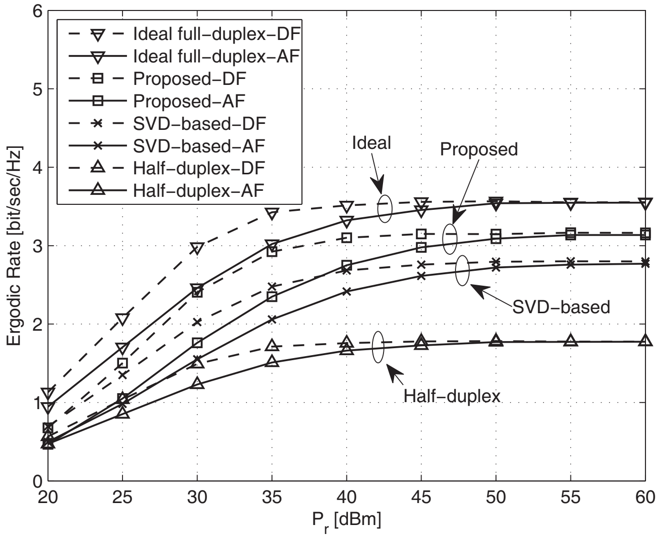

Fig. 2. Ergodic rates for AF (solid) and DF (dashed) against relay transmission power (P_r) with fixed source power (P_s). $N = 2$.

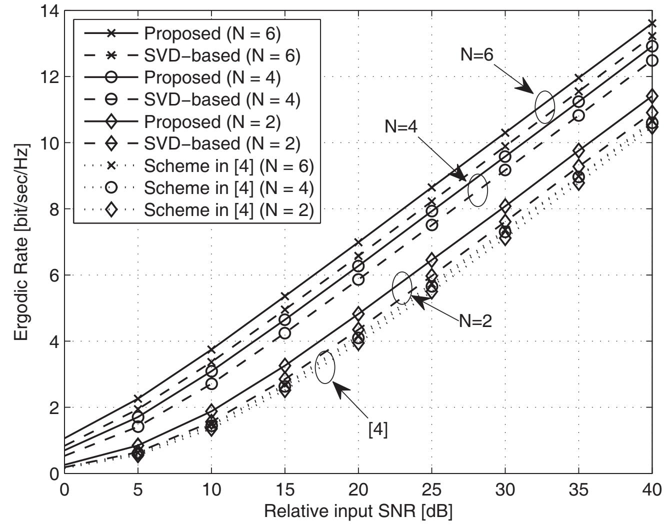

Fig. 3. Ergodic rate v.s. relay input SNR ($\sim \sigma_{sr}^2 P_s/\sigma_r^2 = 1/\sigma_r^2$) with fixed P_r as the number of relay antennas (N-by-N) increases. AF case.

In Step 2, $[\cdot]^+$ means projection onto the constraint set, and $\frac{\partial f(\mathbf{d})}{\partial \mathbf{d}^*}$ is calculated as $\frac{(\mathbf{X}'Y+X\mathbf{Y}')(X+Y+1)-XY(\mathbf{X}'+\mathbf{Y}')}{(X+Y+1)^2}$ for AF, where $\mathbf{X}' = \frac{P_s}{\sigma_r^2}(\mathbf{h}_{sr}\mathbf{h}_{sr}^H)\mathbf{d}$, $\mathbf{Y}' = -\frac{P_r}{\sigma_d^2}\frac{\mathbf{Sd}(\mathbf{d}^H\mathbf{Td})-(\mathbf{d}^H\mathbf{Sd})\mathbf{Td}}{(\mathbf{d}^H\mathbf{Td})^2}$, $\mathbf{S} = \mathbf{H}_{rr}\mathbf{h}_{rd}\mathbf{h}_{rd}^H\mathbf{H}_{rr}^H$, and $\mathbf{T} = \mathbf{H}_{rr}\mathbf{H}_{rr}^H$. For DF, $\frac{\partial f(\mathbf{d})}{\partial \mathbf{d}^*}$ is $\mathbf{X}'$ if $X(\mathbf{d}_k) < Y(\mathbf{d}_k)$, and $\mathbf{Y}'$ otherwise.

Once $\mathbf{d}$ is fixed, $\mathbf{p}$ is obtained from (8) and (11), where $\mathbf{P}_{\mathbf{z}}^{\perp}$ is calculated from (7) and (9).

V. Simulation Results

In this section, the theories presented in the previous sections are verified through computer simulations. The achievable rate results based on each static channel realization are averaged over 10^4 independent realizations. The simulation conditions are as follows: $\sigma_{sr}^2 = \sigma_{rd}^2 = 10^{-3}$, $\sigma_{rr}^2 = 10^{-1}$, $\sigma_r^2 = \sigma_d^2 = -10$ dBm, $P_s = 30$ dBm, and P_r varies in a range from 20 to 60 dBm. Thus, the self-interference power ($\sim \sigma_{rr}^2 P_r$) is at a level from 10 to 50 dBm, which is at least 20 dB greater than σ_r^2. This self-interference will be nullified by the joint-nulling scheme. An initial value $\mathbf{d}_1 = [1\ 0 \cdots 0]^T$ is used for the proposed algorithm.

In Fig. 2, where $N = 2$, the curves for both AF and DF show a general trend that the ergodic rates increase as P_r increases, and get saturated since the system becomes bottlenecked by the SS-to-RS channel. The proposed method shows better performance than the SVD-based method for both AF and DF (by about 0.4 [bit/sec/Hz] for $P_r \geq 40$ dBm), confirming the effect of the continuous domain optimization of the relay matrix. Note that the proposed scheme shows performance closer to that of the ideal full-duplex relay without self-interference (i.e., double the rate of the half-duplex relay), but with some performance loss (about 0.4∼0.5 [bit/sec/Hz]). The loss is inevitable because a portion of the d.o.f. of the relay gain (both $\mathbf{d}$ and $\mathbf{p}$) is devoted to the interference nulling while the ideal full-duplex relay would not need to account for the interference nulling. From the figure, we can verify that DF outperforms AF. This is prominent for small levels of P_r, but the difference in performance becomes marginal as P_r increases.

In Fig. 3, the effect of the number of relay antennas ($N = 2, 4, 6$) on the ergodic rate for AF is demonstrated. P_r is fixed to 30 dBm, and the abscissa is the relay input SNR where the signal power is fixed at the level of 0 dBm as before, and $\sigma_r^2 (= \sigma_d^2)$ varies from 0 to -40 dBm. Again, we can visually verify that the proposed scheme outperforms the SVD-based scheme. Moreover, we can see that the proposed scheme outperforms that of [4]. This is due to the fact that, in the SISO case with $N \geq 2$, the scheme in [4] is reduced to the interference nulling system, and the proposed scheme is optimized in the interference nulling conditions.

VI. Conclusions

We designed the relay processing matrix, maximizing the rate while jointly nullifying the self-interference. The joint-nulling can be made either by a selection of basis vectors based on the SVD of the echo channel or by an optimization of the relay processing vectors over the continuous domain. It was shown that the proposed continuous domain approach is more effective than the SVD-based method.

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