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Published in IET Communications
Received on 18th January 2010
Revised on 26th April 2010
doi: 10.1049/iet-com.2010.0056

ISSN 1751-8628

Full-duplex relay based on block diagonalisation in multiple-input multiple-output relay systems

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Abstract: The authors propose a full-duplex relay (FDR) based on a block diagonalisation (BD) method in multiple-input multiple-output (MIMO) relay channels. The BD method is employed to suppress the self-interference in MIMO FDR as well as to transmit multiple data streams to multiple users at the same time. The authors also derive the optimal power allocation strategy for MIMO FDR operation. Numerical results show that the use of the proposed FDR with BD can provide enhanced system performance compared with the FDR with zero-forcing beamforming.

1 Introduction

Relay transmission is a promising technique for next generation wireless communication systems, owing to its inherent capability of extending the cell coverage and increasing the cell capacity. However, most of the works on relay have been restricted to half-duplex relaying [1], with which the relay station (RS) needs to use separate resources when receiving the signal from the source and when forwarding the signal to the destination. Even though the half-duplex relay (HDR) is easy to implement, the resource partitioning between the RS's transmission and reception may undermine the whole system capacity. The half-duplex requirement also complicates the frame structure design as noted in [2].

Considering the drawbacks of the HDR, full-duplex relay (FDR) [3] is emerging as an attractive approach, which allows the RS to transmit and receive at the same time on the same frequency. However, for the full-duplex operation at the RS, it is important to suppress the possibly strong self-interference induced by the RS's transmitter to its own receiver. In [4], the feasibility of simultaneous transmission and reception was studied and transmit beamformers were suggested to reduce the self-interference for the FDR. A training-based channel estimation and echo cancellation scheme was proposed in [5] to enable an amplify-and-forward relay to transmit and receive simultaneously. Recently, the authors have shown that the self-interference in the FDR can be canceled by the zero-forcing beamforming (ZFBF), based on the observation that the self-interference can be interpreted as a multiuser interference in an equivalent multiuser multiple-input multiple-output (MIMO) downlink model. Moreover, they have also verified that the FDR based on the ZFBF provides a remarkable capacity enhancement compared with the conventional HDR. However, they have dealt with an RS equipped with only a single transmit and a single receive antenna, so that only one data stream can be passed through the RS.

In this paper, we consider a downlink MIMO FDR system, where an RS equipped with multiple transmit and receive antennas serves a mobile station (MS) with multiple receive antennas. A precoding strategy based on the block diagonalisation (BD) method [6] is proposed to suppress the self-interference between multiple transmit and receive antennas at the RS as well as to serve multiple MSs at the same time. Using the Karush–Kuhn–Tucker (KKT) conditions [7], we also derive the optimal power allocation scheme for the MIMO FDR system that maximises the sum rate under the given power constraints of the base station (BS) and RS. Numerical results show that the proposed FDR with BD provides significant capacity improvement over the FDR with ZFBF.

The remainder of this paper is organised as follows. Section 2 presents the MIMO FDR system model. In Section 3, we extend the result to the MIMO FDR scenario, and propose a new FDR precoding scheme based on the BD method. In Section 4, we derive the optimal power allocation scheme for the MIMO FDR transmission. Numerical results are provided in Section 5 to validate the performance enhancement of the proposed scheme. Finally, conclusions are drawn in Section 6.

2 System model

Let us consider a downlink multiuser MIMO relaying scenario where the BS has L transmit antennas and an RS has M_r receive antennas and M_t transmit antennas. The decode-and-forward protocol [8, 9] is employed by the RS. MSs, each assumed to be equipped with M receive antennas, are categorised into two groups according to the reachability of the signals from the BS: (i) MSs in the first group is located in the BS coverage. (ii) MSs in the second group is located out of the BS coverage but in the RS coverage. For notational convenience, an MS in the ith group is referred to as MS-i, $i = 1, 2$. Note that the MS-1 receives the signal from the BS and may also receive the signal from the RS, whereas the MS-2 receives the signal only from the RS.

Fig. 1 illustrates the FDR system model in a MIMO relay channel. The $M \times 1$ received signal vector $\boldsymbol{y}_i$ at the MS-i, $i = 1, 2$, is respectively given as

$$\begin{aligned} \boldsymbol{y}_1 &= \boldsymbol{H}_1^{\mathrm{BS}}\boldsymbol{s}^{\mathrm{BS}} + \boldsymbol{H}_1^{\mathrm{RS}}\boldsymbol{s}^{\mathrm{RS}} + \boldsymbol{z}_1 \\ \boldsymbol{y}_2 &= \boldsymbol{H}_2^{\mathrm{RS}}\boldsymbol{s}^{\mathrm{RS}} + \boldsymbol{z}_2 \end{aligned} \tag{1}$$

where $\boldsymbol{H}_i^{\mathrm{BS}}$ denotes an $M \times L$ channel matrix between the BS and MS-i, $\boldsymbol{H}_i^{\mathrm{RS}}$ denotes an $M \times M_t$ channel matrix between the RS and MS-i, $\boldsymbol{s}^{\mathrm{BS}}$ denotes the $L \times 1$ transmit signal vector from the BS, $\boldsymbol{s}^{\mathrm{RS}}$ denotes the $M_t \times 1$ transmit signal vector from the RS and $\boldsymbol{z}_i$ denotes the zero-mean additive white Gaussian noise (AWGN) vector with the covariance matrix of $\sigma^2\boldsymbol{I}$. In (1), $\boldsymbol{H}_1^{\mathrm{RS}}$ may be a zero matrix when the MS-1 cannot receive the signal from the RS. Moreover, the received signal at the RS in full-duplex operation can be expressed as

$$\boldsymbol{y}_{\mathrm{RS}} = \boldsymbol{H}_{\mathrm{RS}}^{\mathrm{BS}}\boldsymbol{s}^{\mathrm{BS}} + \boldsymbol{H}_{\mathrm{RS}}^{\mathrm{RS}}\boldsymbol{s}^{\mathrm{RS}} + \boldsymbol{z}_{\mathrm{RS}} \tag{2}$$

where $\boldsymbol{H}_{\mathrm{RS}}^{\mathrm{BS}}$ denotes an $M_\mathrm{r} \times L$ channel matrix between the BS and RS, $\boldsymbol{H}_{\mathrm{RS}}^{\mathrm{RS}}$ denotes an $M_\mathrm{r} \times M_\mathrm{t}$ channel matrix between the RSs transmit antennas and receive antennas and $\boldsymbol{z}_{\mathrm{RS}}$ is the AWGN vector with the covariance matrix of $\sigma^2\boldsymbol{I}$. The second term in the right-hand side of (2) denotes the self-interference induced by the full-duplex operation of the RS.

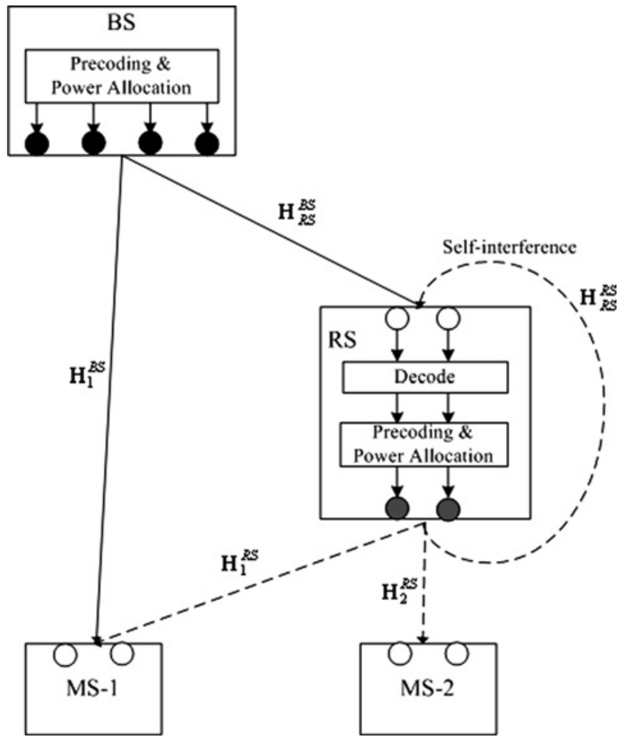

Figure 1 *MIMO FDR system model with $L = 4$ and $M_t = M_r = M = 2$*

3 Precoding for MIMO FDR

In this section, we first extend the result to a MIMO FDR system. Then, we develop a precoding strategy for the MIMO FDR based on the BD method. In the following analysis, we assume that $M_\mathrm{t} = M_\mathrm{r} = M$ and $L = 2M$ for simplicity of presentation. Note that the proposed scheme can easily be extended to the case of $L = kM$ for an integer $k > 2$. The MSs and RS are assumed to receive up to M data streams. Then, the received signals at the MS-1, MS-2 and RS in (1) and (2) can be rearranged into a matrix form as

$$\begin{bmatrix} \boldsymbol{y}_1 \\ \boldsymbol{y}_{\mathrm{RS}} \\ \boldsymbol{y}_2 \end{bmatrix} = \boldsymbol{H}\begin{bmatrix} \boldsymbol{s}^{\mathrm{BS}} \\ \boldsymbol{s}^{\mathrm{RS}} \end{bmatrix} + \begin{bmatrix} \boldsymbol{z}_1 \\ \boldsymbol{z}_{\mathrm{RS}} \\ \boldsymbol{z}_2 \end{bmatrix} \tag{3}$$

where the $3M \times 3M$ channel matrix $\boldsymbol{H}$ is defined as

$$\boldsymbol{H} \triangleq \begin{bmatrix} \boldsymbol{H}_1^{\mathrm{BS}} & \boldsymbol{H}_1^{\mathrm{RS}} \\ \boldsymbol{H}_{\mathrm{RS}}^{\mathrm{BS}} & \boldsymbol{H}_{\mathrm{RS}}^{\mathrm{RS}} \\ \boldsymbol{0}_{M\times 2M} & \boldsymbol{H}_2^{\mathrm{RS}} \end{bmatrix} \tag{4}$$

Now, we will develop FDR transmission strategy based on the BD method using the received signal models in (3) and (4). Hereafter, we assume that the total available transmit

IET Commun., 2010, Vol. 4, Iss. 15, pp. 1817–1826
doi: 10.1049/iet-com.2010.0056

power at the BS and RS is assumed to be $P_{\mathrm{T}}^{\mathrm{BS}}$ and $P_{\mathrm{T}}^{\mathrm{RS}}$, respectively. All the channel state informations (CSIs) are assumed to be available at the BS. Note that the BS can collect all the CSIs of the BS-to-RS link and the BS-to-MSs links in either time-division duplex or frequency-division duplex systems by using CSI feedback schemes [10]. In particular, the MSs located in the RS coverage may feedback the CSIs between the RS and themselves to the RS, which then transfers the CSIs to the BS using an analog linear modulation as suggested in [10].

3.1 Zero-forcing beamforming

The FDR with ZFBF can be extended to a MIMO relay system where the RS receives and transmits multiple data streams. The $3M \times 1$ transmit signal vector in (3) can be expressed as

$$\begin{bmatrix} \boldsymbol{s}^{\mathrm{BS}} \\ \boldsymbol{s}^{\mathrm{RS}} \end{bmatrix} = \boldsymbol{W}_{\mathrm{ZF}} \boldsymbol{P} \begin{bmatrix} \boldsymbol{x}^{(1)} \\ \boldsymbol{x}^{(2)} \\ \tilde{\boldsymbol{x}}^{(2)} \end{bmatrix} \quad (5)$$

where $\boldsymbol{x}^{(i)} = [x_1^{(i)} \cdots x_M^{(i)}]^{\mathrm{T}}$ denotes the $M \times 1$ data symbol vector destined for the MS-i. Particularly for the MS-2, the data symbol vector $\boldsymbol{x}^{(2)}$ at the BS is first destined for the RS. Then, the RS decodes $\boldsymbol{x}^{(2)}$ and forwards $\tilde{\boldsymbol{x}}^{(2)} = [\tilde{x}_1^{(2)} \cdots \tilde{x}_M^{(2)}]^{\mathrm{T}}$ to the MS-2, which is a delayed version of $\boldsymbol{x}^{(2)}$. As shown in Appendix 1, the $3M \times 3M$ ZFBF precoder in (5) can be expressed as

$$\boldsymbol{W}_{\mathrm{ZF}} \triangleq \boldsymbol{H}^{\dagger}(\boldsymbol{H}\boldsymbol{H}^{\dagger})^{-1} = \begin{bmatrix} \boldsymbol{W}_{\mathrm{ZF}}^{(\mathrm{BS}:1)} & \boldsymbol{W}_{\mathrm{ZF}}^{(\mathrm{BS}:\mathrm{RS})} & \boldsymbol{W}_{\mathrm{ZF}}^{(\mathrm{BS}:2)} \\ \boldsymbol{0}_{M\times M} & \boldsymbol{0}_{M\times M} & \boldsymbol{W}_{\mathrm{ZF}}^{(\mathrm{RS}:2)} \end{bmatrix} \quad (6)$$

where $\boldsymbol{H}$ is given in (4) and $(\cdot)^{\dagger}$ denotes the conjugate transpose. In (6), $\boldsymbol{W}_{\mathrm{ZF}}^{(\mathrm{BS}:i)}$ and $\boldsymbol{W}_{\mathrm{ZF}}^{(\mathrm{BS}:\mathrm{RS})}$ denote the $2M \times M$ ZFBF matrices at the BS corresponding to the MS-i and RS, respectively. Moreover, $\boldsymbol{W}_{\mathrm{ZF}}^{(\mathrm{RS}:2)}$ is the $M \times M$ ZFBF matrix at the RS corresponding to the MS-2. These matrices can be expressed as

$$\begin{aligned} \boldsymbol{W}_{\mathrm{ZF}}^{(\mathrm{BS}:i)} &= \begin{bmatrix} \boldsymbol{w}_1^{(\mathrm{BS}:i)} & \cdots & \boldsymbol{w}_M^{(\mathrm{BS}:i)} \end{bmatrix} \\ \boldsymbol{W}_{\mathrm{ZF}}^{(\mathrm{BS}:\mathrm{RS})} &= \begin{bmatrix} \boldsymbol{w}_1^{(\mathrm{BS}:\mathrm{RS})} & \cdots & \boldsymbol{w}_M^{(\mathrm{BS}:\mathrm{RS})} \end{bmatrix} \\ \boldsymbol{W}_{\mathrm{ZF}}^{(\mathrm{RS}:2)} &= \begin{bmatrix} \boldsymbol{w}_1^{(\mathrm{RS}:2)} & \cdots & \boldsymbol{w}_M^{(\mathrm{RS}:2)} \end{bmatrix} \end{aligned} \quad (7)$$

where $\boldsymbol{w}_m^{(\mathrm{BS}:i)}$ and $\boldsymbol{w}_m^{(\mathrm{BS}:\mathrm{RS})}$ denote the $2M \times 1$ ZFBF vectors for the mth data stream at the BS and $\boldsymbol{w}_m^{(\mathrm{RS}:2)}$ is the $M \times 1$ ZFBF vector for the mth data stream at the RS with $m = 1, \ldots, M$. The $3M \times 3M$ diagonal power allocation matrix $\boldsymbol{P}$ is also defined as

$$\boldsymbol{P} = \mathrm{diag}[\boldsymbol{p}^{(\mathrm{BS}:1)}, \boldsymbol{p}^{(\mathrm{BS}:\mathrm{RS})}, \boldsymbol{p}^{(\mathrm{RS}:2)}] \quad (8)$$

where

$$\begin{aligned} \boldsymbol{p}^{(\mathrm{BS}:1)} &= \left[\sqrt{P_1^{(\mathrm{BS}:1)}}, \ldots, \sqrt{P_M^{(\mathrm{BS}:1)}}\right] \\ \boldsymbol{p}^{(\mathrm{BS}:\mathrm{RS})} &= \left[\sqrt{P_1^{(\mathrm{BS}:\mathrm{RS})}}, \ldots, \sqrt{P_M^{(\mathrm{BS}:\mathrm{RS})}}\right] \\ \boldsymbol{p}^{(\mathrm{RS}:2)} &= \left[\sqrt{P_1^{(\mathrm{RS}:2)}}, \ldots, \sqrt{P_M^{(\mathrm{RS}:2)}}\right] \end{aligned} \quad (9)$$

From (5), (7) and (8), the transmit signal vectors at the BS and at the RS can be written as

$$\boldsymbol{s}^{\mathrm{BS}} = \sum_{m=1}^{M} \Bigg[\boldsymbol{w}_m^{(\mathrm{BS}:1)} \sqrt{P_m^{(\mathrm{BS}:1)}} x_m^{(1)} + \boldsymbol{w}_m^{(\mathrm{BS}:\mathrm{RS})} \sqrt{P_m^{(\mathrm{BS}:\mathrm{RS})}} x_m^{(2)} + \boldsymbol{w}_m^{(\mathrm{BS}:2)} \sqrt{P_m^{(\mathrm{RS}:2)}} \tilde{x}_m^{(2)} \Bigg] \quad (10)$$

$$\boldsymbol{s}^{\mathrm{RS}} = \sum_{m=1}^{M} \boldsymbol{w}_m^{(\mathrm{RS}:2)} \sqrt{P_m^{(\mathrm{RS}:2)}} \tilde{x}_m^{(2)} \quad (11)$$

In (10), the BS transmits the beamformed signals of $x_m^{(1)}$, $x_m^{(2)}$ and $\tilde{x}_m^{(2)}$. It should be noted that the BS can transmit the beamformed signal of $\tilde{x}_m^{(2)}$, since $\tilde{x}_m^{(2)}$ is a delayed version of $x_m^{(2)}$ that is originated from the BS. The RS transmit signal in (11) is only the beamformed signal of $\tilde{x}_m^{(2)}$, due to the form of the precoding matrix $\boldsymbol{W}_{\mathrm{ZF}}$ in (6). It is reasonable because the RS can know only $\tilde{x}_m^{(2)}$ after decoding $x_m^{(2)}$. Moreover, the total transmit power constraints of the BS and RS can be expressed as

$$\sum_{m=1}^{M} \Big[\|\boldsymbol{w}_m^{(\mathrm{BS}:1)}\|^2 P_m^{(\mathrm{BS}:1)} + \|\boldsymbol{w}_m^{(\mathrm{BS}:\mathrm{RS})}\|^2 P_m^{(\mathrm{BS}:\mathrm{RS})} + \|\boldsymbol{w}_m^{(\mathrm{BS}:2)}\|^2 P_m^{(\mathrm{RS}:2)} \Big] = P_{\mathrm{T}}^{\mathrm{BS}} \quad (12)$$

and

$$\sum_{m=1}^{M} \|\boldsymbol{w}_m^{(\mathrm{RS}:2)}\|^2 P_m^{(\mathrm{RS}:2)} \le P_{\mathrm{T}}^{\mathrm{RS}} \quad (13)$$

respectively. As shown in (12) and (13), the BS is associated with an equality power constraint, whereas the RS is associated with an inequality power constraint. This is a reasonable assumption from the viewpoint of power saving at the RS, since the transmit power of the RS must be minimised, unless it degrades the relaying performance.

In order to compute the optimal power allocation, the KKT conditions will be presented in Section 4. However, the computation is quite complicated for a large M. We will present analytic derivation of the optimal power allocation and following derivation of sum rate for only

$M = 2$ case, although the method can similarly be applied to the case of $M > 2$. For the case $M = 2$, substitution of (5), (6) and (8) into (3) yields

$$\boldsymbol{y}_1 = \begin{bmatrix} \sqrt{P_1^{(\mathrm{BS}:1)}}x_1^{(1)} \\ \sqrt{P_2^{(\mathrm{BS}:1)}}x_2^{(1)} \end{bmatrix} + \boldsymbol{z}_1 \tag{14}$$

$$\boldsymbol{y}_{\mathrm{RS}} = \begin{bmatrix} \sqrt{P_1^{(\mathrm{BS:RS})}}x_1^{(2)} \\ \sqrt{P_2^{(\mathrm{BS:RS})}}x_2^{(2)} \end{bmatrix} + \boldsymbol{z}_{\mathrm{RS}} \tag{15}$$

$$\boldsymbol{y}_2 = \begin{bmatrix} \sqrt{P_1^{(\mathrm{RS}:2)}}\tilde{x}_1^{(2)} \\ \sqrt{P_2^{(\mathrm{RS}:2)}}\tilde{x}_2^{(2)} \end{bmatrix} + \boldsymbol{z}_2 \tag{16}$$

In (15) and (16), it should be noted that the information rates of the RS's received data streams $x_1^{(2)}$ and $x_2^{(2)}$ are equal to those of the RS's transmit data streams $\tilde{x}_1^{(2)}$ and $\tilde{x}_2^{(2)}$, respectively, which is referred to as the individual data stream matching. Therefore $\tilde{x}_m^{(2)}$ will be constructed with the same modulation and coding scheme (MCS) as $x_m^{(2)}$. Alternatively, we may consider a sum rate matching at the RS, where the sum rate of the RS's received data streams is equal to that of the RS's transmit data streams. Then, each transmit data stream may have different information rate from the corresponding received data stream, so that the MCS of each transmit data stream will be different from that of the received data stream. To realise the sum rate matching, however, the RS needs to change the MCS of each transmit data stream and regenerate a new MAP message to inform the MS of the MCS, which may increase the complexity of the RS.

In the individual data stream matching, there are two different mappings between the RS's transmit and receive data streams. The corresponding relationship between transmit powers associated with each mapping is given as

$$P_1^{(\mathrm{BS:RS})} = P_1^{(\mathrm{RS}:2)}, \quad P_2^{(\mathrm{BS:RS})} = P_2^{(\mathrm{RS}:2)} \tag{17}$$

and

$$P_1^{(\mathrm{BS:RS})} = P_2^{(\mathrm{RS}:2)}, \quad P_2^{(\mathrm{BS:RS})} = P_1^{(\mathrm{RS}:2)} \tag{18}$$

Then, the sum rate of the FDR with ZFBF in a MIMO relay system can be computed as

$$\begin{aligned} C_{\mathrm{ZF}} = {} & \log_2\left(1 + \frac{P_1^{(\mathrm{BS}:1)}}{\sigma^2}\right) + \log_2\left(1 + \frac{P_2^{(\mathrm{BS}:1)}}{\sigma^2}\right) \\ & + \log_2\left(1 + \frac{P_1^{(\mathrm{BS:RS})}}{\sigma^2}\right) + \log_2\left(1 + \frac{P_2^{(\mathrm{BS:RS})}}{\sigma^2}\right) \end{aligned} \tag{19}$$

The total transmit power constraints of the BS and RS in (12) and (13) can be rewritten as

$$\begin{aligned} & \gamma_1^{(\mathrm{BS}:1)}P_1^{(\mathrm{BS}:1)} + \gamma_2^{(\mathrm{BS}:1)}P_2^{(\mathrm{BS}:1)} + \gamma_1^{(\mathrm{BS:RS})}P_1^{(\mathrm{BS:RS})} \\ & \quad + \gamma_2^{(\mathrm{BS:RS})}P_2^{(\mathrm{BS:RS})} = P_{\mathrm{T}}^{\mathrm{BS}} \end{aligned} \tag{20}$$

$$\delta_1^{(\mathrm{BS:RS})}P_1^{(\mathrm{BS:RS})} + \delta_2^{(\mathrm{BS:RS})}P_2^{(\mathrm{BS:RS})} \le P_{\mathrm{T}}^{\mathrm{RS}} \tag{21}$$

where

$$\gamma_k^{(\mathrm{BS}:1)} = \|\boldsymbol{w}_k^{(\mathrm{BS}:1)}\|^2, \quad k = 1, 2 \tag{22}$$

and $\gamma_k^{(\mathrm{BS:RS})}$ and $\delta_k^{(\mathrm{BS:RS})}$ are defined differently according to the two stream mappings. For the stream mapping corresponding to (17), we obtain

$$\begin{aligned} \gamma_1^{(\mathrm{BS:RS})} &= \|\boldsymbol{w}_1^{(\mathrm{BS:RS})}\|^2 + \|\boldsymbol{w}_1^{(\mathrm{BS}:2)}\|^2 \\ \gamma_2^{(\mathrm{BS:RS})} &= \|\boldsymbol{w}_2^{(\mathrm{BS:RS})}\|^2 + \|\boldsymbol{w}_2^{(\mathrm{BS}:2)}\|^2 \\ \delta_1^{(\mathrm{BS:RS})} &= \|\boldsymbol{w}_1^{(\mathrm{RS}:2)}\|^2 \\ \delta_2^{(\mathrm{BS:RS})} &= \|\boldsymbol{w}_2^{(\mathrm{RS}:2)}\|^2 \end{aligned} \tag{23}$$

whereas for the stream mapping corresponding to (18), we have

$$\begin{aligned} \gamma_1^{(\mathrm{BS:RS})} &= \|\boldsymbol{w}_1^{(\mathrm{BS:RS})}\|^2 + \|\boldsymbol{w}_2^{(\mathrm{BS}:2)}\|^2 \\ \gamma_2^{(\mathrm{BS:RS})} &= \|\boldsymbol{w}_2^{(\mathrm{BS:RS})}\|^2 + \|\boldsymbol{w}_1^{(\mathrm{BS}:2)}\|^2 \\ \delta_1^{(\mathrm{BS:RS})} &= \|\boldsymbol{w}_2^{(\mathrm{RS}:2)}\|^2 \\ \delta_2^{(\mathrm{BS:RS})} &= \|\boldsymbol{w}_1^{(\mathrm{RS}:2)}\|^2 \end{aligned} \tag{24}$$

3.2 Block diagonalisation

In this subsection, we apply the BD method [6] to propose a new form of MIMO FDR. The precoding matrix based on the BD method can be decomposed as

$$\boldsymbol{W}_{\mathrm{BD}} = \left[\, \boldsymbol{W}_{\mathrm{BD}}^{(\mathrm{MS}-1)} \quad \boldsymbol{W}_{\mathrm{BD}}^{(\mathrm{RS})} \quad \boldsymbol{W}_{\mathrm{BD}}^{(\mathrm{MS}-2)} \,\right] \tag{25}$$

According to the BD principle, it is seen from (4) that the precoding matrix $\boldsymbol{W}_{\mathrm{BD}}^{(\mathrm{MS}-1)}$ corresponding to the MS-1 should be chosen to nullify the interference such that

$$\tilde{\boldsymbol{H}}_1\boldsymbol{W}_{\mathrm{BD}}^{(\mathrm{MS}-1)} = \boldsymbol{0} \tag{26}$$

where $\tilde{\boldsymbol{H}}_1$ stacks all the channel matrices except for the channel matrices for the MS-1, that is

$$\tilde{\boldsymbol{H}}_1 = \begin{bmatrix} \boldsymbol{H}_{\mathrm{RS}}^{\mathrm{BS}} & \boldsymbol{H}_{\mathrm{RS}}^{\mathrm{RS}} \\ \boldsymbol{0}_{M\times 2M} & \boldsymbol{H}_2^{\mathrm{RS}} \end{bmatrix} \tag{27}$$

The singular value decomposition (SVD) of the $M \times 2M$ channel matrix $\boldsymbol{H}_{\mathrm{RS}}^{\mathrm{BS}}$ yields

$$\boldsymbol{H}_{\mathrm{RS}}^{\mathrm{BS}} = \tilde{\boldsymbol{U}}_1\tilde{\boldsymbol{\Lambda}}_1[\tilde{\boldsymbol{V}}_1^{(1)}\tilde{\boldsymbol{V}}_1^{(0)}]^{\dagger} \tag{28}$$

IET Commun., 2010, Vol. 4, Iss. 15, pp. 1817–1826
doi: 10.1049/iet-com.2010.0056

where $\tilde{\boldsymbol{\Lambda}}_1$ contains the singular values of $\boldsymbol{H}_{\text{RS}}^{\text{BS}}$ and $\tilde{\boldsymbol{U}}_1$ is the left singular matrix. $\tilde{\boldsymbol{V}}_1^{(1)}$ and $\tilde{\boldsymbol{V}}_1^{(0)}$ include the right singular vectors corresponding to non-zero and zero singular values, respectively. Then, it can be easily shown that

$$\tilde{\boldsymbol{H}}_1\begin{bmatrix}\tilde{\boldsymbol{V}}_1^{(0)}\\ \boldsymbol{0}_{M\times M}\end{bmatrix} = \boldsymbol{0}_{2M\times M} \tag{29}$$

If we define

$$\hat{\boldsymbol{H}}_1 = \begin{bmatrix}\boldsymbol{H}_1^{\text{BS}} & \boldsymbol{H}_1^{\text{RS}}\end{bmatrix}\begin{bmatrix}\tilde{\boldsymbol{V}}_1^{(0)}\\ \boldsymbol{0}_{M\times M}\end{bmatrix} \tag{30}$$

the SVD of $\hat{\boldsymbol{H}}_1$ yields

$$\hat{\boldsymbol{H}}_1 = \hat{\boldsymbol{U}}_1\hat{\boldsymbol{\Lambda}}_1\hat{\boldsymbol{V}}_1^{\dagger} \tag{31}$$

where $\hat{\boldsymbol{\Lambda}}_1 = \text{diag}[\lambda_1^{(\text{BS:1})}, \ldots, \lambda_M^{(\text{BS:1})}]$. From (29) and (31), the precoding matrix $\boldsymbol{W}_{\text{BD}}^{(\text{MS}-1)}$ is found as

$$\boldsymbol{W}_{\text{BD}}^{(\text{MS}-1)} = \begin{bmatrix}\tilde{\boldsymbol{V}}_1^{(0)}\\ \boldsymbol{0}_{M\times M}\end{bmatrix}\hat{\boldsymbol{V}}_1 \tag{32}$$

As derived in Appendix 2, we can compute $\boldsymbol{W}_{\text{BD}}^{(\text{RS})}$, $\hat{\boldsymbol{\Lambda}}_{\text{RS}} = \text{diag}[\lambda_1^{(\text{BS:RS})}, \cdots, \lambda_M^{(\text{BS:RS})}]$ and $\hat{\boldsymbol{U}}_{\text{RS}}$ corresponding to the RS, and $\boldsymbol{W}_{\text{BD}}^{(\text{MS}-2)}$, $\hat{\boldsymbol{\Lambda}}_2 = \text{diag}[\lambda_1^{(\text{BS:2})}, \ldots, \lambda_M^{(\text{BS:2})}]$ and $\hat{\boldsymbol{U}}_2$ corresponding to the MS-2.

Consequently, each component submatrix of (25) can be rewritten as

$$\begin{aligned}\boldsymbol{W}_{\text{BD}}^{(\text{MS}-1)} &= \begin{bmatrix}\bar{\boldsymbol{w}}_1^{(\text{BS:1})} & \cdots & \bar{\boldsymbol{w}}_M^{(\text{BS:1})}\\ \boldsymbol{0}_{M\times 1} & \cdots & \boldsymbol{0}_{M\times 1}\end{bmatrix}\\ \boldsymbol{W}_{\text{BD}}^{(\text{RS})} &= \begin{bmatrix}\bar{\boldsymbol{w}}_1^{(\text{BS:RS})} & \cdots & \bar{\boldsymbol{w}}_M^{(\text{BS:RS})}\\ \boldsymbol{0}_{M\times 1} & \cdots & \boldsymbol{0}_{M\times 1}\end{bmatrix}\\ \boldsymbol{W}_{\text{BD}}^{(\text{MS}-2)} &= \begin{bmatrix}\bar{\boldsymbol{w}}_1^{(\text{BS:2})} & \cdots & \bar{\boldsymbol{w}}_M^{(\text{BS:2})}\\ \bar{\boldsymbol{w}}_1^{(\text{RS:2})} & \cdots & \bar{\boldsymbol{w}}_M^{(\text{RS:2})}\end{bmatrix}\end{aligned} \tag{33}$$

Note that the BD precoder matrix in (33) has the same form of the ZFBF precoder matrix in (6). Since each column of the BD precoder matrix is extracted from right singular vectors as shown in (32), it has unit norm

$$\|\bar{\boldsymbol{w}}_m^{(\text{BS:1})}\|^2 = \|\bar{\boldsymbol{w}}_m^{(\text{BS:RS})}\|^2 = 1, \quad m = 1, \ldots, M \tag{34}$$

$$\|\bar{\boldsymbol{w}}_m^{(\text{BS:2})}\|^2 + \|\bar{\boldsymbol{w}}_m^{(\text{RS:2})}\|^2 = 1, \quad m = 1, \ldots, M \tag{35}$$

Using the precoder matrix in (25), we can express the transmit signal vector in (3) as

$$\begin{bmatrix}\boldsymbol{s}^{\text{BS}}\\ \boldsymbol{s}^{\text{RS}}\end{bmatrix} = \boldsymbol{W}_{\text{BD}}\boldsymbol{P}\begin{bmatrix}\boldsymbol{x}^{(1)}\\ \boldsymbol{x}^{(2)}\\ \tilde{\boldsymbol{x}}^{(2)}\end{bmatrix} \tag{36}$$

where $\boldsymbol{P}$ is given in (8). From (33) to (35), the transmit signal vectors at the BS and the RS can be written as

$$\begin{aligned}\boldsymbol{s}^{\text{BS}} = \sum_{m=1}^{M}\Big[&\bar{\boldsymbol{w}}_m^{(\text{BS:1})}\sqrt{P_m^{(\text{BS:1})}}x_m^{(1)} + \bar{\boldsymbol{w}}_m^{(\text{BS:RS})}\sqrt{P_m^{(\text{BS:RS})}}x_m^{(2)}\\ &+ \bar{\boldsymbol{w}}_m^{(\text{BS:2})}\sqrt{P_m^{(\text{RS:2})}}\tilde{x}_m^{(2)}\Big]\end{aligned} \tag{37}$$

$$\boldsymbol{s}^{\text{RS}} = \sum_{m=1}^{M}\bar{\boldsymbol{w}}_m^{(\text{RS:2})}\sqrt{P_m^{(\text{RS:2})}}\tilde{x}_m^{(2)} \tag{38}$$

Note that the RS transmits only the beamformed signal of $\tilde{x}_m^{(2)}$ owing to the form of $\boldsymbol{W}_{\text{BD}}$ in (25) and (33). The total transmit power constraints of the BS and RS in this case are given as

$$\sum_{m=1}^{M}[P_m^{(\text{BS:1})} + P_m^{(\text{BS:RS})} + \|\bar{\boldsymbol{w}}_m^{(\text{BS:2})}\|^2 P_m^{(\text{RS:2})}] = P_{\text{T}}^{\text{BS}} \tag{39}$$

and

$$\sum_{m=1}^{M}\|\bar{\boldsymbol{w}}_m^{(\text{RS:2})}\|^2 P_m^{(\text{RS:2})} \le P_{\text{T}}^{\text{RS}} \tag{40}$$

As in Section 3.1, we consider only $M = 2$ case in this work to present a simple analytic solution. Substituting (36) into (3) and assuming that the MS-1 uses $\hat{\boldsymbol{U}}_1$ in (31) as a receive filter, we can obtain the received signal vector at the MS-1 as

$$\begin{aligned}\bar{\boldsymbol{y}}_1 &= \hat{\boldsymbol{U}}_1^{\dagger}\boldsymbol{y}_1\\ &= \begin{bmatrix}\lambda_1^{(\text{BS:1})}\sqrt{P_1^{(\text{BS:1})}}x_1^{(1)}\\ \lambda_2^{(\text{BS:1})}\sqrt{P_2^{(\text{BS:1})}}x_2^{(1)}\end{bmatrix} + \bar{\boldsymbol{z}}_1\end{aligned} \tag{41}$$

Similarly, the received signal vectors at the RS and at the MS-2 are expressed as

$$\bar{\boldsymbol{y}}_{\text{RS}} = \begin{bmatrix}\lambda_1^{(\text{BS:RS})}\sqrt{P_1^{(\text{BS:RS})}}x_1^{(2)}\\ \lambda_2^{(\text{BS:RS})}\sqrt{P_2^{(\text{BS:RS})}}x_2^{(2)}\end{bmatrix} + \bar{\boldsymbol{z}}_{\text{RS}} \tag{42}$$

$$\bar{\boldsymbol{y}}_2 = \begin{bmatrix}\lambda_1^{(\text{BS:2})}\sqrt{P_1^{(\text{BS:2})}}\tilde{x}_1^{(2)}\\ \lambda_2^{(\text{BS:2})}\sqrt{P_2^{(\text{BS:2})}}\tilde{x}_2^{(2)}\end{bmatrix} + \bar{\boldsymbol{z}}_2 \tag{43}$$

As in Section 3.1, $x_1^{(2)}$ and $x_2^{(2)}$ can be mapped to $\tilde{x}_1^{(2)}$ and $\tilde{x}_2^{(2)}$ in two different ways. The corresponding relationship between transmit powers associated with each mapping is

doi: 10.1049/iet-com.2010.0056

given as

$$(\lambda_1^{(\mathrm{BS:RS})})^2 P_1^{(\mathrm{BS:RS})} = (\lambda_1^{(\mathrm{BS:2})})^2 P_1^{(\mathrm{BS:2})}$$
$$(\lambda_2^{(\mathrm{BS:RS})})^2 P_2^{(\mathrm{BS:RS})} = (\lambda_2^{(\mathrm{BS:2})})^2 P_2^{(\mathrm{BS:2})} \tag{44}$$

and

$$(\lambda_1^{(\mathrm{BS:RS})})^2 P_1^{(\mathrm{BS:RS})} = (\lambda_1^{(\mathrm{BS:2})})^2 P_2^{(\mathrm{BS:2})}$$
$$(\lambda_2^{(\mathrm{BS:RS})})^2 P_2^{(\mathrm{BS:RS})} = (\lambda_2^{(\mathrm{BS:2})})^2 P_1^{(\mathrm{BS:2})} \tag{45}$$

Then, the sum rate of the FDR based on the BD method can be computed as

$$\begin{aligned} C_{\mathrm{BD}} = \log_2\left(1 + \frac{(\lambda_1^{(\mathrm{BS:1})})^2 P_1^{(\mathrm{BS:1})}}{\sigma^2}\right) &+ \log_2\left(1 + \frac{(\lambda_2^{(\mathrm{BS:1})})^2 P_2^{(\mathrm{BS:1})}}{\sigma^2}\right) \\ &+ \log_2\left(1 + \frac{(\lambda_1^{(\mathrm{BS:RS})})^2 P_1^{(\mathrm{BS:RS})}}{\sigma^2}\right) \\ &+ \log_2\left(1 + \frac{(\lambda_2^{(\mathrm{BS:RS})})^2 P_2^{(\mathrm{BS:RS})}}{\sigma^2}\right) \end{aligned} \tag{46}$$

The total transmit power constraints of the BS and RS in (39) and (40) can be rewritten as

$$P_1^{(\mathrm{BS:1})} + P_2^{(\mathrm{BS:1})} + \tau_1^{(\mathrm{BS:RS})} P_1^{(\mathrm{BS:RS})} + \tau_2^{(\mathrm{BS:RS})} P_2^{(\mathrm{BS:RS})} = P_{\mathrm{T}}^{\mathrm{BS}} \tag{47}$$

$$\phi_1^{(\mathrm{BS:RS})} P_1^{(\mathrm{BS:RS})} + \phi_2^{(\mathrm{BS:RS})} P_2^{(\mathrm{BS:RS})} \le P_{\mathrm{T}}^{\mathrm{RS}} \tag{48}$$

where

$$\begin{aligned} \tau_1^{(\mathrm{BS:RS})} &= 1 + \|\bar{\boldsymbol{w}}_1^{(\mathrm{BS:2})}\|^2 \left(\frac{\lambda_1^{(\mathrm{BS:RS})}}{\lambda_1^{(\mathrm{BS:2})}}\right)^2 \\ \tau_2^{(\mathrm{BS:RS})} &= 1 + \|\bar{\boldsymbol{w}}_2^{(\mathrm{BS:2})}\|^2 \left(\frac{\lambda_2^{(\mathrm{BS:RS})}}{\lambda_2^{(\mathrm{BS:2})}}\right)^2 \\ \phi_1^{(\mathrm{BS:RS})} &= \|\bar{\boldsymbol{w}}_1^{(\mathrm{RS:2})}\|^2 \left(\frac{\lambda_1^{(\mathrm{BS:RS})}}{\lambda_1^{(\mathrm{BS:2})}}\right)^2 \\ \phi_2^{(\mathrm{BS:RS})} &= \|\bar{\boldsymbol{w}}_2^{(\mathrm{RS:2})}\|^2 \left(\frac{\lambda_2^{(\mathrm{BS:RS})}}{\lambda_2^{(\mathrm{BS:2})}}\right)^2 \end{aligned} \tag{49}$$

for the stream mapping corresponding to (45) and

$$\begin{aligned} \tau_1^{(\mathrm{BS:RS})} &= 1 + \|\bar{\boldsymbol{w}}_2^{(\mathrm{BS:2})}\|^2 \left(\frac{\lambda_1^{(\mathrm{BS:RS})}}{\lambda_2^{(\mathrm{BS:2})}}\right)^2 \\ \tau_2^{(\mathrm{BS:RS})} &= 1 + \|\bar{\boldsymbol{w}}_1^{(\mathrm{BS:2})}\|^2 \left(\frac{\lambda_2^{(\mathrm{BS:RS})}}{\lambda_1^{(\mathrm{BS:2})}}\right)^2 \\ \phi_1^{(\mathrm{BS:RS})} &= \|\bar{\boldsymbol{w}}_2^{(\mathrm{RS:2})}\|^2 \left(\frac{\lambda_1^{(\mathrm{BS:RS})}}{\lambda_2^{(\mathrm{BS:2})}}\right)^2 \\ \phi_2^{(\mathrm{BS:RS})} &= \|\bar{\boldsymbol{w}}_1^{(\mathrm{RS:2})}\|^2 \left(\frac{\lambda_2^{(\mathrm{BS:RS})}}{\lambda_1^{(\mathrm{BS:2})}}\right)^2 \end{aligned} \tag{50}$$

for the stream mapping corresponding to (45).

4 Power allocation for MIMO FDR

In (19)–(21) for the ZFBF precoding and (46)–(48) for the BD precoding, we need to determine transmit powers to compute the sum rate. In this section, we derive the optimal power allocation that maximises the sum rate under the given power constraints. As mentioned in Section 3, we present the analytic derivation of the optimal power allocation for $M = 2$, whereas the proposed scheme can easily be extended to the case of $M > 2$. The optimisation problem for both precoding schemes can be formulated in the generalised form as

$$(\check{P}_1^{\star}, \check{P}_2^{\star}, \check{P}_3^{\star}, \check{P}_4^{\star}) = \underset{\check{P}_k, k=1,2,3,4}{\operatorname{argmax}} \sum_{k=1}^{4} \log_2(1 + \alpha_k \check{P}_k) \tag{51}$$

subject to

$$\check{P}_1 + \check{P}_2 + \check{P}_3 + \check{P}_4 = P_{\mathrm{T}}^{\mathrm{BS}} \tag{52}$$

$$\beta_3 \check{P}_3 + \beta_4 \check{P}_4 \le P_{\mathrm{T}}^{\mathrm{RS}} \tag{53}$$

For the case of the ZFBF precoding in (19)–(21), the coefficients in (51)–(53) are defined as

$$\begin{aligned} \alpha_1 &= \frac{1}{\sigma^2 \gamma_1^{(\mathrm{BS:1})}}, \quad \alpha_2 = \frac{1}{\sigma^2 \gamma_2^{(\mathrm{BS:1})}}, \quad \alpha_3 = \frac{1}{\sigma^2 \gamma_1^{(\mathrm{BS:RS})}} \\ \alpha_4 &= \frac{1}{\sigma^2 \gamma_2^{(\mathrm{BS:RS})}}, \quad \beta_3 = \frac{\delta^{(\mathrm{BS:RS})}{}_1}{\gamma_1^{(\mathrm{BS:RS})}}, \quad \beta_4 = \frac{\delta_2^{(\mathrm{BS:RS})}}{\gamma_2^{(\mathrm{BS:RS})}} \end{aligned} \tag{54}$$

whereas for the case of the BD precoding in (46)–(48), the

doi: 10.1049/iet-com.2010.0056

coefficients are defined as

$$\alpha_1 = \frac{(\lambda_1^{(\mathrm{BS:1})})^2}{\sigma^2}, \quad \alpha_2 = \frac{(\lambda_2^{(\mathrm{BS:1})})^2}{\sigma^2}, \quad \alpha_3 = \frac{(\lambda_1^{(\mathrm{BS:RS})})^2}{\sigma^2 \tau_1^{(\mathrm{BS:RS})}}$$
$$\alpha_4 = \frac{(\lambda_2^{(\mathrm{BS:RS})})^2}{\sigma^2 \tau_2^{(\mathrm{BS:RS})}}, \quad \beta_3 = \frac{\phi_1^{(\mathrm{BS:RS})}}{\tau_1^{(\mathrm{BS:RS})}}, \quad \beta_4 = \frac{\phi_2^{(\mathrm{BS:RS})}}{\tau_2^{(\mathrm{BS:RS})}} \tag{55}$$

The optimisation problem can be solved from the KKT conditions [7]. Assuming $\check{P}_k \geq 0$, $k = 1, 2, 3, 4$, we compute $\check{P}_k$'s that satisfy the KKT conditions given as

$$\begin{aligned} \check{P}_1 &= \frac{\log_2 e}{\eta - \mu_1} - \frac{1}{\alpha_1} \\ \check{P}_2 &= \frac{\log_2 e}{\eta - \mu_2} - \frac{1}{\alpha_2} \\ \check{P}_3 &= \frac{\log_2 e}{\eta - \mu_3 + \beta_3 \mu} - \frac{1}{\alpha_3} \\ \check{P}_4 &= \frac{\log_2 e}{\eta - \mu_4 + \beta_4 \mu} - \frac{1}{\alpha_4} \end{aligned} \tag{56}$$

where η is introduced to reflect the constraint in (52) and $\mu_k \geq 0$, $\mu \geq 0$. Moreover, the KKT conditions also give $\mu_k \check{P}_k = 0$ and

$$\mu(\beta_3 \check{P}_3 + \beta_4 \check{P}_4 - P_{\mathrm{T}}^{\mathrm{RS}}) = 0 \tag{57}$$

To obtain the solutions of (56), we first assume that $\mu_k = \mu = 0$. Then, (52) leads to

$$\eta^\star = \frac{4 \log_2 e}{P_{\mathrm{T}}^{\mathrm{BS}} + \sum_{k=1}^{4} (1/\alpha_k)} \tag{58}$$

Substituting $\mu_k = \mu = 0$ and (58) into (56), we obtain $\check{P}_k^\star$ with $k = 1, 2, 3, 4$, which should satisfy $\check{P}_k^\star \geq 0$ and $\beta_3 \check{P}_3^\star + \beta_4 \check{P}_4^\star - P_{\mathrm{T}}^{\mathrm{RS}} \leq 0$.

Moreover, we consider the other case where $\mu > 0$ and $\mu_k = 0$, $k = 1, 2, 3, 4$. In that case, we have

$$\begin{aligned} \check{P}_1 &= \frac{\log_2 e}{\eta} - \frac{1}{\alpha_1} \\ \check{P}_2 &= \frac{\log_2 e}{\eta} - \frac{1}{\alpha_2} \\ \check{P}_3 &= \frac{\log_2 e}{\eta + \beta_3 \mu} - \frac{1}{\alpha_3} \\ \check{P}_4 &= \frac{\log_2 e}{\eta + \beta_4 \mu} - \frac{1}{\alpha_4} \end{aligned} \tag{59}$$

With the assumption of $\mu > 0$ and (57), we obtain

$$\beta_3 \check{P}_3 + \beta_4 \check{P}_4 - P_{\mathrm{T}}^{\mathrm{RS}} = 0 \tag{60}$$

Substituting (59) into (52) and (60), we can compute $\eta^\star$ and $\mu^\star$ as shown in Appendix 3. If $\mu^\star > 0$, we can evaluate $\check{P}_k^\star$, $k = 1, 2, 3, 4$ by substituting $\eta^\star$ and $\mu^\star$ into (59). Note that $\check{P}_k^\star$ should be non-negative.

When we set some of $\check{P}_k$ to zeroes, the power allocation that satisfies the KKT conditions can also be derived in a similar way described above. After finding the power allocation that satisfies the KKT conditions for each case, we choose the power allocation that provides the maximum sum rate. As mentioned earlier, the solution can be applied to both the ZFBF and BD precoding schemes with appropriate coefficients in (54) and (55).

5 Numerical results

In this section, we evaluate and compare the ergodic sum rate performance [11] of the FDR with ZFBF in Section 3.1 and FDR with BD in Section 3.2. For both ZFBF and BD cases, the power allocation is determined according to the algorithm in Section 4. We assume $L = 4$, $M = 2$, $P_{\mathrm{T}}^{\mathrm{BS}} = 100$, $P_{\mathrm{T}}^{\mathrm{RS}} = 20$, and $\sigma^2 = 1$. Moreover, for every MS, channel coefficients of the BS-to-MS and RS-to-MS links are assumed to follow independent and identically distributed (i.i.d.) complex Gaussian distribution with zero mean and unit variance. It is also assumed that the average channel gain of each entry in $\boldsymbol{H}_{\mathrm{RS}}^{\mathrm{BS}}$ and that of $\boldsymbol{H}_{\mathrm{RS}}^{\mathrm{RS}}$ is G-dB and I-dB larger than that of the BS-to-MS link, respectively. Assuming that the number of MSs in the first and second groups is the same, we denote the number of MSs in each group as N. When selecting an MS in each group of N MSs, we choose the MS that maximises the overall sum rate.

Fig. 2 compares the sum rate of the FDR with ZFBF and FDR with BD when the number of users N varies. It is seen that the FDR with BD outperforms the FDR with ZFBF in all ranges of N. When $N = 5$, $G = 0$ and $I = 0$, the performance gain of the FDR with BD is 12% compared

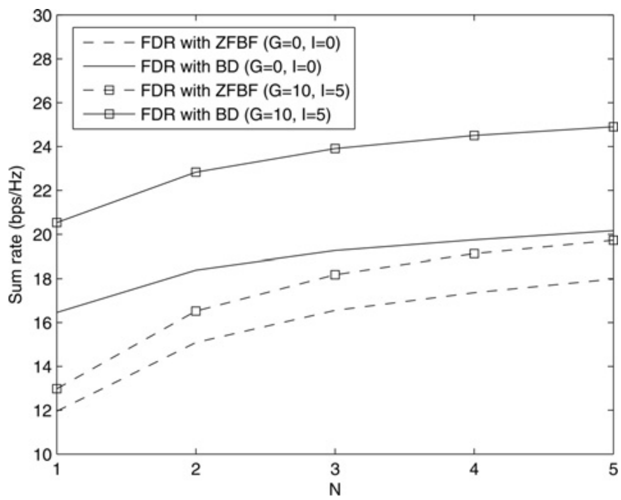

Figure 2 *Sum rate of FDR with ZFBF and FDR with BD against N (G = 0, I = 0 and G = 10, I = 5)*

doi: 10.1049/iet-com.2010.0056

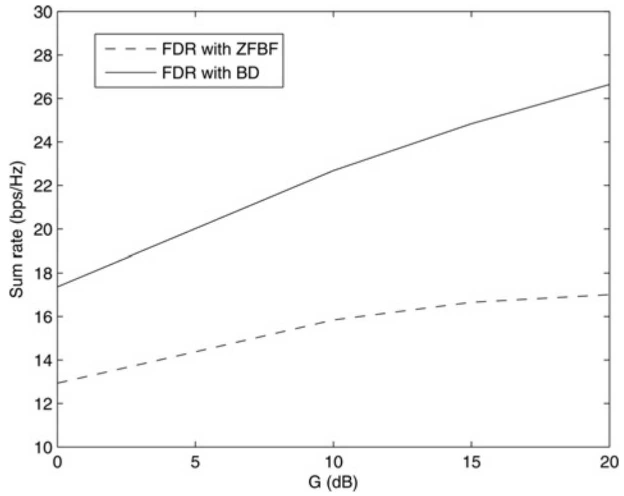

Figure 3 *Sum rate of FDR with ZFBF and FDR with BD against G (N = 2 and I = 10)*

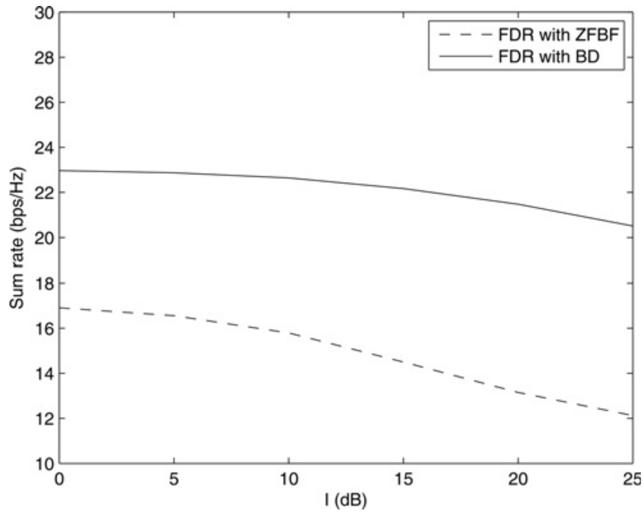

Figure 4 *Sum rate of FDR with ZFBF and FDR with BD against I (N = 2 and G = 10)*

to the FDR with ZFBF. The performance gain becomes as much as 26% when $N = 5$, $G = 10$ and $I = 5$. The ZFBF precoder in (6) suppresses both the inter-stream interference and inter-user interference so that $\boldsymbol{H}\boldsymbol{W}_{\mathrm{ZF}}$ becomes completely diagonal. Note that the inter-user interference is translated into the interference signals associated with $\boldsymbol{H}_1^{\mathrm{RS}}$ and $\boldsymbol{H}_{\mathrm{RS}}^{\mathrm{RS}}$ in (4). Each antenna at the RS or MS receives only one interference-free data stream and the receiver structure becomes quite simple, whereas both the BS and RS need to consume large amount of transmit power to nullify the interference; $\|\boldsymbol{w}_m^{(\mathrm{BS}:i)}\|^2$, $\|\boldsymbol{w}_m^{(\mathrm{BS:RS})}\|^2$ and $\|\boldsymbol{w}_m^{(\mathrm{RS}:2)}\|^2$ in (12) and (13) become large. On the other hand, the BD precoder in (33) suppresses only the inter-user interference. Hence, the BD precoder is expected to require relatively less transmit power for interference cancellation than the ZFBF precoder, so that the BS and RS can allocate more power for sending data streams.

Fig. 3 shows how the sum rate of the FDR with ZFBF and FDR with BD varies with G, when $N = 2$ and $I = 10$. Note that the larger value of G makes the BS-to-RS link more reliable. As shown in Fig. 3, the sum rate of the FDR with BD increases almost linearly with G, whereas that of the FDR with ZFBF is saturated when $G > 15$. Therefore it is obvious that the proposed FDR with BD becomes more effective than the FDR with ZFBF, as the BS-to-RS channel provides more reliable link quality. In Fig. 4, we compare the sum rate of the FDR with ZFBF and FDR with BD, when the isolation factor I varies. Here, an increase of I incurs the self-interference to become severer. As shown in Fig. 4, the FDR with BD outperforms the FDR with ZFBF in all ranges of I. It is remarkable that the decreasing rate of the sum rate in response to increasing I is smaller for the FDR with BD than for the FDR with ZFBF.

6 Conclusion

In this paper, we have proposed an FDR transmission scheme, referred to as FDR with BD, for MIMO relay channels. The proposed FDR scheme comprises a precoding scheme and a power allocation scheme. The precoding scheme has been developed based on the BD method to suppress the self-interference for the FDR operation as well as to support multiple data streams per MS. The power allocation scheme has been designed to maximise the sum rate under the given transmit power constraints of the BS and RS. Numerical results have demonstrated that the proposed FDR with BD provides significant performance enhancement compared with the FDR with ZFBF in MIMO relay systems.

7 Acknowledgment

This work was supported by the Korea Research Foundation Grant funded by the Korean Government [KRF-2008-357-D00177].

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9 Appendix 1

In this Appendix, we prove that the inverse matrix of the $3M \times 3M$ matrix with the form of

$$\begin{bmatrix} \boldsymbol{A}_{M\times 2M} & \boldsymbol{B}_{M\times M} \\ \boldsymbol{C}_{M\times 2M} & \boldsymbol{D}_{M\times M} \\ \boldsymbol{0}_{M\times 2M} & \boldsymbol{E}_{M\times M} \end{bmatrix} \tag{61}$$

takes the following form

$$\begin{bmatrix} \boldsymbol{Q}_{2M\times M} & \boldsymbol{R}_{2M\times M} & \boldsymbol{S}_{2M\times M} \\ \boldsymbol{0}_{M\times M} & \boldsymbol{0}_{M\times M} & \boldsymbol{T}_{M\times M} \end{bmatrix} \tag{62}$$

To prove the above argument, let us assume the following equality is true

$$\begin{bmatrix} \boldsymbol{\Gamma}_{2M\times 2M} & \boldsymbol{\Omega}_{2M\times M} \\ \boldsymbol{0}_{M\times 2M} & \boldsymbol{E}_{M\times M} \end{bmatrix} \begin{bmatrix} \boldsymbol{Q}_{2M\times M} & \boldsymbol{R}_{2M\times M} & \boldsymbol{S}_{2M\times M} \\ \boldsymbol{0}_{M\times M} & \boldsymbol{0}_{M\times M} & \boldsymbol{T}_{M\times M} \end{bmatrix} = \boldsymbol{I}_{3M} \tag{63}$$

where

$$\boldsymbol{\Gamma}_{2M\times 2M} = \begin{bmatrix} \boldsymbol{A}_{M\times 2M} \\ \boldsymbol{C}_{M\times 2M} \end{bmatrix} \qquad \boldsymbol{\Omega}_{2M\times M} = \begin{bmatrix} \boldsymbol{B}_{M\times M} \\ \boldsymbol{D}_{M\times M} \end{bmatrix} \tag{64}$$

and $\boldsymbol{I}_K$ denotes the $K \times K$ identity matrix. Then, we have

$$\boldsymbol{\Gamma}_{2M\times 2M}[\boldsymbol{Q}_{2M\times M}\boldsymbol{R}_{2M\times M}] = \boldsymbol{I}_{2M} \tag{65}$$

$$\boldsymbol{\Gamma}_{2M\times 2M}\boldsymbol{S}_{2M\times M} + \boldsymbol{\Omega}_{2M\times M}\boldsymbol{T}_{M\times M} = \boldsymbol{0}_{2M\times M} \tag{66}$$

$$\boldsymbol{E}_{M\times M}\boldsymbol{T}_{M\times M} = \boldsymbol{I}_M \tag{67}$$

From (65), it is seen that $\boldsymbol{Q}_{2M\times M}$ and $\boldsymbol{R}_{2M\times M}$ can be determined by the inverse of $\boldsymbol{\Gamma}_{2M\times 2M}$. Also, from (67), $\boldsymbol{T}_{M\times M} = \boldsymbol{E}^{-1}_{M\times M}$. Then, in (66), we can compute $\boldsymbol{S}_{2M\times M}$ as

$$\boldsymbol{S}_{2M\times M} = -\boldsymbol{\Gamma}^{-1}_{2M\times 2M}\boldsymbol{\Omega}_{2M\times M}\boldsymbol{E}^{-1}_{M\times M} \tag{68}$$

Therefore, it is obvious that the inverse of (61) takes the form of (62) and the ZFBF precoder for the channel matrix in (4) takes the form of (6).

10 Appendix 2

We derive $\boldsymbol{W}^{(\mathrm{RS})}_{\mathrm{BD}}$ and $\boldsymbol{W}^{(\mathrm{MS-2})}_{\mathrm{BD}}$ that constitue the BD precoding matrix in (25). In the same way of computing the BD precoder corresponding to the MS-1 in Section 3.2, we define

$$\tilde{\boldsymbol{H}}_{\mathrm{RS}} = \begin{bmatrix} \boldsymbol{H}^{\mathrm{BS}}_1 & \boldsymbol{H}^{\mathrm{RS}}_1 \\ \boldsymbol{0}_{M\times 2M} & \boldsymbol{H}^{\mathrm{RS}}_2 \end{bmatrix} \tag{69}$$

and compute the SVD of $\boldsymbol{H}^{\mathrm{BS}}_1$ as

$$\boldsymbol{H}^{\mathrm{BS}}_1 = \tilde{\boldsymbol{U}}_{\mathrm{RS}}\tilde{\boldsymbol{\Lambda}}_{\mathrm{RS}}[\tilde{\boldsymbol{V}}^{(1)}_{\mathrm{RS}}\tilde{\boldsymbol{V}}^{(0)}_{\mathrm{RS}}]^{\dagger} \tag{70}$$

where $\tilde{\boldsymbol{V}}^{(0)}_{\mathrm{RS}}$ spans the null space of $\boldsymbol{H}^{\mathrm{BS}}_1$. So, it can be observed that

$$\tilde{\boldsymbol{H}}_{\mathrm{RS}}\begin{bmatrix} \tilde{\boldsymbol{V}}^{(0)}_{\mathrm{RS}} \\ \boldsymbol{0}_{M\times M} \end{bmatrix} = \boldsymbol{0}_{2M\times M} \tag{71}$$

If we define

$$\hat{\boldsymbol{H}}_{\mathrm{RS}} = [\boldsymbol{H}^{\mathrm{BS}}_{\mathrm{RS}}\boldsymbol{H}^{\mathrm{RS}}_{\mathrm{RS}}]\begin{bmatrix} \tilde{\boldsymbol{V}}^{(0)}_{\mathrm{RS}} \\ \boldsymbol{0}_{M\times M} \end{bmatrix} \tag{72}$$

the SVD of $\hat{\boldsymbol{H}}_{\mathrm{RS}}$ yields

$$\hat{\boldsymbol{H}}_{\mathrm{RS}} = \hat{\boldsymbol{U}}_{\mathrm{RS}}\hat{\boldsymbol{\Lambda}}_{\mathrm{RS}}\hat{\boldsymbol{V}}^{\dagger}_{\mathrm{RS}} \tag{73}$$

where $\hat{\boldsymbol{\Lambda}}_{\mathrm{RS}} = \mathrm{diag}[\lambda^{(\mathrm{BS:RS})}_1, \ldots, \lambda^{(\mathrm{BS:RS})}_M]$. Therefore the precoding matrix $\boldsymbol{W}_{\mathrm{BD}}^{(\mathrm{RS})}$ can be determined as

$$\boldsymbol{W}_{\mathrm{BD}}^{(\mathrm{RS})} = \begin{bmatrix} \tilde{\boldsymbol{V}}_{\mathrm{RS}}^{(0)} \\ \boldsymbol{0}_{M\times M} \end{bmatrix} \hat{\boldsymbol{V}}_{\mathrm{RS}} \tag{74}$$

For the BD precoder corresponding to the MS-2, we also define

$$\tilde{\boldsymbol{H}}_2 = \begin{bmatrix} \boldsymbol{H}_{\mathrm{RS}}^{\mathrm{BS}} & \boldsymbol{H}_{\mathrm{RS}}^{\mathrm{RS}} \\ \boldsymbol{H}_1^{\mathrm{BS}} & \boldsymbol{H}_1^{\mathrm{RS}} \end{bmatrix} \tag{75}$$

where the SVD of $\tilde{\boldsymbol{H}}_2$ yields

$$\tilde{\boldsymbol{H}}_2 = \tilde{\boldsymbol{U}}_2 \tilde{\boldsymbol{\Lambda}}_2 [\tilde{\boldsymbol{V}}_2^{(1)} \tilde{\boldsymbol{V}}_2^{(0)}]^{\dagger} \tag{76}$$

Then, we obtain

$$\begin{aligned} \hat{\boldsymbol{H}}_2 &= [\boldsymbol{0}_{M\times 2M} \boldsymbol{H}_2^{\mathrm{RS}}] \tilde{\boldsymbol{V}}_2^{(0)} \\ &= \hat{\boldsymbol{U}}_2 \hat{\boldsymbol{\Lambda}}_2 \hat{\boldsymbol{V}}_2^{\dagger} \end{aligned} \tag{77}$$

where $\hat{\boldsymbol{\Lambda}}_2 = \mathrm{diag}[\lambda_1^{(\mathrm{BS:2})}, \ldots, \lambda_M^{(\mathrm{BS:2})}]$. The BD precoder corresponding to the MS-2 is found as $\boldsymbol{W}_{\mathrm{BD}}^{(\mathrm{MS-2})} = \tilde{\boldsymbol{V}}_2^{(0)} \hat{\boldsymbol{V}}_2$.

11 Appendix 3

We present how to compute $\eta^\star$ and $\mu^\star$ from (52), (59) and (60). Substituting (59) into (52) and (60), we have

$$\begin{aligned} \frac{2}{\eta} + \frac{1}{\eta + \beta_3\mu} + \frac{1}{\eta + \beta_4\mu} &= A \\ \frac{\beta_3}{\eta + \beta_3\mu} + \frac{\beta_4}{\eta + \beta_4\mu} &= B \end{aligned} \tag{78}$$

where

$$\begin{aligned} A &= \frac{P_{\mathrm{T}}^{\mathrm{BS}} + \sum_{k=1}^{4} (1/\alpha_k)}{\log_2 e} \\ B &= \frac{P_{\mathrm{T}}^{\mathrm{RS}} + (\beta_3/\alpha_3) + (\beta_4/\alpha_4)}{\log_2 e} \end{aligned} \tag{79}$$

After some manipulation in (78), we obtain

$$\begin{aligned} \eta + \beta_3\mu &= \frac{(\beta_4 - \beta_3)\eta}{(\beta_4 A - B)\eta - 2\beta_4} \\ \eta + \beta_4\mu &= \frac{(\beta_3 - \beta_4)\eta}{(\beta_3 A - B)\eta - 2\beta_3} \end{aligned} \tag{80}$$

which yield

$$\begin{aligned} \mu &= \frac{\eta}{\beta_3}\left(\frac{\beta_4 - \beta_3}{(\beta_4 A - B)\eta - 2\beta_4} - 1\right) \\ &= \frac{\eta}{\beta_4}\left(\frac{\beta_3 - \beta_4}{(\beta_3 A - B)\eta - 2\beta_3} - 1\right) \end{aligned} \tag{81}$$

After rearranging (81), we obtain a quadratic equation on η given as

$$\begin{aligned} &(\beta_3 A - B)(\beta_4 A - B)\eta^2 - 3(\beta_3(\beta_4 A - B) + \beta_4(\beta_3 A - B))\eta \\ &\quad + 8\beta_3\beta_4 = 0 \end{aligned} \tag{82}$$

Note that the solutions of (82) yield $\eta^\star$ and we can compute $\mu^\star$ by substituting $\eta^\star$ into (81).

IET Commun., 2010, Vol. 4, Iss. 15, pp. 1817–1826
doi: 10.1049/iet-com.2010.0056