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Physical-Layer Security for Full Duplex Communications with Self-interference Mitigation

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***Abstract*—In this paper, we design transmit beamforming for a full-duplex base station (FD-BS) considering both self-interference mitigation and physical-layer security. The proposed design is formulated as minimizing the power consumption of FD-BS under different signal-to-interference-and-noise-ratio (SINR) constraints. Semi-definite relaxation (SDR) is used to convert the initial non-convex optimization to be a convex semi-definite programming (SDP) problem. Then the optimality of SDR is strictly proved by showing the existence of the rank-one optimal solutions. To reduce the computational complexity, we develop zero forcing beamforming based suboptimal algorithms, where the solutions can be obtained using golden search and closed-form solutions can be derived in each step. Simulation results are then provided to verify the efficiency of the proposed algorithms.**

***Index Terms*—Full duplex (FD), self-interference mitigation, physical-layer security, semidefinite relaxation (SDR), beamforming.**

I. Introduction

Full duplex (FD) communication has attracted lots of attention because of its potential to double the spectrum efficiency and is currently a hot topic for 5G wireless communications, which require higher spectral utilization to support the quality of service (QoS) for a wide variety of multimedia applications [1]. In fact, FD has already been marked as a key technology under EURASIP METIS-2020 project and under China IMT-2020 project [1]. However, due to signal leakage during the transmission, FD systems suffer from strong self-interference which could be as high as 100 dB [2]. To make FD communication feasible, self-interference cancellation for FD system is crucial and has been studied in recent years. It was shown in [3] [4] that effective self-interference cancellation can be achieved by different approaches, e.g., the combination of antenna separation and digital cancellation, the combination of antenna separation and analog cancellation, and the combination of antenna separation, analog and digital cancellation, etc. In [5], the authors designed a new self-interference cancellation architecture using a single radio frequency (RF) and baseband (BB) chain, while [6] presented a full duplex radio design using signal inversion and adaptive cancellation. Moreover, [7] analyzed a wide range of self-interference mitigation schemes in multiple-input multiple-output (MIMO) systems, including natural isolation, time-domain cancellation and spatial domain suppression, etc. In addition, [8] applied digital beamforming system to cancel self-interference, which could achieve up to 50 dB additional isolation.

On the other side, physical layer security for wireless communication is also an important research area, whose basic idea is to exploit the physical characteristics of the wireless channel to transmit confidential messages. The information-theoretic approach to guarantee physical layer security was initiated by Wyner [9] and was later generalized to various channel models, e.g., the broadcast channels [10], the single-input single-output (SISO) fading channels [11], the multiple access channels (MAC) [12] and the MIMO channels [13]. An new way to enhance physical layer security was proposed as applying the simultaneous information and artificial noise (AN) transmission, where AN is sent from a multiple-antenna transmitter to degrade the eavesdropping channels [14]. Motivated by [14], a vast works studying the physical layer security have been developed based on AN. For example [15]–[17] required AN to spread in the null-space of the legitimate receiver's channel; [18]–[20] cooperatively generated AN from different users; [21]–[23] discussed the joint optimization of the covariances of both the confidential information and AN.

As a result, physical layer security for FD system is a promising research area that has attracted much attention recently. The authors in [24] and [25] adopted a full duplex transceiver to improve the physical layer security for a transmitter, and the authors in [26] designed the transmit beamforming to guarantee the physical layer security for an FD base station. An important assumption in [24]–[26] is that the residual self-interference can be rebuilt and eliminated by digital interference cancellation. However, the digital interference cancellation requires expensive circuits and two rounds of pilots [27] which significantly increases the system complexity in practice. In addition, the authors of [28] designed a self-protection mechanism at the full duplex transceiver to improve the secret receive rate, where the self-interference is mitigated by spatial beamforming. However,

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[28] assumes that the full duplex transceiver does not have its own information to transmit (only transmits AN), and there is only one passive Eavesdropper (Eve).

In this paper, we consider a more general scenario where the full duplex base station (FD-BS) simultaneously receives confidential information from a transmitter (Tx) and transmits confidential information to a legitimate receiver (Rx) under passive multiple Eves. Aiming to mitigate the self-interference and guarantee the physical layer security, we design joint information beamforming and AN beamforming to satisfy the intended signal-to-interference-plus-noise ratio (SINR) for different users. The contributions of proposed work are summarized as follows:

1) The residual self-interference is mitigated in the spatial domain, and thus, the extra expensive and complex digital interference cancellation circuits could be omitted.
2) FD-BS can simultaneously receive and transmit confidential information surrounded by multiple Eves, while both physical layer security and spectrum efficiency can be guaranteed.
3) Remarkably, the proposed design can be converted into an equivalent convex optimization, which can be efficiently solved using the standard CVX tools [30].
4) Efficient suboptimal algorithms are further proposed to reduce the computational complexity, where closed-form solutions are derived and the performance loss is small compared to the optimal algorithm.

The rest of this paper is organized as follows: Section II describes the system model and formulates the proposed problem; Section III proves the optimality of SDR by showing the existence of rank-one optimal solutions; A zero-forcing beamforming suboptimal solution for the original problem is derived in Section IV; Simulation results are provided in Section V and conclusions are drawn in Section VI.

Notation: Vectors and matrices are boldface small and capital letters, respectively; the Hermitian of $\boldsymbol{A}$ is denoted by $\boldsymbol{A}^{\mathrm{H}}$; $\mathrm{Tr}(\boldsymbol{A})$ defines the trace; $\boldsymbol{I}$ and $\boldsymbol{0}$ represent an identity matrix and an all-zero matrix, respectively, with appropriate dimensions; $\boldsymbol{A} \succeq \boldsymbol{0}$ and $\boldsymbol{A} \succ \boldsymbol{0}$ mean that $\boldsymbol{A}$ is positive semi-definite and positive definite, respectively; The unit-norm vector of a vector $\boldsymbol{x}$ is described as $\vec{\boldsymbol{x}} = \boldsymbol{x}/\|\boldsymbol{x}\|$; The distribution of a circularly symmetric complex Gaussian (CSCG) random variable with zero mean and variance σ^2 is defined as $\mathcal{CN}(0,\sigma^2)$, and $\sim$ means "distributed as"; $\mathbb{C}^{a\times b}$ denotes the space of $a \times b$ matrices with complex entries; $\|\boldsymbol{x}\|$ is the Euclidean norm of a vector $\boldsymbol{x}$; $\max(x, y)$ denotes the maximum between two real numbers, x and y.

II. System Model and Problem Formulation

A. System Model

An FD system consisted of an FD-BS, a legitimate transmitter (Tx), a legitimate receiver (Rx) and multiple passive Eves, is shown in Fig. 1. It is assumed that FD-BS is equipped with M receive antennas and N transmit antennas, while all the other nodes are equipped with a single antenna. The M receive antennas are defined with index set $\mathcal{K_M} = \{1,\ldots,M\}$. Moreover, there are L passive Eves with index set $\mathcal{K_L} = \{1,\ldots,L\}$. The baseband equivalent channels from Tx to FD-BS, Rx, and Eves are denoted as $\boldsymbol{h}_t \in \mathbb{C}^{M\times 1}$, $g_r \in \mathbb{C}^{1\times 1}$ and $g_{e,l} \in \mathbb{C}^{1\times 1}, \forall l \in \mathcal{K_L}$, respectively. The channels from FD-BS to Rx and Eves are denoted as $\boldsymbol{h}_r \in \mathbb{C}^{N\times 1}$ and $\boldsymbol{h}_{e,l} \in \mathbb{C}^{N\times 1}, \forall l \in \mathcal{K_L}$, respectively. From [7], [28], the residual self-interference channel can be denoted as $\sqrt{\rho}\boldsymbol{H}_b$, where $\boldsymbol{H}_b \in \mathbb{C}^{N\times M}$ is a fading loop channel and $0 \le \rho \le 1$ is used for parameterizing the effect of passive self-interference suppression. Specifically, $\boldsymbol{H}_b$ can be expressed as

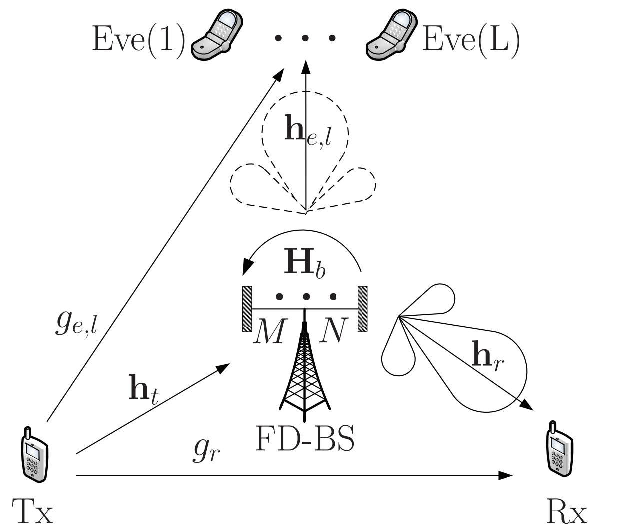

Fig. 1. Guarantee physical layer security for FD system under multiple passive Eves.

$$\boldsymbol{H}_b = [\boldsymbol{h}_{b,1},\ldots,\boldsymbol{h}_{b,M}], \quad (1)$$

where $\boldsymbol{h}_{b,m} \in \mathbb{C}^{N\times 1}$, $m = 1,\ldots,M$, is the channel from the FD-BS's transmit antennas to its mth receive antenna. It is assumed that the channel gains directly connected to the nodes are perfectly known [28], [29].

To provide the strongest distortion to Eves, we apply AN at FD-BS, and the baseband signal from FD-BS can be expressed as

$$\boldsymbol{x}_b = \boldsymbol{s}v_s + \boldsymbol{w}v_w, \quad (2)$$

where $v_s \sim \mathcal{CN}(0,1)$ is the information signal; $v_w \sim \mathcal{CN}(0,1)$ is the AN signal; $\boldsymbol{s} \in \mathbb{C}^{N\times 1}$ and $\boldsymbol{w} \sim \mathcal{CN}(\boldsymbol{0},\boldsymbol{W})$ are information beamforming vector and AN beamforming vector, respectively. Denote the transmit power of Tx as $P_t > 0$ and the baseband transmitted signal at Tx as $v_t \sim \mathcal{CN}(0,1)$, the received signal at FD-BS, Rx and Eves can be expressed respectively as

$$\boldsymbol{y}_b = \sqrt{P_t}\boldsymbol{h}_t v_t + \sqrt{\rho}\boldsymbol{H}_b^{\mathrm{H}}\boldsymbol{x}_b + \boldsymbol{z}_b, \quad (3)$$

$$y_r = \boldsymbol{h}_r^{\mathrm{H}}\boldsymbol{x}_b + \sqrt{P_t}g_r v_t + z_r, \quad (4)$$

$$y_{e,l} = \sqrt{P_t}g_{e,l}v_t + \boldsymbol{h}_{e,l}^{\mathrm{H}}\boldsymbol{x}_b + z_{e,l}, \quad (5)$$

where $\boldsymbol{z}_b \sim \mathcal{CN}(\boldsymbol{0},\boldsymbol{R}_b)$, $z_r \sim \mathcal{CN}(0,\sigma_r^2)$ and $z_{e,l} \sim \mathcal{CN}(0,\sigma_{e,l}^2)$ are the corresponding receive noises. It is assumed that Tx and Rx communicate with FD-BS, respectively, and the information signals v_t and v_s are independent. Here, Rx only needs to decode v_s while treats v_t as a noise.

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B. The SINR Measurement

From (3), the receive SINR of FD-BS can be computed as

$$\mathrm{SINR}_b(\boldsymbol{x}_b, \boldsymbol{q}) = \frac{P_t \|\boldsymbol{q}^{\mathrm{H}} \boldsymbol{h}_t\|^2}{\boldsymbol{q}^{\mathrm{H}}(\rho \boldsymbol{H}_b^{\mathrm{H}} \boldsymbol{x}_b \boldsymbol{x}_b^{\mathrm{H}} \boldsymbol{H}_b + \boldsymbol{R}_b)\boldsymbol{q}}, \tag{6}$$

where $\boldsymbol{q} \in \mathbb{C}^{M\times 1}$ is the receive beam vector of FD-BS and satisfies $\|\boldsymbol{q}\| = 1$. We assume that the maximal-ratio-combining (MRC) principle is used at FD-BS, and thus $\boldsymbol{q} = \boldsymbol{h}_t/\|\boldsymbol{h}_t\|$. Substituting (2) and $\boldsymbol{q}$ into (6) we obtain

$$\mathrm{SINR}_b(\boldsymbol{s}, \boldsymbol{W}) = \frac{P_t\|\boldsymbol{h}_t\|^4}{\Xi + \mathrm{Tr}(\boldsymbol{R}_b \boldsymbol{H}_t)}, \tag{7}$$

where $\boldsymbol{H}_t \triangleq \boldsymbol{h}_t \boldsymbol{h}_t^{\mathrm{H}}$, $\Xi = \boldsymbol{h}_t^{\mathrm{H}}(\rho \boldsymbol{H}_b^{\mathrm{H}} \boldsymbol{s}\boldsymbol{s}^{\mathrm{H}} \boldsymbol{H}_b)\boldsymbol{h}_t + \mathrm{Tr}\left((\rho \boldsymbol{H}_b \boldsymbol{h}_t \boldsymbol{h}_t^{\mathrm{H}} \boldsymbol{H}_b^{\mathrm{H}})\boldsymbol{W}\right)$. From (4), the receive SINR at Rx can be computed as

$$\mathrm{SINR}_r(\boldsymbol{s}, \boldsymbol{W}) = \frac{\|\boldsymbol{h}_r^{\mathrm{H}} \boldsymbol{s}\|^2}{\mathrm{Tr}(\boldsymbol{H}_r \boldsymbol{W}) + P_t\|g_r\|^2 + \sigma_r^2}, \tag{8}$$

where $\boldsymbol{H}_r \triangleq \boldsymbol{h}_r \boldsymbol{h}_r^{\mathrm{H}}$.

From (5), when Eves aim to decode the information transmitted from Tx, the SINRs at the $l \in \mathcal{K}_{\mathcal{L}}$ Eves can be formulated as [21]

$$\mathrm{SINR}_{e,b,l}(\boldsymbol{s}, \boldsymbol{W}) = \frac{P_t\|g_{e,l}\|^2}{\|\boldsymbol{h}_{e,l}^{\mathrm{H}} \boldsymbol{s}\|^2 + \mathrm{Tr}(\boldsymbol{H}_{e,l}\boldsymbol{W}) + \sigma_{e,l}^2}, \tag{9}$$

where $\boldsymbol{H}_{e,l} \triangleq \boldsymbol{h}_{e,l}\boldsymbol{h}_{e,l}^{\mathrm{H}}$; while when Eves aim to decode the information transmitted from FD-BS, the SINRs at the $l \in \mathcal{K}_{\mathcal{L}}$ Eves can be formulated as

$$\mathrm{SINR}_{e,r,l}(\boldsymbol{s}, \boldsymbol{W}) = \frac{\|\boldsymbol{h}_{e,l}^{\mathrm{H}} \boldsymbol{s}\|^2}{\mathrm{Tr}(\boldsymbol{H}_{e,l}\boldsymbol{W}) + P_t\|g_{e,l}\|^2 + \sigma_{e,l}^2}. \tag{10}$$

C. The Proposed Design

In this subsection, we design joint information and AN transmit beamforming in order to minimize the power consumption of FD-BS[1], at the same time provide FD-BS, Rx and Eves with different requirements of SINR. The optimization problem can be formulated as [21]

$$\begin{aligned}
\mathbf{P1}: \min_{\boldsymbol{s}, \boldsymbol{W}\succeq \mathbf{0}} \quad & \|\boldsymbol{s}\|^2 + \mathrm{Tr}(\boldsymbol{W}) && (11)\\
\text{s.t.} \quad & \mathrm{SINR}_b(\boldsymbol{s}, \boldsymbol{W}) \geq \gamma_b, && (12)\\
& \mathrm{SINR}_{e,b,l}(\boldsymbol{s}, \boldsymbol{W}) \leq \gamma_{e,b,l}, \quad \forall l \in \mathcal{K}_{\mathcal{L}}, && (13)\\
& \mathrm{SINR}_r(\boldsymbol{s}, \boldsymbol{W}) \geq \gamma_r, && (14)\\
& \mathrm{SINR}_{e,r,l}(\boldsymbol{s}, \boldsymbol{W}) \leq \gamma_{e,r,l}, \quad \forall l \in \mathcal{K}_{\mathcal{L}}, && (15)
\end{aligned}$$

where $\gamma_b > 0$ is the minimum receive SINR requirement for FD-BS; $\gamma_{e,b,l} > 0$ is the maximum allowable SINR threshold for the lth Eve to eavesdrop the information transmitted by Tx; $\gamma_r > 0$ is the minimum receive SINR requirement for Rx; $\gamma_{e,r,l} > 0$ is the maximum allowable SINR threshold for the lth Eve to eavesdrop the information transmitted by FD-BS.

[1]If cooperation between legal transmitters is available, we may also jointly minimize the power minimization of Tx and FD-BS as did in [31], [32].

In $\mathbf{P1}$, the SINR constraints can be reformulated as secret rate constraints. The secret rate for uplink and downlink transmission of FD-BS can be similarly expressed as [23]

$$\begin{aligned}
R_u = \max\Big(0, &\log_2(1 + \mathrm{SINR}_b(\boldsymbol{s}, \boldsymbol{W}))\\
&- \log_2\Big(1 + \max\Big\{\mathrm{SINR}_{e,b,l}(\boldsymbol{s}, \boldsymbol{W})\Big\}\Big)\Big), \quad (16)\\
R_d = \max\Big(0, &\log_2(1 + \mathrm{SINR}_r(\boldsymbol{s}, \boldsymbol{W}))\\
&- \log_2\Big(1 + \max\Big\{\mathrm{SINR}_{e,r,l}(\boldsymbol{s}, \boldsymbol{W})\Big\}\Big)\Big), \quad (17)
\end{aligned}$$

respectively. For given $\gamma_b > 0$, $\gamma_{e,b,l} > 0$ and $\gamma_b > \gamma_{e,b,l}, \forall l \in \mathcal{K}_{\mathcal{L}}$, we know from (12), (13) and (16) that

$$\begin{aligned}
R_u \geq &\log_2(1 + \mathrm{SINR}_b(\boldsymbol{s}, \boldsymbol{W}))\\
&- \log_2\Big(1 + \max\Big\{\mathrm{SINR}_{e,b,l}(\boldsymbol{s}, \boldsymbol{W})\Big\}\Big)\\
\geq &\log_2(1 + \gamma_b) - \log_2(1 + \max\{\gamma_{e,b,l}\}). \quad (18)
\end{aligned}$$

Similarly, from (14), (15) and (17), we obtain

$$R_d \geq \log_2(1 + \gamma_r) - \log_2(1 + \max\{\gamma_{e,r,l}\}). \tag{19}$$

Combing (18) and (19), we know that for given SINR limits, the minimum secret rates R_u and R_d can be guaranteed up a predefined bound by solving $\mathbf{P1}$.

III. Optimization of the Proposed Design

$\mathbf{P1}$ can be reformulated as

$$\begin{aligned}
\mathbf{P1}: \min_{\boldsymbol{s}, \boldsymbol{W}\succeq \mathbf{0}} \quad & \|\boldsymbol{s}\|^2 + \mathrm{Tr}(\boldsymbol{W}) && (20)\\
\text{s.t.} \quad & \boldsymbol{h}_t^{\mathrm{H}}(\rho \boldsymbol{H}_b^{\mathrm{H}} \boldsymbol{s}\boldsymbol{s}^{\mathrm{H}} \boldsymbol{H}_b)\boldsymbol{h}_t + \mathrm{Tr}\left((\rho \boldsymbol{H}_b \boldsymbol{H}_t \boldsymbol{H}_b^{\mathrm{H}})\boldsymbol{W}\right)\\
& \leq \frac{P_t\|\boldsymbol{h}_t\|^4}{\gamma_b} - \mathrm{Tr}(\boldsymbol{R}_b\boldsymbol{H}_t) && (21)\\
& \|\boldsymbol{h}_{e,l}^{\mathrm{H}}\boldsymbol{s}\|^2 + \mathrm{Tr}(\boldsymbol{H}_{e,l}\boldsymbol{W}) \geq \frac{P_t\|g_{e,l}\|^2}{\gamma_{e,b,l}} - \sigma_{e,l}^2, \ \forall l \in \mathcal{K}_{\mathcal{L}} && (22)\\
& \|\boldsymbol{h}_r^{\mathrm{H}}\boldsymbol{s}\|^2 - \gamma_r \mathrm{Tr}(\boldsymbol{H}_r\boldsymbol{W}) \geq \gamma_r(P_t\|g_r\|^2 + \sigma_r^2) && (23)\\
& \|\boldsymbol{h}_{e,l}^{\mathrm{H}}\boldsymbol{s}\|^2 - \gamma_{e,r,l}\mathrm{Tr}(\boldsymbol{H}_{e,l}\boldsymbol{W})\\
& \leq \gamma_{e,r,l}(P_t\|g_{e,l}\|^2 + \sigma_{e,l}^2), \quad \forall l \in \mathcal{K}_{\mathcal{L}}. && (24)
\end{aligned}$$

A. Optimization for $\mathbf{P1}$

Using the semidefinite relaxation (SDR) method [33], $\mathbf{P1}$ can be relaxed to a convex optimization problem:

$$\begin{aligned}
&\mathbf{P1{-}SDR}:\\
&\min_{\boldsymbol{S}, \boldsymbol{W}} \ \mathrm{Tr}(\boldsymbol{S}) + \mathrm{Tr}(\boldsymbol{W}) && (25)\\
\text{s.t. } & \mathrm{Tr}((\rho \boldsymbol{H}_b \boldsymbol{H}_t \boldsymbol{H}_b^{\mathrm{H}})\boldsymbol{S}) + \mathrm{Tr}((\rho \boldsymbol{H}_b \boldsymbol{H}_t \boldsymbol{H}_b^{\mathrm{H}})\boldsymbol{W})\\
& \leq \frac{P_t\|\boldsymbol{h}_t\|^4}{\gamma_b} - \mathrm{Tr}(\boldsymbol{R}_b\boldsymbol{H}_t) && (26)\\
& \mathrm{Tr}(\boldsymbol{H}_{e,l}\boldsymbol{S}) + \mathrm{Tr}(\boldsymbol{H}_{e,l}\boldsymbol{W})\\
& \geq \frac{P_t\|g_{e,l}\|^2}{\gamma_{e,b,l}} - \sigma_{e,l}^2, \quad \forall l \in \mathcal{K}_{\mathcal{L}} && (27)\\
& \mathrm{Tr}(\boldsymbol{H}_r\boldsymbol{S}) - \gamma_r\mathrm{Tr}(\boldsymbol{H}_r\boldsymbol{W}) \geq \gamma_r(P_t\|g_r\|^2 + \sigma_r^2) && (28)\\
& \mathrm{Tr}(\boldsymbol{H}_{e,l}\boldsymbol{S}) - \gamma_{e,r,l}\mathrm{Tr}(\boldsymbol{H}_{e,l}\boldsymbol{W})\\
& \leq \gamma_{e,r,l}(P_t\|g_{e,l}\|^2 + \sigma_{e,l}^2), \quad \forall l \in \mathcal{K}_{\mathcal{L}} && (29)\\
& \boldsymbol{S} \succeq \mathbf{0}, \quad \boldsymbol{W} \succeq \mathbf{0}, && (30)
\end{aligned}$$

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where the rank-one constraint $\mathrm{Rank}(\boldsymbol{S}) = 1$ is omitted for the time being. Note that SDR does not guarantee rank-one solutions [33]. Thus, the optimal solutions for $\mathbf{P1-SDR}$ are suboptimal for $\mathbf{P1}$ in general. Nevertheless, we next show the existence of rank-one solutions for $\mathbf{P1-SDR}$, which proves the optimality of SDR.

Consider the following problem:

$$\mathbf{P1-EQV}: \max_{\boldsymbol{S},\boldsymbol{W}} \frac{\mathrm{Tr}(\boldsymbol{H}_r\boldsymbol{S})}{\mathrm{Tr}(\boldsymbol{H}_r\boldsymbol{W}) + P_t\|g_r\|^2 + \sigma_r^2} \tag{31}$$

$$\text{s.t. } \mathrm{Tr}(\boldsymbol{S}) + \mathrm{Tr}(\boldsymbol{W}) \leq P_b \tag{32}$$

$$(26), (27), (29), (30), \tag{33}$$

where P_b is the optimal value of (25). It is not difficult to find that $\mathbf{P1-EQV}$ aims to maximize $\mathrm{SINR}_r(\boldsymbol{S}, \boldsymbol{W})$ subject to FD-BS transmit power constraints and other SINR constraints.

Lemma 1: The optimal solutions of $\mathbf{P1-EQV}$ are also optimal for $\mathbf{P1-SDR}$.

Proof: Denote $(\hat{\boldsymbol{S}}, \hat{\boldsymbol{W}})$ and $(\boldsymbol{S}^*, \boldsymbol{W}^*)$ as the optimal solutions of $\mathbf{P1-SDR}$ and $\mathbf{P1-EQV}$, respectively. Since $(\hat{\boldsymbol{S}}, \hat{\boldsymbol{W}})$ satisfies all the constraints in $\mathbf{P1-EQV}$, it follows that $(\hat{\boldsymbol{S}}, \hat{\boldsymbol{W}})$ is a feasible solution of $\mathbf{P1-EQV}$. Moreover, since $(\boldsymbol{S}^*, \boldsymbol{W}^*)$ maximizes (31), there must be

$$\begin{aligned} &\frac{\mathrm{Tr}(\boldsymbol{H}_r\boldsymbol{S}^*)}{\mathrm{Tr}(\boldsymbol{H}_r\boldsymbol{W}^*) + P_t\|g_r\|^2 + \sigma_r^2} \\ \geq &\frac{\mathrm{Tr}(\boldsymbol{H}_r\hat{\boldsymbol{S}})}{\mathrm{Tr}(\boldsymbol{H}_r\hat{\boldsymbol{W}}) + P_t\|g_r\|^2 + \sigma_r^2} \geq \gamma_r. \end{aligned} \tag{34}$$

From (34) and $\mathbf{P1-EQV}$, we know that $(\boldsymbol{S}^*, \boldsymbol{W}^*)$ is a feasible solution of $\mathbf{P1-SDR}$. Thus, we have

$$\mathrm{Tr}(\boldsymbol{S}^*) + \mathrm{Tr}(\boldsymbol{W}^*) \geq \mathrm{Tr}(\hat{\boldsymbol{S}}) + \mathrm{Tr}(\hat{\boldsymbol{W}}) = P_b. \tag{35}$$

Combing (32) with (35) yields

$$\mathrm{Tr}(\boldsymbol{S}^*) + \mathrm{Tr}(\boldsymbol{W}^*) = P_b. \tag{36}$$

Hence, $(\boldsymbol{S}^*, \boldsymbol{W}^*)$ is also the optimal solution of $\mathbf{P1-SDR}$. ■

Lemma 2: There exists an optimal $\boldsymbol{S}^*$ for $\mathbf{P1-EQV}$ that satisfies $\mathrm{Rank}(\boldsymbol{S}^*) = 1$.

Proof: See Appendix A. ■

Lemma 1 says that we can obtain the same optimal solutions by solving $\mathbf{P1-EQV}$, while Lemma 2 implies that $\mathbf{P1-EQV}$ guarantees the existence of rank-one solutions. Thus, combing Lemma 1 with Lemma 2, the existence of rank-one solutions for $\mathbf{P1-SDR}$ can be readily proved. Note that $\mathbf{P1-SDR}$ is convex and can be efficiently solved using the CVX tools. If the derived $\boldsymbol{S}^*$ has rank greater than 1, we can refer to the approach in Appendix A to obtain an equivalent rank-1 solution.

To summarize, the optimal solutions for $\mathbf{P1}$ can be obtained as follows:

1) Solve $\mathbf{P1-SDR}$ to obtain the optimal objective function value P_b^*.
2) Solve $\mathbf{P1-EQV}$ by using P_b^*.
3) Construct rank-1 optimal solutions using (A.30) in Appendix A.

It is seen from the above discussions that the proposed optimal algorithm needs to solve two SDP problems ($\mathbf{P1-SDR}$ and $\mathbf{P1-EQV}$) that contain $N^2 + 2N$ unknown complex variables.

B. Optimization When $\mathbf{P1}$ is Infeasible

From (26), we know that $\mathbf{P1}$ is infeasible when

$$\frac{P_t\|\boldsymbol{h}_t\|^4}{\gamma_b} - \mathrm{Tr}(\boldsymbol{R}_b\boldsymbol{H}_t) < 0, \tag{37}$$

or equivalently $\gamma_b > P_t\|\boldsymbol{h}_t\|^2/\mathrm{Tr}(\boldsymbol{R}_b\boldsymbol{H}_t)$. All the other constraints like (22), (23) and (24) can be satisfied by using enough power, i.e., $\|\boldsymbol{s}\|^2 + \mathrm{Tr}(\boldsymbol{W})$ is big enough. For this case, we enforce the self-interference term be

$$\mathrm{Tr}((\rho\boldsymbol{H}_b\boldsymbol{H}_t\boldsymbol{H}_b^{\mathrm{H}})\boldsymbol{S}) + \mathrm{Tr}((\rho\boldsymbol{H}_b\boldsymbol{H}_t\boldsymbol{H}_b^{\mathrm{H}})\boldsymbol{W}) = 0, \tag{38}$$

which can eliminate the infeasible case and guarantee that we can provide FD-BS with the maximum receive SINR as

$$\mathrm{SINR}_b(\boldsymbol{S}, \boldsymbol{W}) = \frac{P_t\|\boldsymbol{h}_t\|^4}{\mathrm{Tr}(\boldsymbol{R}_b\boldsymbol{H}_t)}, \tag{39}$$

while from (27) and (29), we can still provide non-zero AN to Eves by designing $\mathbf{W}$.

Then $\mathbf{P1-SDR}$ under the infeasible case can be reformulated as

$$\mathbf{P1-INF}:$$

$$\min_{\boldsymbol{S},\boldsymbol{W}} \mathrm{Tr}(\boldsymbol{S}) + \mathrm{Tr}(\boldsymbol{W}) \tag{40}$$

$$\text{s.t. } \mathrm{Tr}((\rho\boldsymbol{H}_b\boldsymbol{H}_t\boldsymbol{H}_b^{\mathrm{H}})\boldsymbol{S}) + \mathrm{Tr}((\rho\boldsymbol{H}_b\boldsymbol{H}_t\boldsymbol{H}_b^{\mathrm{H}})\boldsymbol{W}) = 0 \tag{41}$$

$$(27) \sim (30). \tag{42}$$

Lemma 3: There exists an optimal $\boldsymbol{S}^*$ for $\mathbf{P1-INF}$ that satisfy $\mathrm{Rank}(\boldsymbol{S}^*) = 1$.

Proof: The proof is similar to that of Lemma 2 and is omitted here for brevity. ■

Lemma 3 implies that SDR is still optimal for $\mathbf{P1-INF}$, which can also be efficiently solved by standard convex tools.

IV. Suboptimal Solutions

A suboptimal algorithm with zero-forcing based information beamforming and AN beamforming is proposed here to reduce the computational complexity for $\mathbf{P1}$.

A. Information Zero-forcing Beamforming

The information beamforming vector $\boldsymbol{s}$ can be reexpressed as $\boldsymbol{s} = \sqrt{P_{\boldsymbol{s}}}\vec{\boldsymbol{s}}$, where $P_{\boldsymbol{s}}$ is the power allocated for information beamforming and $\vec{\boldsymbol{s}}$ is the information beamforming direction vector that satisfies $\|\vec{\boldsymbol{s}}\|^2 = 1$.

Let us first enforce $\vec{\boldsymbol{s}}$ be in the null space of $\boldsymbol{H}_b^H$, i.e.,

$$\boldsymbol{H}_b^{\mathrm{H}}\vec{\boldsymbol{s}} = \mathbf{0}. \tag{43}$$

Then from (26) there must be $\mathrm{Tr}((\rho\boldsymbol{H}_b\boldsymbol{H}_t\boldsymbol{H}_b^{\mathrm{H}})\boldsymbol{S}) = 0$ which means that $\vec{\boldsymbol{s}}$ will not bring self-interference.

Secondly, enforce $\vec{\boldsymbol{s}}$ be as close to $\vec{\boldsymbol{h}}_r$ as possible, i.e.,

$$\vec{\boldsymbol{s}} = \frac{\boldsymbol{h}_r}{\|\boldsymbol{h}_r\|}, \tag{44}$$

namely, the information beamforming direction should point to Rx.

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Lastly, we will find an $\vec{\boldsymbol{s}}$ that satisfies (43) and (44) simultaneously. Let $\boldsymbol{V} \in \mathbb{C}^{N\times(N-M)}$ be the null space matrix of $\boldsymbol{H}_b^{\rm H}$ obtained from singular value decomposition (SVD). From (43), it is obvious that $\vec{\boldsymbol{s}}$ must be in the form of $\vec{\boldsymbol{s}} = \boldsymbol{V}\tilde{\boldsymbol{s}}$, where $\tilde{\boldsymbol{s}} \in \mathbb{C}^{(N-M)\times 1}$ is an arbitrary complex vector of unit norm. Then from (44), it can be shown that $\tilde{\boldsymbol{s}}$ should lie in the same direction as $\boldsymbol{V}^{\rm H}\boldsymbol{h}_r$, i.e.,

$$\tilde{\boldsymbol{s}} = \frac{\boldsymbol{V}^{\rm H}\boldsymbol{h}_r}{\|\boldsymbol{V}^{\rm H}\boldsymbol{h}_r\|}. \tag{45}$$

Consequently, $\vec{\boldsymbol{s}}$ can be derived as

$$\vec{\boldsymbol{s}} = \frac{\boldsymbol{V}\boldsymbol{V}^{\rm H}\boldsymbol{h}_r}{\|\boldsymbol{V}^{\rm H}\boldsymbol{h}_r\|}, \tag{46}$$

which is the information zero-forcing beamforming direction.

B. AN Zero-forcing Beamforming

To design AN zero-forcing beamforming vectors, the following lemma is needed.

Lemma 4: The optimal AN transmit covariance $\boldsymbol{W}^*$ for $\mathbf{P1}-\mathbf{SDR}$ must satisfy $\mathrm{Rank}(\boldsymbol{W}^*) \leq L$.

Proof: See Appendix B. ■

Using Lemma 4, we know that $\boldsymbol{W}$ can be expressed as

$$\boldsymbol{W} = \sum_{l=1}^{L}\boldsymbol{w}_l\boldsymbol{w}_l^{\rm H} = \sum_{l=1}^{L}P_{\boldsymbol{w},l}\vec{\boldsymbol{w}}_l\vec{\boldsymbol{w}}_l^{\rm H}, \tag{47}$$

where $\{\boldsymbol{w}_l, l\in\mathcal{K}_{\mathcal{L}}\}$ is the AN beamforming vector; $\{P_{\boldsymbol{w},l}, l \in \mathcal{K}_{\mathcal{L}}\}$ is the power used for the lth AN beamforming vector; $\{\vec{\boldsymbol{w}}_l, l\in\mathcal{K}_{\mathcal{L}}\}$ is the AN beamforming direction vector that satisfies $\|\vec{\boldsymbol{w}}_l\|^2 = 1, \forall l \in \mathcal{K}_{\mathcal{L}}$.

Firstly, enforce $\{\vec{\boldsymbol{w}}_l, l\in\mathcal{K}_{\mathcal{L}}\}$ be in the null space of $\boldsymbol{H} = [\boldsymbol{H}_b, \boldsymbol{h}_r]^{\rm H} \in \mathbb{C}^{(M+1)\times N}$, i.e.,

$$\boldsymbol{H}\vec{\boldsymbol{w}}_l = \mathbf{0},\ \forall l\in\mathcal{K}_{\mathcal{L}}, \tag{48}$$

which means that $\{\vec{\boldsymbol{w}}_l, l \in \mathcal{K}_{\mathcal{L}}\}$ will not affect FD-BS's receive SINR (cf. Eq. (12)) and Rx's SINR (cf. Eq. (14)).

Secondly, enforce $\{\vec{\boldsymbol{w}}_l, l \in \mathcal{K}_{\mathcal{L}}\}$ be as close to $\vec{\boldsymbol{h}}_{e,l}$ as possible, i.e.,

$$\vec{\boldsymbol{w}}_l = \frac{\boldsymbol{h}_{e,l}}{\|\boldsymbol{h}_{e,l}\|},\ \forall l\in\mathcal{K}_{\mathcal{L}} \tag{49}$$

namely, the lth AN beamforming direction should point to the lth Eve.

Lastly, we will find some $\{\vec{\boldsymbol{w}}_l, l\in\mathcal{K}_{\mathcal{L}}\}$ that satisfy (48) and (49) simultaneously. Let $\boldsymbol{V}' \in \mathbb{C}^{N\times(N-M-1)}$ be the null space basis matrix of $\boldsymbol{H} = [\boldsymbol{H}_b, \boldsymbol{h}_r]^{\rm H}$ obtained from SVD. From (48) it obvious that $\vec{\boldsymbol{w}}_l$ must be in the form of $\vec{\boldsymbol{w}}_l = \boldsymbol{V}_2'\tilde{\boldsymbol{w}}_l$, where $\tilde{\boldsymbol{w}}_l \in \mathbb{C}^{(N-M-1)\times 1}$ is an arbitrary complex vector of unit norm. Then from (49), it can be shown that $\tilde{\boldsymbol{w}}_l$ should lie in the same direction as $\boldsymbol{V}'^{\rm H}\boldsymbol{h}_{e,l}$, i.e.,

$$\tilde{\boldsymbol{w}}_l^* = \frac{\boldsymbol{V}'^{\rm H}\boldsymbol{h}_{e,l}}{\|\boldsymbol{V}'^{\rm H}\boldsymbol{h}_{e,l}\|}. \tag{50}$$

Consequently, $\vec{\boldsymbol{w}}_l$ can be derived as

$$\vec{\boldsymbol{w}}_l^* = \frac{\boldsymbol{V}'\boldsymbol{V}'^{\rm H}\boldsymbol{h}_{e,l}}{\|\boldsymbol{V}'^{\rm H}\boldsymbol{h}_{e,l}\|}, \tag{51}$$

which is the lth AN zero-forcing beamforming direction.

C. Joint Information and AN Zero-forcing Beamforming

Substituing (46) and (51) into $\mathbf{P1}-\mathbf{SDR}$, we obtain the following optimization

$\mathbf{P1}-\mathbf{SUB}$:

$$\min_{P_{\boldsymbol{s}},\{P_{\boldsymbol{w},l}\}} \quad P_{\boldsymbol{s}} + \sum_{l=1}^{L}P_{\boldsymbol{w},l} \tag{52}$$

$$\text{s.t.}\quad P_{\boldsymbol{s}}\|\boldsymbol{h}_{e,l}^{\rm H}\vec{\boldsymbol{s}}\|^2 + P_{\boldsymbol{w},l}\|\boldsymbol{h}_{e,l}^{\rm H}\vec{\boldsymbol{w}}_l\|^2 \geq \frac{P_t\|g_{e,l}\|^2}{\gamma_{e,b,l}} - \sigma_{e,l}^2,\quad \forall l\in\mathcal{K}_{\mathcal{L}}, \tag{53}$$

$$P_{\boldsymbol{s}}\|\boldsymbol{h}_r^{\rm H}\vec{\boldsymbol{s}}\|^2 \geq \gamma_r(P_t\|g_r\|^2 + \sigma_r^2), \tag{54}$$

$$P_{\boldsymbol{s}}\|\boldsymbol{h}_{e,l}^{\rm H}\vec{\boldsymbol{s}}\|^2 - \gamma_{e,r,l}P_{\boldsymbol{w},l}\|\boldsymbol{h}_{e,l}^{\rm H}\vec{\boldsymbol{w}}_l\|^2 \leq \gamma_{e,r,l}(P_t\|g_{e,l}\|^2 + \sigma_{e,l}^2),\ \forall l\in\mathcal{K}_{\mathcal{L}}. \tag{55}$$

It can be shown that $\mathbf{P1}-\mathbf{SUB}$ is convex with respect to $P_{\boldsymbol{s}}$ and $\{P_{\boldsymbol{w},l}\}$. Then for $\mathbf{P1}-\mathbf{SUB}$, the optimal $P_{\boldsymbol{s}}^*$ and $\{P_{\boldsymbol{w},l}^*\}$ can be obtained using golden section search over $P_{\boldsymbol{s}}$. From (46) and (54), we know that the following search range

$$P_{\boldsymbol{s}} \in \left[\frac{\gamma_r(P_t\|g_r\|^2+\sigma_r^2)}{\|\boldsymbol{V}_2^{\rm H}\boldsymbol{h}_r\|^2},\ \infty\right) \tag{56}$$

should be satisfied. Note that in each searching step, from (53) and (55), $\{P_{\boldsymbol{w},l}^*\}$ can be derived as

$$P_{\boldsymbol{w},l}^* = \max\left(P_{l,1}, P_{l,2}\right), \tag{57}$$

where

$$P_{l,1} = \frac{\frac{P_t\|g_{e,l}\|^2}{\gamma_{e,b,l}} - \sigma_{e,l}^2 - P_{\boldsymbol{s}}\|\boldsymbol{h}_{e,l}^{\rm H}\vec{\boldsymbol{s}}^*\|^2}{\|\boldsymbol{V}_2'^{\rm H}\boldsymbol{h}_{e,l}\|^2}, \tag{58}$$

$$P_{l,2} = \frac{P_{\boldsymbol{s}}\|\boldsymbol{h}_{e,l}^{\rm H}\vec{\boldsymbol{s}}^*\|^2 - \gamma_{e,r,l}(P_t\|g_{e,r,l}\|^2 + \sigma_{e,r,l}^2)}{\gamma_{e,r,l}\|\boldsymbol{V}_2'^{\rm H}\boldsymbol{h}_{e,l}\|^2}. \tag{59}$$

Then the AN beamforming vector can be derived as $\boldsymbol{w}_l^* = \sqrt{P_{\boldsymbol{w},l}^*}\vec{\boldsymbol{w}}_l^*$. From (52), the overall power consumption at FD-BS is, thus, derived as

$$P_b^* = P_{\boldsymbol{s}}^* + \sum_{l=1}^{L}P_{\boldsymbol{w},l}^*. \tag{60}$$

In summary, the golden section search algorithm for $\mathbf{P1}-\mathbf{SUB}$ with a target accuracy parameter ϵ is listed as follows:

1) Initialization: $P_{\boldsymbol{s}}^{\min} = \frac{\gamma_r(P_t\|g_r\|^2+\sigma_r^2)}{\|\boldsymbol{V}_2^{\rm H}\boldsymbol{h}_r\|^2}$; $P_{\boldsymbol{s}}^{\max} = \bar{P}$ where $\bar{P}$ is a large constant value; $P_s = P_{\boldsymbol{w},l} = \max(P_{l,1}, P_{l,2})$ can be derived using (58) and (59); $P_b' = P_{\boldsymbol{s}} + \sum_{l=1}^{L}P_{\boldsymbol{w},l}$.
2) Repeat
   - $P_{\boldsymbol{s}} \leftarrow \frac{1}{2}(P_{\boldsymbol{s}}^{\min} + P_{\boldsymbol{s}}^{\max})$.
   - $P_b = P_{\boldsymbol{s}} + \sum_{l=1}^{L}P_{\boldsymbol{w},l}$ where $P_{\boldsymbol{w},l} = \max(P_{l,1}, P_{l,2})$. If $\mathbf{P1}-\mathbf{SUB}$ is feasible and $P_b \leq P_b'$, let $P_{\boldsymbol{s}}^{\max} \leftarrow P_{\boldsymbol{s}}$ and $P_b' \leftarrow P_b$; otherwise, let $P_{\boldsymbol{s}}^{\min} \leftarrow P_{\boldsymbol{s}}$.

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- Stop when $P_{\boldsymbol{s}}^{\max} - P_{\boldsymbol{s}}^{\min} \le \epsilon$

3) The optimal objective value of $\mathbf{P1}-\mathbf{SUB}$ is taken as P_b.

Note that there is only one unknown variable in the algorithm of $\mathbf{P1}-\mathbf{SUB}$ where the efficient golden section search algorithm can be used. Thus, we can achieve a remarkable complexity reduction compared to $\mathbf{P1}$.

V. Simulation Results

In this section, computer simulations are presented to evaluate the performance of the proposed optimal and suboptimal algorithms. The FD-BS is equipped with $M = 2$ receive antennas and $N = 4$ transmit antennas. The entries of the channel vectors $\boldsymbol{h}_t$, $\boldsymbol{h}_r$, g_r $\boldsymbol{h}_{e,l}$, $\forall l \in \mathcal{K}_{\mathcal{L}}$, $g_{e,l}$, $\forall l \in \mathcal{K}_{\mathcal{L}}$ and $\boldsymbol{h}_{b,m}$, $\forall m \in \mathcal{K}_{\mathcal{M}}$ are all generated as independent CSCG random variables distributed with $\mathcal{CN}(0,1)$. The received noises per antenna for all receivers are generated as $\mathcal{CN}(0,1)$ [35]. The parameter ρ is set as $\rho = 1$. The Tx transmit power P_t sweeps from 10^0 mW to 10^5 mW, which is equivalent to the range from 0 dBm to 50 dBm. The minimum SINR requirements γ_b and γ_r vary from $10^{0.5}$ to $10^{2.5}$, which is equivalent to the range from 5 dB to 25 dB. The simulation results are derived by averaging over 10000 simulation trails. Denote the infeasible probability as

$$p_{in} = \text{Prob}\left\{ \gamma_b > \frac{P_t \|\boldsymbol{h}_t\|^4}{\text{Tr}(\boldsymbol{R}_b \boldsymbol{H}_t)} \right\}. \tag{61}$$

The average p_{in} are plotted with dashed line in the simulation results.

A. Simulations for Physical Layer Security and Self-interference Mitigation

We consider two cases with single Eve and 3 Eves, respectively.

We first show the average FD-BS transmit power P_b versus the Tx transmit power P_t in Fig. 2, for $\gamma_{e,b,l} = \gamma_{e,r,l} = 3$ dB and $\gamma_b = \gamma_r = 15$ dB. It is seen that p_{in} monotonically decreases with P_t increasing. When $P_t = 0$ dBm, p_{in} approaches 0.9, which means that $\mathbf{P1}$ is almost infeasible. When $P_t > 20$ dBm, we know $\mathbf{P1}$ is almost feasible since p_{in} approaches zero. Note that for both the optimal and suboptimal algorithms, with decreasing number of Eves we can obtain decreasing FD-BS power consumptions. The reason lies in the fact that with decreasing number of Eves, the number of SINR constraints in (22) and (24) will decrease (i.e., Eq.(22) and Eq.(24) can be easier satisfied). Also note that with decreasing number of Eves, the gap between the optimal and suboptimal solutions becomes small. This is mainly due to the fact that the suboptimal algorithm for $\mathbf{P1}$ is obtained by separate AN beamforming for each Eve (cf. Eq.(47) and Eq.(49)). Thus, with decreasing number of Eves, the FD-BS can save more power.

The average P_b versus γ_b are demonstrated in Fig. 3 for $P_t = 50$ dBm, $\gamma_{e,b,l} = \gamma_{e,r,l} = 3$ dB and $\gamma_r = 15$ dB. We see that P_b obtained by the optimal algorithm for both single Eve and 3 Eves monotonically increases with the increase of γ_b.

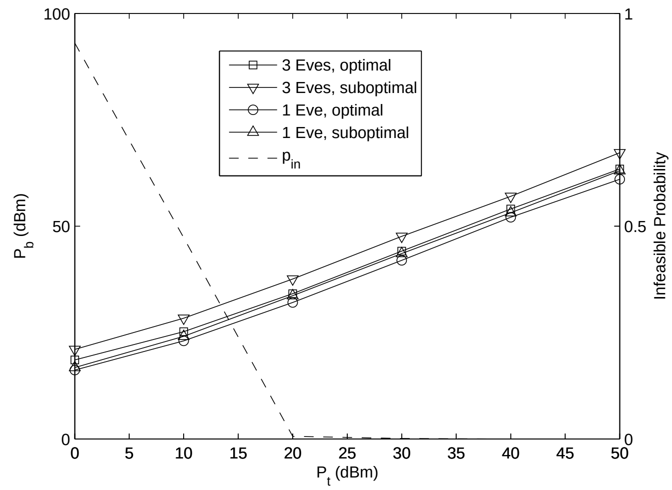

Fig. 2. FD-BS transmit power versus P_t with $\gamma_{e,b,l} = \gamma_{e,r,l} = 3$ dB and $\gamma_b = \gamma_r = 15$ dB.

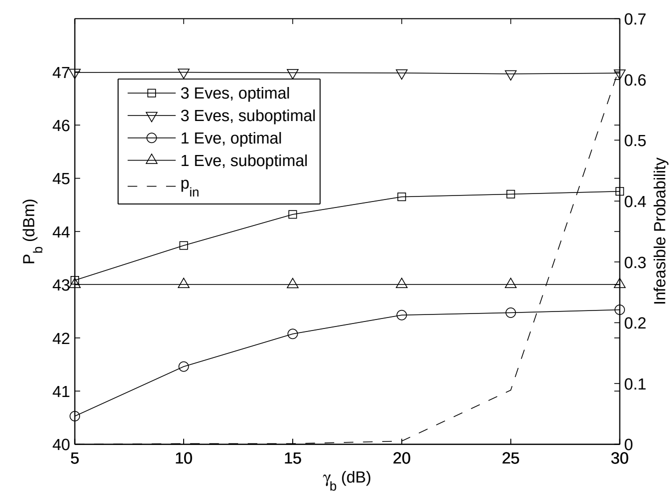

Fig. 3. FD-BS transmit power versus γ_b with $P_t = 50$ dBm, $\gamma_{e,b,l} = \gamma_{e,r,l} = 3$ dB and $\gamma_r = 15$ dB.

Note that such increase of P_b is small, which says the effect of increasing γ_b on P_b is mild. While P_b obtained by the suboptimal algorithm for both 1 Eve and 3 Eves is approximately constant. In fact, the solutions of the suboptimal algorithm are derived without the influence of γ_b (cf. Eq.(57$\sim$60)). It can be seen from Fig. 3 that p_{in} monotonically increases with the increase of γ_b and approaches 0.6 when $\gamma_b = 30$ dB.

In the last example, we plot P_b versus γ_r in Fig. 4 for $P_t = 50$ dBm, $\gamma_{e,b,l} = \gamma_{e,r,l} = 3$ dB and $\gamma_b = 15$ dB. It is clear that p_{in} is approaches 0.8, which means $\mathbf{P1}$ is almost feasible. Note that P_b obtained from both the optimal and suboptimal algorithms monotonically increases with the increase of γ_r. With 3 Eves, the gap between the optimal and suboptimal solutions approaches 3 dBm. While with 1 Eve, such gap approaches 1 dBm, which demonstrates the efficiency of the proposed suboptimal algorithm for $\mathbf{P1}$.

Indicated from Fig. 2, Fig. 3 and Fig. 4, we know that

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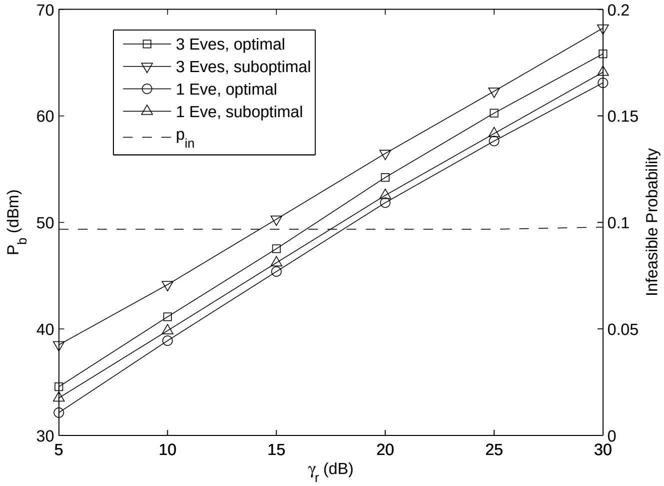

Fig. 4. FD-BS transmit power versus γ_r with $P_t = 50$ dBm, $\gamma_{e,b,l} = \gamma_{e,r,l} = 3$ dB and $\gamma_b = 15$ dB.

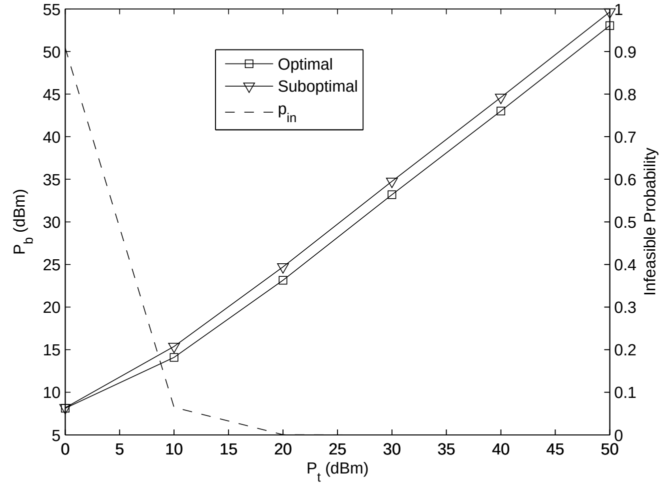

Fig. 5. FD-BS transmit power versus P_t with $\gamma_b = 9$ dB and $\gamma_r = 9$ dB.

with the increase number of antenna or the decrease number of eavesdropper, both optimal and suboptimal algorithms will have more degrees of freedom to design beamforming vectors. Consequently, both optimal and suboptimal algorithms will obtain a decreased power consumption.

B. Self-interference Cancellation

The proposed optimal and suboptimal algorithms can also be easily used for self-interference cancellation in full-duplex systems[2]. For the special case with only self-interference mitigation, we assume that there is no Eve. Then the SINR constraints (13) and (15) can be omitted in $\mathbf{P1}$. The proposed design of self-interference mitigation that aims to minimize the transmit power and provide both FD-BS and Rx with different SINR, which has not been examined to the best of our knowledge. The effectiveness of the proposed algorithms for self-interference mitigation is justified in the following simulations.

The average FD-BS transmit powers P_b versus the Tx transmit power P_t are shown in Fig. 5, for $\gamma_b = 9$ dB and $\gamma_r = 9$ dB. It is seen that both the power consumptions obtained by the proposed optimal and suboptimal algorithms are monotonically increasing functions of P_t. While the infeasible probability p_{in} which is plotted with dashed line is a monotonically decreasing function of P_t. Note that the suboptimal algorithm can obtain a similar P_b as that obtained by the optimal algorithm when P_t is small, i.e., $P_t = 0$ dBm. The reason lies in the fact that with small P_t, $\mathbf{P1}$ is almost infeasible. Moreover, we see that p_{in} approaches 0.9 when $P_t = 0$ dBm. The increase of P_b is slow when $0 < P_t < 10$ dBm and becomes fast when $P_t > 10$ dBm. This is mainly due to the fact that when $0 < P_t < 10$ dBm, p_{in} decreases fast and approaches 0.1 when $P_t = 10$ dBm. On the one hand, with the fast decrease of p_{in}, FD-BS can have more degrees of freedom for beamforming (i.e., the beamforming direction can be closer to the self-interference channel, cf. Eq.(21)). Thus, P_b can be decreased appropriately. However, with the increasing of P_t, more power is required for beamforming to satisfy γ_r (cf. Eq.(23)), and P_b will increase appropriately. Thus, P_b increases slowly with P_t when $0 < P_t < 10$ dBm. When $P_t > 10$ dBm, it is seen that p_{in} approaches zero, and P_b increases fast with P_t. Note that the gap between the optimal and suboptimal solutions is small and approaches 1.5 dBm, which demonstrates the efficiency of the proposed suboptimal algorithm.

[2]The self-interference cancellation problem can be efficiently solved using the proposed algorithms. The details are omitted here for brief.

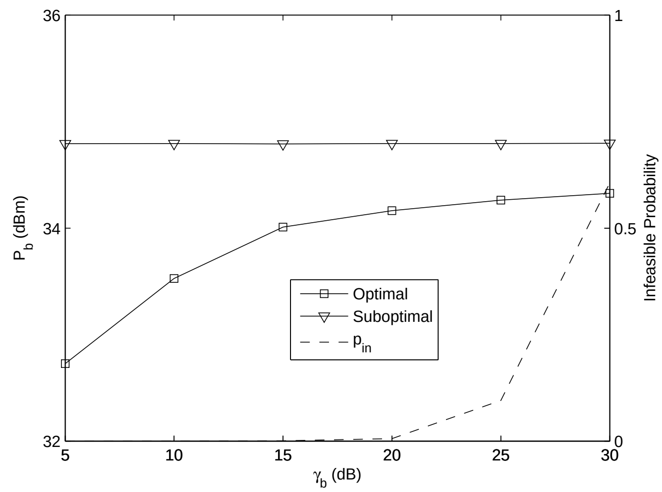

Fig. 6. FD-BS transmit power versus γ_b with $P_t = 30$ dBm and $\gamma_r = 9$ dB.

Another example is shown in Fig. 6, where P_b versus γ_b are plotted with $P_t = 30$ dBm and $\gamma_r = 9$ dB. It is seen that p_{in} monotonically increases with γ_b. For $\gamma_b \le 20$ dB, p_{in} approaches zero. While for $\gamma_b > 20$ dB, p_{in} increases fast and approaches 0.6 when $\gamma_b = 30$ dB. It is clear that P_b obtained by the optimal algorithm monotonically increases with γ_b. In fact, with the increase of γ_b, the degrees of freedom

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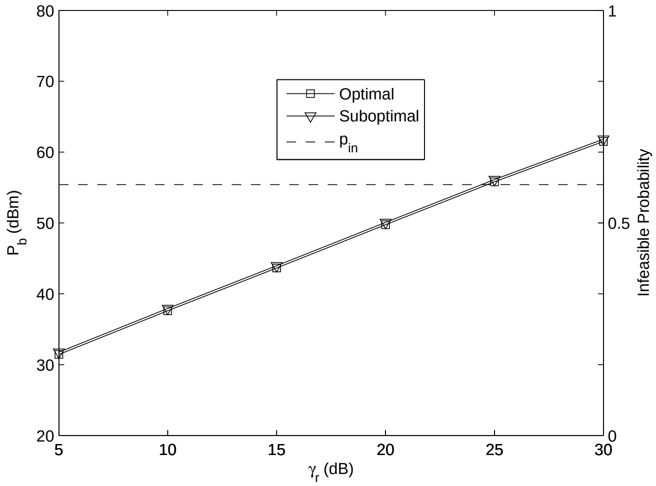

Fig. 7. FD-BS transmit power versus γ_r with $P_t = 30$ dBm and $\gamma_b = 30$ dB.

for beamforming will decrease, i.e., the beamforming direction must be kept away from the self-interference channel to satisfy γ_b. Thus, for constant P_t and γ_r, P_b will increase with γ_b. Note that the increase of optimal P_b is slow, which varies from 32.8 dBm to 34.2 dBm. This phenomenon says the change of γ_b or equivalently p_{in} has small effect on P_b, which further explains the results in Fig. 5 that P_b will increase slowly with P_t when $0 < P_t < 10$ dBm. Also note that P_b obtained by the suboptimal algorithm is almost constant and approaches 34.8 dBm. The reason is that in the suboptimal algorithm, P_b is derived without the influence of γ_b (cf. Eq.(57∼60)).

In the last example, we plot P_b versus γ_r in Fig. 7 for $P_t = 30$ dBm and $\gamma_b = 30$ dB. Obviously, both the FD-BS power consumptions from the optimal and suboptimal algorithms monotonically increase with γ_r. This is mainly due to the fact that with the increase of γ_r, the FD-BS will use more power for beamforming to satisfy γ_r. From (61), we know that p_{in} does not change with γ_r. Thus, we obtain a constant p_{in} as shown in Fig. 7. Note that we obtain a similar P_b from both the optimal and suboptimal algorithms, which again demonstrates the efficiency of the proposed suboptimal algorithm.

VI. Conclusions

In this paper, we proposed a new transmission strategy for an FD-BS that could guarantee both self-interference mitigation and physical layer security. Joint information beamforming and AN beamforming is designed to eliminate the self-interference and guarantee both the transmit and receive security for the FD-BS. Specifically, we formulate the problem as minimizing the FD-BS transmit power under the SINR constraints for legitimate users and illegal Eves. For the initial design, we proved that SDR could serve as the optimal strategy by showing the existence of rank-one optimal solutions. Suboptimal algorithms are also proposed based on zero-forcing beamforming, where closed-form solutions are derived in each golden search step. Simulation results are provided to corroborate the proposed studies.

Appendix A
Proof of Lemma 2

Let us first change the variables of $\mathbf{P1}-\mathbf{EQV}$ as

$$\mathbf{\Phi}_{\boldsymbol{s}} = \varphi \boldsymbol{S}, \quad \mathbf{\Phi}_{\boldsymbol{w}} = \varphi \boldsymbol{W}, \quad \varphi > 0. \tag{A.1}$$

From the Charnes-Cooper transformation [30], $\mathbf{P1}-\mathbf{EQV}$ can be equivalently expressed as

$$\begin{aligned}
&\mathbf{P1}-\mathbf{EQV}-\mathbf{A}: \\
&\max_{\mathbf{\Phi}_{\boldsymbol{s}},\mathbf{\Phi}_{\boldsymbol{w}},\varphi} \ \mathrm{Tr}(\boldsymbol{H}_r\mathbf{\Phi}_{\boldsymbol{s}}) && \text{(A.2)}\\
&\text{s.t. } \mathrm{Tr}(\boldsymbol{H}_r\mathbf{\Phi}_{\boldsymbol{w}}) + \varphi(P_t\|g_r\|^2 + \sigma_r^2) = 1 && \text{(A.3)}\\
&\quad \mathrm{Tr}(\mathbf{\Phi}_{\boldsymbol{s}}) + \mathrm{Tr}(\mathbf{\Phi}_{\boldsymbol{w}}) \le \varphi P_b && \text{(A.4)}\\
&\quad \mathrm{Tr}((\rho\boldsymbol{H}_b\boldsymbol{H}_t\boldsymbol{H}_b^{\mathrm{H}})\mathbf{\Phi}_{\boldsymbol{s}}) + \mathrm{Tr}((\rho\boldsymbol{H}_b\boldsymbol{H}_t\boldsymbol{H}_b^{\mathrm{H}})\mathbf{\Phi}_{\boldsymbol{w}}) \\
&\qquad \le \varphi\left(\frac{P_t\|\boldsymbol{h}_t\|^4}{\gamma_b} - \mathrm{Tr}(\boldsymbol{R}_b\boldsymbol{H}_t)\right) && \text{(A.5)}
\end{aligned}$$

$$\begin{aligned}
&\mathrm{Tr}(\boldsymbol{H}_{e,l}\mathbf{\Phi}_{\boldsymbol{s}}) + \mathrm{Tr}(\boldsymbol{H}_{e,l}\mathbf{\Phi}_{\boldsymbol{w}}) \\
&\qquad \ge \varphi\left(\frac{P_t\|g_{e,l}\|^2}{\gamma_{e,b,l}} - \sigma_{e,l}^2\right), \quad \forall l \in \mathcal{K}_{\mathcal{L}} && \text{(A.6)}\\
&\mathrm{Tr}(\boldsymbol{H}_{e,l}\mathbf{\Phi}_{\boldsymbol{s}}) - \gamma_{e,r,l}\mathrm{Tr}(\boldsymbol{H}_{e,l}\mathbf{\Phi}_{\boldsymbol{w}}) \\
&\qquad \le \varphi\gamma_{e,r,l}(P_t\|g_{e,l}\|^2 + \sigma_{e,l}^2), \quad \forall l \in \mathcal{K}_{\mathcal{L}} && \text{(A.7)}\\
&\mathbf{\Phi}_{\boldsymbol{s}} \succeq \mathbf{0}, \quad \mathbf{\Phi}_{\boldsymbol{w}} \succeq \mathbf{0}, \quad \varphi > 0. && \text{(A.8)}
\end{aligned}$$

Note that $\mathbf{P1}-\mathbf{EQV}-\mathbf{A}$ is convex and the Lagrange function is defined as

$$\begin{aligned}
&\mathcal{L}(\mathbf{\Phi}_{\boldsymbol{s}}, \mathbf{\Phi}_{\boldsymbol{w}}, \varphi, \alpha, \beta, \lambda, \{\mu_l\}, \{\nu_l\}) \\
&\quad = -\mathrm{Tr}(\mathbf{\Sigma}_1\mathbf{\Phi}_{\boldsymbol{s}}) - \mathrm{Tr}(\mathbf{\Sigma}_2\mathbf{\Phi}_{\boldsymbol{w}}) - \Sigma_3\varphi + \alpha, && \text{(A.9)}
\end{aligned}$$

where α, $\beta \ge 0$, $\lambda \ge 0$, $\{\mu_l \ge 0\}$ and $\{\nu_l \ge 0\}$ denote the dual variables of $\mathbf{P1}-\mathbf{EQV}-\mathbf{A}$ associated with the constraints in (A.2) to (A.7), respectively;

$$\begin{aligned}
\mathbf{\Sigma}_1 &= -\boldsymbol{H}_r + \beta\boldsymbol{I} + \lambda\rho\boldsymbol{H}_b\boldsymbol{H}_t\boldsymbol{H}_b^{\mathrm{H}} \\
&\quad - \sum_{l\in\mathcal{K}_{\mathcal{L}}} \mu_l\boldsymbol{H}_{e,l} + \sum_{l\in\mathcal{K}_{\mathcal{L}}} \nu_l\boldsymbol{H}_{e,l} && \text{(A.10)}\\
\mathbf{\Sigma}_2 &= \alpha\boldsymbol{H}_r + \beta\boldsymbol{I} + \lambda\rho\boldsymbol{H}_b\boldsymbol{H}_t\boldsymbol{H}_b^{\mathrm{H}} \\
&\quad - \sum_{l\in\mathcal{K}_{\mathcal{L}}} \mu_l\boldsymbol{H}_{e,l} - \sum_{l\in\mathcal{K}_{\mathcal{L}}} \nu_l\gamma_{e,r,l}\boldsymbol{H}_{e,l} && \text{(A.11)}\\
\Sigma_3 &= \alpha(P_t\|g_r\|^2 + \sigma_r^2) - \beta P_b \\
&\quad - \lambda\left(\frac{P_t\|\boldsymbol{h}_t\|^4}{\gamma_b} - \mathrm{Tr}(\boldsymbol{R}_b\boldsymbol{H}_t)\right) \\
&\quad + \sum_{l\in\mathcal{K}_{\mathcal{L}}} \mu_l\left(\frac{P_t\|g_{e,l}\|^2}{\gamma_{e,b,l}} - \sigma_{e,l}^2\right) \\
&\quad - \sum_{l\in\mathcal{K}_{\mathcal{L}}} \nu_l\gamma_{e,r,l}(P_t\|g_{e,l}\|^2 + \sigma_{e,l}^2). && \text{(A.12)}
\end{aligned}$$

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Then the Karush-Kuhn-Tucker (KKT) conditions [30] that are directly related to $\boldsymbol{\Phi}_{\boldsymbol{s}}^*$ and $\boldsymbol{\Phi}_{\boldsymbol{w}}^*$ can be expressed as

$$\begin{aligned} -\boldsymbol{H}_r + \beta^*\boldsymbol{I} + \lambda^*\rho\boldsymbol{H}_b\boldsymbol{H}_t\boldsymbol{H}_b^{\mathrm{H}} \\ - \sum_{l\in\mathcal{K}_{\mathcal{L}}}\mu_l^*\boldsymbol{H}_{e,l} + \sum_{l\in\mathcal{K}_{\mathcal{L}}}\nu_l^*\boldsymbol{H}_{e,l} =& \boldsymbol{\Sigma}_1^* \succeq \boldsymbol{0}, \quad \text{(A.13)} \\ \alpha^*\boldsymbol{H}_r + \beta^*\boldsymbol{I} + \lambda^*\rho\boldsymbol{H}_b\boldsymbol{H}_t\boldsymbol{H}_b^{\mathrm{H}} \\ - \sum_{l\in\mathcal{K}_{\mathcal{L}}}\mu_l^*\boldsymbol{H}_{e,l} - \sum_{l\in\mathcal{K}_{\mathcal{L}}}\nu_l^*\gamma_{e,r,l}\boldsymbol{H}_{e,l} =& \boldsymbol{\Sigma}_2^* \succeq \boldsymbol{0}, \quad \text{(A.14)} \\ \boldsymbol{\Sigma}_1^*\boldsymbol{\Phi}_{\boldsymbol{s}}^* = \boldsymbol{0}, \quad \boldsymbol{\Sigma}_2^*\boldsymbol{\Phi}_{\boldsymbol{w}}^* =& \boldsymbol{0}, \quad \text{(A.15)} \end{aligned}$$

where α^*, β^*, λ^*, $\{\mu_l^*\}$ and $\{\nu_l^*\}$ are the optimal dual variables.

Define

$$\boldsymbol{C} = \beta^*\boldsymbol{I} + \lambda^*\rho\boldsymbol{H}_b\boldsymbol{H}_t\boldsymbol{H}_b^{\mathrm{H}} - \sum_{l\in\mathcal{K}_{\mathcal{L}}}\mu_l^*\boldsymbol{H}_{e,l} + \sum_{l\in\mathcal{K}_{\mathcal{L}}}\nu_l^*\boldsymbol{H}_{e,l}. \quad \text{(A.16)}$$

Case 1: $\mathrm{Rank}(\boldsymbol{C}) = N$: It follows that $\boldsymbol{C} \succ \boldsymbol{0}$. Since $\mathrm{Rank}(\boldsymbol{H}_r) = 1$, from (52) we obtain

$$\mathrm{Rank}(\boldsymbol{\Sigma}_1^*) = \mathrm{Rank}(\boldsymbol{C} - \boldsymbol{H}_r) \geq N - 1. \quad \text{(A.17)}$$

From (A.15) there is

$$\mathrm{Rank}(\boldsymbol{\Phi}_{\boldsymbol{s}}^*) + \mathrm{Rank}(\boldsymbol{\Sigma}_1^*) \leq N. \quad \text{(A.18)}$$

Thus, $\mathrm{Rank}(\boldsymbol{\Phi}_{\boldsymbol{s}}^*) \leq 1$ should hold.

Case 2: $\mathrm{Rank}(\boldsymbol{C}) = J < N$: Let $\boldsymbol{\pi}_j \in \mathbb{C}^{N\times 1}$ denote the jth basis of the null space of $\boldsymbol{C}$, where $1 \leq j \leq N - J$. Then left and right multiplying both sides of (A.13) by $\boldsymbol{\pi}_j^H$ and $\boldsymbol{\pi}_j$, respectively, yields

$$\boldsymbol{\pi}_j^{\mathrm{H}}\boldsymbol{\Sigma}_1^*\boldsymbol{\pi}_j = \boldsymbol{\pi}_j^{\mathrm{H}}\left(\boldsymbol{C} - \boldsymbol{H}_r\right)\boldsymbol{\pi}_j = -\boldsymbol{\pi}_j^{\mathrm{H}}\boldsymbol{H}_r\boldsymbol{\pi}_j, \forall j. \quad \text{(A.19)}$$

Since $\boldsymbol{\Sigma}_1^* \succeq \boldsymbol{0}$, there must be $\boldsymbol{\pi}_j^{\mathrm{H}}\boldsymbol{\Sigma}_1^*\boldsymbol{\pi}_j = 0, \forall j$ and

$$\boldsymbol{\pi}_j^{\mathrm{H}}\boldsymbol{H}_r\boldsymbol{\pi}_j = 0, \forall j. \quad \text{(A.20)}$$

Thus $\boldsymbol{\pi}_j, \forall j$ must lie in the null space of $\boldsymbol{\Sigma}_1^*$, and must also lie in the null space of $\boldsymbol{H}_r$. Moreover, we can obtain

$$\mathrm{Rank}(\boldsymbol{\Sigma}_1^*) = \mathrm{Rank}(\boldsymbol{C} - \boldsymbol{H}_r) \geq J - 1, \quad \text{(A.21)}$$

in a similar way as (A.17). Let $\boldsymbol{\Gamma}$ denote the orthogonal basis of the null space of $\boldsymbol{\Sigma}_1^*$, it then follows from (A.21) that

$$\mathrm{Rank}(\boldsymbol{\Gamma}) = N - \mathrm{Rank}(\boldsymbol{\Sigma}_1^*) \leq N - J + 1. \quad \text{(A.22)}$$

Next, let us show that $\mathrm{Rank}(\boldsymbol{\Gamma}) = N - J + 1$ must be satisfied. Since $\boldsymbol{\pi}_j, 1 \leq j \leq N - J$ spans $N - J$ orthogonal dimensions of the null space of $\boldsymbol{\Sigma}_1^*$, there is

$$\mathrm{Rank}([\boldsymbol{\pi}_1, \ldots, \boldsymbol{\pi}_{N-J}]) = N - J. \quad \text{(A.23)}$$

Thus, there must be

$$\mathrm{Rank}(\boldsymbol{\Gamma}) \geq N - J. \quad \text{(A.24)}$$

Assuming $\mathrm{Rank}(\boldsymbol{\Gamma}) = N - J$, it follows that

$$\boldsymbol{\Gamma} = [\boldsymbol{\pi}_1, \ldots, \boldsymbol{\pi}_{N-J}]. \quad \text{(A.25)}$$

From (A.15), we know that $\boldsymbol{\Phi}_{\boldsymbol{s}}^*$ must lie in the null space of $\boldsymbol{\Sigma}_1^*$, and can be expressed as

$$\boldsymbol{\Phi}_{\boldsymbol{s}}^* = \sum_{i=1}^{N-J}\sum_{j=1}^{N-J}\tau_{i,j}\boldsymbol{\pi}_i\boldsymbol{\pi}_j^{\mathrm{H}}, \quad \text{(A.26)}$$

where $\{\tau_{i,j}\}$ are the corresponding coefficients. Moreover, from (A.20) we know $\boldsymbol{\pi}_j, \forall j$ must lie in the null space of $\boldsymbol{H}_r$. Substituting (A.26) into (A.2), we obtain $\mathrm{Tr}(\boldsymbol{H}_r\boldsymbol{\Phi}_{\boldsymbol{s}}^*) = 0$. Since $\varphi > 0$, we know that $\mathrm{Tr}(\boldsymbol{H}_r\boldsymbol{S}^*) = 0$ holds. Substituting $\mathrm{Tr}(\boldsymbol{H}_r\boldsymbol{S}^*) = 0$ into (31) of $\mathbf{P1}-\mathbf{EQV}$, we know that the optimal value of (31) is equal to zero, which contradicts (34). Thus, from (A.24) there must be $\mathrm{Rank}(\boldsymbol{\Gamma}) > N - J$. Then using (A.22), we know that

$$\mathrm{Rank}(\boldsymbol{\Gamma}) = N - J + 1, \quad \text{(A.27)}$$

must be satisfied. As a result, there exists only one single orthogonal basis $\boldsymbol{\pi}_{N-J+1}$ which is orthogonal to $\boldsymbol{\Gamma}$ and is not orthogonal to $\boldsymbol{H}_r$. Consequently, $\boldsymbol{\Gamma}$ can be expressed as

$$\boldsymbol{\Gamma} = \left[\boldsymbol{\pi}_1, \ldots, \boldsymbol{\pi}_{N-J}, \boldsymbol{\pi}_{N-J+1}\right]. \quad \text{(A.28)}$$

Since $\boldsymbol{\Phi}_{\boldsymbol{s}}^*$ must lie in the null space of $\boldsymbol{\Sigma}_1^*$, then $\boldsymbol{\Phi}_{\boldsymbol{s}}^*$ can be expressed as

$$\begin{aligned} \boldsymbol{\Phi}_{\boldsymbol{s}}^* &= \sum_{i=1}^{N-J+1}\sum_{j=1}^{N-J+1}\tau_{i,j}\boldsymbol{\pi}_i\boldsymbol{\pi}_j^{\mathrm{H}} \quad \text{(A.29)} \\ &= \tau_{N-J+1,N-J+1}\boldsymbol{\pi}_{N-J+1}\boldsymbol{\pi}_{N-J+1}^{\mathrm{H}} + \sum_{i=1}^{N-J}\sum_{j=1}^{N-J+1}\tau_{i,j}\boldsymbol{\pi}_i\boldsymbol{\pi}_j^{\mathrm{H}}. \end{aligned}$$

Finally, let us show

$$\begin{cases} \boldsymbol{\Phi}_{\boldsymbol{s}}^{\prime *} = \boldsymbol{\Phi}_{\boldsymbol{s}}^* - \sum_{i=1}^{N-J}\sum_{j=1}^{N-J+1}\tau_{i,j}\boldsymbol{\pi}_i\boldsymbol{\pi}_j^{\mathrm{H}}, \\ \boldsymbol{\Phi}_{\boldsymbol{w}}^{\prime *} = \boldsymbol{\Phi}_{\boldsymbol{w}}^* + \sum_{i=1}^{N-J}\sum_{j=1}^{N-J+1}\tau_{i,j}\boldsymbol{\pi}_i\boldsymbol{\pi}_j^{\mathrm{H}}, \\ \varphi^{\prime *} = \varphi^*, \end{cases} \quad \text{(A.30)}$$

are also the optimal solutions for $\mathbf{P1}-\mathbf{EQV}-\mathbf{A}$, where $\mathrm{Rank}(\boldsymbol{\Phi}_{\boldsymbol{s}}^{\prime *}) = 1$ holds. Substituting (A.30) into (A.2)∼(A.8) of $\mathbf{P1}-\mathbf{EQV}-\mathbf{A}$, we obtain

$$\begin{aligned} &\mathrm{Tr}(\boldsymbol{H}_r\boldsymbol{\Phi}_{\boldsymbol{s}}^{\prime *}) = \mathrm{Tr}(\boldsymbol{H}_r\boldsymbol{\Phi}_{\boldsymbol{s}}^*) && \text{(A.31)} \\ &\mathrm{Tr}(\boldsymbol{H}_r\boldsymbol{\Phi}_{\boldsymbol{w}}^{\prime *}) + \varphi^{\prime *}(P_t\|g_r\|^2 + \sigma_r^2) \\ &\quad = \mathrm{Tr}(\boldsymbol{H}_r\boldsymbol{\Phi}_{\boldsymbol{w}}^*) + \varphi^*(P_t\|g_r\|^2 + \sigma_r^2) = 1 && \text{(A.32)} \\ &\mathrm{Tr}(\boldsymbol{\Phi}_{\boldsymbol{s}}^{\prime *}) + \mathrm{Tr}(\boldsymbol{\Phi}_{\boldsymbol{w}}^{\prime *}) = \mathrm{Tr}(\boldsymbol{\Phi}_{\boldsymbol{s}}^*) + \mathrm{Tr}(\boldsymbol{\Phi}_{\boldsymbol{w}}^*) \leq \varphi^* P_b && \text{(A.33)} \\ &\mathrm{Tr}((\rho\boldsymbol{H}_b\boldsymbol{H}_t\boldsymbol{H}_b^{\mathrm{H}})\boldsymbol{\Phi}_{\boldsymbol{s}}^{\prime *}) + \mathrm{Tr}((\rho\boldsymbol{H}_b\boldsymbol{H}_t\boldsymbol{H}_b^{\mathrm{H}})\boldsymbol{\Phi}_{\boldsymbol{w}}^{\prime *}) \\ &\quad = \mathrm{Tr}((\rho\boldsymbol{H}_b\boldsymbol{H}_t\boldsymbol{H}_b^{\mathrm{H}})\boldsymbol{\Phi}_{\boldsymbol{s}}^*) + \mathrm{Tr}((\rho\boldsymbol{H}_b\boldsymbol{H}_t\boldsymbol{H}_b^{\mathrm{H}})\boldsymbol{\Phi}_{\boldsymbol{w}}^*) \\ &\quad \leq \varphi^*\left(\frac{P_t\|\boldsymbol{h}_t\|^4}{\gamma_b} - \mathrm{Tr}(\boldsymbol{R}_b\boldsymbol{H}_t)\right) && \text{(A.34)} \\ &\mathrm{Tr}(\boldsymbol{H}_{e,l}\boldsymbol{\Phi}_{\boldsymbol{s}}^{\prime *}) + \mathrm{Tr}(\boldsymbol{H}_{e,l}\boldsymbol{\Phi}_{\boldsymbol{w}}^{\prime *}) \\ &\quad = \mathrm{Tr}(\boldsymbol{H}_{e,l}\boldsymbol{\Phi}_{\boldsymbol{s}}^*) + \mathrm{Tr}(\boldsymbol{H}_{e,l}\boldsymbol{\Phi}_{\boldsymbol{w}}^*) \\ &\quad \geq \varphi^*\left(\frac{P_t\|g_{e,l}\|^2}{\gamma_{e,b,l}} - \sigma_{e,l}^2\right), \quad \forall l \in \mathcal{K}_{\mathcal{L}} && \text{(A.35)} \end{aligned}$$

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$$\begin{aligned}&\mathrm{Tr}(\boldsymbol{H}_{e,l}\boldsymbol{\Phi}_{\boldsymbol{s}}'^{*})-\gamma_{e,r,l}\mathrm{Tr}(\boldsymbol{H}_{e,l}\boldsymbol{\Phi}_{\boldsymbol{w}}'^{*})\\&\quad\leq\mathrm{Tr}(\boldsymbol{H}_{e,l}\boldsymbol{\Phi}_{\boldsymbol{s}}^{*})-\gamma_{e,r,l}\mathrm{Tr}(\boldsymbol{H}_{e,l}\boldsymbol{\Phi}_{\boldsymbol{w}}^{*})\\&\quad\leq\varphi^{*}\gamma_{e,r,l}(P_t\|g_{e,l}\|^2+\sigma_{e,l}^2),\quad\forall l\in\mathcal{K}_{\mathcal{L}}\end{aligned}\tag{A.36}$$

$$\boldsymbol{\Phi}_{\boldsymbol{s}}'^{*}\succeq\mathbf{0},\quad\boldsymbol{\Phi}_{\boldsymbol{w}}'^{*}\succeq\mathbf{0},\quad\varphi'^{*}>0.\tag{A.37}$$

where the property that $\boldsymbol{\pi}_j^{\mathrm{H}}\boldsymbol{H}_r\boldsymbol{\pi}_j=0$ for $1\leq j\leq N-J$ is utilized in the above derivations. From (A.31), we know that $\boldsymbol{\Phi}_{\boldsymbol{s}}'^{*}$, $\boldsymbol{\Phi}_{\boldsymbol{w}}'^{*}$ and φ'^{*} will provide the same optimal objective value of $\mathbf{P1}-\mathbf{EQV}-\mathbf{A}$. Moreover, from (A.32) to (A.37), we can conclude that $\boldsymbol{\Phi}_{\boldsymbol{s}}'^{*}$, $\boldsymbol{\Phi}_{\boldsymbol{w}}'^{*}$ and φ'^{*} satisfy all the constraints of $\mathbf{P1}-\mathbf{EQV}-\mathbf{A}$. Consequently, $\boldsymbol{\Phi}_{\boldsymbol{s}}'^{*}$, $\boldsymbol{\Phi}_{\boldsymbol{w}}'^{*}$ and φ'^{*} are also optimal solutions of $\mathbf{P1}-\mathbf{EQV}-\mathbf{A}$.

From all the above discussions, we know that there always exists the optimal solution $\boldsymbol{\Phi}_{\boldsymbol{s}}^{*}$ with $\mathrm{Rank}(\boldsymbol{\Phi}_{\boldsymbol{s}}^{*})=1$.

APPENDIX B
PROOF OF LEMMA 4

Since $\mathbf{P1}-\mathbf{SDR}$ is a convex SDP, the Lagrange function of $\mathbf{P1}-\mathbf{SDR}$ can be expressed as

$$\begin{aligned}&\mathcal{L}(\boldsymbol{S},\boldsymbol{W},\alpha',\{\beta_l',l\in\mathcal{K}_{\mathcal{L}}\},\lambda',\{\mu_l',l\in\mathcal{K}_{\mathcal{L}}\})\\&\qquad=\mathrm{Tr}(\boldsymbol{\Sigma}_1'\boldsymbol{S})+\mathrm{Tr}(\boldsymbol{\Sigma}_2'\boldsymbol{W})+\Sigma_3',\end{aligned}\tag{B.1}$$

where $\alpha'\geq0$, $\{\beta_l'\geq0,l\in\mathcal{K}_{\mathcal{L}}\}$, $\lambda'\geq0$ and $\{\mu_l'\geq0,l\in\mathcal{K}_{\mathcal{L}}\}$ are dual variables associated with the constraints in (26)–(29), respectively; Moreover,

$$\begin{aligned}\boldsymbol{\Sigma}_1'=\boldsymbol{I}+\alpha'\rho\boldsymbol{H}_b\boldsymbol{H}_t\boldsymbol{H}_b^{\mathrm{H}}-\sum_{l\in\mathcal{K}_{\mathcal{L}}}\beta_l'\boldsymbol{H}_{e,l}\\-\lambda'\boldsymbol{H}_r+\sum_{l\in\mathcal{K}_{\mathcal{L}}}\mu_l'\boldsymbol{H}_{e,l};\\\boldsymbol{\Sigma}_2'=\boldsymbol{I}+\alpha'\rho\boldsymbol{H}_b\boldsymbol{H}_t\boldsymbol{H}_b^{\mathrm{H}}-\sum_{l\in\mathcal{K}_{\mathcal{L}}}\beta_l'\boldsymbol{H}_{e,l}\\+\lambda'\gamma_r\boldsymbol{H}_r-\sum_{l\in\mathcal{K}_{\mathcal{L}}}\mu_l'\gamma_{e,r,l}\boldsymbol{H}_{e,l};\end{aligned}$$

$$\begin{aligned}\Sigma_3'=-\alpha'\left(\frac{P_t\|\boldsymbol{h}_t\|^4}{\gamma_b}-\mathrm{Tr}(\boldsymbol{R}_b\boldsymbol{H}_t)\right)\\+\sum_{l\in\mathcal{K}_{\mathcal{L}}}\beta_l'\left(\frac{P_t\|g_{e,l}\|^2}{\gamma_{e,b,l}}-\sigma_{e,l}^2\right)\\+\lambda'\gamma_r(P_t\|g_r\|^2+\sigma_r^2)\\-\sum_{l\in\mathcal{K}_{\mathcal{L}}}\mu_l'\gamma_{e,r,l}(P_t\|g_{e,l}\|^2+\sigma_{e,l}^2).\end{aligned}$$

The Lagrange dual function of $\mathbf{P1}-\mathbf{SDR}$ is defined as

$$\begin{aligned}&\zeta(\alpha',\{\beta_l',l\in\mathcal{K}_{\mathcal{L}}\},\lambda',\{\mu_l',l\in\mathcal{K}_{\mathcal{L}}\})\\&=\min_{\boldsymbol{S},\boldsymbol{W}}\mathcal{L}(\boldsymbol{S},\boldsymbol{W},\alpha',\{\beta_l',l\in\mathcal{K}_{\mathcal{L}}\},\lambda',\{\mu_l',l\in\mathcal{K}_{\mathcal{L}}\}),\end{aligned}\tag{B.2}$$

where $\boldsymbol{\Sigma}_1'\succeq\mathbf{0}$ and $\boldsymbol{\Sigma}_2'\succeq\mathbf{0}$ must be satisfied to guarantee that (B.2) is not unbounded to infinity. Then the KKT conditions [30] that are directly related to $\boldsymbol{W}^*$ can be expressed as

$$\begin{aligned}\boldsymbol{I}+\alpha'^{*}\rho\boldsymbol{H}_b\boldsymbol{H}_t\boldsymbol{H}_b^{\mathrm{H}}-\sum_{l\in\mathcal{K}_{\mathcal{L}}}\beta_l'^{*}\boldsymbol{H}_{e,l}+\lambda'^{*}\gamma_r\boldsymbol{H}_r\\-\sum_{l\in\mathcal{K}_{\mathcal{L}}}\mu_l'^{*}\gamma_{e,r,l}\boldsymbol{H}_{e,l}=\boldsymbol{\Sigma}_2'^{*}\succeq\mathbf{0},\end{aligned}\tag{B.3}$$

$$\boldsymbol{\Sigma}_2'^{*}\boldsymbol{W}^*=\mathbf{0},\tag{B.4}$$

where $\alpha'^{*}\geq0$, $\{\beta_l'^{*}\geq0,l\in\mathcal{K}_{\mathcal{L}}\}$, $\lambda'^{*}\geq0$ and $\{\mu_l'^{*}\geq0,l\in\mathcal{K}_{\mathcal{L}}\}$ are the optimal dual variables.

Since $\alpha'^{*}\geq0$, $\rho\geq0$, $\lambda'^{*}\geq0$ and $\gamma_r>0$, from (B.3) there must be

$$\boldsymbol{I}+\alpha'^{*}\rho\boldsymbol{H}_b\boldsymbol{H}_t\boldsymbol{H}_b^{\mathrm{H}}+\lambda'^{*}\gamma_r\boldsymbol{H}_r\succ\mathbf{0}.\tag{B.5}$$

It follows from (B.5) that

$$\mathrm{Rank}\left(\boldsymbol{I}+\alpha'^{*}\rho\boldsymbol{H}_b\boldsymbol{H}_t\boldsymbol{H}_b^{\mathrm{H}}+\lambda'^{*}\gamma_r\boldsymbol{H}_r\right)=N.\tag{B.6}$$

Moreover, it follows from (B.3) that

$$\begin{aligned}&\mathrm{Rank}\left(-\sum_{l\in\mathcal{K}_{\mathcal{L}}}\beta_l'^{*}\boldsymbol{H}_{e,l}-\sum_{l\in\mathcal{K}_{\mathcal{L}}}\mu_l'^{*}\gamma_{e,r,l}\boldsymbol{H}_{e,l}\right)\\&=\mathrm{Rank}\left(-\sum_{l\in\mathcal{K}_{\mathcal{L}}}\left(\beta_l'^{*}+\mu_l'^{*}\gamma_{e,r,l}\right)\boldsymbol{H}_{e,l}\right)\leq L.\end{aligned}\tag{B.7}$$

Combing (B.6) and (B.7), there must be

$$\mathrm{Rank}(\boldsymbol{\Sigma}_2'^{*})\geq N-L.\tag{B.8}$$

From (B.4), we can obtain

$$\mathrm{Rank}(\boldsymbol{\Sigma}_2'^{*})+\mathrm{Rank}(\boldsymbol{W}^*)\leq N.\tag{B.9}$$

Combing (B.8) and (B.9), there must be $\mathrm{Rank}(\boldsymbol{W}^*)\leq L$.

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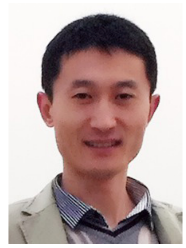

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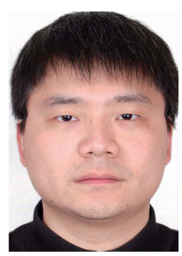

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