```
> restart: with(plots):
```

Mode representation for stability analysis:

```
> mode := (x,t) -> xi^(t) * exp(I*k*x);
```

$$mode := (x, t) \mapsto \xi^{t} e^{I k x} \quad (1)$$

Mode values on next, current, and previous slices:

```
> up := mode(0,1); hr := mode(0,0); dn := mode(0,-1);
```

$$up := \xi$$

$$hr := 1$$

$$dn := \frac{1}{\xi} \quad (2)$$

Neighbours for finite difference scheme:

```
> lt := mode(-1,0); rt := mode(1,0);
```

$$lt := e^{-I k}$$

$$rt := e^{I k} \quad (3)$$

Discretized wave equation (leapfrog scheme):

```
> (lt + rt - 2*hr)/dx^2 = (up + dn - 2*hr)/dt^2; dt := sqrt(alpha)*
  dx;
```

$$\frac{e^{-I k} + e^{I k} - 2}{dx^{2}} = \frac{\xi + \frac{1}{\xi} - 2}{dt^{2}}$$

$$dt := \sqrt{\alpha}\, dx \quad (4)$$

Two roots for growth factor xi:

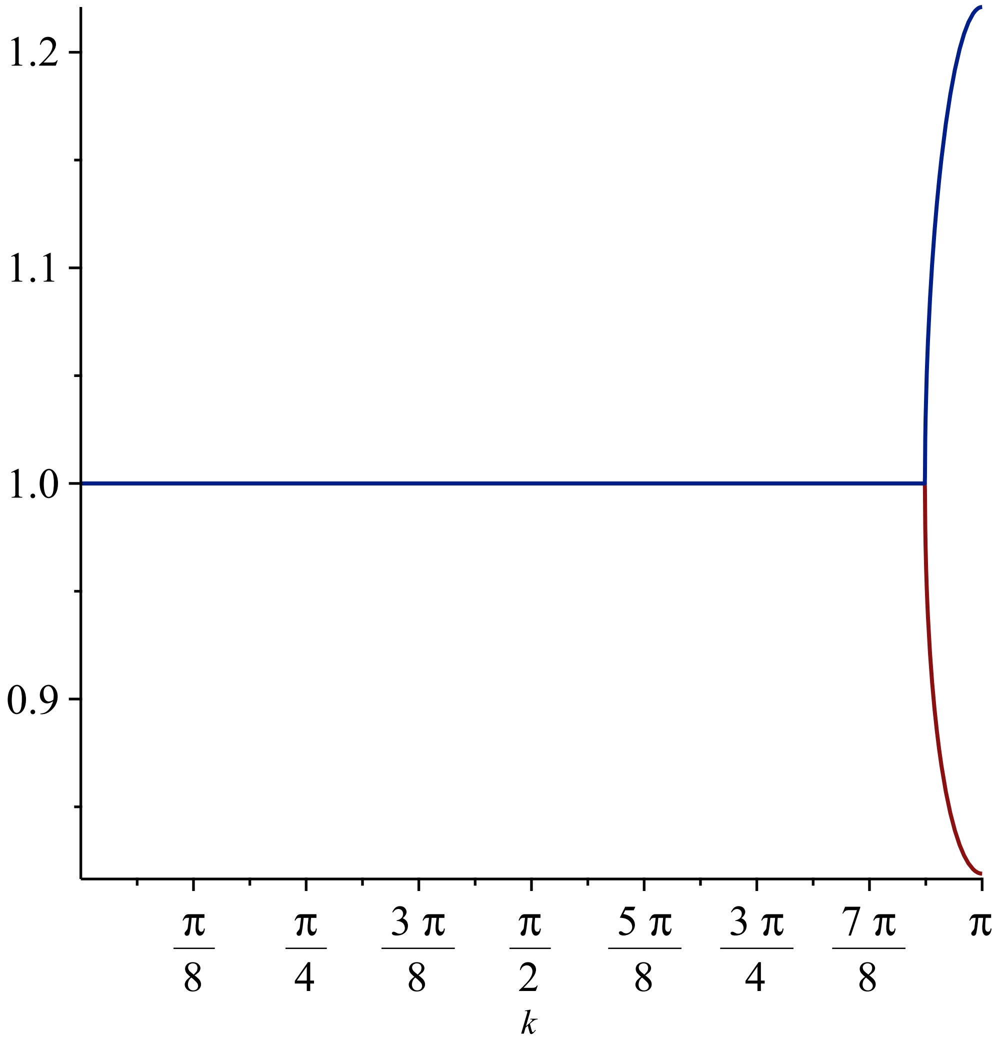
```
> X:=solve(%%,xi): xi[1] := simplify(X[1]); xi[2] :=simplify(X[2]);
```

$$\xi_{1} := -\alpha + 1 + \sqrt{\alpha\,(\cos(k) - 1)\,(\cos(k)\,\alpha - \alpha + 2)} + \cos(k)\,\alpha$$

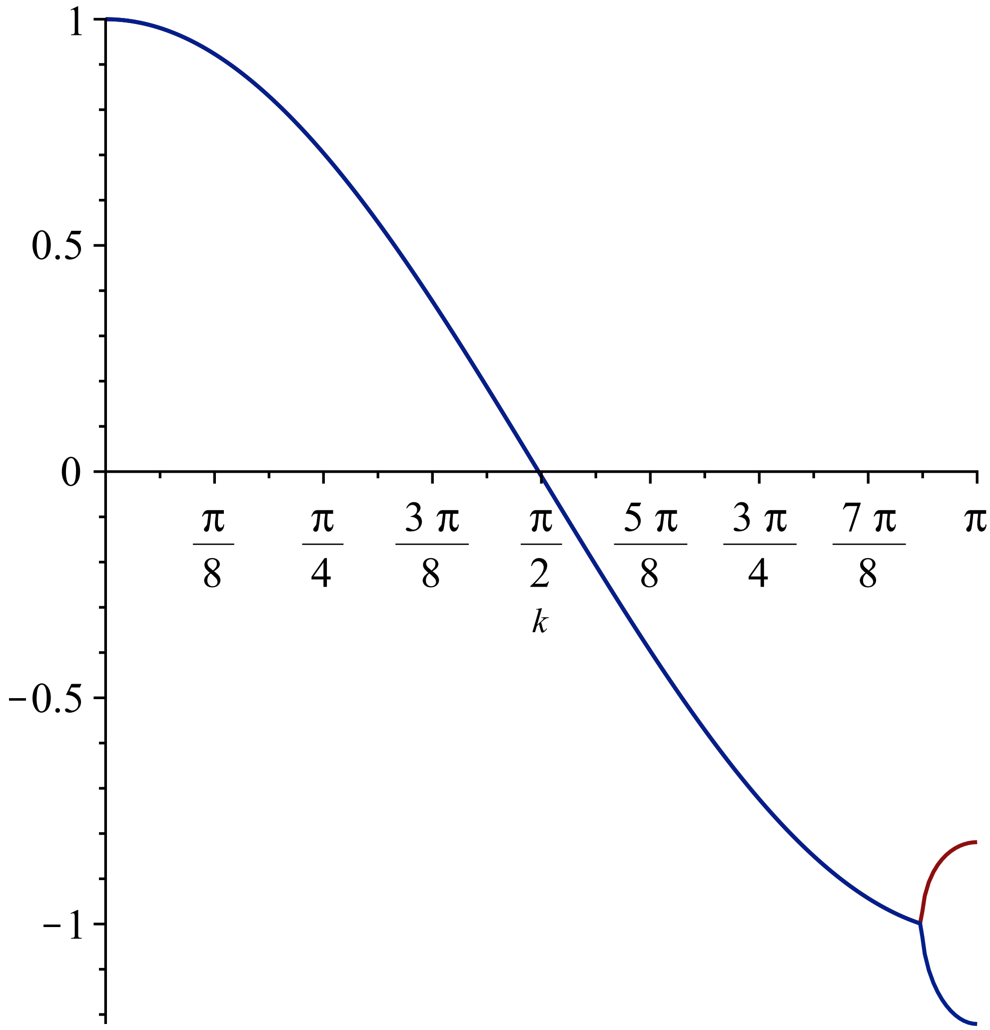
$$\xi_{2} := -\alpha + 1 - \sqrt{\alpha\,(\cos(k) - 1)\,(\cos(k)\,\alpha - \alpha + 2)} + \cos(k)\,\alpha \quad (5)$$

If absolute value of xi is greater than one, the mode is unstable:

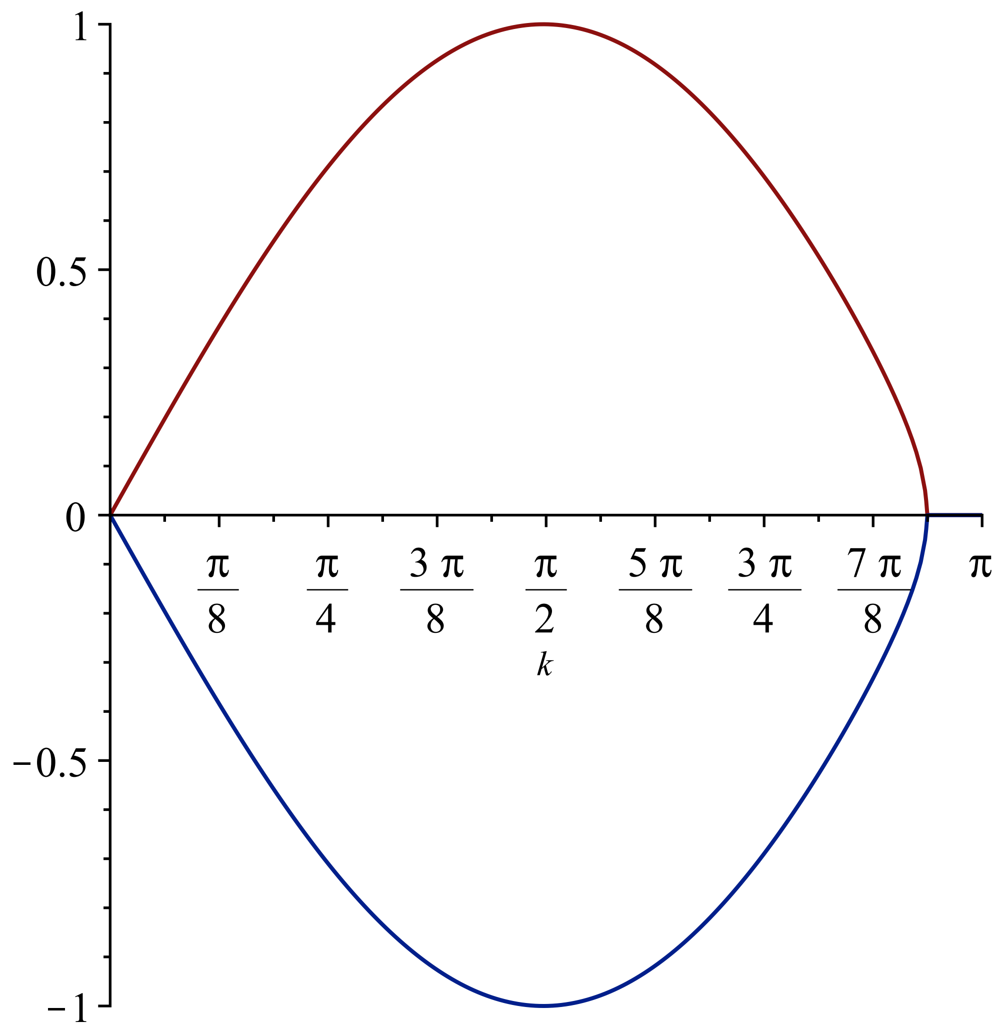
```
> eval([xi[1],xi[2]],alpha=1.01): plot(map(abs,%), k = 0..Pi);
```




```
> eval([xi[1],xi[2]],alpha=1.01): plot(map(Re,%), k = 0..Pi);
```




```
> eval([xi[1],xi[2]],alpha=1.01): plot(map(Im,%), k = 0..Pi);
```



1
0.5
0
−0.5
−1
$\frac{\pi}{8}$
$\frac{\pi}{4}$
$\frac{3\pi}{8}$
$\frac{\pi}{2}$
$\frac{5\pi}{8}$
$\frac{3\pi}{4}$
$\frac{7\pi}{8}$
π
k