```
> restart: with(plots):
```

Mode representation for stability analysis:

```
> mode := (x,t) -> xi^(t) * exp(I*k*x);
```

$$mode := (x, t) \mapsto \xi^{t} e^{I k x} \quad (1)$$

Mode values on next, current, and previous slices:

```
> up := mode(0,1); hr := mode(0,0); dn := mode(0,-1);
```

$$up := \xi$$

$$hr := 1$$

$$dn := \frac{1}{\xi} \quad (2)$$

Neighbours for finite difference scheme:

```
> lt := mode(-1,0); rt := mode(1,0);
```

$$lt := e^{-I k}$$

$$rt := e^{I k} \quad (3)$$

Discretized diffusion equation (forward time difference):

```
> (lt + rt - 2*hr)/dx^2 = (up - hr)/dt; dt := alpha*dx^2;
```

$$\frac{e^{-I k} + e^{I k} - 2}{dx^{2}} = \frac{\xi - 1}{dt}$$

$$dt := \alpha\, dx^{2} \quad (4)$$

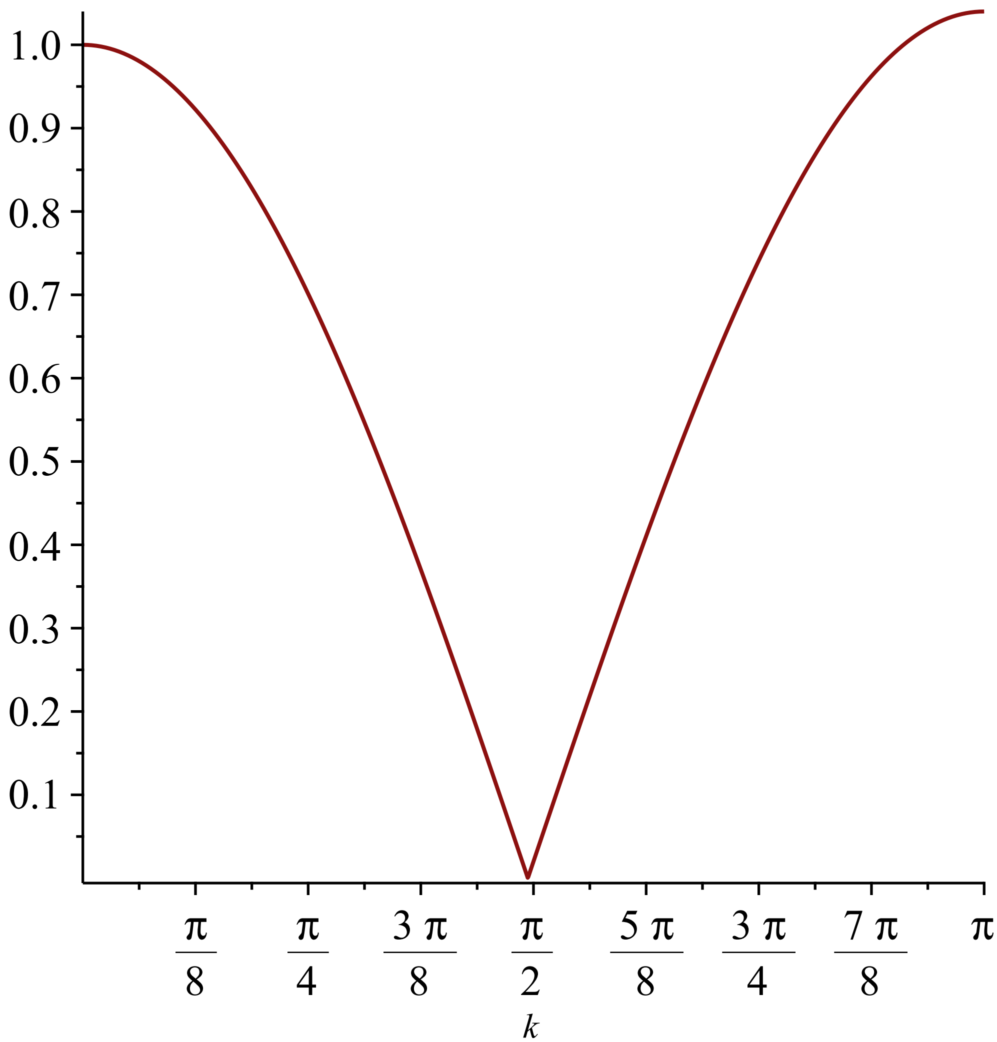
Single root for growth factor xi:

```
> X:=solve(%%,xi): xi[1] := simplify(X);
```

$$\xi_{1} := -2\,\alpha + 1 + 2\,\alpha \cos(k) \quad (5)$$

If absolute value of xi is greater than one, the mode is unstable:

```
> eval(xi[1],alpha=0.51): plot(abs(%), k = 0..Pi); xi
```



$$\xi \tag{6}$$

Discretized diffusion equation (backward time difference):

```
> (lt + rt - 2*hr)/dx^2 = (hr - dn)/dt; dt := alpha*dx^2;
```

$$\frac{e^{-\mathrm{I}\,k} + e^{\mathrm{I}\,k} - 2}{dx^2} = \frac{1 - \frac{1}{\xi}}{\alpha\, dx^2}$$

$$dt := \alpha\, dx^2 \tag{7}$$

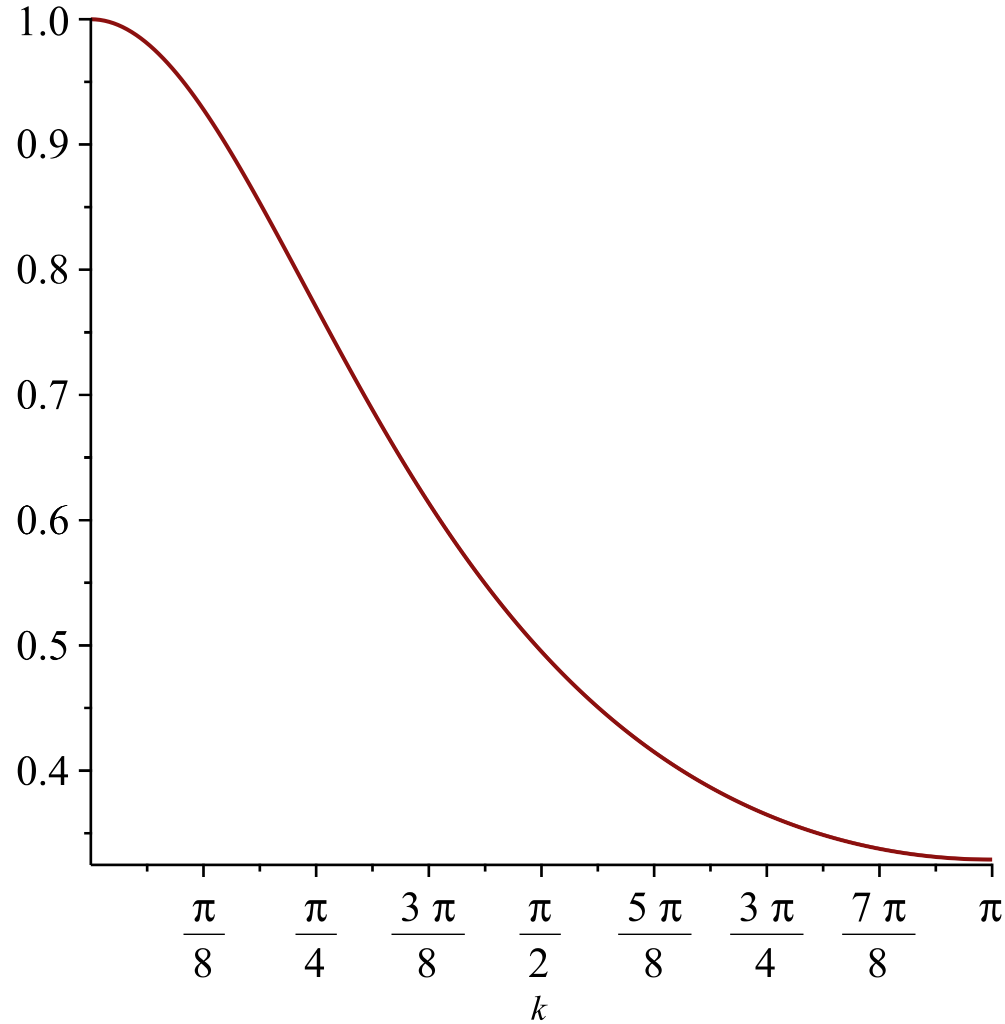
Single root for growth factor xi:

```
> X:=solve(%%,xi): xi := simplify(X);
```

$$\xi := -\frac{1}{-2\,\alpha - 1 + 2\,\alpha\cos(k)} \tag{8}$$

If absolute value of xi is greater than one, the mode is unstable:

```
> eval(xi,alpha=0.51): plot(abs(%), k = 0..Pi);
```


1.0
0.9
0.8
0.7
0.6
0.5
0.4
π/8
π/4
3 π/8
π/2
5 π/8
3 π/4
7 π/8
π
k