media unei variabile aliatoare = expectation for a random variable (E X = LINIE * COLOANA)

dispersia unei variabile aliatoare = variance for a random variable ($VAR[X] = E[X^2] - (E[X])^2$)

distributia unei variabile aliatoare = distribution of a random variable (TABEL)

I. 1.

X:	-1	1	2	3
	1/4	1/4	3/16	5/16

Y:	0	1	2
	1/3	5/9	1/9

Z:	-2	0	2
	1/7	4/7	2/7

$E X = (-1) * 1/4 + 1 * 1/4 + 2 * 3/16 + 3 * 5/16 = 21/16$

$E Y = 0 * 1/3 + 1 * 5/9 + 2 * 1/9 = 7/9$

$E Z = (-2) * 1/7 + 0 * 4/7 + 2 * 2/7 = 2/7$

X^2:	1	1	4	9
	1/4	1/4	3/16	5/16

Y^2:	0	1	4
	1/3	5/9	1/9

Z^2:	4	0	4
	1/7	4/7	2/7

$E X^2 = 1 * 1/4 + 1 * 1/4 + 4 * 3/16 + 9 * 5/16 = 65/16$

$E Y^2 = 0 * 1/3 + 1 * 5/9 + 4 * 1/9 = 9/9$

$E Z^2 = 4 * 1/7 + 0 * 4/7 + 4 * 2/7 = 12/7$

$Var [X] = 65/16 - 441/256 = 599/256$

$Var [Y] = 1 - 49/81 = 32/81$

$Var [Z] = 12/7 - 4/49 = 80/49$

I. 4.

{t,t,t} = 0-3 = -3

{h,t,t} = {t,h,t} = {t,t,h} = 1-2 = -1

{h,h,t} = {t,h,h} = {h,t,h} = 2-1 = 1

{h,h,h} = 3-0 = 3

X:	-3	-1	1	3
	1/8	3/8	3/8	1/8

X^2:	9	1	1	9
	1/8	3/8	3/8	1/8

$E X = -3 * 1/8 + -1 * 3/8 + 1 * 3/8 + 3 * 1/8 = 0$

$E X^2 = 9 * 1/8 + 1 * 3/8 + 1 * 3/8 + 9 * 1/8 = 24/8$

$Var [X] = 24/8 - 0 = 3$

I. 7.

$P(X=1) = \{1,1\}\{1,2\}\{1,3\}\{1,4\}\{1,5\}\{1,6\}\{2,1\}\{3,1\}\{4,1\}\{5,1\}\{6,1\}$

$P(X=2) = \{2,2\}\{2,3\}\{2,4\}\{2,5\}\{2,6\}\{3,2\}\{4,2\}\{5,2\}\{6,2\}$

$P(X=3) = \{3,3\}\{3,4\}\{3,5\}\{3,6\}\{4,3\}\{5,3\}\{6,3\}$

$P(X=4) = \{4,4\}\{4,5\}\{4,6\}\{5,4\}\{6,4\}$

$P(X=5) = \{5,5\}\{5,6\}\{6,5\}$

$P(X=6) = \{6,6\}$

X:	1	2	3	4	5	6
	11/36	9/36	7/36	5/36	3/36	1/36

I. 8.

$U_1 = 1w, 1b$; $U_2 = 2w, 6b$; $U_3 = 1w, 3b$;

X = nr of w

X:	0	1	2	3
	28/90	37/90	19/90	6/90

X^2:	0	1	4	9
	28/90	37/90	19/90	6/90

$$P(X=0) = P(\bar{A}_1 \cap \bar{A}_2 \cap \bar{A}_3) = \frac{1}{2} \cdot \frac{7}{9} \cdot \frac{4}{5} = \frac{28}{90}$$

$$P(X=1) = P(\bar{A}_1 \cap \bar{A}_2 \cap A_3) + P(\bar{A}_1 \cap A_2 \cap \bar{A}_3) + P(A_1 \cap \bar{A}_2 \cap \bar{A}_3) = \frac{1}{2} \cdot \frac{7}{9} \cdot \frac{1}{5} + \frac{1}{2} \cdot \frac{2}{9} \cdot \frac{3}{5} + \frac{1}{2} \cdot \frac{6}{9} \cdot \frac{4}{5}$$

$$P(X=2) = P(A_1 \cap A_2 \cap \bar{A}_3) + P(A_1 \cap \bar{A}_2 \cap A_3) + P(\bar{A}_1 \cap A_2 \cap A_3) = \frac{1}{2} \cdot \frac{3}{9} \cdot \frac{3}{5} + \frac{1}{2} \cdot \frac{6}{9} \cdot \frac{1}{5} + \frac{1}{2} \cdot \frac{2}{9} \cdot \frac{2}{5} = \frac{19}{90}$$

$$P(X=3) = P(A_1 \cap A_2 \cap A_3) = \frac{1}{2} \cdot \frac{3}{9} \cdot \frac{2}{5} = \frac{6}{90}$$

$E X = 0 * 28/90 + 1 * 37/90 + 2 * 19/90 + 3 * 6/90 = 93/90$

$E X^2 = 0 * 28/90 + 1 * 37/90 + 4 * 19/90 + 9 * 6/90 = 167/90$

$Var[X] = 167/90 – 8649/8100 = 6381/8100$

I. 9.

X = nr of girls

X:	0	1	2	3
	4/8	2/8	1/8	1/8

{b, fb, ffb, fff}

$P(X=0) = 1/2$

$P(X=1) = 1/2 * 1/2 = 1/4$

$P(X=2) = 1/2 * 1/2 * 1/2 = 1/8$

$P(X=3) = 1/2 * 1/2 * 1/2 = 1/8$

X^2:	0	1	4	9
	4/8	2/8	1/8	1/8

$E X = 0 * 4/8 + 1 * 2/8 + 2 * 1/8 + 3 * 1/8 = 7/8$

$E X^2 = 0 * 4/8 + 1 * 2/8 + 4 * 1/8 + 9 * 1/8 = 15/8$

$Var[X] = 15/8 - 49/64 = 71/64$

I. 15.

P_1, P_2 ; 2 sets won by one of the players $P(P_1) = 1/3$;

X = nr of sets played by P_1; Y = nr of sets P_2 wins

X:	2	3
	5/9	4/9

Y:	0	1	2
	3/27	4/27	20/27

$P(X=2) = 1/9 + 4/9 = 5/9$

$\{P_1, P_1\} = 1/3 * 1/3 = 1/9$

$\{P_2, P_2\} = 2/3 * 2/3 = 4/9$

$P(X=3) = 2/27 + 2/27 + 4/27 + 4/27 = 12/27 = 4/9$

$\{P_1, P_2, P_1\} = \{P_2, P_1, P_1\} = 1/3 * 2/3 * 1/3 = 2/27$

$\{P_2, P_1, P_2\} = \{P_1, P_2, P_2\} = 2/3 * 1/3 * 2/3 = 4/27$

$P(Y=0) = \{P_1, P_1\} = 1/3 * 1/3 = 1/9 = 3/27$

$P(Y=1) = \{P_1, P_2, P_1\} + \{P_2, P_1, P_1\} = 2/27 + 2/27 = 4/27$

$P(Y=2) = \{P_2, P_1, P_2\} + \{P_1, P_2, P_2\} + \{P_2, P_2\} = 4/27 + 4/27 + 4/9 = 20/27$

$X \sim B(n,p)$: X = "nr de succese"

n = nr de repetari

$E[X] = n * p$ p = probabilitate de succese

$Var[X] = n * p * (1 - p)$

II. 1.

2 coins flipped 7 times each; P (h,h)

$P(h,h) = 1/2 * 1/2 = 1/4$

$X \sim B(7,1/4)$

$E X = 7 * 1/4 = 7/4$

$Var[X] = 7 * 1/4 * 3/4 = 21/16$

II. 2.

$P(0) = 6/10$; $P(1) = 4/10$;

a.) {x,x,x,x,x,x,x} P of five 0's and two 1's

$(6/10)^5 * (4/10)^2 * C^5{}_7$

b.) X = "nr of 0"

$X \sim B(5,6/10)$

$E X = 5 * 3/5 = 3$

II. 6.

10 cards (out of 52); X = nr of clubs

$P(club) = 13/52$

$X \sim B(10, 1/4)$

$E X = 10 * 1/4 = 5/2$

X~G (p); X = “nr experientei la care obtinem 1-a oara succes”

X:	1	2	3	…	n	…
	…	…	…	…	$p * (1-p)^{n-1}$	…

E X = 1/p

Var [X] = $1\text{-}p/p^2$

II. 9.

2 dice, 8 times each; X = even product

P (even product) = 1-P (odd product) = 1 – {1,1; 1,3; 1,5; 3,1; 3,3; 3,5; 5,1; 5,3; 5,5} = 1 – 9/36 = 27/36

X~B (8,27/36)

E X = 8 * 27/36 = 6

III. 1.

nr of rolls of 2 dice to get a product < 7

{1,1; 1,2; 1,3; 1,4; 1,5; 1,6; 2,1; 2,2; 2,3; 3,1; 3,2; 4,1; 5,1; 6,1} = 14/36

X~G (14/36)

E X = 36/14

III. 4.

nr of withdraws from a deck to get a carp which is not diamond

P (not diamond) = 3/4

X~G (3/4)

E X = 4/3

III. 6.

P (0) = 4/10; pairs of bits; nr of pairs until we get 0-0

P (1) = 6/10

P (0-0) = 4/10 * 4/10 = 16/100

X~G (16/100)

E X = 100/16

Var [X] = …

X = nr of accedents between 7-8

X~Poisson (lam)

P (X=n) = $e^{-l} * l^n/n!$

E X = lam

a.) P (X>=3) = 1 – P (X=0) – P(X=2) = 1-$e^{-0.7}$ * (1 + 0.7 + 0.49/2)
b.) P (X>=1) = 1 – P (X=0) = 1-$e^{-0.7}$