Aplicații la formula lui Taylor

P1. Să se scrie formula lui MacLaurin pentru funcţia $f:[a,\infty]\to\mathbb{R},\ f(x)=\sqrt{a+x},\ a>0$.

```
syms a x f expo xi
assume(a>0)
expo=sym(1)/sym(2);
f=a^expo*(1+x/a)^expo;
taylor(f,x,0,'Order',11)
```

ans =

$$\frac{x}{2\ \sqrt{a}}+\sqrt{a}-\frac{x^2}{8\ a^{3/2}}+\frac{x^3}{16\ a^{5/2}}-\frac{5\ x^4}{128\ a^{7/2}}+\frac{7\ x^5}{256\ a^{9/2}}-\frac{21\ x^6}{1024\ a^{11/2}}+\frac{33\ x^7}{2048\ a^{13/2}}-\frac{429\ x^8}{32768\ a^{15/2}}+\frac{715\ x^9}{65536\ a^{17/2}}\cdot$$

```
taylor(f,x,0,'Order',11)+subs(diff(f,x,11),x,xi)
```

ans =

$$\frac{654729075}{2048\ a^{21/2}\ \left(\frac{\xi}{a}+1\right)^{21/2}}+\frac{x}{2\ \sqrt{a}}+\sqrt{a}-\frac{x^2}{8\ a^{3/2}}+\frac{x^3}{16\ a^{5/2}}-\frac{5\ x^4}{128\ a^{7/2}}+\frac{7\ x^5}{256\ a^{9/2}}-\frac{21\ x^6}{1024\ a^{11/2}}+\frac{33\ x^7}{2048\ a^{13/2}}-\frac{}{3}$$

P2. Să se determine numărul natural n, astfel ca pentru $a=0$ şi $f:\mathbb{R}\to\mathbb{R},\ f(x)=e^x\ (T_nf)$ să aproximeze f în $[-1,1]$ cu trei zecimale exacte.

```
syms n rest(n)
rest(n)=3/factorial(n+1);
for k=3:10
    r=vpa(rest(k));
    disp([k,r])
    if abs(double(r))<1e-3, break; end
end
```

$(3\quad 0.125)$
$(4\quad 0.025)$
$(5\quad 0.0041666666666666666666666666666667)$
$(6\quad 0.00059523809523809523809523809523810952381)$

Deci, $n=6$ este suficient

```
dezvT=taylor(exp(x),x,0,'Order',k+1)
```

dezvT =

$$\frac{x^6}{720}+\frac{x^5}{120}+\frac{x^4}{24}+\frac{x^3}{6}+\frac{x^2}{2}+x+1$$

```
vpa(subs(dezvT,x,1.0),10)
```

ans = 2.718055556

```
exp(1)
```

```
ans = 2.7183
```

P3. Să se aproximeze $\sqrt[3]{999}$ cu 12 zecimale exacte.

```
dg=digits;
digits(13);
syms f(x) R(n) ex
ex=sym(1)/sym(3);
f(x)=10*(1-x)^(sym(1)/sym(3));
R(n)=vpa(nchoosek(ex,n)*10^(-3*(n+1)));
R(2)
```

ans = -0.000000001111111111111

```
R(3)
```

ans = $6.172839506173e\text{-}14$

$n = 4$ este suficient

```
P=taylor(f,x,0,'Order',4)
```

P(x) =

$$-\frac{50\,x^3}{81} - \frac{10\,x^2}{9} - \frac{10\,x}{3} + 10$$

```
vpa(subs(P,x,0.001))
```

ans(x) = 9.996665554938

```
vpa(999^ex)
```

ans = 9.996665554938

```
digits(dg)
```