The Vandermonde Matrix given in the exam paper:

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 |
| 1 | 4 | 9 | 16 | 8 | 2 | 15 | 13 | 13 | 15 | 2 | 8 | 16 | 9 | 4 | 1 |
| 1 | 8 | 10 | 13 | 6 | 12 | 3 | 2 | 15 | 14 | 5 | 11 | 4 | 7 | 9 | 16 |

R2 → R2 – 13 · R1

R3 → R3 – 16 · R1

R4 → R4 – 4 · R1

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | **1** | 1 | 1 | 1 |
| 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | **0** | 1 | 2 | 3 |
| 2 | 5 | 10 | 0 | 9 | 3 | 16 | 14 | 14 | 16 | 3 | 9 | **0** | 10 | 5 | 2 |
| 14 | 4 | 6 | 9 | 2 | 8 | 16 | 15 | 11 | 10 | 1 | 7 | **0** | 3 | 5 | 12 |

R1 → R1 – R2

R3 → R3 – 10 · R2

R4 → R4 – 3 · R2

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| 13 | 12 | 11 | 10 | 9 | 8 | 7 | 6 | 5 | 4 | 3 | 2 | **1** | **0** | 16 | 15 |
| 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | **0** | **1** | 2 | 3 |
| 3 | 13 | 8 | 5 | 4 | 5 | 8 | 13 | 3 | 12 | 6 | 2 | **0** | **0** | 2 | 6 |
| 16 | 3 | 2 | 2 | 9 | 12 | 0 | 13 | 6 | 2 | 7 | 10 | **0** | **0** | 16 | 3 |

R3 → 9 · R3

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| 13 | 12 | 11 | 10 | 9 | 8 | 7 | 6 | 5 | 4 | 3 | 2 | **1** | **0** | 16 | 15 |
| 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | **0** | **1** | 2 | 3 |
| 10 | 15 | 4 | 11 | 2 | 11 | 4 | 15 | 10 | 6 | 3 | 1 | **0** | **0** | 1 | 3 |
| 16 | 3 | 2 | 2 | 9 | 12 | 0 | 13 | 6 | 2 | 7 | 10 | **0** | **0** | 16 | 3 |

R1 → R1 – 16 · R3

R2 → R2 – 2 · R3

R4 → R4 – 16 · R3

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| 6 | 10 | 15 | 4 | 11 | 2 | 11 | 4 | 15 | 10 | 6 | 3 | **1** | **0** | **0** | 1 |
| 2 | 10 | 16 | 3 | 5 | 5 | 3 | 16 | 10 | 2 | 9 | 14 | **0** | **1** | **0** | 14 |
| 10 | 15 | 4 | 11 | 2 | 11 | 4 | 15 | 10 | 6 | 3 | 1 | **0** | **0** | **1** | 3 |
| 9 | 1 | 6 | 13 | 11 | 6 | 4 | 11 | 16 | 8 | 10 | 11 | **0** | **0** | **0** | 6 |

R4 → 3 · R4

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| 6 | 10 | 15 | 4 | 11 | 2 | 11 | 4 | 15 | 10 | 6 | 3 | **1** | **0** | **0** | 1 |
| 2 | 10 | 16 | 3 | 5 | 5 | 3 | 16 | 10 | 2 | 9 | 14 | **0** | **1** | **0** | 14 |
| 10 | 15 | 4 | 11 | 2 | 11 | 4 | 15 | 10 | 6 | 3 | 1 | **0** | **0** | **1** | 3 |
| 10 | 3 | 1 | 5 | 16 | 1 | 12 | 16 | 14 | 7 | 13 | 16 | **0** | **0** | **0** | 1 |

R1 → R1 – R4

R2 → R2 – 14 · R4

R3 → R3 – 3 · R4

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| 13 | 7 | 14 | 16 | 12 | 1 | 16 | 5 | 1 | 3 | 10 | 4 | **1** | **0** | **0** | **0** |
| 15 | 2 | 2 | 1 | 2 | 8 | 5 | 13 | 1 | 6 | 14 | 11 | **0** | **1** | **0** | **0** |
| 14 | 6 | 1 | 13 | 5 | 8 | 2 | 1 | 2 | 2 | 15 | 4 | **0** | **0** | **1** | **0** |
| 10 | 3 | 1 | 5 | 16 | 1 | 12 | 16 | 14 | 7 | 13 | 16 | **0** | **0** | **0** | **1** |

The Parity Check Matrix obtained is below:

Text

Description automatically generated

Text

Description automatically generated

We get the check digits as follows:

The Generator Matrix is as follows:

b) Implementation and checking results:

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| x | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 |
| -x | 16 | 15 | 14 | 13 | 12 | 11 | 10 | 9 | 8 | 7 | 6 | 5 | 4 | 3 | 2 | 1 |
| x2 | 1 | 4 | 9 | 16 | 8 | 2 | 15 | 13 | 13 | 15 | 2 | 8 | 16 | 9 | 4 | 1 |
|  | 1 | 6 |  | 2 |  |  |  | 5 | 3 |  |  |  | 8 |  | 7 | 4 |
| x-1 | 1 | 9 | 6 | 13 | 7 | 3 | 5 | 15 | 2 | 12 | 14 | 10 | 4 | 11 | 8 | 16 |