The Vandermonde Matrix given in the exam paper:

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 |
| 1 | 4 | 9 | 16 | 8 | 2 | 15 | 13 | 13 | 15 | 2 | 8 | 16 | 9 | 4 | 1 |
| 1 | 8 | 10 | 13 | 6 | 12 | 3 | 2 | 15 | 14 | 5 | 11 | 4 | 7 | 9 | 16 |

R2 → R2 – 13 · R1

R3 → R3 – 16 · R1

R4 → R4 – 4 · R1

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | **1** | 1 | 1 | 1 |
| 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | **0** | 1 | 2 | 3 |
| 2 | 5 | 10 | 0 | 9 | 3 | 16 | 14 | 14 | 16 | 3 | 9 | **0** | 10 | 5 | 2 |
| 14 | 4 | 6 | 9 | 2 | 8 | 16 | 15 | 11 | 10 | 1 | 7 | **0** | 3 | 5 | 12 |

R1 → R1 – R2

R3 → R3 – 10 · R2

R4 → R4 – 3 · R2

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| 13 | 12 | 11 | 10 | 9 | 8 | 7 | 6 | 5 | 4 | 3 | 2 | **1** | **0** | 16 | 15 |
| 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | **0** | **1** | 2 | 3 |
| 3 | 13 | 8 | 5 | 4 | 5 | 8 | 13 | 3 | 12 | 6 | 2 | **0** | **0** | 2 | 6 |
| 16 | 3 | 2 | 2 | 9 | 12 | 0 | 13 | 6 | 2 | 7 | 10 | **0** | **0** | 16 | 3 |

R3 → 9 · R3

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| 13 | 12 | 11 | 10 | 9 | 8 | 7 | 6 | 5 | 4 | 3 | 2 | **1** | **0** | 16 | 15 |
| 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | **0** | **1** | 2 | 3 |
| 10 | 15 | 4 | 11 | 2 | 11 | 4 | 15 | 10 | 6 | 3 | 1 | **0** | **0** | 1 | 3 |
| 16 | 3 | 2 | 2 | 9 | 12 | 0 | 13 | 6 | 2 | 7 | 10 | **0** | **0** | 16 | 3 |

R1 → R1 – 16 · R3

R2 → R2 – 2 · R3

R4 → R4 – 16 · R3

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| 6 | 10 | 15 | 4 | 11 | 2 | 11 | 4 | 15 | 10 | 6 | 3 | **1** | **0** | **0** | 1 |
| 2 | 10 | 16 | 3 | 5 | 5 | 3 | 16 | 10 | 2 | 9 | 14 | **0** | **1** | **0** | 14 |
| 10 | 15 | 4 | 11 | 2 | 11 | 4 | 15 | 10 | 6 | 3 | 1 | **0** | **0** | **1** | 3 |
| 9 | 1 | 6 | 13 | 11 | 6 | 4 | 11 | 16 | 8 | 10 | 11 | **0** | **0** | **0** | 6 |

R4 → 3 · R4

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| 6 | 10 | 15 | 4 | 11 | 2 | 11 | 4 | 15 | 10 | 6 | 3 | **1** | **0** | **0** | 1 |
| 2 | 10 | 16 | 3 | 5 | 5 | 3 | 16 | 10 | 2 | 9 | 14 | **0** | **1** | **0** | 14 |
| 10 | 15 | 4 | 11 | 2 | 11 | 4 | 15 | 10 | 6 | 3 | 1 | **0** | **0** | **1** | 3 |
| 10 | 3 | 1 | 5 | 16 | 1 | 12 | 16 | 14 | 7 | 13 | 16 | **0** | **0** | **0** | 1 |

R1 → R1 – R4

R2 → R2 – 14 · R4

R3 → R3 – 3 · R4

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| 13 | 7 | 14 | 16 | 12 | 1 | 16 | 5 | 1 | 3 | 10 | 4 | **1** | **0** | **0** | **0** |
| 15 | 2 | 2 | 1 | 2 | 8 | 5 | 13 | 1 | 6 | 14 | 11 | **0** | **1** | **0** | **0** |
| 14 | 6 | 1 | 13 | 5 | 8 | 2 | 1 | 2 | 2 | 15 | 4 | **0** | **0** | **1** | **0** |
| 10 | 3 | 1 | 5 | 16 | 1 | 12 | 16 | 14 | 7 | 13 | 16 | **0** | **0** | **0** | **1** |

The Parity Check Matrix obtained is below:

Text

Description automatically generated

Text

Description automatically generated

We get the check digits as follows:

The Generator Matrix is as follows:

b) Implementation and checking results:

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| x | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 |
| -x | 16 | 15 | 14 | 13 | 12 | 11 | 10 | 9 | 8 | 7 | 6 | 5 | 4 | 3 | 2 | 1 |
| x2 | 1 | 4 | 9 | 16 | 8 | 2 | 15 | 13 | 13 | 15 | 2 | 8 | 16 | 9 | 4 | 1 |
|  | 1 | 6 |  | 2 |  |  |  | 5 | 3 |  |  |  | 8 |  | 7 | 4 |
| x-1 | 1 | 9 | 6 | 13 | 7 | 3 | 5 | 15 | 2 | 12 | 14 | 10 | 4 | 11 | 8 | 16 |

(b1) when input is (1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 2), what are the check digits?

**Check digits: 2 5 4 6**

**(S1 S2 S3 S4) = (0 0 0 0)**

**p = 0 q = 0 r = 0**

**No error**

(b2) when input is (10, 3, 7, 5, 9, 1, 4, 1, 11, 8, 6, 5), what are the check digits?

**Check digits: 7 10 2 13**

**(S1 S2 S3 S4) = (0 0 0 0)**

**p = 0 q = 0 r = 0**

**No error**

(b3) correct\_if\_needed (2, 3, 3, 4, 5, 6, 7, 8, 9, 10, 11, 2, 6, 7, 7, 13)

**(S1 S2 S3 S4) = (1 2 4 8)**

**p = 0 q = 0 r = 0**

**2**

**One error present. Corrected code: 2234567891011267713**

(b4) correct\_if\_needed (5, 3, 2, 8, 6, 9, 1,1, 2, 6, 7, 4, 15, 1, 14, 9)

**(S1 S2 S3 S4) = (8 13 2 16)**

**p = 0 q = 0 r = 0**

**1**

**One error present. Corrected code: 5328691102674151149**

(b5) correct\_if\_needed (10, 11, 3, 5, 3, 9, 8, 8,6, 11, 14, 15, 6, 6, 15, 8)

**(S1 S2 S3 S4) = (2 5 16 11)**

**p = 10 q = 10 r = 14**

**Two errors present. Corrected Code: 71135398861114156616**

(b6) correct\_if\_needed (1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 5, 12, 12, 5)

**(S1 S2 S3 S4) = (12 10 7 1)**

**p = 16 q = 10 r = 5**

**Two errors present. Corrected Code: 1111111111110512**

(b7) correct\_if\_needed (10, 11, 3, 5, 2, 9, 8, 6, 6, 11, 14, 15, 6, 6, 15, 9),

**(S1 S2 S3 S4) = (0 0 0 0)**

**p = 0 q = 0 r = 0**

**No error**

(b8) correct\_if\_needed (7, 11, 3, 5, 3, 9, 8, 8, 6, 11, 14, 15, 6, 6, 16, 8),

**(S1 S2 S3 S4) = (0 0 0 0)**

**p = 0 q = 0 r = 0**

**No error**

(b9) correct\_if\_needed (1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 5, 5, 5, 5, 5)

**(S1 S2 S3 S4) = (2 8 16 1)**

**p = 15 q = 10 r = 10**

**Two errors present. Corrected Code: 1111111151151655**

(b10) correct\_if\_needed (5, 5, 5, 5, 5, 5, 4, 4, 4, 4, 4, 4, 5, 5, 5, 5)

**(S1 S2 S3 S4) = (6 11 2 1)**

**p = 7 q = 1 r = 10**

**Two errors present. Corrected Code: 5512555448444555**