Artificial Intelligence: Answers to exercises for Tutorial 1 on Propositional Logic

Exercises

1. Consider the proposition:

   $$R \Rightarrow (\neg R \Rightarrow W)$$

   How many models are there for this proposition? (Do model checking, make a truth table.)

   (a) 2
   (b) 4
   (c) 6
   (d) 8

   Answer: b (4), i.e. the formula is a tautology (always true, a valid statement). Note: a model for propositional formula α is a truth assignment to all atomic propositions that makes the formula α true. The formula is true in that model, or the possible model is a model for the formula. With only 2 atomic propositions (R and W) there are 4 possible models. In this case all of them are models for the given proposition.

2. Consider the proposition:

   $$R \Rightarrow (\neg S \Rightarrow W)$$

   How many models are there for this proposition? (Do model checking, make a truth table.)

   (a) 1
   (b) 3
   (c) 5
   (d) 7

Answer: d (7). Out of the possible 8 models (since the formula has three atoms, we have 2^3 possible models) in one of them, namely when $R = 1$, $S = 0$, $W = 0$, the proposition is false.

3. We are given the following premises:

    1. $bread \vee earlyMeeting$
    2. $(tea \vee \text{coffee}) \wedge juice$
    3. $earlyMeeting \Rightarrow yoghurt$
    4. $yoghurt \Rightarrow \neg coffee$
    5. $\neg yoghurt$

    Important note: the slides leave implicit how to go from a formula in CNF, e.g., $(tea \vee \text{coffee}) \wedge juice$, to formulas to which one can apply the resolution rule, i.e., disjunctions of literals. The step which is left implicit is elimination of conjunction: if $p \wedge q$ holds, we can conclude from that that both p and q hold. Thus if we have a CNF formula like the former, we can apply conjunction elimination and use the formulas $tea \vee \text{coffee}$ and $juice$ separately in the proof.

    The question is whether we can prove *bread* from the premises above using resolution. Which of the following answers is correct?

    (a) Yes, the conclusion follows

    (b) No, the conclusion does not follow, but if you add the premise *juice* the conclusion can be derived

    (c) No, the conclusion does not follow, but if you add the premise *tea* the conclusion can be derived

    Answer: a, yes the conclusion follows. We can derive the empty clause using the following steps, based on the given premises. In brackets behind each step you can find the numbers of the premises and the proof rule based on which the formula can be derived:

    6. $\neg bread$ (add negation of conclusion)
    7. $earlyMeeting$ (1,6, Resolution rule)
    8. $\neg earlyMeeting \vee yoghurt$ (3, Implication elimination)
    9. $\neg earlyMeeting$ (8,5, Resolution rule)
    10. empty clause (7,9, Resolution rule)

    We have added the negation of our desired conclusion to our knowledge base, and proven that in this case a contradiction (the empty clause) can be derived. Therefore the opposite must be the case, i.e., we can derive that bread follows from our premises.

Note that in this case we did not need formula 2, i.e., we would not have needed elimination of conjunction.

4. A KB contains the following logical sentences:

   (i) $K \vee L$
   (ii) $K \Rightarrow M$
   (iii) $\neg L$

   Prove that M follows from this KB using resolution.

   Answer:
   Step 1: add negation of desired conclusion, i.e., $\neg M$, to KB
   Step 2: Translate formulas to CNF:
   (i) $K \vee L$ (already CNF)
   (ii) $K \Rightarrow M \equiv \neg K \vee M$
   (iii) $\neg L$ (already CNF)

   Step 3: apply resolution rule to derive the empty clause; if this can be done, i.e., a contraduction is derived from assuming the opposite of the conclusion, we can conclude that indeed the conclusion follows from the KB.

   1. $\neg M$ (negation of desired conclusion)
   2. $K \vee L$ (i)
   3. $\neg K \vee M$ (ii)
   4. $\neg L$ (iii)
   5. K (2,4, resolution rule)
   6. M (3,5, resolution rule)
   7. empty clause (1,6, resolution rule)

5. We are given the following premises:

   - $(P \vee Q) \wedge (P \vee T)$
   - $(Q \wedge T) \Rightarrow (V \Rightarrow W)$
   - $\neg[(T \Rightarrow S) \Rightarrow \neg(S \Rightarrow W)]$

   The question is whether we can prove $V \Rightarrow S$ from these premises. Which of the following answers is correct?

   (a) Yes, the conclusion follows.

(b) No, the conclusion does not follow, but if you add the premise T the conclusion can be derived.

(c) No, the conclusion does not follow, but if you add the premise $\neg S$ the conclusion can be derived.

(d) No, the conclusion does not follow, but if you add the premise V the conclusion can be derived.

The way to solve this type of problems is as follows. Apply resolution either until you have found a contradiction (and then alternative (a) is apparently correct) or until there is no valid application of the resolution rule that produces a new sentence. In the latter case, comparing the remaining three alternatives with the list of sentences you have constructed by means of resolution will quickly tell you the correct alternative.

Answer: b.

1. $P \vee Q$ (From first premise)
2. $P \vee T$ (From first premise)
3. $\neg Q \vee \neg T \vee \neg V \vee W$ (From second premise)
4. $\neg T \vee S$ (From third premise)
5. $\neg S \vee W$ (From third premise)
6. V (From negated conclusion)
7. $\neg S$ (From negated conclusion)
8. $\neg Q \vee \neg T \vee W$ (Resolvent from 6 and 3)
9. $\neg T$ (Resolvent from 7 and 4)
10. P (Resolvent from 2 and 9)
11. $\neg T \vee W$ (Resolvent from 4 and 5)
12. $P \vee \neg T \vee W$ (Resolvent from 1 and 8)
13. $P \vee S$ (Resolvent from 2 and 4)
14. $W \vee P$ (Resolvent from 5 and 13)

That is all we can resolve. Since we can't derive a contradiction (a) is not true. If we add T we have a contradiction with $\neg T$ (9). Hence, answer (b) is correct. (The other two alternatives are not correct.)

6. Of the following formulas, only one is NOT a Horn clause. Which one?

(a) $\neg p \vee \neg q$

(b) p

(c) $\neg p \vee \neg q \vee r \vee s$

(d) $\neg tea \vee coffee$

Answer: c. A Horn clause is a disjunction of literals with at most one positive literal. (c) does not adhere to this, since it has two positive literals (r and s). (b) is also a Horn clause, since it is a disjunction, albeit with only one disjunct, and this disjunct is a positive literal (and thus a disjunct with at most one positive literal). This can also be concluded by recalling that Prolog statements are restricted to Horn clauses. We know that we can state facts in Prolog which is atomic proposition, such as p. Thus also from that we can conclude that p must be a Horn clause.

7. (For discussion. There is not one correct answer, I guess.)
   In order to let a machine reason logically it is required to use a language that has a clear semantics, i.e. names denote one single object and predicates and function symbols have unambiguous meanings. Do you think this makes a machine a better reasoner than people? What about vague predicates like “young”, “large”, “friendly”, should we try to avoid them to become more “logical” and “rational” or is it possible to make the machine also be competent to use these fuzzy terms in a “logical” way?