Artificial Intelligence: Exercises for Tutorial 1 on Propositional Logic

Exercises

1. Consider the proposition:

   $$R \Rightarrow (\neg R \Rightarrow W)$$

   How many models are there for this proposition? (Do model checking, make a truth table.)

   (a) 2

   (b) 4

   (c) 6

   (d) 8

2. Consider the proposition:

   $$R \Rightarrow (\neg S \Rightarrow W)$$

   How many models are there for this proposition? (Do model checking, make a truth table.)

   (a) 1

   (b) 3

   (c) 5

   (d) 7

3. We are given the following premises:

   1. $bread \vee earlyMeeting$
   2. $(tea \vee \text{coffee}) \wedge juice$
   3. $earlyMeeting \Rightarrow yoghurt$
   4. $yoghurt \Rightarrow \neg coffee$

5. $\neg yoghurt$

Important note: the slides leave implicit how to go from a formula in CNF, e.g., $(tea \vee \text{coffee}) \wedge juice$, to formulas to which one can apply the resolution rule, i.e., disjunctions of literals. The step which is left implicit is elimination of conjunction: if $p \wedge q$ holds, we can conclude from that that both p and q hold. Thus if we have a CNF formula like the former, we can apply conjunction elimination and use the formulas $tea \vee \text{coffee}$ and *juice* separately in the proof.

The question is whether we can prove *bread* from the premises above using resolution. Which of the following answers is correct?

(a) Yes, the conclusion follows

(b) No, the conclusion does not follow, but if you add the premise *juice* the conclusion can be derived

(c) No, the conclusion does not follow, but if you add the premise *tea* the conclusion can be derived

4. A KB contains the following logical sentences:

(i) $K \vee L$
(ii) $K \Rightarrow M$
(iii) $\neg L$

Prove that M follows from this KB using resolution.

5. We are given the following premises:

- $(P \vee Q) \wedge (P \vee T)$
- $(Q \wedge T) \Rightarrow (V \Rightarrow W)$
- $\neg[(T \Rightarrow S) \Rightarrow \neg(S \Rightarrow W)]$

The question is whether we can prove $V \Rightarrow S$ from these premises. Which of the following answers is correct?

(a) Yes, the conclusion follows.

(b) No, the conclusion does not follow, but if you add the premise T the conclusion can be derived.

(c) No, the conclusion does not follow, but if you add the premise $\neg S$ the conclusion can be derived.

(d) No, the conclusion does not follow, but if you add the premise V the conclusion can be derived.

6. Of the following formulas, only one is NOT a Horn clause. Which one?

   (a) $\neg p \vee \neg q$

   (b) p

   (c) $\neg p \vee \neg q \vee r \vee s$

   (d) $\neg tea \vee coffee$

7. (For discussion. There is not one correct answer, I guess.)
   In order to let a machine reason logically it is required to use a language that has a clear semantics, i.e. names denote one single object and predicates and function symbols have unambiguous meanings. Do you think this makes a machine a better reasoner than people? What about vague predicates like "young", "large", "friendly", should we try to avoid them to become more "logical" and "rational" or is it possible to make the machine also be competent to use these fuzzy terms in a "logical" way?