2) 1L = 1000 cm³

Fórmula do Vol. cilindro reto

$\pi.R^2.h = 1000$ (I)

$h = \frac{1000}{\pi.R^2}$

Fórmula Área total de um cilindro reto

$A = 2.\pi.R^2 + 2.\pi.R.h$ (II)

$A = 2.\pi.R^2 + 2.\pi.R.\frac{1000}{\pi.R^2}$ $\quad A(R) = 2.\pi.R^2 + 2000.R^{-1}$

Derivando a equação

$A'(R) = 4.\pi.R - 2000.R^{-2}$

Igualando a zero para acharmos o valor R que minimiza

$4.\pi.R - 2000.R^{-2} = 0$

$R^3 = \frac{1000}{2.\pi}$

Achar valor da altura

$h = \frac{1000}{\pi.\left(\frac{10}{\sqrt[3]{2.\pi}}\right)^2}$

$R = \sqrt[3]{\frac{1000}{2.\pi}}$

$\boxed{R = \frac{10}{\sqrt[3]{2.\pi}} \text{ cm}}$ $\quad \boxed{h = \frac{10}{\pi.\frac{1}{\sqrt[3]{4.\pi^2}}} \text{ cm}}$

Área mínima para um raio $R = \sqrt[3]{\frac{1}{2\pi}}$ altura $= \frac{1}{\pi\sqrt[3]{\frac{1}{2\pi}}}$

/ /

5) $y = x^2 - 2x$, $y = x + 4$

$x^2 - 2x = x + 4$

$x^2 - 2x - x - 4 = 0$

$x^2 - 3x - 4 = 0$

$\Delta = b^2 - 4ac$

$\Delta = 9 - 4 \cdot 1 \cdot (-4)$

$\Delta = 9 + 16$

$\Delta = 25$

$x = \dfrac{(-b \pm \sqrt{\Delta})}{2a}$

$x = \dfrac{3 \pm 5}{2}$ $x' = \dfrac{3 + 5}{2} = 4$

$x'' = \dfrac{3 - 5}{2} = -1$

$-1 \leq x \leq 4$

$y = x^2 - 2x \rightarrow y = 0$

$y = x + 4 \rightarrow y = 4$

$A = \int_{-1}^{4} [(x + 4) - (x^2 - 2x)] \, dx$

$A = \int_{-1}^{4} (-x^2 + 3x + 4) \, dx$

$A = \int_{-1}^{4} x^2 \, dx = \dfrac{65}{3}$ $A = \int_{-1}^{4} 3x \, dx = \dfrac{45}{2}$

$A = \int_{-1}^{4} 4 \, dx = 20$

$= -\dfrac{65}{3} + \dfrac{45}{2} + 20 = \dfrac{125}{6}$

a)

4) $\int x^2 \cos(2x)\, dx$

$\{x^2 \int \cos 2x\, dx - \int \frac{d}{dx}(x^2) . \int \cos 2x\, dx\} dx$

$= x^2 . \frac{\sin 2x}{2} - \int 2x . \frac{\sin 2x}{2}\, dx$

$= \frac{1}{2} x^2 \sin 2x - \int x \sin 2x\, dx$

$= \frac{1}{2} x^2 \sin 2x - \{x \int \sin 2x\, dx - \int \frac{d}{dx}(x) . \int \sin 2x\, dx\}$

$= \frac{1}{2} x^2 \sin x - \{x - \frac{\cos 2x}{2} - \int 1x - \frac{\cos 2x}{2}\, dx\}$

$= \frac{x^2 \sin 2x}{2} + \frac{x \cos 2x}{2} - \frac{1}{2} \int \cos 2x\, dx$

$= \boxed{\frac{x^2 \sin 2x}{2} + \frac{x \cos 2x}{2} - \frac{\sin 2x}{4} + c}$

b) $\int 2x e^x\, dx$

$\int f(x)\, g'(x)\, dx = f(x) g(x) - \int f'(x) g(x)\, dx$

let $f(x) = 2x \qquad g'(x) = e^x$
$f'(x) = 2 \qquad g(x) = e^x$

$\int 2x e^x\, dx = -2x e^x - \int 2e^x\, dx$

$= 2x e^x + 2 \int e^x\, dx = 2x e^x + \frac{2e^x}{2} + c$

$= \boxed{2x e^x - e^x + c}$