5.12

1) a) $S = \pi r^2 + a^2$

r: Raio do círculo

a: lado do quadrado

$2\pi r + 4a = l \qquad a = \dfrac{l - 2\pi r}{4}$

$S = \pi r^2 + \left(\dfrac{l - 2\pi r}{4}\right)^2$

$S = \pi r^2 + \dfrac{l^2 - 4l\pi r + 4\pi^2 r^2}{16}$

$S' = 2\pi r + \dfrac{-4l\pi + 4\pi^2 \cdot 2r}{16}$

$2\pi r + \dfrac{-4l\pi + 8\pi^2 r}{16} = 0$

$32\pi r - 4l\pi + 8\pi^2 r = 0$

$r(32\pi + 8\pi^2) = 4l\pi$

$r = \dfrac{4l\pi}{32\pi + 8\pi^2} = \dfrac{l}{8 + 2\pi}$

$S'' = 2\pi + \dfrac{8\pi^2}{16}$

$S''\left(\dfrac{l}{8 + 2\pi}\right) = 2\pi + \dfrac{8\pi^2}{16} > 0$ ∴ ponto mínimo

$r = \dfrac{l}{8 + 2\pi}$ e $a = \dfrac{l}{4 + \pi}$

b) $A(\text{círculo}) = \pi r^2 = \pi \dfrac{l^2}{2\pi} = \dfrac{l^2}{2}$

$A(\text{quadrado}) = \dfrac{l^2}{16}$

$A(\text{círculo}) > A(\text{quadrado})$

Sendo assim, usaremos o comprimento do fio para fazer um círculo de raio $r = \dfrac{l}{2\pi}$

5.14

5) $\lim\limits_{x \to 3} \frac{6 - 2x + 3^2 - x^3}{x^4 - 3x^3 - x + 3} = \lim\limits_{x \to 3} \frac{-2 + 6x - 3x^2}{4x^3 - 9x^2 - 1} = \frac{-11}{26}$

10) $\lim\limits_{x \to \infty} \frac{5 - x + x^2}{2 - x - 2x^2} = \lim\limits_{x \to \infty} \frac{-1 + 2x}{-1 - 4x} = \lim\limits_{x \to \infty} \frac{2}{4} = -\frac{1}{2}$

13) $\lim\limits_{x \to 0} \frac{x}{e^x - \cos x} \quad \lim\limits_{x \to 0} \frac{1}{e^x + \operatorname{sen} x} = \frac{1}{1+0} = 1$

14) $\lim\limits_{x \to +\infty} x^2\left(e^{\frac{1}{x}} - 1\right) \quad 0 \cdot \infty$

$\lim\limits_{x \to +\infty} \left[\frac{e^{\frac{1}{x}} - 1}{\frac{1}{x^2}}\right] = \lim\limits_{x \to +\infty} \left[\frac{e^{\frac{1}{x}}\left(-\frac{1}{x^2}\right)}{\frac{-2x}{x^4}}\right]$

$\lim\limits_{x \to +\infty} e^{\frac{1}{x}} \cdot \frac{1}{x^2} \cdot \frac{x^4}{2x} = \lim\limits_{x \to +\infty} \frac{x\, e^{\frac{1}{x}}}{2} = +\infty$

22) $\lim\limits_{x \to \infty} \frac{\ln x}{\sqrt[3]{x}}$

$\lim\limits_{x \to \infty} \frac{\frac{1}{x}}{\frac{1}{3}x^{-\frac{2}{3}}} = \lim\limits_{x \to \infty} \frac{3\sqrt[3]{x^2}}{x}$

$= \lim \frac{3 \cdot \frac{2}{3} x^{-\frac{1}{3}}}{1} = \lim\limits_{x \to \infty} \frac{2}{\sqrt[3]{x}} = 0$

33) $\lim\limits_{x \to +\infty} \frac{x^{2/3}}{(x^2+2)^{1/3}}$

$= \left(\lim\limits_{x \to \infty} \frac{x^2}{x^2+2}\right)^{\frac{1}{3}} = \left(\lim\limits_{x \to \infty} \frac{2x}{2x}\right)^{\frac{1}{3}} = 1^{\frac{1}{3}} = 1$

data . .

23) $L = 60x - 12x^2 - 2x^3 - 6x^2 - 18x - 60$

$L = -2x^3 - 18x^2 + 42x - 60$

$L' = -6x^2 - 36x + 42$

$L' = 0$

$-6x^2 - 36x + 42 = 0$

$x^2 + 6x - 7 = 0$

$x_1 = 1 \qquad x_2 = -7$

$L'' = -12x - 36$

$L'' = -12 - 36 < 0$

$x_1 = 1$ ponto Máximo

26) $V = 2xy = 3 \qquad y = \frac{3}{2x}$

$A = (4+2x)y + 4x$

$A = (4+2x) \cdot \frac{3}{2x} + 4x$

$A = \frac{12 + 6x + 8x^2}{2x}$

$A' = \frac{2x(6 + 16x) - (12 + 6x + 8x^2) \cdot 2}{4x^2}$

$A' = 0 \qquad 12x + 32x^2 - 24 - 12x - 16x^2 = 0$

$16x^2 - 24 = 0$

$16x^2 = 24$

$x^2 = \frac{24}{16} = \frac{12}{8} = \frac{3}{2} \quad \therefore \quad x = \sqrt{\frac{3}{2}}$

$A' = \frac{16x^2 - 24}{4x^2} = 4 - \frac{6}{x^2}$

$A'' = \frac{12}{x^3}$

$A''\left(\sqrt{\frac{3}{2}}\right) > 0 \qquad x = \sqrt{\frac{3}{2}}$

ponto máximo

Dimensões $2m \times \frac{\sqrt{6}}{2} m \times \frac{\sqrt{6}}{2} m$

18) $T = \frac{\sqrt{40^2 + (100-x)^2}}{18} + \frac{x}{50}$

$T' = \frac{-2(100-x)}{2\sqrt{40^2 + (100-x)^2}} \cdot \frac{1}{18} + \frac{1}{50}$

$T' = 0 \quad \frac{-(100-x)}{18\sqrt{40^2 + (100-x)^2}} + \frac{1}{50} = 0$

$-50(100-x) + 18\sqrt{40^2 + (100-x)^2} = 0$

$-25(100-x) + 9\sqrt{40^2 + (100-x)^2} = 0$

$x_1 = 115,43$

$x_2 = 84,56$

$9\sqrt{40^2 + (100-x)^2} = 25(100-x)$

$81(40^2 + (100-x)^2) = 625(100-x)^2$

$81(11600 - 200x + x^2) = 625(10000 - 200x + x^2)$

$939600 - 16200x + 81x^2 = 6250000 - 125000x + 625x^2$

$544x^2 - 108800x + 5310400 = 0$

$34x^2 - 6800x + 331900 = 0$

$17x^2 - 3400x + 165950 = 0$

$T'' = \frac{1}{18} \cdot \frac{\sqrt{40^2 + (100-x)^2} + (100-x) \cdot \frac{-2(100-x)}{2\sqrt{40^2 + (100-x)^2}}}{40^2 + (100-x)^2}$

$T'' = \frac{1}{18} \cdot \frac{\sqrt{40^2 + (100-x)^2} - \frac{(100-x)^2}{\sqrt{40^2 + (100-x)^2}}}{40^2 + (100-x)^2}$

$T'' = \frac{1}{18} \cdot \frac{40^2 + (100-x)^2 - (100-x)^2}{\sqrt{40^2 + (100-x)^2}\; 40^2 + (100-x)^2}$

$T'' = \frac{1}{18} \cdot \frac{40^2}{[40^2 + (100-x)^2]\sqrt{40 + (100-x)^2}}$

$T''|_{84,56} > 0 \quad x = 84,56$ ponto mínimo

$0 \leq x \leq 100$ mínimo é em $x \cong 84,56$km

$T(84,56) = \frac{\sqrt{40^2 + (100-84,56)^2}}{18} + \frac{84,56}{50}$

$= \frac{\sqrt{1600 + 238,3936}}{18} + 1,6912$

$\cong \frac{42,87}{18} + 1,6912$

$\cong 2,38 + 1,6912 \cong 4,07$ horas

$T(0) = \frac{\sqrt{1600 + 10000}}{18} + 0 = 5,98$

$T(100) = \frac{\sqrt{1600 + 0}}{18} + \frac{100}{50} = 2,22 + 2 = 4,22$

data

6) $V = (a-2x)^2 x = (a^2 - 4ax + 4x^2)x$

$V = a^2x - 4ax^2 + 4x^3$

$V' = a^2 - 8ax + 12x^2$

$a^2 - 8ax + 12x^2 = 0$

$x = a/2$ ou $x = a/6$

$V'' = -8a + 24x$

$V''(a/2) = -8a + 12a = 4a > 0$ $a/2$ ponto de mínimo

$V''\left(\frac{a}{6}\right) = -8a + 24 \cdot \frac{a}{6} = -4a < 0$ $a/6$ ponto de máximo

7) $\overline{AR} = \sqrt{a^2 + (c-x)^2}$

$\overline{BR} = \sqrt{b^2 + x^2}$

$L(x) = \sqrt{a^2 + (c-x)^2} + \sqrt{b^2 + x^2}$

$L(x) = \sqrt{16 + (12-x)^2} + \sqrt{4 + x^2}$

$L'(x) = \frac{1}{2} \frac{2(12-x)(-1)}{\sqrt{16+(12-x)^2}} + \frac{2x}{2\sqrt{4+x^2}}$

$= \frac{x-12}{\sqrt{16+(12-x)^2}} + \frac{x}{\sqrt{4+x^2}}$

$L''(x) \frac{16}{(x^2-24x+160)^{3/2}} + \frac{4}{(x^2+4)^{3/2}}$

$L' = 0$ $\frac{x-12}{\sqrt{16+(12-x)^2}} + \frac{x}{\sqrt{4 \times x}} = 0$

$(x-12)\sqrt{4+x^2} + x\sqrt{16+(12-x)^2} = 0$

$(x-12)^2(4+x^2) = x^2(16 + (12-x)^2)$

$(x^2 - 24x + 144)(4 + x^2) = x^2(16 + 144 - 24x + x^2)$

$12x^2 + 96x - 576 = 0$

$x^2 + 8x - 48 = 0$

$x_1 = 4$ $x_2 = -12$ não interessa

$L''(4) = \frac{3\sqrt{5}}{100} > 0$ $x = 4$ ponto mínimo

$L(0) = 12,64 + 2 = 14,64$

$L(12) = 4 + 12,16 = 16,16$

$L(4) = 8,94 + 4,47 = 13,41$

$x = 4$ é o ponto de mínimo procurado