

6.2) 7)

$$\int \left(\sqrt{2x} - \frac{1}{\sqrt{2x}} \right) dx$$

$$\int \left(\sqrt{2} \cdot x^{\frac{1}{2}} - \frac{1}{\sqrt{2}} \cdot x^{-\frac{1}{2}} \right) dx = \frac{\sqrt{2} x^{\frac{3}{2}}}{\frac{3}{2}} - \frac{1}{\sqrt{2}} \cdot \frac{x^{\frac{1}{2}}}{\frac{1}{2}} + C$$

$$\frac{2\sqrt{2} x^{\frac{3}{2}}}{3} - \frac{2}{\sqrt{2}} x^{\frac{1}{2}} + C = \sqrt{2x} \left(\frac{2}{3} x - 1 \right) + C$$

$$\frac{d}{dx} \left(\frac{2\sqrt{2} x^{\frac{3}{2}}}{3} - \frac{2}{\sqrt{2}} x^{\frac{1}{2}} + C \right) = \frac{2\sqrt{2}}{3} \cdot \frac{3}{2} x^{\frac{1}{2}} - \frac{2}{\sqrt{2}} \cdot \frac{1}{2} x^{-\frac{1}{2}} = \sqrt{2x} - \frac{1}{\sqrt{2x}}$$

$$= \sqrt{2x} - \frac{1}{\sqrt{2x}}$$

$$b) \int \frac{x^3 + 2x^2 - 1}{x^4} dx$$

$$\int (x + 2x^{-2} - x^{-4}) dx = \frac{x^2}{2} + 2 \cdot \frac{x^{-1}}{-1} - \frac{x^{-3}}{-3} + C = \frac{x^2}{2} - 2 + \frac{1}{3x^3} + C$$

$$\frac{d}{dx} \left(\frac{x^2}{2} - 2 + \frac{1}{3x^3} + C \right) = \frac{2x}{2} - \frac{-2}{x^2} + \frac{-1 \cdot 3x^2}{9x^6} = x + \frac{2}{x^2} - \frac{1}{x^4}$$

$$= \frac{x^3 + 2x^2 - 1}{x^4}$$

$$17) \int \left(\frac{e^x}{2} + \sqrt{x} + \frac{1}{x} \right) dx$$

$$\frac{1}{2} e^x + \frac{x^{\frac{3}{2}}}{\frac{3}{2}} + \ln|x| + C = \frac{1}{2} e^x + \frac{2}{3} x^{\frac{3}{2}} + \ln|x| + C$$

36) $f'(x) + \tan x = 0$

$f'(x) = -\tan x$

$\int -\tan x \, dx = -\ln |\cos x| + C$

$f(x) = -\ln |\cos x| + C$

$f(0) = \ln 1 + C = 2 \quad C = 2 - \ln 1 = 2$

$\therefore f(x) = -\ln |\cos x| + 2$

4(1) 2) $\int (x^3 - 2)^{1/3} x^2 \, dx$

$u = x^3 - 2 \quad du = 3x^2 \, dx$

$\int (x^3 - 2)^{1/3} x^2 \, dx = \frac{1}{3} \int (u)^{1/3} du + C$

$\frac{1}{3} \cdot \frac{3}{4} (x^3 - 2)^{4/3} + C$

6) $\int (e^{2x} + 2)^{1/3} e^{2x} \, dx$

$u = e^{2x} + 2 \quad du = 2e^{2x} \, dx$

$\int (e^{2x} + 2)^{1/3} e^{2x} \, dx = \frac{1}{2} \int (u)^{1/3} du + C$

$\frac{3}{8} (e^{2x} + 2)^{4/3} + C$

8) $\int \frac{e^{1/x} + 2}{x^2} \, dx$

$\int \frac{e^{1/x}}{x^2} \, dx + \int \frac{2}{x^2} \, dx = -e^{1/x} + 2 \cdot \frac{x^{-1}}{-1} + C = -e^{1/x} - 2 + C$

$u = e^{1/x}$

$du = e^{1/x} \cdot \frac{-1}{x^2} \, dx$

10) $\int \sin^4 x \cos x \, dx$

$u = \sin x$

$du = \cos x \, dx \quad \int \sin^3 x \cos x \, dx = \frac{\sin^4 x}{4} + C$

$$20) \int \sin^3 \theta \cos \theta d\theta$$

$$\int (\sin \theta)^2 \cos \theta d\theta = \frac{(\sin \theta)^3}{3} + C = \frac{3}{4} \sin^{\frac{4}{3}} \theta + C$$

$$31) \int x e^{-x^2} dx$$

$$u: x^2$$

$$\frac{1}{6} e^{-x^2} + C$$

$$du: 2x dx$$

$$66) 21) \int \ln(1-x) dx$$

$$u: \ln(1-x) \Rightarrow du: \frac{-1}{1-x} dx$$

$$dv: dx = v: x$$

$$I = \ln(1-x) x - \int x \frac{-1}{1-x} dx$$

$$I = x \ln(1-x) + \int \left(-1 + \frac{1}{1-x} \right) dx$$

$$I = x \ln(1-x) - x - \ln(1-x) + C$$

$$I = (x-1) \ln(1-x) - x + C$$

$$41) \int (x+1) \cos 2x dx$$

$$u: x+1 \Rightarrow du: dx$$

$$dv: \cos 2x dx$$

$$v: \int \cos 2x dx = \frac{1}{2} \sin 2x$$

$$I = (x+1) \frac{1}{2} \sin 2x - \int \frac{1}{2} \sin 2x dx$$

$$x+1 \sin 2x + \frac{1}{4} \cos 2x + C$$

$$6) \int \cos^3 x dx$$

$$u: \cos^2 x \Rightarrow du: -2 \cos x \sin x dx$$

$$dv: \cos x dx \Rightarrow v: \sin x$$

$$I = \cos^2 x \sin x - \int \sin x (-2) \cos x \sin x dx$$

$$\cos^2 x \sin x + 2 \int \sin^2 x \cos x dx$$

$$\cos^2 x \sin x + 2 \frac{\sin^3 x}{3} + C$$

$$7) \int e^x \cos x \, dx$$

$$u: e^x \quad du: e^x dx$$

$$dv: \cos x \, dx \quad v: \int \cos x \, dx = \sin x$$

$$I: e^x \sin x - \int \sin x e^x \, dx$$

$$u: e^x \quad du: e^x dx$$

$$dv: \sin x \, dx \quad v: -\cos x$$

$$I: e^x \sin x - 2 \left[e^x (-\cos x) - \int -\cos x e^x \, dx \right]$$

$$2e^x \sin x + 4e^x \cos x - 4I$$

$$6I: 2e^x \sin x + 4e^x \cos x$$

$$\frac{1}{6} \left(2e^x \sin x + 4e^x \cos x \right) + C$$

$$10) \int x^2 \cos ax \, dx$$

$$u: x^2 \quad du: 2x \, dx$$

$$dv: \cos ax \, dx \quad v: \int \cos ax \, dx = \frac{1}{a} \sin ax$$

$$I: x^2 \cdot \frac{1}{a} \sin ax - \int \frac{1}{a} \sin ax \cdot 2x \, dx \quad I: x^2 \sin ax - 2 \int x \sin ax \, dx$$

$$u: x \quad du: dx \quad dv: \sin ax \, dx \quad v: -\frac{1}{a} \cos ax$$

$$I: x^2 \sin ax - 2 \left[x \cdot \left(-\frac{1}{a} \cos ax \right) - \int -\frac{1}{a} \cos ax \, dx \right]$$

$$\int \frac{1}{a} \sin ax + \frac{2x}{a^2} \cos ax - \frac{2}{a^2} \sin ax \frac{1}{a} + C$$

$$\frac{1}{a} \sin ax + \frac{2x}{a^2} \cos ax - \frac{2}{a^3} \sin ax + C$$

6. 11) 1b)

$$\int_1^2 [2x(x+1)] dx = \int_1^2 (2x^2 + 2x) dx = \left[\frac{2x^3}{3} + 2 \frac{x^2}{2} \right]_1^2$$

$$2 \cdot \frac{7}{3} + 2 \cdot \frac{3}{2} = \frac{14}{3} + 3 = \frac{14+9}{3} = \frac{23}{3}$$

$$6c) \int_2^3 (2x+1) dx$$

$$f(x) = 2x+1 \quad 2x+1 \geq 0 \quad 2x \geq -1$$

$$x \geq -\frac{1}{2}$$

$$f(x) = 2x+1 \quad \text{é} \quad \text{paralela} \quad \text{para} \quad 2$$

$$x \in [2, 3]$$

$$12) \int_{-1}^2 x(1+x^2) dx = \int_{-1}^2 (x+x^3) dx = \left[\frac{x^2}{2} + \frac{x^4}{4} \right]_{-1}^2 = 2 + \frac{16}{4} - \frac{1}{2} - \frac{1}{4} = 2 + 4 - \frac{1}{2} - \frac{1}{4} = 6 - \frac{3}{4} = \frac{24-3}{4} = \frac{21}{4}$$

$$\frac{21}{4}$$

$$10$$

$$17) \int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \sin x \cos x dx$$

$$\frac{\sin^2 x}{2} \Big|_{\frac{\pi}{4}}^{\frac{3\pi}{4}} = \frac{1}{2} \left(\frac{1}{2} - \frac{1}{2} \right) = 0$$

$$20) \int_{-2}^5 |2x-4| dx$$

$$\int_{-2}^2 (-2x+4) dx + \int_2^5 (2x-4) dx$$

$$\left(-2 \frac{x^2}{2} + 4x \right) \Big|_{-2}^2 + \left(\frac{2x^2}{2} - 4x \right) \Big|_2^5$$

$$-4 + 8 + 4 + 8 + 25 - 20 - 4 + 8 = 25$$

$$6.13 \ 2) y^2 = 2x \Rightarrow x^2 = 2y$$

$$A_1 = \int_0^2 \sqrt{2x} dx = \sqrt{2} \cdot \frac{x^{\frac{3}{2}}}{\frac{3}{2}} \Big|_0^2 = \sqrt{2} \cdot \frac{2}{3} \sqrt{2^3} = \frac{8}{3}$$

$$A_2 = \int_0^2 \frac{x^2}{2} dx = \frac{1}{2} \cdot \frac{x^3}{3} \Big|_0^2 = \frac{1}{6} (2)^3 = \frac{8}{6}$$

$$A = A_1 - A_2 = \frac{8}{3} - \frac{8}{6} = \frac{4}{3} \text{ u.a.}$$

$$3) y = 5 - x^2 \Rightarrow y = x + 3 \quad A_1 = \int_{-2}^1 (5 - x^2) dx = 5x - \frac{x^3}{3} \Big|_{-2}^1 = 5(1+2) - \frac{1}{3}(1+8)$$

$$5 - x^2 = x + 3$$

$$x^2 + x - 2 = 0$$

$$x = \frac{-1 \pm \sqrt{1+8}}{2}$$

$$x' = \frac{-1+3}{2} = 1$$

$$x'' = \frac{-1-3}{2} = -2$$

$$A_2 = \int_{-2}^1 (x+3) dx = \frac{x^2}{2} + 3x \Big|_{-2}^1 = \frac{1}{2}(1-4) + 3(1+2) = \frac{1}{2}(-3) + 3 \cdot 3$$

$$= \frac{-3}{2} + 9 = \frac{-3+18}{2} = \frac{15}{2}$$

$$A = \frac{12}{2} - \frac{15}{2} = \frac{24-15}{2} = \frac{9}{2} \text{ u.a.}$$

8) $y = x^3 - x$ & $p = 0$

$$A = 2 \int_{-1}^0 (x^3 - x) dx = 2 \left[\frac{x^4}{4} - \frac{x^2}{2} \right]_{-1}^0$$

$$2 \left(\frac{-1}{4} + \frac{1}{2} \right) = \frac{1}{2} \text{ u.a.}$$

16) $y = e^x$, $x = 0$, $x = 1$ & $p = 0$

$$\int_0^1 e^x dx = e^x \Big|_0^1 = e - 1 \text{ u.a.}$$

$$A = e - 1 - \frac{1}{2} = e - \frac{3}{2} \text{ u.a.}$$

17) $y = e^x$, $y = x + 1$ & $p = -1$

$$A_1 = \int_{-1}^0 e^x dx = -e^x \Big|_{-1}^0 = -1 + e$$

$$A_2 = \int_{-1}^0 (x+1) dx = \frac{x^2}{2} + x \Big|_{-1}^0 = \frac{1}{2} + 1 = \frac{3}{2}$$

23) $y = 4 - x^2$ & $y = x^2 - 14$

$$A = 2 \int_0^3 [(4-x^2) - (x^2-14)] dx = 2 \int_0^3 (18-2x^2) dx = 2 \left(\frac{18x}{1} - \frac{2x^3}{3} \right) \Big|_0^3$$

$$2(54-18) = 72$$

29) $y = e^x - 1$, $y = -x$ & $p = 1$

$$\int_0^1 (e^x - 1) dx = e^x - x \Big|_0^1 = e - 1 - 1 = e - 2$$

$$- \int_0^1 -x dx = \frac{x^2}{2} \Big|_0^1 = \frac{1}{2}$$

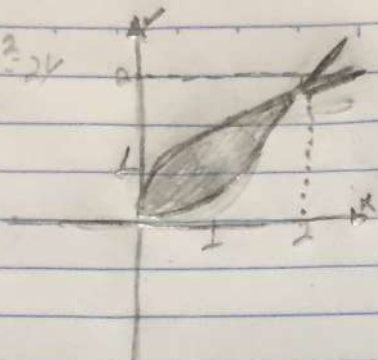
$$A = e - 2 + \frac{1}{2} = \frac{2e - 4 + 1}{2} = \frac{2e - 3}{2} \text{ u.a.}$$

30) SL $A_1 = \int_0^1 x dx = \frac{x^2}{2} \Big|_0^1 = \frac{1}{2}$ $A_2 = \int_1^2 \frac{1}{x} dx = \ln x \Big|_1^2 = \ln 2$

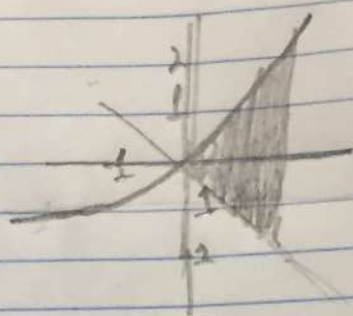
$$A_3 = \int_0^2 \frac{x}{4} dx = \frac{1}{4} \frac{x^2}{2} \Big|_0^2 = \frac{1}{4} \cdot \frac{4}{2} = \frac{1}{2}$$

$$SL: A = \frac{1}{2} + \ln 2 - \frac{1}{2} = \ln 2$$

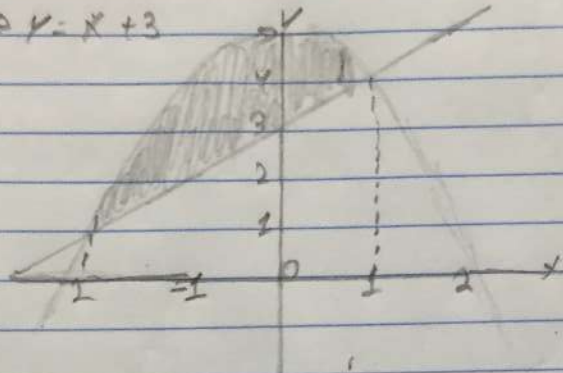
2) $y^2 = 2x$ & $x^2 = 2y$



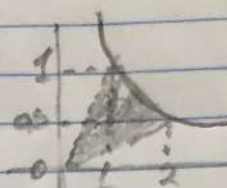
29) $y = e^x - 1$, $y = -x$ & $x = 1$



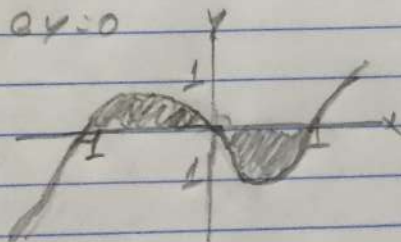
3) $y = 5 - x^2$ & $y = x + 3$



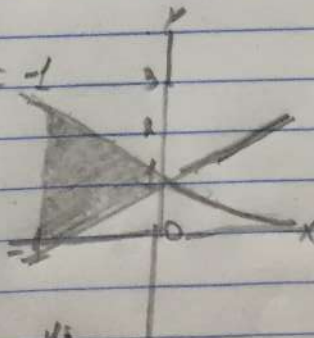
30)



8) $y = x^3 - x$ & $y = 0$



17) $y = e^{-x}$, $y = x + 1$ & $x = -1$



23) $y = 4 - x^2$ & $y = x^2 - 14$

