

# Database Management Systems

Lecture 7

Evaluating Relational Operators

Query Optimization

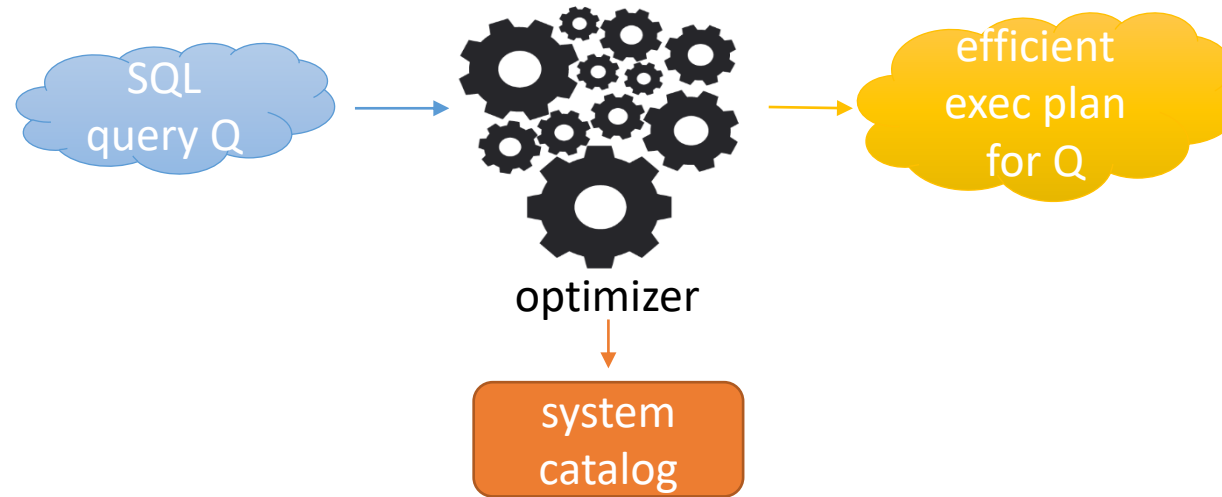
- \* queries – composed of relational operators:
  - selection ( $\sigma$ )
    - selects a subset of records from a relation
  - projection ( $\pi$ )
    - eliminates certain columns from a relation
  - join ( $\otimes$ )
    - combines data from two relations
  - cross-product ( $R1 \times R2$ )
    - returns every record in R1 concatenated with every record in R2
  - set-difference ( $R1 - R2$ )
    - returns records that belong to R1 and don't belong to R2
  - union ( $R1 \cup R2$ )
    - returns all records in relations R1 and R2

\*Review lecture notes on *Relational Algebra* (Databases course)

- \* queries – composed of relational operators:
  - intersection ( $R1 \cap R2$ )
    - returns records that belong to both R1 and R2
  - grouping and aggregate operators (algebra extensions)
  - every operation returns a relation => operations can be composed
  - an operator can have several implementation algorithms

\* optimizer

- input: SQL query Q
- output: an efficient execution plan for evaluating Q



\* algorithms for operators - based on 3 techniques:

- iteration:
  - examine iteratively:
    - all tuples in input relations
  - or
  - data entries in indexes, provided they contain all the necessary fields (data entries are smaller than data records)
- indexing:
  - used when the query contains a selection condition or a join condition
  - examine only the tuples that meet the condition, using an index
- partitioning:
  - partition the tuples
  - decompose operation into collection of cheaper operations on partitions

\* algorithms for operators - based on 3 techniques:

- partitioning:
  - partitioning techniques
    - sorting
    - hashing

\* access paths

- *access path* = way of retrieving tuples from a relation
  - file scan
- or
- an index  $I$  + a matching selection condition  $C$
- condition  $C$  matches index  $I$  if  $I$  can be used to retrieve just the tuples satisfying  $C$
- if relation  $R$  has an index  $I$  that matches selection condition  $C$ , then there are at least 2 access paths for  $R$  (file scan; index)

\*Review lecture notes on *Indexes* (*Databases* course)

\* access paths - example:

- relation *Students*[*SID*, *Name*, *City*]
- *I* - tree index on *Students* with search key  $\langle \textit{Name} \rangle$
- query Q:  
SELECT \*  
FROM *Students*  
WHERE *Name* = 'Ionescu'
- condition *C*: *Name* = 'Ionescu'
- *C* matches *I*, i.e., index *I* can be used to retrieve only the *Students* tuples satisfying *C*
- the following condition also matches the index: *Name* > 'Ionescu'



\* access paths - example:

- relation *Students*[*SID*, *Name*, *City*]
- *I* - hash index on *Students* with search key  $\langle \textit{Name} \rangle$
- query Q:  
SELECT \*  
FROM *Students*  
WHERE *Name* = 'Ionescu'
- condition *C*: *Name* = 'Ionescu'
- *C* matches *I*, i.e., index *I* can be used to retrieve only the *Students* tuples satisfying *C*
- condition *Name* > 'Ionescu' doesn't match *I* (since *I* is a hash index; it cannot be used to retrieve just the tuples satisfying *Name* > 'Ionescu')

\* access paths

- to sum up:
  - condition  $C: attr\ op\ value$ ,  $op \in \{<, <=, =, <>, >=, >\}$
  - condition  $C$  matches index  $I$  if:
  - the search key of  $I$  is  $attr$  and:
    - $I$  is a tree index or
    - $I$  is a hash index and  $op$  is  $=$

\* access paths

- index  $I$ , selection condition  $C$
- $I$  - hash index
- condition  $C$  of the form:
  - $\bigwedge_{i=1}^n T_i$
  - term  $T_i$ :  $attr = value$(i.e.,  $C$  in CNF - defined on page 15)
- $I$  matches  $C$  if  $C$  has one term for each attribute in the search key of  $I$

Condition	Hash index with search key <a, b, c>
a = 10 AND b = 5 AND c = 2	Yes
a = 10 AND b = 5	No
b = 5	No
b = 5 AND c = 2	No

\* access paths

- index  $I$ , selection condition  $C$  (CNF)
- $I$  - tree index
- condition  $C$  of the form:
  - $\bigwedge_{i=1}^n T_i$
  - term  $T_i$ : *attr op value*;  $op \in \{<, <=, =, <>, >=, >\}$
- $I$  matches  $C$  if  $C$  has one term for each attribute in a prefix of the search key of  $I$
- examples of prefixes for search key  $\langle a, b, c, d \rangle$ :  $\langle a \rangle$ ,  $\langle a, b, c \rangle$ ;  $\langle a, c \rangle$  and  $\langle b, c \rangle$ , on the other hand, are not prefixes for this search key

Condition	B+ tree index with search key $\langle a, b, c \rangle$
$a = 10 \text{ AND } b = 5 \text{ AND } c = 2$	Yes
$a = 10 \text{ AND } b = 5$	Yes
$b = 5$	No
$b = 5 \text{ AND } c = 2$	No

## \* access paths

- selectivity of an access path
  - the number of retrieved pages when using the access path to obtain the desired tuples
  - both data pages and index pages are counted
- example:  
SELECT \*  
FROM Students  
WHERE Name = 'Ionescu'
  - there are 2 access paths for *Students*:
    - file scan – selectivity could be 1000
    - matching index I with search key <Name> – selectivity could be 3
- most selective access path
  - retrieves the fewest pages, i.e., data retrieval cost is minimized

\* general selection conditions

- in general, a selection condition can contain one or several terms of the form:
  - *attr op constant*
  - *attr1 op attr2*,combined with  $\wedge$  and  $\vee$

```
SELECT *  
FROM Exams  
WHERE SID = 7 AND EDate = '04-01-2021'
```

$$\sigma_{SID=7 \wedge EDate='04-01-2021'}(Exams)$$

\* general selection conditions

- process a selection operation with a general selection condition  $C \rightarrow$  express  $C$  in CNF (conjunctive normal form)
- condition in CNF:
  - collection of conjuncts connected with the  $\wedge$  operator
  - a *conjunct* has one or more terms connected with the  $\vee$  operator
  - *term*:
    - *attr op constant*
    - *attr1 op attr2*
- example:  
condition  $(EDate < '4-1-2021' \wedge Grade = 10) \vee CID = 5 \vee SID = 3$   
is rewritten in CNF:  
 $(EDate < '4-1-2021' \vee CID = 5 \vee SID = 3) \wedge (Grade = 10 \vee CID = 5 \vee SID = 3)$

\* general selection conditions matching an index

- relation  $R[a, b, c, d, e]$ , index  $I$  with search key  $\langle a, b, c \rangle$

Condition	B+ tree index	Hash index
$a = 10 \text{ AND } b = 5 \text{ AND } c = 2$	Yes	Yes
$a = 10 \text{ AND } b = 5$	Yes	No
$b = 5$	No	No
$b = 5 \text{ AND } c = 2$	No	No
$d = 2$	No	No
$a = 20 \text{ AND } b = 10 \text{ AND } c = 5 \text{ AND } d = 11$	Partly	Partly

*Condition* – CNF selection condition

*B+ tree index / Hash index* – *B+ tree / hash index*  $I$  matches (Yes) / doesn't match (No) / matches a part of (Partly) the selection condition

- for the condition in the last row ( $a = 20 \text{ AND } b = 10 \text{ AND } c = 5 \text{ AND } d = 11$ ):
  - use index  $I$  to retrieve tuples satisfying  $a = 20 \text{ AND } b = 10 \text{ AND } c = 5$ , then apply  $d = 11$  to each retrieved tuple



\* general selection conditions matching an index

- relation R[a, b, c, d]
- index I1 with search key <a, b>
- B+ tree index I2 with search key <c>

Condition	Indexes
$c < 100 \text{ AND } a = 3 \text{ AND } b = 5$	<ul style="list-style-type: none"><li>- use I1 or I2 to retrieve tuples</li><li>- then check terms in the selection condition that do not match the index for each retrieved tuple</li><li>- e.g., use the B+ tree index to retrieve tuples where <math>c &lt; 100</math>; then apply <math>a = 3 \text{ AND } b = 5</math> to each retrieved tuple</li></ul>

\* running example - schema

- Students (SID: integer, SName: string, Age: integer)
- Courses (CID: integer, CName: string, Description: string)
- Exams (SID: integer, CID: integer, EDate: date, Grade: integer, FacultyMember: string)
  
- Students
  - every record has 50 bytes
  - there are 80 records / page
  - 500 pages of Students tuples
- Courses
  - every record has 50 bytes
  - there are 80 records / page
  - 100 pages of Courses tuples

\* running example - schema

- Students (SID: integer, SName: string, Age: integer)
- Courses (CID: integer, CName: string, Description: string)
- Exams (SID: integer, CID: integer, EDate: date, Grade: integer, FacultyMember: string)
- Exams
  - every record has 40 bytes
  - there are 100 records / page
  - 1000 pages of Exams tuples

## \* joins

SELECT \*

FROM Exams E, Students S

WHERE E.SID = S.SID

- algebra:  $E \bowtie_{E.SID=S.SID} S$ 
  - to be carefully optimized
  - size of  $E \times S$  is large, so computing  $E \times S$  followed by selection is inefficient
- E
  - M pages
  - $p_E$  records / page
- S
  - N pages
  - $p_S$  records / page
- evaluation: number of I/O operations

\* joins – implementation techniques

- iteration
  - Simple/Page-Oriented Nested Loops Join
  - Block Nested Loops Join
- indexing
  - Index Nested Loops Join
- partitioning
  - Sort-Merge Join
  - Hash Join
- equality join, one join column
  - join condition:  $E_i = S_j$

## Simple Nested Loops Join

```
foreach tuple e ∈ E do
    foreach tuple s ∈ S do
        if ei == sj then add <e, s> to the result
```

- for each record in the outer relation E, scan the entire inner relation S
- cost
  - $M + p_E * M * N = 1000 + 100 * 1000 * 500 \text{ I/Os} = 1000 + (5 * 10^7) \text{ I/Os}$ 
    - M I/Os – cost of scanning E
    - N I/Os – cost of scanning S
    - S is scanned  $p_E * M$  times (there are  $p_E * M$  records in the outer relation E)

\* E - M pages,  $p_E$  records / page \*

\* 1000 pages \* \* 100 records / page\*

\* S - N pages,  $p_S$  records / page \*

\* 500 pages \* \* 80 records / page \*

## Page-Oriented Nested Loops Join

```
foreach page  $pe \in E$  do
    foreach page  $ps \in S$  do
        if  $e_i == s_j$  then add  $\langle e, s \rangle$  to the result
```

- for each page in  $E$  read each page in  $S$
- pairs of records  $\langle e, s \rangle$  that meet the join condition are added to the result (where record  $e$  is on page  $pe$ , and record  $s$  – on page  $ps$ )
- refinement of Simple Nested Loops Join

## Page-Oriented Nested Loops Join

```
foreach page  $p_e \in E$  do
    foreach page  $p_s \in S$  do
        if  $e_i == s_j$  then add  $\langle e, s \rangle$  to the result
```

- cost

- **$M + M*N = 1000 + 1000*500$  I/Os = 501.000 I/Os**
  - M I/Os – cost of scanning E; N I/Os – cost of scanning S
  - S is scanned M times
  - significantly lower than the cost of Simple Nested Loops Join (improvement - factor of  $p_E$ )
- if the smaller table (S) is chosen as outer table:  
 $\Rightarrow \text{cost} = 500 + 500 * 1000 \text{ I/Os} = 500.500 \text{ I/Os}$

\* E - M pages,  $p_E$  records / page \*      \* 1000 pages \*      \* 100 records / page\*

\* S - N pages,  $p_S$  records / page \*      \* 500 pages \*      \* 80 records / page \*



## Block Nested Loops Join

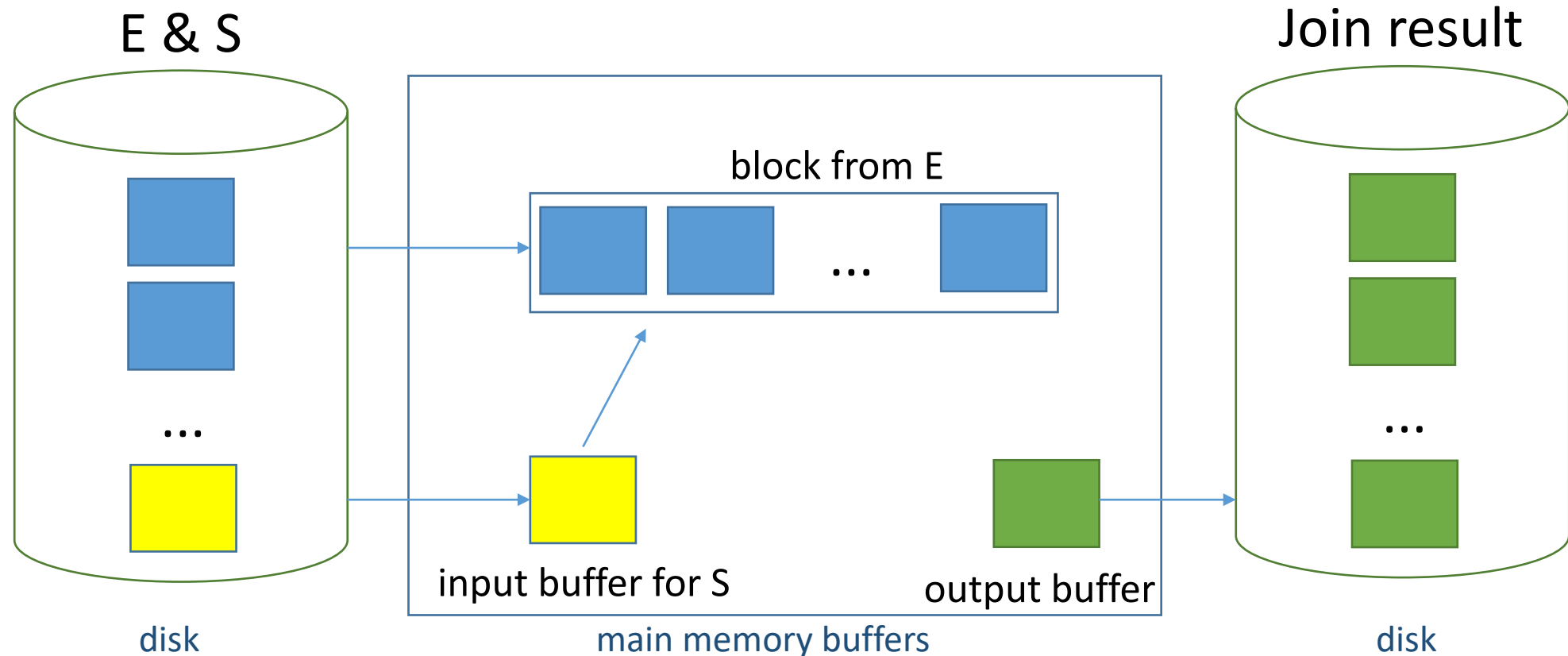
- previously presented join algorithms do not use buffer pages effectively
  - join relations R1 and R2; R1 – the smaller relation
  - assumption – the smaller relation fits in main memory
  - improvement:
    - store smaller relation R1 in memory
    - keep at least 2 extra buffer pages B1 and B2
    - use B1 to read the larger relation R2 (one page at a time)
    - use B2 as the output buffer (i.e., for tuples in the result of the join)
    - for each tuple in R2, search R1 for matching tuples
- => optimal cost: *number of pages in R1 + number of pages in R2*, since R1 is scanned only once, R2 is also scanned only once

## Block Nested Loops Join

- refinement
  - don't store the smaller relation in main memory as is, build an in-memory hash table for it instead
  - the I/O cost remains unchanged, but the CPU cost is usually much lower (since for each tuple in the larger relation, the smaller relation is examined to find matching tuples)

## Block Nested Loops Join

- if there isn't enough main memory to hold one of the input relations:
  - use one buffer page to scan the inner table (e.g., S)
  - use one page for the result
  - use all remaining pages to read a *block* from the outer table (e.g., E)
    - block – set of pages from E that fit in main memory



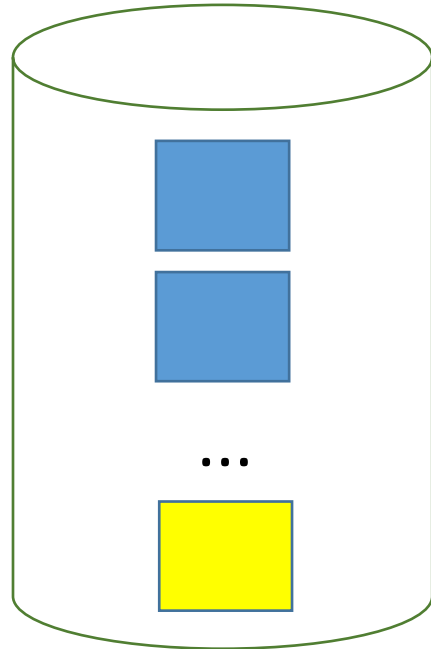
## Block Nested Loops Join

```
foreach block be ∈ E do  
  foreach page ps ∈ S do  
  {
```

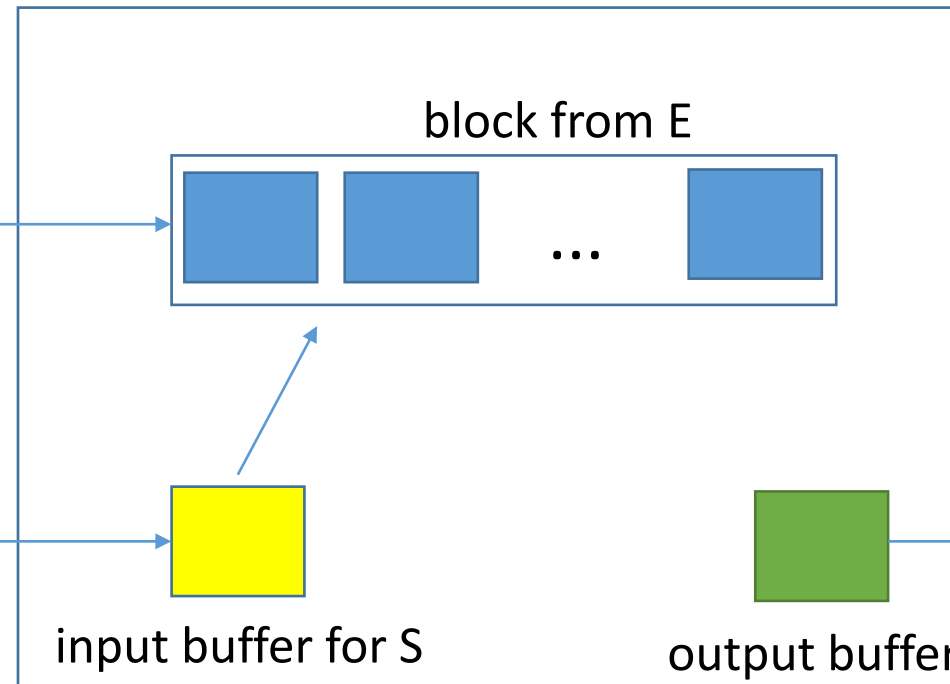
```
    for all pairs of tuples <e, s> that meet the join  
      condition, where e ∈ be and s ∈ ps,  
        add <e, s> to the result
```

```
  }
```

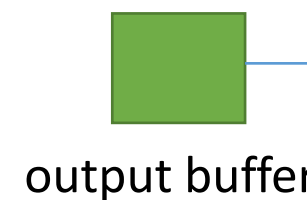
**E & S**



disk

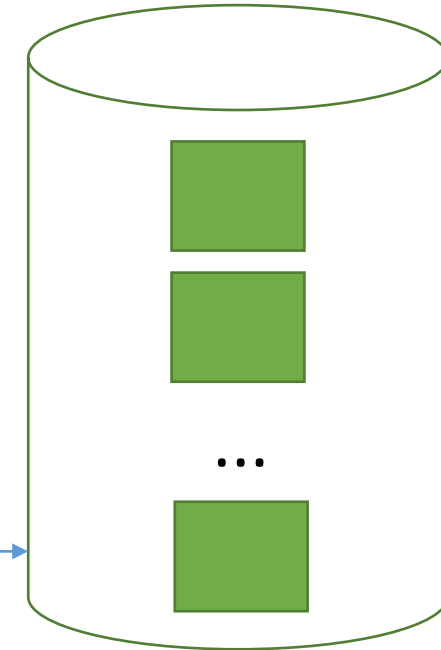


main memory buffers



output buffer

**Join result**

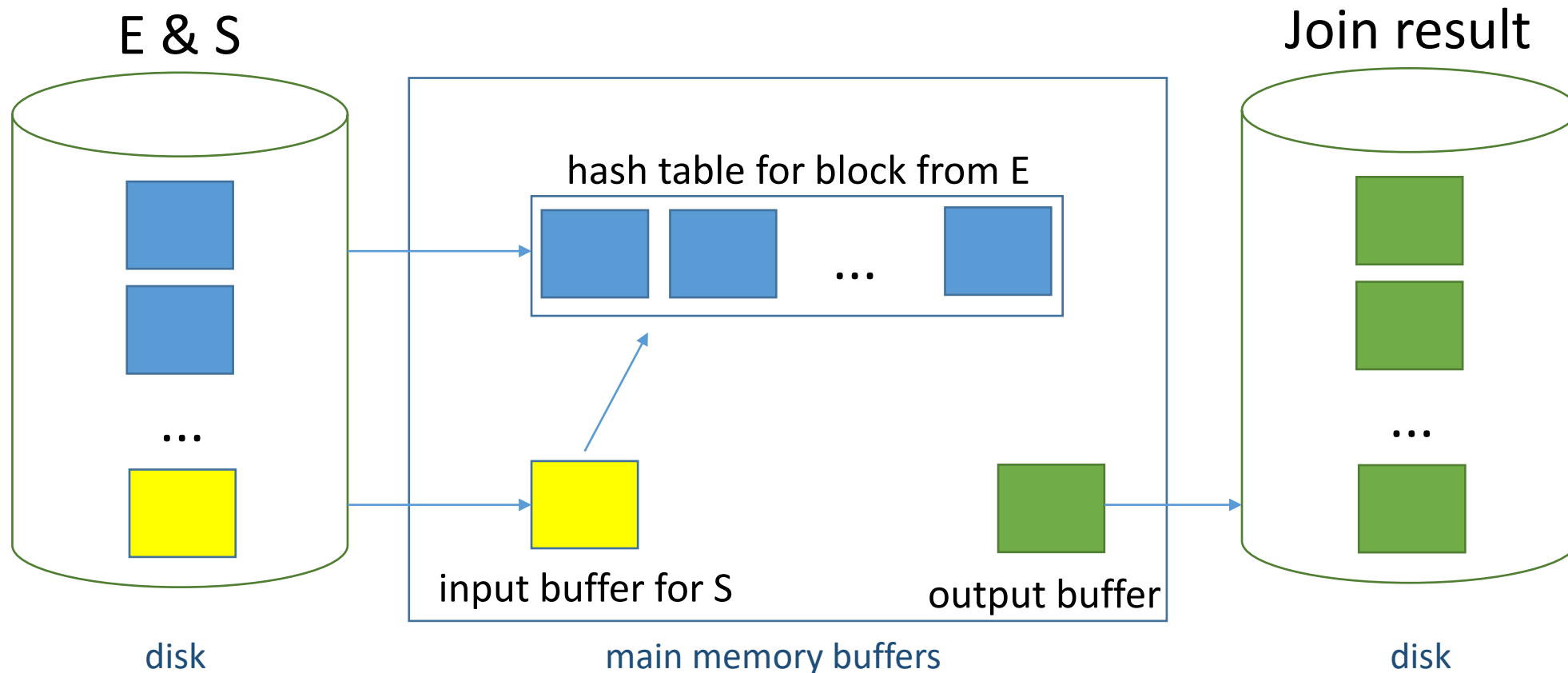


disk

- inner relation S is scanned once for each block in outer relation E
- outer relation E is scanned once

## Block Nested Loops Join

- refinement to efficiently find matching tuples
  - build main-memory hash table for the block of E
  - trade-off: reduce size of E block



## Block Nested Loops Join

- cost
    - scan of outer table + number of blocks in outer table \* scan of inner table
    - number of outer blocks =  $\left\lceil \frac{\text{number of pages in outer table}}{\text{size of block}} \right\rceil$
    - outer table: Exams (E), a block can hold 100 pages
      - scan cost for E: 1000 I/Os
      - number of blocks:  $\left\lceil \frac{1000}{100} \right\rceil = 10$
      - foreach block in E, scan Students (S): 10\*500 I/Os
- => total cost = 1000 + 10 \* 500 = **6000 I/Os**

\* E - M pages,  $p_E$  records / page \*

\* 1000 pages \* \* 100 records / page \*

\* S - N pages,  $p_S$  records / page \*

\* 500 pages \* \* 80 records / page \*

## Block Nested Loops Join

- cost
    - scan of outer table + number of blocks in outer table \* scan of inner table
    - number of outer blocks =  $\left\lceil \frac{\text{number of pages in outer table}}{\text{size of block}} \right\rceil$
    - outer table: Exams (E)
      - suppose the buffer has 90 pages available for E, i.e., block of 90 pages
- => number of blocks:  $\left\lceil \frac{1000}{90} \right\rceil = 12$
- => S is scanned 12 times
- scan cost for E: 1000 I/Os
  - foreach block in E, scan Students (S): 12\*500 I/Os
- => total cost = 1000 + 12 \* 500 = **7000 I/Os**

* E - M pages, $p_E$ records / page *	* 1000 pages *	* 100 records / page *
* S - N pages, $p_S$ records / page *	* 500 pages *	* 80 records / page *

## Block Nested Loops Join

- cost
    - scan of outer table + number of blocks in outer table \* scan of inner table
    - number of outer blocks =  $\left\lceil \frac{\text{number of pages in outer table}}{\text{size of block}} \right\rceil$
    - outer table: Students (S), block of 100 pages
      - scan cost for S: 500 I/Os
      - number of blocks:  $\left\lceil \frac{500}{100} \right\rceil = 5$
      - for each block in S, scan E: 5 \* 1000 I/Os
- => total cost = 500 + 5 \* 1000 = **5500 I/Os**

\* E - M pages,  $p_E$  records / page \*

\* 1000 pages \* \* 100 records / page \*

\* S - N pages,  $p_S$  records / page \*

\* 500 pages \* \* 80 records / page \*



## Index Nested Loops Join

```
foreach tuple e in E do
    foreach tuple s in S where  $e_i == s_j$ 
        add <e, s> to the result
```

- if there is an index on the join column of S, S can be considered as inner table and the index can be used
- cost
  - $M + (M * p_E) * \text{cost of finding corresponding records in S}$

\* E - M pages,  $p_E$  records / page \*

\* S - N pages,  $p_S$  records / page \*

\* 1000 pages \* \* 100 records / page \*

\* 500 pages \* \* 80 records / page \*

## Index Nested Loops Join

- for a record  $e$  in  $E$ :
  - the cost of examining the index on  $S$  is:
    - approx. 1.2 for a hash index (typical cost for hash indexes)
    - typically 2-4 for a B+-tree index
  - the cost of reading corresponding records in  $S$ :
    - for a clustered index:
      - plus one I/O for each outer tuple in  $E$  (typically)
    - for an unclustered index:
      - up to one I/O for each corresponding record in  $S$   
(worst case – there are  $n$  matching records in  $S$  located on  $n$  different pages!)

## Index Nested Loops Join

- hash index on SID in Students (Students – inner table)
- scan Exams:
  - cost = 1000 I/Os, with a total of 100\*1000 records
- for each record in Exams:
  - (on average) 1.2 I/Os to obtain the page in the hash index (i.e., the page containing the rid of the matching Students tuple)  
and
  - 1 I/O to retrieve the page in Students that contains the matching tuple (exactly one! – since SID is a key in Students, i.e., there is one matching Students tuple for an exam)

=> cost to retrieve matching Students tuples:  $1000 * 100 * (1.2 + 1) = 220.000$

- total cost:  $1000 + 220.000 = 221.000$  I/Os

\* E - M pages,  $p_E$  records / page \*      \* 1000 pages \*      \* 100 records / page\*

\* S - N pages,  $p_S$  records / page \*      \* 500 pages \*      \* 80 records / page \*

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