

Lab 8

Quadrature formulas (1)

Trapezium formula for double integral

Applying successively trapezium formula with respect to y , and with respect to x , we get

$$\begin{aligned} \int_a^b \int_c^d f(x, y) dy dx \approx & \frac{(b-a)(d-c)}{16} [f(a, c) + f(a, d) + f(b, c) + f(b, d) \\ & + 2f\left(\frac{a+b}{2}, c\right) + 2f\left(\frac{a+b}{2}, d\right) + 2f\left(a, \frac{c+d}{2}\right) \\ & + 2f\left(b, \frac{c+d}{2}\right) + 4f\left(\frac{a+b}{2}, \frac{c+d}{2}\right)] \end{aligned} \quad (1)$$

Problems

1. a) Approximate the integral

$$I = \int_0^1 f(x) dx, \quad \text{for } f(x) = \frac{2}{1+x^2},$$

using trapezium formula.

b) Plot the graph of the function f and the graph of the trapezium with vertices $(0, 0)$, $(0, f(0))$, $(1, f(1))$ and $(1, 0)$.

c) Approximate the integral I using Simpson's formula.

2. Approximate the following double integral

$$\int_{1.4}^2 \int_1^{1.5} \ln(x+2y) dy dx$$

using trapezium formula for double integrals, given in (1). (*Result:* 0.4295545)

3. Evaluate the integral that arises in electrical field theory:

$$H(p, r) = \frac{60r}{r^2 - p^2} \int_0^{2\pi} \left[1 - \left(\frac{p}{r} \right)^2 \sin x \right]^{1/2} dx,$$

for $r = 110$, $p = 75$, using the repeated trapezium formula for two given values of n . (*Result:* 6.3131)

4. Find the smallest value of n that gives an approximation of the integral $\int_1^2 x \ln(x) dx$ which is correct to three decimals, using the repeated trapezium formula. Apply the repeated trapezium formula for the obtained value of n to approximate the integral. (*Result: 0.636294368858383*)

5. Evaluate the integral

$$\int_0^\pi \frac{dx}{4 + \sin 20x}$$

using the repeated Simpson's formula for $n = 10$ and 30 . (*Result: 0.8111579*)

6. The error function $\text{erf}(x)$ is defined by

$$\text{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2} dt.$$

Use the repeated Simpson's formula to evaluate $\text{erf}(0.5)$ with $n = 4$ and $n = 10$. Estimate the accuracy of your result and compare with the correct value $\text{erf}(0.5) = 0.520499876$.

Facultative:

7. The volume of a solid is given by $\int_{0.1}^{0.5} \int_{0.01}^{0.25} e^{\frac{y}{x}} dy dx$. Approximate this volume applying Simpson's algorithm for double integrals considering 21 equidistant points in $[0.1, 0.5]$, respectively in $[0.01, 0.25]$. See the algorithm below. (*Result: 0.178571*)

Simpson's formula for double integral

Consider the integral $I = \int_a^b \int_c^d f(x, y) dy dx$. Let $m, n \in \mathbb{N}$ and the equidistant points x_0, \dots, x_{2m} in $[a, b]$, with step $h = \frac{b-a}{2m}$, respectively y_0, \dots, y_{2n} in $[c, d]$, with step $k = \frac{d-c}{2n}$.

We apply the repeated Simpson's formula to the integral $\int_c^d f(x, y) dy$ and then to the integral $\int_a^b \int_c^d f(x, y) dy dx$.

Algorithm:

INPUT: a,b,c,d,m,n

OUTPUT: the approximant J of the integral I

$h = (b-a)/(2*n);$

$j1=0; j2=0; j3=0$

for $i=0, 1, \dots, 2*n$

$Let\ x=a+i*h;$

$h1=(d-c)/(2*m);$

$k1=f(x,c)+f(x,d);$

$k2=0;$

$k3=0;$

for $j=1, 2, \dots, 2*m-1$

$y=c+j*h1;$

$z=f(x,y);$

if j is even do $k2=k2+z;$

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else k3=k3+z;
end{if}
end{for}
l=(k1+2*k2+4*k3)*h1/3;
if (i==0)| (i==2*n) do j1=j1+l;
else if i is even do j2=j2+l;
else j3=j3+1;
end{if}
end{if}
end
J=(j1+2*j2+4*j3)*h/3

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