Lab 3

Lagrange interpolation

Using *the barycentric form* of the Lagrange interpolation polynomial, solve the following problems:

Problems:

1. The table below contains the population of the USA from 1930 to 1980 (in thousands of inhabitants):

1930	1940	1950	1960	1970	1980
123203	131669	150697	179323	203212	226505.

Approximate the population in 1955 and 1995.

2. Approximate $\sqrt{115}$ with Lagrange interpolation, using the known values for three given nodes.

3. Plot the graphics of the function $f : [0, 10] \to \mathbb{R}$, $f(x) = \frac{1+\cos(\pi x)}{1+x}$ and of the Lagrange interpolation polynomial that interpolates the function f at 21 equally spaced points in the interval $[0, 10]$.

4. Consider the function $f : [-5, 5] \to \mathbb{R}$, $f(x) = \frac{1}{1+x^2}$. For $n = 2, 4, ..., 8$, compute Lagrange polynomial of degree n which interpolates $f(x)$ at the $n+1$ equally spaced points $x_i = i\frac{10}{n} - 5$, $i = 0, ..., n$. Then estimate the maximum interpolation error

$$E_n : \max_{-5 \leq x \leq 5} |f(x) - P_n(x)|, \qquad n = 2, 4, ..., 8$$

on the interval $[-5, 5]$ by computing

$$E_n \approx \max_i |f(y_i) - P_n(y_i)|,$$

where $y_i = \frac{i}{10} - 5, \quad i = 0, ..., 100.$

Facultative:

1. Consider the function $f : [-\frac{\pi}{4}, \frac{\pi}{2}] \to \mathbb{R}$, $f(x) = \cos(x)$ and the given nodes $0, \frac{\pi}{4}, \frac{\pi}{3}$.

a) Plot the fundamental interpolation polynomials $\ell_i(x) = \frac{u_i(x)}{u_i(x_i)}$, $i = 0, .., m$.

b) Compute the value of Lagrange interpolation polynomial at $x = \frac{\pi}{6}$ using both the classical formula $(L_m f)(x) = \sum\limits_{i=0}^{m} \ell_i(x) f(x_i)$ and the barycentric formula.

c) Plot the graphs of the function f and of the corresponding Lagrange interpolation polynomial.

d) Give two other sets of nodes in $[-\frac{\pi}{4}, \frac{\pi}{2}]$ and plot the correponding Lagrange interpolation polynomials.