Lab 7

Least squares approximation

1. The following table list the temperatures of a room recorded during the time interval $[1:00, 7:00]$. Find the best liniar least squares function $\varphi(x) = ax + b$ that approximates the table, using the normal equations (equations (2) from Course 6). Use your result to predict the temperature of the room at $8:00$. Find the minimum value $E(a, b)$ (equation (1) from Course 6), for the obtained a and b. In the same figure, plot the points (Time, Temperature) and the least squares function.

Time	$1:00$	$2:00$	$3:00$	$4:00$	$5:00$	$6:00$	$7:00$
Temperature	13	15	20	14	15	13	10

2. The vapor pressure P of the water (in bars) as a function of temperature T (in $°C$) is:

T (temperature)	0	10	20	30	40	60	80	100
P (pressure)	0.0061	0.0123	0.0234	0.0424	0.0738	0.1992	0.4736	1.0133

a) Obtain two least squares approximations for the given data, using *polyfit* for 2 different degrees of the polynomials. Find their values for $T = 45$ using *polyval*. Compute the approximation errors, knowing that the exact value is $P(45) = 0.095848$.

b) Plot the interpolation points, the least squares approximants and the interpolation polynomial, in the same figure.

3. Consider 10 random points in the plane $[0, 3] \times [0, 5]$ using Matlab function *ginput*. Plot the points and the least squares polynomial of 2nd degree that best fits these points.

Facultative:

4. Consider 12 random points in the interval $[0, 10]$. Find the discret least squares approximant of n-th degree for the function $f(x) = e^x$ using the least squares approximation method with weight function $w(x) = 1$ and the basis $1, x, x^2, ..., x^n$. (The least squares approximant is of the form $\varphi(x) = \sum_{i=1}^{n} a_i g_i(x)$,

where $\{g_i, \ i = 1, ..., n\}$ is a basis of the space and the coefficients a_i are obtained solving the normal equations: $\sum_{i=1}^{n} a_i \langle g_i, g_k \rangle = \langle f, g_k \rangle, \quad k = 1, ..., n,$ with $\langle f, g \rangle = \sum_{i=0}^{m} f(x_k)g(x_k))$. Plot the function and the obtained least squares approximant.