Lab 2

Orthogonal and Taylor polynomials. Finite and divided differences

1. The first 4 Legendre polynomials are given by:

$$\begin{aligned} l_1(x) &= x \\ l_2(x) &= \frac{3}{2}x^2 - \frac{1}{2} \\ l_3(x) &= \frac{5}{2}x^3 - \frac{3}{2}x \\ l_4(x) &= \frac{35}{8}x^4 - \frac{15}{4}x^2 + \frac{3}{8}, \qquad x \in [0,1]. \end{aligned}$$

   Divide the display in 4 parts and plot in each part the Legendre polynomial l_i, $i = 1, ..., 4$. (Use the *subplot* command).

2. a) Chebyshev polynomials of the first kind are defined by

$$T_n(t) = \cos(n \arccos t), \qquad t \in [-1,1].$$

   Plot, in the same figure, the polynomials T_1, T_2, T_3.

   b) Plot, in the same figure, the first n Chebyshev polynomials of the first kind, using the following reccurence formula:

$$\begin{aligned} &T_{n+1}(x) = 2xT_n(x) - T_{n-1}(x), \qquad x \in [-1,1], \\ &\text{with } T_0(x) = 1 \text{ and } T_1(x) = x. \end{aligned}$$

3. Taylor polynomial of n-th degree, associated to the function f and the point x_0, is given by $P_n(x) = \sum\limits_{k=0}^{n} \frac{(x-x_0)^k}{k!} f^{(k)}(x_0)$. Plot, in the same figure, the first six Taylor polynomials for $f(x) = e^x$ and $x_0 = 0$, on the interval $[-1, 3]$.

4. Considering $h = 0.25$, $x_i = 1+ih$, $i = \overline{0,6}$, and $f(x) = \sqrt{5x^2 + 1}$, construct the finite differences table.

5. For $x_0 = 2$, $x_1 = 4$, $x_2 = 6$, $x_3 = 8$ and $f_0 = 4$, $f_1 = 8$, $f_2 = 14$, $f_3 = 16$ construct the divided differences table.