Lab 4

1. Using the data from the following table

x	1	1.5	2	3	4
$\lg x$	0	0.17609	0.30103	0.47712	0.60206

approximate $\lg 2.5$ and $\lg 3.25$ using the Newton interpolation polynomial. Estimate the maximum interpolation error $E = max|f(y_i) - (N_4 f)(y_i)|$, with $y_i = \frac{i}{10}$, for $i = \overline{10, 35}$.

2. To investigate the relationship between yield of potatoes, y, and level of fertilizer, x, an experimenter divided a field into 5 plots of equal size and applied differing amounts of fertilizer to each. The recorded data are given in the table (in pounds).

x	1	2	3	4	5
y	22	23	25	30	28

a) According to Newton interpolation polynomial, approximate how many pounds of potatoes are expected from a plot to which 2.5 pounds of fertilizer had been applied.

b) Plot the data given in the table and the corresponding Newton interpolation polynomial.

3. Consider the function $f : [0, 6] \to \mathbb{R}$, $f(x) = e^{\sin x}$ and 13 equidistant interpolation points. Plot the interpolation points, the function f and the Newton intepolation polynomial $N_{12} f$.

4. Approximate $\sqrt{115}$ with precision $\varepsilon = 10^{-3}$, using Aitken's algorithm.

5. Use Neville's algorithm to approximate $\sqrt{3}$ for:

a) the function $f_1(x) = 3^x$ and the nodes $x_0 = -2,\ x_1 = -1,\ x_2 = 0,\ x_3 = 1,\ x_4 = 2$.

b) the function $f_2(x) = \sqrt{x}$ and the nodes $x_0 = 0,\ x_1 = 1,\ x_2 = 2,\ x_3 = 4,\ x_4 = 5$.

Facultative:

1. Consider the function $f : [-5, 5] \to \mathbb{R}$, $f(x) = \sin x$ and 20 equidistant interpolation points. Plot the interpolation points, the function f and the Lagrange intepolation polynomial obtained using Aitken's algorithm with precision $\varepsilon = 10^{-3}$.

2. Consider the function $f : [-1, 1] \to \mathbb{R}$ and $n = 10$ nodes in the inteval $[-1, 1]$.

a) Compute the Lebesque function $g(x) = \sum_{i=1}^{n} |l_i(x)|$ at a given point x, where $l_i(x)$, $i = 1, ..., n$ are the fundamental Lagrange polynomials.

b) Plot the function g for n equidistant nodes in $[-1, 1]$.

c) Plot the function g for n Chebyshev nodes of the first kind, $x_i = \cos(\frac{(2i-1)\pi}{2n}) \in [-1, 1]$, $i = 1, ..., n$.

d) Plot the function g for n Chebyshev nodes of the second kind, $x_i = \cos(\frac{i\pi}{n}) \in [-1, 1]$, $i = 0, ..., n-1$.