

Media variabilei aleatoare exponențiale este dată de

$$\begin{aligned}
 M(X) &= \int_{-\infty}^{\infty} x f(x) dx \\
 &= \int_0^{\infty} x (\lambda e^{-\lambda x}) dx \\
 &= \int_0^{\infty} x (-e^{-\lambda x})' dx \\
 &= -x e^{-\lambda x} \Big|_0^{\infty} - \int_0^{\infty} -e^{-\lambda x} dx \\
 &= \int_0^{\infty} e^{-\lambda x} dx \\
 &= -\frac{e^{-\lambda x}}{\lambda} \Big|_0^{\infty} \\
 &= -\frac{e^{-\infty}}{\lambda} - \left(-\frac{e^0}{\lambda}\right) \\
 &= \frac{1}{\lambda},
 \end{aligned}$$

iar dispersia este dată de

$$\begin{aligned}
 \sigma^2(X) &= M[(X - M(X))^2] \\
 &= \int_{-\infty}^{\infty} \left(x - \frac{1}{\lambda}\right)^2 (-\lambda e^{-\lambda x}) dx \\
 &= \int_{-\infty}^{\infty} \left(x - \frac{1}{\lambda}\right)^2 (e^{-\lambda x})' dx \\
 &= \left(x - \frac{1}{\lambda}\right)^2 e^{-\lambda x} \Big|_0^{\infty} - \int_0^{\infty} 2 \left(x - \frac{1}{\lambda}\right) e^{-\lambda x} dx \\
 &= \frac{1}{\lambda^2} - 2 \int_0^{\infty} \left(x - \frac{1}{\lambda}\right) \left(-\frac{e^{-\lambda x}}{\lambda}\right)' dx \\
 &= \frac{1}{\lambda^2} - 2 \left(x - \frac{1}{\lambda}\right) \left(-\frac{e^{-\lambda x}}{\lambda}\right) \Big|_0^{\infty} + 2 \int_0^{\infty} -\frac{e^{-\lambda x}}{\lambda} dx \\
 &= \frac{1}{\lambda^2} - \frac{2}{\lambda^2} - \frac{2}{\lambda} \int_0^{\infty} e^{-\lambda x} dx \\
 &= -\frac{1}{\lambda^2} - \frac{2}{\lambda} \left(-\frac{e^{-\lambda x}}{\lambda}\right) \Big|_0^{\infty} \\
 &= -\frac{1}{\lambda^2} + \frac{2}{\lambda^2} \\
 &= \frac{1}{\lambda^2}
 \end{aligned}$$